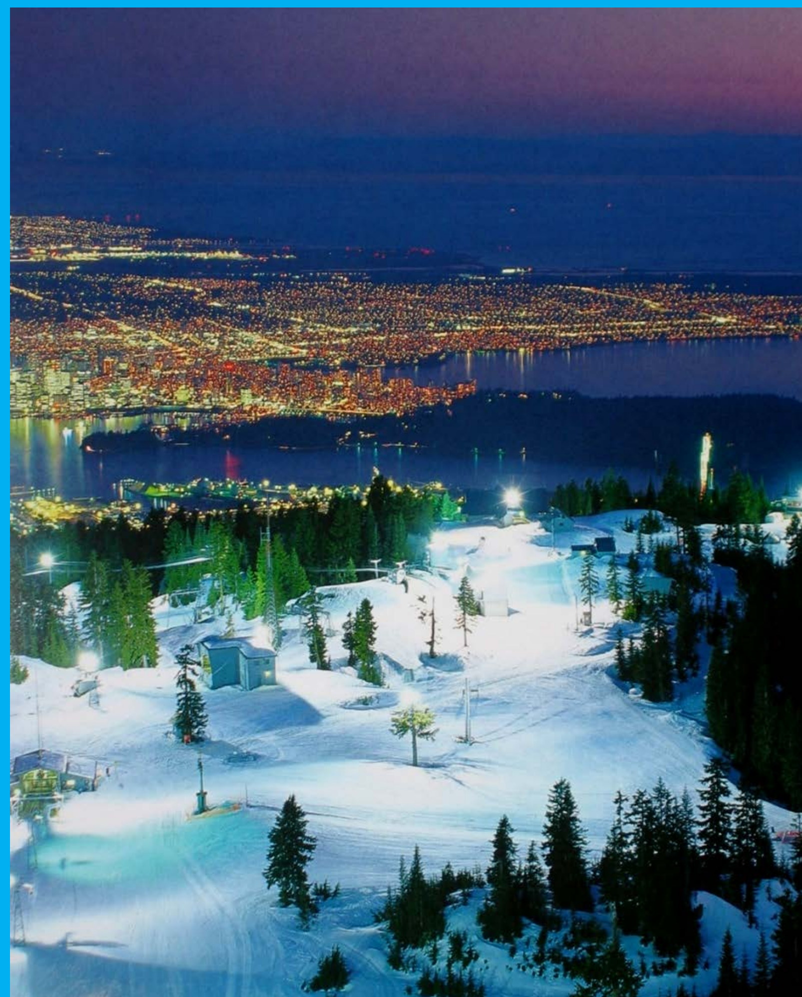
CORRELATED WORLDLINE THEORY OF QUANTUM GRAVITY: LOW-ENERGY CONSEQUENCES & TABLE-TOP TESTS

P.C.E. STAMP

Madrid, Sep 22nd 2015



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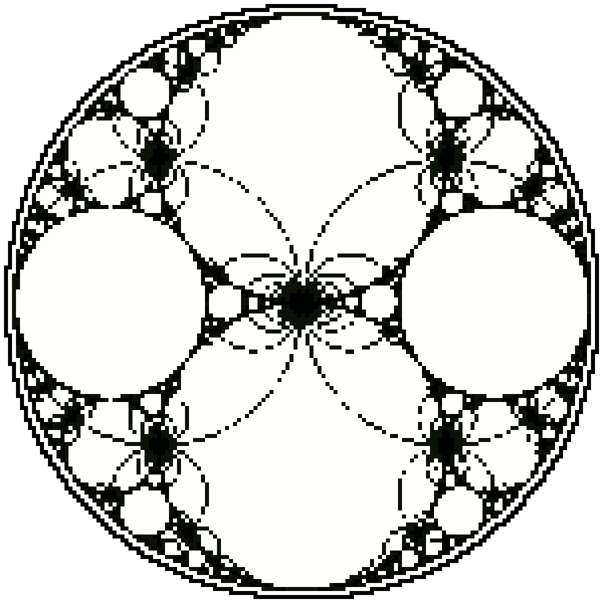
The talk will address the following themes:

(i) The Correlated Worldline Theory: basic structure
(ii) Thought experiments
(iii) Real Experiments



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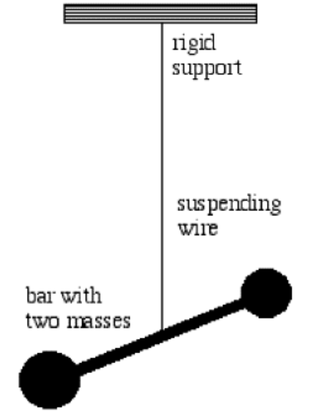
The low-E INCOMPATIBILITY of QM & GR



It is commonly asserted (usually by high-energy theorists) that the conflict between QM and gravity only exists at high energy (at energies ~ the Planck scale), where it is supposedly resolved in favour of QFT.

We will argue that this is wrong.

Feynman 1957, Karolhazy 1966, Eppley-Hannah 1977, Kibble 1978-82, Page 1981, Unruh 1984, Penrose 1996, showed there is a basic conflict between the superposition principle & GR at ordinary 'table-top' energies.

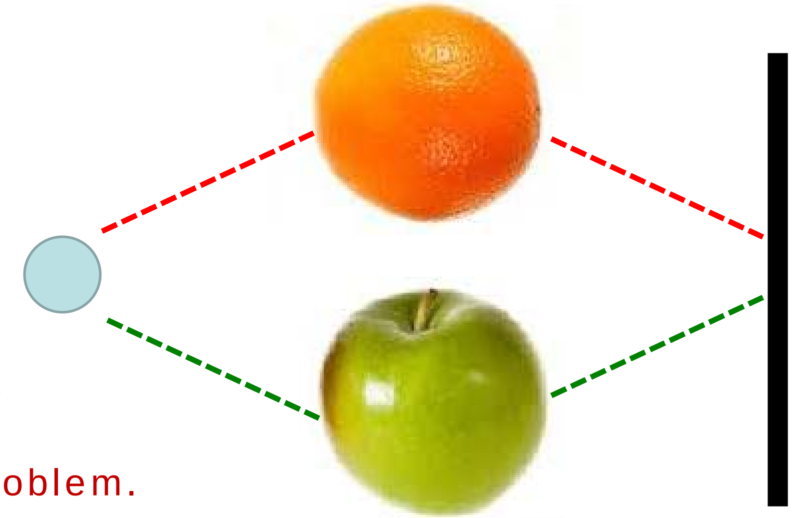
Consider a 2-slit experiment with a mass M.
Suppose we assume a 'wave-fn':

$$|\Psi\rangle \;=\; a_1|\Phi_1;\tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2;\tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$

In a non-relativistic treatment we write

$$\Phi(\mathbf{r},t) \equiv \langle \mathbf{r}|\Phi(t)\rangle \;=\; a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

and then: $\langle \Phi_1(t)|\Phi_2(t)\rangle \;=\; \int d^3r \;\langle \Phi_1^*(\mathbf{r},t)|\Phi_2(\mathbf{r},t)\rangle$



But now we have both a formal and a physical problem.

(i) FORMAL PROBLEM: There are 2 different coordinate systems, $(\mathbf{r}_j, t_j)$, defined by the 2 different metrics: $\tilde{g}^{\mu\nu}_{(j)}(x)$, & in general we cannot relate these.

(ii) PHYSICAL PROBLEM: A "wave-function collapse" causes non-local changes, which if linked to the metric cause drastically unphysical changes in the metric.

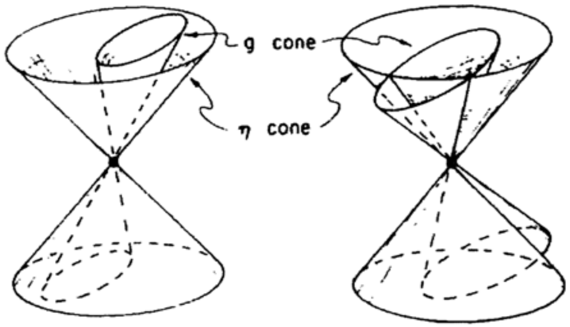
This is quite apart from all the usual problems of Quantum Gravity !

We need to drop something from either QM or GR to find a new theory

(1) WHAT is the ESSENCE of GRAVITY/GENERAL RELATIVITY ?

The ***general theory of relativity*** was established by Einstein (and finally formulated by him in 1916), and represents probably the most beautiful of all existing physical theories.

L.D. Landau, E.M. Lifshitz "The Classical Theory of Fields", sec.82



(1) CAUSAL STRUCTURE: As field strength goes up (eg., add gravitons), spacetime causal structure *changes*. The original gravitons become superluminal.

Causal structure is essential

(2) WEAK PRINCIPLE of EQUIVALENCE: identical coupling of all forms of energy to gravity, as expressed in the "minimal substitution", has overwhelming support from weak field tests and strong field observations.

So: we must use a metric structure to define spacetime – ie., $g^{\mu\nu}(x)$

(3) WORLDLINES & CONNECTION FIELDS: We will assume what is at the heart of relativity – and also in QM – the idea of worldliness or worldsheets. In addition we assume that in curved spacetime the connection field can be defined in the usual way for a worldline. NB: a metric-affine formulation is OK.

(4) LOW-ENERGY EFFECTIVE THEORY: General Relativity is assumed to be good for quantization at low-energy. If spacetime is coupled to a quantized matter field, it must also go into a superposition

So: spacetime must also be quantized.

So: The essence of GR is to be found in the metric, the connection, the associated causal structure, & the association with Quantum Mechanics

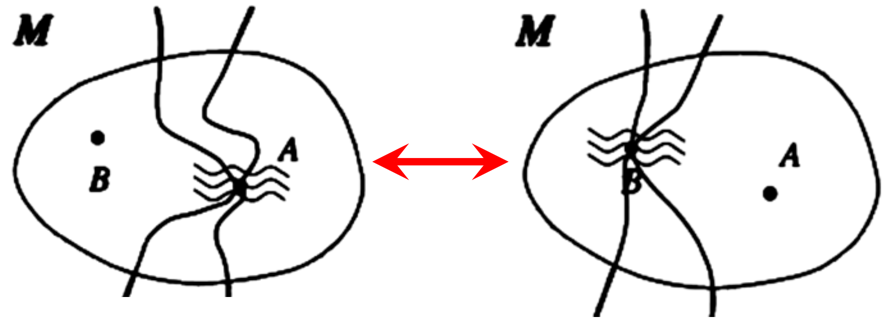
MORE on GENERAL RELATIVITY

Einstein (1879-1955)

A key question concerns the nature of spacetime – is spacetime a field or simply a 'background' dynamic Geometry? Einstein took different views at different times: the question is now central. Key points are:

(1) NATURE of SPACETIME We are NOT given 'spacetime' as a Newtonian absolute receptacle. In Einstein's view we were given 'events' as 'spacetime coincidences' between the intersecting worldlines of classical objects.

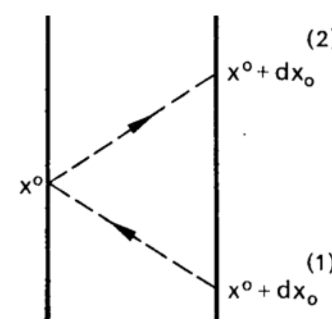
But there is a crucial arbitrariness here – one can 'relabel' the points of spacetime, & still describe the same physics → general covariance of physical laws/equations, ie., invariance under diffeomorphisms



(2) METRIC & CONNECTION: Cartan (1922) made it clear to Einstein that one needed 2 independent fundamental entities in his theory to describe spacetime:

(i) The spacetime metric $g^{\mu\nu}(x)$ This is determined by light signals passing between objects, to determine intervals.

(ii) The connection $\Gamma^{\alpha}{}_{\mu\nu}(x)$ This tells us how vectors and phases evolve along worldliness (again, one uses chronometry)



In Einstein's theory these are not independent – one has the Christoffel relation

$$\Gamma^{a}{}_{bc} = \frac{1}{2} g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc})$$

(3) PRINCIPLE of WEAK EQUIVALENCE: Whether we think of spacetime as a geometry or as a field, it couples in exactly the same way to every other field. Thus, eg., for a scalar field we have

$$S = \int_{\mathcal{R}} \left[\tfrac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) - V(\phi) \right] \sqrt{-g}\, d^4 x.$$

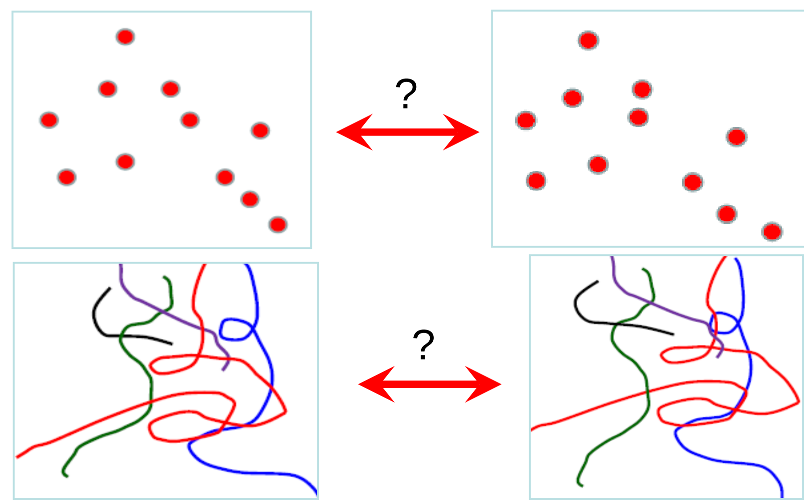
which is independent of the connection and depends on the metric in a universal way.

Let's give an intuitive view of this. Consider some arbitrary set of points (eg., a random 'glassy' set of atomic nuclei), and of lines (eg., a random polymer) and now <u>take away the spacetime receptacle, & the labels</u>. How do we COMPARE 2 configurations?

The answer is that ALL WE HAVE is

(i) The chronometry along the worldlines (uses QM ,via E = mc^2), to give us the connection

(ii) The chronometry of signalling, which gives us the metric properties – defines our spacetime

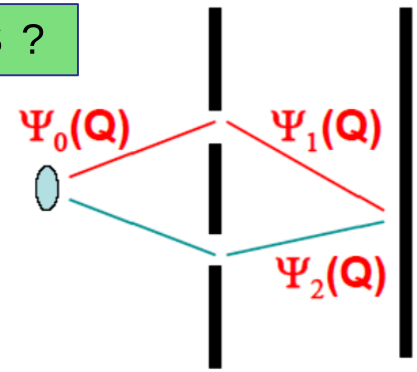


In addition to this, weak equivalence tells us that the lines have no "colour", ie., that the only thing that matters in gravity is the stress-energy tensor – we can't distinguish the different lines (or points). Generlizing to field theory, we can talk of field configurations filling spacetime – but they all have the same colour as far as gravity is concerned.

It is really important to understand how successful this theory has been: extending our understanding of spacetime & matter to unprecedented scales, and making outlandish predictions – all verified.

(2) WHAT is the ESSENCE of QUANTUM MECHANICS ?

(1) **According to Feynman (1965), ' the fundamental mystery of QM' is encapsulated in the '2-slit' experiment:**



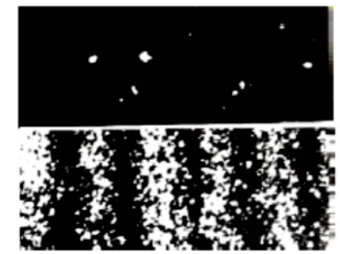
$\Psi_0(q)$ **evolves according to**

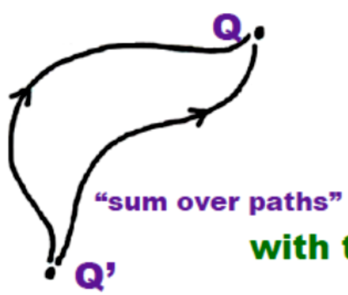
$$\Psi_0(q) \rightarrow [a_1\Psi_1(q) + a_2\Psi_2(q)]$$

The probability of seeing particle at position Q on screen:

$$P(Q) = |a_1\Psi_1(Q) + a_2\Psi_2(Q)|^2 = P_1 + P_2 + 2P_{12}$$

with cross-term $P_{12}(Q) - |a_1a_2\Psi_1(Q)\,\Psi_2(Q)|$





Feynman gave a beautiful formulation of QM, perfectly encapsulating this 'superposition'. He writes

$$\psi(Q,t) = \int dQ'\, G(Q,Q';t,t')\,\psi(Q',t')$$

with the 'path integral' sum:

$$G(Q,Q';t,t') = \int_{q(t')=Q'}^{q(t)=Q} \mathcal{D}q(\tau)\, e^{\frac{i}{\hbar}S[q,\dot{q}]}$$

Notice that the path integral captures the relation between phase & action along the worldline

Actually, the path integral formulation gives us much more than the wave-function description:

$$G_o(2,1) = \int_1^2 \mathcal{D}\mathbf{r}(\tau)\, e^{\frac{i}{\hbar}S_{21}[\mathbf{r}(\tau)]} = \sum_\alpha \chi(\alpha) G_o^\alpha(2,1)$$

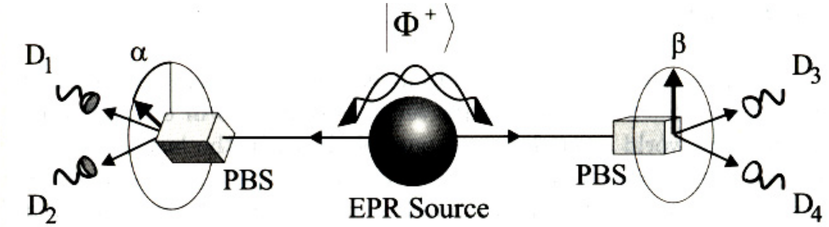
(fractional statistics!)

Thus, Feynman's formalism gives directly an unambiguous answer to global problems. Other formalisms use ad hoc, extraneous conditions to deal with global problems, such as boundary conditions on wave functions, symmetry or antisymmetry property of the wave function, etc. ... and their answers are not necessarily identical with Feynman's.

C. Morette-DeWitt,
Comm. Math. Phys. 28, 47 (1972)

(2) Long before Feynman, Einstein & Schrodinger (1935) fingered "ENTANGLEMENT" as the real essence of QM – embodied in states like

$$\Psi = [\,\phi_+(A)\,\phi_-(B) + \phi_-(A)\,\phi_+(B)\,]$$



for which the quantum state of either individual system is literally meaningless!
NB: In the path integral formulation, entanglement is a CONSEQUENCE of superposition.

(3) Another thing that is often forgotten, but is also essentially quantum-mechanical, is the idea of INDISTINGUISHABILITY, which leads to particle statistics. Laidlaw & Morette-deWitt (1977) showed we need path integrals to truly understand this (for example, for fractional statistics, or any topological quantum state)

(4) The flat space field generating functional is a generalization of sourced QM (path integral form). Thus, eg., for QED, we have the 'in-out' functional:

$$\mathcal{Z}[\bar\eta,\eta;j^\mu] = \int_{in}^{out} \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}A^\mu \quad \exp\frac{i}{\hbar}\int d^4x\ [(\bar\psi\eta+\bar\eta\psi+j_\mu A^\mu)]$$
$$\times\ \exp\frac{i}{\hbar}\int d^4x\ \left[L_A^o(A^\mu)-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2+\bar\psi(i\gamma_\mu\partial^\mu-m_o-e\gamma_\mu A^\mu)\psi\right]$$

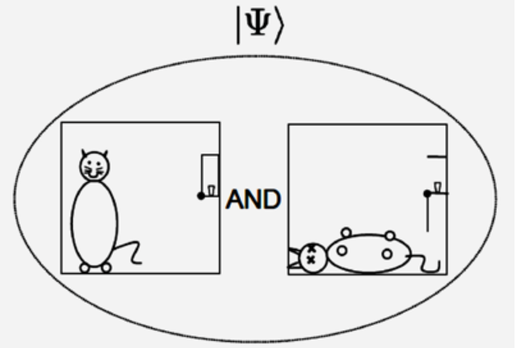
These are the basis of contemporary QFT – they are NECESSARY TO CAPTURE GLOBAL EFFECTS. Again, we need a path integral. Likewise in curved spacetime.

So – we conclude that the essence of QM can be captured by path integrals over worldlines, incorporating indistinguishability

WHY CONVENTIONAL QM/QFT is a REAL PAIN

(a) the Wave-Function: We have no real clue in QM or QFT what the state vector $|\psi>$ is supposed to represent physically ...

Let's entangle a microscopic object with a macroscopic object in a macroscopic superposition of states. What then does the state vector refer to? To "Live Cat + Dead Cat" ???



(i) Now $|\psi>$ can't represent a real physical object, because changes in $|\psi>$ can happen non-locally (cf the EPR & related paradoxes). But (ii) if $|\psi>$ only represents 'information', different observers can assign different $|\psi>$. Moreover, we then lose all reference to the physical world. SO THE WAVE-FUNCTION IS A REAL PAIN

(b) Measurements & Operators: $|\psi>$ changes discontinuously during a measurement. But a Mmt. is a physical process, not some extra-physical operation! We write

$$\langle M_j \rangle = Tr\{\hat{M}_j \hat{\rho}\}$$

where the density matrix acts as a 'projection' operator – these operators represent EXTRANEOUS non-QUANTUM AGENTS.

We have ARTIFICIALLY divided the world into systems and 'observers' (likewise in conventional QFT). So MEASUREMENTS & OPERATORS ARE A REAL PAIN.

(c) Time & Boundary Conditions: QM & QFT are formulated with very strange B.C.s., giving a central role to 'experimental observers', who also impose time-asymmetry – it would be better do things globally. So, BOUNDARY CONDITIONS in QM VIOLATE ITS NON-LOCAL SPIRIT, & ARE A REAL PAIN.

Conventional QM is mediaeval in its anthropocentrism

"To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory." JS Bell (1988)

Yet QM/QFT has also been incredibly successful. QUO VADIS ??

INGREDIENTS for a NEW THEORY

We would like to set up a theory which overcomes the clash between GR & QM. Both work really well, and have never been falsified. So we need to decide what to keep, & what to throw away.

MAKE FOLLOWING ASSUMPTIONS

ASSUMPTION 1. The existence of world-lines in spacetime is fundamental. Spacetime is then defined by the world-lines of objects or fields, and by its interaction with their stress-energy (so that spacetime is also self-interacting).

ASSUMPTION 2. Superpositions and interference exist in Nature (along with entanglement); and the phase ϕ along world-lines or world-surfaces is given by $\phi = hS$ where S is the worldline/world-surface action. Indistinguishability is also incorporated at this point.

These first two lead us to a path integral formulation

ASSUMPTION 3. The comparison/communication between different spacetimes in a superposition is achieved – indeed defined - by gravity itself. This is why it couples universally to matter. The comparison is one of accumulated phase along the worldlines (cf. assumptions 1 and 2).

What we wish now is to argue that this leads to a picture in which paths are correlated – so that the superposition principle is no longer valid

For more detail: PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
" , New J. Phys. 17, 06517 (2015)
& to be published

RULES of the GAME

Q1: What is the most general modification we can make to QM/QFT, consistent with those features we wish to keep?

These features are:

(i) connection between phase (+ connection), and action on worldlines (paths)

(ii) indistinguishability for multiple particles and/or fields

(iii) fully relativistic – obeying the weak principle of equivalence, no violation of causal structure, well-defined metric.

(iv) gravity/spacetime is treated as a quantum field as well as matter

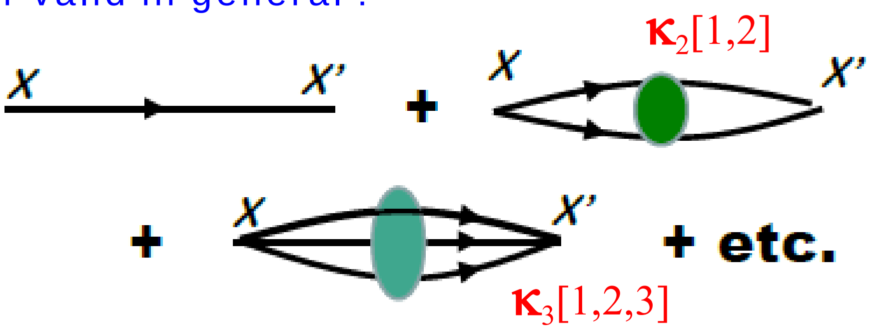
The answer goes roughly as follows; we change the mathematics to:

$$G_o(2,1) = \int_1^2 \mathcal{D}q(\tau) e^{\frac{i}{\hbar}S(2,1)} \longrightarrow \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\, \kappa_n[\{q_k\}]\, e^{\frac{i}{\hbar}S[q_k;2,1]}$$

In other words, we allow arbitrary correlations between any number of different paths. Since the paths are no longer independent, the superposition principle is no longer valid in general !

A diagrammatic view of this is:

G(x,x') =



But – this is only a mathematical framework with almost infinite freedom to choose different correlators – so far it is almost useless.

Q2: If the correlation between paths is "gravitational", what does this imply for the correlators $\kappa_n[q_1,....q_n]$?

The key here is that we treat the quantum phase along each path as physical – & the relationship between them is inevitably 'seen' by gravity. Because of indistinguishability & the equivalence principle, there is no way of distinguishing between gravitational interactions between 2 paths for 2 different particles/fields, and 2 paths for the same particle/field.

We thus arrive at the following prescription:

(1) Use the action: $S_G = \frac{1}{\lambda^2}\int d^4x\,[\tilde{g}^{\mu\nu}R_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu \tilde{g}^{\mu\nu})^2]$ with gauge-fixing term

(2) Use the correlator:

$$\kappa_n = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)\; e^{\frac{i}{\hbar}S_G}\,\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$$

ie., integrate over different spacetimes with a weighting factor

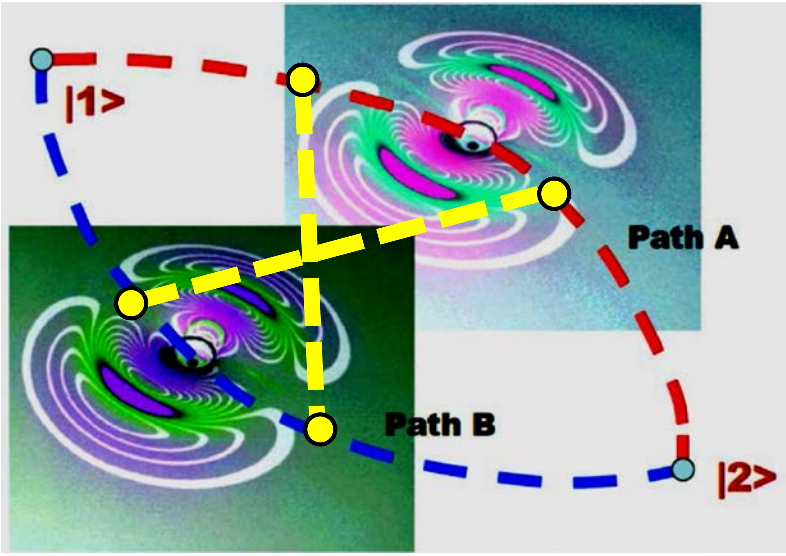
metric density

gravitational action

Faddeev-Popov determinant

Now what this does is COMMUNICATE BETWEEN PATHS the information about each path's spacetime status (and what the object is doing to spacetime).

We then get a PREDICTIVE THEORY with NO ADJUSTABLE PARAMETERS



CWL THEORY: FORMAL STRUCTURE

I. GENERATING FUNCTIONAL

Start with the generating functional

$$\boxed{\mathcal{Q}[j] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}q_k \; \kappa_n[\{q_k\}] e^{\frac{i}{n\hbar}\sum_k (S[q_k] + \int j q_k)}}$$

with normalization

$$\mathcal{N} = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}q_k \; \kappa_n[\{q_k\}] e^{\frac{i}{n\hbar}\sum_k S[q_k]}$$

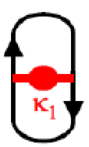
The inter-path correlator is given by

$$\kappa_n[q] \;=\; \frac{1}{n!} \int'' \mathcal{D}g^{\mu\nu} e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \Delta[g^{\mu\nu}] \qquad (\forall\, n > 0)$$

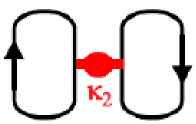
Thus, for the generating functional of a single particle we have

$$\mathcal{Q}[j] \;=\; \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int'' \mathcal{D}\tilde{g}^{\mu\nu} \int \mathcal{D}q_k$$

$$\times \; \frac{1}{n!} e^{iS_G/\hbar} \; \Delta[\tilde{g}^{\mu\nu}] \; e^{\frac{i}{n\hbar}\sum_k (S[g, q_k] + \int ds\; j(s) q_k(s))}$$



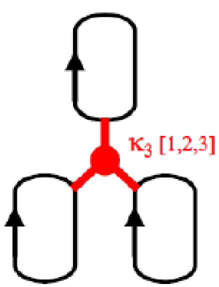
Q$_1$



Q$_2$

where the particle action is

$$S_M \;=\; \int d^4x \, \frac{m}{2} g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \; \delta(x^\lambda - q^\lambda(s))$$



Q$_3$

For a scalar field we have the simple generalization

$$\mathcal{Q}[J] \;=\; \frac{1}{\mathcal{N}} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int \mathcal{D}\Phi_k \; \kappa_n[\{\Phi_k\}]$$

$$\times \; e^{\frac{i}{n\hbar}(S[\Phi_k] + \int d^4x J(x) \Phi_k(x))}$$

II. CORRELATION FUNCTIONS

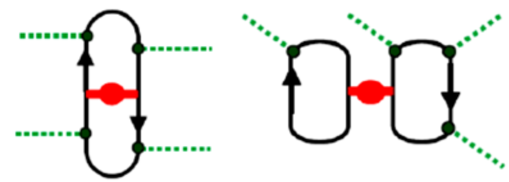
For a single particle we define the CWL correlator

$$\boxed{\mathcal{G}_n^{\sigma_1,..\sigma_n}(s_1,..s_n) \;=\; \left(\frac{-i}{\hbar}\right)^n \lim_{j(s)\to 0}\left[\frac{\delta^n \mathcal{Q}[j]}{\delta j(s_1\sigma_1)..\delta j(s_n\sigma_n)}\right]}$$

$$= \sum_{r=1}^{\infty}\prod_{\alpha=1}^{r}\int \mathcal{D}q^{\alpha}(\tau)\, e^{\frac{i}{r\hbar}\sum_\alpha S_o[q^\alpha]}\;\kappa_r(\{q^\alpha\})\prod_{j=1}^{n}\left(\sum_{\alpha=1}^{r} q^{\alpha}(s_j,\sigma_j)\right)$$

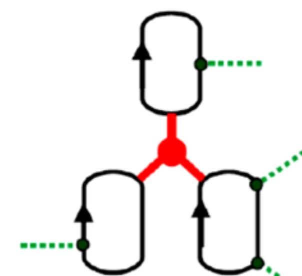
$$= \sum_{r=1}^{\infty}\frac{1}{r!}\int'' \mathcal{D}g^{\mu\nu} e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]}\Delta[g^{\mu\nu}]\prod_{\alpha=1}^{r}\int \mathcal{D}q^{\alpha}(\tau)\, e^{\frac{i}{r\hbar}\sum_\alpha S_o[q^\alpha,g^{\mu\nu}]}\prod_{j=1}^{n}\left(\sum_{\alpha=1}^{r} q^{\alpha}(s_j,\sigma_j)\right)$$

We can represent this messy formula diagrammatically by the sum shown at right, where the green hashed lines represent current insertions – we sum over all combinatoric possibilities.



(a) (b)

The same structure exists for a set of fields. Thus, eg., for a single scalar field we have the explicit expansion, for the 4-point correlator, given by



(c)

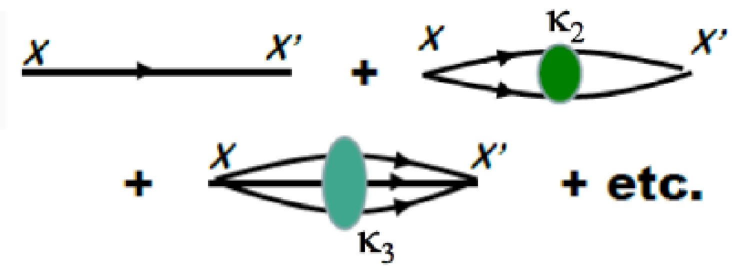
$$\mathbb{G}_4^{\sigma_1,..\sigma_4}(x_1,..x_4) \;=\; \oint \mathcal{D}\phi(x)\, e^{\frac{i}{\hbar}S[\phi]}\prod_{j=1}^{4}\phi(x_j,\sigma_j)$$

$$+\;\oint \mathcal{D}\phi(x)\oint \mathcal{D}\phi'(x)\, e^{\frac{i}{\hbar}(S[\phi]+S[\phi'])}\;\kappa_2[\phi,\phi']\prod_{j=1}^{4}\left[\phi(x_j,\sigma_j)+\phi'(x_j,\sigma_j)\right]$$

III. STRUCTURE of PROPAGATORS

Recall that in ordinary QM we have the 1-particle propagator:

$$K(x,x') \;=\; \int_{x'}^{x} \mathcal{D}q \; e^{\frac{i}{\hbar} S_M[q]}$$



In CWL theory we have the generalization:

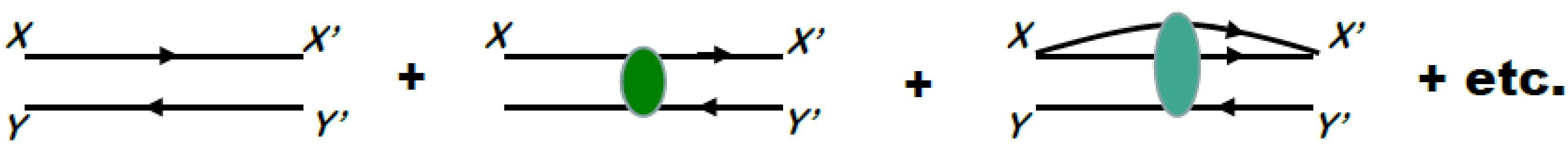
$$\boxed{\mathcal{K}(x,x') \;=\; \int \mathcal{D}g^{\mu\nu} \Delta(g) \; e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{x'}^{x} \mathcal{D}q_k \; e^{\frac{i}{n\hbar} \sum_k \; S_M[q_k, g^{\mu\nu}]}}$$

which is shown diagrammatically at top right.

For a many-body system we can define the N-particle propagator

$$\mathcal{K}_N(x_1, ...x_N; x'_1, ...x'_N) \;=\; \prod_{j=1}^{N} \int \mathcal{D}g^{\mu\nu} \Delta(g) \; e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \sum_{n_j=1}^{\infty} \prod_{k_j=1}^{n_j} \int_{x'_j}^{x_j} \mathcal{D}q_{k_j}^{(j)} \; e^{\frac{i}{n_j\hbar} \sum_{k_j=1}^{n_j} S_M[q_{k_j}^{(j)}, g^{\mu\nu}]}$$

Diagrammatically we have:



All of this has an obvious generalization to fields – for propagation between initial and final field configurations

IV. CONDITIONAL / COMPOSITE PROPAGATORS

Let's first recall that in conventional QFT we can define the composite propagator/correlator:

$$\chi_1^{(p)}(x, x'|\{q(t_\alpha)\}) = \langle x|\hat{T}\{q(t_1), ...q(t_p)\}|x'\rangle$$

which has the path integral representation:

$$\chi_1^{(p)}(x, x'|\{q(t_\alpha)\}) = \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar}S[q]} \prod_{\alpha=1}^{p} q(t_\alpha)$$

$$= (-i\hbar)^p \frac{\delta^p}{\delta j(t_1)....\delta j(t_p)} K_1(x, x'|j(t)) \Big|_{j=0}$$

where we have defined the external current-dependent propagator:

$$K_1(x, x'|j) = \int_{x'}^{x} \mathcal{D}q \, e^{\frac{i}{\hbar}(S[q]+\int dt j(t) q(t))}$$

Now in CWL theory we have

$$\boxed{\chi_1^{(p)}(x, x'|\{q(t_\alpha)\}) = (-i\hbar)^p \frac{\delta^p}{\delta j(t_1)....\delta j(t_p)} \mathcal{K}_1(x, x'|j(t)) \Big|_{j=0}}$$

where now the propagator involves the CWL sum:

$$\boxed{\mathcal{K}_1(x, x'|j) = \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{x'}^{x} \mathcal{D}q_k \, e^{\frac{i}{n\hbar}\sum_k \left(S_M[q_k, g^{\mu\nu}] + \int dt j(t) q_k(t)\right)}}$$

Working this out we get:

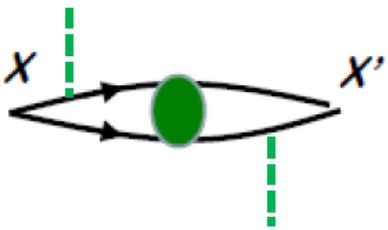
$$\chi_1^{(p)}(x, x'|\{q(t_\alpha)\}) = \int \mathcal{D}g^{\mu\nu} \Delta(g) \, e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_{x'}^{x} \mathcal{D}q_k \, e^{\frac{i}{n\hbar}\sum_k S_M[q_k, g^{\mu\nu}]} \prod_{\alpha=1}^{p} \left(\sum_{k'=1}^{n} q'_k(t_\alpha) \right)$$

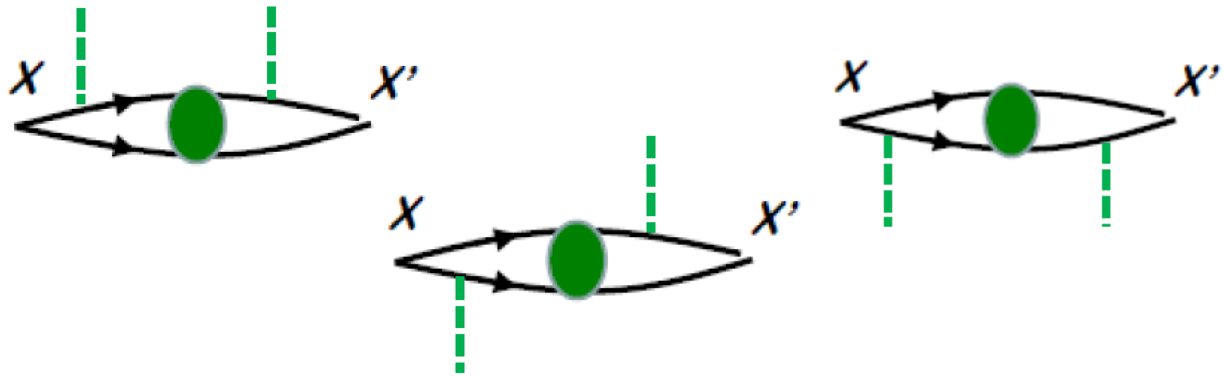
which has the diagrammatic interpretation shown on the next page

DIAGRAMMATIC INTERPRETATION

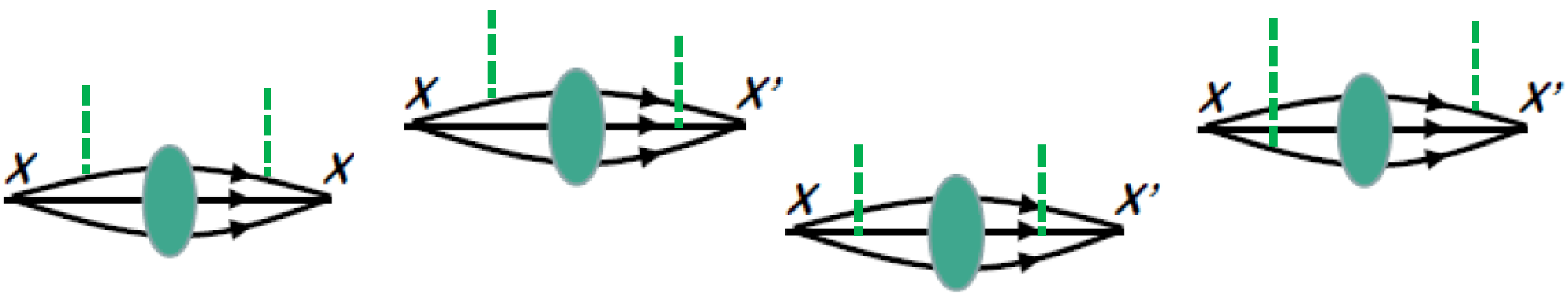
Consider for example a 1-particle propagator with 2 current insertions. Then the conventional QFT result is

The first set of CWL corrections looks like:





The next set of CWL corrections looks like:



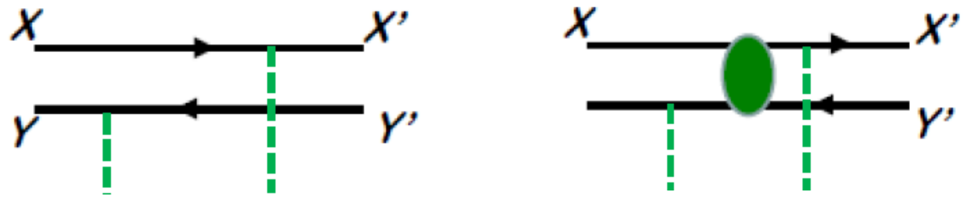
and so on....

HIGHER CONDITIONAL / COMPOSITE PROPAGATORS

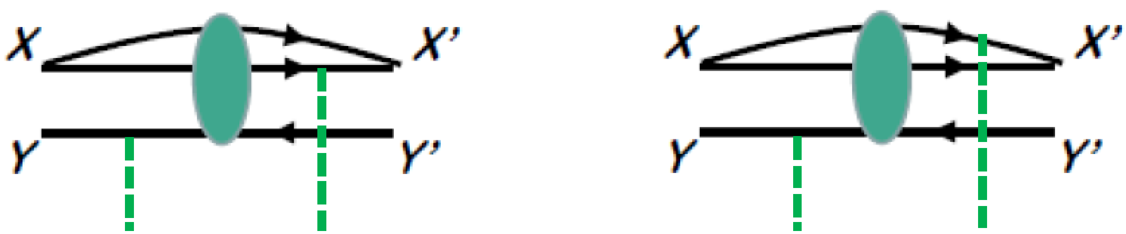
Consider, eg., the 2-particle propagator. Without writing down the formulas, it is obvious what we will get.

Thus, eg., if we have 2 external insertions, and the 2 particles are distinguishable, we have

(a) Conventional QFT:

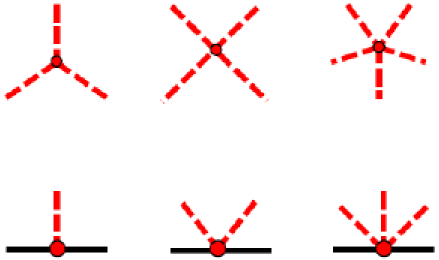


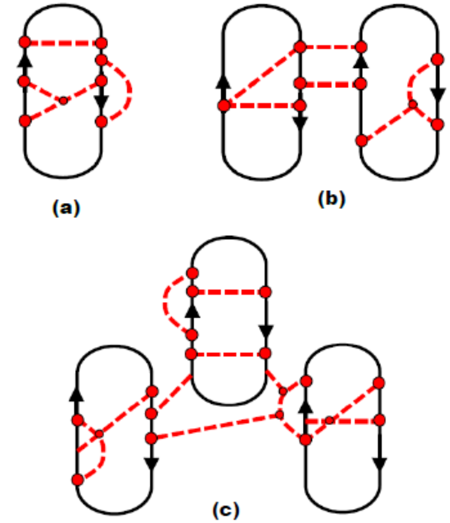
(b) CWL corrections: The lowest-order terms are:



It is fairly obvious where one goes on from here.

V. GRAVITON EXPANSIONS





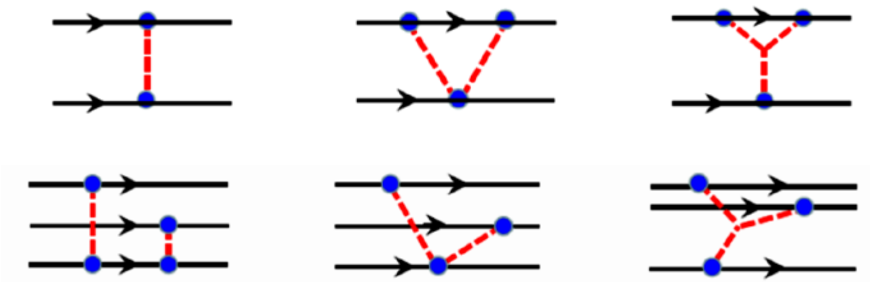
Suppose we make an expansion about a background spacetime – in this case flat space. Then:

$$\tilde{\mathfrak{g}}^{\mu\nu}(x) = \eta^{\mu\nu} + \lambda h^{\mu\nu}(x)$$

The Lagrangian is written as a graviton expansion:

$$L_G = L_o - \int d^4x U(h^{\mu\nu})$$

The CWL generating functional then has the form shown at right, and the correlators have terms like those shown below.



$\kappa_2[q_1(\tau), q_2(\tau)]$

$\kappa_3[q_1(\tau), q_2(\tau), q_3(\tau)]$

EXAMPLE: DENSITY MATRIX PROPAGATOR

Define $\hat{h}(x) = h^{\mu\nu}(x)$

and $\hat{D}(x) \doteq D^{\mu\nu\lambda\rho}(x)$

Then

$$\boxed{\begin{aligned} \mathcal{K}_{2,2';1,1'} &= lim_{\hat{h}=0}\ \{e^{\frac{i}{2\hbar}(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'})} \\ &\times\ e^{\frac{-i}{\hbar}\int U(\hat{h})}\ \mathcal{K}_{2,1}[\hat{h}(x)]\ \mathcal{K}_{1',2'}[\hat{h}(x')]\} \end{aligned}}$$

where $(\delta_{\hat{h}}|\hat{D}|\delta_{\hat{h}'}) = \int d^4x d^4x' \frac{\delta}{\delta \hat{h}(x)} \hat{D}(x,x') \frac{\delta}{\delta \hat{h}(x')}$

and where $\mathcal{K}_{2,1}[\hat{h}(x)]$ is the CWL propagator in a field $\hat{h}(x)$

WEAK FIELD EXPANSON for an INTERFERENCE EXPERIMENT

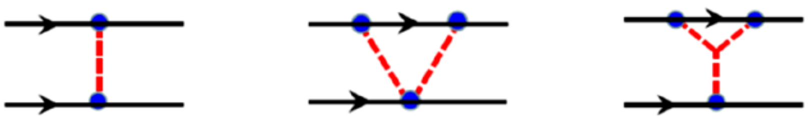
We can calculate the 4-point correlator for the density matrix dynamics, but it is easier to just find the 2-point propagator. Again, recall the form this will take – after integrating over the field *h*(x) we have

$$\mathcal{G}(2,1) = \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\ \kappa_n[\{q_k\}]\ e^{\frac{i}{\hbar} S[q_k;2,1]}$$

The lowest correction to QM goes like:

$$\Delta\mathcal{G}(2,1) = \int_1^2 \mathcal{D}q \int_1^2 \mathcal{D}q'\ \kappa_2[q,q'] e^{\frac{i}{\hbar}(S[q]+S[q'])} + \ \ldots$$

The lowest order irreducible diagrams for this first correction are at right. In de Donder gauge the graviton propagator is



$\kappa_2[q_1(\tau), q_2(\tau)]$

$$\mathcal{D}^o_{\mu\nu\lambda\rho}(q) \ = \ \frac{1}{2q^2}[\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}]$$

and we get: $\kappa_2[x,x'] \ = \ \exp\left[\frac{i\lambda^2}{2\hbar}\int d^4x \int d^4x'\ T^{\mu\nu}(x)\mathcal{D}^o_{\mu\nu\lambda\rho}(x-x')T^{\lambda\rho}(x')\right] - 1 \qquad + \ etc.$

Let's write this as $\kappa_2[q,q'] = e^{i\chi_2[q,q']} - 1$ and take the 'slow-moving' limit where v << c.

Then $q \rightarrow (\mathbf{q}, t)$; define the relative coordinate $\mathbf{r} = \mathbf{q} - \mathbf{q}'$

and we find $\chi_2[q,q'] \ = \ \int^t \frac{d\tau}{\hbar} \frac{m^2\lambda^2}{r(\tau)} \left[1 - \frac{R_s}{2r(\tau)} - \frac{17}{20}\frac{L_p^2}{r^2(\tau)} + \ldots\right]$

$L_p = (\hbar G/c^3)^{1/2}$

$R_s \ = 2Gm/c^2$

SLOW DYNAMICS

In any lab experiment involving massive objects, we will also be able to assume velocities << c. The correlator then simplifies further, to

$$\kappa_2[\mathbf{q},\mathbf{q}'] \;=\; \exp\frac{i}{\hbar}\int^t d\tau\,\frac{4\pi G m^2}{|\mathbf{q}(\tau)-\mathbf{q}'(\tau)|}\;-\;1$$

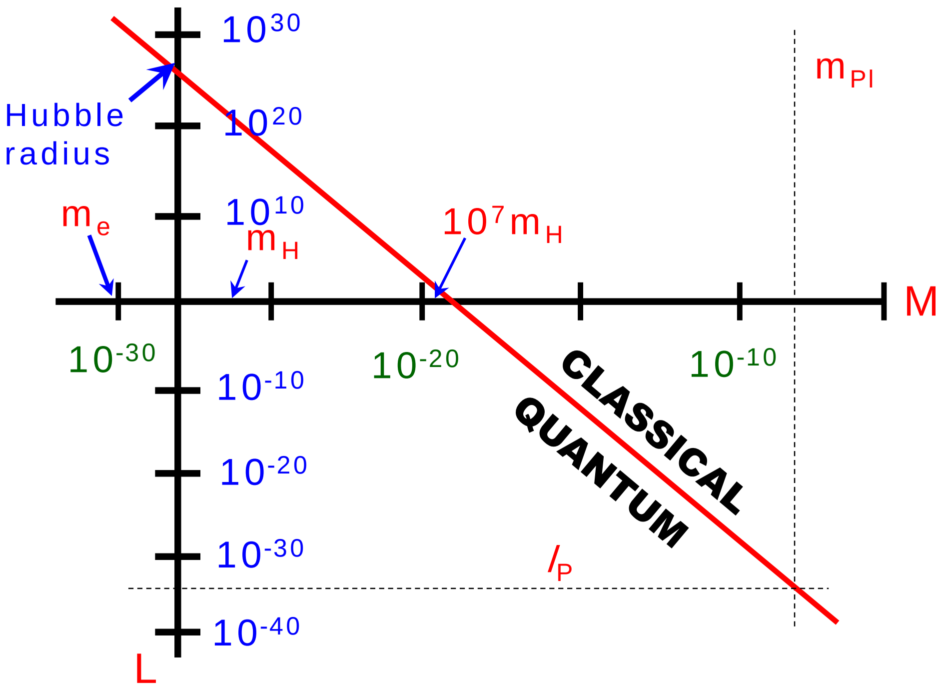
so the path integral looks like that for a Coulomb attraction, with charges m. The key scales are

$$l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$$ Newton radius (gravitational analogue of the Bohr radius)

$$\epsilon_G(m) = G^2m^2/l_G(m) \equiv E_p(m/M_p)^5$$ Mutual binding energy for paths

(QM)

$$R_s \;=\; 2Gm/c^2$$ Schwarzchild radius for the particle (Classical)



The potential well created by this 'Coulomb-Newton' attraction causes a 'path-bunching'. 2 paths will bind if

$$\varepsilon_G > E_Q$$

where E_Q is the energy scale associated with any other perturbations from impurities, phonons, photons, imperfections in any controlling potentials in the systems, and, worst of all, dynamical localized modes like defects, dislocations, paramagnetic or nuclear spins, etc.

N-PARTICLE SYSTEM (SLOW-MOVING)

We write positions around the centre of mass $\mathbf{R_o}(t) = \frac{1}{N}\sum^{N} \mathbf{q}_i(t)$ so that $\mathbf{q}_i = \mathbf{R_o} + \mathbf{r}_j$

The effective action is then $$S_o[\mathbf{R}_o, \{\mathbf{r}_j\}] \;=\; \int d\tau \left[\frac{M_o}{2}\dot{\mathbf{R}}_o^2 + \sum_{j=1}^{N} \frac{m_j}{2}\dot{\mathbf{r}}_j^2 - \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j) \right]$$

Define sum & difference coordinates:
$$\begin{aligned} \mathbf{r_j} + \mathbf{r_j'} &= \mathbf{x}_j \qquad\qquad & \mathbf{R_o} + \mathbf{R_o'} &= \mathbf{X_o} \\ \mathbf{r_j} - \mathbf{r_j'} &= \xi_j & \mathbf{R_o} - \mathbf{R_o'} &= \mathbf{\Xi_o} \end{aligned}$$

Then we have a propagator
$$\begin{aligned} \Delta\mathcal{G}(2,1) \;&=\; \int \mathcal{D}\mathbf{X}_o \int \mathcal{D}\mathbf{\Xi_o} \prod_j \int \mathcal{D}\mathbf{x}_j \int \mathcal{D}\xi_j \\ &\times\; \kappa_2^N[\mathbf{\Xi}_o; \{\xi_j\}] \int d\mathbf{P}d\mathbf{K}\, e^{\frac{i}{\hbar N}(\mathbf{P}\cdot\mathbf{x}_j + \mathbf{K}\cdot\xi_j)}\; e^{i\Psi_2[\mathbf{\Xi}_o,\{\xi_j\};\mathbf{X_o},\{\mathbf{x}_j\}]} \end{aligned}$$

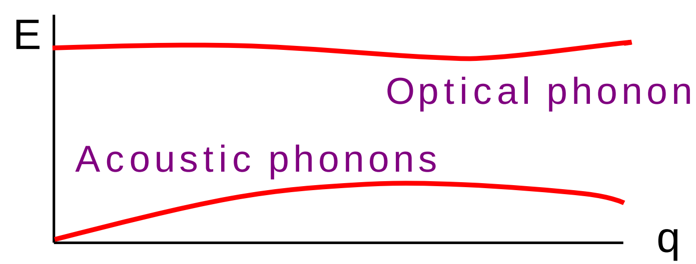
where the C.o.m. correlates gravitationally with the individual particles according to
$$\kappa_2^N[\mathbf{\Xi}_o, \{\xi_j\}] \;=\; \left(\exp\left[\frac{i\lambda^2}{4\pi\hbar} \int d\tau \sum_{j=1}^{N} \frac{m_j^2}{|\mathbf{\Xi}_o + \xi_j|} \right] \;-\; \delta_{\mathbf{\Xi}_o}\delta_{\xi_\mathbf{j}} \right)$$

We now want to analyze this for a real solid

PHONON EFFECTS

We can understand the main effect by looking at the displacement correlator

$$\langle u_i^\alpha(t_1) u_j^\beta(t_1)\rangle \;=\; \frac{1}{N}\sum_{\mathbf{Q}\mu} \frac{\hat{e}^\alpha_{\mathbf{Q}\mu}\hat{e}^\beta_{\mathbf{Q}\mu}}{2m\omega_{\mathbf{Q}\mu}}\; e^{i[\mathbf{Q}\cdot\mathbf{r_{ij}^{(o)}} - \omega_{\mathbf{Q}\mu}(t_1 - t_2)]}$$



Typical displacements: 10^{-12} -- 6×10^{-12} m

TABLE-TOP EXPERIMENTS

Only wimps specialize in the general case. Real scientists pursue examples. MV Berry (1995)

I. MECHANICAL OSCILLATOR

Now we add a term to the action: $S_U[\mathbf{R_o};\mathbf{F_o}] \;=\; -\int d\tau \left[\frac{1}{2} U_o \mathbf{R_o^2}(\tau) \;+\; \mathbf{F_o}(\tau)\cdot\mathbf{R_o}(\tau)\right]$

In the absence of any coupling between the phonons and the centre of mass, we get

$$\begin{aligned}\mathcal{G}(2,1) \;&=\; G_{osc}(2,1)\,G_c(2,1)\\ &\equiv\; G_{osc}(\mathbf{X_o}^{(2)},\mathbf{X_o}^{(1)}|\mathbf{F_o}(t))\;\; G_c(\mathbf{\Xi_o}^{(2)},\mathbf{\Xi_o}^{(1)};\{\xi_\mathbf{j}^{(2)},\xi_\mathbf{j}^{(1)}\})\end{aligned}$$

where the latter term incorporates the reduction of the path-bunching coming from individual ion dynamics.

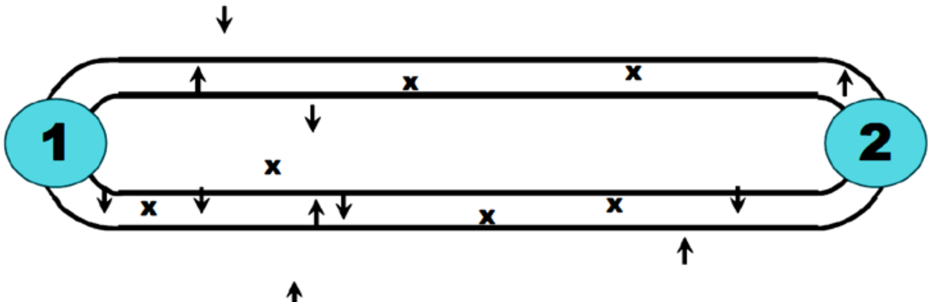
The final result depends strongly on both the phonon dynamics and on the coupling of phonons to defects and spin impurities. The onset of path bunching is now at mass scales $M \sim 10^{18}\ m_H$ with an effective path-bunching length $\sim 10^{-16}$ m.

Such an experiment has many attractive features.

II. 2-SLIT EXPERIMENT

This is at first glance a very attractive experiment to analyse – but to realize it will be very difficult. For an extended mass the numbers come out similarly to those for the oscillator – but the influence of defects and impurities is much greater.

Such an experiment is likely impossible – even if one could do interference for such large objects.



CRUCIAL RESULT: The CWL CORRELATIONS & PATH BUNCHING MECHANISM DO NOT INVOLVE DECOHERENCE !!

COMPARISON with OTHER PREDICTIONS

COMPARISON with PENROSE RESULT: Penrose argues that the 2 proper times elapsed in a 2-branch superposition cannot be directly compared; there is a time uncertainty, related to an energy uncertainty given in weak field by

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}.$$

$$E_{i,j} = -G \int\int d\vec{r}_1 d\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

There are 2 problems here:

(i) The density is fed in by hand – it should be calculated from the theory itself, and will depend on the UV cutoff

(ii) It is only the first term in an exponential.

R Penrose Gen Rel Grav 28, 581 (1996)

W Marshall et al., PRL 91, 130401 (2003)
D Kleckner et al., NJ Phys 10, 095020 (2008)

To understand this, note that each individual term in our correlator is meaningless. It is not permissible to expand the exponential – if we do, each term gives a divergent contribution:

$$\kappa_2[\mathbf{r},\mathbf{r}'] = \sum_{n=1}^{\infty}\prod_{j=1}^{n}\int^{t_j} d\tau_j\, \theta(\tau_j - \tau_{j-1})\delta(t-\tau_n)\, \frac{(4\pi i G m^2)^n}{|\mathbf{r}(\tau_j) - \mathbf{r}'(\tau_j)|}$$

$$= \int^t d\tau\, \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} + \int^t d\tau \int^\tau d\tau'\, \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|}\frac{4\pi i G m^2}{|\mathbf{r}(\tau') - \mathbf{r}'(\tau')|} + \ldots.$$

If we feed in the density by hand, the role of a UV cutoff is obvious from the results:

$$\Delta E = \frac{Gmm_1}{x_0}\left(\frac{24}{5} - \frac{1}{\sqrt{2\kappa}}\right)$$

"Zero point" estimate

$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right)$$

"nuclear radius" estimate

These numbers differ by roughly 1000 !

THANK YOU TO:

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
“ , New J. Phys. 17, 06517 (2015)

WG Unruh
R Penrose
M Aspelmeyer

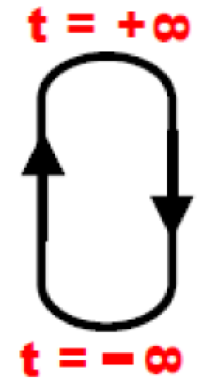
H Brown
D Carney
A Gomez

G Semenoff
RM Wald
C Gooding

Non-RELATIVISTIC QUANTUM MECHANICS in terms of 'RINGS'

We now reformulate ordinary QM in terms of what we will call 'ring' diagrams. The basic object is defined as:

$$\mathcal{Q}_o = \oint \mathcal{D}q(s) e^{\frac{i}{\hbar} S_o[q]}$$



and it sums over all closed rings; eg., for a single particle we have

$$\mathcal{Q}_o \rightarrow \oint \mathcal{D}x(s) e^{-\frac{i}{\hbar} \int ds \frac{m}{2} g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu}$$

We can also "fix" the ring using an external source:

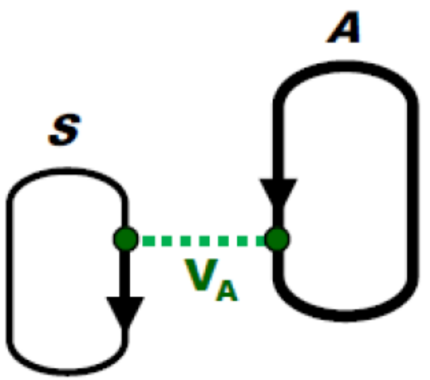
$$\mathcal{Q}_o[j] = \oint \mathcal{D}q(s) e^{\frac{i}{\hbar}(S_o[q] + jq)}$$

which for a single particle is:

$$\rightarrow \oint \mathcal{D}x(s) e^{-\frac{i}{\hbar} \int ds [\frac{m}{2} g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu - j_\mu(x(s)) x^\mu(s)]}$$

In what follows we will assume that these rings go asymptotically out to very large (or infinite) positive and negative times (in practise this may be governed by horizons in the past and future).

For a pair of systems we just get a double ring integral. Of particular interest is the case where one of these is the 'system' S, and the other is the 'apparatus' A.



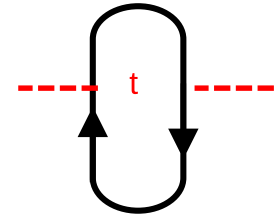
We then go immediately to a field theory; system S is described by a field $\phi(x)$, and the apparatus A by a field $\chi(x)$. Adding external source currents we have the ring functional

$$\mathcal{Q}_o[j, I] = \oint \mathcal{D}\phi \oint \mathcal{D}\chi \, \exp \frac{i}{\hbar} \left[S[\phi, \chi] + \int d^4x j(x)\phi(x) + I(x)\chi(x) \right]$$

CORRELATIONS & DENSITY MATRIX

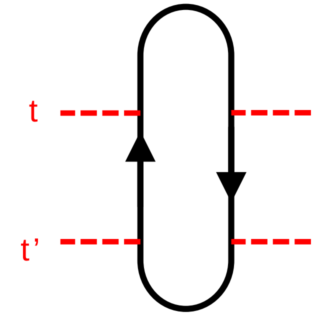
We can define all correlation functions, etc., as well as the density matrix, in terms of functional derivatives of the ring functional – the development is the same as standard QFT. Thus, the 1-particle density matrix is

$$\rho(\mathbf{r},\mathbf{r}';t) \;=\; -\frac{1}{\hbar^2}\frac{\delta^2 \mathcal{Q}_o[j]}{\delta j(x_+)\delta j(x'_-)}\delta(t_+ - t)\delta(t'_- - t)$$



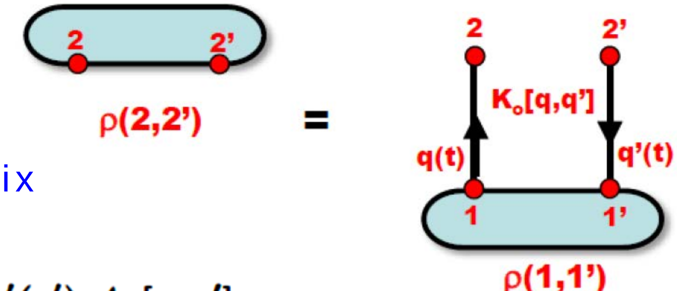
More generally we simply define higher correlators in the usual way:

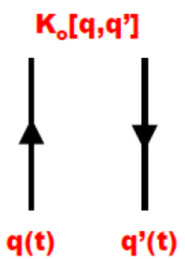
$$G_n^{\sigma_1,..\sigma_n}(x_1,..x_n) \;=\; \left(\frac{-i}{\hbar}\right)^n \frac{\delta^n \mathcal{Q}_o[j]}{\delta j(x_1\sigma_1)..\delta j(x_n\sigma_n)}$$



The time evolution of the non-relativistic density matrix is then given by 'cutting open the ring' (functionally differentiating) to get the usual result:

$$\rho_o(2,2') = \int d1 \int d1' K_o(2,2';1,1')\rho_o(1,1')$$

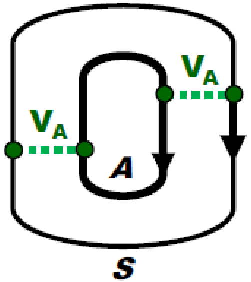




where the propagator K_o for the density matrix is the usual

$$K_o(2,2';1,1') \;=\; \int_1^2 \mathcal{D}q(s) \int_{1'}^{2'} \mathcal{D}q'(s')\; A_o[q,q']$$

where $\quad A_o[x(s),x'(s)] = e^{\frac{i}{\hbar}(S_o[x(s)] - S_o[x'(s)])}$



EXPECTATION VALUES & PROBABILITIES

So far we have not discussed either wave-functions or 'projection operators' or 'measurements'. Actually there is no need to discuss these explicitly, since all we require are the correlations set up between system & apparatus – 2[nd] order perturbation theory is at left.

Suppose however that the apparatus is coupled to an environment; eg.,

$$L(\phi,\chi) = [\partial_\mu\phi\partial^\mu\phi - m^2\phi^2] - \frac{1}{2}\phi^2(x)\chi(x) + [\partial_\mu\chi\partial^\mu\chi - M^2\chi^2]$$

so that then $\mathcal{Q}_o[j,I] \rightarrow \oint \mathcal{D}\phi\, F[\phi,I]\, \exp\frac{i}{\hbar}\left[\mathcal{S}_\phi + \int d^4x\, j(x)\phi(x)\right]$

with 'decoherence functional': $F[\phi,I] = \oint \mathcal{D}\chi\, \exp\frac{i}{\hbar}\left[S^o_\chi + \int d^4x\, [I(x)+\lambda\phi(x)]\chi(x)\right]$

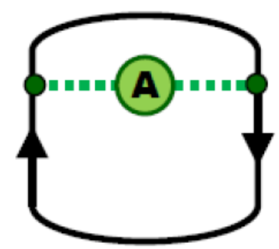
$$\rightarrow \Omega_o[\phi]\, \exp\frac{-i}{2\hbar}\left[\int d^4x \int d^4x'\, I(x)\tilde{\mathcal{D}}(x,x'|\phi)I(x')\right]$$

where $\Omega_o[\phi] = \exp -\frac{1}{2}[Tr \ln|\tilde{\mathcal{D}}_\chi(x,x'|\phi)/D^o_\chi(x,x')|]$,

with $[(D^o_\chi)^{-1}(x,x') + \lambda\phi^2(x)]\,\tilde{\mathcal{D}}_\chi(x,x'|\phi) = -\delta(x-x')$

The net effect of this is to reduce the coupling to a trace; in the non-relativistic limit:

$$< A(t) > = \int dx_1^+ \int dx_2^-\, A(x_1^+,x_2^-)\rho(x_2^-,x_1^+)\delta(t_1^+ - t)\delta(t_2^- - t)$$



More generally: $\langle M_j\rangle = Tr\{M_j\rho\}$

$$= \int dq \int dq'\, M_j(q,q')\rho(q',q).$$

Notice that at no point here have we written down either wave-functions, or state vectors, or operators.