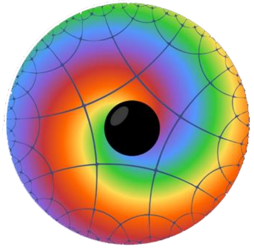


Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Alexander von Humboldt
Stiftung/Foundation

Thermodynamics of an anisotropic brane

Daniel Fernández

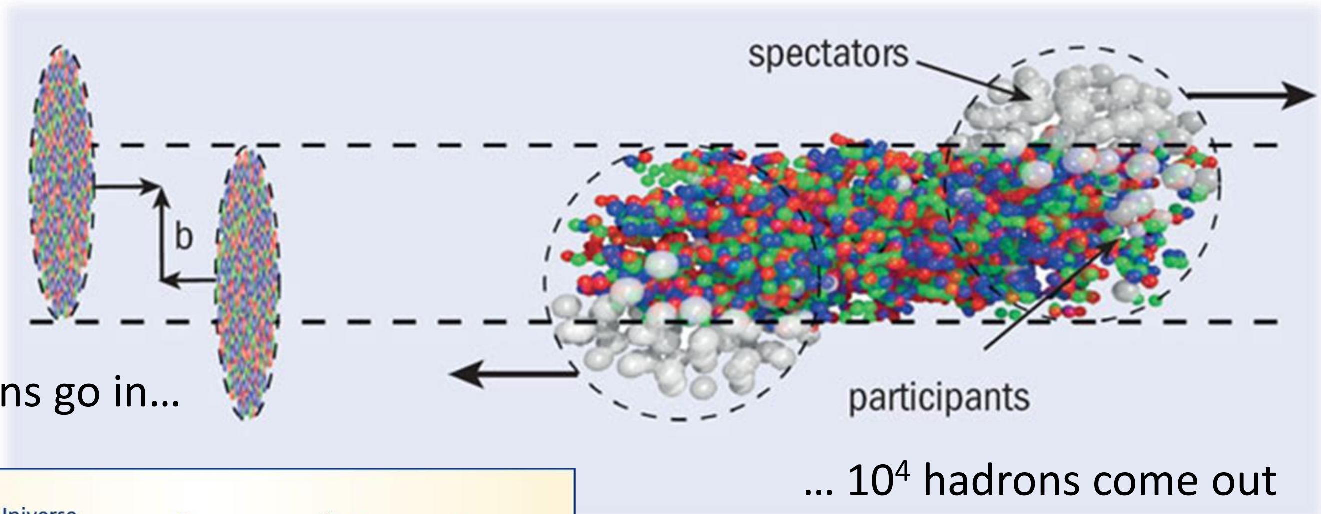
Max Planck Institute for Physics in Munich

In collaboration with Diego Trancanelli & Leonardo Patiño

23 June 2016 - Madrid IFT

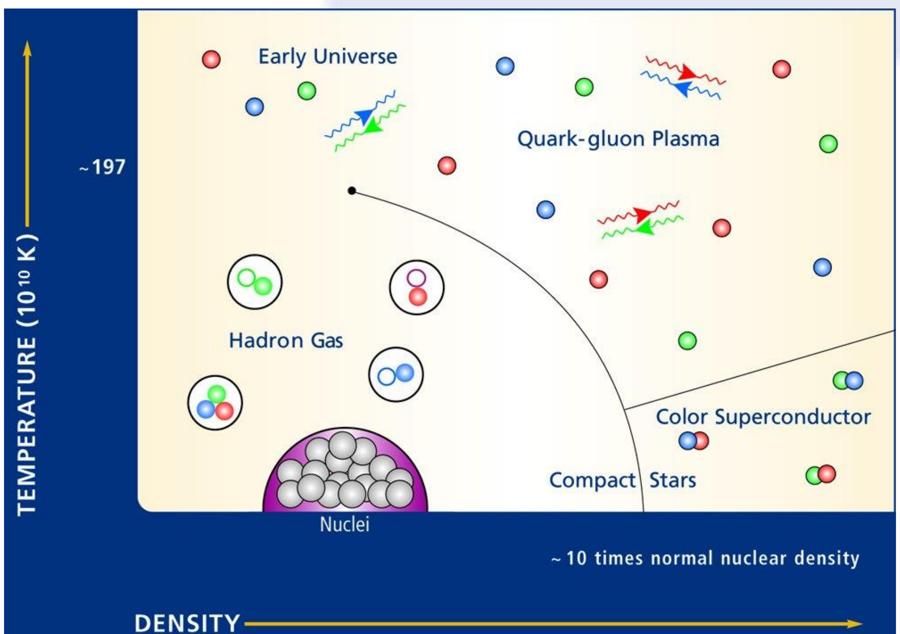
Motivation

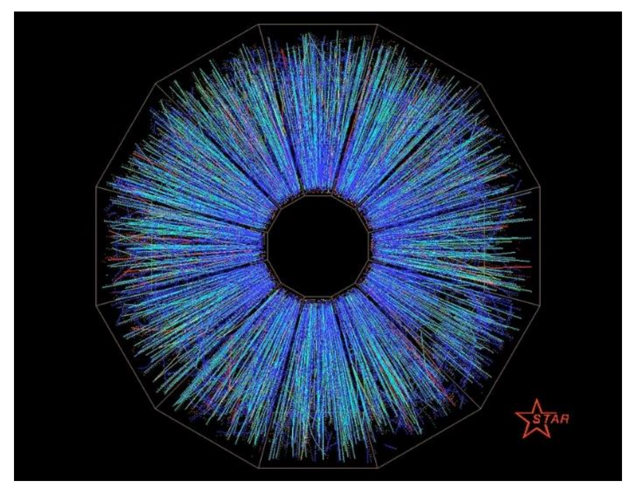
In a heavy ion collision at RHIC and LHC:



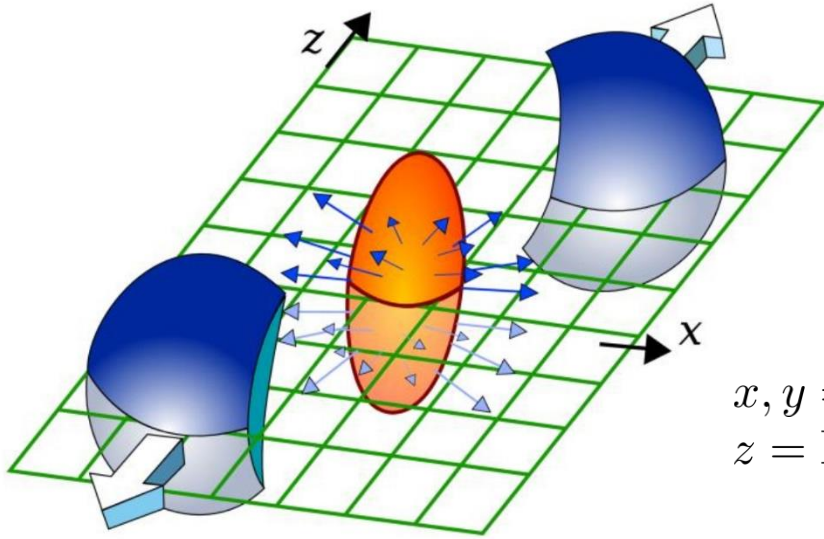
~ 400 nucleons go in…

… 10^4 hadrons come out





Motivation



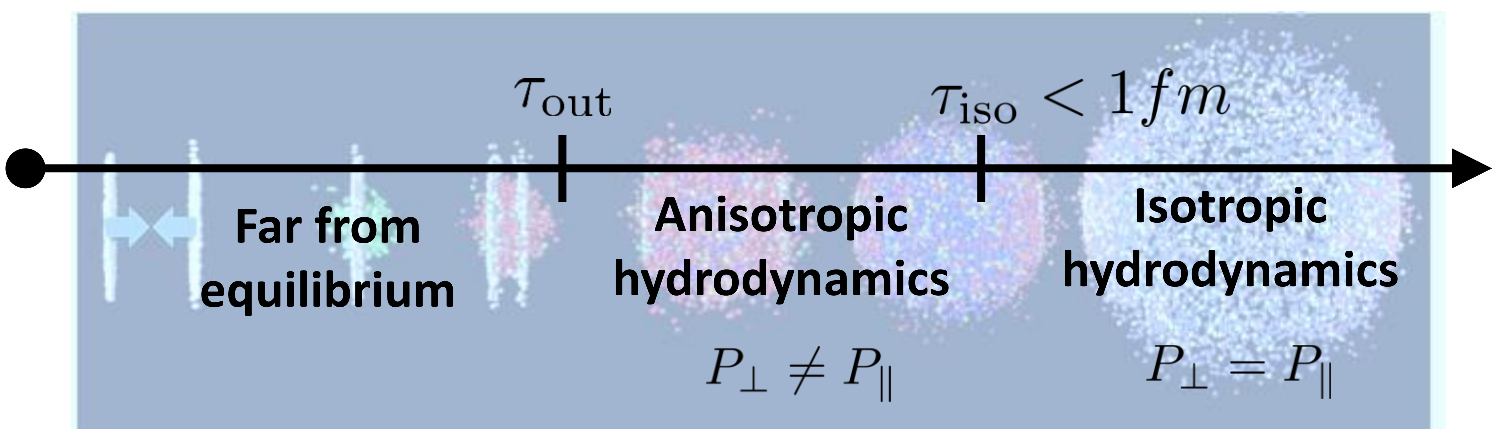
Picture credit: BNLJ

- The **quark-gluon plasma** created in heavy ion collisions is time-dependent and anisotropic.

- Collisions are generically non-central

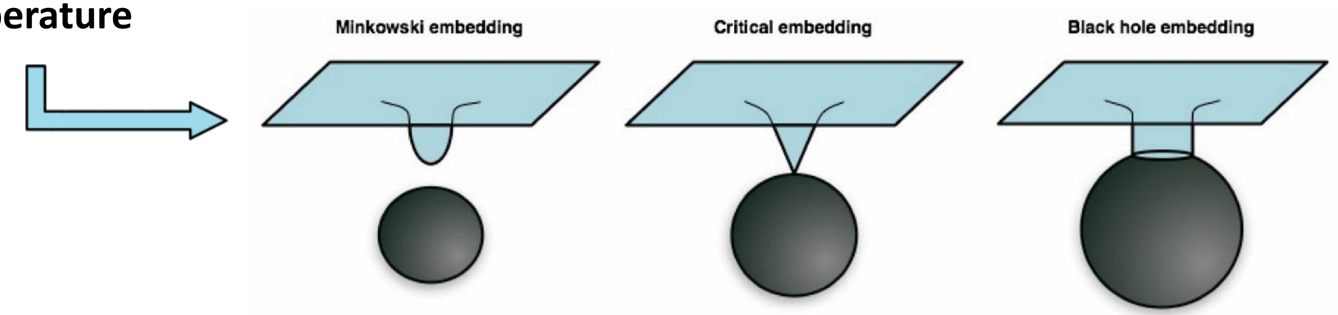
$x, y = \text{Transverse plane}$
$z = \text{Longitudinal (beam) direction}$

- Evolution scheme:



Static anisotropic plasma

- This is a good approximation if $t_{char} \ll t_{expansion}$
- Observables influenced by anisotropy:
  - ❖ Energy loss and momentum broadening
  - ❖ Quarkonium dissociation
  - ❖ **Critical temperature**



- Characteristics of the solution:
  - ❖ Static and spatially anisotropic
  - ❖ With a horizon and regular on and outside the horizon
  - ❖ With AdS boundary conditions in the UV

Holographic model?

Anisotropic dual

- ***FT side***: Include a position-dependent theta term: *[Azeyanagi, Li, Takayanagi '09]*

$$\int \theta(\vec{x})\ \mathrm{Tr} F \wedge F\ , \quad \text{where} \quad \theta(\vec{x}) \propto a\ z$$

- ***Interpretation***: RG flow to a Lifshitz IR fixed point: $z \to k^{2/3} z$

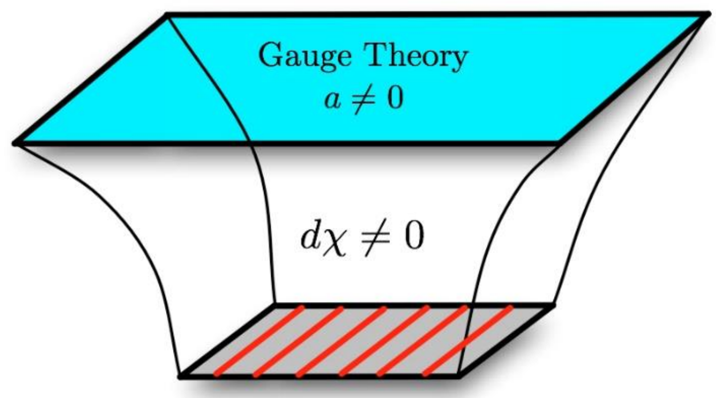
- ***Realization***: System of disolved D7-branes behind the horizon.

- Supergravity solution: Fields turned on:

$$F_{(5)} = 4(\Omega_5 + *\Omega_5), \qquad F_{(1)} = a\, dz$$

→ ***Probes***: Additional set of D7-branes!

	t	x	y	z	u	Ω_3	ϕ	φ
$D3$:	$\times$	$\times$	$\times$	$\times$				
$D7_b$:	$\times$	$\times$	$\times$			$\times$	$\times$	$\times$
$D7_p$:	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		



[Mateos, Trancanelli '11]

Features of the background

- Anisotropic, non-traceless Stress Tensor: $\mathcal{A} = \langle T^i_i \rangle = \frac{N_c^2}{48\pi^2} a^2, \quad P_{xy} \neq P_z$

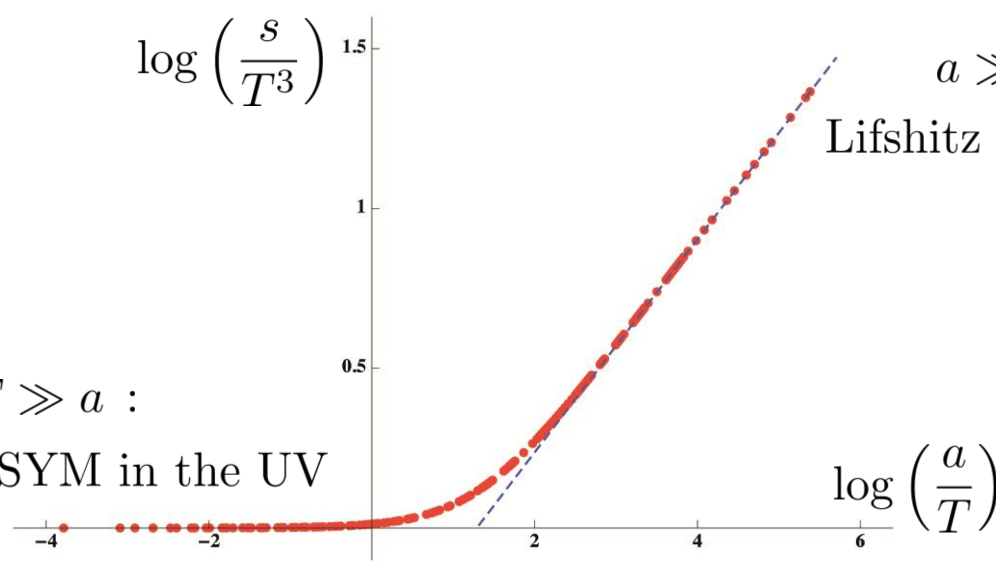
- The **conformal anomaly** has consequences:

$$E(a,T) = a^4 f\left(\frac{T}{a}\right) + a^4 \frac{N_c^2}{48\pi^2} \log\left(\frac{a}{\mu}\right)$$

One must introduce a reference scale μ

[Papadimitriou, Skenderis '05]

- The entropy density interpolates:



$a \gg T$:

Lifshitz scaling in the IR

$$\{t,x,y\} \to k\{t,x,y\}$$
$$z \to k^{2/3} z$$
$$r \to k^{-1} r$$

$T \gg a$:

$\mathcal{N} = 4$SYM in the UV

Geometry of the flavor branes

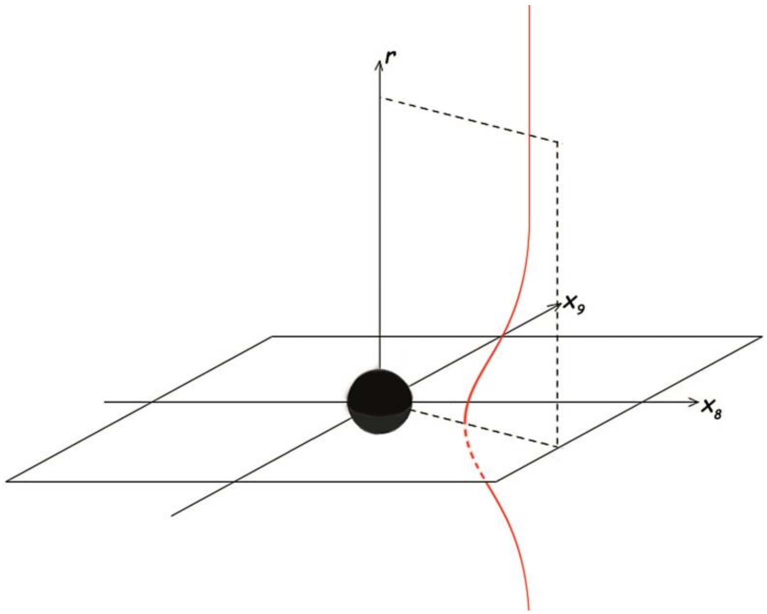
- Ansatz for the metric turns into:

$$ds^2 = \frac{1}{u(\rho)^2}\left(-\mathcal{F}(\rho)\mathcal{B}(\rho)\,dt^2 + dx^2 + dy^2 + e^{-\phi(\rho)}dz^2\right) + \frac{1}{\rho^2}e^{\phi(\rho)/2}\left(d\rho^2 + \rho^2 d\Omega_5^2\right)$$

The wrapping of the D7-branes is obtained by $d\Omega_5^2 = d\vartheta^2 + \cos^2\vartheta\, d\varphi^2 + \sin^2\vartheta\, d\Omega_3^2$

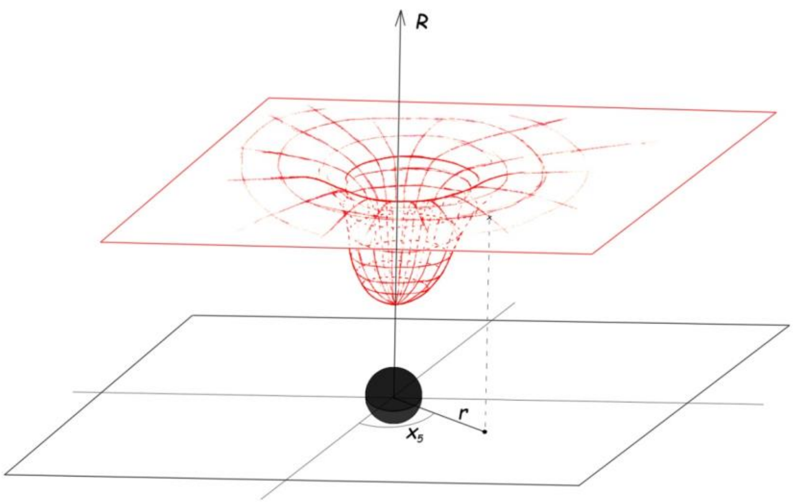
- **Embedding function** (BH embedding):

$$\psi(\rho) = \cos\vartheta(\rho)$$



- For Minkowski embedding:

$R(r)$, where $R = \rho\,\cos\vartheta, \quad r = \rho\,\sin\vartheta$



Previous results

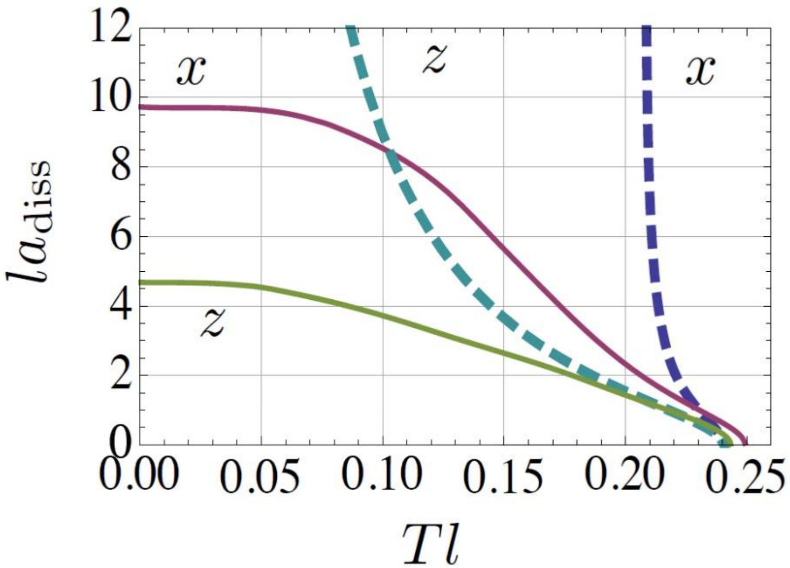
- ***Drag force:*** Misalignment of quark velocity, dragforce & trailing of gluon cloud.

- ***Jet quenching:*** Greater range of values, depending on orientations.

- Real world QGP: Agreement in some cases, disagreement in others.

[Chernicoff, Fernández, Mateos, Trancanelli ,13]

- Anisotropy acts like Temperature → A ***critical anisotropy*** can be defined:

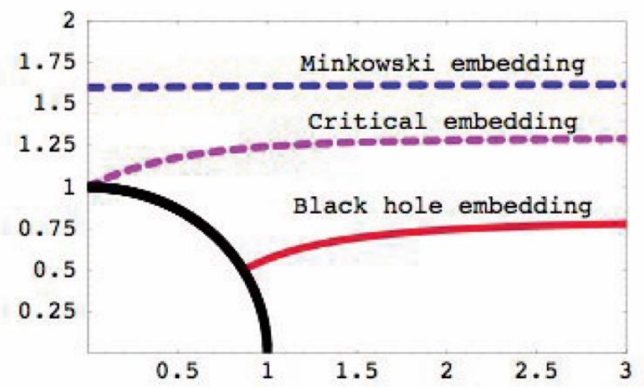


- Mesons dissociate even at zero temperature, if *a* is large enough.

- There is a limiting velocity for mesons in the plasma, even at zero temperature.

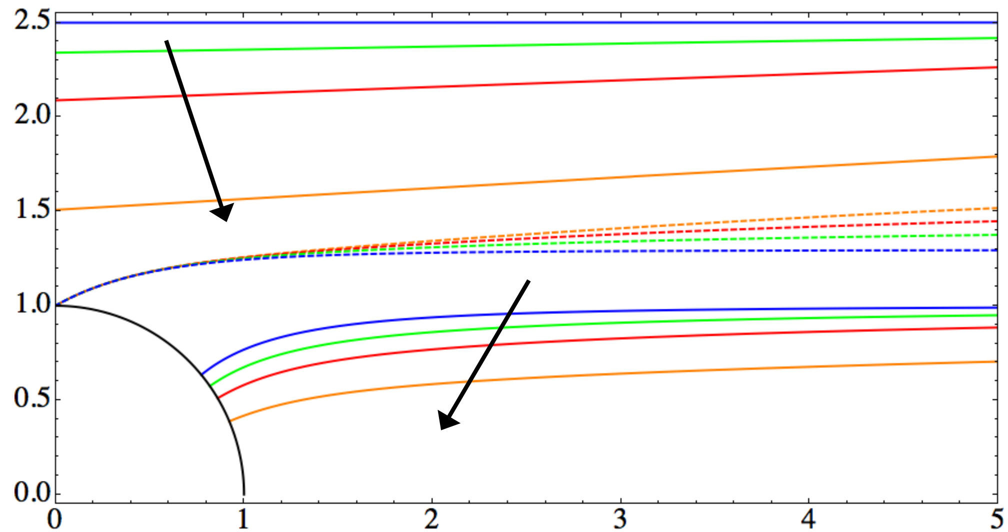
$$L_s \sim (1-v^2)^\epsilon \quad \begin{cases} a \neq 0 \Rightarrow \epsilon = 1/2 \\ a = 0 \Rightarrow \epsilon = 1/4 \end{cases}$$

Embedding of the flavor branes



Isotropic case

Effect of anisotropy:



1) The anisotropy bends the branes towards the horizon.
   → Increasing *a* has similar effect to increasing the BH's temperature.

2) Anisotropic profiles converge to the flat geometry at long distances.

3) ***Critical embedding:*** Greater asymptotic distance ≈ M_q / T
   → T_c is smaller in the anisotropic case.

Thermodynamics

- Wick rotation t --> it_E → Euclidean path integral yields thermal Partition Function.
- Gravity action evaluated at saddle point: the classical solution.

Free Energy: $I_E = \beta F$

Entropy:

$$S = -\frac{\partial F}{\partial T}, \qquad E = F + TS$$

- For the D3-brane isotropic background,

$$F = -\frac{\pi^2}{8} N_c^2 T^4$$

[Hawking, Ross '05]

- To introduce a chemical potential for the background's D7 branes, we need to dualize to an electric description:

$dC_8 \sim *d\chi \longrightarrow$ From asymptotic fall-off, read off charge density a and the potential Φ.

$$dF = -s\, dT + \Phi\, da$$

Holographic Renormalization

The flavor branes action: *[Papadimitriou '11]*

$$S_{\mathrm{DBI}} = -T_{D7}\int_V d^8\xi\, e^{\phi}\sqrt{-g_{\mathrm{ind}}}$$

...in the background of the axion-dilaton-gravity theory, with fields: $\{g_{\mu\nu},\phi,\chi\}$

with an embedding given by $\psi(v)$

Asymptotic solution near the boundary:

$$\psi = mv + \left(c + \frac{5}{24}a^2 m\log v\right)v^3 + \mathcal{O}(v^5)$$

Counterterms are required:

$$\boxed{S_{\mathrm{reg}} = -\int d^4x\, e^{\phi_{(0)}}\sqrt{-g_{(0)}}\left(\frac{1}{\epsilon^4}a_{(0)} + \frac{1}{\epsilon^2}a_{(1)} + a_{(2)}\log\epsilon + a_{(3)}\log^2\epsilon + \mathcal{O}\left(\epsilon^0\right)\right)}$$

where $a_{(3)} = -\frac{1}{4}\left(g^{ij}_{(0)}\tilde{g}_{(4)ij} + 2\tilde{\phi}_{(4)}\right)$

From expansions like for the dilaton:

$$\phi = \phi_{(0)} + \phi_{(2)}v^2 + \left(\phi_{(4)} + \tilde{\phi}_{(0)}\log v\right)v^4 + \mathcal{O}(v^6)$$

with metric in FG coordinates:

$$ds^2 = G_{vv}\,dv^2 + \frac{1}{v^2}g_{ij}\,dxdx^j + d\Omega_5^2$$

Holographic Renormalization

- There is a $\log^2$ divergence, with coefficient

$$a_{(3)} = -\frac{1}{4}\left(g^{ij}_{(0)}\tilde{g}_{(4)ij} + 2\tilde{\phi}_{(4)}\right)$$ but our EOM dictate $$g^{ij}_{(0)}\tilde{g}_{(4)ij} = 0$$

  - Origin: interaction of dilaton with D7-brane.
  - Counterterm related to conformal anomaly: $$\boxed{S^{\log 2}_{\rm ct} = \frac{2}{5}\log^2\epsilon\int d^4x\sqrt{-\gamma}\,e^{\phi}\mathcal{A}}$$

- Usual procedure results in:

$$\frac{S_{\rm sus}}{T_{D7}V_{S^3}} = \int d^4x\, e^{\phi_{(0)}}\sqrt{-g_{(0)}}\left[\frac{\phi_{(4)}}{4} - \psi_{(1)}\psi_{(3)} + \frac{5}{48}\psi^2_{(1)}\, e^{2\phi_{(0)}} g^{ij}_{(0)}\partial_i\chi_{(0)}\partial_j\chi_{(0)} + \frac{55}{1152}e^{4\phi_{(0)}}\left(g^{ij}_{(0)}\partial_i\chi_{(0)}\partial_j\chi_{(0)}\right)^2\right] + \log\epsilon\int d^4x e^{\phi_{(0)}}\sqrt{-g_{(0)}}\,\phi_{(4)}$$

$\phi_{(4)}$ not expressable in terms of original field!

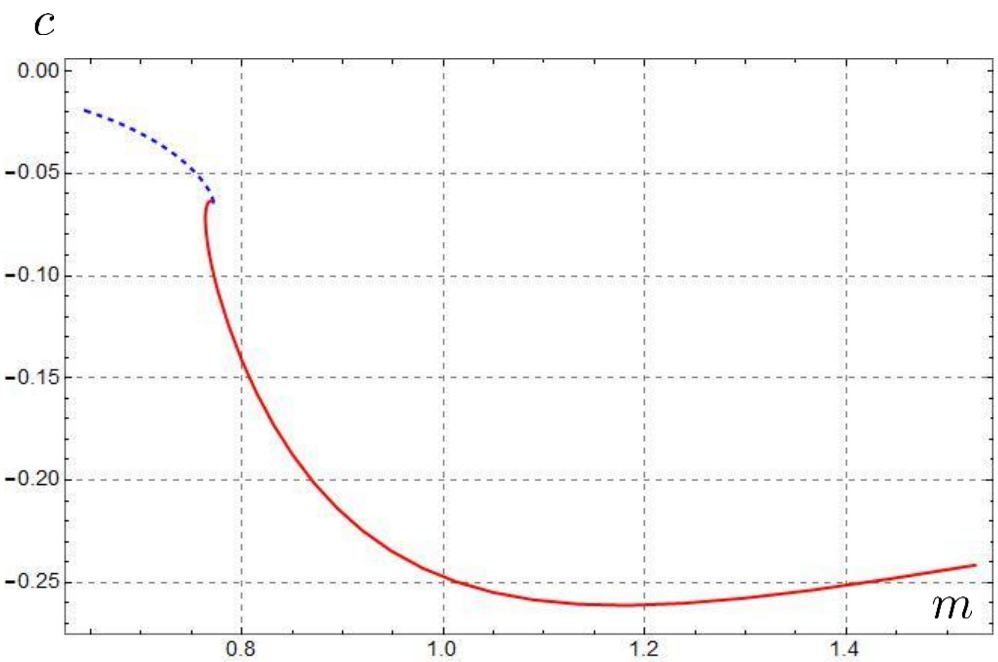
→ Additional set of counterterms.

- Background action rescales under:

$$\tilde{\phi} = \phi + C, \quad \tilde{\chi} = e^{-C}\chi$$

so explicit dependence on $\phi_{(0)}$ is required.

Results – *quark condensate*



$$\psi \sim mv + \left(c + \frac{5}{24}a^2 m \log v\right) v^3$$

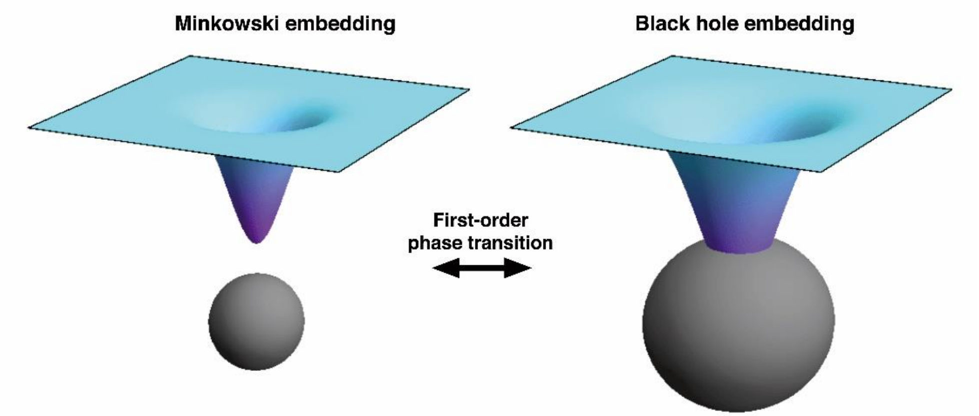
Dictionary: m → quark mass,

c → this expectation value:

$$c \propto \langle \bar{\Psi}\Psi + q^\dagger \Phi q + M_q q^\dagger q \rangle$$

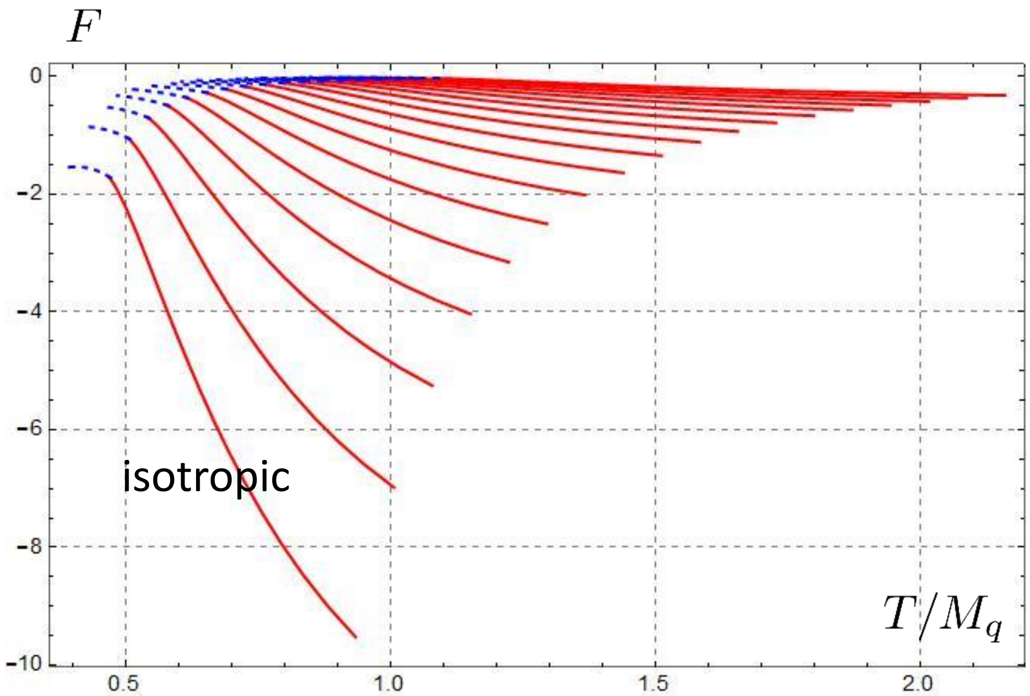
<u>1st order phase transition:</u>

Gauge Theory: Transition between discrete gapped meson spectrum & continuous gapless distr. Excitations.



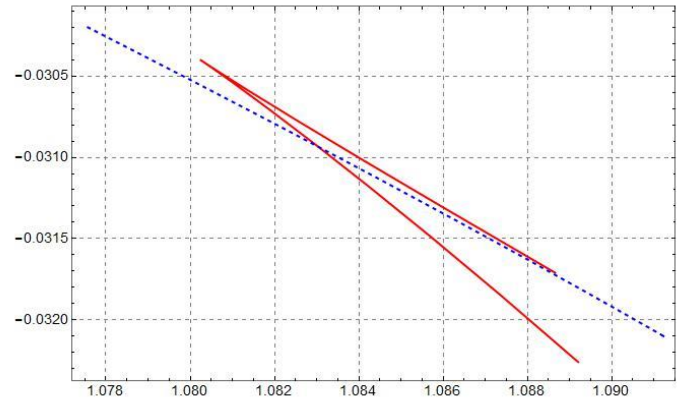
- **Red:** Black Hole phase
- **Blue, dashed:** Minkowski phase

Results – *Free Energy*



→ Curves at fixed a/T.

- Here, the position of the horizon is fixed to $u_H = 1$.



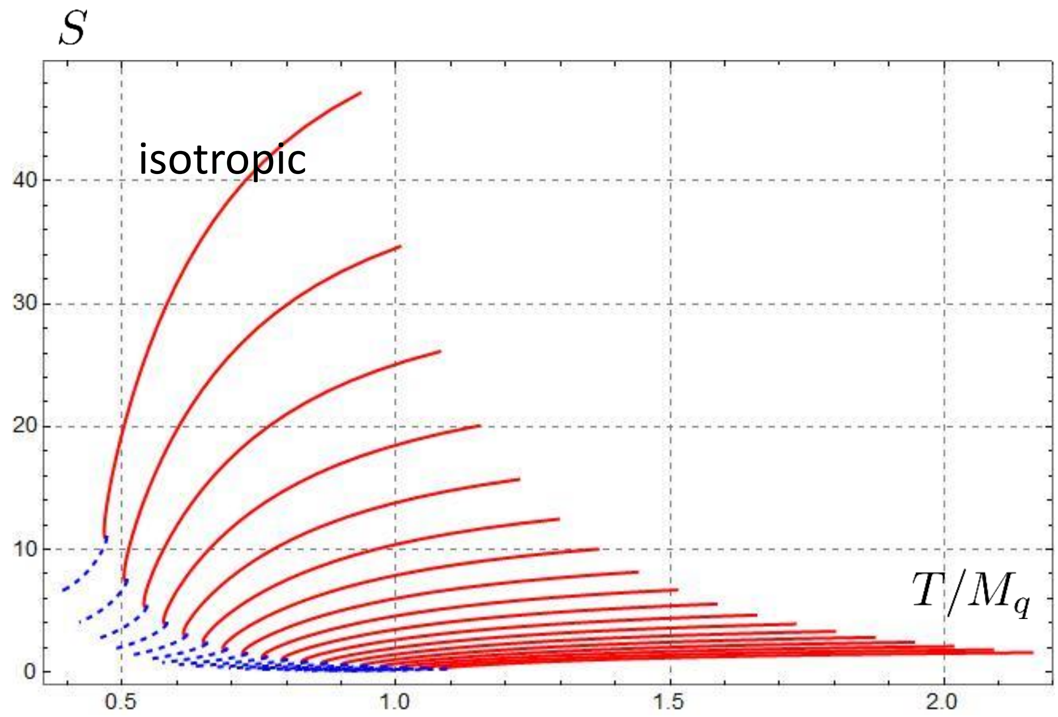
Scaling of the Free Energy:

$$F(ka, kT) = k^4 F(a,T) + k^4 \log\, k \frac{N_c^2 a^4}{48\pi^2}$$

Close-up near critical temp.

- **Red:** Black Hole phase
- **Blue, dashed:** Minkowski phase

Results – *Entropy*

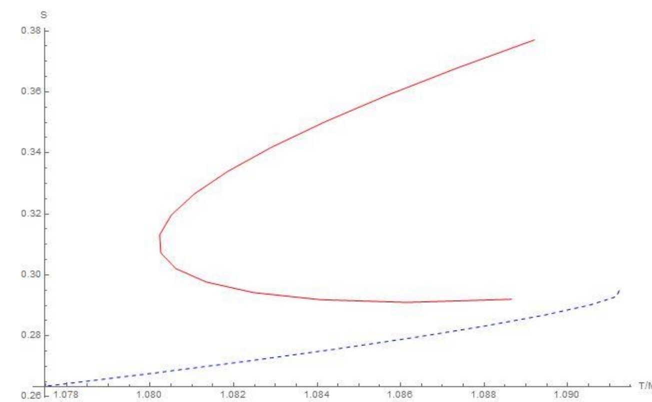


Computable from derivative of F at fixed a, since:

$$dF = -s\,dT + \Phi\,da$$

<u>Check from two computations:</u>

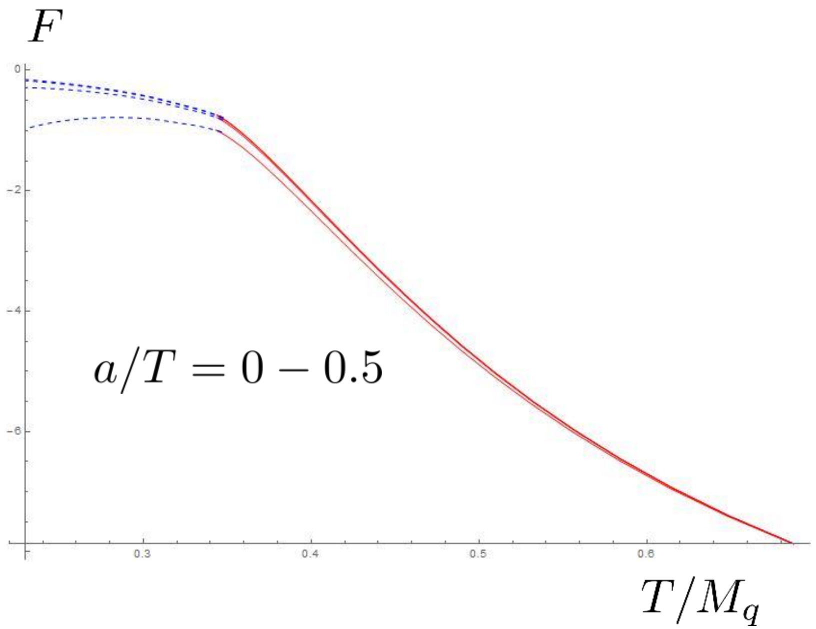
- Analytical expression (incl. Counterterms)
- Numerical derivation w.r.t. a

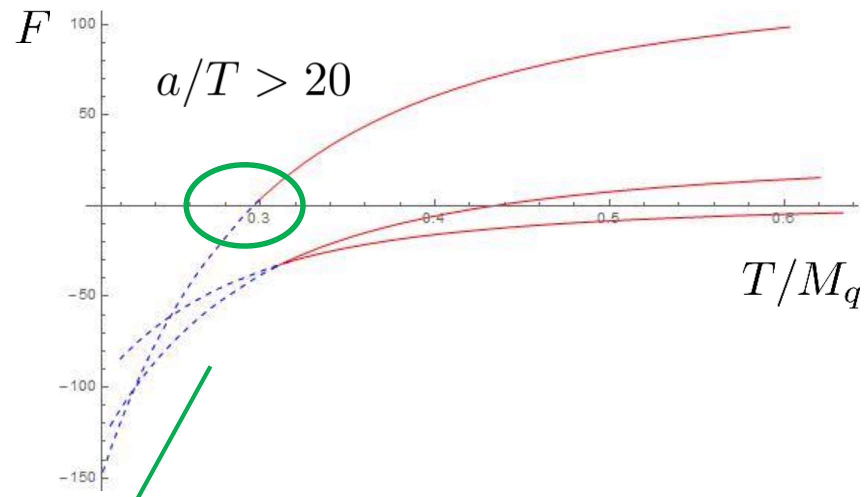


Close-up near critical temp.

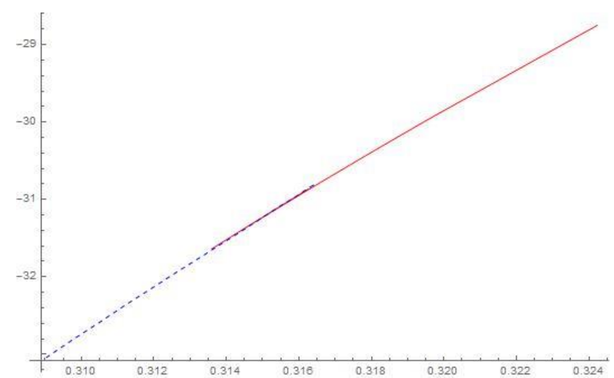
- **Red:** Black Hole phase
- **Blue, dashed:** Minkowski phase

Small / Large anisotropy limits





BH and Minkowski curves overlap, not only intersecting at one point:



2nd order phase transition for a critical a/T?

- **Red:** Black Hole phase
- **Blue, dashed:** Minkowski phase

Thank you
for your attention