Standard Model Effective Theory and Dimension 6 Operators

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Outline

- SMEFT formalism
- Dimension six operators
- Power Counting and Equations of Motion
- Naive Dimensional Analysis
- Tree-Loop Mixing
- $h \to \gamma\gamma$ and the S parameter
- MFV
- Holomorphy

Rodrigo Alonso, Elizabeth Jenkins, Michael Trott, AM

Experimental Summary

- Standard Model provides a good description of all observations
- A particle has been seen with a mass $M_h \sim 126$ GeV consistent with the Higgs boson of the standard model
- 0^+ quantum numbers favored
- Production rate times branching ratios consistent with the standard model, but with large error bars.
- No evidence for any BSM physics up to energies of ~ 1 TeV

Precision EW data consistent with SM and a light Higgs.

Precision measurements at BaBar and Belle, including rare decays such as $B \to X_s\gamma$, are consistent with SM
(CKM and the GIM mechanism).

Simplest theory that contains everything we know up to the electroweak scale is the SM.

Dark matter, neutrino masses, baryon asymmetry point to new physics, but no indication that this is at the electroweak scale.

If there are no new particles at the $\sim$ 500 GeV scale, then can use the SM and parametrize new physics by higher dimension operators. New physics effects $\propto p^2/\Lambda^2$.

Main assumption:

$SU(2) \times U(1)$ symmetry broken by a scalar H — SMEFT

$$\langle H \rangle = \frac{v}{\sqrt{2}}$$

is the only dimensionful parameter in the SM at the classical level. $M_{W,Z}, m_f \propto v$. Quantum level $\Lambda_{\rm QCD}$.

$$H = \begin{bmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\varphi^0\right) \end{bmatrix}$$

$SU(2) \times U(1)$ linearly realized, and broken by $\langle H \rangle$. SM plus some heavy particles, e.g. extra gauge bosons.

$$D_\mu H^\dagger D_\mu H \to Z^2(v+h)^2 = Z^2(v^2 + 2vh + h^2)$$

A lot of work on theories with $SU(2) \times U(1)$ broken symmetry and an additional light scalar h added "by hand." h couplings such as $h \to ZZ$ not constrained to be their SM values. — Higgs EFT

$$H \to e^{i\tau\cdot\varphi/v} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix}, \qquad\qquad h \text{ (unrelated field)}$$

Technicolor with a light dilaton

Generalize SM to SMEFT

Fields are three generations of fermions

$$\mathrm{L}: q_i,\ l_i, \qquad \mathrm{R}: u_i,\ d_i,\ e_i \qquad i = 1, \ldots, n_g = 3$$

the scalar doublet H, and $SU(3) \times SU(2) \times U(1)$ gauge fields.

$$L = L_{SM} + \sum \frac{1}{\Lambda^n} L^{(4+n)} = L_{SM} + \frac{1}{\Lambda^2} L^{(6)} + \ldots \quad \textit{note the dots}$$

Λ is the scale of new physics, and assume $\Lambda > v$

Power Counting

$$L^{(6)} \sim \frac{m_H^2}{\Lambda^2} \qquad L^{(8)} \sim \frac{m_H^4}{\Lambda^4} \qquad L^{(6)} \times L^{(6)} \sim \frac{m_H^4}{\Lambda^4}$$

m_H (or v, m_Z, m_t is a characteristic EW scale).

Baryon and Lepton Number Violation

Dimension five operator:

$$\frac{1}{\Lambda_5}(H\,\ell)(H\,\ell)$$

$\Delta L = 2$ operator which gives neutrino masses.

Not relevant for 1 TeV LHC processes, i.e. can have $\Lambda_5 \gg \Lambda$ since Λ_5 violates lepton number, but Λ does not.

Similarly, baryon number violating operators can be dropped.

SMEFT Operators

- Leading higher dimension operators are $d = 6$.
- Assuming B and L conservation, there are 59 independent dimension-six operators (not including flavor indices) which form complete basis of $d = 6$ operators.
- 59 operators divided into eight operator classes.

$1 : X^3$	$2 : H^6$	$3 : H^4 D^2$	$4 : X^2 H^2$
$5 : \psi^2 H^3$	$6 : \psi^2 XH$	$7 : \psi^2 H^2 D$	$8 : \psi^4$

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e$$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

Dimension Six Operators

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

S, T parameter operators are Q_{WB} and Q_{HD}. U parameter operator $\left(H^\dagger W_{\mu\nu} H\right)^2$ is dimension eight.

Dimension Six Operators

8 : $(\bar{L}L)(\bar{L}L)$		8 : $(\bar{R}R)(\bar{R}R)$		8 : $(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

8 : $(\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

8 : $(\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Buchmuller & Wyler 1986

Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

$$\psi^4 \to JJ,\ (\bar{L}R)(\bar{L}R),\ (\bar{L}R)(\bar{R}L)$$

Field redefinitions (equations of motion) used to eliminate operators.

59 baryon number conserving operators, not including flavor indices.

2499 independent coefficients for $n_g = 3$:
1350 CP-even and 1149 CP-odd terms

156 different irreducible flavor representations: $\otimes_{q,l,u,d,e} SU(n_g)$

$$Q_{\substack{He \\ pr}} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$$

under $SU(n_g)_e$ has both singlet and adjoint pieces,

$$\left(\frac{1}{n_g} Q_{\substack{He \\ ss}} \delta_{pr}\right) + \left(Q_{\substack{He \\ pr}} - \frac{1}{n_g} Q_{\substack{He \\ ss}} \delta_{pr}\right)$$

Four-quark operators have more complicated representations.

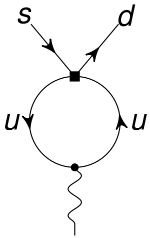
Field Redefinitions (Equations of Motion)

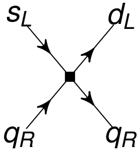
Used to eliminate operators with derivatives:

$$D^\mu F_{\mu\nu} = g\, j_\nu$$

$$g\, \overline{d}\gamma^\mu T^A P_L s\, D^\mu F^A_{\mu\nu} \to g^2\, \overline{d}\gamma^\mu T^A P_L s\;\; \overline{q}\gamma^\mu T^A q$$

Penguin operators give LL and LR operators; no 1PI diagram for LR





Gaillard and Lee; Gilman and Wise

Equations of Motion (contd)

H.D. Politzer: NPB172 (1980) 349

Operator conversions done by making field redefinitions, since

$$L(\phi + \epsilon f(\phi)) = L(\phi) + \epsilon \frac{\delta L}{\delta \phi} f(\phi) + \ldots$$

- Change of variables in a (functional) integral
- *S*-matrix unchanged
- Green's functions can change
- Have to be a bit careful, since usually one computes 1PI graphs, and the *S*-matrix includes non-1PI graphs.
- Can induce operators for which there is no direct 1PI graph such as the *LR* four-quark operators.

EOM:

$$E_i = 0$$

RG equations:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} O_i = -\gamma_{ji}\, O_j + \zeta_{ri}\, E_r \qquad \mu \frac{\mathrm{d}}{\mathrm{d}\mu} E_i = -\Gamma_{ji}\, E_j$$

ζ_r can be gauge and scheme dependent.

RG evolution consistent with equations of motion — can evolve and use EOM or use EOM and evolve.

Dawson, Hagelin, Hall; Bauer et al.

Power Counting for the RGE

Amplitudes and anomalous dimensions obey power counting:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2$$

In the SM, because of the dimension two operator $H^\dagger H$, have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto C^{(4)} + m_H^2 C^{(6)} + \ldots$$

⋆ SM parameter RG evolution affected by dim 6 terms at order m_H^2/Λ^2. Just as important as dim 6 operators. Jenkins, AM, Trott

RGE for SM parameters from Dim 6

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\lambda = \frac{m_H^2}{16\pi^2}\Big[12C_H + \left(-32\lambda + \frac{10}{3}g_2^2\right)C_{H\Box} + \left(12\lambda - \frac{3}{2}g_2^2 + 6g_1^2 y_H^2\right)C_{HD} + 2\eta_1 + 2\eta_2 + 12g_2^2 c_{F,2}C_{HW} + 12g_1^2 y_H^2 C_{HB} + 6g_1 g_2 y_H C_{HWB} + \frac{4}{3}g_2^2 C^{(3)}_{\substack{Hl\\tt}} + \frac{4}{3}g_2^2 N_c C^{(3)}_{\substack{Hq\\tt}}\Big],$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_H^2 = \frac{m_H^4}{16\pi^2}\left[-4C_{H\Box} + 2C_{HD}\right],$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}[Y_d]_{rs} = \frac{m_H^2}{16\pi^2}\Big[3C^*_{\substack{dH\\sr}} - C_{H\Box}[Y_d]_{rs} + \frac{1}{2}C_{HD}[Y_d]_{rs} + [Y_d]_{rt}\left(C^{(1)}_{\substack{Hq\\ts}} + 3C^{(3)}_{\substack{Hq\\ts}}\right) - C_{\substack{Hd\\rt}}[Y_d]_{ts} - [Y_u]_{ts}C^*_{\substack{Hud\\tr}} - 2\left(C^{(1)*}_{\substack{qd\\sptr}} + c_{F,3}C^{(8)*}_{\substack{qd\\sptr}}\right)[Y_d]_{tp} + C_{\substack{ledq\\ptrs}}[Y_e]^*_{pt} + N_c C^{(1)*}_{\substack{quqd\\ptsr}}[Y_u]^*_{tp} + \frac{1}{2}\left(C^{(1)*}_{\substack{quqd\\sptr}} + c_{F,3}C^{(8)*}_{\substack{quqd\\sptr}}\right)[Y_u]^*_{tp}\Big]$$

$$\mu\frac{\mathrm{d}g_3}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_3 C_{HG}, \qquad \mu\frac{\mathrm{d}g_2}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_2 C_{HW}, \qquad \mu\frac{\mathrm{d}g_1}{\mathrm{d}\mu} = -4\frac{m_H^2}{16\pi^2}g_1 C_{HB},$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_3 = -\frac{4m_H^2}{g_3^2}C_{H\widetilde{G}}, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_2 = -\frac{4m_H^2}{g_2^2}C_{H\widetilde{W}}, \qquad \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\theta_1 = -\frac{4m_H^2}{g_1^2}C_{H\widetilde{B}},$$

Anomalous Dimension Matrix

Alonso, Jenkins, AM, Trott: 1308.2627, 1310.4838, 1312.2014

Computed the running of the SM dimension-four terms and the full dimension-six anomalous dimension at one loop, including all Yukawa couplings for general n_g.
γ: 2499×2499, or at least $156 \times 156 = 24336$.

J. Elias Miro, J.R. Espinosa, E. Masso, A. Pomarol, 1308.1879, 1302.5661

Use $n_g = 1$ and only keep y_t.
Compute γ for 5 classes of operators — 23 rows $\times \leq 59$ columns.

Anomalous Dimension Matrix

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	3	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

Structure of anomalous dimension matrix.

$$\frac{g}{4\pi}, \ \frac{y}{4\pi}, \ \frac{\lambda}{16\pi^2}$$

Jenkins, Trott, AM: 1309.0819

Many entries exist because of EOM

Naive Dimensional Analysis

Georgi, AM: NPB234 (1984) 189

Jenkins, Trott, AM: 1309.0819

$$f^2\Lambda^2\left(\frac{\psi}{f\sqrt{\Lambda}}\right)^a\left(\frac{H}{f}\right)^b\left(\frac{yH}{\Lambda}\right)^c\left(\frac{D}{\Lambda}\right)^d\left(\frac{gX}{\Lambda^2}\right)^e$$

with $\Lambda \sim 4\pi f$.

$$\begin{array}{llll} \frac{f^2}{\Lambda^4}g^3X^3, & \frac{\Lambda^2}{f^4}H^6, & \frac{1}{f^2}\,H^4D^2, & \frac{1}{\Lambda^2}\,g^2X^2H^2, \\ \frac{1}{f^2}\,y\psi^2H^3, & \frac{1}{\Lambda^2}\,y\psi^2gXH, & \frac{1}{f^2}\psi^2H^2D, & \frac{1}{f^2}\psi^4 \end{array}$$

Differs from just using $1/\Lambda^2$ for all operators by factors of 4π.

Naive Dimensional Analysis

NDA weight $w \equiv$ powers of f^2 in denominator.

$$L = f^2 \Lambda^2 \left(\frac{H}{f}\right)^6 = \Lambda^2 \frac{(H^\dagger H)^3}{f^4}, \qquad w = 2$$

$$\gamma_{ij} \propto \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{g^2}{16\pi^2}\right)^{n_g}, \qquad N = n_\lambda + n_y + n_g$$

$$N = 1 + w_i - w_j$$

Familiar Example: $b \to s\gamma$

Use

$$O_q = \overline{b}\gamma^\mu P_L u \, \overline{u}\gamma_\mu P_L s$$
$$O_g = \frac{g}{16\pi^2} m_b \, \overline{b}\sigma^{\mu\nu} G_{\mu\nu} P_L s$$

Then

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} C_q \\ C_g \end{bmatrix} = \begin{bmatrix} L & L+1 \\ L-1 & L \end{bmatrix} \begin{bmatrix} C_q \\ C_g \end{bmatrix}$$

where L is the number of loops of the diagram.

To get all terms to order $g^2/(16\pi^2)$, need γ_{gq} at two loops.

Features of RG evolution

There are some big numbers:

The evolution of the H^6 coefficient is

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_H = \frac{1}{16\pi^2} \left[108\,\lambda\,C_H - 160\,\lambda^2\,C_{H\Box} + 48\,\lambda^2\,C_{HD} \right] + \ldots$$

For $m_H \sim 126$ GeV, $108\,\lambda/(16\pi^2) \approx 0.1$.

Independent of normalization of C_H.

Many other terms with coefficients of order 30–40.

Tree-Loop Mixing

Claim:

- Operators can be classified into “tree” and “loop”
- Based on some notion of “minimal coupling”
- The mixing of “tree” and “loop” operators vanishes at one loop

counterexample:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_{\substack{eB\\pr}} = \frac{1}{16\pi^2}\left[4 g_1 N_c \left(\mathsf{y}_u + \mathsf{y}_q\right) C^{(3)}_{\substack{lequ\\prst}} \left[Y_u\right]_{ts}\right] + \ldots$$

“loop” magnetic dipole operators:

$$Q_{\substack{eB\\pr}} = (\bar{l}_{p,a} \sigma^{\mu\nu} e_r) H^a B_{\mu\nu}$$

“tree” four-fermion operators

$$Q^{(3)}_{lequ} = (\bar{l}^j_p \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}^k_s \sigma^{\mu\nu} u_t)$$

Bauer et al. for HQET provides a counterexample

X^2H^2 (Gauge-Higgs) Operators

Look at only 4 operators (since there are 59 operators)

$$\mathcal{O}_G = \frac{g_3^2}{2\,\Lambda^2}\, H^\dagger\, H\, G^A_{\mu\,\nu} G^{A\,\mu\,\nu}, \qquad \mathcal{O}_B = \frac{g_1^2}{2\,\Lambda^2}\, H^\dagger\, H\, B_{\mu\,\nu} B^{\mu\,\nu},$$

$$\mathcal{O}_W = \frac{g_2^2}{2\,\Lambda^2}\, H^\dagger\, H\, W^a_{\mu\,\nu} W^{a\,\mu\,\nu}, \qquad \mathcal{O}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2}\, H^\dagger\, \tau^a\, H\, W^a_{\mu\,\nu} B^{\mu\,\nu},$$

Also CP-odd versions, which do not interfere with the SM amplitude.

When you expand them out in the broken phase, $H \to h + v$,
get $h \to \gamma\gamma$, $h \to \gamma Z$ and $gg \to h$.

Considered these operators in an earlier work, and an explicit model that produced these operators.

AM, M.B. Wise, PLB636 (206) 107, PRD74 (2006) 035009

An exactly solvable model that produces the O_W, O_{WB} and O_B operators and the W^3 operator:

AM: PLB 726 (2013) 347

$$c_W = \frac{(\lambda_1/\lambda_3)}{48 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_B = \frac{(\lambda_1/\lambda_3) Y_S^2}{12 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_{WB} = \frac{(\lambda_2/\lambda_4) Y_S}{24 \log \frac{\Lambda_4^2}{\langle \Phi \rangle}}$$

$$c_{W^3} = \frac{N g_2^3}{2880 \pi^2 \langle \Phi \rangle}.$$

$\star$ some claims that these are "loop-supressed operators, and smaller than other contributions such as four-fermion operators.

The $gg \to h$ amplitude gets contribution from c_G

For $h \to \gamma\gamma$,

$$c_{\gamma\gamma} = c_W + c_B - c_{WB}$$

For $h \to \gamma Z$,

$$c_{\gamma Z} = c_W \cot\theta_W - c_B \tan\theta_W - c_{WB} \cot 2\theta_W,$$

$$S = -\frac{8\,\pi\,v^2}{\Lambda^2} c_{WB}(M_h)\,.$$

Contribution to the Higgs Decay Rate

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} \simeq \left| 1 - \frac{4\pi^2 v^2\ c_{\gamma\gamma}}{\Lambda^2\ I^\gamma} \right|^2$$

and $I^\gamma \approx -1.64$. Note the $4\pi^2$.

Similar expressions for $gg \to h$ and $h \to \gamma Z$.

Brief Experimental Summary

$$\text{ATLAS:} \quad \mu_{\gamma\gamma} = 1.6 \pm 0.3 \qquad \text{CMS:} \quad \mu_{\gamma\gamma} = 0.77 \pm 0.27$$

Naive combination of these results (not recommended) gives

$$\mu_{\gamma\gamma} \simeq 1.14 \pm 0.2$$

If due to $c_{\gamma\gamma}$:

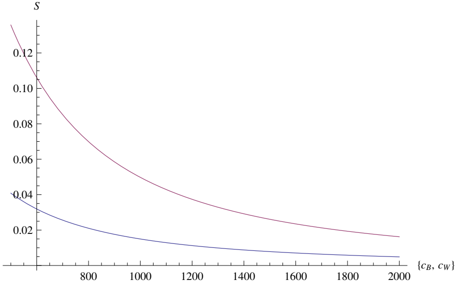
$$\frac{v^2}{\Lambda^2} c_{\gamma\gamma}(M_h) \simeq -0.08, \; 0.003 \pm 0.003$$

The second solution is preferred. The first solution is when $c_{\gamma\gamma}$ switches the sign of the standard model $h \to \gamma\gamma$ amplitude.

The experiments are sensitive to these effects if the new physics scale Λ is near a few TeV.

$$\mu_{\gamma\gamma} \simeq 1 - 0.02\, S \log \frac{\Lambda}{M_h} + 2.7 \left(\frac{1\,\text{TeV}}{\Lambda} \right)^2 c_{\gamma\gamma}(\Lambda)$$

Experimental limit $|S| \lesssim 0.1$.



Plot of S at $\mu = m_H$ assuming $S = 0$ at Λ, from c_B and c_W
Largest contribution from top quark.

Minimal Flavor Violation

MFV: An $SU(3)$ for q, l, u, d, e. Chivukula, Georgi

- Interesting mixing betweeen different flavor sectors
- The SM respects MFV (by definition)
- Is there a MFV symmetry in the UV?
- Dimension-Six RGE preserves MFV

RGE feeds MFV violation from one sector to another.

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu}$$

$$MFV \Rightarrow \; \underset{pr}{C_{eW}} \propto [Y_e]_{pr}$$

Magnetic Dipole Operator

$$\mathscr{C}_{\underset{rs}{e\gamma}} = \frac{1}{g_1} C_{\underset{rs}{eB}} - \frac{1}{g_2} C_{\underset{rs}{eW}} \qquad \mathcal{L} = \frac{ev}{\sqrt{2}} \mathscr{C}_{\underset{rs}{e\gamma}} \, \overline{e}_r \sigma^{\mu\nu} P_R e_s F_{\mu\nu} + h.c.$$

where r and s are flavor indices ($\{e_e\,,\ e_\mu\,,\ e_\tau\} \equiv \{e\,,\ \mu\,,\ \tau\}$) and

$$\begin{aligned}
\dot{\mathscr{C}}_{\underset{rs}{e\gamma}} = &\left\{ Y(s) + e^2 \left(12 - \frac{9}{4} \csc^2\theta_W + \frac{1}{4} \sec^2\theta_W \right) \right\} \mathscr{C}_{\underset{rs}{e\gamma}} \\
&+ 2 \mathscr{C}_{\underset{rv}{e\gamma}} [Y_e Y_e^\dagger]_{vs} + \left(\frac{1}{2} + 2\cos^2\theta_W \right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\underset{ws}{e\gamma}} + e^2 \left(12 \cot 2\theta_W \right) \mathscr{C}_{\underset{rs}{eZ}} \\
&- \left(2 \sin\theta_W \cos\theta_W \right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\underset{ws}{eZ}} - \cot\theta_W [Y_e^\dagger]_{rs} \left(C_{HWB} + i C_{H\widetilde{W}B} \right) \\
&+ \frac{8}{3} e^2 [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma\gamma} + i \widetilde{\mathscr{C}}_{\gamma\gamma} \right) + e^2 \left(\cot\theta_W - \frac{5}{3} \tan\theta_W \right) [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma Z} + i \widetilde{\mathscr{C}}_{\gamma Z} \right) \\
&+ 16 [Y_u]_{wv} C^{(3)}_{\underset{rsvw}{lequ}}
\end{aligned}$$

as corrected by Signer and Pruna, arXiv:1408:3565

Constraints at $\Lambda \sim 1 - 10$ TeV level from $\mu \to e\gamma$, EDM, $g - 2$, etc.

The current experimental limit on BR($\mu \to e\gamma$) is 5.7×10^{-13} from the MEG experiment, which implies

$$\frac{v}{\sqrt{2}\, m_e} \mathscr{C}_{\substack{e\gamma \\ \mu e}} \lesssim \ 2.7 \times 10^{-4} \text{ TeV}^{-2}$$

at the low energy scale $\mu \sim m_\mu$.

This bound implies

$$\frac{m_t}{m_e} C^{(3)}_{\substack{lequ \\ \mu ett}} \lesssim 1.4 \times 10^{-3} \text{ TeV}^{-2}$$

using the estimate $\ln(\Lambda/m_H)/(16\pi^2) \sim 0.01$ for the renormalization group evolution, and assuming that this term is the only contribution to $\mathscr{C}_{\substack{e\gamma \\ \mu e}}$ at low energies.

The anomalous magnetic moment of the muon is

$$\delta a_\mu = -\frac{4 m_\mu v}{\sqrt{2}} \mathrm{Re}\, \mathscr{C}_{\substack{e\gamma \\ \mu\mu}}$$

which yields the limits

$$|C_{HWB}| \lesssim 0.6\,\mathrm{TeV}^{-2}, \qquad |\mathscr{C}_{\gamma\gamma}| \lesssim 4\,\mathrm{TeV}^{-2}, \qquad \left| \frac{m_t}{m_\mu} \mathrm{Re}\, C^{(3)}_{\substack{lequ \\ \mu\mu tt}} \right| \lesssim 7\ \mathrm{TeV}^{-2},$$

assuming that each of these is the only contribution to $\mathscr{C}_{\substack{e\gamma \\ \mu\mu}}$.

The bound on the electric dipole moment of the electron translates to the limits

$$\left|C_{H\widetilde{W}B}\right| \lesssim 2 \times 10^{-3}\,\mathrm{TeV}^{-2}, \quad \left|\widetilde{\mathscr{C}}_{\gamma\gamma}\right| \lesssim 2 \times 10^{-2}\,\mathrm{TeV}^{-2}, \quad \left| \frac{m_t}{m_e} \mathrm{Im}\, C_{\substack{lequ \\ eett}} \right| \lesssim 3 \times 10^{-4}\ \mathrm{TeV}^{-}$$

using the recently measured upper bound $d_e < 1.05 \times 10^{-27}\, e\,\mathrm{cm}$ from the ACME collaboration, again assuming each of these terms is the only contribution.

Empirical Observation

$$\dot{\mathscr{C}}_{e\gamma} \propto \left(\mathscr{C}_{\gamma\gamma} + i\widetilde{\mathscr{C}}_{\gamma\gamma}\right),\ \mathscr{C}_{e\gamma}$$

no

$$\left(\mathscr{C}_{\gamma\gamma} - i\widetilde{\mathscr{C}}_{\gamma\gamma}\right),\ \mathscr{C}^{*}_{e\gamma}$$

Find this after adding all graphs and using the EOM. Individual contributions are not holomorphic, and only the total respects holomorphy.

- Observation: 1-loop anomalous dimension matrix respects holomorphy to a large extent.
- Using EOM, so equivalent to computing (on-shell) *S*-matrix elements.

Holomorphy

Divide $d = 6$ Operators into Holomorphic, Antiholomorphic and Non-Holomorphic Operators

$$X^{\pm}_{\mu\nu} = \frac{1}{2}\left(X_{\mu\nu} \mp i\widetilde{X}_{\mu\nu}\right), \qquad \widetilde{X}^{\pm}_{\mu\nu} = \pm i X^{\pm}_{\mu\nu},$$

$$\widetilde{X}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}/2 \qquad \widetilde{\widetilde{X}}_{\mu\nu} = -X_{\mu\nu}$$

Complex self-duality condition in Minkowski space.

Holomorphy

R. Alonso, E. Jenkins, AM: 1409.0868

Definition

The holomorphic part of the Lagrangian, $\mathcal{L}_{\mathfrak{h}}$, is the Lagrangian constructed from the fields X^+, R, $\overline{L}$, but none of their hermitian conjugates. These transform as $(0, \frac{1}{2})$ or $(0, 1)$ under the Lorentz group, i.e. only under the $SU(2)_R$ part of $SU(2)_L \times SU(2)_R$.

$$\mathcal{L}^{d=6} = \mathcal{L}_{\mathfrak{h}} + \mathcal{L}_{\overline{\mathfrak{h}}} + \mathcal{L}_{\mathfrak{n}} = C_{\mathfrak{h}} Q_{\mathfrak{h}} + C_{\overline{\mathfrak{h}}} Q_{\overline{\mathfrak{h}}} + C_{\mathfrak{n}} Q_{\mathfrak{n}}$$

$$Q_{\mathfrak{h}} \subset \left\{ X^{+3},\ X^{+2} H^2,\ \left(\overline{L} \sigma^{\mu\nu} R\right) X^+ H,\ (\overline{L} R)(\overline{L} R) \right\}$$

$$Q_{\overline{\mathfrak{h}}} \subset \left\{ X^{-3},\ X^{-2} H^2,\ \left(\overline{R} \sigma^{\mu\nu} L\right) X^- H,\ (\overline{R} L)(\overline{R} L) \right\} = Q_{\mathfrak{h}}^\dagger$$

$$Q_{\mathfrak{n}} \subset \left\{ H^6,\ H^4 D^2,\ \psi^2 H^3,\ \psi^2 H^2 D,\ (\overline{L} R)(\overline{R} L),\ JJ \right\}$$

Holomorphy

	$(X^+)^3$	$(X^+)^2H^2$	ψ^2X^+H	$(\bar{L}R)(\bar{L}R)$	$(\bar{L}R)(\bar{R}L)$	JJ	ψ^2H^3	H^6	H^4D^2	ψ^2H^2D
$(X^+)^3$	$\to \mathfrak{h}$	$\to 0$	0	0	0	0	0	0	0	0
$(X^+)^2H^2$	$\to \mathfrak{h}$	$\to \mathfrak{h}$	$\to \mathfrak{h}$	0	0	$\nexists$	0	0	$\to 0$	$\to 0$
ψ^2X^+H	$\to \mathfrak{h}$	$\to \mathfrak{h}$	$\to \mathfrak{h}$	$\mathfrak{h}_F$	$\to 0$	$\to 0$	$\to 0$	0	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{L}R)$	$\to 0$	$\nexists$	$\mathfrak{h}_F$	$\mathfrak{h}_F$	$Y_u^\dagger Y_{e,d}^\dagger$	$Y_u^\dagger Y_{e,d}^\dagger$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
$(\bar{L}R)(\bar{R}L)$	$\to 0$	$\nexists$	$\to 0$	$Y_u Y_d,\ Y_u^\dagger Y_e^\dagger$	$\mathfrak{h}_F$	$*$	$\nexists$	$\nexists$	$\nexists$	$\to 0$
JJ	$\to 0$	$\nexists$	$\to 0$	$Y_u Y_{e,d}$	$*$	$*$	$\nexists$	$\nexists$	$\nexists$	$*$
ψ^2H^3	$\to 0$	$\to \mathfrak{h}$	$\to \mathfrak{h}$	$\to \mathfrak{h}$	$*$	$*$	$*$	$\nexists$	$*$	$*$
H^6	$\to 0$	$\boxed{\star}$	$\nexists$	$\nexists$	$\nexists$	$\nexists$	$*$	$*$	$*$	$*$
H^4D^2	$\to 0$	$\to 0$	$\to 0$	$\nexists$	$\nexists$	$\nexists$	$\to 0$	$\nexists$	$*$	$*$
ψ^2H^2D	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$\to 0$	$*$	$\to 0$	$\nexists$	$*$	$*$

- 0: Vanishes by NDA, i.e. NDA gives a negative loop order
- $\nexists$: There is no one-loop diagram (including from EOM)
- $\mathfrak{h}_F$: Holomorphic. Nonholomorphic terms forbidden by NDA and flavor symmetry
- $\to 0$: Vanishes by explicit computation, after adding all contributions. Individual graphs need not vanish.
- $\to \mathfrak{h}$: Holomorphic, by explicit computation
- $\star$: Non-zero

- The 1 1 block is holomorphic

$$\gamma_{\mathfrak{h}\bar{\mathfrak{h}}} = 0$$

- The 1 2 block vanishes except for the red terms proportional to $Y_u Y_e$ or $Y_u Y_d$.

$$\mathcal{L}_Y = -\overline{q}^j \, Y_d^\dagger \, d \, H_j - \overline{q}^j \, Y_u^\dagger \, u \, \widetilde{H}_j - \overline{l}^j \, Y_e^\dagger \, e \, H_j + \text{h.c.}$$
$$\widetilde{H}_j = \epsilon_{ij} H^{\dagger j}$$

- one entry $\star$ present even if Yukawa couplings set to zero.

RGE of SM parameters

Recall that

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto m_H^2 C^{(6)}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tau = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \left(\frac{4\pi}{g_X^2} - i \frac{\theta_X}{2\pi} \right) = \frac{2 m_H^2}{\pi g_X^2} C_{HX,+}$$

τ is the SUSY holmorphic gauge coupling

Clifford Cheung and Chia-Hsien Shen have an explanation. arXiv:1505.01844

⋆ Entry: Some Numerology

$$\dot{C}_H = -3g_2^2\left(g_1^2 + 3g_2^2 - 12\lambda\right)\mathrm{Re}(C_{HW,+})$$
$$- 3g_1^2\left(g_1^2 + g_2^2 - 4\lambda\right)\mathrm{Re}(C_{HB,+})$$
$$- 3g_1 g_2\left(g_1^2 + g_2^2 - 4\lambda\right)\mathrm{Re}(C_{HWB,+}) + \ldots$$

The $C_{HB,+}$ and $C_{HWB,+}$ terms vanish if $g_1^2 + g_2^2 = 4\lambda$:

$$m_H^2 = 2m_Z^2 = (129\,\mathrm{GeV})^2\,,$$

and the $C_{HW,+}$ term vanishes if $g_1^2 + 3g_2^2 = 12\lambda$:

$$m_H^2 = \frac{2}{3}m_Z^2 + \frac{4}{3}m_W^2 = (119\,\mathrm{GeV})^2\,,$$

$$\frac{2}{3}(129\,\mathrm{GeV})^2 + \frac{1}{3}(119\,\mathrm{GeV})^2 = (125.7\,\mathrm{GeV})^2$$

Summary

- EFT prvoides an efficient way to parametrize deviations from the SM.
- Complete RGE of dimension-six operators of SM EFT has been computed, including contribution of dimension-six operators to running of SM parameters.
- RG evolution of dimension-six operators important for Higgs processes at the $\sim 10 - 15\%$ level.
- Flavor mixing allows for a test of MFV hypothesis.
- Holomorphy of 1-loop anomalous dimension matrix of dimension-six operators.
  - Does it hold in a more general gauge theory?
  - Does any of it extend beyond one loop?
  - Does it hold at dim 8?