Toward Verifying Leptonic Unitarity

Hisakazu Minakata IFT
UAM/CSIC, Madrid

Chee Sheng Fong, HM, Hiroshi Nunokawa,
arXiv:1609.08623, 1712.02798

We see things converge: $\delta \sim 3\pi/2$ is the best case

- T2K see more and more preference of $\delta \sim 3\pi/2$ (or $-\pi/2$), and NOvA agrees

See slide

- T2K (everybody) – NOvA discrepancy about θ_{23} seems resolved (best fit near maximal)

See slide

- T2K II (proposed, run until 2026) they could see CPV at ~3 σ (expected for ~15×10^{21} POT)

14.7×10^{20} POT (nu) **7.6×10^{20}** POT (antinu) → **20×10^{21}** POT (2026)

- $\delta \sim 3\pi/2$ is the best case for NOvA for mass ordering: (see bi-P plot)

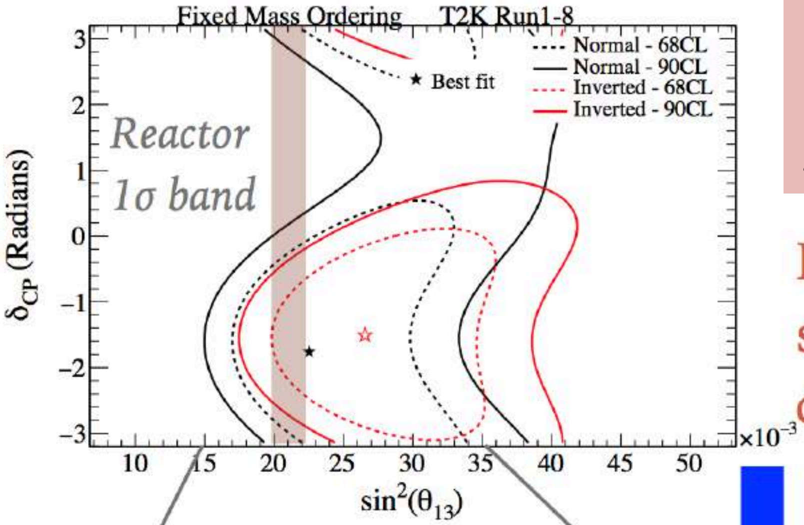
See slide

- NOvA could see mass hierarchy at ~3 σ ?
- Implication to the next generation projects ?

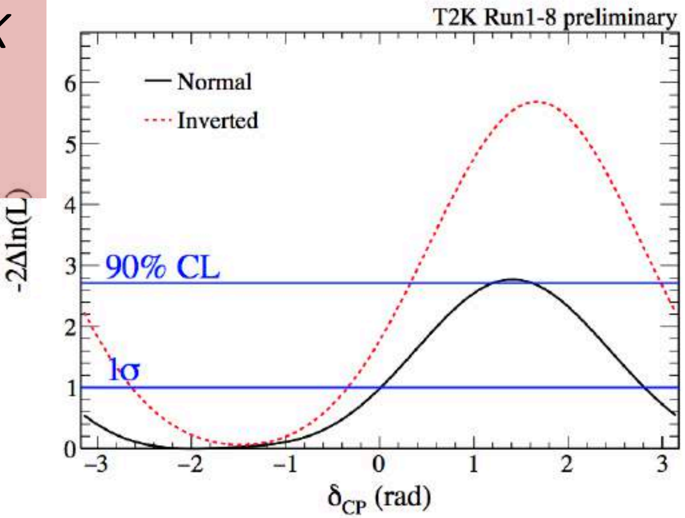
OSCILLATION PARAMETER SENSITIVITIES (2017)

Without the reactor experiment constraint on $\sin^2 2\theta_{13}$

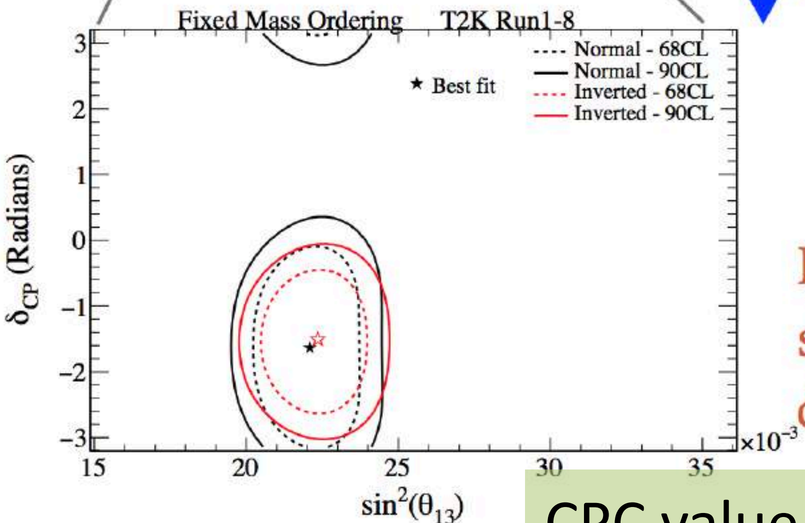
Mark Hartz, KEK Colloquium Aug 2017



Integrate out $\sin^2\theta_{13}$ dependence



Reactor constraint on $\sin^2(2\theta)_{13}$ (PDG2016)



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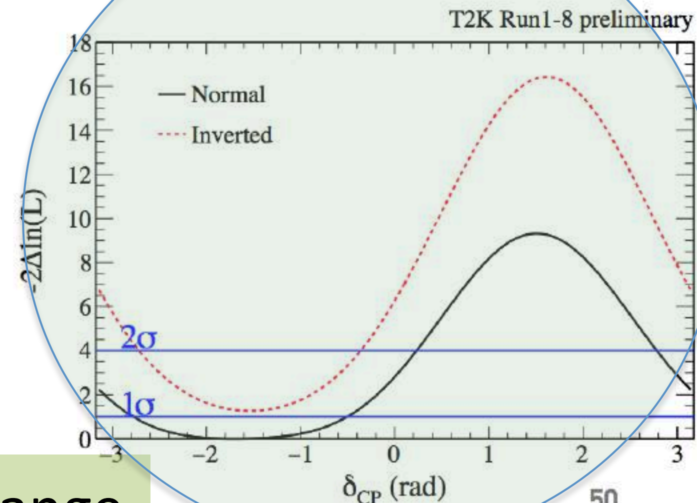
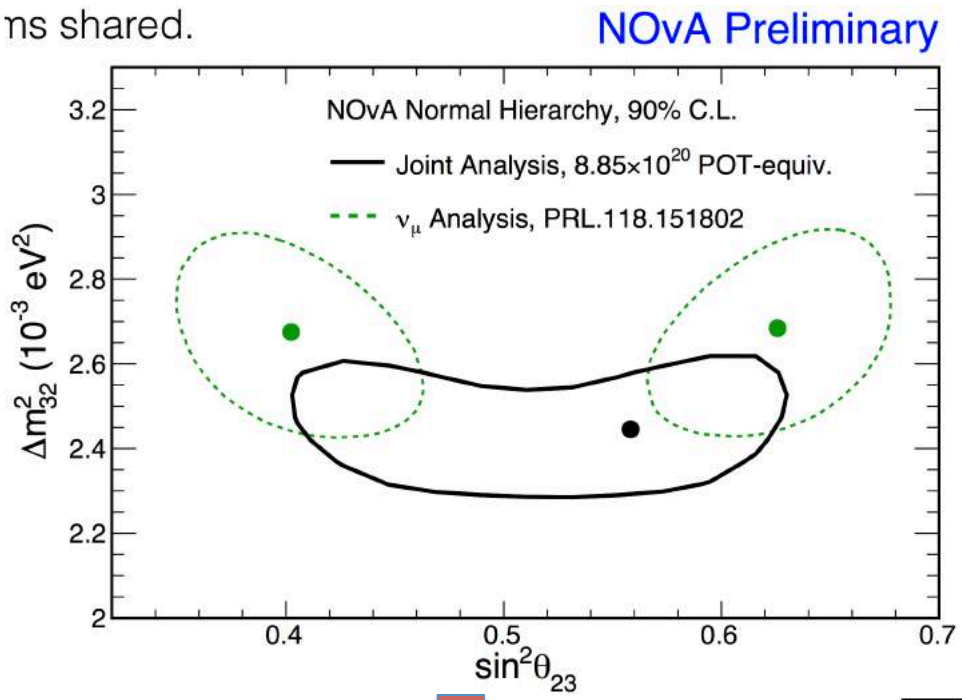


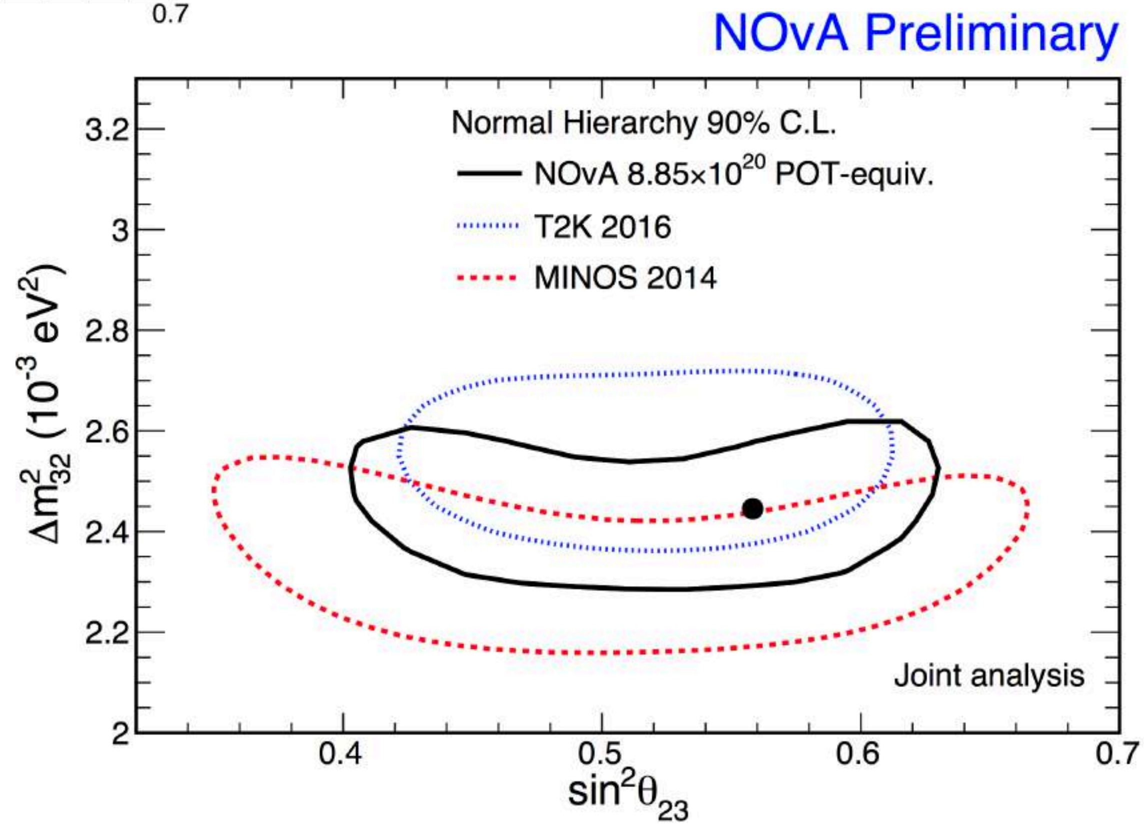
CPC value outside 2σ range

ns shared.

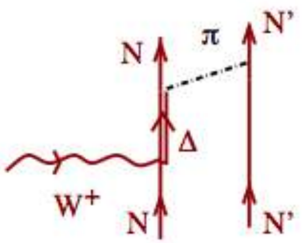


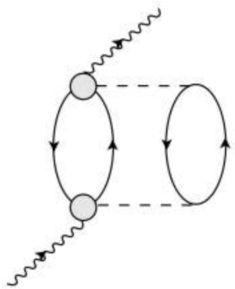
Agree on near maximal θ_{23}

Alexander Radovic
Wine&Cheese@ Fermilab Jan 2018



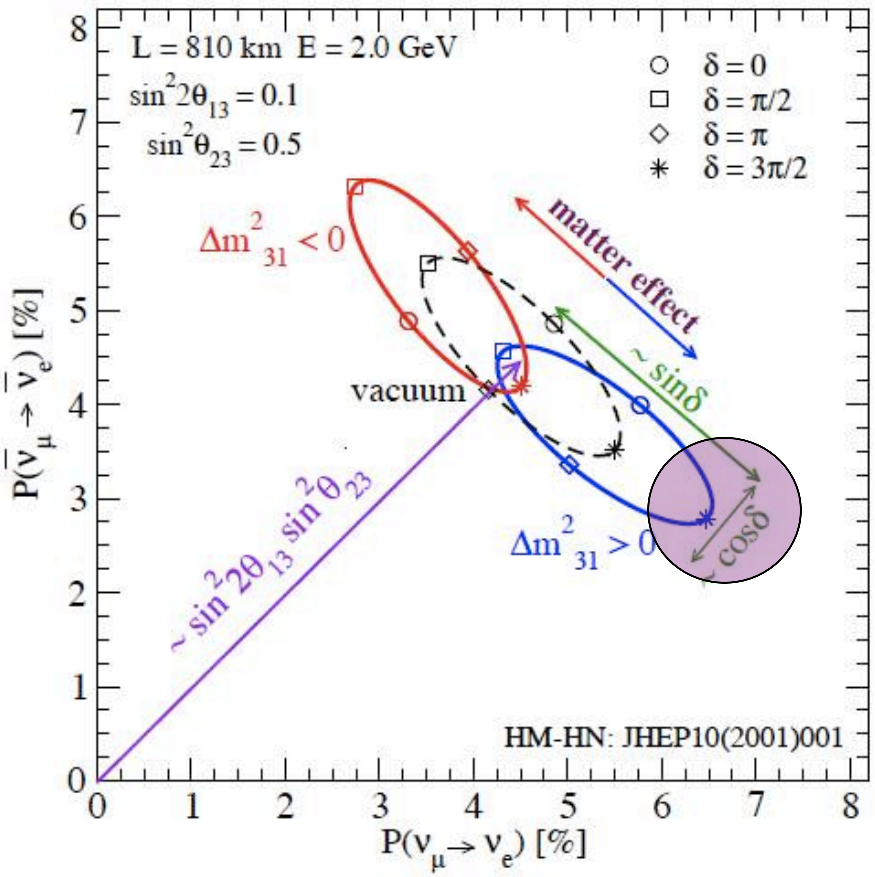
Improving on nuclear physics models, etc

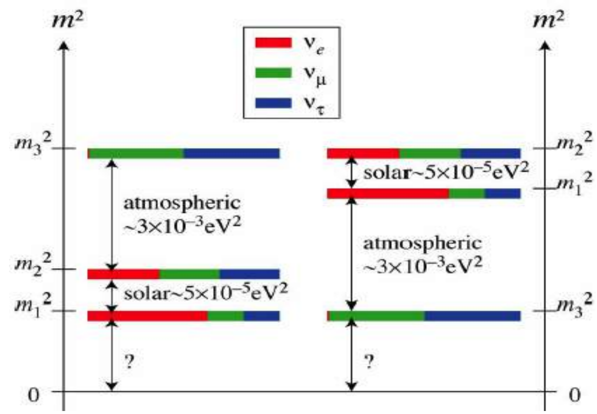




$\delta=3\pi/2$ (or $-\pi/2$) implies that we are at the tip of the ellipse → the best case for NOvA

P-\bar{P} bi-probability diagram, proposed by HM-H.Nunokawa, JHEP 2001





Sign of Δm^2_{31} distinguishes normal vs inverted mass ordering

δ and sign Δm^2_{31} couple because ($\Delta m^2_{31} \to -\Delta m^2_{31}$, $\delta \to \pi-\delta$) symmetry in vacuum (JHEP 2001)

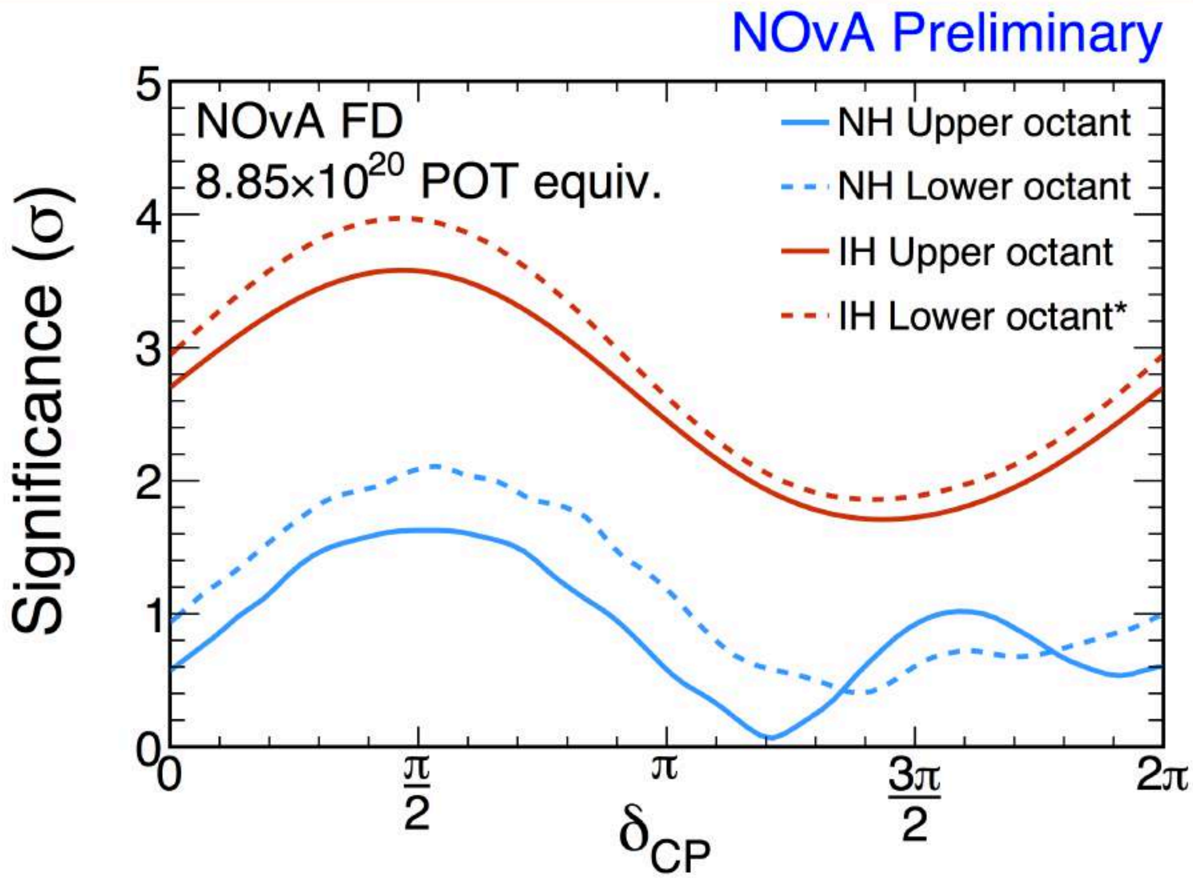
NOvA starts to see preference of NH

Alexander Radovic
Wine&Cheese@ Fermilab Jan 2018

Joint Best Fits

IH at $\delta_{cp} = \pi/2$ disfavored at greater than 3σ.

Approaching IH rejection at 2σ.



Assuming all these go through well, the key question is "What is left?"

and

"what is most important among them?"

My answer is:

- Paradigm Test !

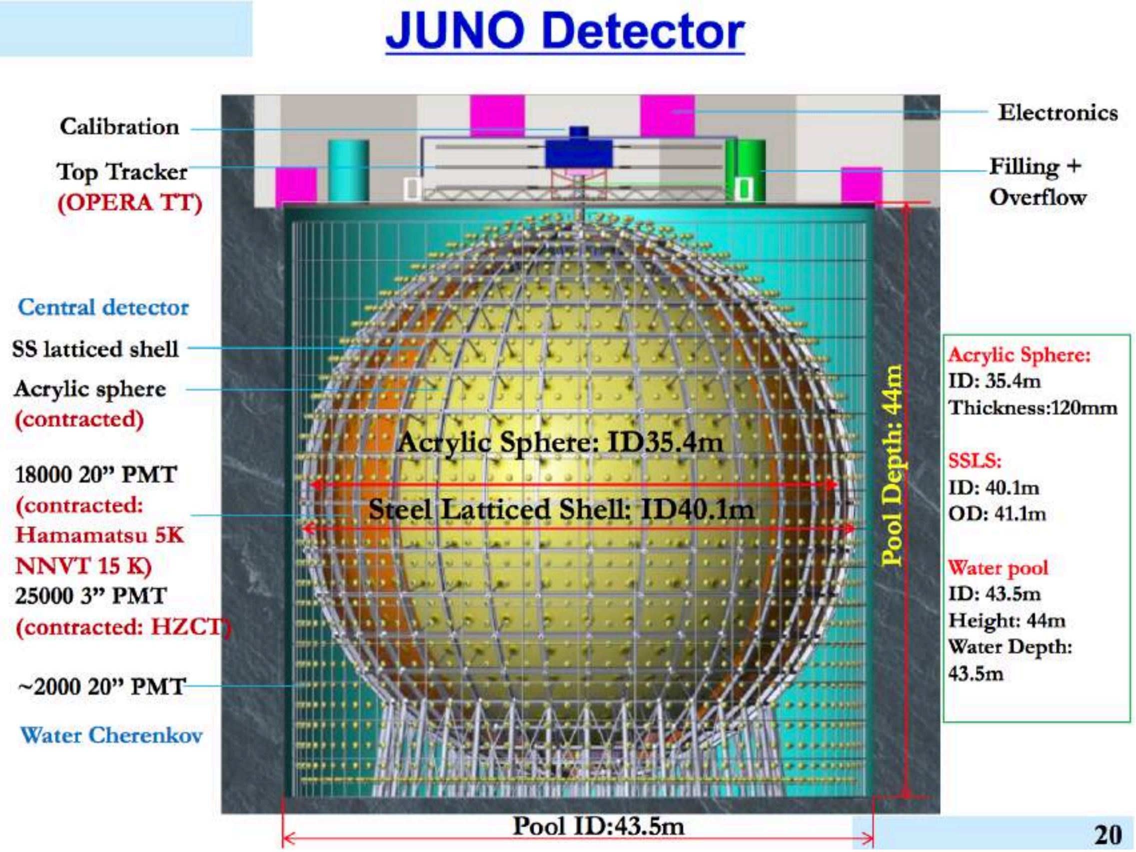
Unitarity test: 2 ways

#1: Unitarity triangle

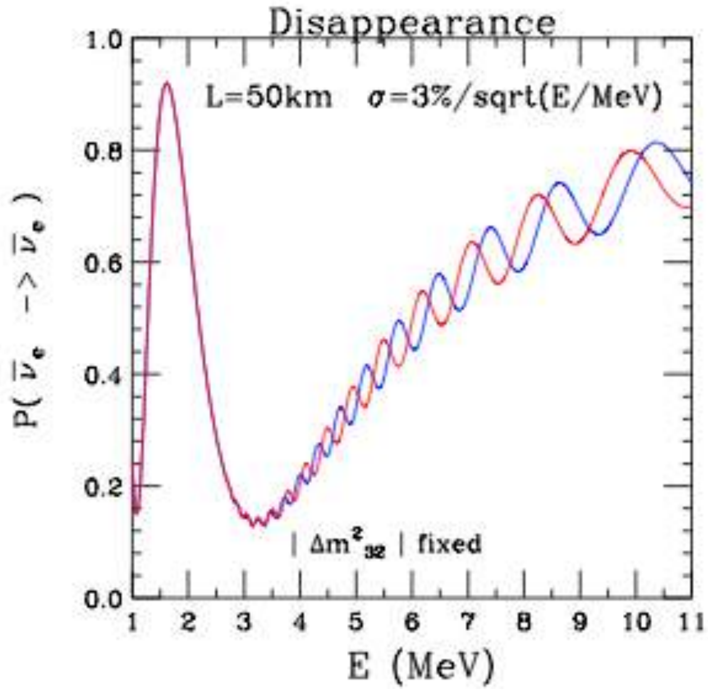
$$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3}=0,$$

Model-independent !

- Determine $|U_{e1}|$, $|U_{e2}|$, $|U_{e3}|$, separately JUNO (+Daya Bay etc) → See slide
- Determine $|U_{\mu 1}|$, $|U_{\mu 2}|$, $|U_{\mu 3}|$, separately "ν_μ-JUNO" →
- For JPARC beam, L=300x30=9000 km, pretty hard….
- 6×10^{20} ν_e-bar/s (1GW$_e$) → 6×10^{20} pion/kaon decay / s (stopped π/K)



20 kt Liquid Scintillator Detector, receive nu's from (6+4) reactor cores at 53 km, ~ the solar oscillation maximum



Capozzi et al PRD2014

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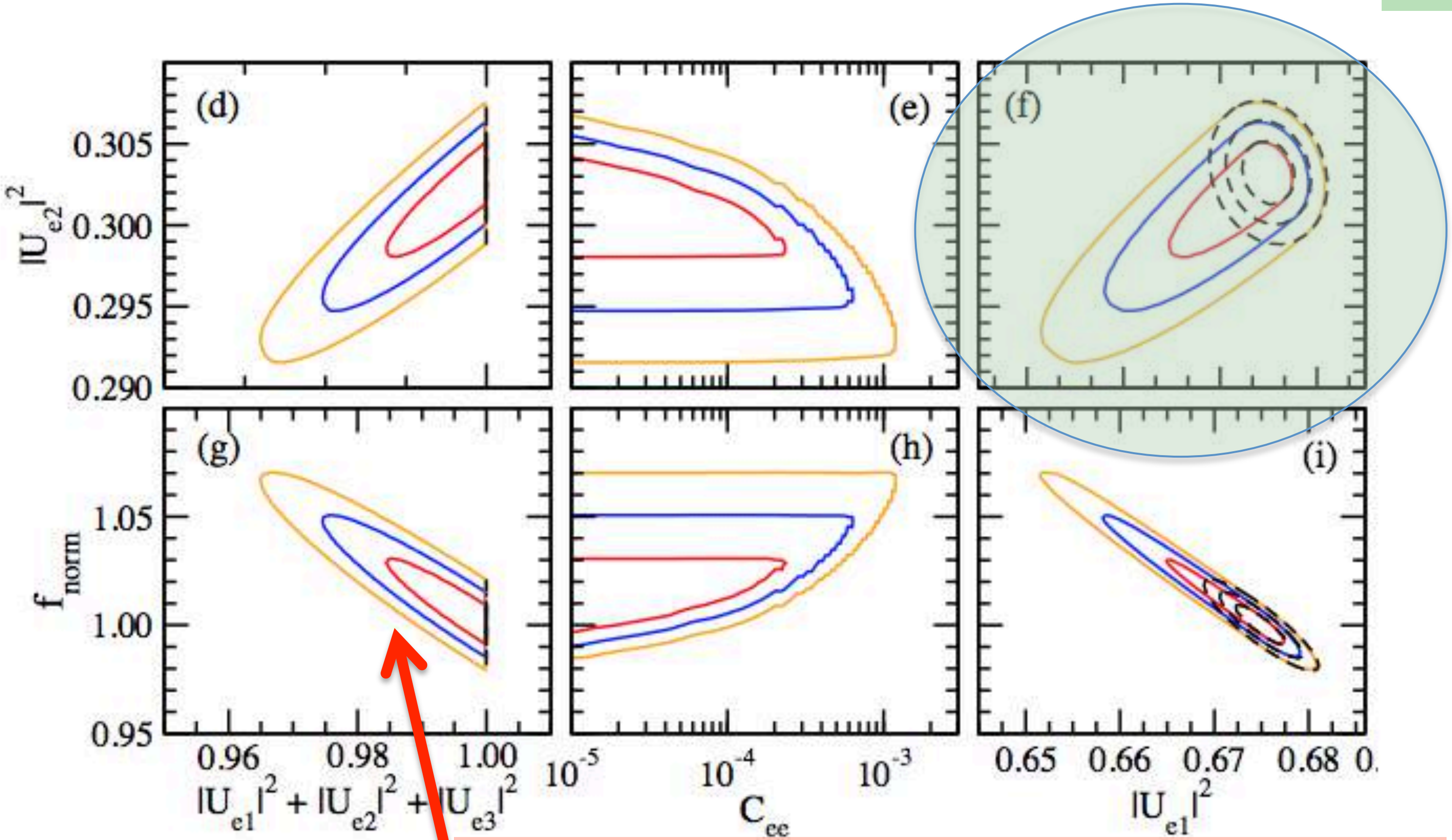
Δm^2_{ee}	2.0	0.26	0.25	0.26	0.26	0.25	0.25
δm^2	3.2	0.22	0.21	0.16	0.21	0.21	0.16
s^2_{12}	5.5	0.49	0.47	0.39	0.49	0.46	0.42
s^2_{13}	10.3	6.95	6.88	6.95	6.84	6.77	6.84
f_R	3.0	0.66	0.66	0.64	0.65	0.65	0.64
$f_{\rm Th}$	20.0	15.3	14.6	...	15.5	15.4	...
$f_{\rm U}$	20.0	13.3	13.3	...	13.3	13.3	...

Separate determination $|U_{e1}|^2$ and $|U_{e2}|^2$: JUNO 5 years

3% flux error assumed



$1 - (|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2)$ can be constrained to ~1% level by JUNO !

Chee Sheng Fong, HM, Hiroshi Nunokawa, JHEP 2017

How can $|U_{e1}|$ and $|U_{e2}|$ be determined separately?

$P(\bar{\nu}_e \to \bar{\nu}_e)$ is given by the non-unitary version of the one derived in ref. [34] ($\alpha = e$ below):

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = \mathcal{C}_{\alpha\alpha} + \left\{|U_{\alpha1}|^2 + |U_{\alpha2}|^2 + |U_{\alpha3}|^2\right\}^2 - 4|U_{\alpha1}|^2|U_{\alpha2}|^2 \sin^2 \frac{\Delta m^2_{21} x}{4E} \tag{5.7}$$
$$-2|U_{\alpha3}|^2 \left(|U_{\alpha1}|^2 + |U_{\alpha2}|^2\right) \left[1 - \sqrt{1 - 4XY \sin^2 \frac{\Delta m^2_{21} x}{4E}} \, \cos\left(\frac{\Delta m^2_{\alpha\alpha} x}{2E} \pm \phi^\alpha_\odot\right)\right],$$

where

$$X \equiv \frac{|U_{\alpha1}|^2}{|U_{\alpha1}|^2 + |U_{\alpha2}|^2}, \qquad Y \equiv \frac{|U_{\alpha2}|^2}{|U_{\alpha1}|^2 + |U_{\alpha2}|^2}, \tag{5.8}$$

and

$$\Delta m^2_{\alpha\alpha} \equiv X|\Delta m^2_{31}| + Y|\Delta m^2_{32}|,$$
$$\phi^\alpha_\odot = \arctan\left[(X - Y)\tan\left(\frac{\Delta m^2_{21} x}{4E}\right)\right] - (X - Y)\left(\frac{\Delta m^2_{21} x}{4E}\right). \tag{5.9}$$

$\phi^\alpha_\odot$ is a slowly varying function of x/E which depends only on the solar parameters, see [34].

#2: Models with unitarity violation

- Prepare model of unitarity violation
- Constrain these models by confronting them with experiments

- Unitarity test is a passive way
- Question: how one can execute a complete job? → I don't know the answer

Natural starting point

- Consider seesaw model of neutrino mass
- M= | 0 m |
-        | m M |
- θ ~ m/M ~ 10^{-14} if m=100 GeV and M=10^{16} GeV
- unitarity violation effect in the light sector at the order of ~$(m/M)^2$
- Too small to observe (high-scale seesaw)
- But suppose M~TeV etc…

Unitarity violation as a natural concept

High vs low scale unitarity

High- vs low-energy scale unitarity violation

- Let us define "high scale" and "low scale" unitarity violation as
- "low scale": heavy leptons/neutrinos do communicate with light ν system, i.e., participate to nu oscillation

My concern today

- "high scale": heavy leptons/neutrinos do not communicate with light ν system

- Note: high scale UV→pioneering work by Antusch, Biggio, Fernandez-Martinez, Gavela, and Lopez-Pavon, JHEP2006

Since then, Enrique has been in the deep mine..

New Physics at low energies: relatively new option

- Various scenarios are proposed which involve "new physics" at low energies
- Motivated by LSND-MiniBoone, DAMA, etc.

[21] A. E. Nelson and J. Walsh, "Short Baseline Neutrino Oscillations and a New Light Gauge Boson," Phys. Rev. D **77** (2008) 033001 doi:10.1103/PhysRevD.77.033001 [arXiv:0711.1363 [hep-ph]].

[22] M. Pospelov and J. Pradler, "Elastic scattering signals of solar neutrinos with enhanced baryonic currents," Phys. Rev. D **85** (2012) 113016 Erratum: [Phys. Rev. D **88** (2013) no.3, 039904] doi:10.1103/PhysRevD.85.113016, 10.1103/PhysRevD.88.039904 [arXiv:1203.0545 [hep-ph]].

[23] R. Harnik, J. Kopp and P. A. N. Machado, "Exploring nu Signals in Dark Matter Detectors," JCAP **1207** (2012) 026 doi:10.1088/1475-7516/2012/07/026 [arXiv:1202.6073 [hep-ph]].

[24] K. S. Babu, A. Friedland, P. A. N. Machado and I. Mocioiu, "Flavor Gauge Models Below the Fermi Scale," JHEP **1712** (2017) 096 doi:10.1007/JHEP12(2017)096 [arXiv:1705.01822 [hep-ph]].

Plus many more !!

- Orthodoxy seems challenged, e.g., WIMP dark matter, low-E SUSY, day one NP, ..

High- vs low-energy unitarity violation

Low-energy UV

- lepton flavor universality: YES
- zero distance neutrino flavor transition: NO
- Kinematical effect of sterile nu emission: YES

High-energy UV

- lepton flavor universality: NO
- zero distance neutrino flavor transition: YES
- Kinematical effect of sterile nu emission: YES (if kinematically allowed)

High-energy unitarity violation

Antusch et al JHEP 2006

- When high mass sector integrated out we have effective Lagrangian of light neutrinos and leptons but with unitarity violation

Aiming at model-independent formulation !!

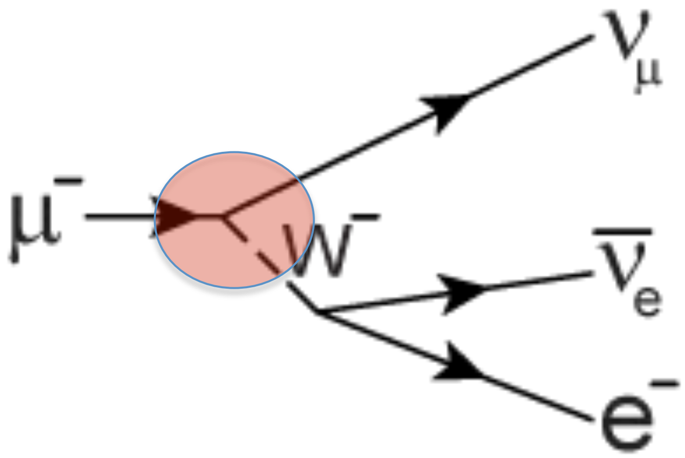
$$\mathcal{L}^{eff} = \frac{1}{2}\left(\bar{\nu}_i i \partial\!\!\!/ \nu_i - \overline{\nu^c}_i m_i \nu_i + h.c.\right) - \frac{g}{2\sqrt{2}}\left(W_\mu^+ \bar{l}_\alpha \gamma_\mu (1-\gamma_5) N_{\alpha i} \nu_i + h.c.\right)$$
$$- \frac{g}{2\cos\theta_W}\left(Z_\mu \bar{\nu}_i \gamma^\mu (1-\gamma_5)(N^\dagger N)_{ij} \nu_j + h.c.\right) + \ldots$$

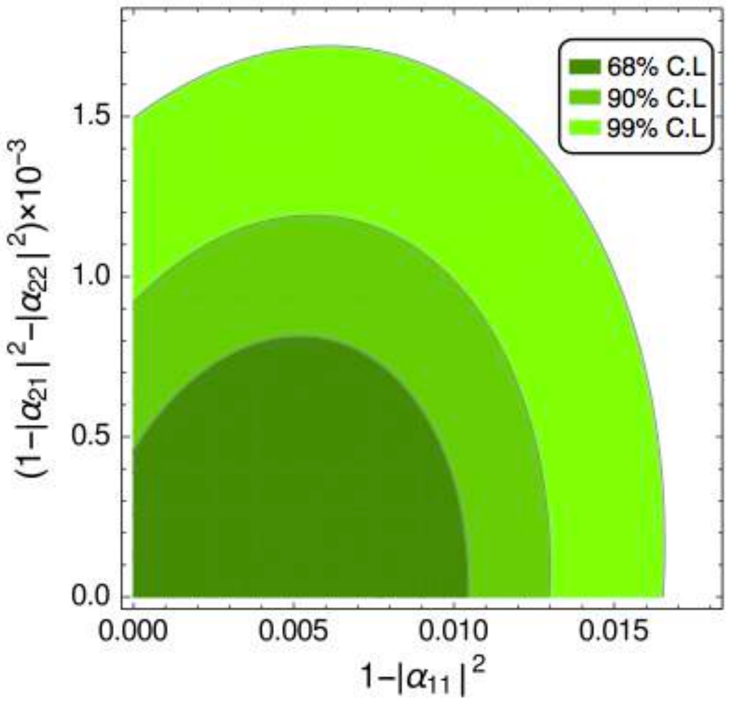
$$\nu_\alpha = N_{\alpha i} \nu_i \,. \qquad \langle \nu_i | \nu_j \rangle = \delta_{ij} \,, \qquad G_F = \frac{G_F^M}{\sqrt{(NN^\dagger)_{ee}(NN^\dagger)_{\mu\mu}}} \,.$$

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}}} \sum_i N^*_{\alpha i} |\nu_i\rangle \equiv \sum_i \tilde{N}^*_{\alpha i} |\nu_i\rangle \,,$$

Flavor nu states NOT orthogonal with each other

Flavor non-universality





Escrihuela etal PRD2015

- $\mu \rightarrow \nu_i$: $N_{\mu i}{}^*$
- $\nu_i \rightarrow \nu_\mu$: $N_{\mu i} / \sqrt{(NN^+)_{\mu\mu}}$ because
- $\langle \nu_i \mid \nu_\mu \rangle = N_{\mu i} / \sqrt{(NN^+)_{\mu\mu}}$
- Then, $= \Sigma_i N_{\mu i} N_{\mu i}{}^* / \sqrt{(NN^+)_{\mu\mu}} = \sqrt{(NN^+)_{\mu\mu}}$

$$G_\mu = G_F \sqrt{(NN^+)_{\mu\mu}} \sqrt{(NN^+)_{ee}}$$

Most probably in high scale unitarity violation, …

- In high scale UV, we assume NP at high scale, M_{NP} > EW scale
- Then, most probably, SU(2) x U(1) holds at the scale
- Unitarity violation is constrained primarily by charged lepton sector

Better probe!, easier to explore experimentally !

- Nothing wrong with this!, but less room for the next nu project playing a crucial role, ….

(3+N) model for Low-E unitarity violation

Other models of Low-E UV?

3 active +N sterile unitary model in vacuum

3x3

$$\nu_\alpha = \mathbf{U}_{\alpha i}\tilde{\nu}_i, \qquad \mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

NxN

$$i\frac{d}{dx}\nu = H\nu.$$

$$H = \mathbf{U}\begin{bmatrix} \Delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{3+N} \end{bmatrix}\mathbf{U}^\dagger$$

$$\Delta_i \equiv \frac{m_i^2}{2E} \quad (i = 1, 2, 3), \qquad \Delta_J \equiv \frac{m_J^2}{2E} \quad (J = 4, \cdots, 3+N).$$

3+N model for low-E UV and modest requests on it

- By 3+N model I mean (3+N) space is unitary, but not in 3 active nu space

Unique? Probably not. General Low-E UV model hard to construct. My strategy is …

- Requirement: The prediction of the 3+N model must be independent of details of N sterile sector
- After fulfilling this criterion we will show what is the *difference* between High-E vs Low-E UV

Probability in vacuum

$$
\begin{aligned}
P(\nu_\beta \to \nu_\alpha) = & \left| \sum_{k=1}^{3} U_{\alpha k} U_{\beta k}^* \right|^2 \qquad (4.5) \\
& -2 \sum_{j \neq k} \mathrm{Re}\,(U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2} + \sum_{j \neq k} \mathrm{Im}\,(U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*) \sin(\Delta_k - \Delta_j)x \\
& + \sum_{J} |W_{\alpha J}|^2 |W_{\beta J}|^2 \\
& + \sum_{J \neq K} [\mathrm{Re}\,(W_{\alpha J}^* W_{\beta J} W_{\alpha K} W_{\beta K}^*) \cos(\Delta_K - \Delta_J)x + \mathrm{Im}\,(W_{\alpha J}^* W_{\beta J} W_{\alpha K} W_{\beta K}^*) \sin(\Delta_K - \Delta_J)x] \\
& + 2 \sum_{j=1}^{3} \sum_{K=4}^{3+N} [\mathrm{Re}\,(U_{\alpha j}^* U_{\beta j} W_{\alpha K} W_{\beta K}^*) \cos(\Delta_K - \Delta_j)x + \mathrm{Im}\,(U_{\alpha j}^* U_{\beta j} W_{\alpha K} W_{\beta K}^*) \sin(\Delta_K - \Delta_j)x] \,.
\end{aligned}
$$

What is this?

- Active-active, active-sterile, sterile-sterile oscillations
- If Δm^2_{as} (Δm^2_{ss}) > 0.1 eV^2, "fast oscillation" due to active-sterile and sterile-sterile Δm^2 are averaged out

$$\left\langle \sin\left(\frac{\Delta m^2_{Ji} x}{2E}\right) \right\rangle \approx \left\langle \sin\left(\frac{\Delta m^2_{JK} x}{2E}\right) \right\rangle \approx 0,$$

Fast oscillation averaged out by decoherence

$$\left|\delta\left(\frac{\Delta m_{ab}^2 x}{2E}\right)\right| = \left|\frac{\Delta m_{ab}^2}{2E}\delta x - \frac{\Delta m_{ab}^2 x}{2E^2}\delta E\right| \gtrsim 2\pi.$$

i. *Spatial resolution.* In this case, decoherence happens if

$$\delta x \gtrsim \frac{4\pi E}{|\Delta m_{ab}^2|}.$$

ii. *Energy resolution.* In this case, decoherence happens if

$$\delta E \gtrsim \frac{4\pi E^2}{|\Delta m_{ab}^2| x}.$$

$$\frac{\Delta m_{21}^2 x}{4E} \sim 1 \qquad \text{or} \qquad \frac{|\Delta m_{31}^2| x}{4E} \sim 1.$$

LBL reactor (JUNO) accelerator

$$|\Delta m_{Ja}^2| \gtrsim \pi\left(\frac{\Delta m_{21}^2}{\delta E/E}\right) \approx 7.9 \times 10^{-3}\text{eV}^2 \left(\frac{\delta E/E}{0.03}\right)^{-1} \qquad \text{(for LBL reactor)},$$

$$|\Delta m_{Ja}^2| \gtrsim \pi\left(\frac{|\Delta m_{31}^2|}{\delta E/E}\right) \approx 7.5 \times 10^{-2}\text{eV}^2 \left(\frac{\delta E/E}{0.1}\right)^{-1} \qquad \text{(for accelerator)}.$$

P looks almost standard one, but there is a new term

Appearance

Probability leakage !

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2 - 2 \sum_{j \neq k} \mathrm{Re} \left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j) x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im} \left(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j) x,$$

U = non-unitary "MNS"

Disappearance

$$P(\nu_\alpha \to \nu_\alpha) = \mathcal{C}_{\alpha\alpha} + \left(\sum_{j}^{3} |U_{\alpha j}|^2 \right)^2 - 4 \sum_{k>j}^{3} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin^2 \frac{(\Delta_k - \Delta_j) x}{2},$$

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^2 |W_{\beta J}|^2, \qquad \mathcal{C}_{\alpha\alpha} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^4$$

Order ~ W^4, small!!

P-leaking term:
It must be obvious to exist, right?

- There is a N sterile sector which can communicate with active nu sector
- So the probability leaks to sterile sector
- Yet, not emphasized before…
- ~ $W^{4,}$ Too small?

$$\delta_{\alpha\beta} = \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} + \sum_{J=4}^{N+3} W_{\alpha J} W^*_{\beta J}.$$

Then, $\left|\sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j}\right|^2 = \left|\sum_{J=4}^{N+3} W_{\alpha J} W^*_{\beta J}\right|^2$ in the appearance channel ($\alpha \neq \beta$),
$\left(\sum_{j=1}^{3} |U_{\alpha j}|^2\right)^2 = \left(1 - \sum_{J=4}^{N+3} |W_{\alpha J}|^2\right)^2 = 1 - \mathcal{O}(W^2)$ in the disappearance channel

Term kept by S. Parke and M. Ross Lonergan, PRD 2017 is also 4^{th} order in W

Summary: There exists sterile-sector model independent P formula if $\Delta m^2_{as} > 0.1$ eV^2

Disappearance

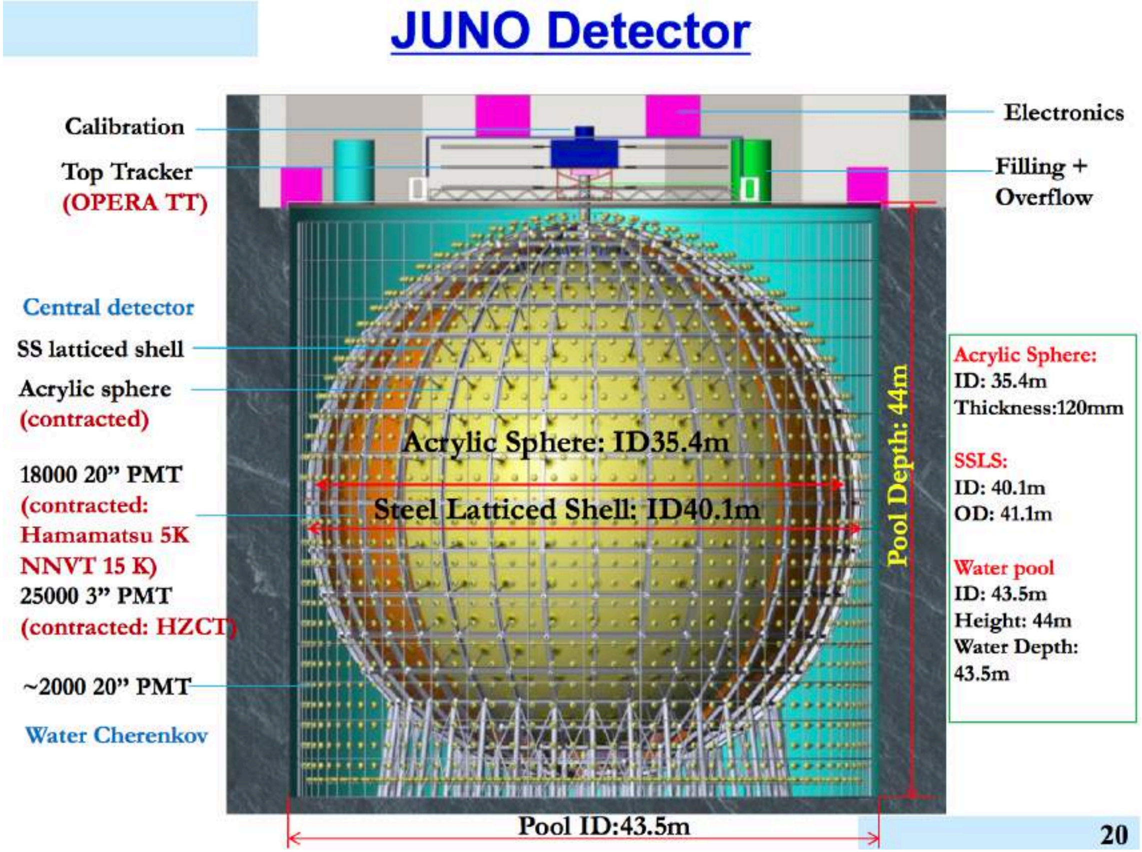
$$P(\nu_\alpha \to \nu_\alpha) = \mathcal{C}_{\alpha\alpha} + \left(\sum_j^3 |U_{\alpha j}|^2 \right)^2 - 4 \sum_{k>j}^3 |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin^2 \frac{(\Delta_k - \Delta_j)x}{2},$$

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=1}^N |W_{\alpha J}|^2 |W_{\beta J}|^2, \qquad \mathcal{C}_{\alpha\alpha} \equiv \sum_{J=1}^N |W_{\alpha J}|^4$$

- A constant leaking term $C_{\alpha\beta}$ (= distinguishes between low-E vs high-E unitarity violation !!)
- Unitary MNS → non-unitarty "U"

UV effect is in: (1) explicit W correction term, (2) non-unitary U matrix

Can one detect C_{ee}? (=P leaking term): JUNO!



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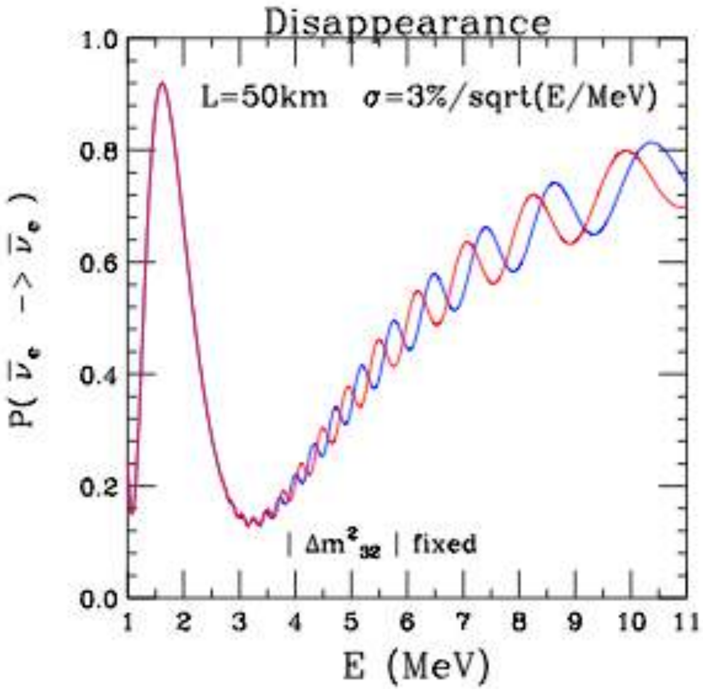

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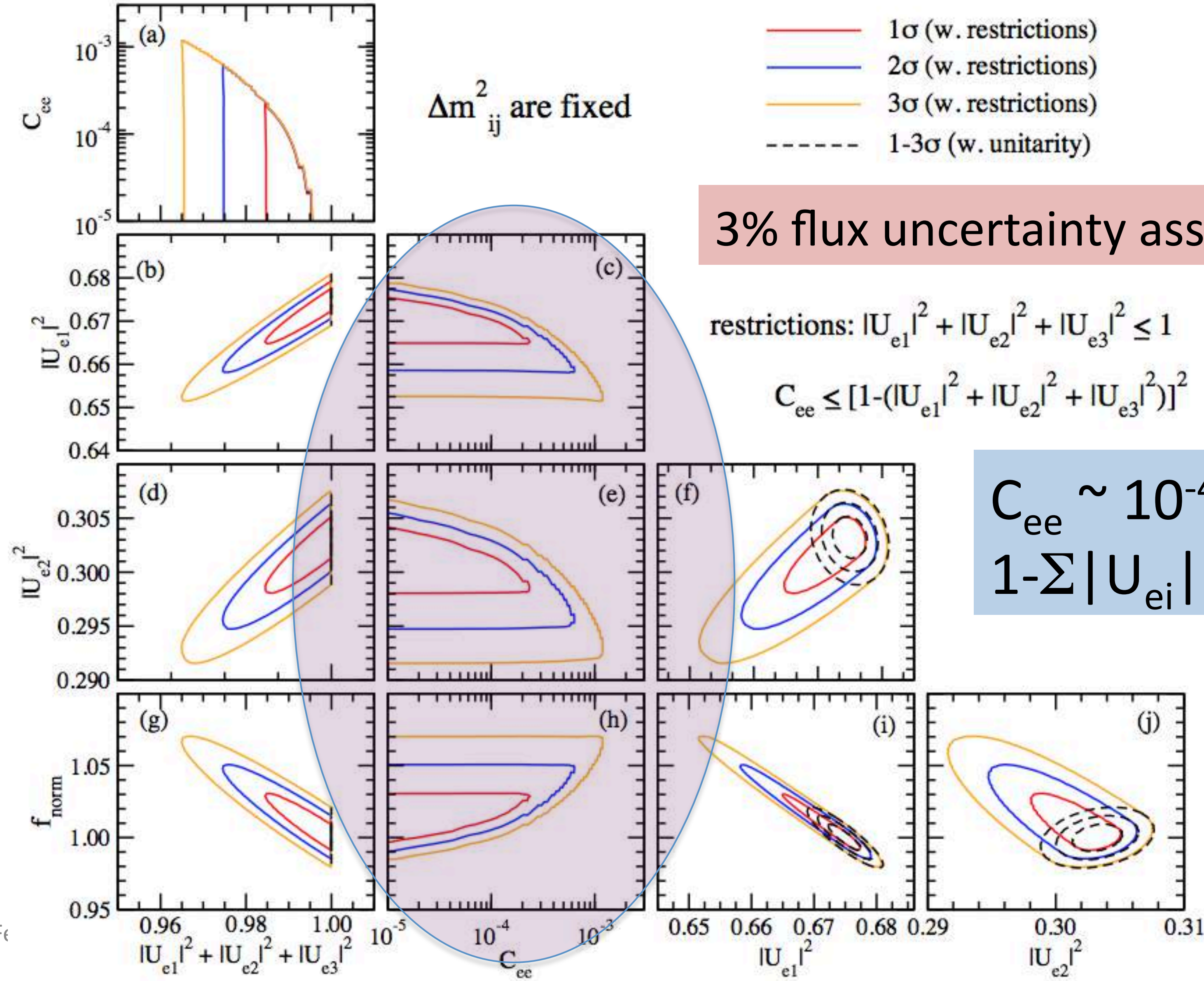
Sensitivity to C_{ee} and $1-\Sigma|U_{ei}|^2$ from JUNO 5 years



3% flux uncertainty assumed

$C_{ee} \sim 10^{-4}$ ($1\,\sigma$)
$1-\Sigma|U_{ei}|^2 \sim 0.01$

Invitation to non-unitary world..

Parke-Ross-Lonergan PRD2016

Without P leaking term!

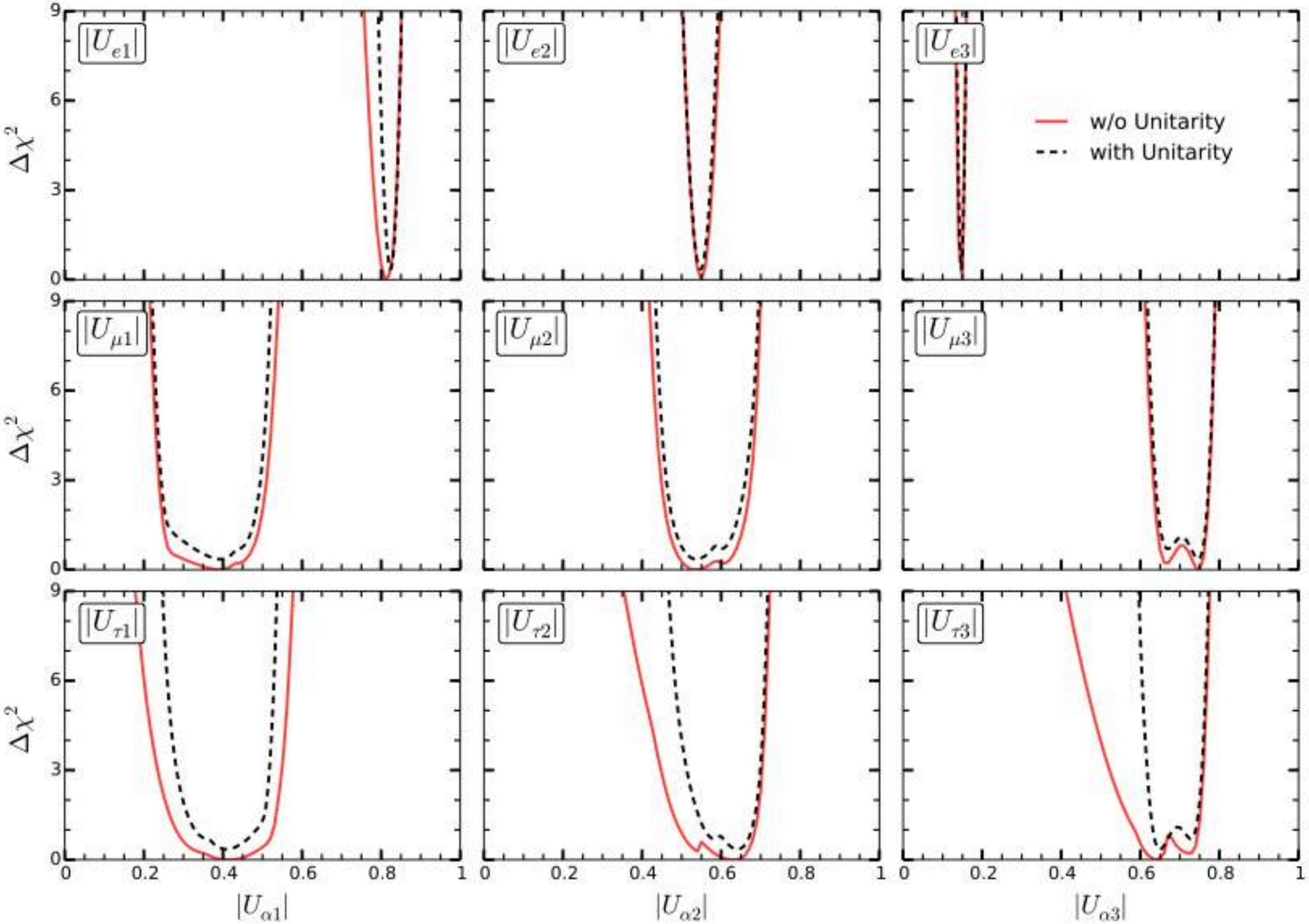


FIG. 1. Marginalized 1-D $\Delta\chi^2$ for each of the magnitudes of the 3×3 neutrino mixing matrix elements, without (red solid) and with (black dashed) the assumption of unitarity. The x-axis is the magnitude of each individual matrix element, and the y-axis is the associated $\Delta\chi^2$ after marginalization over all parameters other than the one in question. This analysis was performed for the normal hierarchy, the inverse hierarchy providing the same qualitative result.

Constraints on unitarity violation (Parke-Ross-Lonergan)

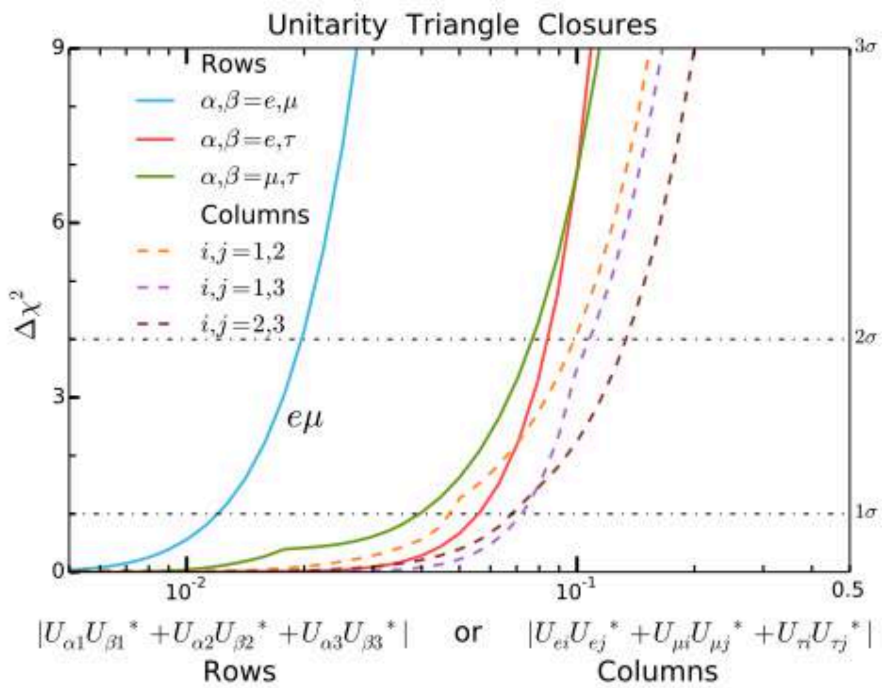


FIG. 2. 1-D $\Delta\chi^2$ for the absolute value of the closure of the three row (solid) and three column (dashed) unitarity triangles when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV^2. There is one unique unitarity triangle, the $\nu_e\nu_\mu$ row unitarity triangle, in that it does not contain any ν_τ elements and hence is constrained to be unitary at a level half an order of magnitude better than the others. By comparison to Fig. 3 one can clearly see that the Cauchy-Schwartz constraints are satisfied.

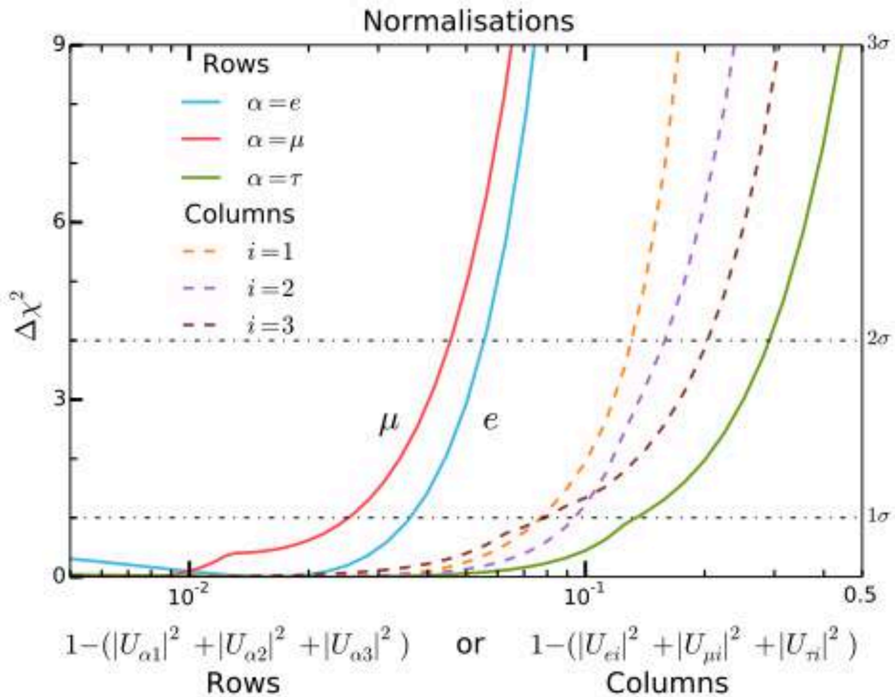


FIG. 3. 1-D $\Delta\chi^2$ for deviation of both $U_{\rm PMNS}$ row (solid) and column (dashed) normalizations, when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV^2.

Sterile model-independent P: Prevail to "in matter" ?

Small-UV perturbation theory

Chee Sheng Fong, HM, Hiroshi Nunokawa, arXiv:1712.02798

$$H = \mathbf{U} \begin{bmatrix} \boldsymbol{\Delta}_{\mathbf{a}} & 0 \\ 0 & \boldsymbol{\Delta}_{\mathbf{s}} \end{bmatrix} \mathbf{U}^{\dagger} + \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \equiv H_{\text{vac}} + H_{\text{matt}}$$

where $\boldsymbol{\Delta}_{\mathbf{a}} = \text{diag}(\Delta_1, \Delta_2, \Delta_3)$ and $\boldsymbol{\Delta}_{\mathbf{s}} = \text{diag}(\Delta_4, \Delta_5, \cdots, \Delta_{N+3})$.

$$A = \begin{bmatrix} \Delta_A - \Delta_B & 0 & 0 \\ 0 & -\Delta_B & 0 \\ 0 & 0 & -\Delta_B \end{bmatrix}$$

$$\Delta_A \equiv \frac{a}{2E}, \qquad \Delta_B \equiv \frac{b}{2E},$$

$$a = 2\sqrt{2} G_F N_e E \approx 1.52 \times 10^{-4} \left(\frac{Y_e \rho}{\text{g}\,\text{cm}^{-3}} \right) \left(\frac{E}{\text{GeV}} \right) \text{eV}^2,$$

$$b = \sqrt{2} G_F N_n E = \frac{1}{2} \left(\frac{N_n}{N_e} \right) a.$$

Small-UV perturbation theory: change to vacuum mass basis

$$\tilde{H} = \tilde{H}_{\rm vac} + \tilde{H}_{\rm matt} = \begin{bmatrix} \mathbf{\Delta_a} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} + \mathbf{U}^\dagger \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}.$$

$$\tilde{H}_0 = \begin{bmatrix} \mathbf{\Delta_a} + U^\dagger A U & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix}, \qquad \tilde{H}_1 = \begin{bmatrix} 0 & U^\dagger A W \\ W^\dagger A U & W^\dagger A W \end{bmatrix}.$$

X=unitary matrix, diagonalize H_0(3x3)

$$\mathbf{X}^\dagger \tilde{H}_0 \mathbf{X} = \begin{bmatrix} X^\dagger \left(\mathbf{\Delta_a} + U^\dagger A U \right) X & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} = \begin{bmatrix} \mathbf{h} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} \equiv \hat{H}_0,$$

$$\hat{H}_1 = \mathbf{X}^\dagger \tilde{H}_1 \mathbf{X} = \begin{bmatrix} 0 & (UX)^\dagger A W \\ W^\dagger A (UX) & W^\dagger A W \end{bmatrix}.$$

Do perturbation theory in hat basis

$$S_{aa} = (UX) \hat{S}_{aa} (UX)^\dagger + (UX) \hat{S}_{aS} W^\dagger + W \hat{S}_{Sa} (UX)^\dagger + W \hat{S}_{SS} W^\dagger,$$

Do W perturbation to 4th order to keep P leaking term

- Did we find ~W^4 P leaking term? ⇒ Yes!
- How about what is the role of the rest?

$$
\begin{aligned}
\left|S^{(2)}_{\alpha\beta}\right|^2_{\rm 1st} &= \sum_{k,K}\sum_{l,L}\frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)} \\
&\times\left[x^2e^{-i(h_k-h_l)x}-(ix)\frac{e^{-i(\Delta_K-h_l)x}-e^{-i(h_k-h_l)x}}{(\Delta_K-h_k)}+(ix)\frac{e^{-i(h_k-\Delta_L)x}-e^{-i(h_k-h_l)x}}{(\Delta_L-h_l)}\right. \\
&+\left.\frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)}\left\{e^{-i(\Delta_K-\Delta_L)x}+e^{-i(h_k-h_l)x}-e^{-i(\Delta_K-h_l)x}-e^{-i(h_k-\Delta_L)x}\right\}\right] \\
&\times(UX)_{\alpha k}(UX)^*_{\beta k}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&\times(UX)^*_{\alpha l}(UX)_{\beta l}\left\{(UX)^\dagger AW\right\}_{lL}\left\{W^\dagger A(UX)\right\}_{Ll} \\
&+\sum_{k\neq m}\sum_{K}\sum_{l\neq n}\sum_{L}\frac{1}{(h_m-h_k)(\Delta_K-h_k)(\Delta_K-h_m)}\frac{1}{(h_n-h_l)(\Delta_L-h_l)(\Delta_L-h_n)} \\
&\times\left[(\Delta_K-h_k)\,e^{-ih_mx}-(\Delta_K-h_m)\,e^{-ih_kx}-(h_m-h_k)e^{-i\Delta_Kx}\right] \\
&\times\left[(\Delta_L-h_l)\,e^{+ih_nx}-(\Delta_L-h_n)\,e^{+ih_lx}-(h_n-h_l)e^{+i\Delta_Lx}\right] \\
&\times(UX)_{\alpha k}(UX)^*_{\beta m}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km} \\
&\times(UX)^*_{\alpha l}(UX)_{\beta n}\left\{(UX)^\dagger AW\right\}_{nL}\left\{W^\dagger A(UX)\right\}_{Ll} \\
&+\sum_{k,K}\sum_{l,L}\frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)}\left(e^{-i\Delta_Kx}-e^{-ih_kx}\right)\left(e^{+i\Delta_Lx}-e^{+ih_lx}\right) \\
&\times\left[(UX)_{\alpha k}W^*_{\beta K}\left\{(UX)^\dagger AW\right\}_{kK}+W_{\alpha K}(UX)^*_{\beta k}\left\{W^\dagger A(UX)\right\}_{Kk}\right] \\
&\times\left[(UX)^*_{\alpha l}W_{\beta L}\left\{W^\dagger A(UX)\right\}_{Ll}+W^*_{\alpha L}(UX)_{\beta l}\left\{(UX)^\dagger AW\right\}_{lL}\right] \\
&+\sum_K|W_{\alpha K}|^2|W_{\beta K}|^2+\sum_{K\neq L}e^{-i(\Delta_K-\Delta_L)x}W_{\alpha K}W^*_{\beta K}W^*_{\alpha L}W_{\beta L}.
\end{aligned}
\tag{C.1}
$$

P leaking term

Typical W^4 term

$$
\begin{aligned}
&P(\nu_\beta \to \nu_\alpha)^{(4)}_{2nd} \equiv 2\mathrm{Re}\left[\left(S^{(0)}_{\alpha\beta}\right)^* S^{(4)}_{\alpha\beta}[4]_{\text{ diag}}\right] \\
&= 2\mathrm{Re}\Bigg\{\sum_n\sum_k\sum_K\Bigg[-\frac{x^2}{2}\frac{1}{(\Delta_K-h_k)^2}e^{-i(h_k-h_n)x} - \frac{2(ix)}{(\Delta_K-h_k)^3}e^{-i(h_k-h_n)x} \\
&- \frac{(ix)}{(\Delta_K-h_k)^3}e^{-i(\Delta_K-h_n)x} - \frac{3}{(\Delta_K-h_k)^4}\left(e^{-i(\Delta_K-h_n)x} - e^{-i(h_k-h_n)x}\right)\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&+ \sum_n\sum_k\sum_K\sum_{m\neq k}\Bigg[\frac{(ix)}{(\Delta_K-h_k)^2(h_m-h_k)}e^{-i(h_k-h_n)x} \\
&- \frac{(ix)}{(\Delta_K-h_k)^2(\Delta_K-h_m)}e^{-i(\Delta_K-h_n)x} + \frac{(h_k+2h_m-3\Delta_K)}{(\Delta_K-h_k)^3(\Delta_K-h_m)^2}e^{-i(\Delta_K-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_m)^2(h_m-h_k)^2}e^{-i(h_m-h_n)x} - \frac{(\Delta_K+2h_m-3h_k)}{(\Delta_K-h_k)^3(h_m-h_k)^2}e^{-i(h_k-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km}\left\{(UX)^\dagger AW\right\}_{mK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&+ \sum_n\sum_k\sum_{K\neq L}\Bigg[-\frac{x^2}{2}\frac{1}{(\Delta_K-h_k)(\Delta_L-h_k)}e^{-i(h_k-h_n)x} - (ix)\frac{(\Delta_K+\Delta_L-2h_k)}{(\Delta_K-h_k)^2(\Delta_L-h_k)^2}e^{-i(h_k-h_n)x} \\
&- \frac{1}{(\Delta_K-h_k)^3(\Delta_L-\Delta_K)}e^{-i(\Delta_K-h_n)x} + \frac{1}{(\Delta_L-h_k)^3(\Delta_L-\Delta_K)}e^{-i(\Delta_L-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_k)^3(\Delta_L-h_k)^3}\left\{\Delta_L^2+\Delta_L\Delta_K+\Delta_K^2-3h_k(\Delta_L+\Delta_K)+3h_k^2\right\}e^{-i(h_k-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk}\left\{(UX)^\dagger AW\right\}_{kL}\left\{W^\dagger A(UX)\right\}_{Lk} \\
&+ \sum_n\sum_k\sum_{K\neq L}\sum_{m\neq k}\Bigg[\frac{(ix)}{(\Delta_K-h_k)(\Delta_L-h_k)(h_m-h_k)}e^{-i(h_k-h_n)x} \\
&- \frac{1}{(\Delta_K-h_k)^2(\Delta_L-h_k)^2(h_m-h_k)^2}\left\{\Delta_K\Delta_L+(h_m-2h_k)(\Delta_K+\Delta_L)+3h_k^2-2h_mh_k\right\}e^{-i(h_k-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_m)(\Delta_L-h_m)(h_m-h_k)^2}e^{-i(h_m-h_n)x} \\
&+ \frac{1}{(\Delta_K-\Delta_L)(\Delta_K-h_k)^2(\Delta_K-h_m)}e^{-i(\Delta_K-h_n)x} - \frac{1}{(\Delta_K-\Delta_L)(\Delta_L-h_k)^2(\Delta_L-h_m)}e^{-i(\Delta_L-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km}\left\{(UX)^\dagger AW\right\}_{mL}\left\{W^\dagger A(UX)\right\}_{Lk}\Bigg\}. \qquad \text{(C.5)}
\end{aligned}
$$

Do W perturbation to 4th order to keep P leaking term

- Did we find ~W^4 P leaking term? → Yes!
- How about what is the role of the rest?
- To answer the question let us first examine W^2 terms

After averaging out fast oscillations..

$$
\begin{aligned}
&P(\nu_\beta \to \nu_\alpha)^{(0+2)} \\
&= \left|\sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j}\right|^2 - 2\sum_{j\neq k} \mathrm{Re}\left[(UX)_{\alpha j}(UX)^*_{\beta j}(UX)^*_{\alpha k}(UX)_{\beta k}\right] \sin^2\frac{(h_k-h_j)x}{2} \\
&- \sum_{j\neq k} \mathrm{Im}\left[(UX)_{\alpha j}(UX)^*_{\beta j}(UX)^*_{\alpha k}(UX)_{\beta k}\right] \sin(h_k-h_j)x \\
&+ 2\mathrm{Re}\Bigg\{\sum_m \sum_{k,K} \frac{1}{\Delta_K - h_k}\left[(ix)e^{-i(h_k-h_m)x} - \frac{e^{-i(h_k-h_m)x}}{(\Delta_K-h_k)}\right] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&- \sum_m \sum_{k\neq l} \sum_K \frac{1}{(h_l-h_k)(\Delta_K-h_k)(\Delta_K-h_l)} \\
&\times \left[(\Delta_K-h_k)\,e^{-i(h_l-h_m)x} - (\Delta_K-h_l)\,e^{-i(h_k-h_m)x}\right] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta l}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kl} \\
&- \sum_m \sum_{k,K} \frac{e^{-i(h_k-h_m)x}}{(\Delta_K-h_k)}\Bigg[(UX)_{\alpha k} W^*_{\beta K}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger AW\right\}_{kK} \\
&+ W_{\alpha K}(UX)^*_{\beta k}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{W^\dagger A(UX)\right\}_{Kk}\Bigg]\Bigg\},
\end{aligned}
$$

Zeroth order in W

Do it: W perturbation to 4^{th} order to keep P leaking term

- Did we find $\sim W^4$ P leaking term? ⇒ Yes!
- How about what is the role of the rest?
- To answer the question let us first examine W^2 terms

Always comes with matter potential

- If we impose $\Delta m^2_{jK} > 0.1\ \text{eV}^2$, then all the $\sim W^2$ correction terms are small ⇒ negligible
- Then, all the $\sim W^4$ terms can be ignored except for P leaking term

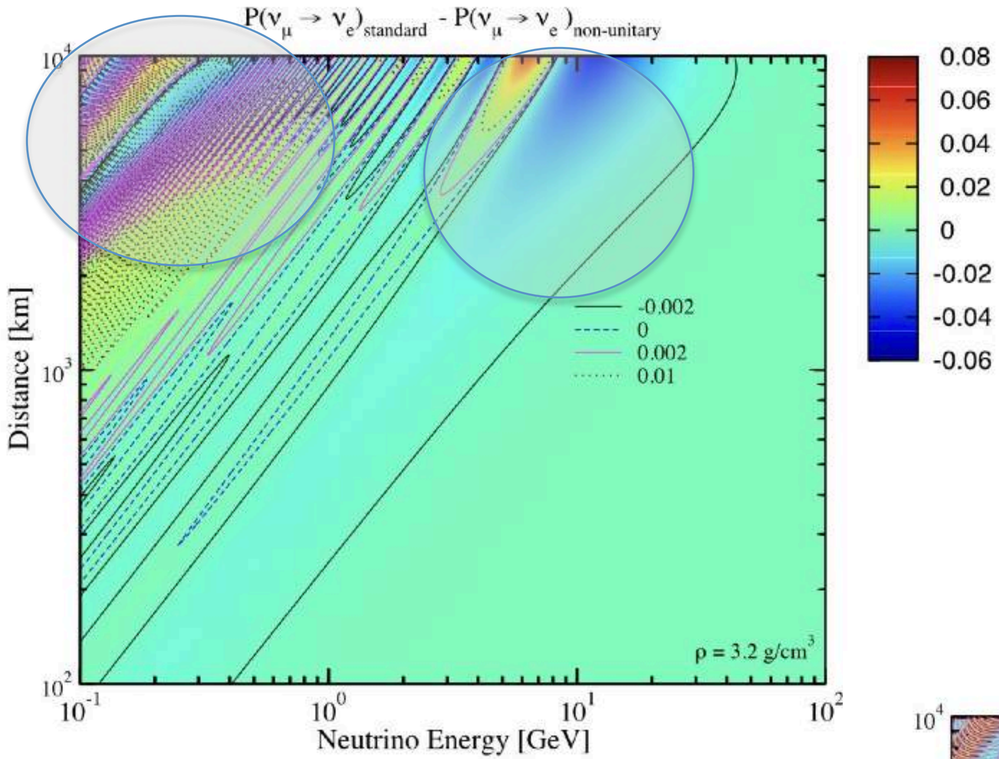
$$\frac{|A|}{\Delta m^2_{Jk}} = 2.13 \times 10^{-3} \left(\frac{\Delta m^2_{Jk}}{0.1\text{eV}^2}\right)^{-1} \left(\frac{\rho}{2.8\,\text{g/cm}^3}\right) \left(\frac{E}{1\ \text{GeV}}\right),$$

A simple formula for oscillation probability in matter w/o unitarity

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2$$
$$- 2 \sum_{j \neq k} \mathrm{Re}\left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin^2 \frac{(h_k - h_j)x}{2}$$
$$- \sum_{j \neq k} \mathrm{Im}\left[(UX)_{\alpha j} (UX)^*_{\beta j} (UX)^*_{\alpha k} (UX)_{\beta k} \right] \sin(h_k - h_j)x,$$

- Apart from P leaking term, leading (zeroth) order terms only !
- For constant matter density, one can write down an exact expression for $P_{\beta\alpha}$
- UV effect is in: (1) explicit W correction term, (2) non-unitary U matrix

Where is the region of large UV?

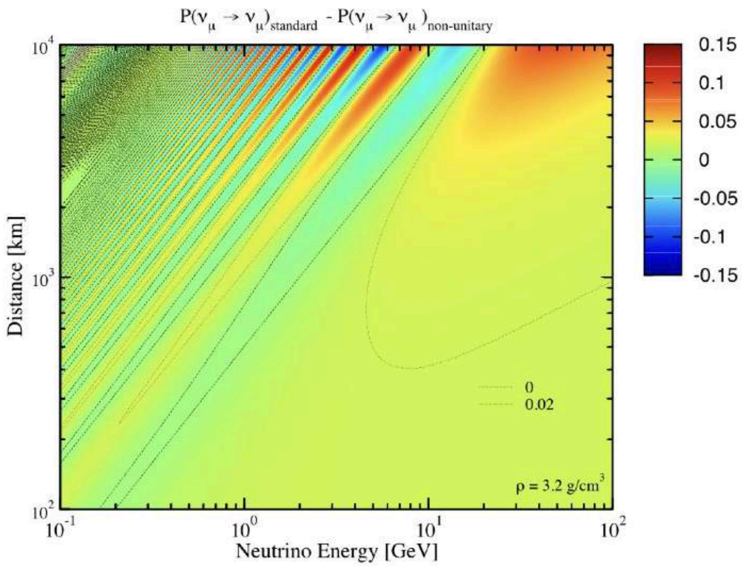


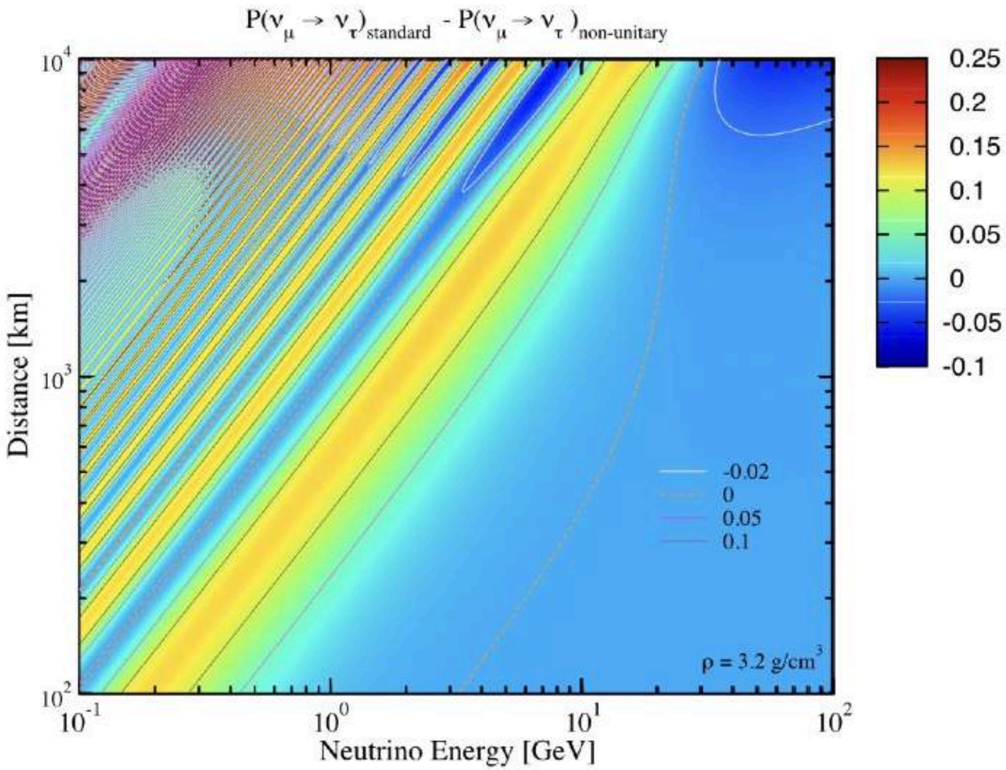
- Leading order terms = Zeroth order in W

ΔP large at solar- and atm-MSW enhanced regions

$$\alpha_{11} = 0.990,\ \alpha_{21} = -0.0141,\ \alpha_{22} = 0.995,\ \alpha_{31} = -0.0445,$$

$$\alpha_{32} = -0.0316,\ \alpha_{33} = 0.949.$$





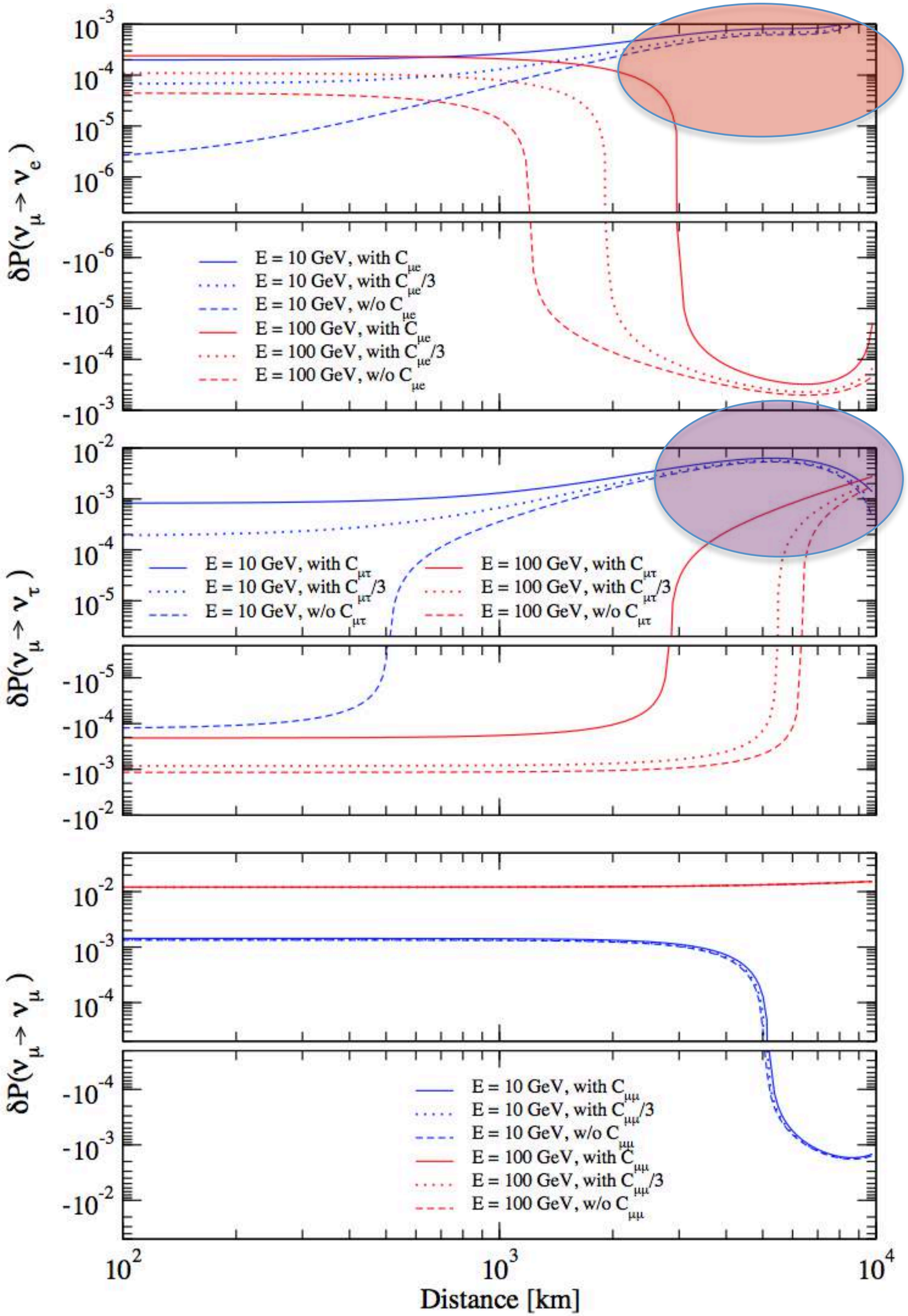
Sen

Large ~W^2 corrections?

- Order W2 correction terms

  → small in most of the regions of L-E, but sizeable in limited places

- High energy, long baseline → IceCube, PINGU, Hyper-K



How to proceed?

How to proceed?

- Find hint for non-unitarity → can be done with leading order P, e.g., (implicit) order W^2 correction in disappearance channels

$$\left(\sum_{j=1}^{3} |U_{\alpha j}|^2\right)^2 = \left(1 - \sum_{J=4}^{N+3} |W_{\alpha J}|^2\right)^2 = 1 - \mathcal{O}(W^2)$$

- This step is being done by many people: common to high-E and low-E UV

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[39] F. J. Escrihuela, D. V. Forero, O. G. Miranda, M. Tórtola and J. W. F. Valle, "On the description of non-unitary neutrino mixing," Phys. Rev. D **92** (2015) no.5, 053009 doi:10.1103/PhysRevD.92.053009 [arXiv:1503.08879 [hep-ph]].

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[41] F. J. Escrihuela, D. V. Forero, O. G. Miranda, M. Tórtola and J. W. F. Valle, "Probing CP violation with non-unitary mixing in long-baseline neutrino oscillation experiments: DUNE as a case study," New J. Phys. **19** (2017) no.9, 093005 doi:10.1088/1367-2630/aa79ec [arXiv:1612.07377 [hep-ph]].

Plus many more !!

How to proceed? 2

- Then, if we see UV, the next step would be:
- Detect P leaking term $C_{\alpha\beta}$
- Detect explicit W^2 corrections

→ To distinguish low-E UV from high-E UV

- So far, we only did "JUNO" with known flux
- Detecting $C_{\alpha\beta}$ (in accelerator) requires near detector measurement
- T2K/T2HK → ND at 300m → OM for Δm^2=3 eV^2 → ND before averaged out, if we limit to Δm^2 <~0.3 eV^2
- W^2 terms → IceCube, Hyper-K atmospheric nu?

Conclusion (1st part)

- Mixing parameter measurement in progress → looks converging
- Accumulating hints for lepton CP violation $\delta \sim 3\pi/2$ ➡ δ could me measured much earlier than we thought?
- $\delta \sim 3\pi/2$ implies NOvA could determine Nu mass ordering
- ~3 σ evidence for both CP and mass ordering before Hyper-K and DUNE? ➡

Conclusion (non-unitarity)

- General structure of nu oscillation in active nu sector of (3+N) unitary system is analyzed in vacuum and in matter in the context of low-E unitarity violation
- A new term, the "probability leaking term" found (leaking to sterile sector)
- Distinguishes between Low-E vs High-E unitarity violation
- Conditions for sterile sector model-independent P in vacuum and in matter are elucidated $m_J^2 > 0.1\ \mathrm{eV}^2$
- Likely to be insensitive to sterile interactions

Conclusion (non-unitarity2)

- JUNO analysis shows one can constrain UV in ν_e row at a high level

$C_{ee} \sim 10^{-4}$, $1-\Sigma|U_{ei}|^2 \sim 0.01$ (both 1 σ)

- Non-unitarity effect in the leading order (W^0) seems sizeable in solar- and atm MSW regions (Probability level)
- generally requires $L \sim 3000\text{-}10^4$ km
- W^2 correction sizeable in limited L-E regions

distinguishes between low-E from high-E UV

$L \sim 3000\text{-}10^4$ km

Thank you Belen (and IFT) for bringing me to Madrid