QCD for the LHC

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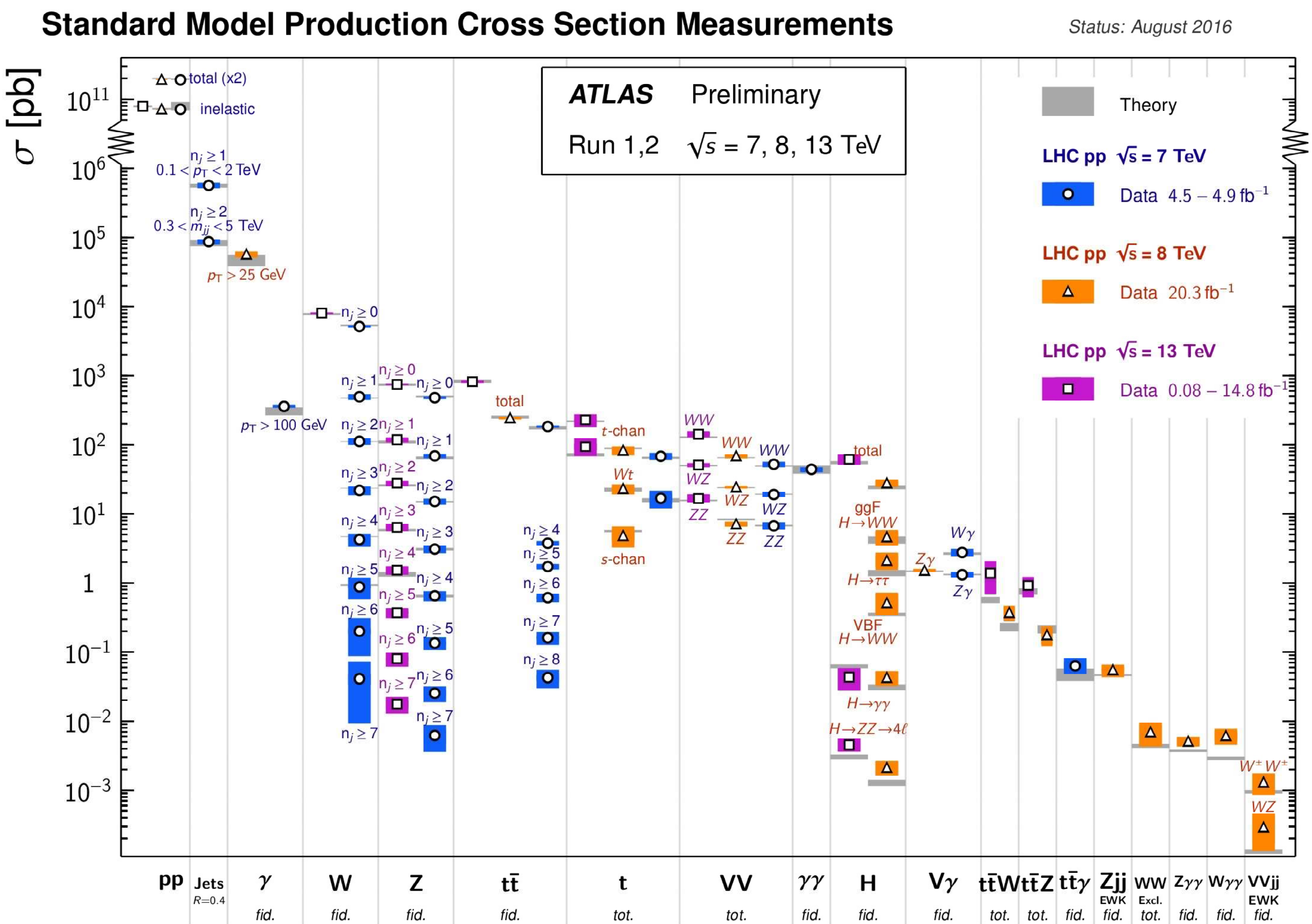
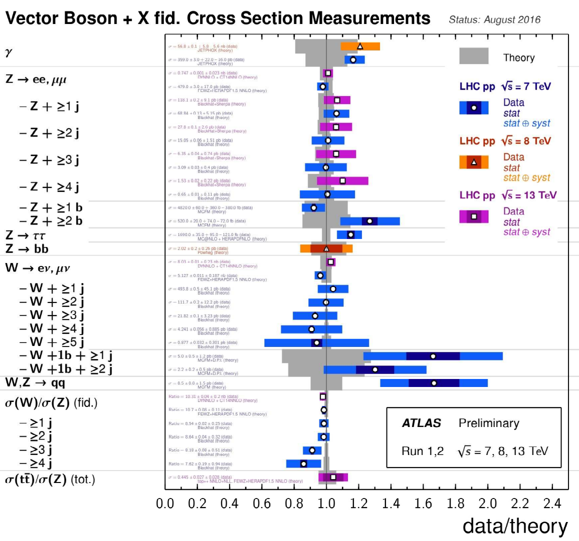
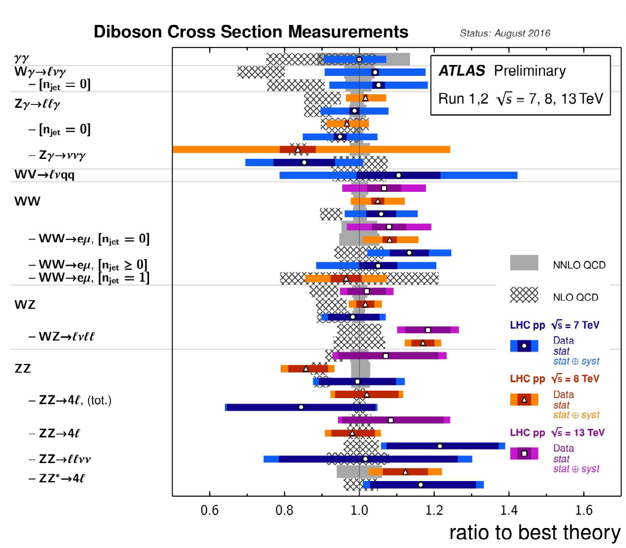
Cross Sections at the LHC

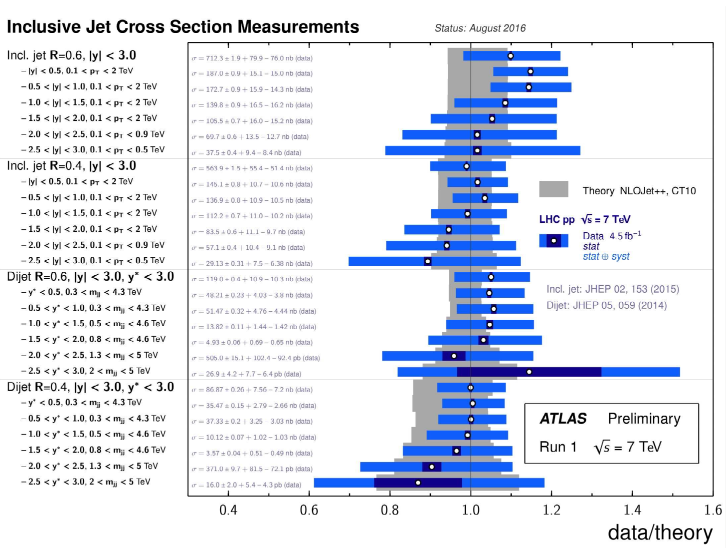


excellent agreement between theory and experiment over a wide range of observables

Discrepancies with data?





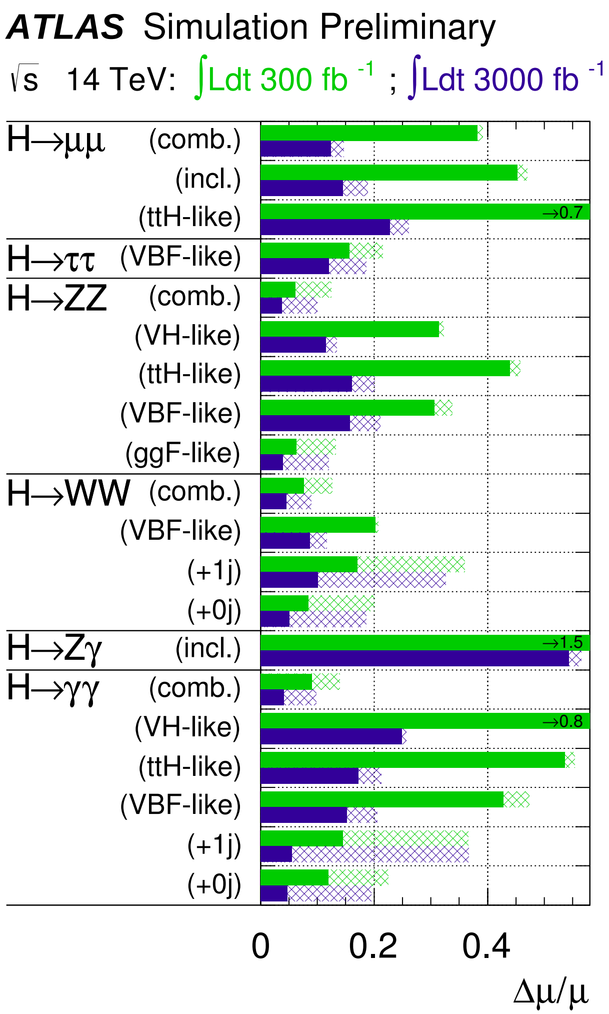


No BSM discovered yet. . . but plenty of BNLO

Motivation for more accurate theoretical calculations

- ✓ Theory uncertainty has big impact on quality of measurement
- ✘ *NLO QCD is clearly insufficiently precise for SM, top (and even Higgs) measurements*, D. Froidevaux, HiggsTools School, 2015
- ➠ Revised wishlist of theoretical predictions for
  - ✚ Higgs processes
  - ✚ Processes with vector bosons
  - ✚ Processes with top or jets

Les Houches 2015, arXiv:1605.04692



Theoretical Uncertainties

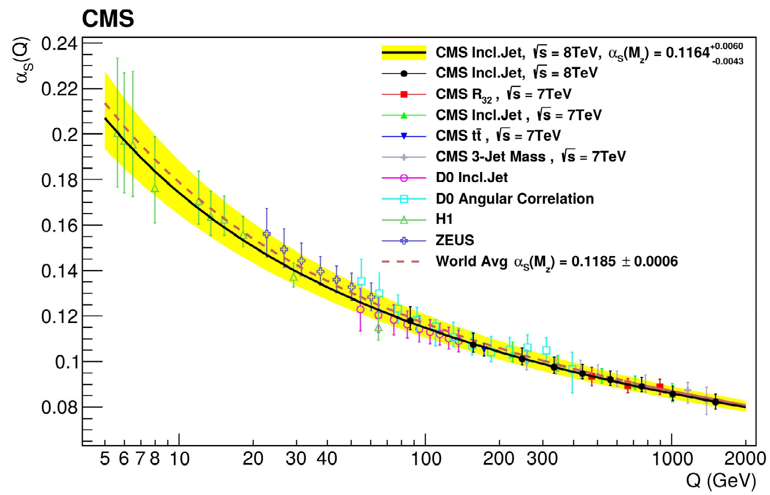
- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
  - ✓ systematically improvable by resummation of large logs
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - (✓) improvable by estimation of non-perturbative effects
  - hadronisation (model)
  - underlying event (tunes)

Goal: Reduce theory uncertainties by a factor of two compared to where we are now in next decade

The strong coupling

World Average

Year	$\alpha_s(M_Z)$
2008	0.1176 ± 0.0009
2012	0.1184 ± 0.0007
2014	0.1185 ± 0.0006
2016	0.1181 ± 0.0011



- ✓ Average of wide variety of measurements
  - ✓ τ-decays
  - ✓ e^+e^- annihilation
  - ✓ Z resonance fits
  - ✓ DIS
  - ✓ Lattice
- ✓ Generally stable to choice of measurements

- ✓ Impressive demonstration of running of α_s past O(1 TeV)
- ✓ …but some outlier values from global PDF fits, e.g.,
  $\alpha_s(M_Z) \sim 0.1136 \pm 0.0004$ (G)JR
  $\alpha_s(M_Z) \sim 0.1147 \pm 0.0008$ ABM16
- ➠ Still need to understand uncertainty and make more precise determination

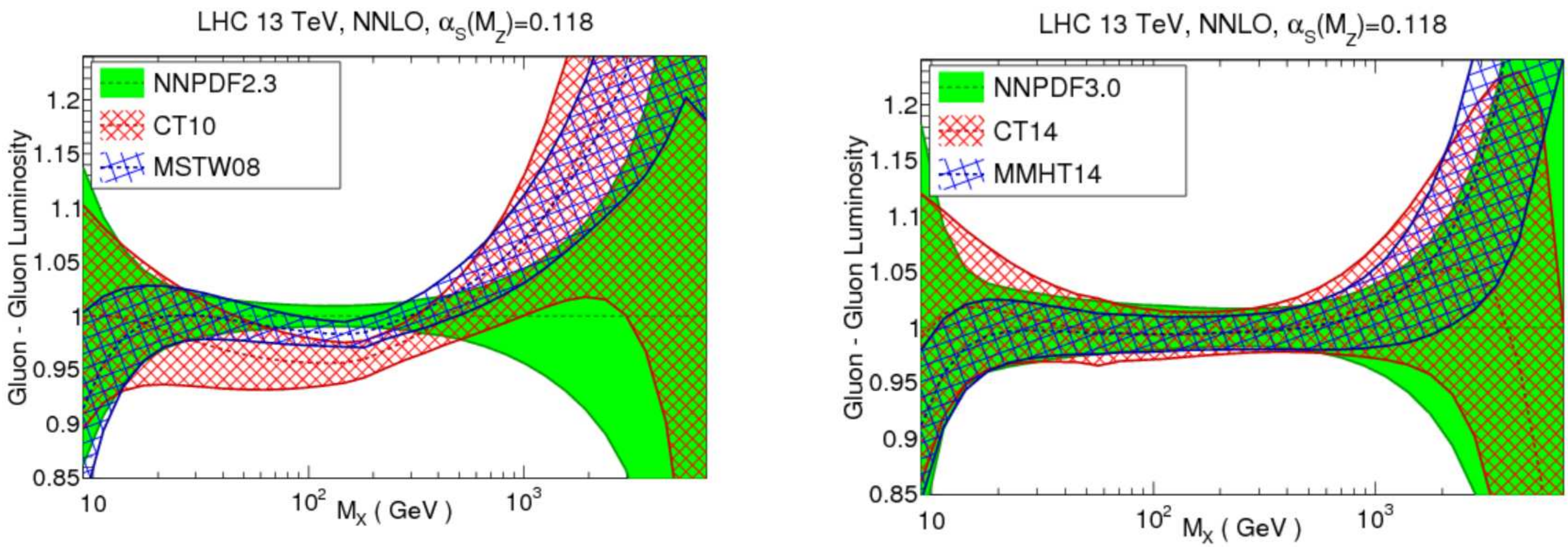
1% on α_s ➠ n% on process of $\mathcal{O}(\alpha_s^n)$

Parton Distribution Functions

All fits NNLO

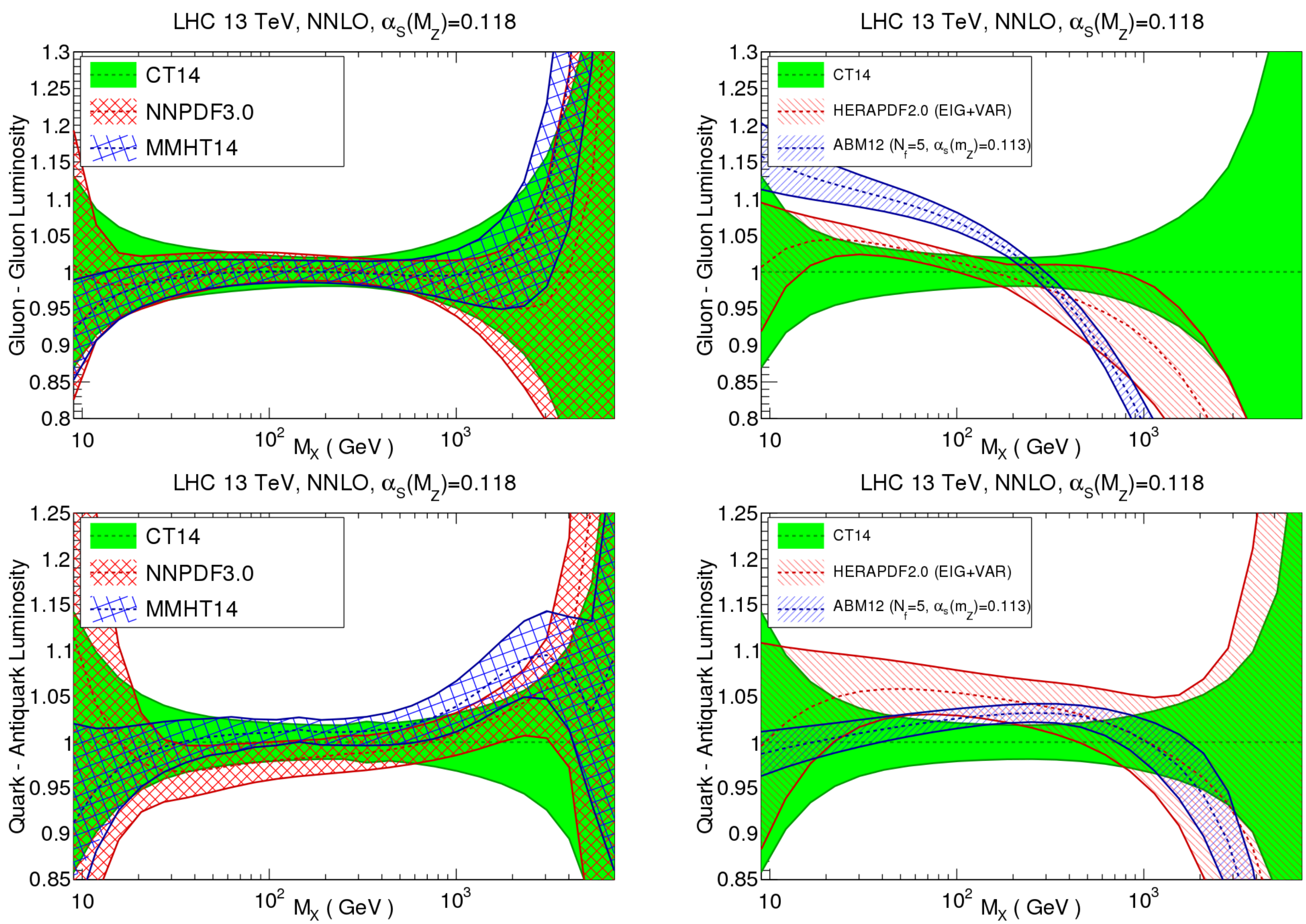
Set	DIS	DY	jets	LHC	errors
MMHT14	✓	✓	✓	✓	hessian
CT14	✓	✓	✓	✓	hessian
NNPDF3.0	✓	✓	✓	✓	Monte Carlo
HeraPDF2.0	✓	✗	✗	✗	hessian
ABM14	✓	✓	✓	✗	hessian
G(JR)	✓	✓	✓	✗	hessian

✓ Clear reduction in gluon-gluon luminosity for $M_X \sim 125$ GeV



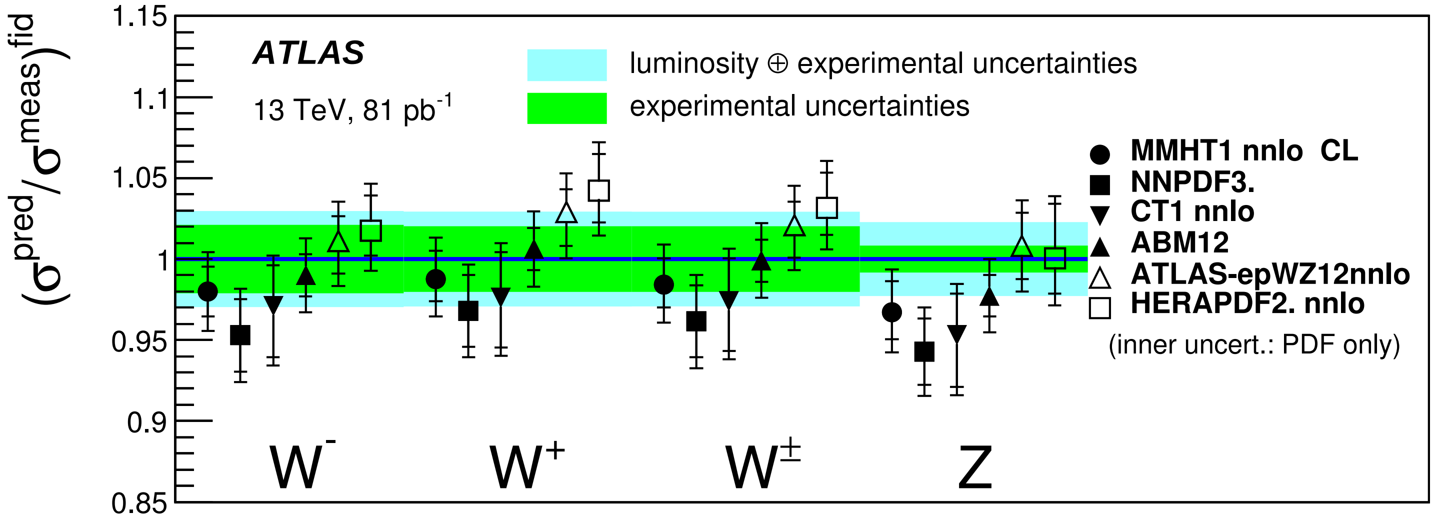
✓ . . . with commensurate reduction in uncertainty on Higgs cross section

Parton Distribution Functions



but still differences of opinion

Parton Distribution Functions



and disagreements even for the best measured cross sections

sensitivity to inputs into the PDF fits

- ✓ strange content of proton
- ✓ mass of charm quark

Partonic cross sections

$$\hat{\sigma} \sim \alpha_s^n \left(\hat{\sigma}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NLO}_{QCD} \quad + \quad \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{NNLO}_{QCD} + \left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}^{N3LO}_{QCD} + \ldots \right.$$

$$+ \quad \left(\frac{\alpha_W}{2\pi}\right) \hat{\sigma}^{NLO}_{EW} + \left(\frac{\alpha_W}{2\pi}\right) \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NNLO}_{QCDxEW} \cdots$$

NLO QCD

- ✓ NLO QCD is the current state of the art

NNLO QCD

- ✓ provides the first serious estimate of the theoretical uncertainty
- ✓ rapid development with many new results in past couple of years

NLO EW

- ✓ naively similar size to NNLO QCD
- ✓ particularly important at high energies/p_T and near resonances

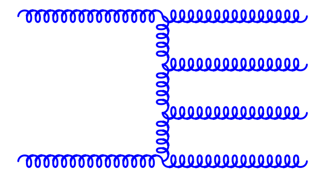
N3LO QCD

- ✓ recent landmark results for Higgs production

Anatomy of a Higher Order calculation

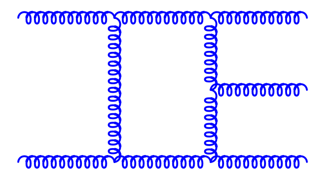
e.g. pp to JJ at NNLO

✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$



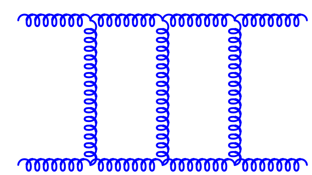
- ✚ implicit poles from double unresolved emission

✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$



- ✚ explicit infrared poles from loop integral
- ✚ implicit poles from soft/collinear emission

✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$



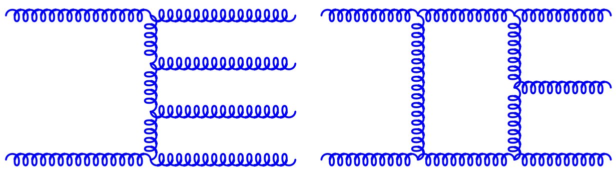
- ✚ explicit infrared poles from loop integral

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

Anatomy of a Higher Order calculation

e.g. pp to JJ at NNLO

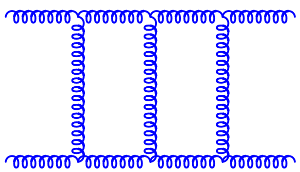
- ✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet



  Can exploit NLO automation

  ...but needs to be evaluated in regions of phase space where extra jet is not resolved

- ✚ Two loop amplitudes - very limited set known



  ...currently far from automation

- ✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

  ...currently not automated

IR cancellation at NNLO

✓ The aim is to recast the NNLO cross section in the form

$$
\begin{aligned}
d\hat{\sigma}_{NNLO} &= \int_{d\Phi_{m+2}} \left[d\hat{\sigma}^{RR}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right] \\
&+ \int_{d\Phi_{m+1}} \left[d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO} \right] \\
&+ \int_{d\Phi_{m}} \left[d\hat{\sigma}^{VV}_{NNLO} - d\hat{\sigma}^{U}_{NNLO} \right]
\end{aligned}
$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

✚ Much more complicated cancellations between the double-real, real-virtual and double virtual contributions

✚ intricate overlapping divergences

NNLO - IR cancellation schemes

Unlike at NLO, we do not have a fully general NNLO IR cancellation scheme

- Antenna subtraction — Gehrmann, Gehrmann-De Ridder, NG (05)
- Colourful subtraction — Del Duca, Somogyi, Trocsanyi (05)
- q_T subtraction — Catani, Grazzini (07)
- STRIPPER (sector subtraction) — Czakon (10); Boughezal et al (11)
  Czakon, Heymes (14)
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello (15)
  Gaunt, Stahlhofen, Tackmann, Walsh (15)
- Projection to Born — Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

Each method has its advantages and disadvantages

	Analytic	FS colour	IS colour	Azimuthal	Approach
Antenna	✓	✓	✓	✘	Subtraction
Colourful	✓	✓	✘	✓	Subtraction
q_T	✓	✘ (✓)	✓	—	Slicing
STRIPPER	✘	✓	✓	✓	Subtraction
N-jettiness	✓	✓	✓	—	Slicing
P2B	✓	✓	✓	—	Subtraction

Slicing v Subtraction example

$$V = \frac{F(0)}{\epsilon}, \qquad R = \int_0^1 dx \frac{F(x)}{x^{1+\epsilon}}$$

Slicing

$$\begin{aligned} \sigma &= V + R \\ &= \frac{F(0)}{\epsilon} \\ &+ \int_0^X dx \frac{F(0)}{x^{1+\epsilon}} + \int_X^1 dx \frac{F(x)}{x} \\ &= F(0)\ln(X) + \int_X^1 dx \frac{F(x)}{x} \end{aligned}$$

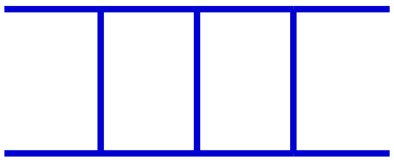
- ✓ Approximation made for $x < X$
- ✓ X should be small, but not so small that numerical errors dominate
- ✓ q_T and N-jettiness schemes related to soft-collinear resummation

Subtraction

$$\begin{aligned} \sigma &= V + R \\ &= \frac{F(0)}{\epsilon} + \int_0^1 dx \frac{S(x)}{x^{1+\epsilon}} \\ &+ \int_0^1 dx \left[\frac{F(x)}{x^{1+\epsilon}} - \frac{S(x))}{x^{1+\epsilon}} \right] \\ &= \text{finite} + \int_0^1 dx \left[\frac{F(x) - S(x)}{x} \right] \end{aligned}$$

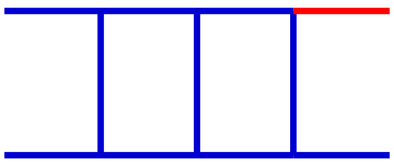
- ✓ $S(x) \to F(0)$ as $x \to 0$
- ✓ integral of $S(x)$ must be computed
- ✓ antenna, STRIPPER, ColorFul, P2B all subtraction schemes

Two Loop Master Integrals - analytic

✓ 

Smirnov (99); Smirnov, Tausk (99)

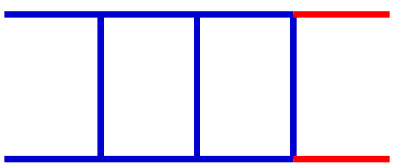
➠ enables $pp \to \gamma\gamma,\, \gamma J,\, JJ$

✓ 

Gehrmann and Remiddi (00,01,02)

➠ enables $pp \to WJ,\, ZJ,\, HJ,\, W\gamma,\, Z\gamma,$

$e^+e^- \to JJJ,\, ep \to JJ(+J)$

✓ 

Gehrmann, Tancredi, Weihs (13);

Gehrmann, von Manteuffel, Tancredi, Weihs (14);

Caola, Henn, Melnikov, Smirnov (14);

Papadopoulos, Tommasini, Wever (14)

➠ enables $pp \to WW,\, ZZ,\, WZ,\, HH$

✓ now intensive work towards two-loop five point integrals

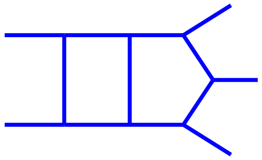
Two Loop Master Integrals - analytic

✓ Basis functions for two-loop pentagon graphs with massless internal propagators known - Goncharov Polylogs

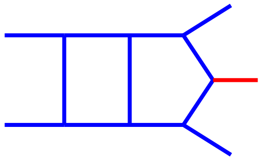
$$G(a_n, a_{n-1}, \ldots, a_1, t) = \int_0^t \frac{dt}{t_n - a_n} G(a_{n-1}, \ldots, a_1, t_n)$$

✓ Canonical (Henn) basis for evaluating integral as series in ϵ

$$\partial_x \vec{f} = \epsilon \hat{A}_x(x, y, z, \ldots) \vec{f}$$

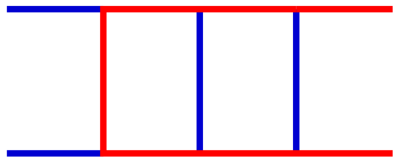
✓  Gehrmann, Henn, Lo Presti (15); Papadopoulos, Tomassini, Wever (15)

➠ enables $pp \to JJJ,\ \gamma\gamma J,\ \gamma\gamma\gamma$

✓  Papadopoulos, Tomassini, Wever (15)

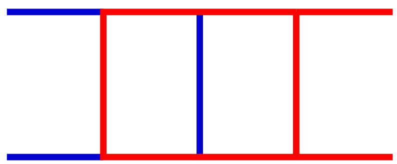
➠ enables $pp \to VJJ,\ HJJ$

Two Loop Master Integrals - numeric

✓ 

Czakon (07); Bonciani, Ferroglia, Gehrmann, Studerus (09)

➠ enables $pp \to t\bar{t}$

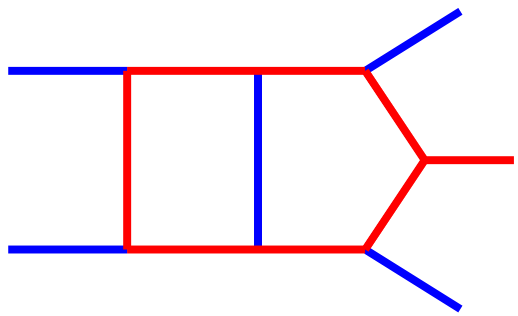
✓ 

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (16)

➠ enables $pp \to HH$ at NLO with massive top loop

✓ now intensive work including additional scales

Two Loop Master Integrals - numeric

- ✓ Integrals with massive propagators much more complicate, new functions
  Tancredi, Remiddi (16); Adams, Bogner, Weinzierl (15,16)
- ✓  e.g. Higgs plus Jet production via massive quark loop
  - ✓ First results as one-fold (elliptic) integrals Bonciani et al (16)
  - ✓ Light quark effects Melnikov et al (16)

Inclusive N3LO

The current best perturbative calculations

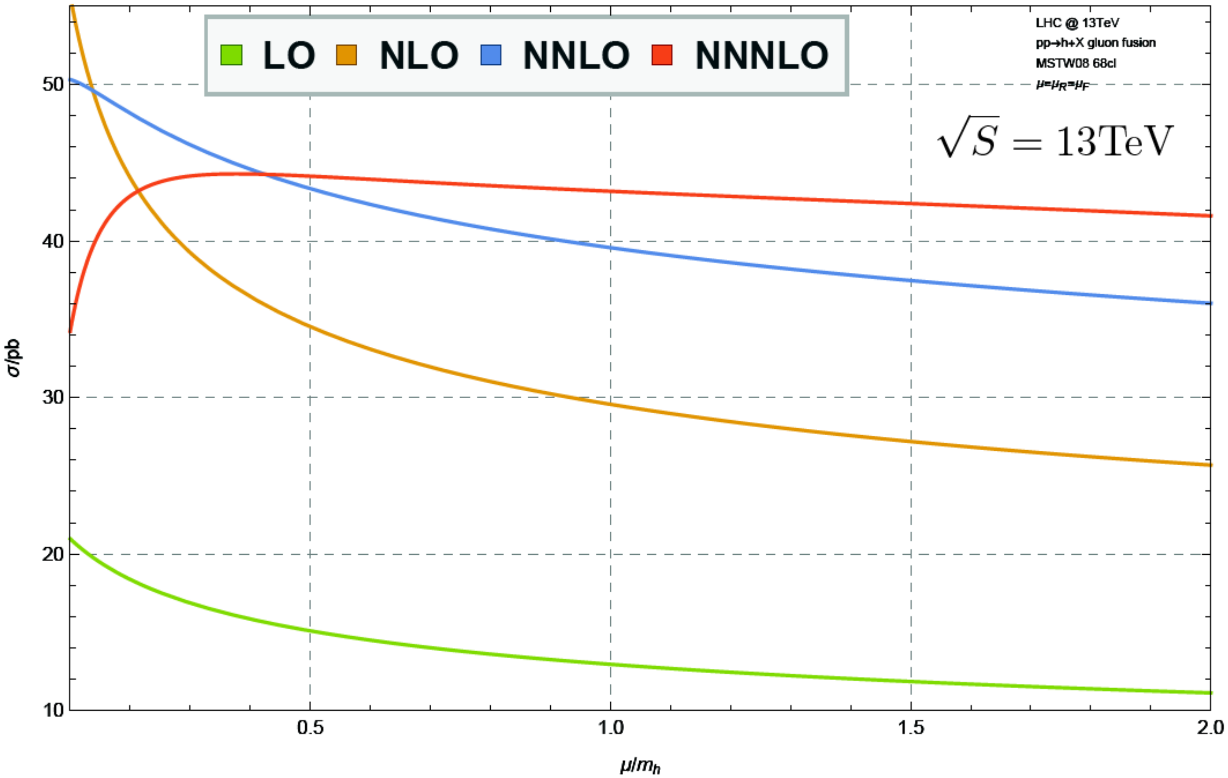
- ✓ Inclusive Higgs cross section via gluon fusion

  Anastasiou, Duhr, Dulat, Herzog, Mistlberger (15);
  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger (16)

- ✓ Inclusive Higgs cross section via vector boson fusion

  Dreyer, Karlberg (16)

Inclusive N3LO Higgs via ggF



Anastasiou, Duhr, Dulat, Herzog, Mistlberger (15)

✓ Stabilisation of scale dependence around $\mu = m_H/2 \sim \pm 2.2\%$

✓ Convergence

Inclusive N3LO Higgs via ggF

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger (16)

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\mathrm{PDF}{+}\alpha_s)\,.}$$

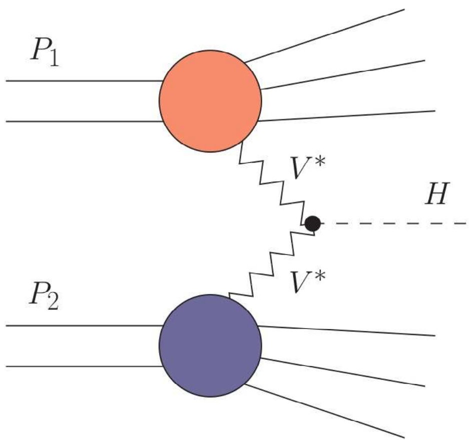
✓ including all known contributions

$$\begin{aligned}
48.58\,\mathrm{pb} = \quad & 16.00\,\mathrm{pb} & (+32.9\%) \quad & (\mathrm{LO,\ rEFT}) \\
& +20.84\,\mathrm{pb} & (+42.9\%) \quad & (\mathrm{NLO,\ rEFT}) \\
& -\ \ 2.05\,\mathrm{pb} & (-4.2\%) \quad & ((t,b,c),\ \mathrm{exact\ NLO}) \\
& +\ \ 9.56\,\mathrm{pb} & (+19.7\%) \quad & (\mathrm{NNLO,\ rEFT}) \\
& +\ \ 0.34\,\mathrm{pb} & (+0.2\%) \quad & (\mathrm{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\mathrm{pb} & (+4.9\%) \quad & (\mathrm{EW,\ QCD\text{-}EW}) \\
& +\ \ 1.49\,\mathrm{pb} & (+3.1\%) \quad & (\mathrm{N^3LO,\ rEFT})
\end{aligned}$$

✓ overall theory uncertainty estimated to be $+5/-7\%$

δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10\ \mathrm{pb}}_{-1.15\ \mathrm{pb}}$	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

Inclusive N3LO Higgs via VBF

✓ 

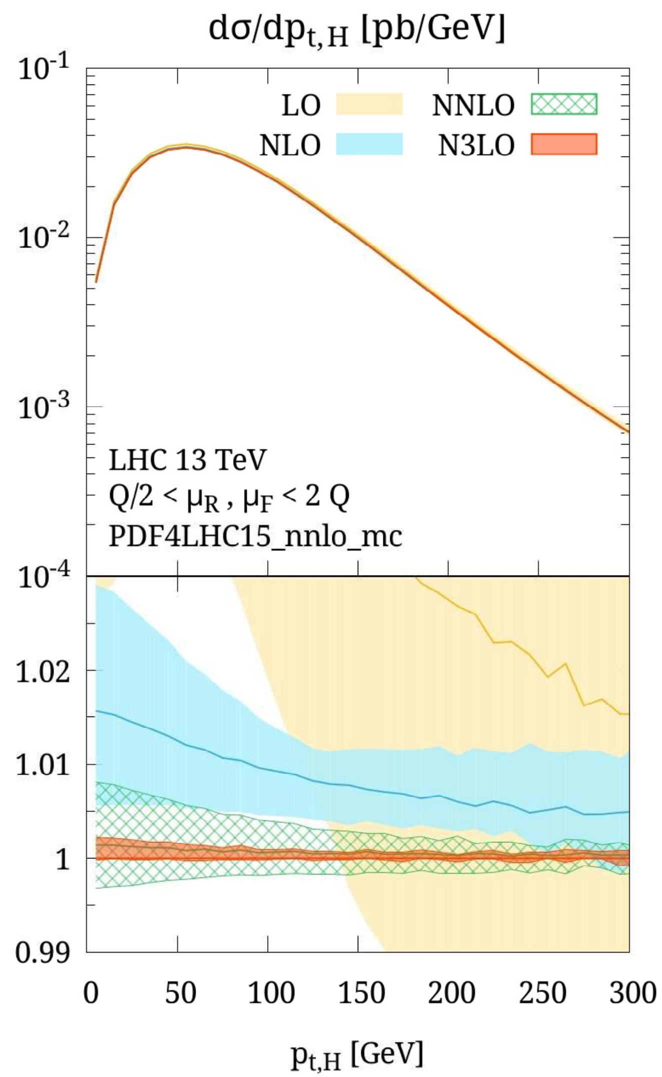
✓ DIS approximation - uncertainty permille level

✓ NNLO PDFs - uncertainty permille level

✓ scale uncertainty $\sim 1.4^o/_{oo}$

✓

	$\sigma^{(13\ \mathrm{TeV})}$ [pb]	$\sigma^{(14\ \mathrm{TeV})}$ [pb]
LO	$4.099\,^{+0.051}_{-0.067}$	$4.647\,^{+0.037}_{-0.058}$
NLO	$3.970\,^{+0.025}_{-0.023}$	$4.497\,^{+0.032}_{-0.027}$
NNLO	$3.932\,^{+0.015}_{-0.010}$	$4.452\,^{+0.018}_{-0.012}$
N^{3}LO	$3.928\,^{+0.005}_{-0.001}$	$4.448\,^{+0.006}_{-0.001}$



Dreyer, Karlberg (16)

Fully Differential NNLO

- ✓ $pp \to X$
  - ✚ MATRIX library using q_T subtraction
  - ✚ MCFM library using N-jettiness subtraction
- ✓ $pp \to X$+J
  - ✚ individual codes based on STRIPPER

    Boughezal, Caola, Melnikov, Petriello (15); Caola, Melnikov, Schulze (15)
  - ✚ NNLOJET library based on Antenna subtraction
  - ✚ MCFM-based with N-jettiness subtraction

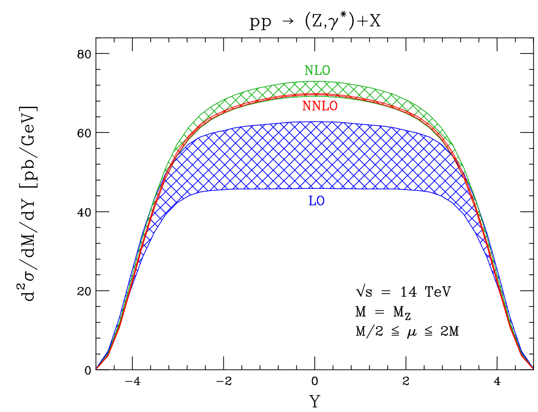
  where X is a colourless final state
- ✓ $pp \to t\bar{t}$, JJ, HJJ
  - ✚ individual codes based on STRIPPER

    Czakon, Heymes, Mitov (15,16); Czakon, Fielder, Heymes, Mitov (16)
  - ✚ NNLOJET library based on Antenna subtraction
  - ✚ individual codes based on Projection to Born

    Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

What to expect from NNLO (1)

✓ Reduced renormalisation scale dependence

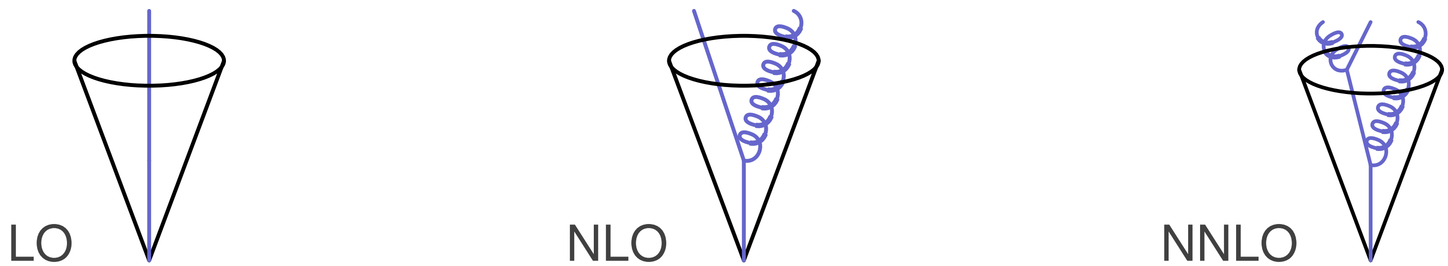


✓ Better able to judge convergence of perturbation series

✓ Fiducial (parton level) cross sections. Fully differential, so that experimental cuts can be applied directly

✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower

➠ better matching of jet algorithm between theory and experiment

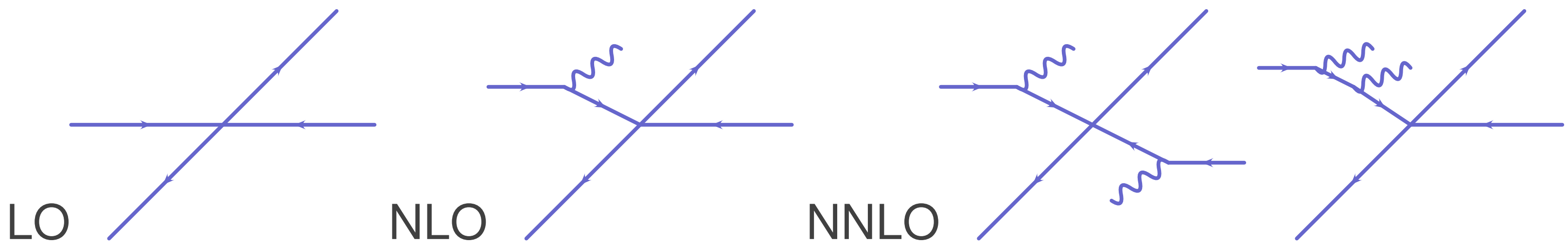


What to expect from NNLO (2)

✓ All channels present at NNLO

LO	NLO	NNLO
gg	gg, qg	gg, qg, qq
$q\bar{q}$	$q\bar{q}$, qg	$q\bar{q}$, qg, gg

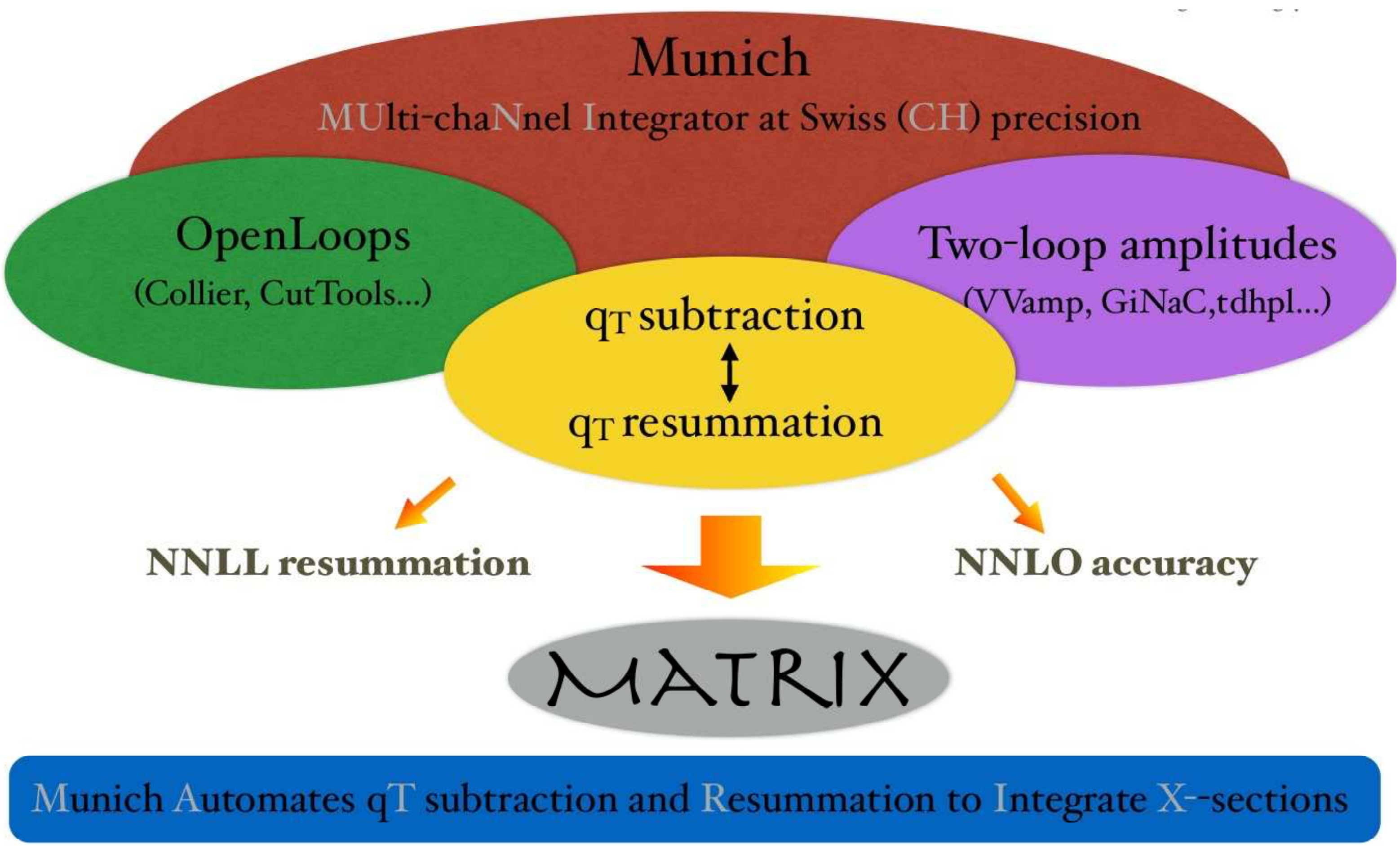
✓ Better description of transverse momentum of final state due to double radiation off initial state



- ✓ At LO, final state has no transverse momentum
- ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
- ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state

MATRIX - q_T subtraction

M. Grazzini, S. Kallweit, D. Rathlev, M. Wiesemann, . . .



MATRIX - q_T subtraction

$$d\sigma^X_{NNLO} = H^X_{NNLO} \otimes d\sigma^X_{LO} + \left[d\sigma^{X+J}_{NLO} - d\sigma^{CT}_{NLO} \right]$$

- ✓ the process dependent hard function H^X_{NNLO} is known for arbitrary colourless final state
- ✓ the counterterm $d\sigma^{CT}_{NLO}$ is universal
- ✓ $d\sigma^{X+J}_{NLO}$ is known

Implementing fully exclusive NNLO corrections including decays for $(2 \to 2)$

✓	$pp \to H, W, Z$	
✓	$pp \to \gamma\gamma$	
✓	$pp \to W\gamma, Z\gamma$	1505.01330,1601.06751
✓	$pp \to ZZ$	1507.06257
✓	$pp \to WW$	1601.06751
✓	$pp \to WZ$	1604.08576
✓	$pp \to HH$	1606.09519
✓	$\ldots$	

MATRIX - q_T subtraction

✚ Fiducial WW cross section

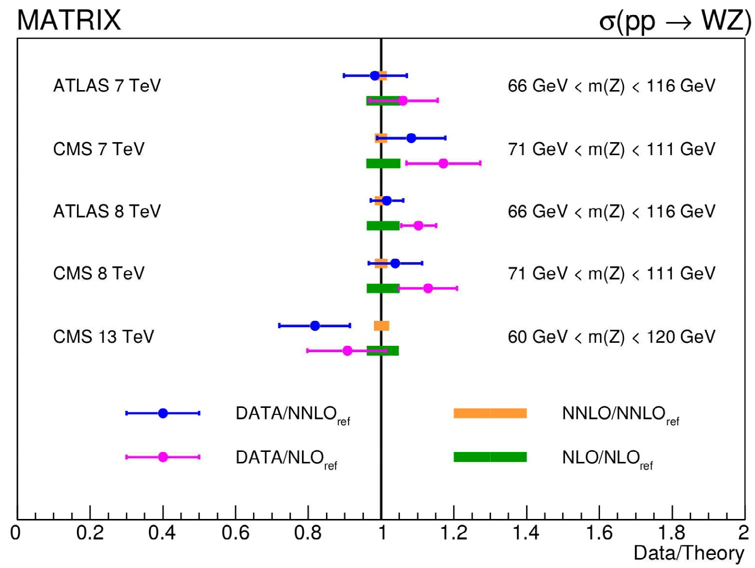
	$\sigma_{\mathrm{fiducial}}(W^+W^-\text{-cuts})$ [fb]		$\sigma/\sigma_{\mathrm{NLO}}-1$	
$\sqrt{s}$	8 TeV	13 TeV	8 TeV	13 TeV
LO	$147.23\ (2)^{+3.4\%}_{-4.4\%}$	$233.04(2)^{+6.6\%}_{-7.6\%}$	-3.8%	$-\ 1.3\%$
NLO	$153.07\ (2)^{+1.9\%}_{-1.6\%}$	$236.19(2)^{+2.8\%}_{-2.4\%}$	0	0
NLO′	$156.71\ (3)^{+1.8\%}_{-1.4\%}$	$243.82(4)^{+2.6\%}_{-2.2\%}$	$+2.4\%$	$+\ 3.2\%$
NLO′$+gg$	$166.41\ (3)^{+1.3\%}_{-1.3\%}$	$267.31(4)^{+1.5\%}_{-2.1\%}$	$+8.7\%$	$+13.2\%$
NNLO	$164.16(13)^{+1.3\%}_{-0.8\%}$	$261.5(2)^{+1.9\%}_{-1.2\%}$	$+7.2\%$	$+10.7\%$

✓ Impact of radiative corrections strongly reduced by the jet veto

✓ Consequently NLO+gg provides good approximation of the fiducial cross sections (but not of the acceptance)

Grazzini, Kallweit, Pozzorini, Rathlev, Wieseman (16)

✚ Inclusive WZ cross section



✓ NNLO corrections nicely improve the agreement with the data (with the exception of CMS at 13 TeV where, however, the uncertainties are still large)

Grazzini, Kallweit, Rathlev, Wieseman (16)

MCFM @NNLO

R. Boughezal, J.Campbell, K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello, C. Williams

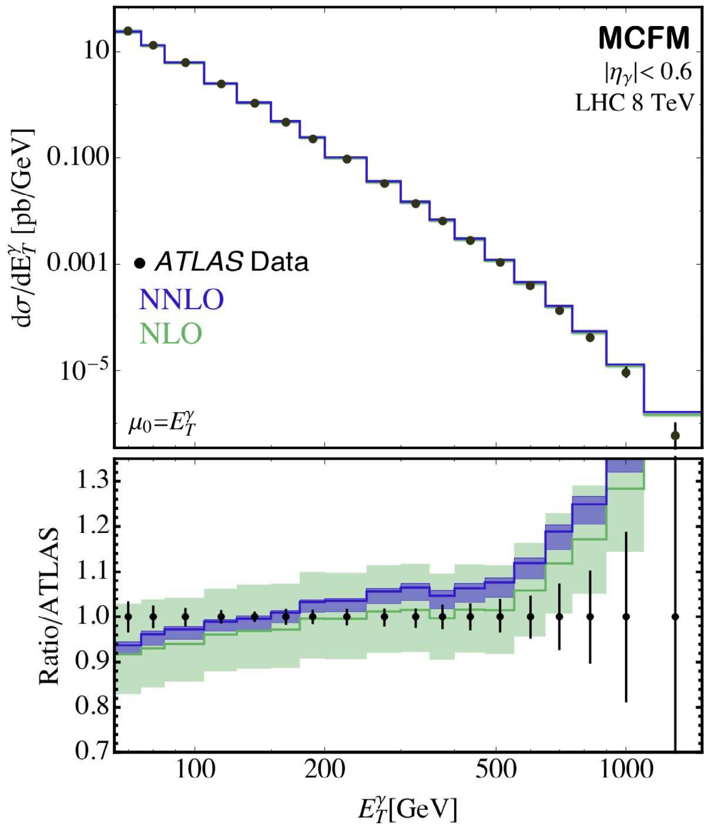
1605.08011

Implementing NNLO corrections using N-jettiness technique including decays for

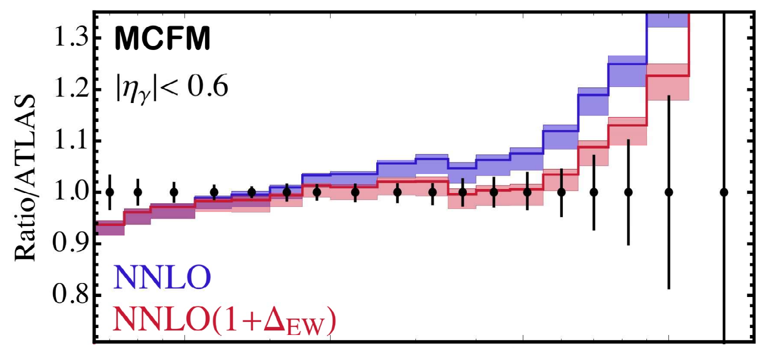
- ✓ $pp \to H, W, Z$
- ✓ $pp \to HW, HZ$ — 1601.00658
- ✓ $pp \to \gamma\gamma$ — 1603.02663
- ✓ $pp \to W + J$ — 1504.02131,1602.05612,1602.06965
- ✓ $pp \to H + J$ — 1505.03893
- ✓ $pp \to Z + J$ — 1512.01291,1602.08140
- ✓ $ep \to J + (J)$ — 1607.04921
- ✓ $pp \to \gamma + J$ — 1612.04333
- ✓ $\ldots$

γ+ J production

Campbell, Ellis, Williams (16)



- ✓ Frixione isolation
- ✓ Significantly reduced scale dependence
- ✓ Inclusion of EW effects improves agreement with data



NNLOJET

X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, NG, A. Huss, M. Jaquier, T. Morgan, J. Niehues, J. Pires

Implementing NNLO corrections using Antenna subtraction including decays for

- ✓ $pp \to H, W, Z$
- ✓ $pp \to H + J$ 1408.5325, 1607.08817
- ✓ $pp \to Z + J$ 1507.02850, 1605.04295, 1610.01843
- ✓ $pp \to JJ$ 1301.7310, 1310.3993, 1611.01460
- ✓ $ep \to JJ + (J)$ 1606.03991
- ✓ $\ldots$

H + J production, large mass limit

Boughezal, Caola, Melnikov, Petriello, Schulze (13,15)
Chen, Gehrmann, NG, Jaquier (14,16)
Boughezal, Focke, Giele, Liu, Petriello (15)
Caola, Melnikov, Schulze (15)

- ✓ phenomenologically interesting
- ✓ large scale uncertainty
- ✓ large K-factor

$$\sigma_{NLO}/\sigma_{LO} \sim 1.6$$
$$\sigma_{NNLO}/\sigma_{NLO} \sim 1.3$$

- ✓ significantly reduced scale dependence $\mathcal{O}(4\%)$

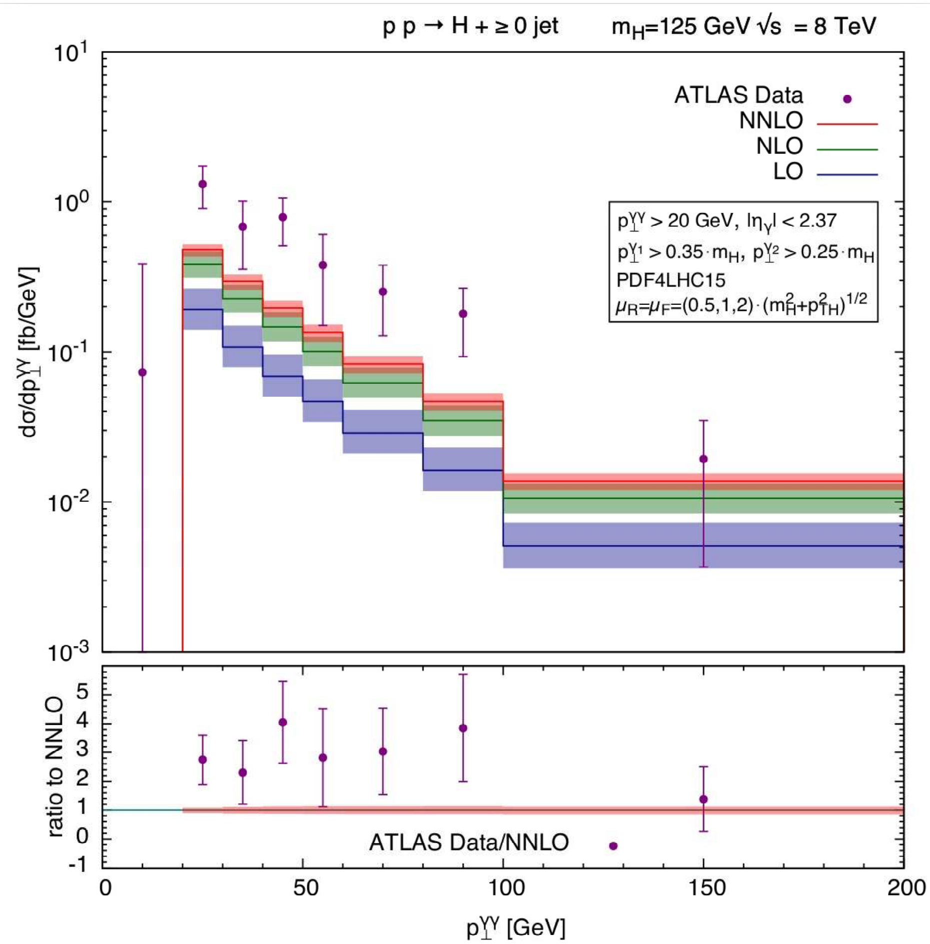
- ✓ Three independent computations:
  - ✚ STRIPPER
  - ✚ Antenna
  - ✚ N-jettiness

- ✓ allows for benchmarking of methods (for gg, qg and $\bar{q}g$ processes)
  - ✚ $\sigma^{NNLO} = 9.45^{+0.58}_{-0.82}$ fb

    Caola, Melnikov, Schulze (15)
  - ✚ $\sigma^{NNLO} = 9.44^{+0.59}_{-0.85}$ fb

    Chen, Gehrmann, NG, Jaquier (16)

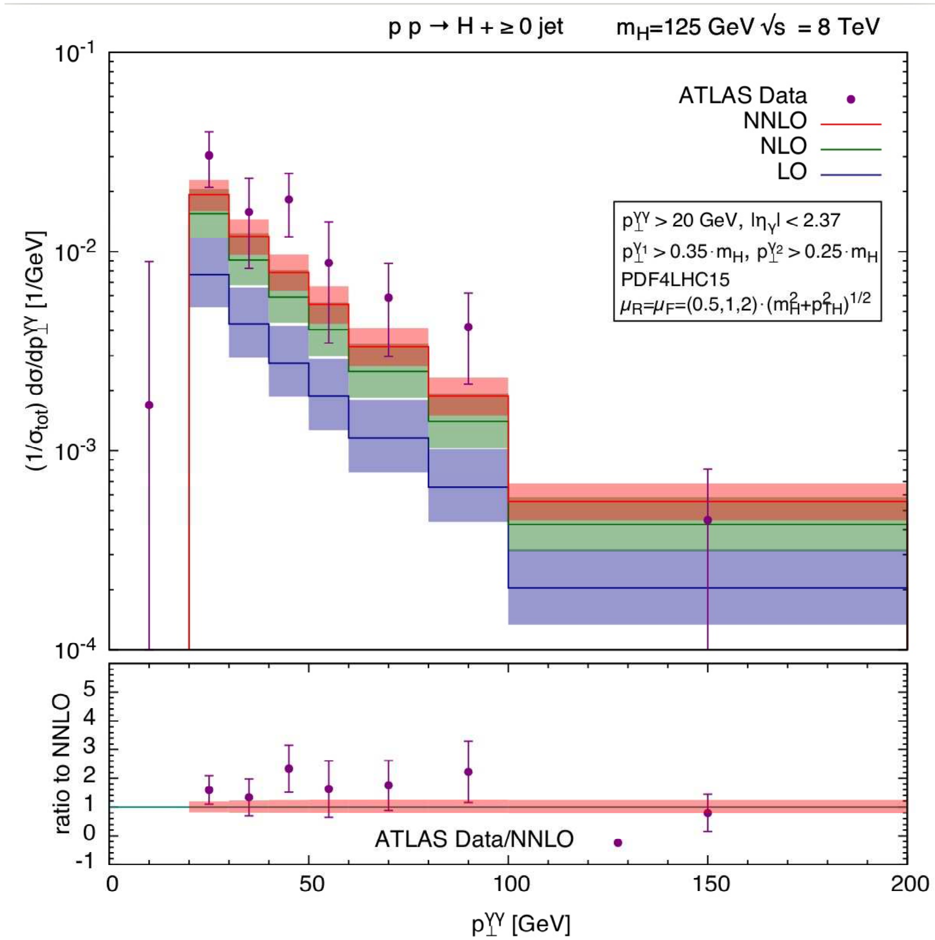
ATLAS H p_T distribution



ATLAS setup

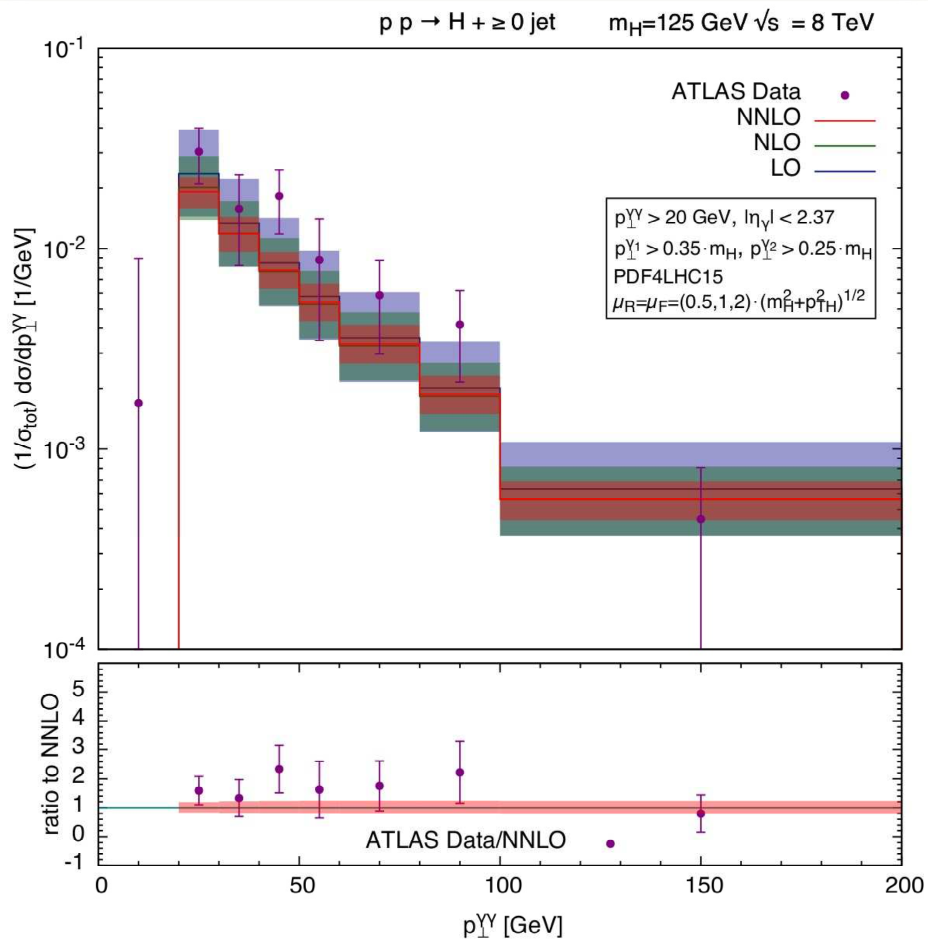
arXiv:1407.4222

ATLAS H p_T distribution



Normalised by σ_H^{NNLO}

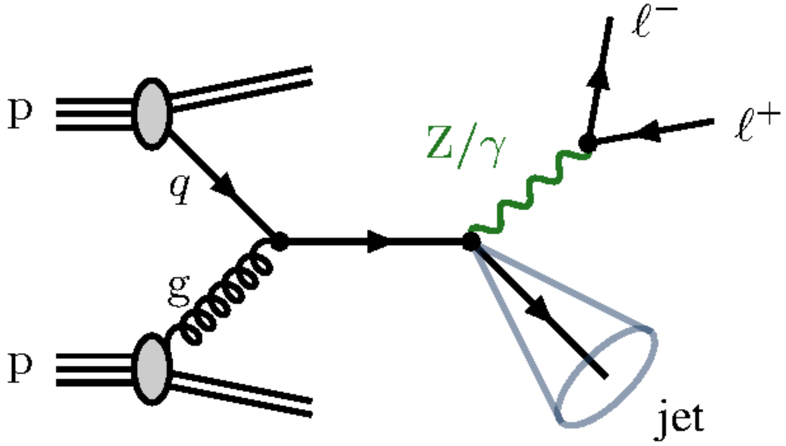
ATLAS H p_T distribution



Normalised by σ_H^{LO} at corresponding order - convergence

Z + J production

Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15,16)
Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)
Boughezal, Liu, Petriello (16)

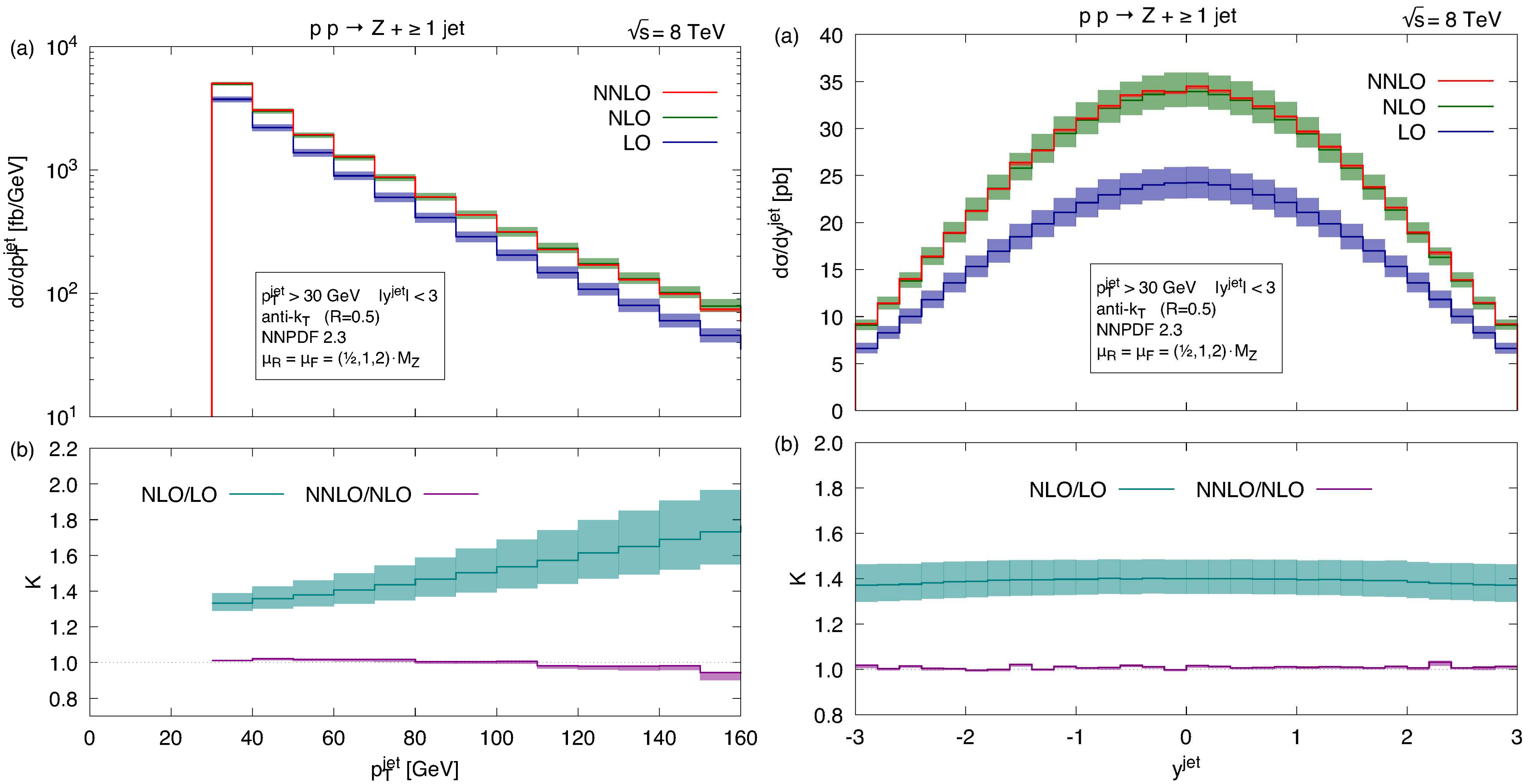


- ✓ clean leptonic signature
- ✓ good handle on jet energy scale
- ✓ significant NLO K-factor and scale uncertainty

$$\sigma_{NLO}/\sigma_{LO} \quad \sim \quad 1.4$$

- ✓ Two independent computations:
- ✓ allows for benchmarking of methods
  - ✚ $\sigma^{NNLO} = 135.6^{+0.0}_{-0.4}$ fb
    Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15)
  - ✚ $\sigma^{NNLO} = 135.6^{+0.0}_{-0.4}$ fb
    Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)

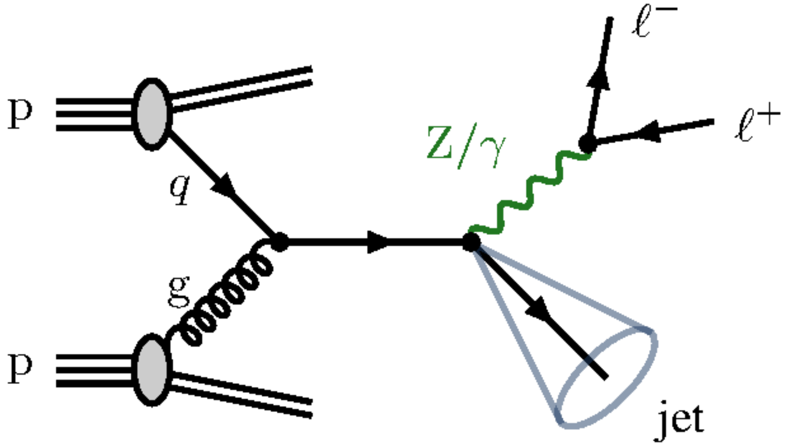
Jet p_T and rapidity



Leading jet p_T and rapidity distributions

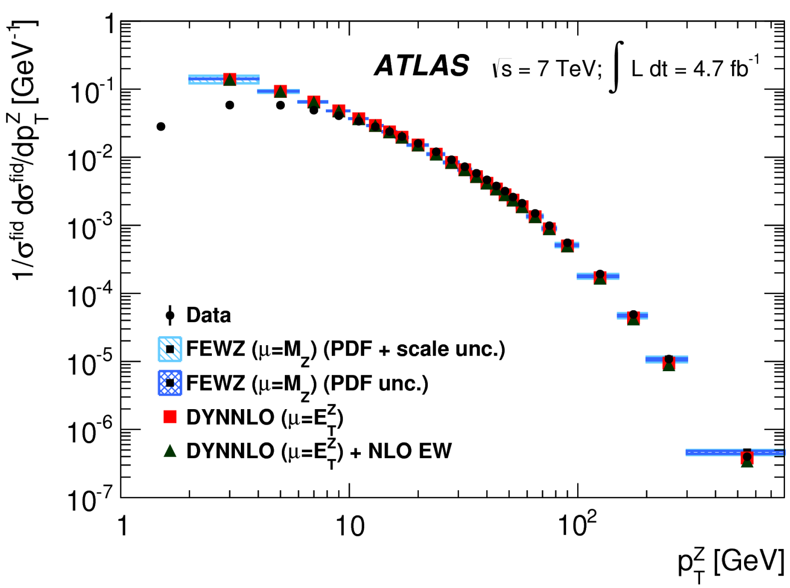
$\sqrt{s} = 8$ TeV, NNPDF2.3, $p_T^{jet} > 30$ GeV, $|y^{jet}| < 3$, anti-k_T, $R = 0.5$, $80\text{ GeV} < m_{\ell\ell} < 100\text{ GeV}$, $\mu_F = \mu_R = (0.5, 1, 2)m_Z$

Inclusive p_T spectrum of Z



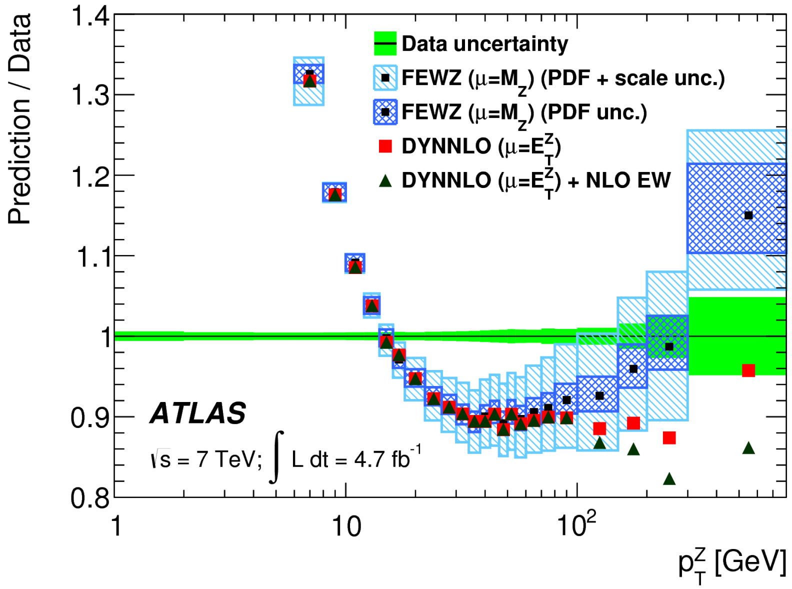
$$pp \to Z/\gamma^* \to \ell^+\ell^- + X$$

- large cross section
- clean leptonic signature



- fully inclusive wrt QCD radiation
- only reconstruct ℓ^+, ℓ^- so clean and precise measurement
- potential to constrain gluon PDFs

Inclusive p_T spectrum of Z

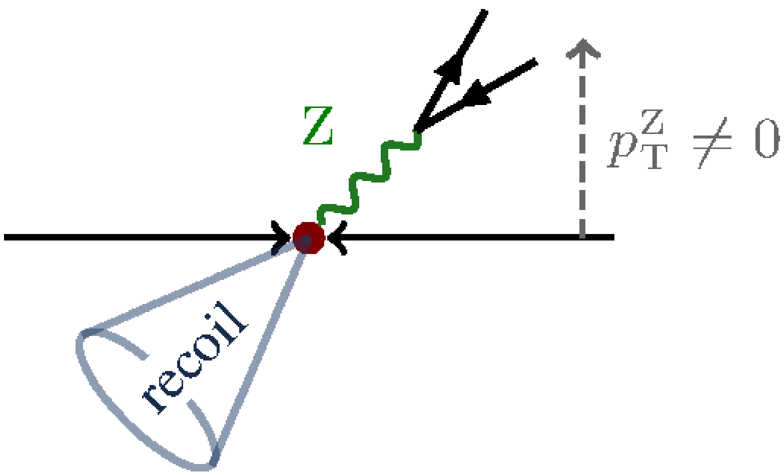


✚ low $p_T^Z \leq 10$ GeV, resummation required

✚ $p_T^Z \geq 20$ GeV, fixed order prediction about 10% below data

✘ *Very precise measurement of Z p_T poses problems to theory*,

D. Froidevaux, HiggsTools School



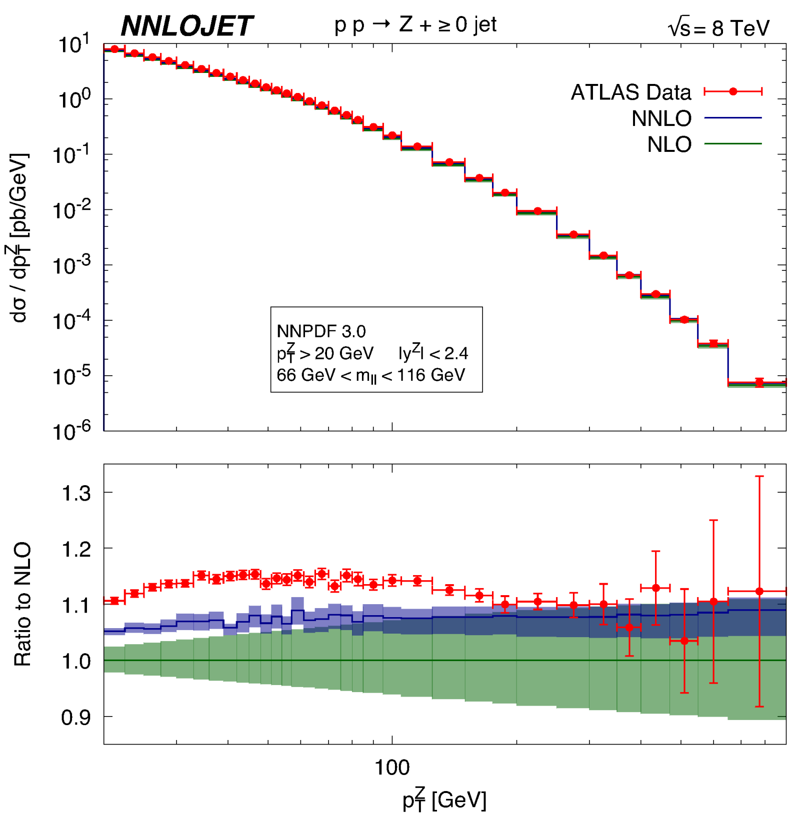
FEWZ/DYNNLO are Z + 0 jet @ NNLO

✘ Only NLO accurate in this distribution

✓ Requiring recoil means Z + 1 jet @ NNLO required

Inclusive p_T spectrum of Z

(1) $$\left.\frac{d\hat{\sigma}}{dp_T^Z}\right|_{p_T^Z>20\ \mathrm{GeV}} \equiv \frac{d\hat{\sigma}^{ZJ}_{LO}}{dp_T^Z} + \frac{d\hat{\sigma}^{ZJ}_{NLO}}{dp_T^Z} + \frac{d\hat{\sigma}^{ZJ}_{NNLO}}{dp_T^Z}$$

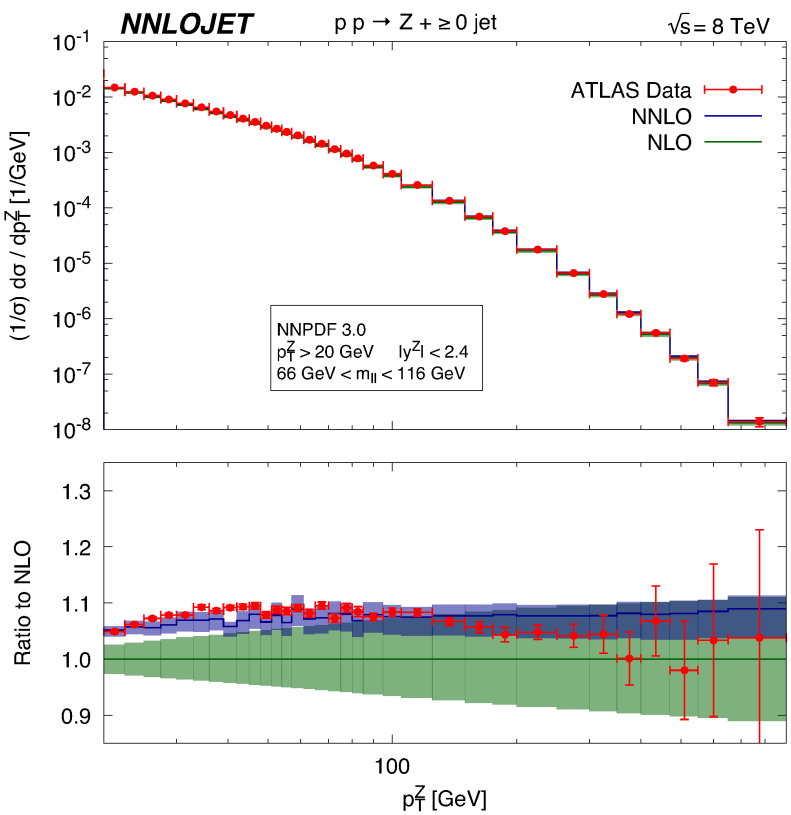


- ✓ NLO corrections $\sim 40-60\%$
- ✓ significant reduction of scale uncertainties NLO $\rightarrow$ NNLO
- ✓ NNLO corrections relatively flat $\sim 4-8\%$
- ✓ improved agreement, but not enough
- ✓ Note that for $66\ \mathrm{GeV} < m_{\ell\ell} < 116\ \mathrm{GeV}$

$$\sigma_{\mathrm{exp}} = 537.1 \pm 0.45\% \pm 2.8\%\ \mathrm{pb}$$

$$\sigma_{\mathrm{NNLO}} = 507.9^{+2.4}_{-0.7}\ \mathrm{pb}$$

Normalised Z p_T spectrum



$$\frac{1}{\sigma} \cdot \left. \frac{\mathrm{d}\hat{\sigma}}{dp_T^Z} \right|_{p_T^Z > 20 \text{ GeV}}$$

with

$$\sigma = \int_0^\infty \frac{\mathrm{d}\hat{\sigma}}{dp_T^Z} dp_T^Z \equiv \sigma_{LO}^Z + \sigma_{NLO}^Z + \sigma_{NNLO}^Z.$$

- ✓ Much improved agreement
- ✓ luminosity uncertainty cancels
- ✓ dependence on EW parameters reduced
- ✓ dependence on PDFs reduced ➡ study

Normalised ϕ^*_η spectrum

$$\phi^*_\eta \equiv \tan\left(\frac{\phi_{acop}}{2}\right)\cdot\sin(\theta^*_\eta)$$

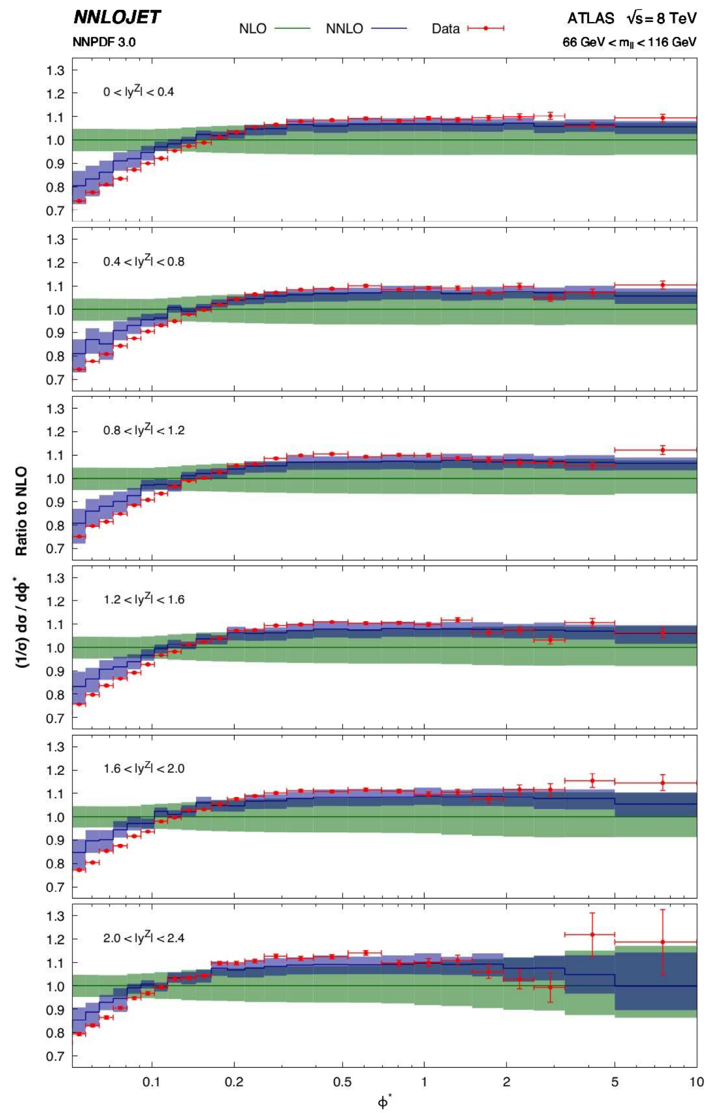
$$\phi_{acop} = 2\arctan\left(\sqrt{\frac{1+\cos\Delta\phi}{1-\cos\Delta\phi}}\right)$$

$$\cos(\theta^*_\eta) = \tanh\left(\frac{\eta^{\ell^-}-\eta^{\ell^+}}{2}\right)$$

✓ In the small ϕ^*_η region,

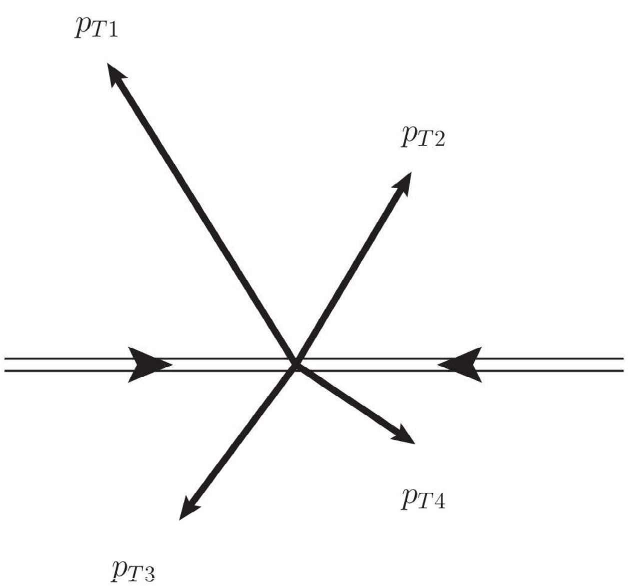
$$\phi^*_\eta \sim \frac{2p_T^\ell}{m_{\ell\ell}}$$

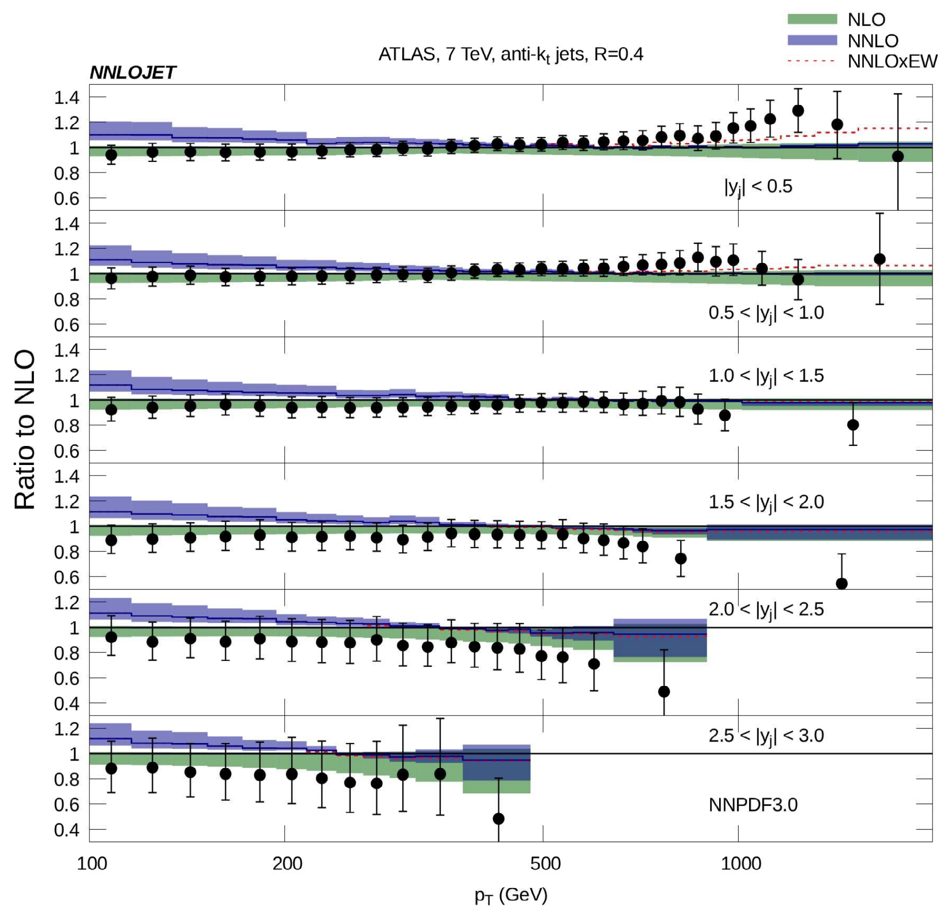
✓ NNLO is significant improvement over NLO



Single Jet Inclusive Distribution

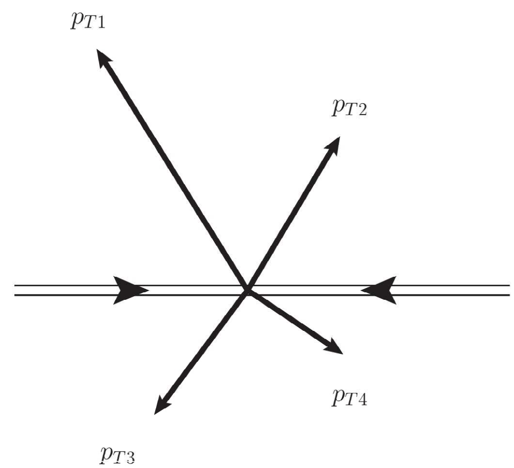
Currie, NG, Pires (16)



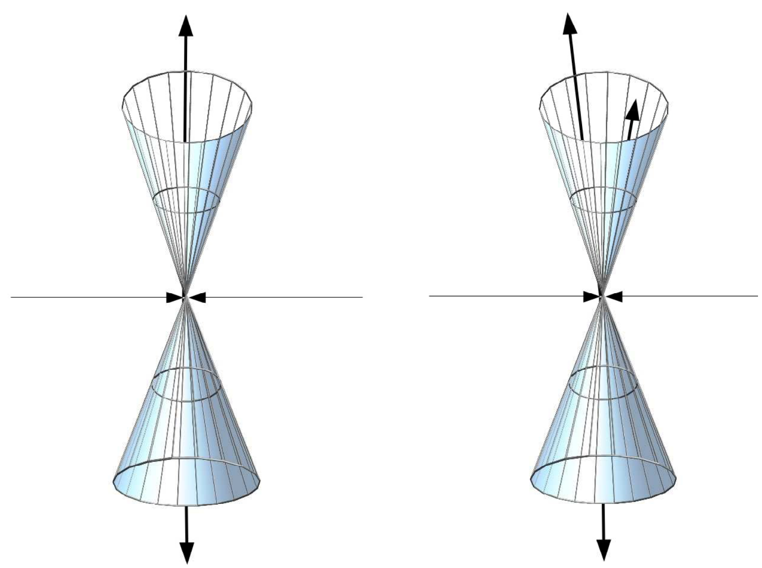


- ✓ Classic jet observable
- ✓ Every jet in the event enters in the distribution
- ✓ Expect sensitivity to PDFs

Scale Choice



- ✓ no fixed hard scale for jet production
- ✓ two widely used scale choices
  - ➠ leading jet p_T (p_{T1})
  - ➠ individual jet p_T (p_T)

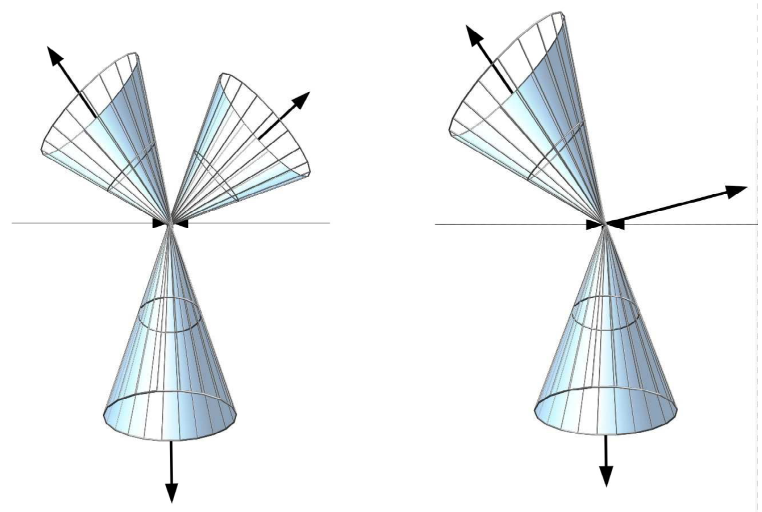


- ✓ different scale changes PDF and α_s
- ✓ no difference for back-to-back jet configurations (only arises at higher orders)

Scale Choice

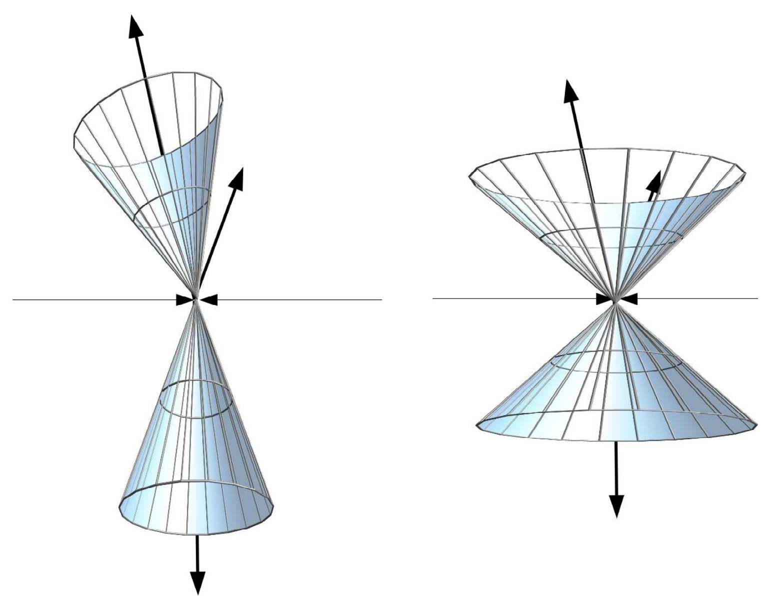
At NLO, $p_T \neq p_{T1}$ for

- ✓ 3-jet rate (small effect)
- ✓ 2-jet rate (3rd parton falls outside jet)

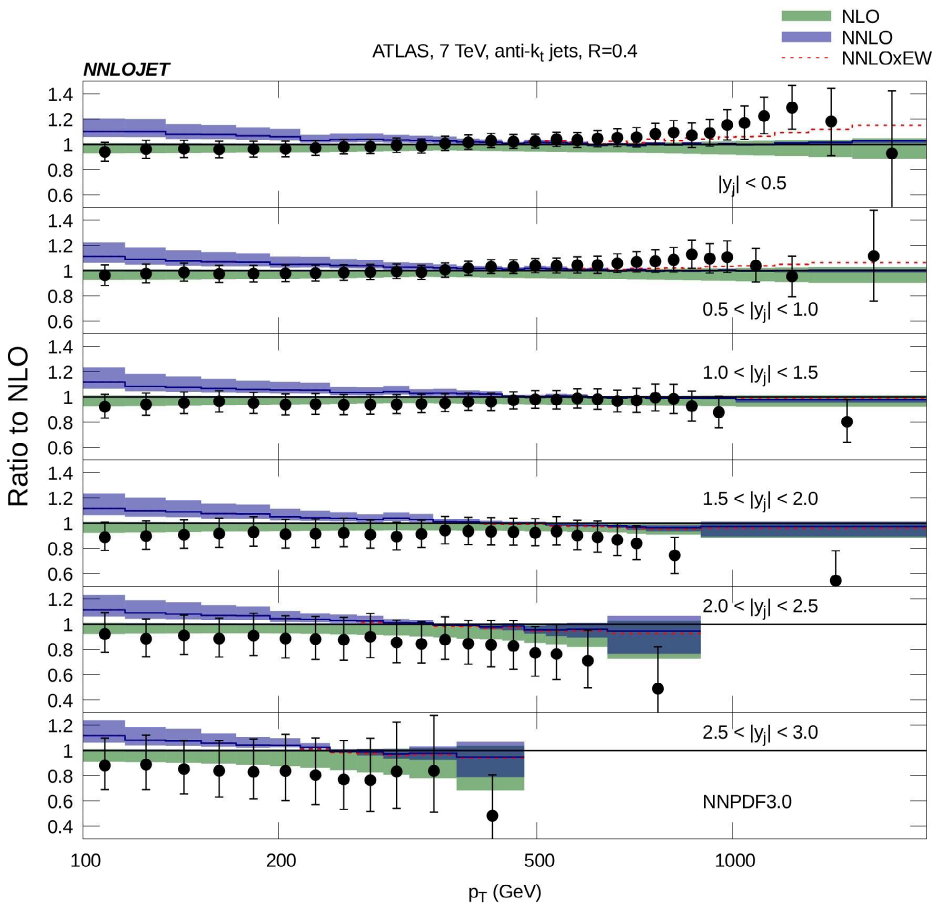


Changing R has an effect on the cross section, but also on the scale choice:

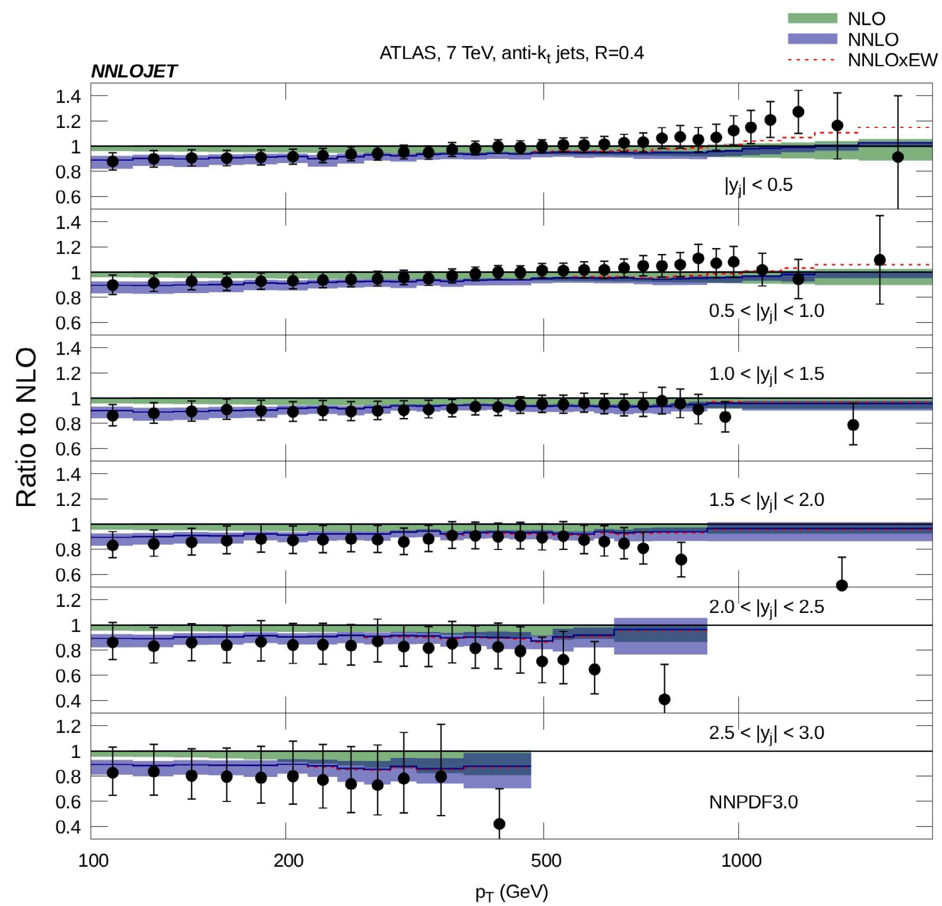
- ✓ introduces spurious R-dependence in scale choice
- ✓ p_{T1} scale has no R-dependence at NLO, unlike p_T
- ✓ at NNLO even p_{T1} scale choice has R- dependence in some four-parton configurations



Single Jet Inclusive Distribution



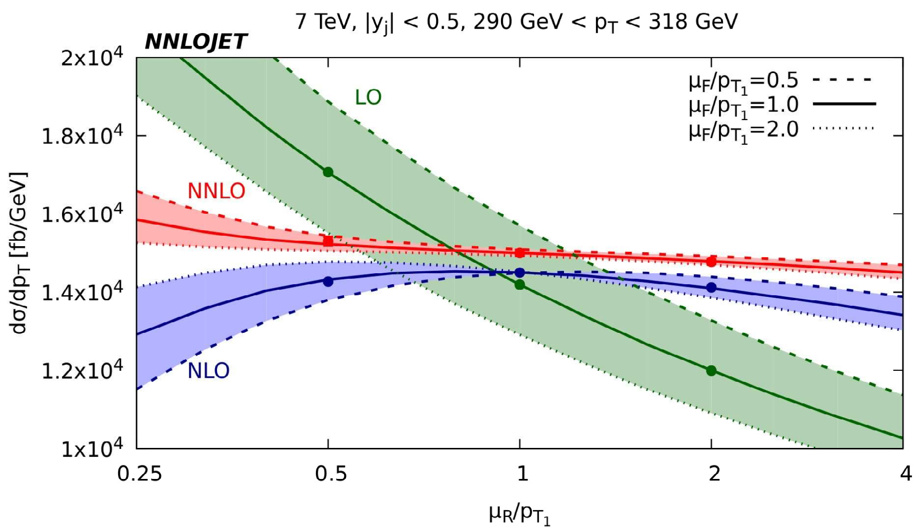
$$\mu_R = \mu_F = p_{T1}$$



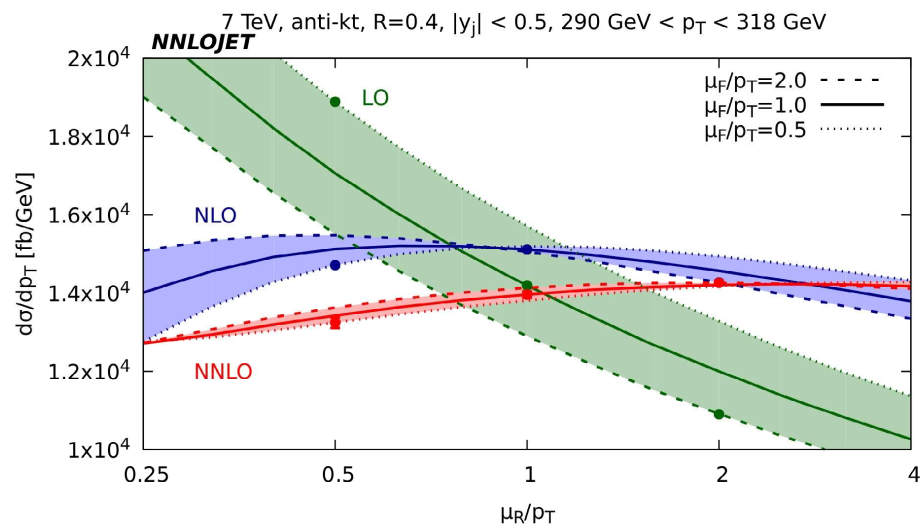
$$\mu_R = \mu_F = p_T$$

- ✘ Quite different behaviour!

Single Jet Inclusive Distribution



$$\mu_R = \mu_F = p_{T1}$$



$$\mu_R = \mu_F = p_T$$

✘ Quite different behaviour!

➠ scale uncertainty much smaller than difference between scale choices ➠ explore alternative scale choices

Maximising the impact of NNLO calculations

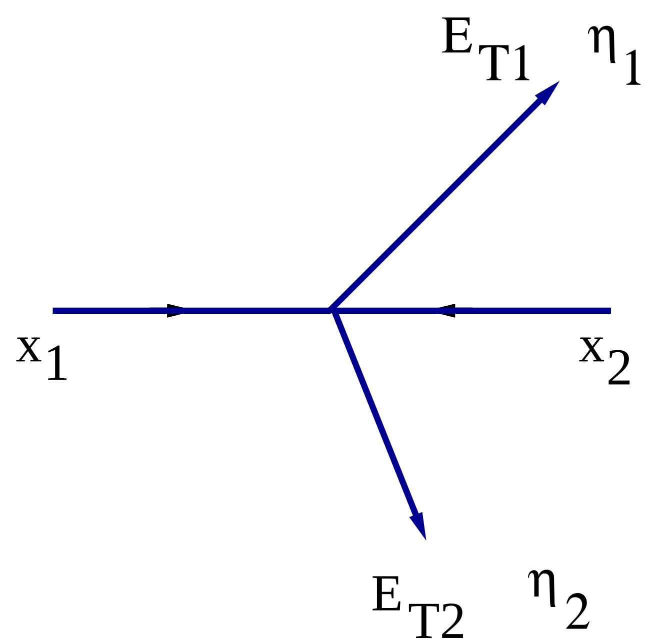
Triple differential form for a $2 \to 2$ cross section

$$\frac{d^3\sigma}{dE_T d\eta_1 d\eta_2} = \frac{1}{8\pi} \sum_{ij} x_1 f_i(x_1, \mu_F)\, x_2 f_j(x_2, \mu_F)\, \frac{\alpha_s^2(\mu_R)}{E_T^3} \frac{|\mathcal{M}_{ij}(\eta^*)|^2}{\cosh^4 \eta^*}$$

✓ Direct link between observables E_T, η_1, η_2 and momentum fractions/parton luminosities

$$x_1 = \frac{E_T}{\sqrt{s}} \left(\exp(\eta_1) + \exp(\eta_2)\right),$$

$$x_2 = \frac{E_T}{\sqrt{s}} \left(\exp(-\eta_1) + \exp(-\eta_2)\right)$$

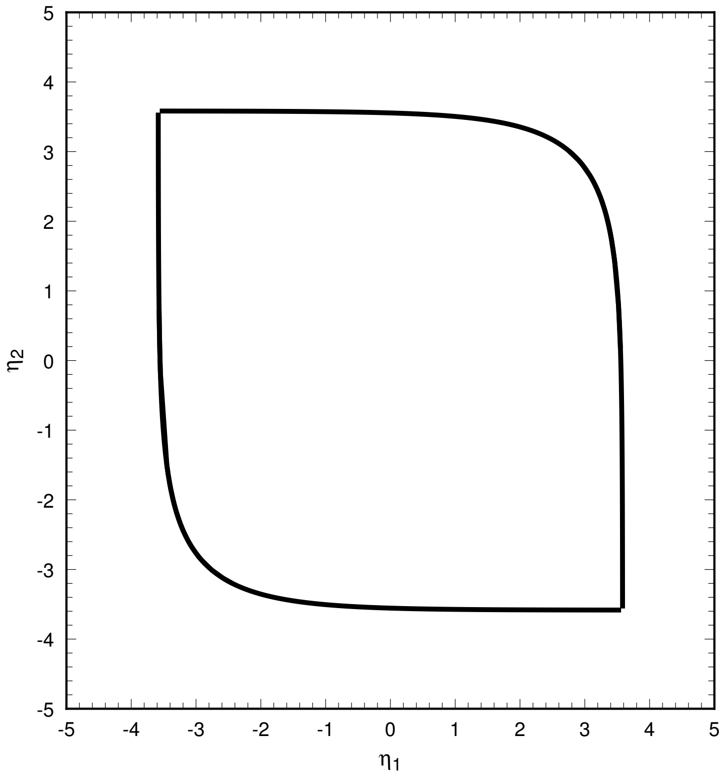


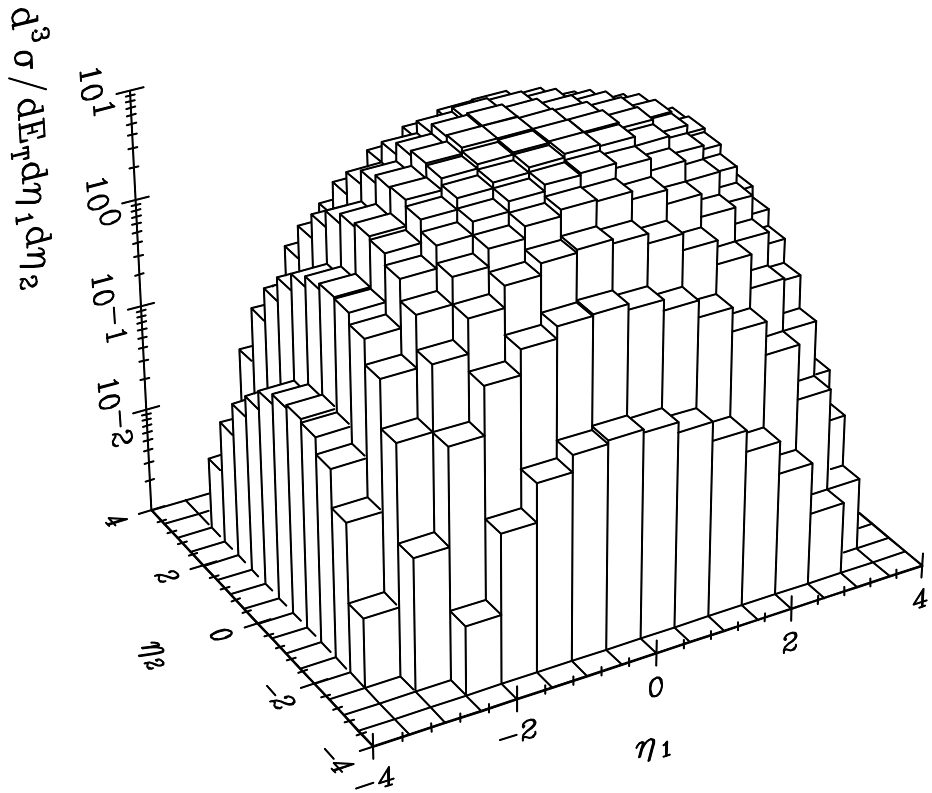
✓ and matrix elements that only depend on

$$\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$$

Triple differential distribution

✓ Range of x_1 and x_2 fixed allowed LO phase space for jets
$E_T \sim 200$ GeV at $\sqrt{s} = 7$ TeV





✓ Shape of distribution can be understood by looking at parton luminosities and matrix elements (in for example the single effective subprocess approximation)

Giele, NG, Kosower, hep-ph/9412338

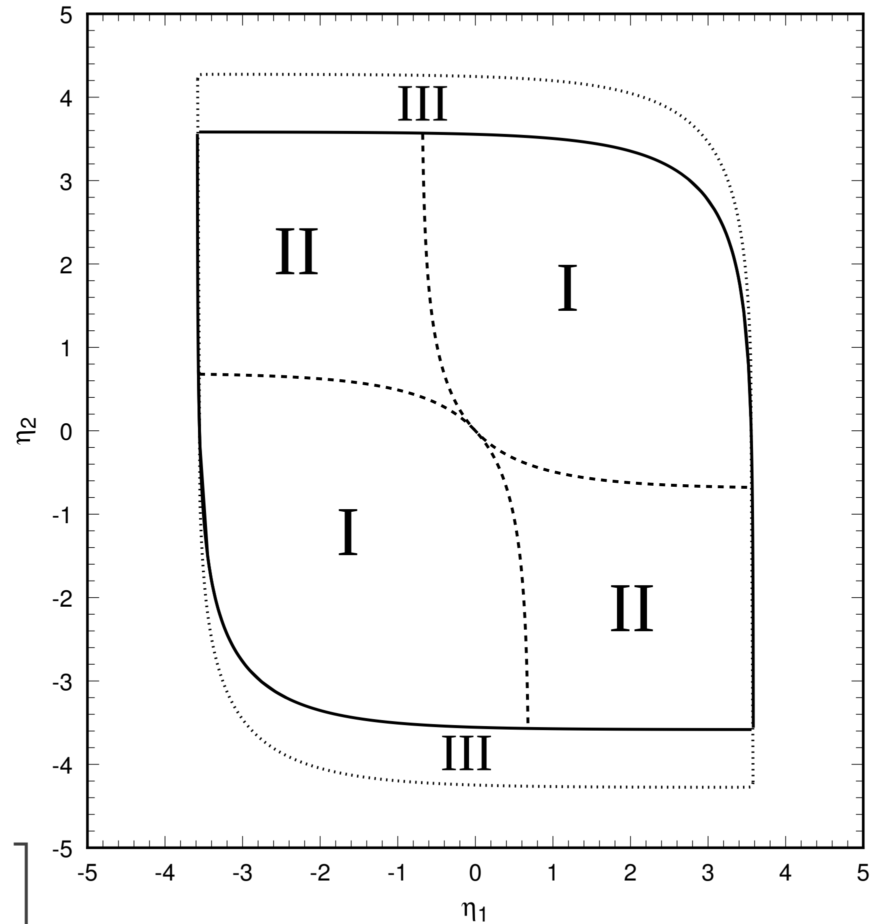
Phase space considerations

✓ Phase space boundary fixed when one or more parton fractions $\rightarrow 1$.

I $\eta_1 > 0$ and $\eta_2 > 0$ OR $\eta_1 < 0$ and $\eta_2 < 0$
➠ one x_1 or x_2 is less than x_T
- small x

II $\eta_1 > 0$ and $\eta_2 < 0$ OR $\eta_1 < 0$ and $\eta_2 > 0$
➠ both x_1 and x_2 are bigger than x_T
- large x

III growth of phase space at NLO (if $E_{T1} > E_{T2}$)



$$\left[\, x_T^2 < x_1 x_2 < 1 \qquad \text{and} \qquad x_T = 2E_T/\sqrt{s} \,\right]$$

Measuring PDF's at the LHC?

Should be goal of LHC to be as self sufficient as possible!

Study triple differential distribution for as many $2 \to 2$ processes as possible!

- ✓ Medium and large x gluon and quarks
  - ✓ $pp \to$ di-jets — dominated by gg scattering
  - ✓ $pp \to \gamma$ + jet — dominated by qg scattering
  - ✓ $pp \to \gamma\gamma$ — dominated by $q\bar{q}$ scattering
- ✓ Light flavours and flavour separation at medium and small x
  - ✓ Low mass Drell-Yan
  - ✓ W lepton asymmetry
  - ✓ $pp \to Z$+jet
- ✓ Strangeness and heavy flavours
  - ✓ $pp \to W^{\pm} + c$ — probes $s,\ \bar{s}$ distributions
  - ✓ $pp \to Z + c$ — probes c distribution
  - ✓ $pp \to Z + b$ — probes b distribution

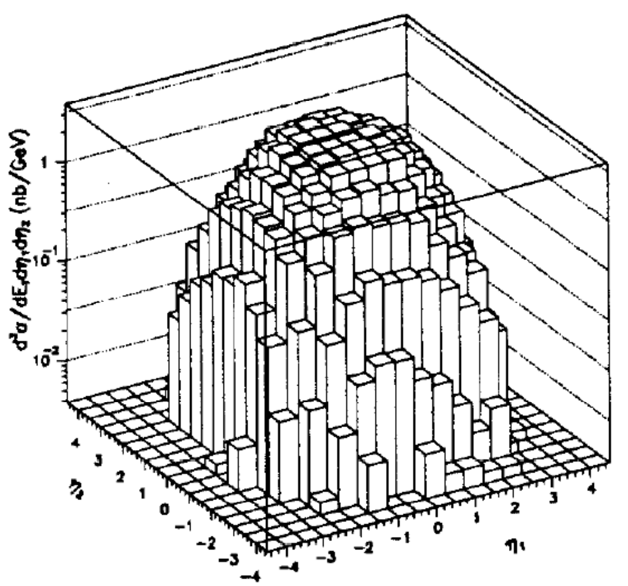
Measurements of strong coupling

- ✓ With incredible jet energy resolution, the LHC can do better!!
- ✓ by simultaneously fitting the parton density functions and strong coupling
- ✓ If the systematic errors can be understood, the way to do this is via the triple differential cross section

Giele, NG, Yu, hep-ph/9506442

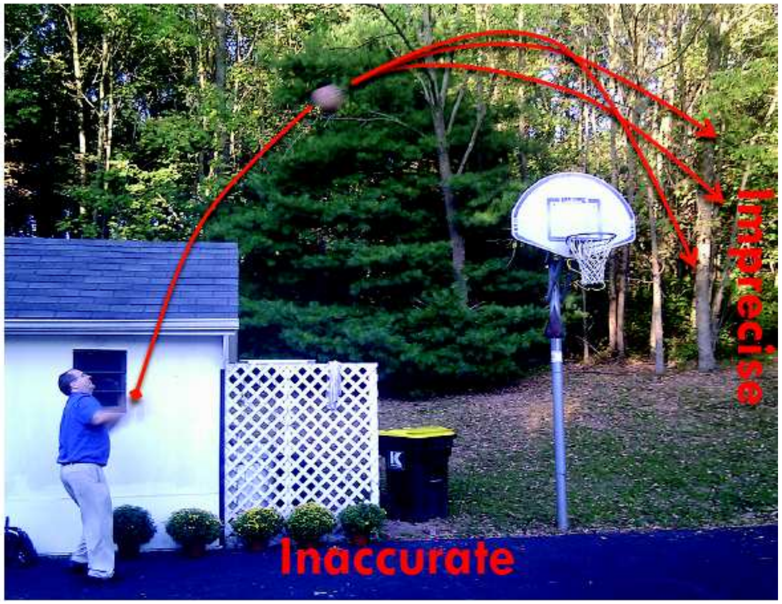
- ✓ and add NNLO $W^{\pm}$+jet, Z+jet, γ+jet calculations (with flavour tagging) as they become available

D0 preliminary, 1994



Accuracy and Precision (A. David)

Accuracy and Precision (A. David)



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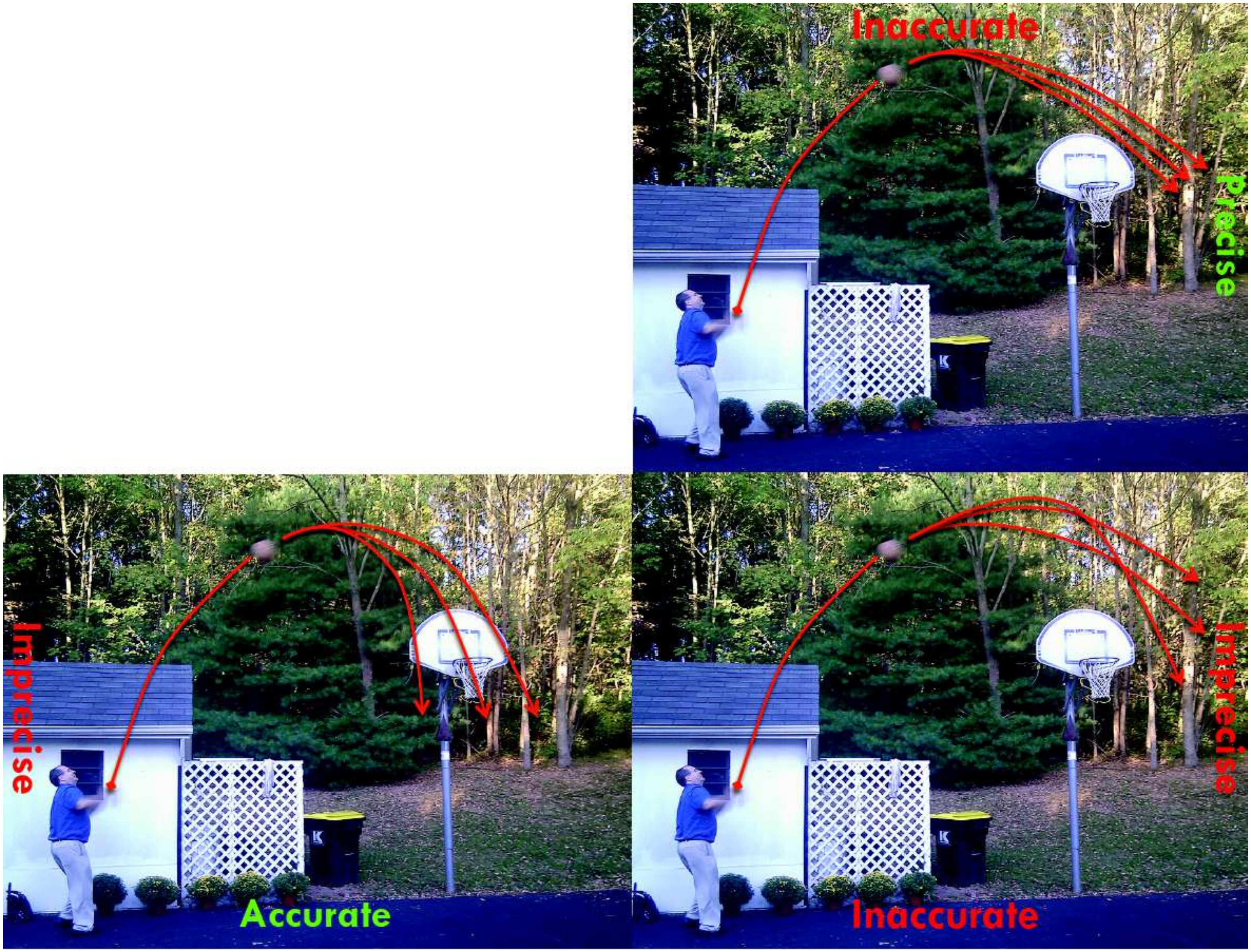


Accuracy and Precision (A. David)

Estimating uncertainties of MHO

✓ Consider a generic observable $\mathcal{O}$ (e.g. σ_H)

$$\mathcal{O}(Q) \quad \sim \quad \mathcal{O}_k(Q,\mu) + \Delta_k(Q,\mu)$$

where

$$\mathcal{O}_k(Q,\mu) \equiv \sum_{n=0}^{k} c_n(Q,\mu)\alpha_s(\mu)^n, \qquad \Delta_k(Q,\mu) \equiv \sum_{n=k+1}^{\cdots} c_n(Q,\mu)\alpha_s(\mu)^n$$

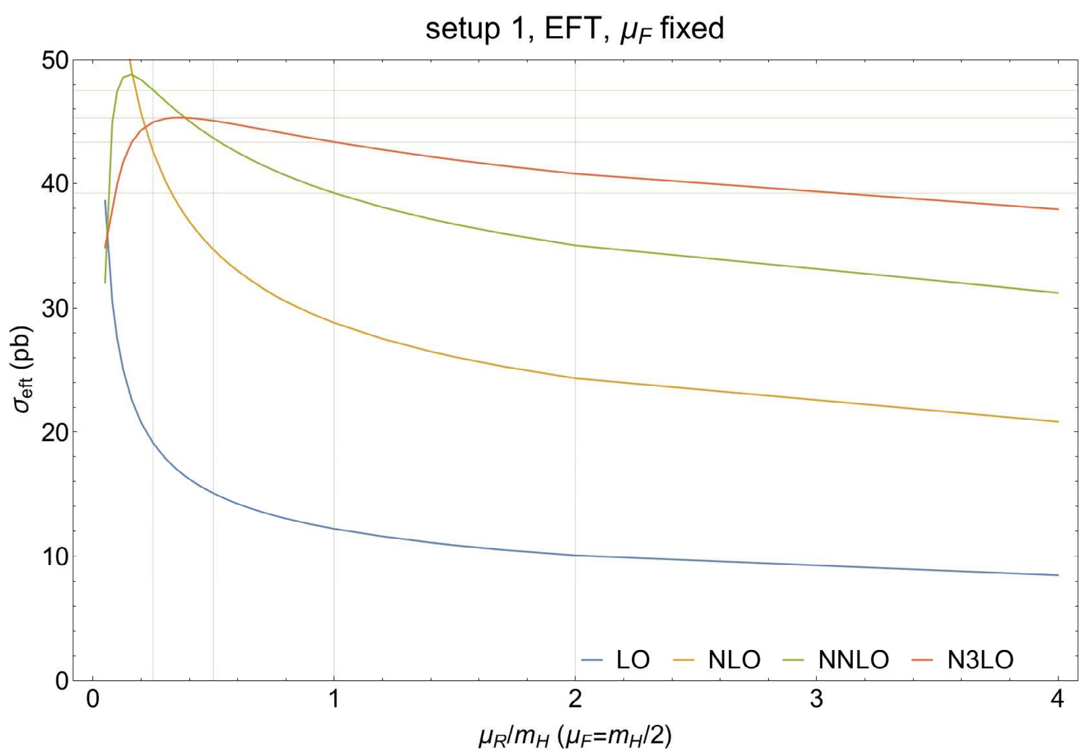
✓ Usual procedure is to use scale variations to estimate Δ_k,

$$\Delta_k(Q,\mu) \quad \sim \quad \max\left[\mathcal{O}_k\left(Q,\frac{\mu}{r}\right), \mathcal{O}_k(Q,r\mu)\right] \sim \alpha_s(\mu)^{k+1}$$

where μ is chosen to be a typical scale of the problem and typically $r=2$.

Choice of μ and $r=2$ is convention

Convergence



Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger (16)

- ✓ Convergence (or not) depends on choice of μ and r
- ✓ and whether inputs (PDF, α_s) are matched to order
- ✓ reduced scale dependence
  - ➠ more precise . . . but is it more accurate?
- ✚ need better way of estimating effect of MHO

Summary - Where are we now?

- ✓ First high precision N3LO calculations available

  could help reduce Missing Higher Order uncertainty by a factor of two

- ✓ Substantial and rapid progress in NNLO
  - ✚ many new calculations available
  - ➠ improved descriptions of experimental data
  - — codes typically require significant CPU resource
  - ✓ NNLO is emerging as standard for benchmark processes such as V+jet production and could lead to improved pdfs etc.

    could help reduce theory uncertainty due to inputs by a factor of two

- ✓ NNLO automation?
  - as we gain analytical and numerical experience with NNLO calculations, can we further exploit the developments at NLO
  - automation of two-loop contributions?
  - automation of infrared subtraction terms?

- ✓ Is there a better way of estimating the theoretical uncertainties?