Exceptional vs superPoincaré algebra as the defining symmetry of maximal Supergravity

Lars Brink

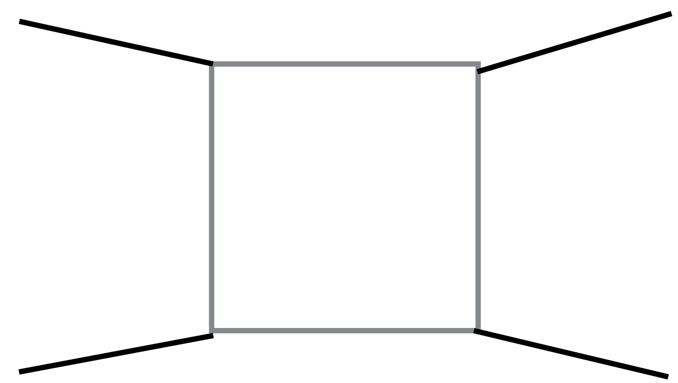
w Pierre Ramond, Sung-Soo Kim,

Sudarshan Ananth, Sucheta Majumdar

The N=8 Supergravity and the N=4 Yang-Mills Theories are very complicated theories with often very simple results.

The Cremmer-Julia preprint was 124 pages full of formulae.
The multiplet consists of 128 bosons and 128 fermions.

Still the 4-graviton one-loop amplitude is a simple box diagram



with kinematical factors and we now know that the same is true up to four loops.

There must be a lot of symmetries in these field theories.

We do want to master these theories since they are the low-energy limit of the Superstring Theory.

The action was constructed using (maximal) supersymmetry in component form.
Cremmer-Julia

A superspace formulation was constructed by Howe and myself in term of a superfield which transforms as a 56 under SU(8) and is a spinor and is a function of 8 4-component spinors.
It has billions of components.

These two formulations have been the starting points for most work in the field.

A third formulation was found by Bengtsson[2] and me.

We studied the theory in the light-cone gauge with only the physical fields remaining. The corresponding superfield is indexfree and has only 256 components.

The price to pay is that the Hamiltonian is obtained as an expansion in the coupling constant and at that time we constructed only the 3-point coupling. Non-local on the light-cone.

One remarkable fact about the D=4 theory is that the Hamiltonian is invariant under an $E_{7(7)}$ symmetry which acts as a

σ -model for the scalars

and a duality symmetry for the vector fields.

It does not touch the other fields.

It comes back in Superstring Theory as the U-duality.

When compactifying further one has found that

in d=3 the symmetry is $E_{8(8)}$ and it has been argued that

in d=2 the symmetry should be E_9

in d=1 the symmetry should be E_{10}

in d=0 the symmetry should be E_{11}

West, Henneaux, Nicolai, Damour, Englert, Houart, Kleinschmidt…… have argued that those symmetries should also exist in higher dimensions.

My question is now what happens to the exceptional symmetries in the light-cone gauge formulation, where every symmetry is a symmetry between the physical fields.

With Ramond and Kim.

N=8, d=4 Supergravity in light-cone superspace

$$x^{\pm} = \frac{1}{\sqrt{2}} \left(x^0 \pm x^3 \right) \; ; \qquad \partial^{\pm} = \frac{1}{\sqrt{2}} \left(-\partial_0 \pm \partial_3 \right) \; ,$$

$$x = \frac{1}{\sqrt{2}} \left(x_1 + i\, x_2 \right) \; ; \qquad \bar{\partial} = \frac{1}{\sqrt{2}} \left(\partial_1 - i\, \partial_2 \right) \; ,$$

$$\bar{x} = \frac{1}{\sqrt{2}} \left(x_1 - i\, x_2 \right) \; ; \qquad \partial = \frac{1}{\sqrt{2}} \left(\partial_1 + i\, \partial_2 \right) \; .$$

$$\theta^m , \;\; m = 1, , , , 8 \qquad \bar{\theta}_m , \;\; m = 1, , , , 8$$

$\boxed{\text{SU(8)}}$

$$y \; = \; \left(x, \, \bar{x}, \, x^+, \, y^- \equiv x^- - \frac{i}{\sqrt{2}} \theta^m \, \bar{\theta}_m \right)$$

x^+ time

All 128 + 128 component fields are included in

$$\phi(y) = \frac{1}{\partial^{+2}} h(y) + \theta^m \frac{1}{\partial^{+2}} \bar{\psi}_m\,(y) + \frac{i}{2} \theta^m \theta^n \frac{1}{\partial^+} \bar{A}_{mn}(y) + \ldots .$$

$$\frac{1}{7!}\,\theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \epsilon_{mnpqrstu} \partial^+ \psi^u(y)$$

$$+ \frac{4}{8!} \theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \theta^u \epsilon_{mnpqrstu} \partial^{+2} \bar{h}(y)$$

$$d^m\,\phi\,(\,y\,)\;=\;0\;\;; \qquad \bar{d}_n\,\bar{\phi}\,(\,y\,)\;=\;0 \qquad \phi\;=\;\frac{1}{4}\,\frac{(d)^8}{\partial^{+4}}\,\bar{\phi}$$

$$d^{\,m}\;=\;-\,\frac{\partial}{\partial\,\bar{\theta}_m}\;-\;\frac{i}{\sqrt{2}}\,\theta^m\,\partial\;\;; \qquad \bar{d}_n\;=\;\frac{\partial}{\partial\,\theta^n}\;+\;\frac{i}{\sqrt{2}}\,\bar{\theta}_n\,\partial\;,$$

We have to find a representation of the N=8 superPoincaré algebra on this superfield.

Free case $\quad x^+ = 0$

Kinematical generators (Poincaré)

$$p^+ \;=\; -\,i\,\partial^+\,, \qquad p \;=\; -\,i\,\partial\,, \qquad \bar p \;=\; -\,i\,\bar\partial\,,$$

$$j \;=\; x\,\bar\partial - \bar x\,\partial + \frac{1}{2}\,(\,\theta^\alpha\,\bar\partial_\alpha \;-\; \bar\theta_\alpha\,\partial^\alpha\,)$$

$$j^+ \;=\; i\,x\,\partial^+\,, \qquad \bar j^+ \;=\; i\,\bar x\,\partial^+$$

$$j^{+-} \;=\; i\,x^-\,\partial^+ - \frac{i}{2}\,(\,\theta^\alpha\bar\partial_\alpha + \bar\theta_\alpha\,\partial^\alpha\,)$$

Dynamical generators (Poincaré)

$$p^{-} \;=\; -\,i\,\frac{\partial\bar{\partial}}{\partial^{+}}$$

Hamiltonian

$$j^{-} \qquad \bar{j}^{-}$$

Kinematical generators Supersymmetry

$$q^{m}_{+} = -\frac{\partial}{\partial\,\bar{\theta}_{m}} + \frac{i}{\sqrt{2}}\,\theta^{m}\,\partial; \qquad \bar{q}_{+\,n} = \frac{\partial}{\partial\,\theta^{n}} - \frac{i}{\sqrt{2}}\,\bar{\theta}_{n}\,\partial\ ,$$

Dynamical generators Supersymmetry

$$q^{m}_{-} = \frac{\bar{\partial}}{\partial^{+}}q^{m}_{+}, \qquad \bar{q}_{-m} = \frac{\partial}{\partial^{+}}\bar{q}_{+m}$$

Supersymmetry algebra

$$\{q^{+m}\,,\,\bar{q}^{+}{}_{n}\} = \delta^{m}{}_{n}\, i\,\sqrt{2}\;\partial^{+}$$

Spectrum generating

$$\{q^{-m}\,,\,\bar{q}^{-}{}_{n}\} = \delta^{m}{}_{n}\, i\,\sqrt{2}\;\frac{\partial\bar{\partial}}{\partial^{+}}$$

Hamiltonian

All the juice in q^{-m}. Will get interactions terms

All dynamical generators will get interaction terms

The action to order κ

$$\int d^4x \int d^8\theta \, d^8\bar{\theta} \bar{\phi} \, \frac{\Box}{\partial^{+4}} \phi - 2\kappa \left(\frac{1}{\partial^2} \bar{\phi} \;\; \bar{\partial} \phi \, \bar{\partial} \phi + \frac{1}{\partial^2} \phi \, \partial \, \bar{\phi} \, \partial \, \bar{\phi} \right)$$

The important generator is

the dynamical supersymmetry generator

$$\bar{Q}_m{}^{(\kappa)} \phi = \frac{1}{\partial^+} \left(\bar{\partial} \, \bar{q}_m \, \phi \;\, \partial^{+\,2} \phi - \partial^+ \, \bar{q}_m \, \phi \, \partial^+ \bar{\partial} \, \phi \right)$$

Next order is known but contains about 50 terms.

In a component covariant formulation about 5000 terms.

The $E_{7(7)}$ symmetry of the theory (w Kim and Ramond)

$$E_{7(7)} = E_{7(7)}/SU(8) \times SU(8)$$

70 non-linear

63 R-symmetry
linear

$$\delta\phi = \; -\frac{2}{\kappa}\,\theta^{klmn}\,\bar{\Xi}_{klmn}$$
$$+\frac{\kappa}{4!}\,\Xi^{mnpq}\,\frac{1}{\partial^{+2}}\left(\bar{d}_{mnpq}\frac{1}{\partial^{+}}\phi\,\partial^{+3}\phi \;-\; 4\,\bar{d}_{mnp}\phi\,\bar{d}_q\partial^{+2}\phi \;+\; 3\,\bar{d}_{mn}\partial^{+}\phi\,\bar{d}_{pq}\partial^{+}\phi\right) +$$

$$\bar{d}_{m_1\ldots m_n} = \bar{d}_{m_1}\ldots\bar{d}_{m_n}$$

σ-model symmetry

All fields in the supermultiplet transform

$$\delta\, h \;=\; -\,\kappa\,\Xi^{ijkl}\left(\frac{1}{8}\bar{A}_{ij}\bar{A}_{kl} \;+\; \frac{1}{4!}\frac{1}{\partial^+}\bar{C}_{ijkl}\partial^+ h \;+\; \frac{i}{6}\frac{1}{\partial^+}\bar{\chi}_{ijk}\bar{\psi}_l\right) \;+\; O(\kappa^2)$$

$$\delta\,\bar{\psi}_i = -\,\kappa\,\Xi^{mnpq}\left(\frac{1}{4!}\frac{1}{\partial^+}\bar{D}_{mnpq}\partial^+\bar{\psi}_i + \frac{1}{3!}\bar{D}_{mnpi}\bar{\psi}_q\right)$$

$$\left(-\frac{1}{4!}\epsilon_{mnpqirst}\frac{1}{\partial^+}\chi^{rst}\partial^+ h \;+\; \frac{1}{4}\bar{\chi}_{imn}\bar{B}_{pq} + \frac{1}{3!}\frac{1}{\partial^+}\bar{\chi}_{mnp}\partial^+\bar{B}_{iq}\right)$$

$$+\; O(\kappa^2)$$

We can see that

$$\left[E_{7(7)/SU(8)}, Q\right] = 0$$

even though

$$[SU(8), Q] \neq 0$$

because of the non-linearity of the quotient.

Hence

$$\left[E_{7(7)}/SU(8), P^{-}\right] = 0$$

Can be used to find the Hamiltonian

Note that the exceptional algebra and the maximal supersymmetry are spanned on the same multiplet.

Both are non-linearly realised.

Both can be used to find the Hamiltonian.

Which is the chicken and which is the egg?

Supergravity in 11 dimensions

We will use the same superfield with spacetime augmented with $x^m \quad \partial^m \quad m = 4...10$

The transverse symmetry we write as

$$SO(9) = \frac{SO(9)}{SO(7) \times SO(2)} \times SO(7) \times SO(2)$$

$j^m \quad \bar{j}^n \qquad j^{mn} \qquad j$

Linearly realized

w. Ananth and Majumdar

Dynamical supersymmetry in d=11

The O(K) should be linear in transverse derivatives

- Terms with $\bar{\partial}$. These must be the same as in d = 4.
- Terms with ∂^n. These are new types of terms associated with the dimensions 4 . . . 10.
- Terms with ∂. These terms do not exist in the d = 4 formulation. The d = 11 case however, has a SO(7) R-symmetry and for a d = 4 theory with SO(7) R-symmetry, instead of SU(8) R-symmetry, such a term can appear.

Let us rewrite the first term

$$\bar{Q}_\alpha{}^{\bar{\partial}}\phi = \frac{1}{\partial^+}(\bar{\partial}\bar{q}_\alpha\phi\partial^{+2}\phi - \partial^+\bar{q}_\alpha\phi\partial^+\bar{\partial}\phi)$$
$$= \frac{1}{\partial^+}(E\partial^+\bar{\partial}\phi E^{-1}\partial^{+2}\phi)|_{\rho^\alpha}$$

where

$$E = exp(\frac{\bar{q}\cdot\rho}{\partial^+})$$

By writing it like this we are assured the correct commutations with all the kinetic generators except the new ones $\bar{j}^m$ and j^m .

The crucial commutator is

$$[\bar{J}^m, \bar{Q}_\alpha] = -\sqrt{2}(\gamma^m)_{\alpha\beta}Q^\beta$$

A looong calculation gives

$$\begin{aligned}\bar{Q}_\alpha\phi =& \frac{1}{\partial^+}(E\partial^+\bar{\partial}\phi E^{-1}{\partial^+}^2\phi)|_{\rho^\alpha} \\ & -\frac{1}{4\sqrt{2}}(\gamma^n)^{\beta\gamma}\frac{1}{{\partial^+}^2}[E{\partial^+}^2\partial^n\phi E^{-1}{\partial^+}^3\phi)|_{\rho^\beta,\rho^\gamma,\rho^\alpha} \\ & +\frac{1}{36\sqrt{2}}(\gamma^n\gamma^m)^{\alpha\delta}(\gamma^m)^{\beta\gamma}\frac{1}{{\partial^+}^2}[E{\partial^+}^2\partial^n\phi E^{-1}{\partial^+}^3\phi)|_{\rho^\beta,\rho^\gamma,\rho^\delta} \\ & -\frac{i}{288}(\gamma^n)^{\beta\,\gamma}(\gamma^n)^{\delta\,\epsilon}\,\frac{1}{{\partial^+}^3}[E{\partial^+}^3\partial\phi E^{-1}{\partial^+}^4\phi]|_{\rho^\beta,\rho^\gamma,\rho^\delta,\rho^\epsilon,\rho^\alpha}\end{aligned}$$

From this we can compute the Hamiltonian.

We do not need to compute the other dynamical generators since the above solution is unique

and we know that there should be a solution.

Is it invariant under the $E_{7(7)}$ transformation?

No!

E_7 invariance in d=11

An alternative way to "oxidize" is to make sure that the Hamiltonian is Poincaré invariant in d=11. If we can have the same derivative structure as in d=4 we can keep the E_7 invariance. We introduce the generalised derivative

$$\bar{\nabla} = \bar{\partial} + \frac{\sigma}{16}\,\bar{d}_\alpha\,(\,\gamma^m\,)^{\alpha\,\beta}\,\bar{d}_\beta\,\frac{\partial^m}{\partial^+}$$

We now try a three-point coupling in H as

$$\mathcal{V} = -\,\frac{3}{2}\,\kappa\int d^{10}x\int d^8\theta\,d^8\bar{\theta}\,\frac{1}{\partial^{+\,2}}\,\bar{\phi}\;\;\bar{\nabla}\,\phi\;\bar{\nabla}\,\phi + c.c.$$

By checking its Poincaré covariance we find that it works if

$$\sigma = 1$$

This is E_7 invariant to this order. R-symmetry SU(8).

The Hamiltonian is different from the previous case.

Still it must be the same theory.

There must be a field redefinition taking you from one formulation to the other.

Maximal Supergravity in d=3 and $E_{8(8)}$ invariance

We can dimensionally reduce to d=3 and by a field redefinition we can find an $E_{8(8)}$ invariance.

In the light-cone gauge we use

$$E_{8(8)} = \frac{E_{8(8)}}{SO(16)} \times SO(16)$$

128 non-linearly realized

120 linearly realized

We use the same superfield. 128 scalars 128 fermions

SO(16) is the largest R-symmetry on this superspace

All scalars participate in the σ -model

$$\delta_{E_{8(8)}/SO(16)}\,\phi \;=\; \frac{1}{\kappa}\,F \,+\, \kappa\,\epsilon^{i_1 i_2 \ldots i_8}\,\sum_{c=-2}^{2}\left(\hat{\bar{d}}_{i_1 i_2 \cdots i_{2(c+2)}}\,\partial^{+c}\,F\right)$$

$$\times\left\{\left(\frac{\partial}{\partial\,\eta}\right)_{i_{2c+5}\cdots i_8}\,\partial^{+(c-2)}\left(e^{\eta\cdot\hat{\bar{d}}}\,\partial^{+(3-c)}\phi\,e^{-\eta\cdot\hat{\bar{d}}}\partial^{+(3-c)}\phi\right)\bigg|_{\eta=0} + O(\kappa^2)\right\}$$

$$F = \frac{1}{{\partial^+}^2}\,\beta\,(y^-) \;+\; i\,\theta^{mn}\,\frac{1}{\partial^+}\,\overline{\beta}_{mn}\,(y^-) - \;\theta^{mnpq}\,\overline{\beta}_{mnpq}\,(y^-) +$$

$$+\; i\widetilde{\theta}_{\;mn}\,\partial^+\,\beta^{mn}\,(y^-) + \;4\,\widetilde{\theta}\,{\partial^+}^2\,\bar{\beta}\,(y^-)$$

Supermultiplet is also a representation of E_8.

We should now be able to oxidise to d=4 in two ways

- Break SO(16) to SU(8) and get the old formalism with E_7 symmetry.
- Keep the derivative structure and keep the E_8 symmetry.

Both should be the same theory and there should be a field redefinition to go from one to the other.

Hence we claim that there is a hidden E_8 symmetry also in d=4.

Hence we should expect better quantum behaviour than standard counterterm arguments would give.

What does it say about quantum finiteness?

We do not know.

Only explicit calculations can give definite answers.

Is this the tip of the iceberg?

What about E_9, E_{10}, E_{11}......?

Do not know but it is intriguing.

Does it have consequences for the string?

Possibly.

I think there is something very deep in d=11 supergravity, the Superstring Theory and the M-theory that we still do not know.