

Current status of the theory predictions for the Higgs inclusive cross section

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Why the Higgs?

- The Standard Model describes the properties and interactions of the fundamental constituents of all visible matter
- It is an highly predictive theory that has been validated by a huge number of collider experiments
- The Higgs boson is essential for the self-consistency of the Standard Model!

Why (still) the Higgs?

- Its discovery, after decades of searches, brought great excitement...



Why (still) the Higgs?

- Its discovery, after decades of searches, brought great excitement...
... followed by a bit of blues..

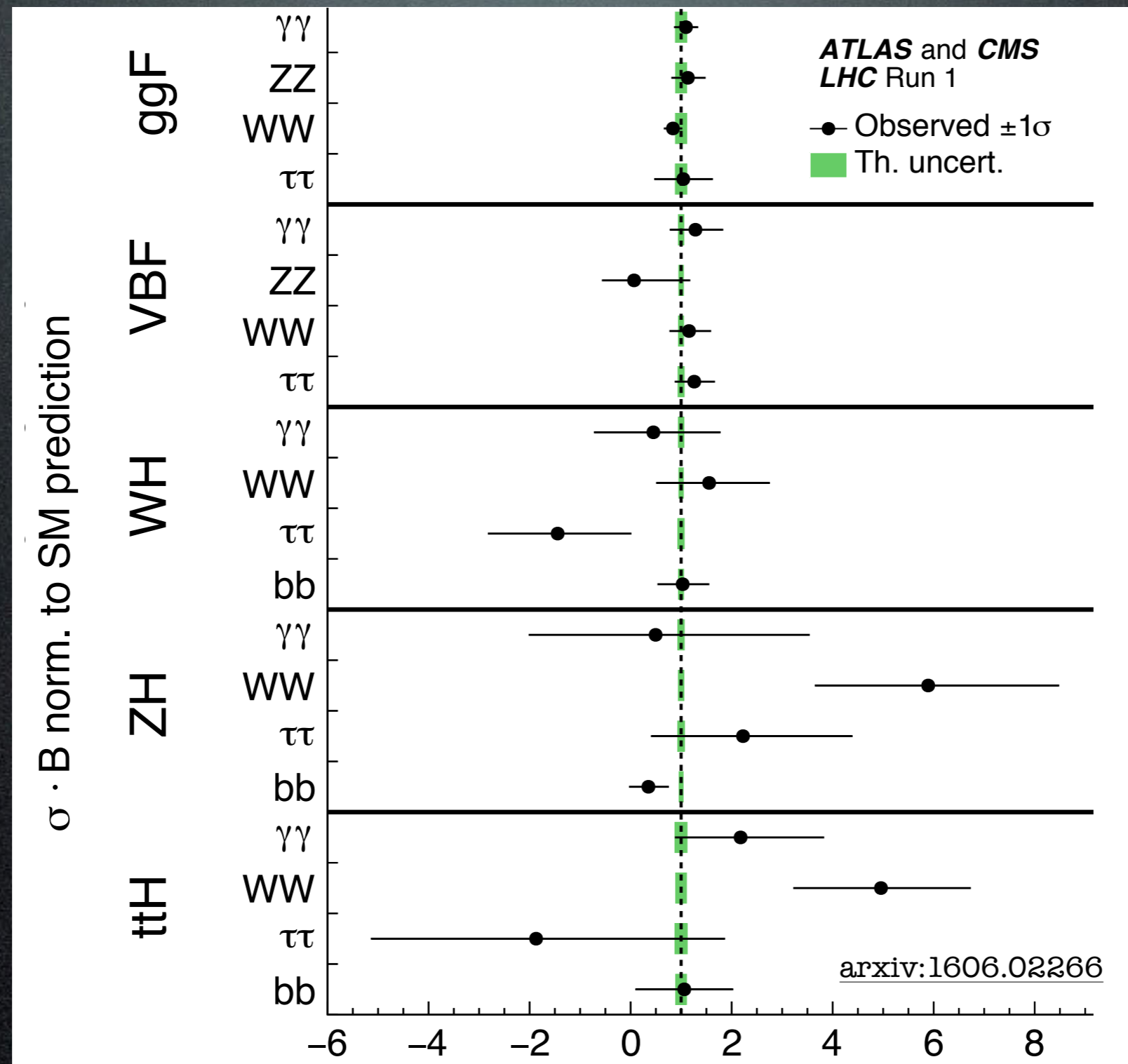


Too predictable?!?

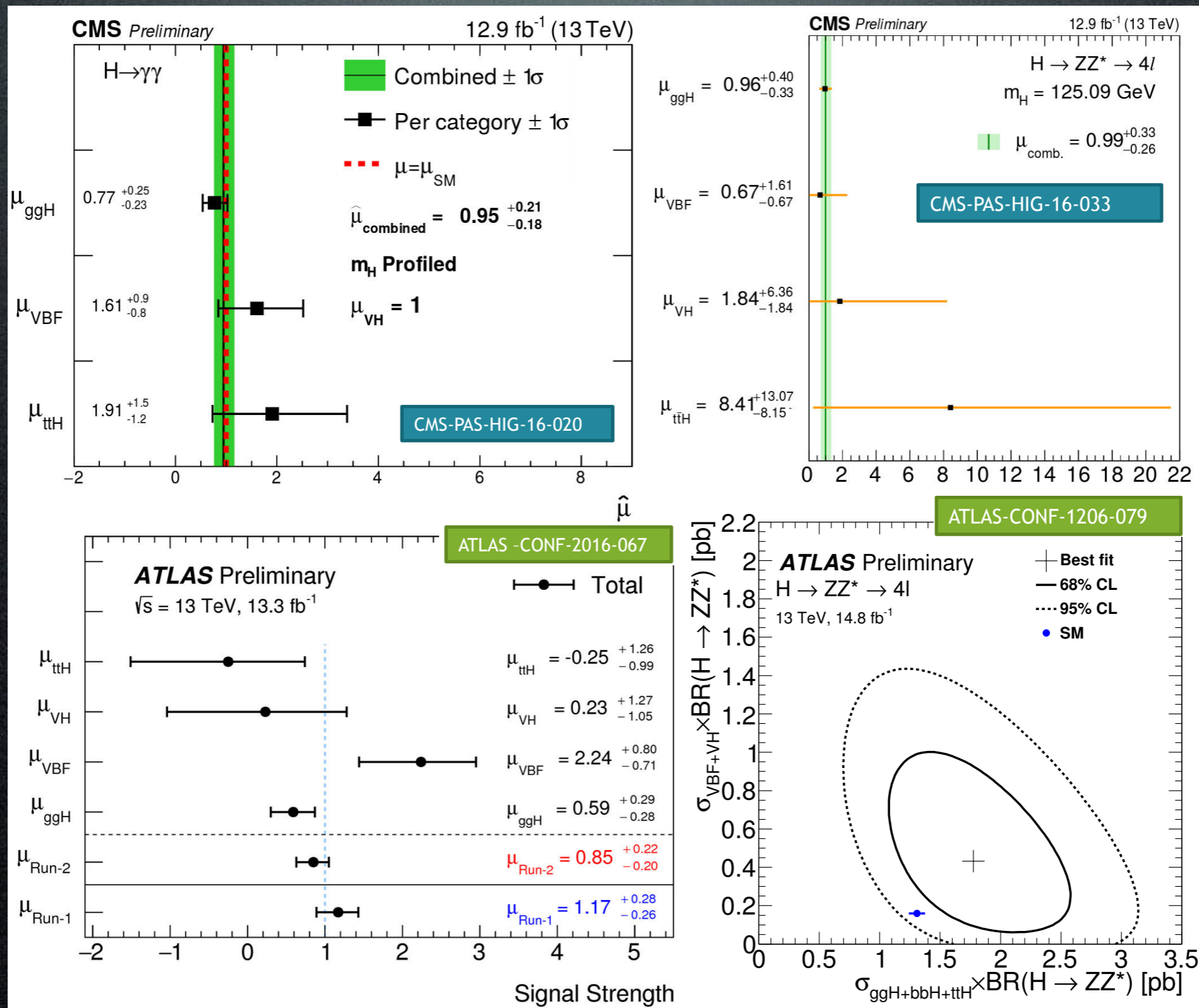
... this Higgs boson looks too “simple”!

- In the “minimal” description of electroweak symmetry breaking, all the couplings of the Higgs are strictly fixed
 - ➡ once its mass is known, all the properties of the Higgs boson (production cross sections, decay rates,..) can be predicted

Too predictable?!?



Too predictable?!?



(see also
summary talk
of F. Canelli at
ICHEP 2016)

So, what is the problem?

- Many observed phenomena (neutrino mass, dark matter, fermion mass hierarchy, inflation, ..) are not described by the Standard Model
 - ▶ can they be related to the origin of electroweak symmetry breaking? Can they affect Higgs physics?
- The Higgs boson is “unnaturally” light
 - ▶ how does the electroweak scale emerge? Is the Higgs sector more complicated than in the Standard Model (new particles/interactions)?

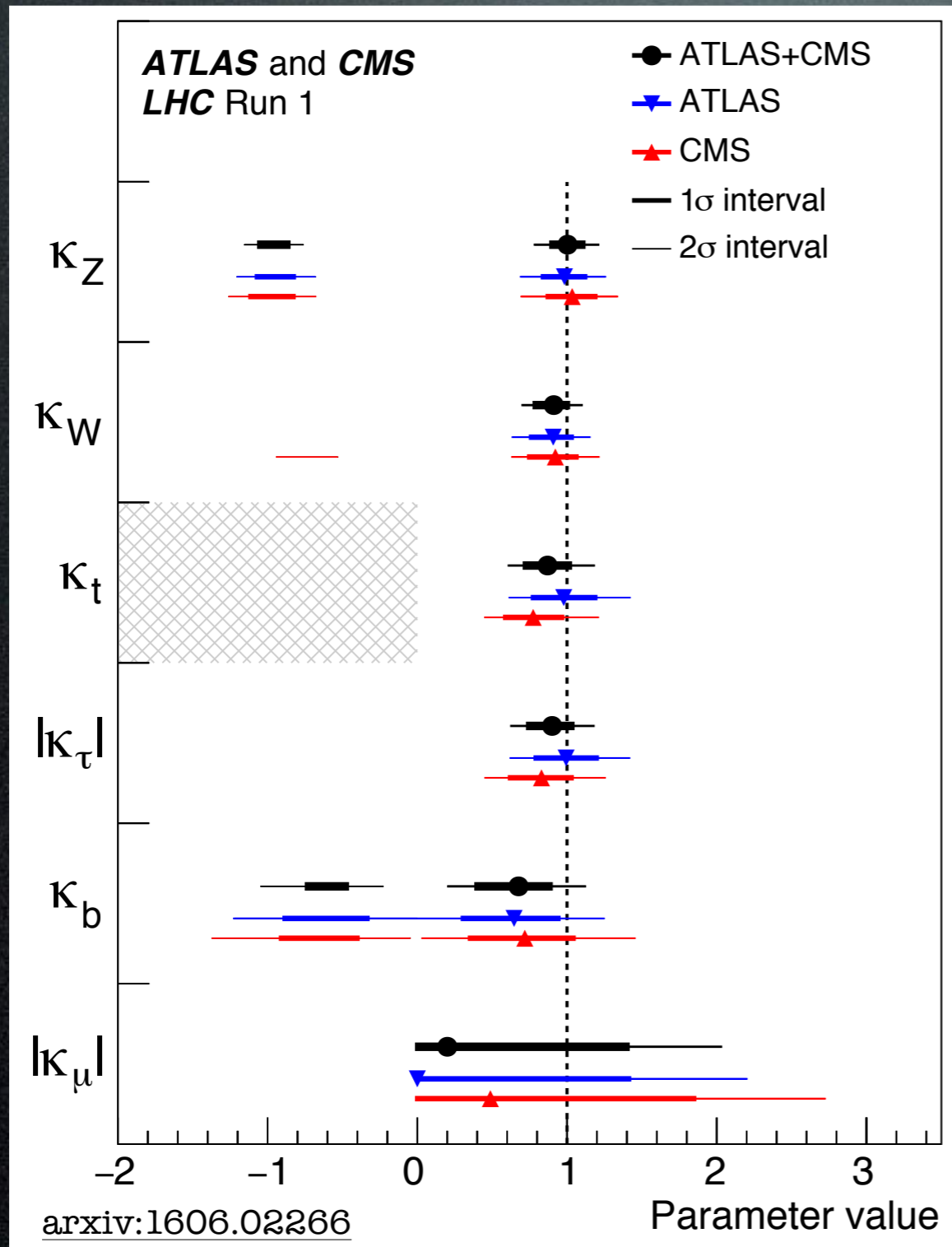
Precision Higgs physics

- Several scenarios of “physics beyond the Standard Model” try to address these questions
- They typically predict modifications in the Higgs phenomenology (new decay modes, modification of the couplings,..)
- These effects can be rather small ($\mathcal{O}(\text{few}\%)$)

Precision Higgs physics

- ➡ to validate the Standard Model (or see hints of new physics) we need
 - ▶ very accurate measurements of the properties of the Higgs boson ..

Precision Higgs physics



Parameter	ATLAS+CMS Measured
κ_Z	$[-1.08, -0.88] \cup [0.94, 1.13]$
κ_W	0.87 $[0.78, 1.00]$
κ_t	$1.40^{+0.24}_{-0.21}$
$ \kappa_\tau $	$0.84^{+0.15}_{-0.11}$
$ \kappa_b $	$0.49^{+0.27}_{-0.15}$

Precision Higgs physics

- ➔ to validate the Standard Model (or see hints of new physics) we need
 - ▶ very accurate measurements of the properties of the Higgs boson ..
 - ▶ .. and equally accurate theoretical predictions of these properties:
the precision in the extraction of the Higgs couplings will soon be limited by the uncertainty in the theory predictions!

Precision Higgs physics

- How can we improve our theory predictions?
 - ➡ which are the largest sources of error in the theory result?
 - ➡ what are the status and the prospects for improving it?

The theory error

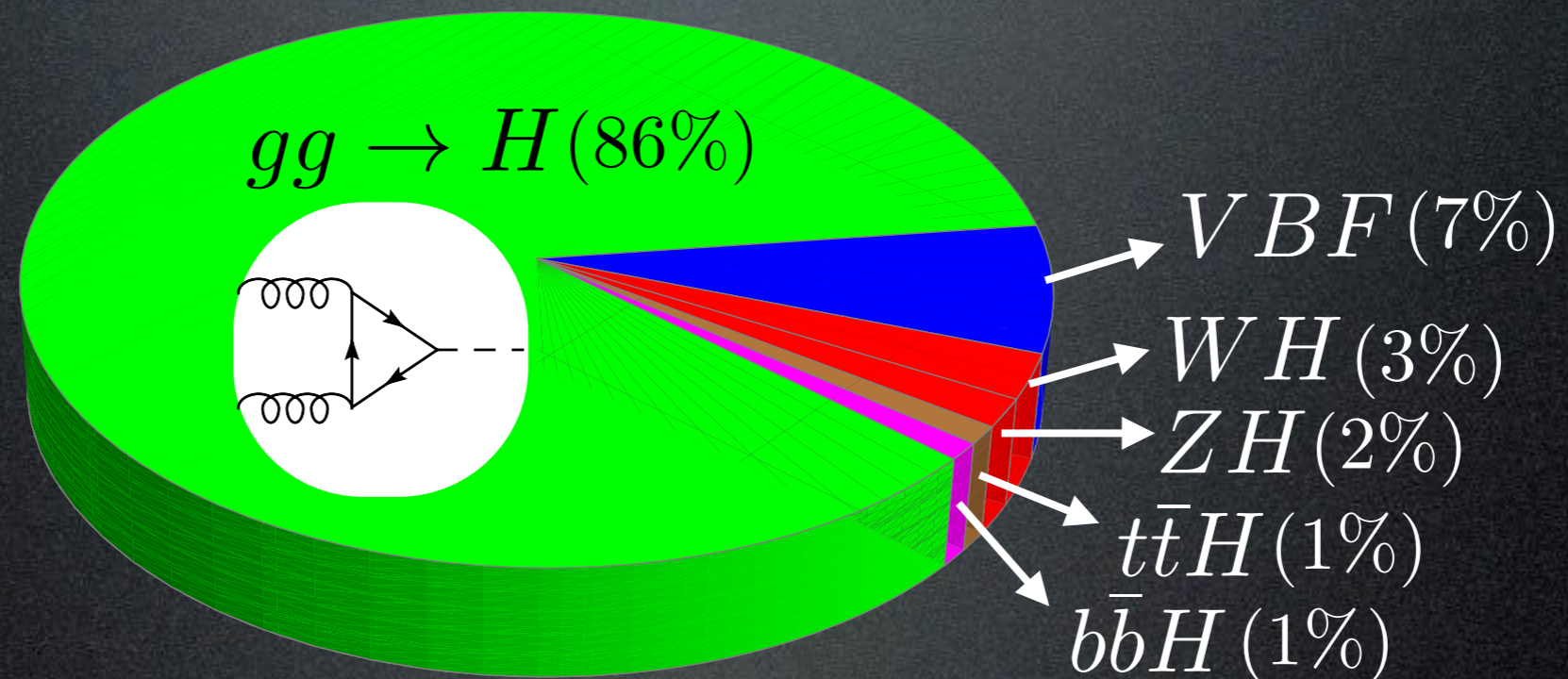
- on the branching ratios: under control

Decay channel	Branching ratio	TU [%]	PU(m_q) [%]	PU(α_s) [%]
$H \rightarrow \gamma\gamma$	2.27×10^{-3}	+1.73 -1.72	+0.93 -0.99	+0.61 -0.62
$H \rightarrow ZZ$	2.62×10^{-2}	+0.99 -0.99	+0.99 -0.98	+0.66 -0.63
$H \rightarrow WW$	2.14×10^{-1}	+0.99 -0.99	+0.99 -0.98	+0.66 -0.63
$H \rightarrow \tau^+\tau^-$	6.27×10^{-2}	+1.17 -1.16	+0.98 -0.99	+0.62 -0.62
$H \rightarrow b\bar{b}$	5.82×10^{-1}	+0.65 -0.65	+0.72 -0.74	+0.78 -0.80
$H \rightarrow Z\gamma$	1.53×10^{-3}	+5.71 -5.71	+0.98 -1.01	+0.58 -0.65
$H \rightarrow \mu^+\mu^-$	2.18×10^{-4}	+1.23 -1.23	+0.97 -0.99	+0.59 -0.64

$m_H = 125 \text{ GeV}$

The theory error

- on the production:
gluon fusion is the main Higgs production mechanism...



The theory error

- on the production:
... and is the channel with the largest theory uncertainty (as of at the end of Run I)

$\sqrt{s} = 13 \text{ TeV}$	$\sigma \text{ [pb]}$	$\delta\sigma^{theo} / \sigma$
ggH	44	+7.4% -7.9%
VBF	3.7	+0.7% -0.7%
WH	1.4	+0.7% -1.5%
ZH	0.87	+3.8% -3.8%
⋮	⋮	⋮

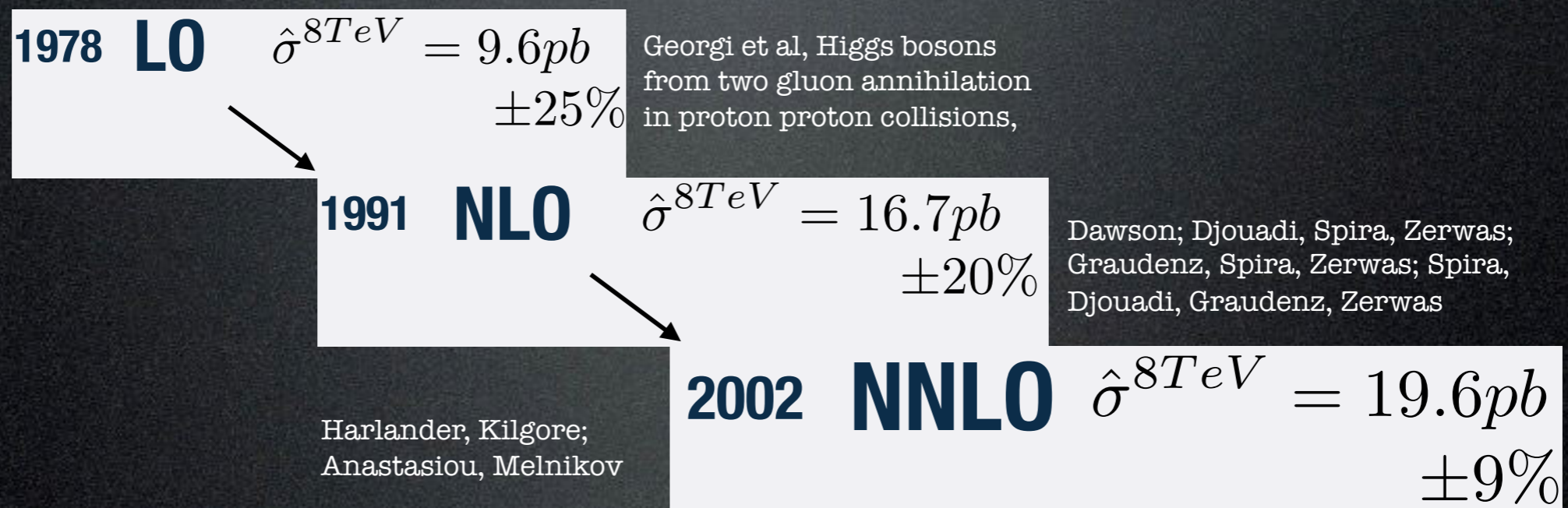
LHC Higgs cross section
WG recommendations, 2014

The theory error

- ➡ the error on the gluon–fusion cross section dominates the theory uncertainty
- ➡ but achieving this level of precision was already extremely hard!

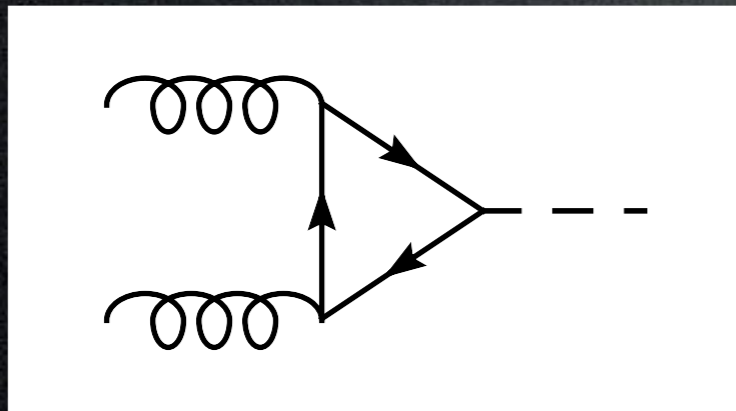
The theory error

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The theory error

- ➡ the error on the gluon–fusion cross section dominates the theory uncertainty
- ➡ but achieving this level of precision was already extremely hard!



leading (lowest) order, “LO”
➡ already one loop
➡ largest contribution from top quark → massive particle
➡ two external momenta

The theory error

- ➡ for Run II, it is necessary to know the gluon–fusion Higgs cross section at $N^3\text{LO}$ in QCD..
- ➡ .. and to include all other contributions beyond pure QCD that change the result by an $\mathcal{O}(5\%)$

Higgs Production at $N^3\text{LO}$

Tools

- heavy-quark effective field theory (EFT)
- threshold expansion
- inverse unitarity
- integration by part identities (IBPs)
- expansion by regions
- differential equation methods
- ...

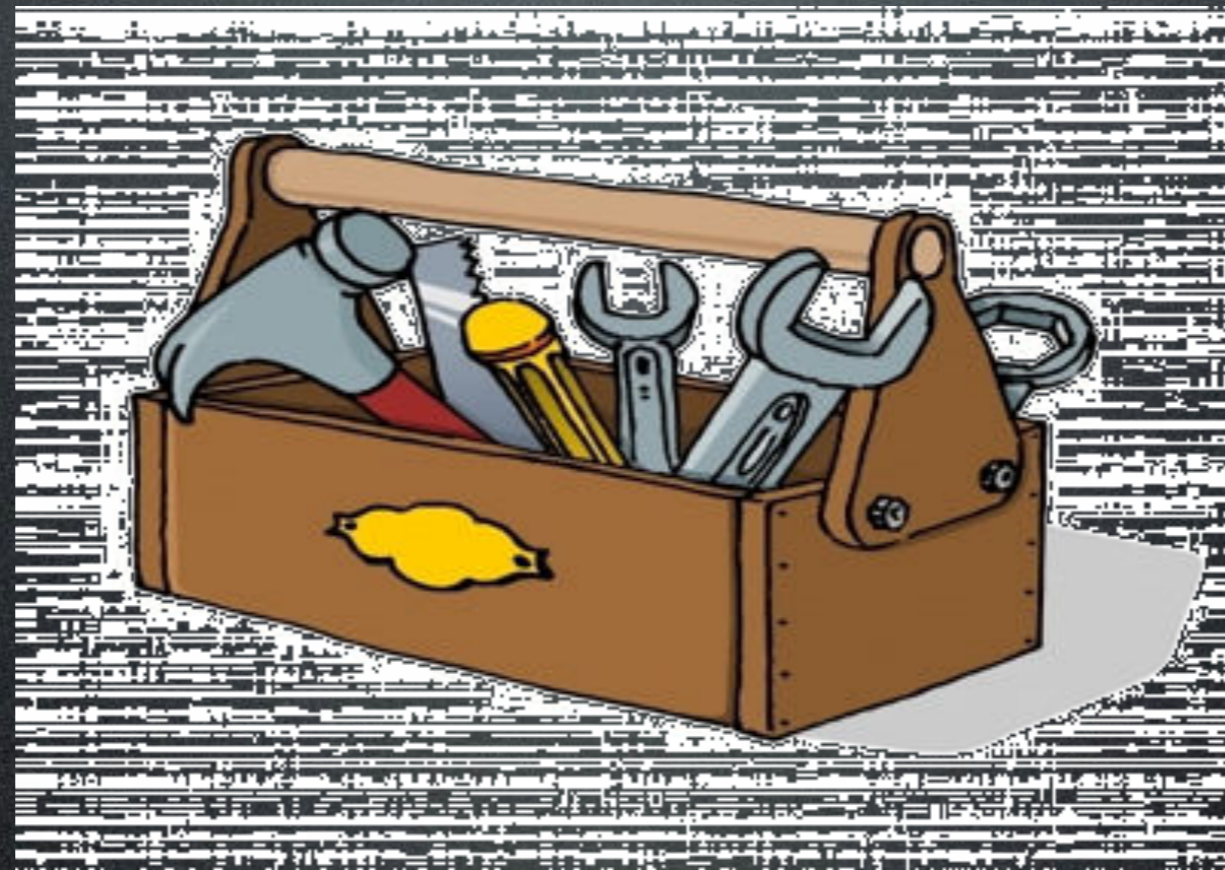
Higgs Production at N³LO

“Additional ingredients”

- full quark–mass effects (from top, bottom, charm) through NLO → go beyond the EFT!
- electroweak (EW) two–loop corrections
three–loop QCD/EW corrections
- convolution with parton distribution functions
- assessment of remaining uncertainties (scale, pdf, α_s , missing contributions, approximations)

Tools

(just a few of them..)



Heavy quark effective theory

- at LO, gluon–fusion Higgs production is mediated by *one* loop of heavy quarks
 - ➡ $N^3LO \rightarrow$ four loops! (~ 15000 diagrams)

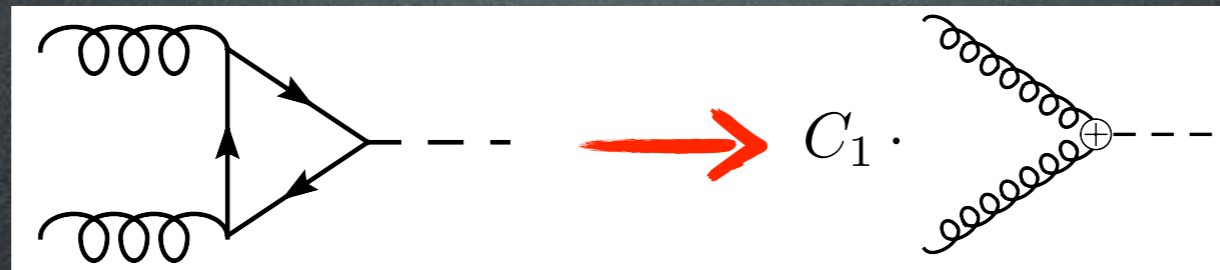
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Heavy quark effective theory

- for a light Higgs boson, the top quark can be integrated out



→ construct an effective theory than only contains the light (massless) Standard Model particles

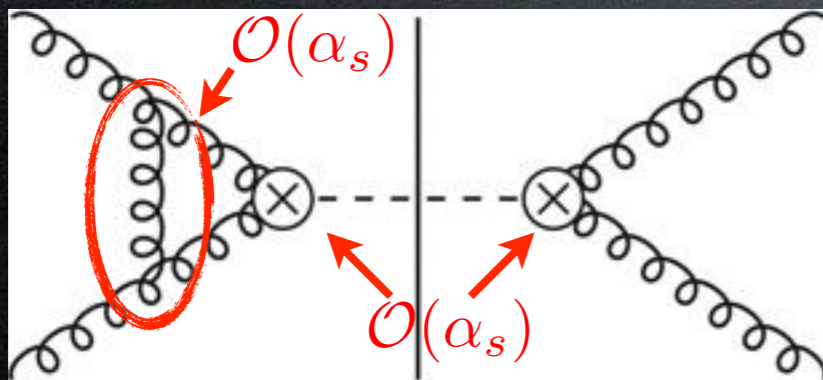
where the top-quark loop is replaced by an effective gluon-Higgs vertex and only the light dof of the SM are present

$$\mathcal{L} \rightarrow \mathcal{L}_{\text{light}} - \frac{\alpha_S}{4v} C_1 H G_{\mu\nu}^a G^{a\mu\nu}$$

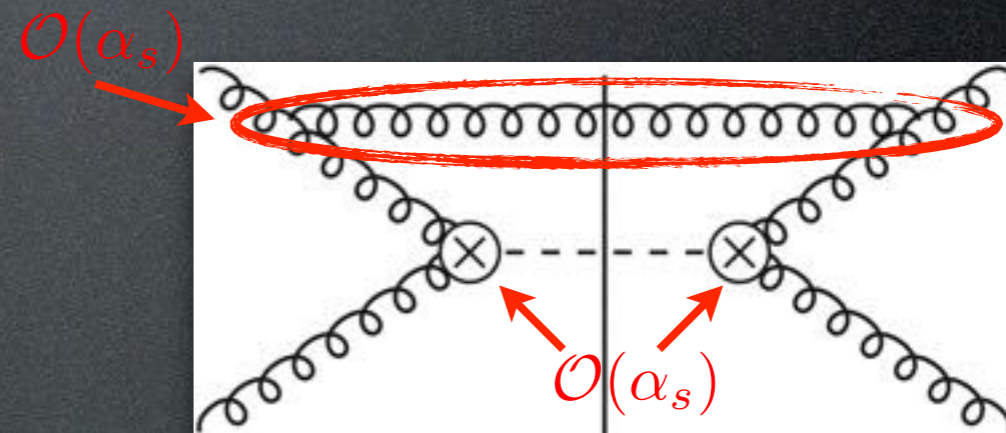
Real radiation

- huge number of contributions from “real” radiation
- at NLO ($\mathcal{O}(\alpha_s^3)$), there are two kind of contribution (in addition to the two loop gluon-Higgs vertex)

Dawson, Nucl. Phys. B 359 (1991) 283;
Djouadi et al., Phys. Lett. B 264 (1991)



Virtual corrections

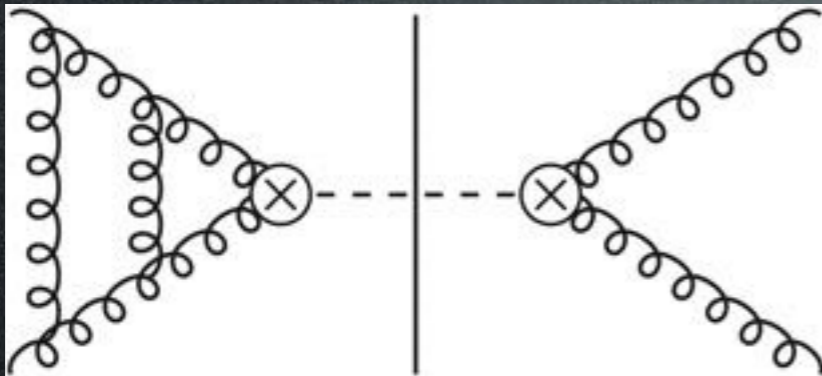


Real radiation

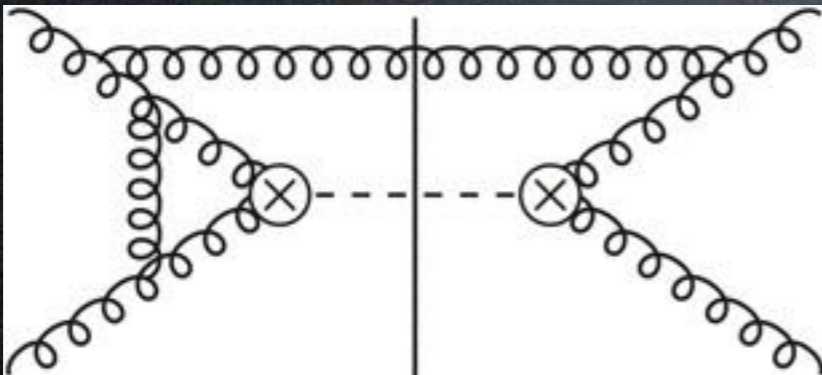
→ ~ 10 diagrams

Real radiation

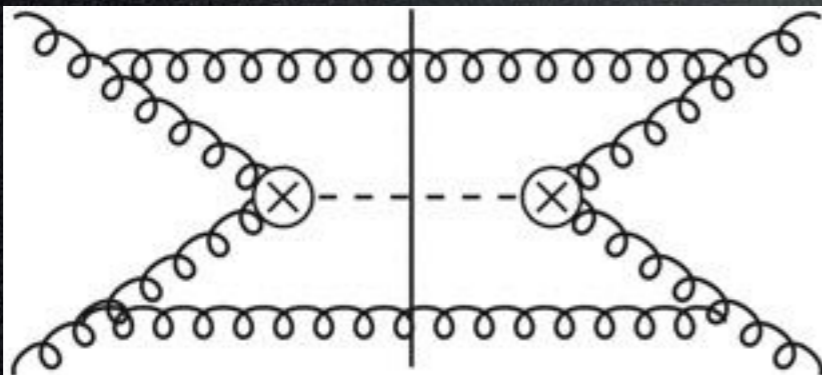
- at NNLO ...



Double-virtual
corrections



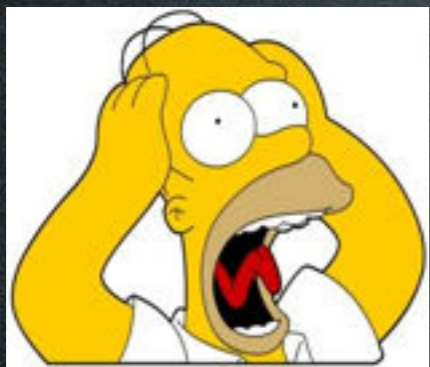
Real-virtual
corrections



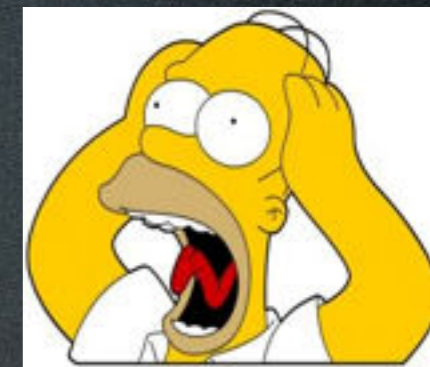
Double-real
corrections

→ ~ 1000 diagrams

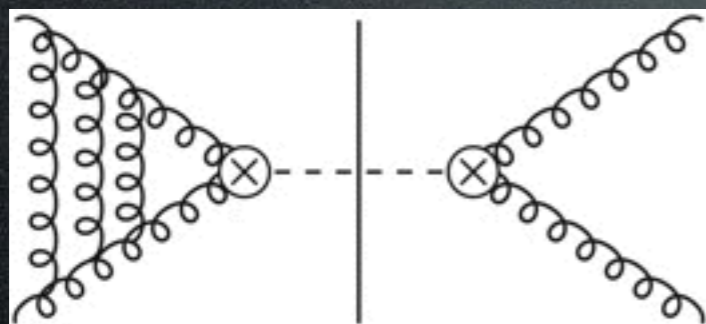
Harlander and Kilgore, PRL 88, 201801 (2002);
Anastasiou and Melnikov, NP B 646 (2002) 220;
Ravindran et al., NP B 665, 325 (2003)



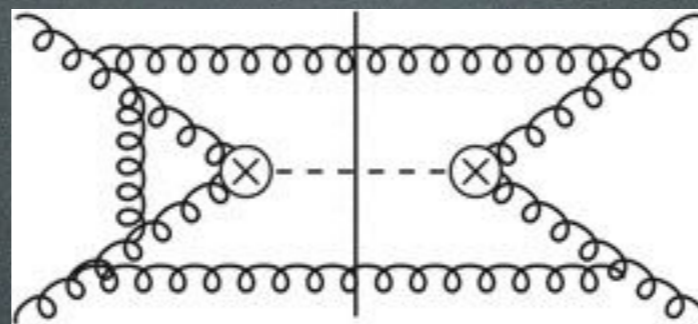
Real radiation



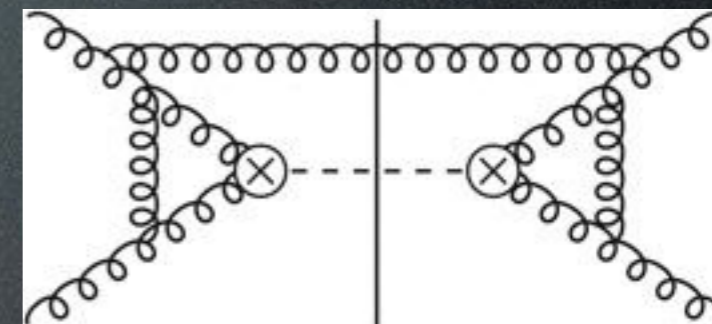
- at $N^3\text{LO}$...



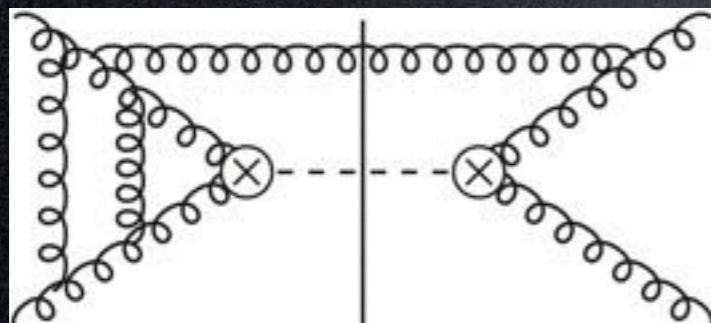
triple-virtual
corrections



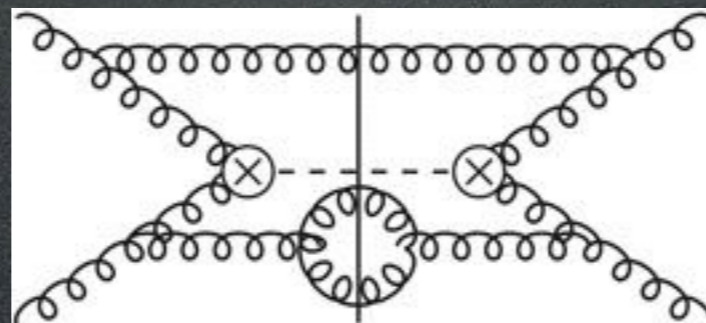
double-real
virtual corrections



real-virtual squared
corrections



double-virtual
real corrections



triple-real
corrections

→ ~ 100000
diagrams



Real radiation



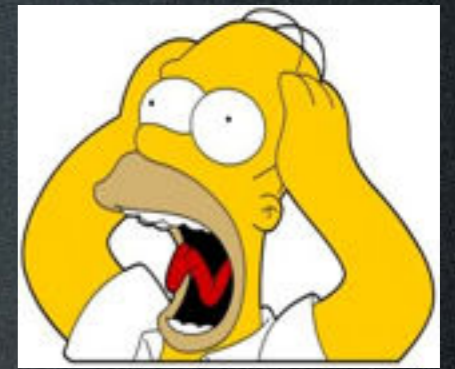
- ➔ the number of interference diagrams increases by a factor of 100 from NNLO to N^3LO
- ➔ the number of integrals increases by 10.000!

NNLO
~ 50.000

N^3LO
~ 500.000.000



Real radiation



- ➔ the number of interference diagrams increases by a factor of 100 from NNLO to N^3LO
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NNLO
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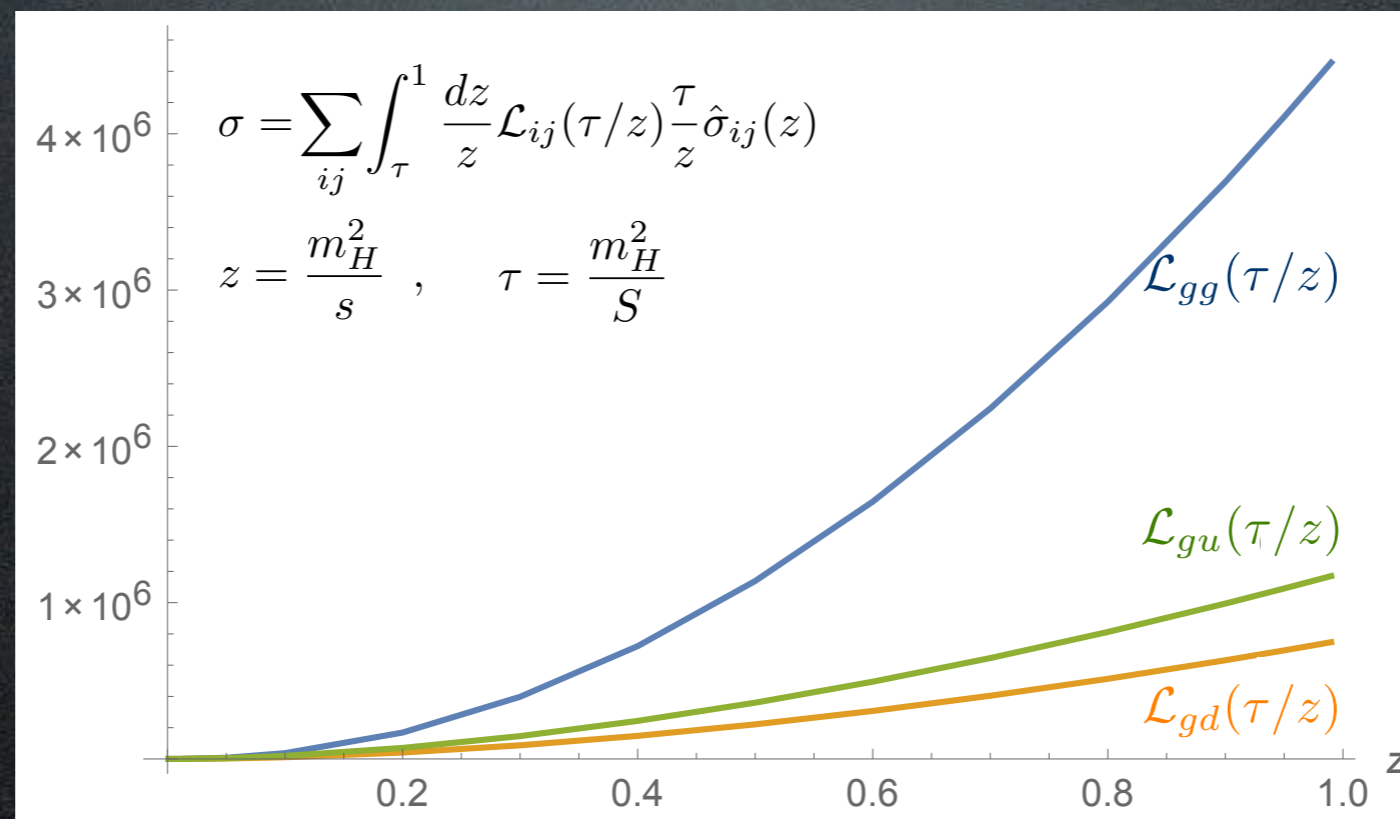
N^3LO
~ 500.000.000

*How can we tackle
this problem?*

Threshold expansion



- As a first approximation, we can consider the production of the Higgs boson at threshold...



Threshold expansion



- As a first approximation, we can consider the production of the Higgs boson at threshold...
- ... and then add subleading terms in the threshold expansion

$$\hat{\sigma}^{N^3LO}(z) = \hat{\sigma}_{SV} + \sum_{n=0}^{N_{trunc}} \sigma^{(n)} (1-z)^n$$

Threshold expansion



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(still, some 20–30.000
integrals to compute)

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integrals to compute)

Anastasiou, Duhr, Dulat, EF, Gehrmann, Herzog,
Mistlberger, JHEP 1503, 091 (2015) (next-to-soft);
PRL 114 (2015) 212001 (up to $N_{trunc} = 37$)

Anastasiou, Duhr, Dulat, EF, Gehrmann, Herzog,
Mistlberger, PLB 737, 325 (2014); Li, von Manteuffel,
Schabinger, Zhu, PRD 90, 053006 (2014)

Integration by part identities

- They allow to handle the huge number of integrals to compute
- They are relations among the integrals that we need to compute
- Solving these equations, we can rewrite all the integrals as linear combinations of a small set of “master integrals”

Tkachov, PLB100, 65 (1981); Chetyrkin, Tkachov, NPB192, 159 (1981); Gehrmann, Remiddi, NPB 580 (2000) 485

Integration by part identities

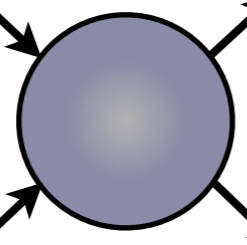
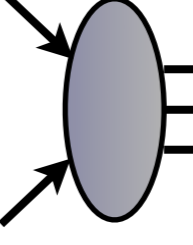
- The system of equation generated is huge itself, but it can be solved in an automated way implementing the Laporta algorithm

Laporta, Int. J. Mod. Phys. A 15, 5087 (2000)

➡ in house software

➡ benefit: reduce to the calculation of ~ 50 soft master integrals!

Reverse unitarity

$$\text{Im}\left(\text{Diagram 1}\right) \propto \sum_f \int d\Pi_f \left| \text{Diagram 2} \right|^2$$



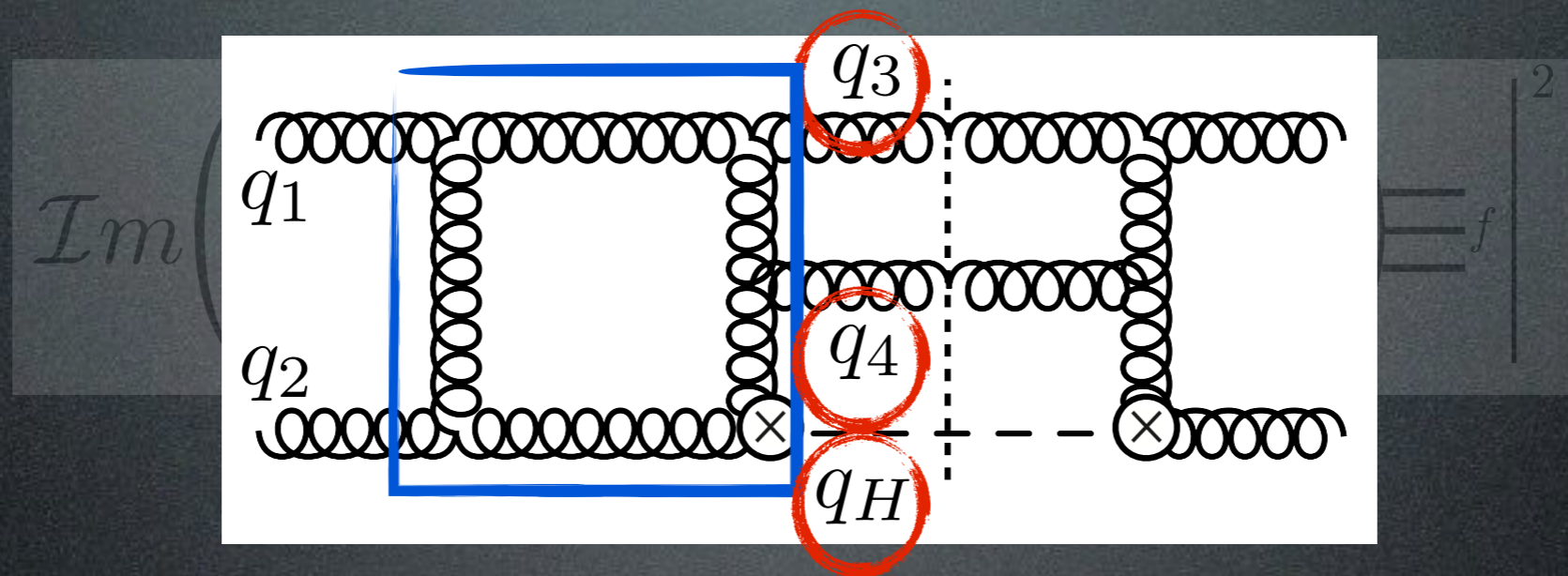
unitarity methods

Bern, Dixon, Kosower, NPB 513, 3 (1998)
 Britto, Cachazo, Feng, NPB 725, 275 (2005)
 Ossola, Papadopoulos, Pittau, NPB 763, 147 (2007)

reverse unitarity

Anastasiou, Melnikov, NPB 646 (2002) 220;
 Anastasiou, Dixon, Melnikov, Petriello, PRL 91 (2003) 182002

Expansion be regions



Bern, Dixon, Kosower, NPB 513, 3 (1998)
 Britto, Cachazo, Feng, NPB 725, 275 (2005)
 Ossola, Papadopoulos, Pittau, NPB 763, 147 (2007)

q_3, q_4 soft \rightarrow scalings

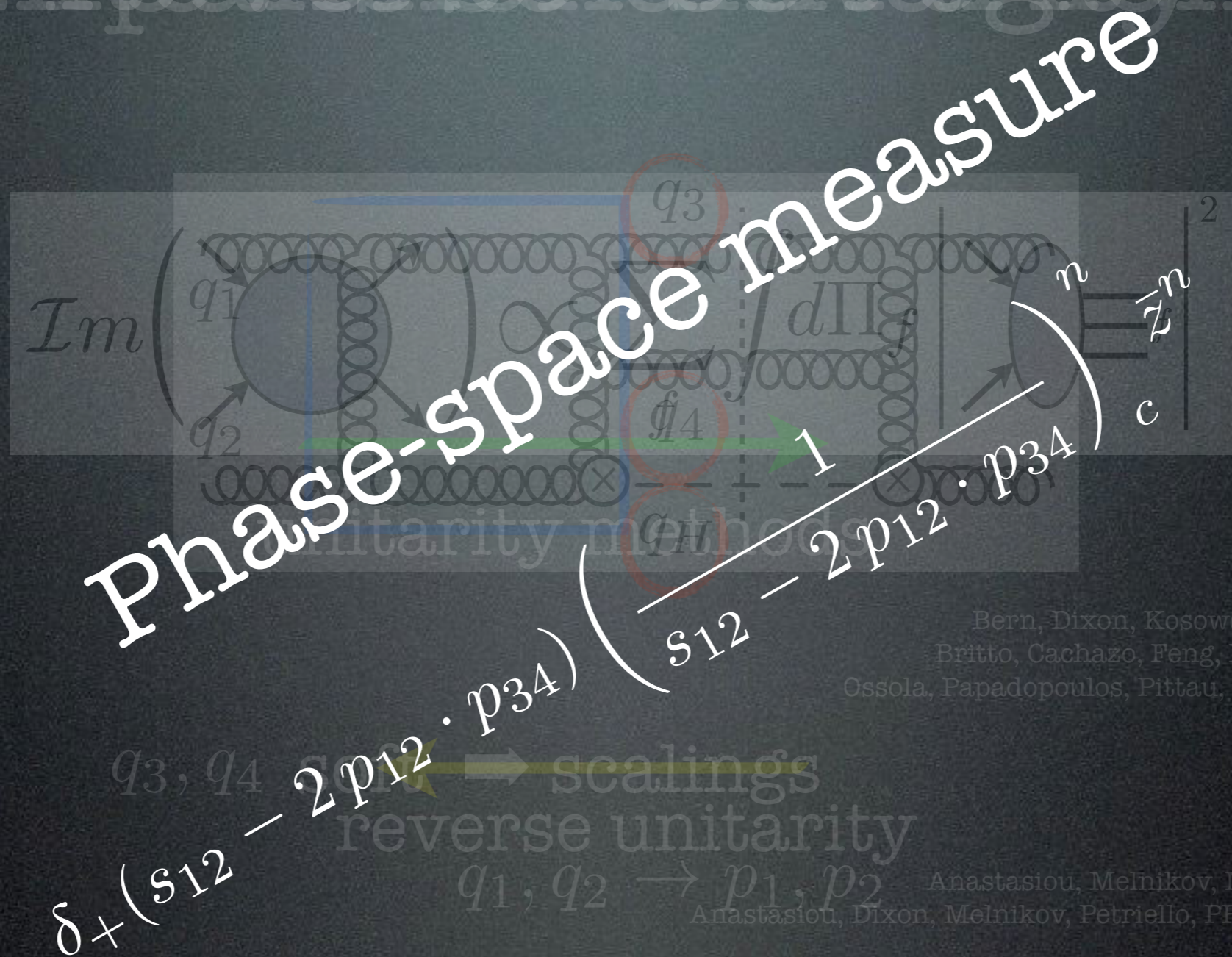
reverse unitarity

$q_1, q_2 \rightarrow p_1, p_2$ Anastasiou, Melnikov, NPB 646 (2002) 220;
 Anastasiou, Dixon, Melnikov, Petriello, PRL 91 (2003) 182002

$q_3, q_4 \rightarrow \bar{z}p_3, \bar{z}p_4$

$$\bar{z} = 1 - z$$

Extrapolation regions



$q_3, q_4 \rightarrow$ scalings
reverse unitarity

$$q_1, q_2 \rightarrow p_1, p_2$$

$$q_3, q_4 \rightarrow \bar{z}p_3, \bar{z}p_4$$

$$\bar{z} = 1 - z$$

Exposition bei regions

Differential equations



Phasitarity methods

Bern, Dixon, Kosower, NPB 513, 3 (1998)
 Britto, Cachazo, Feng, NPB 725, 275 (2005)
 Ossola, Papadopoulos, Pittau, NPB 763, 147 (2007)

$q_3, q_4 \rightarrow p_3, p_4$ scalings
 reverse unitarity

$\delta_+(s_{12} - 2p_1^2)$
 $q_1, q_2 \rightarrow p_1, p_2$ Anastasiou, Melnikov, NPB 646 (2002) 220;
 Anastasiou, Dixon, Melnikov, Petriello, PRL 91 (2003) 182002

$q_3, q_4 \rightarrow \bar{z}p_3, \bar{z}p_4$

$\bar{z} = 1 - z$

(Intermediate)
Results





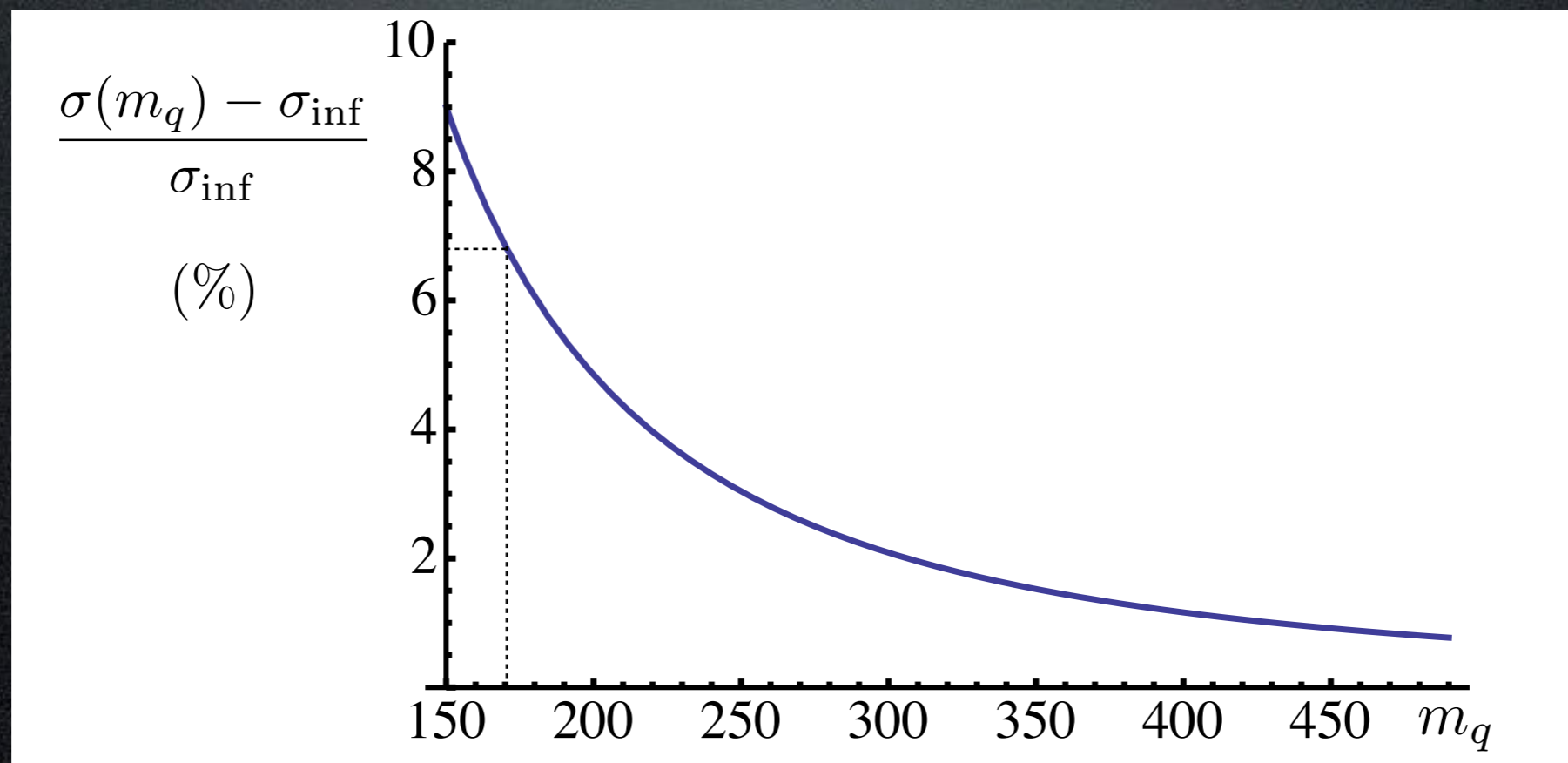
A photograph of two tall glasses filled with bright orange juice. A slice of orange is perched on the rim of the glass in the foreground. To the right of the glasses, there is a whole orange and a large orange slice. The scene is set against a dark background with a light-colored surface in the foreground.

To the juice....

Finite mass effects

(top quark)

- How good is the heavy-top approximation?
 - ▶ at LO



Finite mass effects

(top quark)

- the exact top-mass dependance is known through NLO

Graudenz et al., PRL 70, 1372 (1993);
Spira et al., NP B 453, 17 (1995)

- finite top-mass corrections have been computed at NNLO, confirming the accuracy of the effective theory at the per-mille level

Harlander and Ozeren, JHEP 0911, 088 (2009); Pak et al., JHEP 1002, 025 (2010)

➡ include the $N^3\text{LO}$ cross section in the “rescaled” heavy-quark effective theory

Finite mass effects

(top quark)

More in detail..

- at NLO, “improve” the EFT result by rescaling it with the exact LO cross section:

$$\sigma_{EFT}^{NLO} = 34.66 \text{ pb}$$

$$\sigma_{ex}^{NLO} = 36.60 \text{ pb}$$

$$\sigma_{EFT,r}^{NLO} = R_{LO} \times \sigma_{EFT}^{NLO} = 36.84 \text{ pb}$$

$$R_{LO} = \frac{\sigma_{exact}^{LO}}{\sigma_{EFT}^{LO}} \simeq 1.06$$

0.65%

\sqrt{s}	13TeV
m_h	125GeV
PDF	PDF4LHC15_nnlo_100
$a_s(m_Z)$	0.118
$m_t(m_t)$	162.7 (\overline{MS})
$m_b(4.18\text{GeV})$	4.18 (\overline{MS})
$m_c(3\text{GeV})$	0.986 (\overline{MS})
$\mu = \mu_R = \mu_F$	62.5 ($= m_h/2$)

Finite mass effects

(top quark)

- rescale NNLO and N³LO cross sections, computed in the EFT, by R_{LO}
- at NNLO, include known m_H/m_t correction

which have also been computed factorizing the exact LO XS

$$gg \sim +0.8\%$$

$$qg \sim -0.1\%$$

Harlander, Ozeren; Pak, Rogal, Steinhauser; Mantler, Marzani

- ▶ tiny effect → confirms the validity of the rescaled EFT
- ▶ the error due to unknown top-mass effects at NNLO is estimated as

$$\delta(1/m_t) \sim \pm 1\%$$

Harlander, Ozeren; Pak, Rogal, Steinhauser; Mantler, Marzani

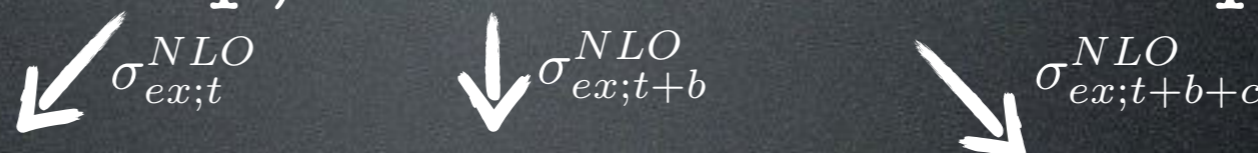
Full NLO mass effects

(top, bottom, charm quark)

- The full dependance of the Higgs production cross section on the quark mass is known exactly through NLO

Spira, Djouadi, Graudenz, Zerwas ; Harlander, Kant; Aglietti, Bonciani, Degrassi, Vicini.

- ▶ include it for top, bottom and charm quarks

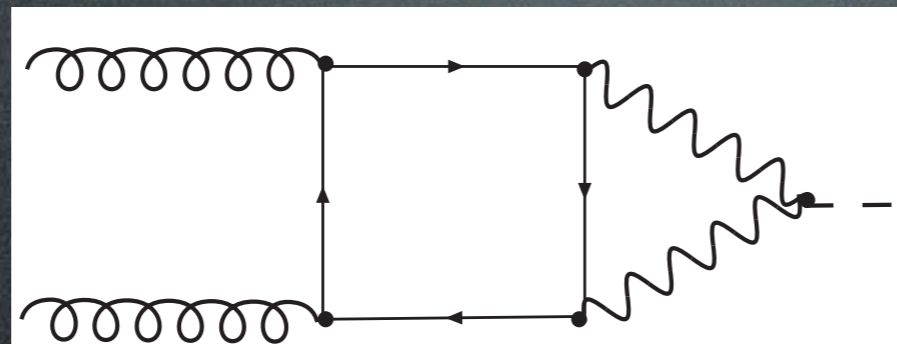

0.65% 5.1% 5.6% on $\sigma_{EFT,r}^{NLO}$

- ▶ estimate an error from unknown light-quark effects at NNLO of $\pm 0.6\%$

Electroweak corrections

- Known exactly at LO in α_s ($\mathcal{O}(\alpha\alpha_s)$)

Aglietti, Bonciani, Degrassi, Vicini;
Actis, Passarino, Sturm, Uccirati



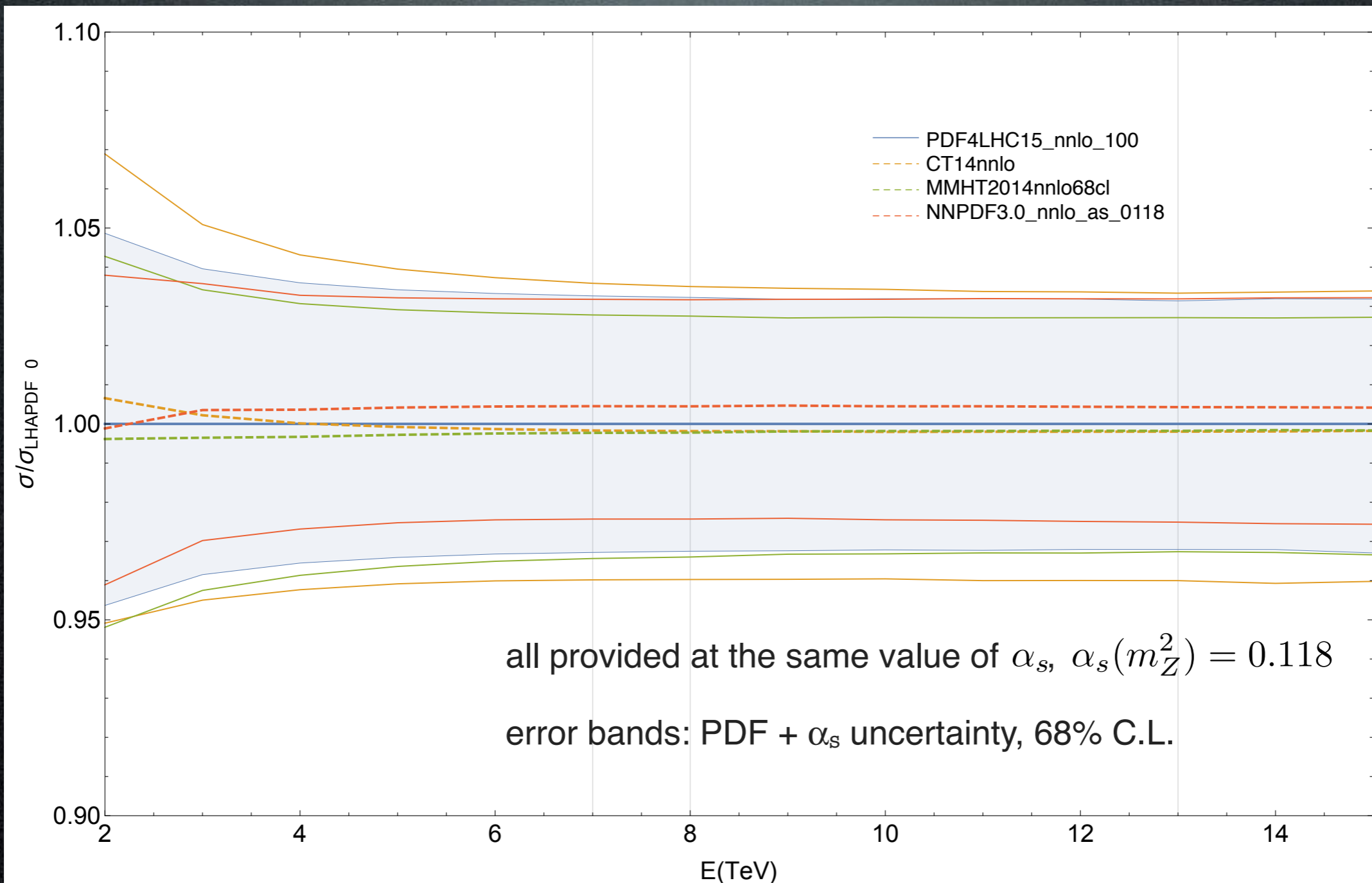
$\longrightarrow +5.2\%$

- At NLO, effects from light quarks are known in an effective theory (heavy t, W, Z) $\longrightarrow +5.1\%$
- Estimate the error from missing QCD/EW contributions by varying the Wilson coefficient of the QCD/EW effective theory $\longrightarrow \delta(EW) \sim \pm 1\%$

Convolution with PDFs

- Various PDF fits are available at NNLO (ABM12, CT14, MMHT2014, NNPDF3.0, PDF4LHC15, HERAPDF2.0)
- (How) does our cross section prediction change when using these different sets?

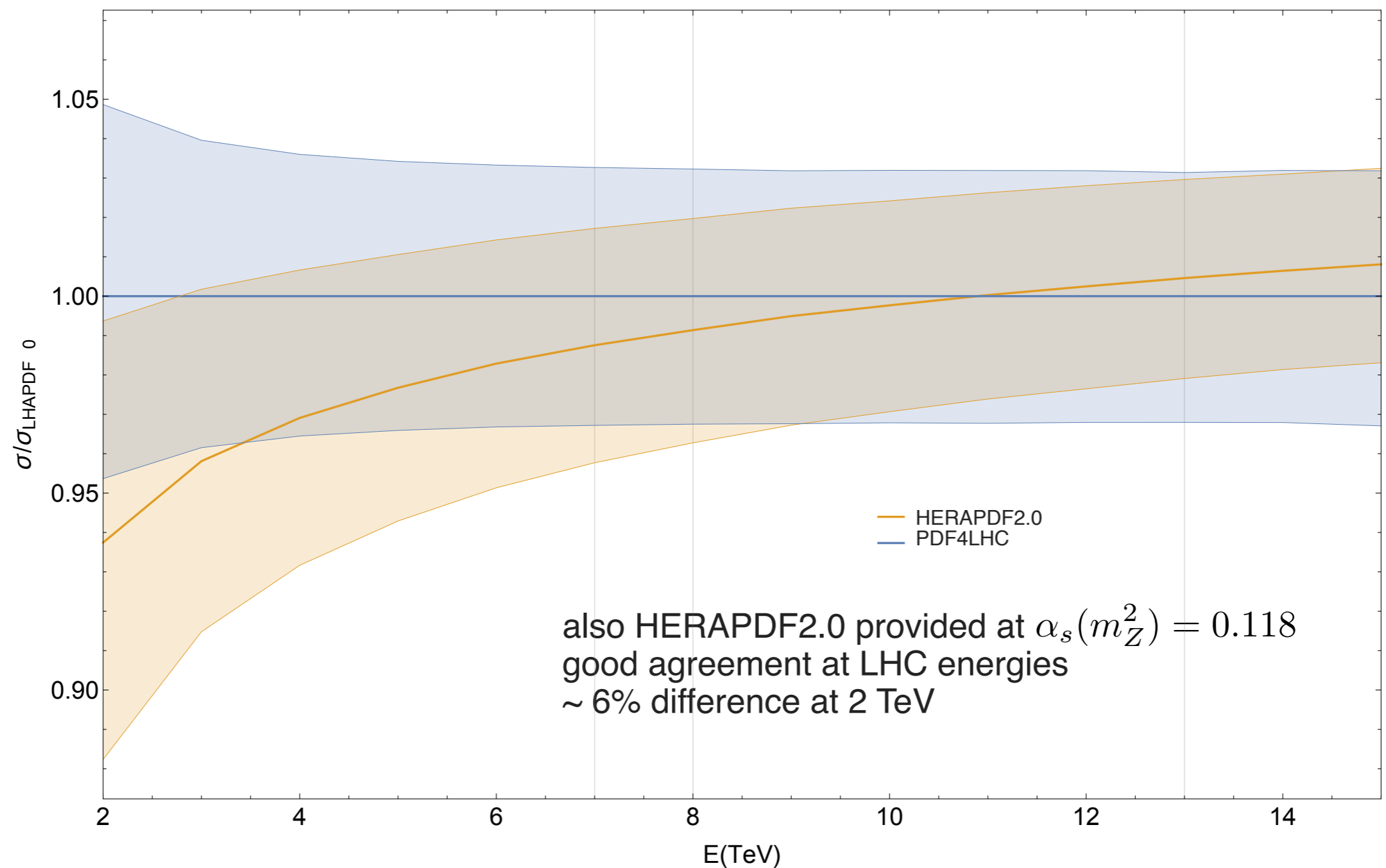
Convolution with PDFs



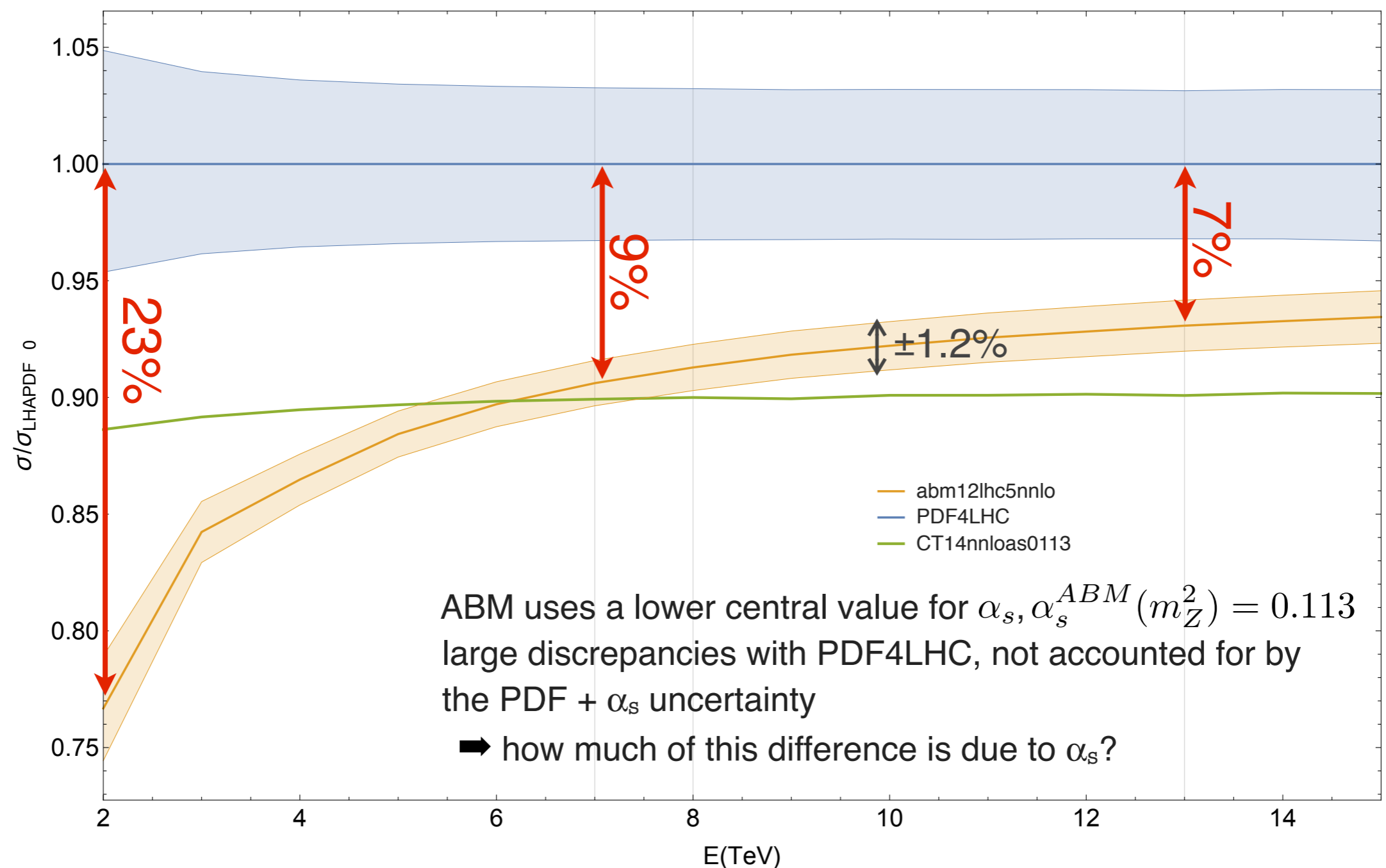
Convolution with PDFs

- The central values lie within 1% of each other (with MMHT2014 and NNPDF3.0 agreeing at the permille level)
- The combined PDF + α_s uncertainty is $\sim 3-4\%$ at LHC energies and captures well the small difference among the sets

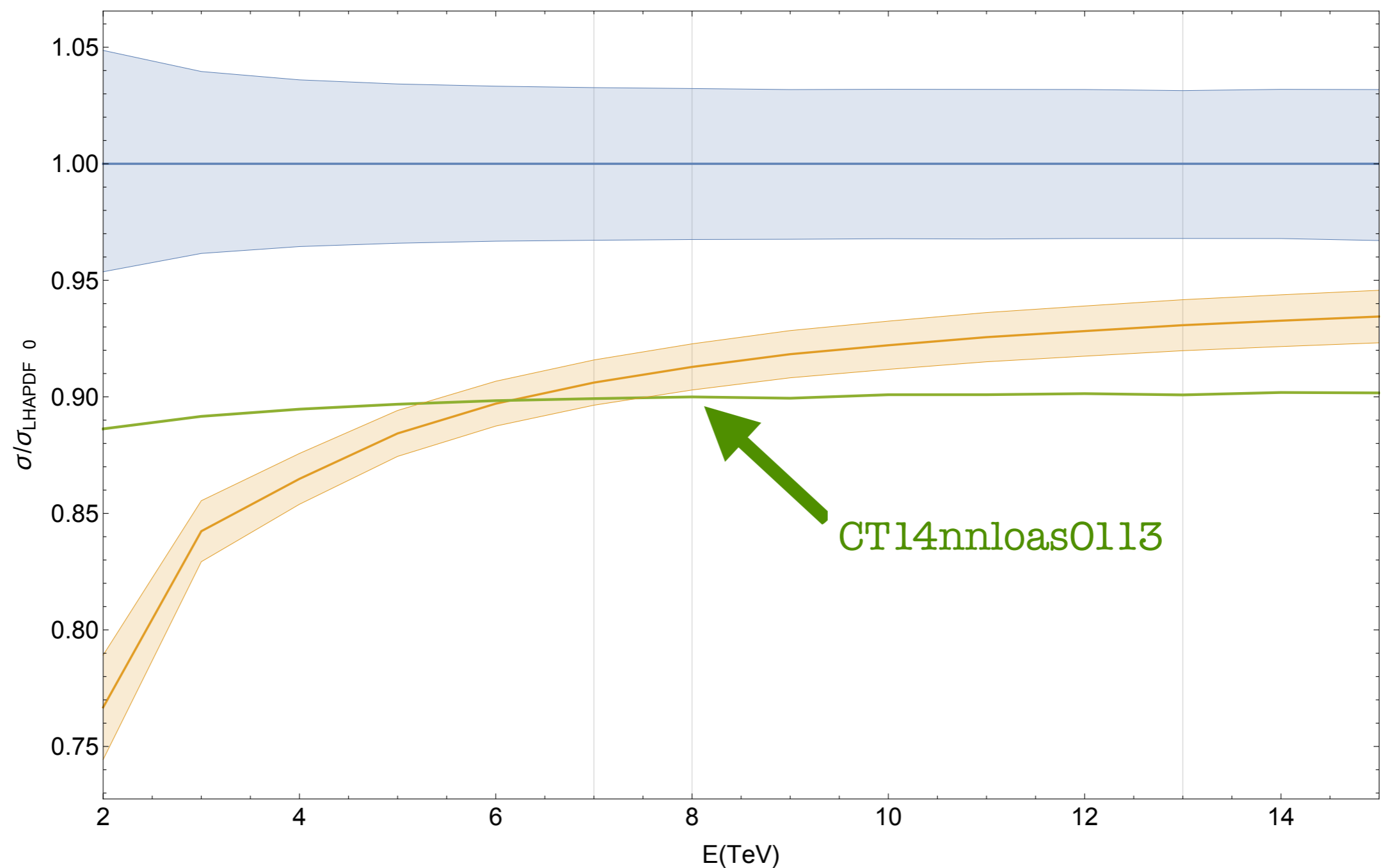
Convolution with PDFs



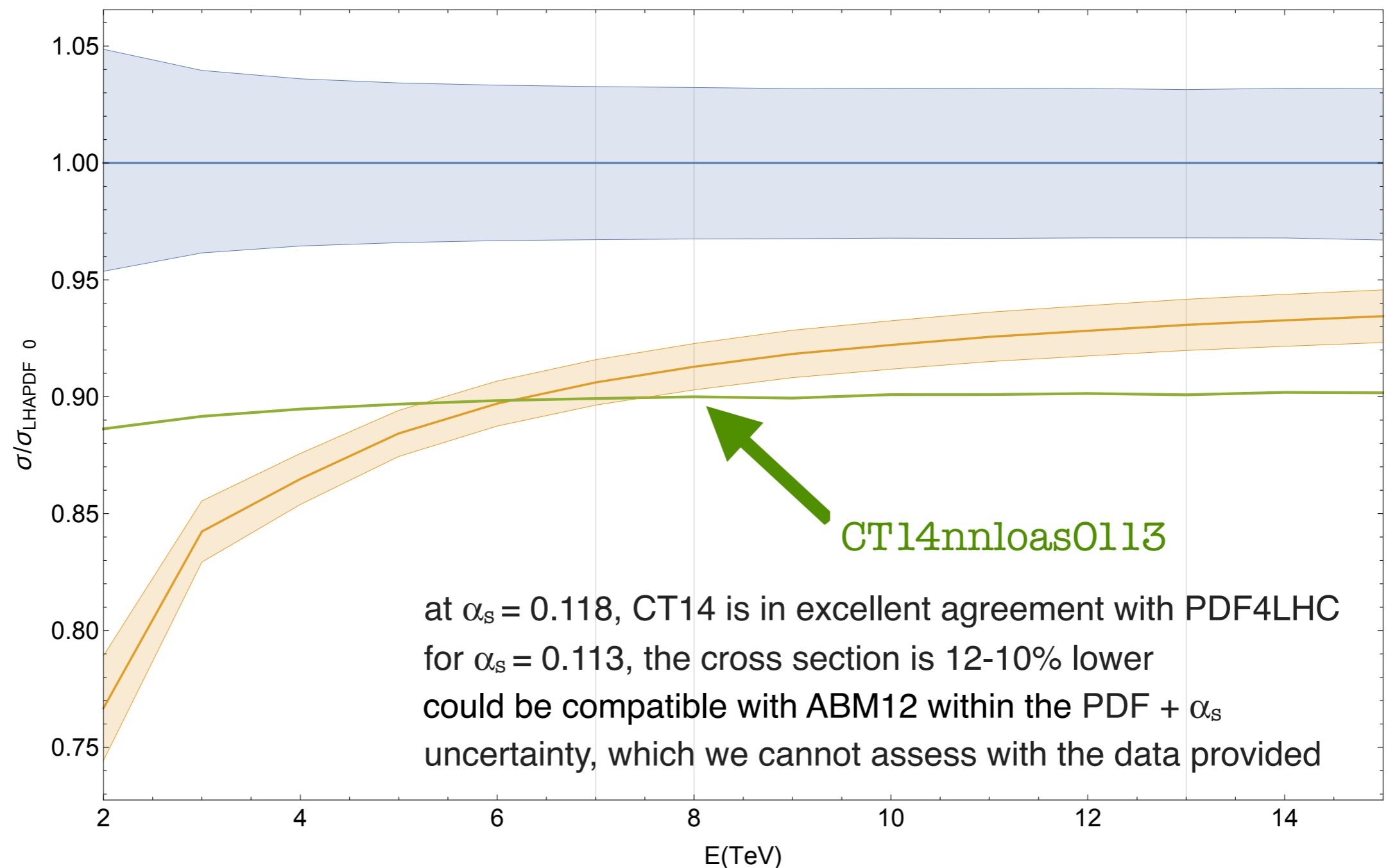
Convolution with PDFs



Convolution with PDFs



Convolution with PDFs



Convolution with PDFs


- Given the good agreement among the various sets that use $\alpha_s = 0.118$ (consistent with the PDG world average), we will use the combined PDF4LHC fit for our final results
- Caveat: we are computing the $N^3\text{LO}$ Higgs cross section using NNLO PDFs.
Is it legitimate to do so?
 - ▶ look at what happens to the NNLO cross section using NNLO or NLO PDFs

N³LO PDF uncertainty

- For the uncertainty from missing N³LO PDFs, we adopt a conservative estimate

$$\delta(\text{PDF} - \text{TH}) = \frac{1}{2} \left| \frac{\sigma_{EFT}^{(2),NNLO} - \sigma_{EFT}^{(2),NLO}}{\sigma_{EFT}^{(2),NNLO}} \right| = 1.16\%$$

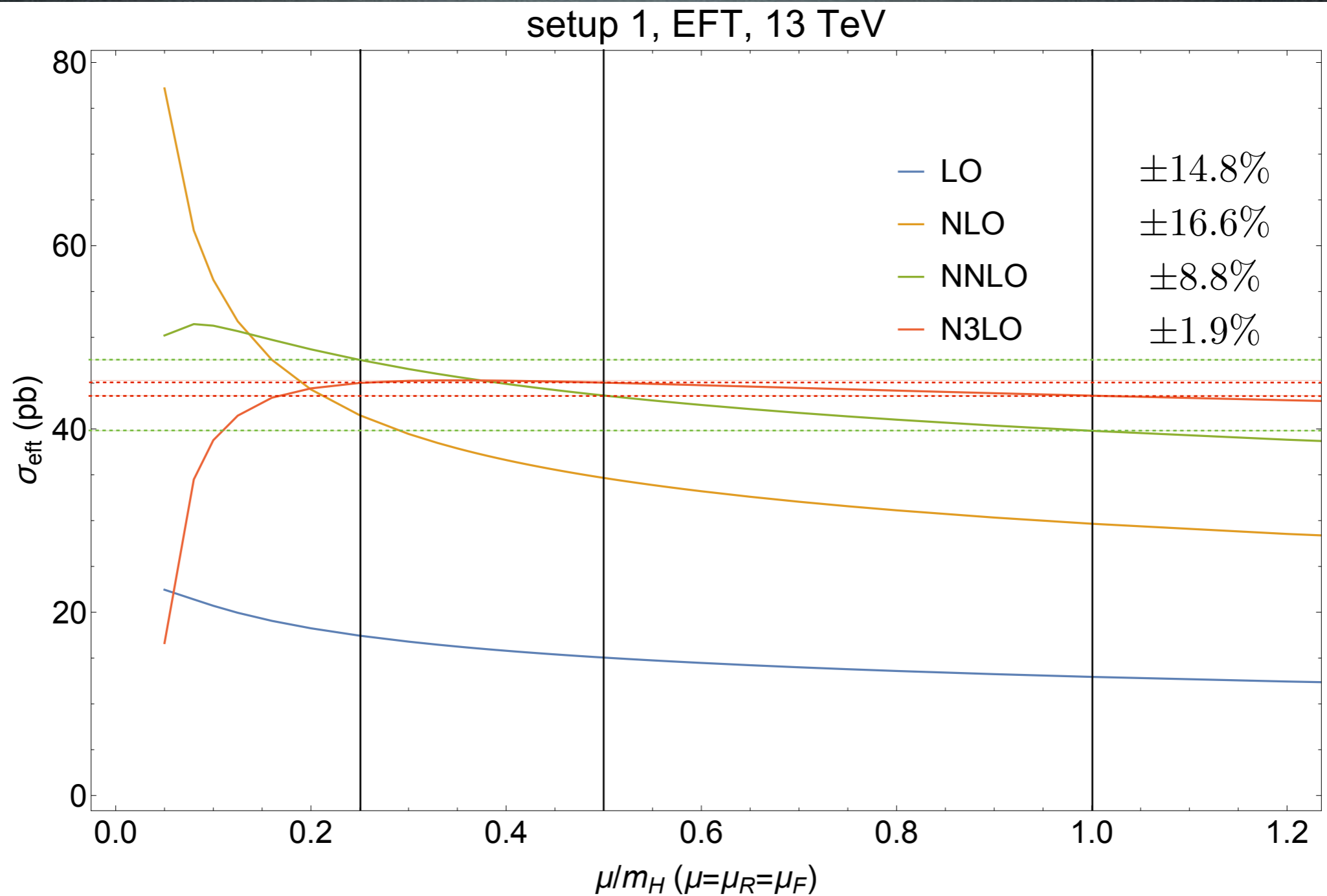
from comparison
with other processes
computed at N³LO



Uncertainties

- We already discussed the uncertainties due to missing finite-mass effects at NNLO ($\delta(1/m_t)$, $\delta(\text{tbc})$), unknown electroweak corrections ($\delta(\text{EW})$) and the lack of N³LO PDFs
- What are the errors associated to missing higher order terms in the α_s expansion and soft approximation?

Effects beyond N³LO

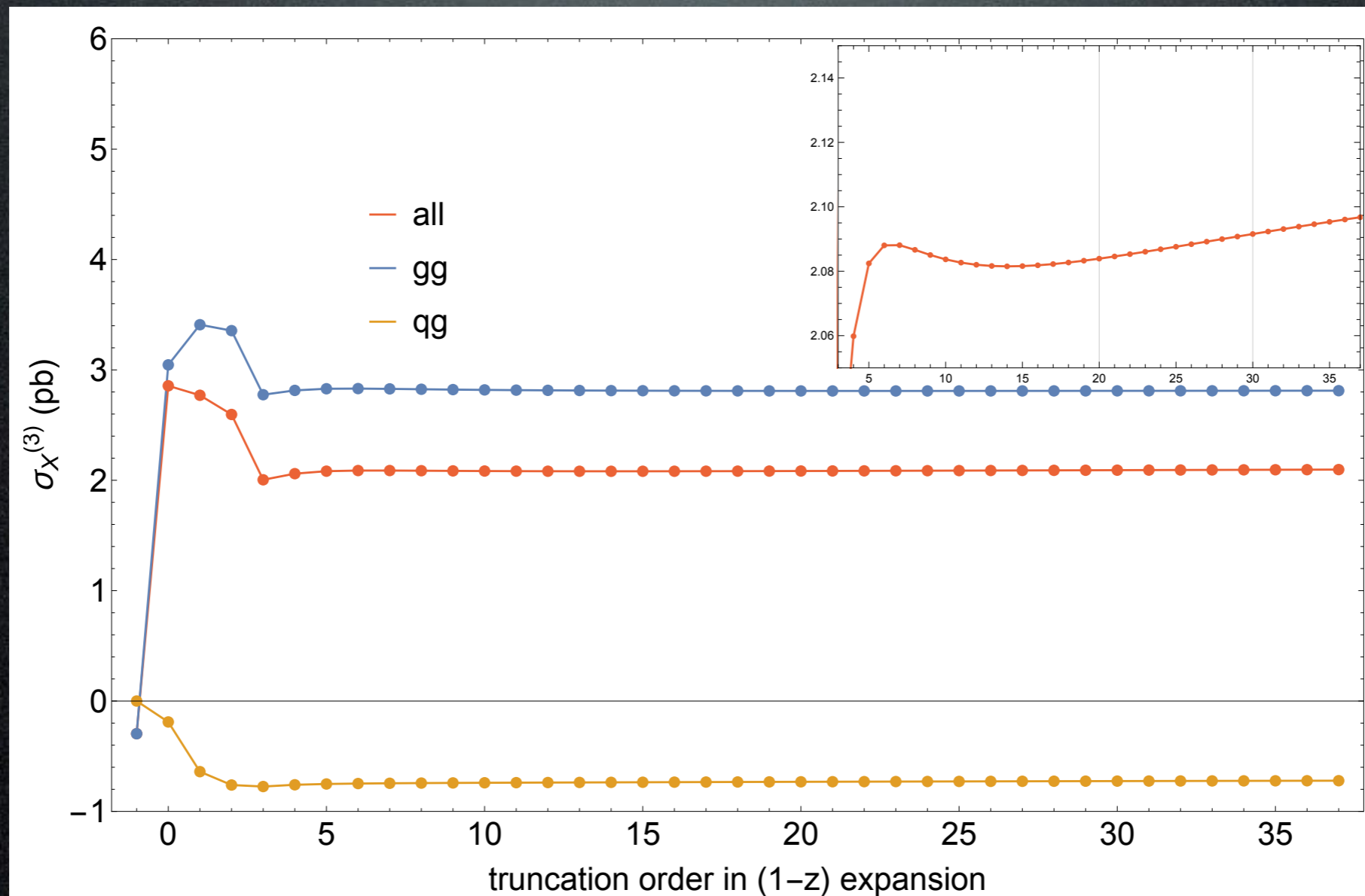


Effects beyond N³LO

- At N³LO, one starts to observe a convergence of the α_s expansion
 - ▶ the cross section in the EFT increases by about 3%, i.e., within the NNLO scale variation error
 - ▶ the scale variation error decreases
 - ▶ seems to indicate the validity of scale variation studies as estimates of missing higher order corrections

Soft approximation

- What is the error associated to the truncation of the expansion in $(1 - z)$?



Soft approximation

- ▶ the cross section increases by 3.7 per mille over the last ten terms in the expansion
- ▶ assume (conservatively) that it will take another 100 terms to converge, and it will converge at the same speed

$$\delta(\text{trunc}) = 10 \times \frac{\sigma_{EFT}^{(3)}(37) - \sigma_{EFT}^{(3)}(27)}{\sigma_{EFT}^{\text{N}^3\text{LO}}} = 0.37\%$$

(consistent with other estimates of the truncation error)

The N³LO cross section

The N³LO Higgs boson production cross section and the associated errors are

σ	$\delta(\text{PDF})$	$\delta(\alpha_s)$	$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(tbc)$	$\delta(1/m_t)$	
48.58	± 0.90	$+1.27$ -1.25	$+0.10$ -1.15	± 0.18	± 0.56	± 0.49	± 0.40	± 0.49	pb
	± 1.86	$+2.61$ -2.58	$+0.21$ -2.37	± 0.37	± 1.16	± 1	± 0.83	± 1	%

in quadrature

linearly

$$\sigma = 48.58 \text{ pb} \begin{matrix} +2.22 \text{ pb } (+4.56\%) \\ -3.27 \text{ pb } (-6.72\%) \end{matrix} (\text{theory}) \pm 1.56 \text{ pb } (3.20\%) (\text{PDF} + \alpha_s)$$

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The N³LO Higgs boson production cross section and the associated errors are

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	± 1.86	$+2.61$ -2.58	$+0.21$ -2.37	± 0.37	± 1.16	± 1	± 0.55	± 0.51	%

“traditionally”

neglected

in quadrature

linearly

(the scale variation error at NNLO is so large that they are not relevant)

$$\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb} (+4.56\%)}_{-3.27 \text{ pb} (-6.72\%)} (\text{theory}) \pm 1.56 \text{ pb} (3.20\%) (\text{PDF} + \alpha_s)$$

$$\sigma = 48.58 \text{ pb}^{+0.84 \text{ pb} (+1.73\%)}_{-1.89 \text{ pb} (-3.90\%)} (\text{theory}) \pm 1.56 \text{ pb} (3.20\%) (\text{PDF} + \alpha_s)$$

Room for improvement

The N³LO Higgs boson production cross section and the associated errors are

σ	$\delta(\text{PDF})$	$\delta(\alpha_s)$	$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(tbc)$	$\delta(1/m_t)$	
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	± 1.86	+2.61 -2.58	+0.21 -2.37	± 0.37	± 1.16	± 1	± 0.83	± 1	%

in quadrature

linearly

$$\sigma = 48.58 \text{ pb} \begin{matrix} +2.22 \text{ pb } (+4.56\%) \\ -3.27 \text{ pb } (-6.72\%) \end{matrix} (\text{theory}) \pm 1.56 \text{ pb } (3.20\%) (\text{PDF} + \alpha_s)$$

Conclusions

- We computed the $N^3\text{LO}$ gluon–fusion production cross section in an EFT
- Added all known effects beyond the EFT (finite top mass, light quarks, EW corrections)
- Studied the dependance on the choice of PDFs
- Provided an accurate estimate of the uncertainties, including errors from missing information and from approximations

Conclusions

- Room for improvement
 - ▶ going beyond the threshold expansion (full kinematics)
 - ▶ exact NNLO cross section/approximate NNLO results for light quarks/exact three-loop mixed QCD/EW corrections

