Searching for P-odd effects in heavy ion collisions

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EXCELENCIA MARÍA DE MAEZTU

Publications

Collaboration with S. Afonin, A. Andrianov, V. Andrianov, X. Planells

Some publications:

- A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.
- A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C 73 (2013) 2294.
- A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C 74 (2014) 2776.
- A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Rev. D 90 (2014) 034024.
- S. Afonin, A. A. Andrianov & D. Espriu, Phys. Lett. B745 (2015) 52.

Outline

- Why P and CP might not be good symmetries in HIC
- Effective meson theory in a medium with LPB

  Scalar mesons: modified chiral lagrangian

  Vector mesons: Maxwell-Chern-Simons electrodynamics
- Possible Manifestations of P-odd effects in HIC

  Dalitz decays in a P-odd environment

  ρ broadening

  In-medium $V \to \ell^+\ell^-$ decays

  Observables sensitive to P-odd effects
- Understanding the phase diagram with $\mu_5 \neq 0$
- A possible bottom-up approach
- Conclusions & beyond

Why considering P- and CP- odd effects in HIC?

Parity is one of the well established global symmetries of strong interactions. Yet there are reasons to believe that it may be broken. No fundamental principle forbids spontaneous parity breaking for $\mu \neq 0$ or out of equilibrium.

- *P- and CP-odd condensates = "pion" or "eta" condensates (finite density)*
  - A. Vilenkin, Phys. Rev. D22, 3080 (1980);
  - A.B. Migdal, Zh. Eksp. Teor. Fiz. 61 (1971);
  - T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974);
  - A. A. Andrianov & D. Espriu, Phys. Lett. B 663 (2008) 450;
  - A. A. Andrianov, V. A. Andrianov & D. Espriu, Phys. Lett. B 678 (2009) 416.
- *Topological fluctuations (finite volume, large T)*
  - D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998);
  - K. Buckley, T. Fugleberg, & A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814;
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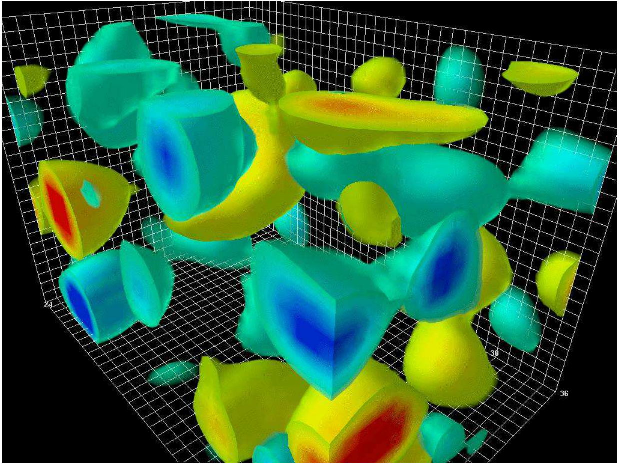


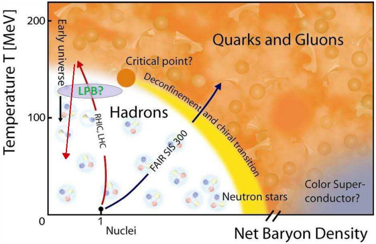
Lattice simulation of fluctuations in the local topological charge [Leinweber]

Motivation of local parity breaking

Local large fluctuations in the topological charge should exist in a hot environment.

For *peripheral* heavy ion collisions: may lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

For *central* collisions (and light quarks): may trigger an ephemeral phase with axial chemical potential $\mu_5 \neq 0$.



Let us see how this comes about and consider possible consequences in the *hadronic* phase

Generating a chiral charge

Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron-like transitions

$$T_5 = \frac{1}{8\pi^2}\int_{\text{vol.}} d^3x \varepsilon_{jkl} \text{Tr}\left(G^j \partial^k G^l - i\frac{2}{3} G^j G^k G^l\right)$$

and survive for a sizeable lifetime in a heavy-ion fireball.

$$\Delta T_5 = T_5(t_f) - T_5(t_i) = \frac{1}{8\pi^2}\int_{t_i}^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right).$$

An induced axial charge is conserved during $\tau_{fireball}$ if $m = 0$

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5\right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle N_R - N_L\rangle$$

- For u, d quarks $1/m_q \sim 1/5$ MeV^{-1} ~ 40 fm $\gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150$ MeV^{-1} ~ 1 fm $\ll \tau_{\text{fireball}}$ and $\langle Q_5^s\rangle \simeq 0$ due to left-right oscillations.

Chiral chemical potential

If Q_5 is conserved along the collision process, in a quasi equilibrium situation we can introduce its conjugated chemical potential μ_5
Then it is possible to write several terms with the right symmetry in the lagrangian e.g.:

$$\Delta\mathcal{L}_{top} = \mu_5 \Delta T_5$$

$$\Delta\mathcal{L}_{CME} = \mu_5 \vec{E}\vec{B}$$

.....

These terms break P and CP (and of course T).
In this situation we speak of 'local parity breaking' because while the total topological charge can be conserved overall, but 'islands' with $\mu_5 \neq 0$ exist. (Glasma picture ?)

Effective meson theory in a medium with LPB

We have two possible isospin structures for μ_5:

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Isotriplet and/or isosinglet pseudoscalar background ($\mu \gg T$) [FAIR, NICA]

Only the first case will be considered in this talk.

We will now build an effective theory for mesons in a P- and CP-odd environment.
We will deal in turn with scalars and vector mesons.

The scalar sector can be estimated by using the spurion technique in the Lagrangian

$$D_\mu \Longrightarrow D_\mu - i\{\mu_5 \delta_{0\mu}, \cdot\} \qquad D = 2$$

and constructing a generalised sigma model with the light scalar mesons $\sigma, \vec{\pi}, \eta, \eta', \vec{a}_0$

Due to the explicit parity breaking there is mixing in the $\sigma - \eta - \eta'$ and $\vec{\pi} - \vec{a}_0$ channels.

The new eigenstates do not have a well defined parity. The resulting $J = 0$ eigenstates are *all* coupled to $\pi\pi$ due to the strong mixing with the σ, and *thermalize* in the HIC fireball.

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$
$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$
$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger H H^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

where

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle\Sigma_{ii}\rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \vec{v}^2 v_i + c\frac{v_u v_d v_s}{v_i} = 0.$$

For further purposes we need the non-strange meson sector and η_s

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

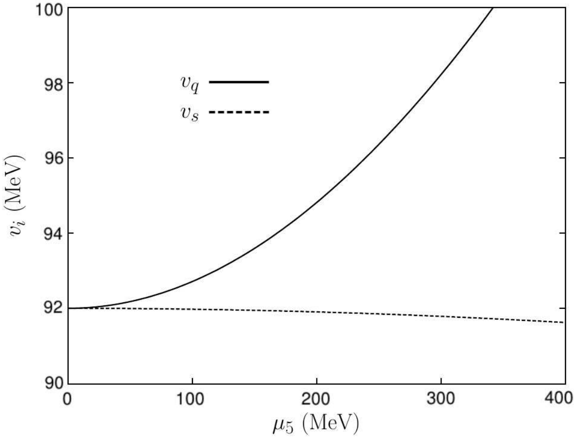
For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV. The coupling constants (in units of MeV) are fitted to phenomenology assuming isospin symmetry via χ^2 minimization (`MINUIT`):

$$b = -3510100/m,\ M^2 = 1255600,\ c = 1252.2,\ \lambda_1 = 67.007,$$

$$\lambda_2 = 9.3126,\ d_1 = -1051.7/m,\ d_2 = 523.21/m,$$

where $m \equiv m_q = (m_u + m_d)/2$ and $m/m_s \simeq 1/25$.

Vacuum: for non-vanishing isosinglet μ_5 we impose our solutions to be $v_u = v_d = v_q \neq v_s$.



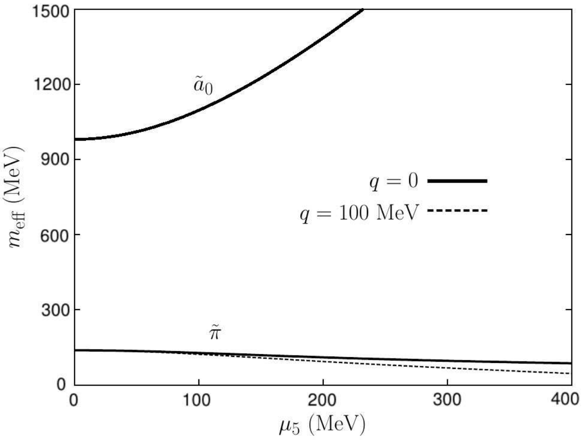
We present a simple case of mixing due to LPB in the isotriplet sector with π and a_0. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2} m_1^2 a_0^2 - \frac{1}{2} m_2^2 \pi^2 - 4\mu_5 a_0 \dot{\pi},$$

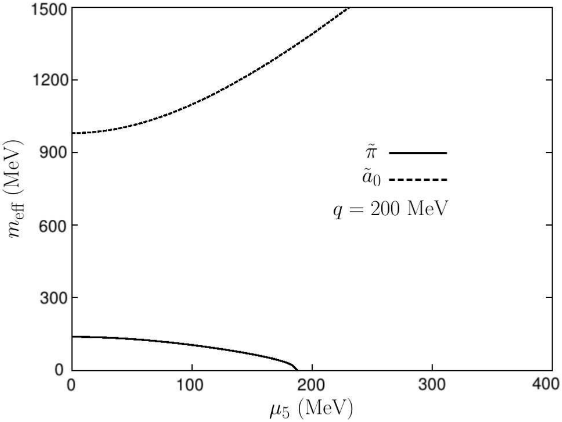
where

$$\begin{aligned} m_1^2 =& -2[M^2 - 2(3\lambda_1 + \lambda_2) v_q^2 - \lambda_2 v_s^2 \\ & - c v_s + 2(3d_1 + 2d_2) m v_q + 2 d_2 m_s v_s + 2\mu_5^2] \\ m_2^2 =& \frac{2m}{v_q} \left[b + (d_1 + 2d_2) v_q^2 + d_2 v_s^2 \right] \end{aligned}$$

After diagonalization in the momentum representation, the new (momentum-dependent) eigenstates are defined $\tilde{\pi}$ and $\tilde{a}_0$.



For high energies $k_0, |\vec{k}| > m_1 m_2/(4\mu_5) \equiv k^c_{\tilde{\pi}}$, in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are always positive (no vacuum instabilities). $\tilde{a}_0$ mass shows an enhancement, but μ_5 has to be understood as a perturbatively small parameter. A better treatment of $\tilde{a}_0$ would require heavier degrees of freedom.



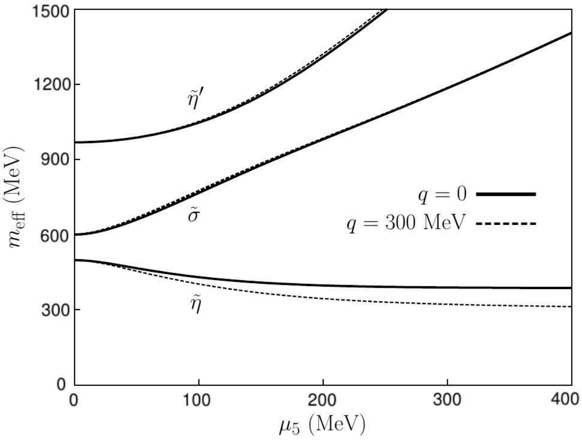
In the isosinglet sector, we show the mixing of η, σ and η'. The kinetic and mixing terms in the Lagrangian are given by

$$\begin{aligned}\mathcal{L} =&\frac{1}{2}[(\partial\sigma)^2 + (\partial\eta_q)^2 + (\partial\eta_s)^2] - \frac{1}{2}m_3^2\sigma^2 - \frac{1}{2}m_4^2\eta_q^2 - \frac{1}{2}m_5^2\eta_s^2\\ &- 4\mu_5\sigma\dot{\eta}_q - 2\sqrt{2}cv_q\eta_q\eta_s,\end{aligned}$$

where

$$\begin{aligned}m_3^2 =& -2(M^2 - 6(\lambda_1+\lambda_2)v_q^2 - \lambda_2 v_s^2 + cv_s\\ &+ 6(d_1+2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2),\\ m_4^2 =&\frac{2m}{v_q}\left[b + (d_1+2d_2)v_q^2 + d_2 v_s^2\right] + 2cv_s,\\ m_5^2 =&\frac{2m_s}{v_s}[b + 2d_2 v_q^2 + (d_1+d_2)v_s^2] + \frac{cv_q^2}{v_s}.\end{aligned}$$

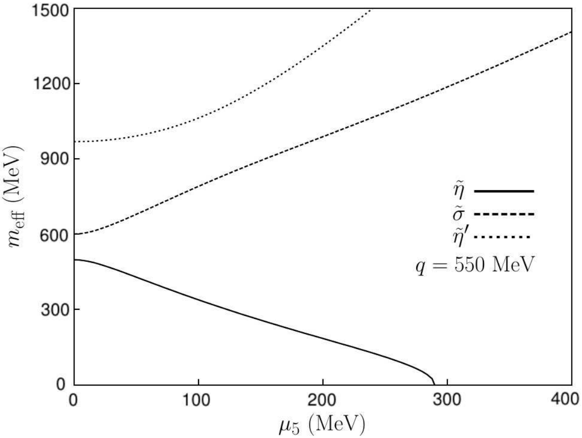
After diagonalization, the new eigenstates are $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$.



Again, for high energies $k_0, |\vec{k}| > k^c_{\tilde{\eta}}$ with

$k^c_{\tilde{\eta}} \equiv \frac{m_3}{4\mu_5 m_5}\sqrt{m_4^2 m_5^2 - 8c^2 v_q^2}$, in-medium $\tilde{\eta}$ goes tachyonic.

$\tilde{\eta}'$ mass also shows an enhancement and a better treatment would require the inclusion of heavier degrees of freedom.



The cubic couplings used to calculate the widths $\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}' \to \tilde{\pi}\tilde{\pi}$ from the Lagrangian are given by

$$\mathcal{L}_{\sigma aa} = 2[(3d_1 + 2d_2)m - 2(3\lambda_1 + \lambda_2)v_q]\sigma \vec{a}_0^2,$$

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{v_q^2}\left[(\partial\vec{\pi})^2 v_q - (b + 3(d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\vec{\pi}^2\right]\sigma,$$

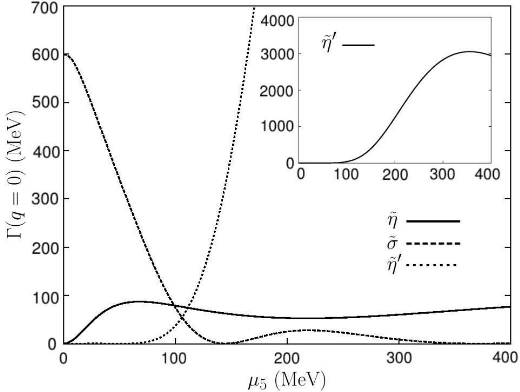
$$\mathcal{L}_{\eta a\pi} = \frac{2}{v_q^2}\vec{a}_0[\partial\eta_q\partial\vec{\pi}v_q - (b + (3d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\eta_q\vec{\pi}],$$

$$\mathcal{L}_{\sigma a\pi} = -\frac{4\mu_5}{v_q}\sigma\vec{a}_0\dot{\vec{\pi}}, \quad \mathcal{L}_{\eta aa} = -\frac{2\mu_5}{v_q}\dot{\eta}_q\vec{a}_0^2, \quad \mathcal{L}_{\eta\pi\pi} = 0.$$

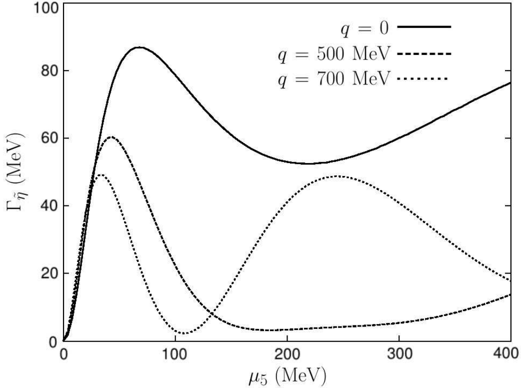
After diagonalization, one replaces the initial $\{\eta_q, \eta_s, \sigma\}$ to $\{\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}'\}$ and $\{\pi, a_0\}$ to $\{\tilde{\pi}, \tilde{a}_0\}$.

The widths are firstly computed at the rest frame of the decaying particle and secondly with a boosted particle.

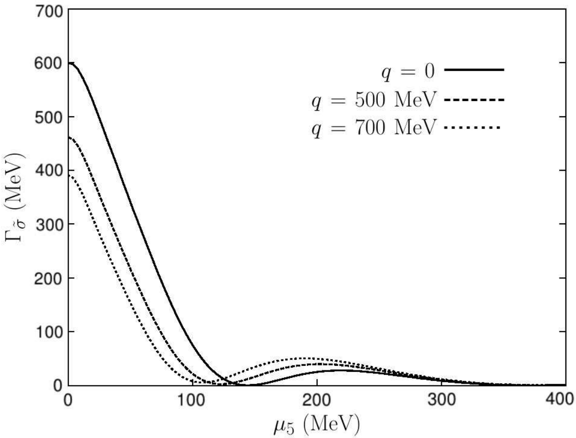
$\tilde{\eta}$ exhibits a smooth behaviour with $\langle \Gamma_{\tilde{\eta}} \rangle \sim 60$ MeV $\leftrightarrow$ mean free path ~ 3 fm $\lesssim L_{\text{fireball}} \sim 5 \div 10$ fm. Possible thermalization! Down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and becomes stable. The visible bumps in these two channels seem to reflect the tachyonic nature of the decaying $\tilde{\pi}$. $\tilde{\eta}'$ width grows up to the GeV scale (violation of unitarity). More degrees of freedom are needed.



Small variations at low 3-momenta: 2 initial bumps slowly separate as one increases q. $\Gamma_{\tilde{\eta}}(\mu_5, q)$ exhibits a saddle point at $\mu_5^* \sim 240$ MeV and $q^* \sim 500$ MeV. For large 3-momenta, a third intermediate bump appears (creation of 2 tachyons) and grows fast as one increases q becoming the global maximum when $q \gtrsim 700$ MeV.



In the $\tilde{\sigma}$ and $\tilde{\eta}'$ channels no huge differences arise when boosting the decaying particle. The most salient behaviour in the $\tilde{\sigma}$ width is the separation of the two minima as one increases q.



What to make out of all this?

It is possible to formulate that includes explicitly the chiral chemical potential μ_5

As long as $\mu_5 \neq 0$ states of different parity mix.

Typically all are strongly coupled to two pion (new pion!) states. If LPB is present this is likely to have a strong influence in the properties of the hadronic phase, particularly if μ_5 is substantial.

The calculation becomes soon unreliable when μ_5 grows. New techniques are called for.

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Effective scalar/pseudosalar meson theory with μ_5

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In this case the most relevant operator has $D = 3$. P- and CP-odd effects will appear through the Chern-Simons term:

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \partial_\mu \hat{a}(\vec{x}, t) = \delta_{\mu 0}\partial_0 \hat{a}(t)$ (for a spatially homogeneous, time dependent background).

We shall assume $\partial_0 \hat{a}(t) \sim \hat{\zeta} \propto \mu_5$.

Vector mesons will be introduced and treated in the conventional way using the Vector Meson Dominance model.

For vector mesons here will be no mixing (at this order) but a distortion of the spectrum.

Note the breaking of Lorentz symmetry in both cases.

Vector Meson Dominance bosonization:

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathbb{I} + \frac{1}{2} g_\rho \rho^0_\mu \tau_3,$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6$.

Maxwell and mass terms

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu}\right) + \frac{1}{2} V_{\mu,a} (\hat{m}^2)_{a,b} V^\mu_b$$

$$\hat{m}^2 \simeq m_V^2 \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}$$

P-odd interaction

$$\mathcal{L}_{P-odd}(k) = -\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(\hat{\zeta}_\mu \hat{V}_\nu \hat{V}_{\rho\sigma}\right) = \frac{1}{2}\zeta\epsilon_{jkl}\, V_{j,a}\, N_{ab}\, \partial_k V_{l,b}$$

For an isosinglet pseudoscalar background:

$$(N^\theta_{ab}) \simeq \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det\left(N^\theta\right) = 0$$

Simultaneous diagonalization of the matrices $\hat{m}^2, N$

$$N = \mathrm{diag}\left[0,\, 1,\, 1 + \frac{10e^2}{9g^2}\right] \simeq \mathrm{diag}\,[0,\, 1,\, 1]$$

$$\hat{m}^2 = m_V^2\,\mathrm{diag}\left[0,\, 1,\, 1 + \frac{10e^2}{9g^2}\right] \simeq \mathrm{diag}\,[0,\, 1,\, 1]$$

After diagonalization the photon itself is unaffected.

Vector mesons exhibit the following dispersion relation:

$$m_{V,\epsilon}^2 = m_V^2 - \epsilon\zeta|\vec{k}|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.

The position of the poles for $\pm$ polarized mesons is moving with wave vector $|\vec{k}|$.

Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$.

This splitting unambiguously signifies P breaking. Can it be measured?

Possible manifestations of P-odd effects in HIC

Look for e.m. probes $\Rightarrow$ 'anomalies' in dilepton production

$$\rho, \omega \to e^+ e^-.$$

The total dilepton production also receives potential contribution from the pseudoscalar Dalitz decays

$$\eta, \eta' \to \gamma e^+ e^-,$$

and the ω Dalitz decay

$$\omega \to \pi^0 e^+ e^-,$$

Results computed using PHENIX experimental cuts: $|y_{ee}| < 0.35$, $|\vec{p}_t^{\,e}| > 200$ MeV and gaussian M_{ee} smearing (width=10 MeV); and an effective temperature $T = 220$ MeV.

In fact dilepton production shows a number of anomalies...

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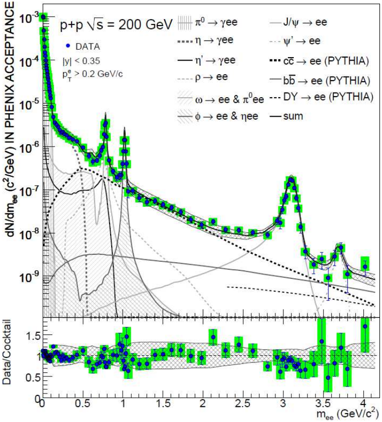
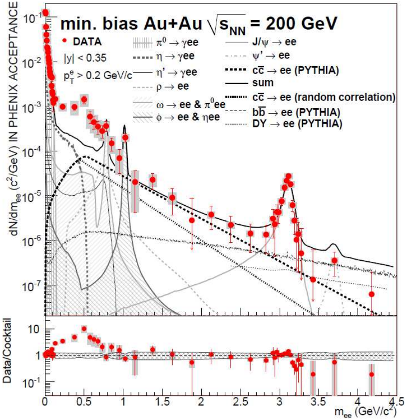
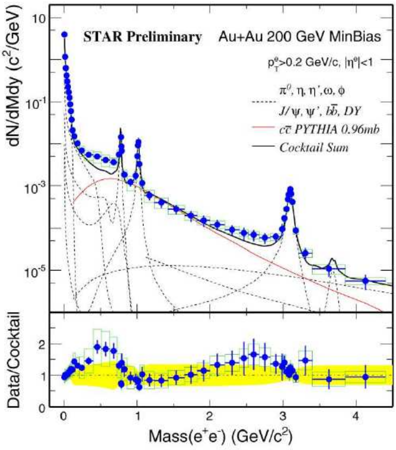
Results computed using PHENIX experimental cuts: $|y_{ee}| < 0.35$, $|\vec{p}_t^{\,e}| > 200$ MeV and gaussian M_{ee} smearing (width=10 MeV); and an effective temperature $T = 220$ MeV.

In fact dilepton production shows a number of anomalies...

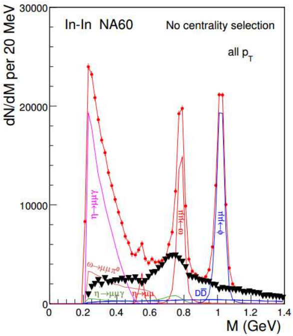

p+p √s = 200 GeV
DATA
|y| < 0.35
p$_T^e$ > 0.2 GeV/c
π⁰ → γee
η → γee
η' → γee
ρ → ee
ω → ee & π⁰ee
φ → ee & ηee
J/ψ → ee
ψ' → ee
cc̄ → ee (PYTHIA)
bb̄ → ee (PYTHIA)
DY → ee (PYTHIA)
sum
dN/dm$_{ee}$ (c²/GeV) IN PHENIX ACCEPTANCE
Data/Cocktail
m$_{ee}$ (GeV/c²)


min. bias Au+Au √s$_{NN}$ = 200 GeV
DATA
|y| < 0.35
p$_T^e$ > 0.2 GeV/c
π⁰ → γee
η → γee
η' → γee
ρ → ee
ω → ee & π⁰ee
φ → ee & ηee
J/ψ → ee
ψ' → ee
cc̄ → ee (PYTHIA)
sum
cc̄ → ee (random correlation)
bb̄ → ee (PYTHIA)
DY → ee (PYTHIA)
dN/dm$_{ee}$ (c²/GeV) IN PHENIX ACCEPTANCE
Data/Cocktail
m$_{ee}$ (GeV/c²)

Manifestation of LPB in heavy ion collisions

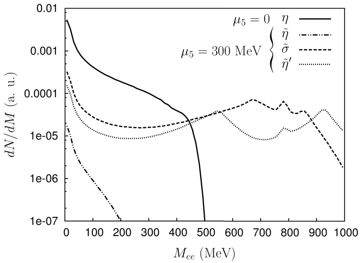


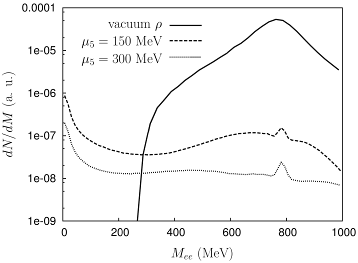
A large broadening of the ρ spectrum was measured by the NA60 collaboration several years ago



It is claimed that the broadening can be understood by 'conventional' physics...

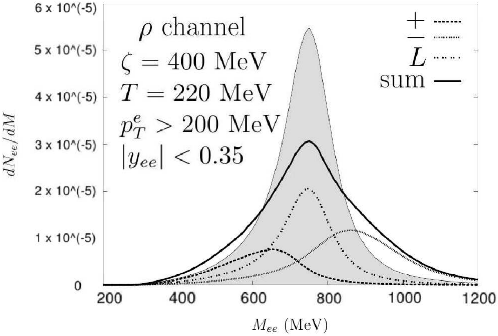
Dalitz decays in a P-odd environment





Clearly insufficient to explain the enhancement in PHENIX in the region 200-500 MeV but it helps a little in STAR data.

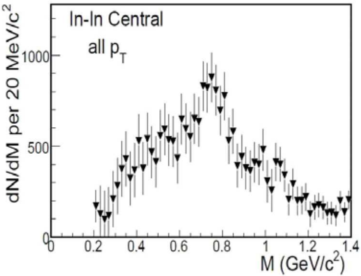
P-odd effects introduce broadening in a natural manner.

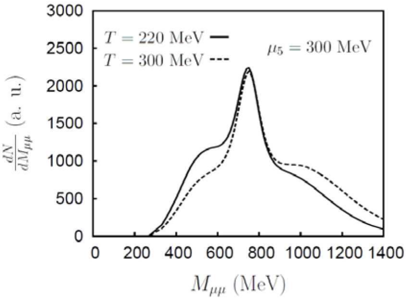


Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

ρ broadening

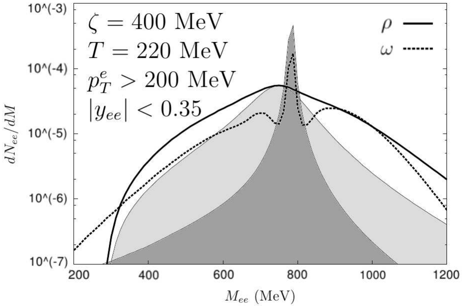
And indeed the peculiar shape of the ρ spectrum measured by NA60 is grossly reproduced:





Only one parameter is fitted — the value of μ_5.

In-medium ρ and ω channels (solid and dashed line) and their vacuum contributions (light and dark shaded regions) for $\mu_5 = 290$ MeV.



Enhancement of the dilepton yield that could (at least partly) explain the anomalous enhancement seen by PHENIX and STAR.

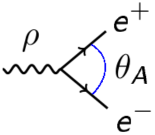
One of the clearest signals of P-odd effects is the separation between polarizations.

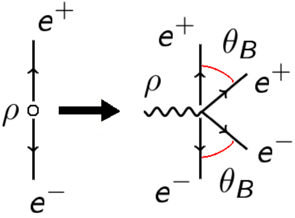
Is there any way to study these decays in order to separate the different polarizations?

It is well known that the angular distribution of leptons carries the information on the polarization. However, current angular distribution studies are not thought to detect possible P-odd effects.

We will define the two following angles:

Case A: θ_A is the angle between the two outgoing leptons in the laboratory frame.



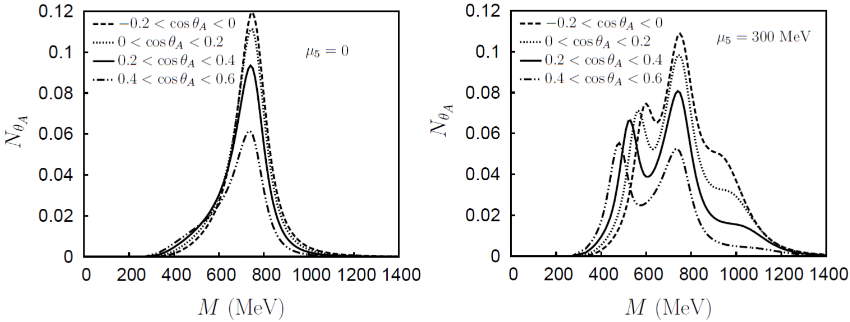


Case B: θ_B is the angle between one of the two outgoing leptons in the laboratory frame and the same lepton in the dilepton rest frame.

In order to isolate the transverse polarizations, we will perform different cuts for each angle and study the variations of the ρ (and ω) spectral function.

Angular analysis of $V \to \ell^+\ell^-$ decays
CASE A

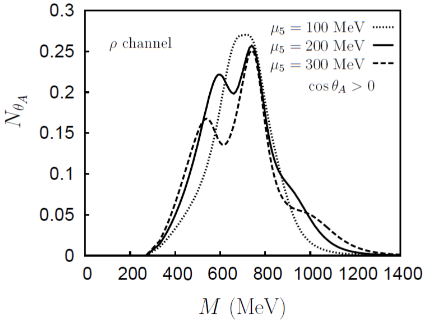
Angle θ_A between the two outgoing leptons in the laboratory frame.

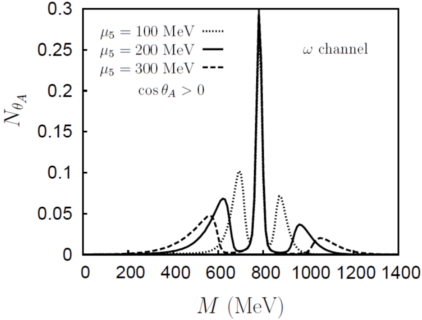


ρ spectral function depending on the dielectron invariant mass M in vacuum ($\mu_5 = 0$) and in a parity-breaking medium with $\mu_5 = 300$ MeV for different ranges of θ_A.

Visible secondary peak in a P-odd medium! Important reduction of the number of events: the vacuum peak shows at most $\simeq 10\%$ of the events one would expect without any cut in θ_A.

If the secondary peak was found for a particular angular coverage, its position would be an unambiguous measurement of μ_5.

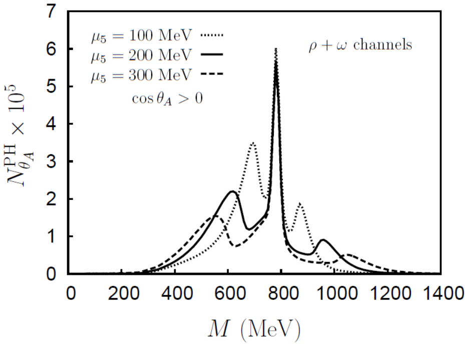




ρ and ω spectral functions depending on the invariant mass M and integrating $\cos\theta_A \geq 0$ for $\mu_5 = 100, 200$ and 300 MeV.

The vacuum ρ peak hides the secondary one for $\mu_5 \simeq 100$ MeV due to its large width. For ω, all the peaks are visible.

We also present the combination of the ρ and ω channels. In this case, the total production is normalized to PHENIX data.

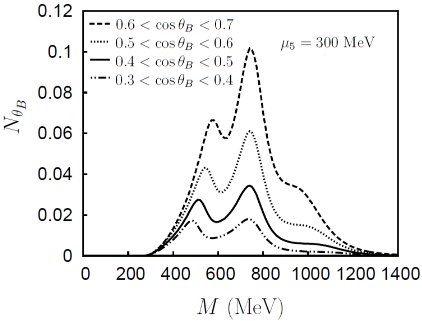


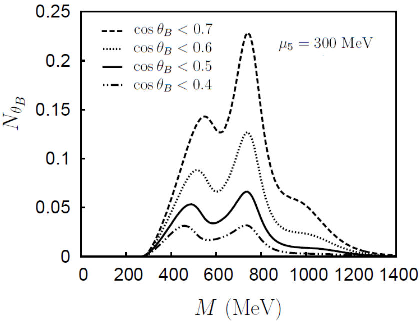
Combination of ρ and ω spectral functions depending on the invariant mass M and integrating $\cos\theta_A \geq 0$ for $\mu_5 = 100, 200$ and 300 MeV.

Angular analysis of $V \to \ell^+\ell^-$ decays

CASE B

Angle θ_B between one of the two outgoing leptons in the laboratory frame and the same lepton in the dilepton rest frame.





ρ spectral function depending on the invariant mass M for different ranges of θ_B for fixed $\mu_5 = 300$ MeV.

The main difference with θ_A is a slightly smaller number of events. The rest of the analysis is completely equivalent.

Checking LPB in QCD with a finite μ is difficult: lattice simulations with $\mu \neq 0$ present serious difficulties.
Simpler models reproducing the main features of the theory may be useful.

NJL vs QCD:

- ✓ Chiral symmetry breaking (CSB) and global symmetries
- × Confinement and CS restoration

Previous studies of NJL + μ or NJL + μ_5, but not including both.
We incorporate both chemical potentials in order to find the different stable phases.

Aiming at: finding a phase where the vacuum itself is substantially modified by μ_5

NJL Lagrangian for 2 flavours and N colours (in Euclidean conventions)

$$\mathcal{L} = \bar\psi(\partial\!\!\!/ + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5)\psi$$
$$- \frac{G_1}{N}[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2] - \frac{G_2}{N}[(\bar\psi\vec\tau\psi)^2 + (\bar\psi i\gamma_5\psi)^2],$$

with $U(2)_L \times U(2)_R$ ($SU(2)_L \times SU(2)_R \times U(1)_V$) in the case that $G_1 = G_2$ ($G_1 \neq G_2$).

2 doublets are introduced: $\{\sigma, \vec\pi\}$ and $\{\eta \equiv \eta_q, \vec a \equiv \vec a_0(980)\}$ and shift in the fields is performed in order to remove the 4-fermion interactions

$$\mathcal{L} = \bar\psi[\partial\!\!\!/ + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + (\sigma + i\gamma_5\vec\tau\vec\pi) + (i\gamma_5\eta + \vec\tau\vec a)]\psi$$
$$+ \frac{N}{4G_1}(\sigma^2 + \vec\pi^2) + \frac{N}{4G_2}(\eta^2 + \vec a^2)$$

Fermions are integrated out and fluctuations are neglected (mean field approximation) [$\sigma = \langle\sigma\rangle$, etc]

$$V_{\text{eff}} = \frac{N}{4G_1}(\sigma^2 + \vec{\pi}^2) + \frac{N}{4G_2}(\eta^2 + \vec{a}^2) - \operatorname{Tr}\log\mathcal{M}(\mu,\mu_5),$$

where the trace is in the isospin and Dirac spaces in addition to a 4-momentum integration.

The fermion operator is defined as

$$\mathcal{M}(\mu,\mu_5) = \not{\partial} + (M + \vec{\tau}\vec{a}) - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + i\gamma_5(\vec{\tau}\vec{\pi} + \eta),$$

with the constituent quark mass $M \equiv m + \sigma$.

In the search for stable configurations of the potential, we will need the derivatives of the fermion determinant, which are divergent in the UV. We use DR and a 3-dimensional cut-off both at $T = 0$.

We will explore the different phases allowed by the previous potential by solving the gap equations and analysing the second derivatives in order to study their stability. For simplicity, we will assume $a = 0$.

Stable phases:

- Chirally symmetric phase (in the chiral limit with $m = 0$ and $\mu = \mu_5 = 0$): $\sigma = 0$ [not discussed]
- Chirally broken phase (CSB): $\sigma \neq 0$
- Parity breaking phase with CSB: $\sigma, \eta \neq 0$

No stable parity breaking phases with $\pi \neq 0$ (with or without CSB) are found.

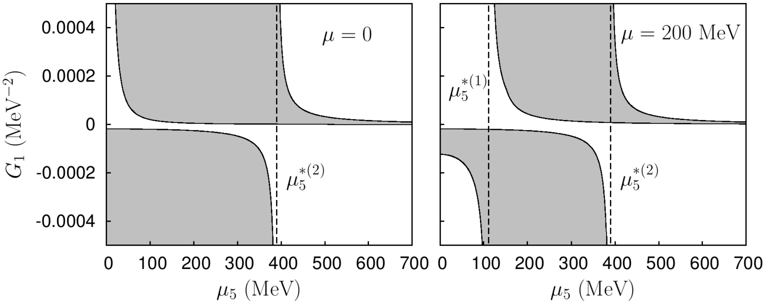
In the CSB phase we find the following mass spectrum relations for any non-trivial μ and μ_5:

$$m_\sigma^2 - m_\pi^2 = m_a^2 - m_\eta^2 > 0,$$

$$m_a^2 - m_\sigma^2 = m_\eta^2 - m_\pi^2 = \frac{N}{2}\left(\frac{1}{G_2} - \frac{1}{G_1}\right).$$

The second equation implies that $\frac{1}{G_2} - \frac{1}{G_1} > 0$ if we want to make an analogy with QCD.

Allowed region of G_1 as a function of μ_5 with fixed μ and $m = 0$ for a stable CSB phase (dark region).



Non-trivial behaviour due to the presence of both chemical potentials.

The only possibility for P-breaking is $\sigma \neq 0$ and $\eta \neq 0$. The main features are:

- Mixing among the $\sigma - \eta$ and $\pi - a$ states. No states with well defined parity.
- $U(1)_A$ is needed to be broken $(G_1 \neq G_2)$ together with $m \neq 0$.
- $M \simeq \sigma =$ ctant and the dependence on μ, μ_5 is absorbed in η.
- Stability requires $\frac{1}{G_2} - \frac{1}{G_1} < 0$!!

Recall that in the CSB phase:

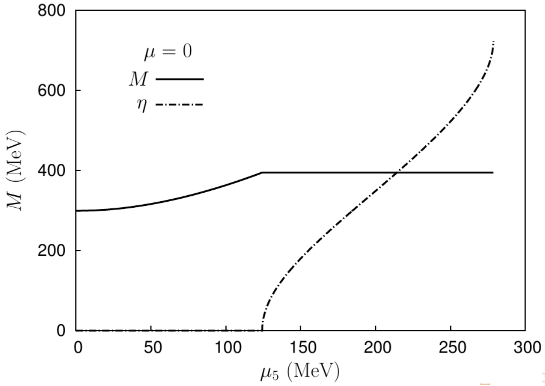
$$m_a^2 - m_\sigma^2 = m_\eta^2 - m_\pi^2 = \frac{N}{2}\left(\frac{1}{G_2} - \frac{1}{G_1}\right)$$

Stable P-breaking phase $\Longrightarrow$ ~~QCD~~

Requiring stability to the CSB and the P-breaking phases implies:

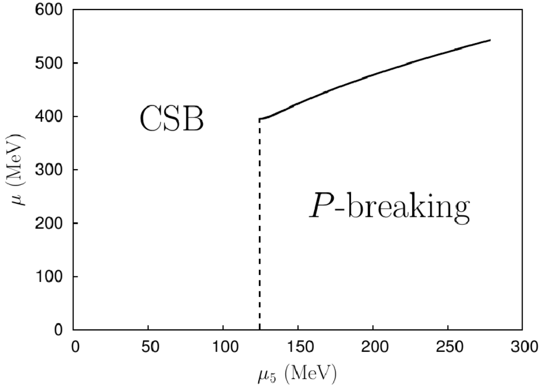
$$\frac{1}{G_1}\left(1-\frac{m}{M_0}\right) < \frac{1}{G_2} < \frac{1}{G_1}$$

where $M_0 = M(G_1, \mu = \mu_5 = 0)$. If $m \to 0$ the inequalities break down! For $\mu < M^c$, we have a 2nd order phase transition.



For $\mu > M^c$ the phase transition is a 1st order one. The behaviour of the condensates is similar to the previous case but with some complications.

Transition line from the CSB to the P-breaking phase.



The vertical dashed line is related to a 2nd order phase transition while the solid one corresponds to a 1st order one.

What do we learn from NJL?

One should not forget this is just a toy model

A rich phase structure appears

A phase with both CSB and η condensate appears, but at large values of μ_5 only. Too large? In QCD?

When this happens the spectrum is very different from QCD

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When this happens the spectrum is very different from QCD

A soft-wall model with 5D Abelian fields L and R dual (on the AdS_5 boundary) to the sources of the left and right 4D vector currents including a CS term

$$S = S_{\text{free}}[L] + S_{\text{free}}[R] + S_{\text{CS}}[L] - S_{\text{CS}}[R]$$

$$S_{\text{free}}[B] = -\frac{1}{8g_5^2}\int d^4x dz e^{\varphi}\sqrt{g} B_{MN}B^{MN}, \qquad B = L, R$$

$$S_{\text{CS}}[B] = -k\int d^4x dz \epsilon^{MNABC} B_M B_{NA} B_{BC}$$

AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad \mu = 0, 1, 2, 3$$

The constants g_5 and k are fixed by matching to the ultraviolet asymptotics of the two-point vector correlator and to the axial anomaly

$$\frac{g_5^2}{R} = \frac{12\pi^2}{N_c}, \qquad k = \frac{N_c}{24\pi^2}$$

A bottom-up approach

In terms of the vector, $V = (L + R)/2$ and axial-vector $A = (R - L)/2$, fields the free and CS parts of the action are

$$S_{\text{free}} = -\frac{1}{4g_5^2} \int d^4x dz \frac{e^{\varphi}}{z} \left(V_{MN}^2 + A_{MN}^2 \right)$$

$$S_{\text{CS}} = -k \int d^4x dz \epsilon^{MNABC} A_M \left(V_{NA} V_{BC} + A_{NA} A_{BC} \right)$$

If one wishes to provide conservation of the 4D vector current the Bardeen surface counterterm is neede

$$S_{\text{B}} = 2k \int d^4x \epsilon^{\mu\nu\lambda\rho} A_\mu V_\nu V_{\lambda\rho}$$

Then one obtains the standard result for the covariant anomaly

$$\partial_\mu J_\mu^V = 0, \qquad \partial_\mu J_\mu^A = 3k V_{\mu\nu} \tilde{V}_{\mu\nu} + k A_{\mu\nu} \tilde{A}_{\mu\nu},$$

We assume for the vacuum expectation value of the axial vector field that

$$\langle A_M \rangle = \langle A_z \rangle = \mu_5 x_0 f(z)$$

where the shape function $f(z)$ will be specified later.
For the vector part of the action

$$S = \frac{R}{g_5^2} \int d^4xdz \left(-\frac{e^{\varphi}}{4z} V_{MN}^2 + \xi \mu_5 f \epsilon^{05ABC} V_A \partial_B V_C \right)$$

$\xi = \frac{2kg_5^2}{R} = 1$. In the axial gauge $V_z = 0$ the equation of motion reads

$$\partial_z \left(\frac{e^{\varphi}}{z} \partial_z V_\mu \right) - \frac{e^{\varphi}}{z} \partial_\mu^2 V_\mu - 2\mu_5 f \epsilon_{mik} \partial_i V_k = 0.$$

Fourier transform and assume the ansatz $V_\mu(p,z) = \varepsilon_\mu v(z)$. Define

$$\hat{F} = \partial_z\left(\frac{e^\varphi}{z}\partial_z\right) + \frac{e^\varphi}{z}p^2$$

The EOM for vector fields reads

$$\left(\hat{F}\vec{\varepsilon} + i2\mu_5 f\,\vec{p}\times\vec{\varepsilon}\right)v = 0$$

The physical spectrum is given by the eigenvalues $p_n^2 = m_n^2$ of normalizable solutions. However, the last term induces mixing between different polarizations.

Diagonalization is possible with the help of the projectors

$$\hat{P}^{\parallel}_{ik} = \frac{p_i p_k}{\vec{p}^2}$$

$$\hat{P}^{\pm}_{ik} = \frac{1}{2}\left[\delta_{ik} - \frac{p_i p_k}{\vec{p}^2} \pm \frac{i}{|\vec{p}|}\epsilon_{ikn}p_n\right]$$

The EOM take the form

$$\hat{F} v^{\parallel} = 0 \qquad \left(\hat{F} \pm 2\mu_5 f |\vec{p}|\right) v^{\pm} = 0$$

Thus, the longitudinal and circular polarizations will have different masses, the latter being dependent on the momentum.
The simplest SW model resulting in Regge-like spectrum is given by

$$\varphi = -\lambda^2 z^2$$

Consider the longitudinal polarization. Define

$$v(z) = e^{\lambda^2 z^2/2} \sqrt{z} \psi(z)$$

$$-\partial_z^2 \psi_n^{\parallel} + \left(\lambda^4 z^2 + \frac{3}{4z^2}\right) \psi_n^{\parallel} = m_n^2 \psi_n^{\parallel}.$$

The mass spectrum is given by the corresponding eigenvalues

$$m_{n,\parallel}^2 = 4\lambda^2 (n+1)$$

the parameter λ controls the slope of the radial Regge trajectory

To obtain the spectrum of circular polarizations we need to specify the function $f(z)$. A simple possibility is given by the ansatz

$$f = \frac{b}{2}\frac{e^{-\lambda^2 z^2}}{z}.$$

b is a dimensionless constant. After the substitution

$$-\partial_y^2 \psi_n^{\pm} + \left(y^2 + \frac{3}{4y^2} \pm \frac{b\mu_5}{\lambda^2}|\vec{p}|\right)\psi_n^{\pm} = \frac{m_{n,\pm}^2}{\lambda^2}\psi_n^{\pm}$$

The eigenfunctions remain the same as before but the mass spectrum is shifted,

$$m_{n,\pm}^2 = 4\lambda^2(n+1) \pm b\mu_5|\vec{p}|.$$

Thus, the massive vector fields split into three polarizations with masses $m_{n,-} < m_{n,\|} < m_{n,+}$.
This formula can be considered as a generalization of our previous results to the radially excited spectrum.

Conclusions

- P- and CP-odd effects not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- P- breaking leads to unexpected modifications of the in-medium properties of scalar and vector mesons.
- LPB has an influence on the observed dilepton spectrum in the LMR of PHENIX and STAR.
- Some observables may allow us to establish P- and CP-breaking unambiguously.

Experimental collaborations should check this possibility.

Thank you for your attention!

The inclusion of the singlet axial-vector meson $h^\mu \equiv h_1(1170)$ coupled to quarks via

$$\Delta\mathcal{L} = -\frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \frac{1}{2}m_h^2 h_\mu h^\mu + \bar{q}\gamma_\mu\gamma_5(g_h h^\mu + \delta^{\mu 0}\mu_5)\mathbf{I}_q q$$

mixes and renormalizes the bare axial chemical potential due to condensation of the time component $h^\mu \simeq \langle h^0\rangle\delta^{0\mu}$ (like condensation of ω for baryon chemical potential). The effective chemical potential now is $\bar{\mu}_5 \equiv \mu_5 + g_h\langle h_0\rangle$.

The stationary point equation allows to relate the bare and effective axial chemical potentials

$$\bar{\mu}_5\left[1 + \frac{4g_h^2}{m_h^2}v_q^2(\bar{\mu}_5)\right] = \mu_5 .$$

In the mass-gap equations for v_q, v_s the effective axial chemical potential $\bar{\mu}_5$ must be used!!

The $L, \pm$ contribution for vector mesons decaying into leptons before applying experimental cuts is given by:

$$\frac{dN_{ee}^{\epsilon}}{dM} \simeq c_V \frac{\alpha^2 \Gamma_V m_V^2}{3\pi^2 g^2 M^2} \left(\frac{M^2 - n_V^2 m_\pi^2}{m_V^2 - n_V^2 m_\pi^2} \right)^{3/2}$$

$$\times \sum_{\epsilon} \int_M^\infty dk_0 \frac{\sqrt{k_0^2 - M^2}}{e^{k_0/T} - 1} \frac{m_{V,\epsilon}^4}{\left(M^2 - m_{V,\epsilon}^2\right)^2 + m_{V,\epsilon}^4 \frac{\Gamma_V^2}{m_V^2}},$$

where $n_V = 2, 0$; $|\vec{k}| = \sqrt{k_0^2 - M^2}$ and $M^2 > n_V^2 m_\pi^2$. c_V absorbs combinatorial factors different for ρ and ω, μ and finite volume suppression. Empirically for $\mu_5 = 0$ the ratio $c_\rho / c_\omega \sim 10$ holds.

LPB polarization analysis

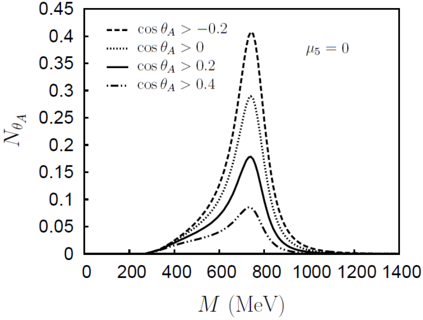
We will perform a two-dimensional study of the decay product with the dielectron invariant mass M_{ee} and one of the angles θ_A or θ_B

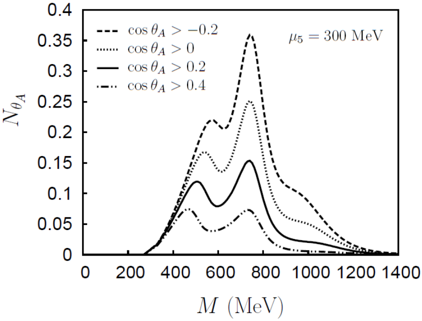
The overall constants c_V are chosen so that after integrating the entire phase space (except for experimental cuts) the total production at the vacuum resonance peak is normalized to 1:

$$N_\theta = \frac{\int_{\Delta\theta} \frac{dN}{dM d\cos\theta}(M, \cos\theta)\, d\cos\theta}{\int_{-1}^{+1} \frac{dN}{dM d\cos\theta}(M = m_V, \cos\theta)\, d\cos\theta}.$$

This choice will help us to quantify the number of events found when the phase space is restricted by our own cuts in θ.

Dealing with quantities smaller than 10% may be tricky. The main information is focused in $\cos\theta_A \approx 1$ so we integrate up to this value in wider bins.





ρ spectral function depending on the dielectron invariant mass M in vacuum ($\mu_5 = 0$) and in a parity-breaking medium with $\mu_5 = 300$ MeV for different ranges of θ_A.

Optimization process required in every different experiment.