Lifshitz dynamics in the UV

with A. O. Barvinsky, D. Blas, G. Perez-Nadal and C. F. Steinwachs

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Motivation

What is a Lifshitz field theory?

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$$S = \int dt\, d^D x \left(\frac{1}{2} \partial_t \phi \partial_t \phi + \phi (-\Delta)^z \phi \right)$$

Poincaré group is only possible when $z = 1$.

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E. M. Lifshitz, 1941

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Poincaré group is only possible when $z = 1$.

For high energy physicist, the important feature is the improved UV regime

$$G(\omega, p^i) \sim \frac{1}{\omega^2 + p^{2z}}$$

Motivation

Condensed matter

It explains tri-critical phenomena known as "Lifshitz points"

$$S = \int dt\, d^D x \left(\frac{1}{2} \partial_t \phi \partial_t \phi + \phi(-\Delta)^z \phi + \alpha_1 \phi (-\Delta)^{z-1} \phi + \ldots + \alpha_{z-1} \phi \Delta \phi \right)$$

The theory can flow from the anisotropic scaling to the isotropic one depending on the RGE of the coupling constants.

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The theory can flow from the anisotropic scaling to the isotropic one depending on the RGE of the coupling constants.

z can then be though as an order parameter.

Lorentz invariance emerges in the IR with a light speed $c^2 = \alpha_{z-1}$

S. Chadha and H. B. Nielsen, 1982

Lifshitz Theories in curved space

We are interested in studying Lifshitz field theories in curved space

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i - \gamma_{ij} dx^i dx^j$$

$$t \to f(t) \qquad x^i \to \tilde{x}^i(x^j, t)$$

$$S = \int dt d^D x \sqrt{|g|} \, \left(\mathcal{L}_n \phi \mathcal{L}_n \phi + \phi(-\Delta)^z \phi + ...\right)$$

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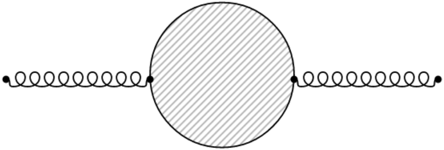

Can produce gravitational counterterms and phenomena analogous to the Weyl anomaly

Motivation

Holography

Lifshitz scalar field theories are conjectured to be holographic duals to Lifshitz space-times

- Massive vector coupled to Einstein-Hilbert gravity in the bulk

  S. Kachru, X. Liu and M. Mulligan, 2008

- Hořava-Lifshitz gravity in the bulk

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Anisotropic Weyl invariance

$$N \to \Omega^{-z} N \qquad \gamma_{ij} \to \Omega^{-2}\gamma_{ij} \qquad \phi \to \Omega^{\frac{D-z}{2}}\phi \qquad zE + T_i^i = 0$$

$$S = \int dt d^D x \sqrt{|g|}\, \left(\mathcal{L}_n\phi\mathcal{L}_n\phi + \phi(-\Delta)^z\phi + ...\right)$$

For $z = D$, this is already invariant

Anisotropic Weyl invariance will be broken at the quantum level by conformal anomalies. Entanglement entropy, general properties of RG flows,...??

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Quantum Gravity

Following this same idea, it is possible to construct an, a priori, renormalizable theory of Quantum Gravity: Hořava-Lifshitz gravity

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The problem with GR can be summarized i na clash between dimensionality and Lorentz invariance

$$[G_n] = -2, \qquad G(k) \propto \frac{1}{k^2}$$

Higher loops will produce new divergent counterterms, driving the non-renormalizability of the theory. Modifying the propagator using Lifshitz scaling is then a solution.

H-L Gravity

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$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i - \gamma_{ij} dx^i dx^j$$

$$S = \frac{1}{16\pi G} \int dt d^3x \sqrt{|g|} \left(K_{ij} K^{ij} - K^2 + R \right)$$

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For example, for $z = D = 2$

$$R^2, \quad R$$

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For example, for $z = D = 2$

$$R^2, \quad R$$

While for $z = D = 3$

$$R^3, \quad R_{\mu\nu} R^{\mu\nu} R, \quad \nabla^2 R, \quad \ldots$$

H-L Gravity

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There are two versions of the theory

- Projectable: The lapse does not depend on the spatial coordinates $N = N(t)$ and thus can be integrated out by fixing the gauge invariance in the time direction

$$N = 1$$

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There are two versions of the theory

- Projectable: The lapse does not depend on the spatial coordinates $N = N(t)$ and thus can be integrated out by fixing the gauge invariance in the time direction

$$N = 1$$

- Non-projectable: The lapse depends on all the coordinates $N = N(t, x)$. Then, the potential must be suplemented with extra terms

$$a_i a^i, \quad \nabla_i a^i, \quad \ldots$$

where $a_i = \frac{\nabla_i N}{N}$.

D. Blas, O. Pujolas and S. Sibiryakov, 2010

The action

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i - \gamma_{ij} dx^i dx^j$$

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$$S = \int dt d^D x N \sqrt{\gamma} \phi \mathcal{D} \phi$$

$$\mathcal{D} = \underbrace{-\mathcal{L}_n^2 - \mathcal{L}_n K}_{\text{Temporal part}} + \underbrace{\mathcal{F}(\nabla_i, \Omega_I) + \Omega_0}_{\text{spatial part}}$$

$$\mathcal{L}_n \phi = \frac{1}{2N} \left(\partial_t - N^i \partial_i \right) \phi \quad (1)$$

Effective action from the Heat Kernel

$$W = \frac{1}{2}\log\left(det(\mathcal{D})\right)$$

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In order to compute it for a riemannian manifold, we assume that there is a base of eigenvectors of $\mathcal{D}$

$$\mathcal{D}\Psi_i = \lambda_i \Psi_i$$

and write

$$\log(\lambda_i) = -\int_0^\infty \frac{dt}{t} e^{-\lambda t} \to W = -\frac{1}{2}\int_0^\infty \frac{dt}{t} K(t,\mathcal{D})$$

where $K(t,\mathcal{D}) = Tr\left(e^{-t\mathcal{D}}\right)$

Effective action from the Heat Kernel

We reglarize the effective action by shifting the power of the proper time s

$$W_{reg} = -\frac{1}{2}\mu^{2s}\int_0^\infty \frac{dt}{t^{1-s}}K(t,\mathcal{D}) \quad (2)$$

Effective action from the Heat Kernel

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One can now relate the UV divergences to the asymptotic behavior of the kernel when $t \to 0$ and show that

$$W_{ren} = -\frac{1}{2}\log(\mu^2)a_u(\mathcal{D}) \tag{3}$$

where a_u is certain coefficient in the asymptotic expansion of the heat kernel

$$K(s,\mathcal{D}) \sim \frac{1}{s^u}\sum_n s^n a_n \tag{4}$$

The short-time expansion

For relativistic operators of the form

$$\mathcal{D} = -\nabla^2 - E$$

the short time expansion is well known

$$K(s, \mathcal{D}) = \frac{1}{(4\pi s)^{d/2}} \sum_n s^{n/2} a_n$$

$$a_0 = \int d^d x \sqrt{|g|}$$

$$a_2 = \frac{1}{6} \int d^d x \sqrt{|g|}\ Tr(R + 6E)$$

$$a_4 = \frac{1}{360} \int d^n x \sqrt{|g|}\ Tr(60\Box E + 60RE + 180E^2 + 12\Box R + 5R^2 - \\ - 2R_{\mu\nu}R^{\mu\nu} + 2R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 30\hat{\mathcal{R}}_{\mu\nu}\hat{\mathcal{R}}^{\mu\nu})$$

The short-time expansion

- In the case of non-minimal operators, there are techniques available

A. O. Barvinsky and G. A. Vilkovisky, 1985

It stands schematically for applying BCH lemma

$$e^{A+B} \sim \mathcal{M} e^{A} e^{B} \tag{5}$$

- For higher order operators, one needs to combine this with the following lemma

$$a_k(\mathcal{O}^n) = \frac{1}{n} \lim_{\epsilon \to 0} \left[\Gamma\left(\frac{d-k+\epsilon}{2}\right)^{-1} \Gamma\left(\frac{d-k+\epsilon}{2n}\right)\right] a_k(\mathcal{O})$$

P. Gilkey, 1985

The short-time expansion

$$K(s,\mathcal{D}) = Tr(f\ e^{-s\mathcal{D}}) = \frac{1}{s^{\alpha}}\sum_{n=0}^{\infty} s^{\frac{n}{2\beta}} a_n(f,\mathcal{D})$$

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The leading divergence is obtained by combining two techniques

- The highest momenta operators appearing in an anisotropic operator are

  $$\mathcal{D}_l = \omega^2 - p^{2z}$$

  and by using dimensionless variables

  $$\tilde{\omega} = \omega/s^{1/2}, \qquad \tilde{p} = p/s^{z/2}$$

  one has

  $$K(s,\mathcal{D}_l) \sim \frac{1}{s^{\frac{1}{2}(1+D/z)}}$$

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The leading divergence is obtained by combining two techniques

- By choosing a separable spacetime

$$\mathcal{M} = \mathcal{M}_1 \otimes \mathcal{M}_D \longrightarrow e^{-s\mathcal{D}} \sim e^{-s\mathcal{D}_1} e^{-s\mathcal{D}_D}$$

one finds

$$K(s,\mathcal{D}) = Tr(f\ e^{-s\mathcal{D}}) = \frac{1}{(4\pi s)^{\frac{1}{2}(1+D/z)}} \sum_{n=0}^{\infty} s^{\frac{n}{2z}} a_n(f,\mathcal{D}) \tag{6}$$

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where the coefficients a_n are constrained by simple relations.

$$[N] = z, \quad [\gamma_{ij}] = 2, \quad [\gamma^{ij}] = -2$$
$$[\nabla_i] = 0, \quad [\mathcal{L}_t] = -z, \quad [a_i] = 0$$

$$2\Delta\gamma - d = 0$$
$$-zL - 2\Delta\gamma = -(D+z)$$
$$L \equiv even$$

where

- $L \equiv (\#of\, \mathcal{L}_t)$, $\Delta\gamma = (\#of\, \gamma^{ij}) - (\#of\, \gamma_{ij})$, $d = (\#of\, \nabla_i)$

The short-time expansion

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$$\begin{aligned} &2\Delta\gamma - d = 0 \\ &-zL - 2\Delta\gamma = -(D+z) \\ &L \equiv even \end{aligned}$$

- No possible term (no possible anomaly) for $D + z \equiv$odd

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- No possible term (no possible anomaly) for $D+z \equiv$odd
- The number of Lie derivatives for $z > D$ is constrained to $L = 0, 2$
- For $z > D$ the counterterm (anomaly) is given only by the spatial part of the action

The critical coefficient

$$\phi\mathcal{D}\phi = \phi(\mathcal{D}_t + \mathcal{D}_x)\phi$$

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$$\mathcal{D}_t = -\mathcal{L}_t^2 - K\mathcal{L}_t$$

$$\mathcal{D}_x = N^{-\frac{D+3z}{2z}}\left[\sum_{l=0}^{z}\Omega_l\left(N^{\frac{D}{z}}\Delta'\right)^l\right]N^{-\frac{z-D}{2z}}, \quad \Delta' = \nabla_i\left(N^{\frac{2-D}{z}}\nabla^i\right)$$

For $z > D$ there is no Lie derivatives in the critical coefficient

$$a_{z+D} = \int dtd^D x N\sqrt{\gamma}\sum_i C_i\mathcal{I}^I(R_{ijkl}, a_i, \nabla_i)$$

The critical coefficient

$$a_{z+D} = \int dtd^D x N \sqrt{\gamma} \sum_i C_i \mathcal{I}^I(R_{ijkl}, a_i, \nabla_i) = \int dt\ a_{z+D}(\mathcal{D}_x)$$

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$$e^{-s\mathcal{D}_x} \sim exp\left[-s\sum_{l=0}\Omega_l\left(N^{\frac{D}{z}}\Delta'\right)^l\right] \sim exp\left[-s\sum_{l=0}\Omega_l\tilde{\Delta}^l\right]$$

$$\tilde{\Delta} = N^{\frac{D}{z}}\Delta' = N^{\frac{2}{z}}\left(\Delta + \frac{2-D}{z}\frac{\nabla_i N}{N}\nabla^i\right)$$

this is just a conformal transformation of the standard laplacian

$$\gamma'_{ij} = N^{-\frac{2}{z}}\gamma_{ij}$$

The critical coefficient

In the case of $z = D$ things are slightly different.

$$a_{z+D} = \int dtd^D x \, N\sqrt{\gamma}\left(C_0 K_{ij}K^{ij} + C_1 K^2 + C_2 \mathcal{L}_t K\right) + \\ + \int dtd^D x N\sqrt{\gamma}\sum_i C_i \mathcal{I}^I(R_{ijkl}, a_i, \nabla_i)$$

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To fix C_0, C_1 and C_2 we use a technique developed by Solodukhin and Nesterov, relying in the BCH formula

$$e^{-s\mathcal{D}_t - s\mathcal{D}_x} \sim \mathcal{M} e^{-s\mathcal{D}_t} e^{-s\mathcal{D}_x}$$

where $\mathcal{M}$ contains comutators.

S. Solodukhin and D. Nesterov, 2010

The other pieces are fixed by going to a spacetime in which $K_{ij} = 0$. There, we have the same situation as with $z > D$.

The critical coefficient

And we end up with

$$a_{2z}(f,\mathcal{D}) = -\frac{1}{(4\pi)^{\frac{1+D}{2}}}\int dt d^D x\, N\sqrt{\gamma}\left(\frac{1}{2(D+2)}\frac{\sqrt{\pi}}{\Gamma\left(\frac{D}{2}\right)}\left[K_{\mu\nu}K^{\mu\nu}-\frac{1}{D}K^2\right]+\right.$$

$$\left.+\frac{\sqrt{\pi}}{3z\Gamma\left(\frac{D}{2}\right)}\left(\mathcal{L}_t K+K^2\right)\right)+\frac{1}{(4\pi)^{\frac{1}{2}}}\int dt\ \ a_{2z}(\tilde{\mathcal{D}}_x)\Big|_{\gamma'_{ij}=N^{-\frac{2}{z}}\gamma_{ij}}$$

So now what?

- We can study the UV structure of the general Lifshitz theory

$$S = \int dt d^D x \, N\sqrt{\gamma} \, \left(\mathcal{L}_t\phi\mathcal{L}_t\phi + \phi\Delta^z\phi\right)$$

- We can also address the issue of Anisotropic Weyl Anomalies

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$$S = \int dtd^D x \; N\sqrt{\gamma} \; (\mathcal{L}_t\phi\mathcal{L}_t\phi + \phi\Delta^z\phi)$$

For even D, it is only possible to construct a counterterm if $z \equiv$even.

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For even D, it is only possible to construct a counterterm if $z\equiv$even. The critical coefficient will take the form

$$a_{D+z}(f,\mathcal{D}) = -\frac{\delta_{D-z}}{(4\pi)^{\frac{1+D}{2}}}\int dtd^D x\, N\sqrt{\gamma}\, \left(\frac{1}{2(D+2)}\frac{\sqrt{\pi}}{\Gamma\left(\frac{D}{2}\right)}\left[K_{\mu\nu}K^{\mu\nu} - \frac{1}{D}K^2\right] + \right.$$
$$\left. + \frac{\sqrt{\pi}}{3z\Gamma\left(\frac{D}{2}\right)}\left(\mathcal{L}_t K + K^2\right)\right) + \frac{1}{(4\pi)^{\frac{1}{2}}}\int dt\;\; a_{D+z}(\tilde{\mathcal{D}}_x)\Big|_{\gamma'_{ij}=N^{-\frac{2}{z}}\gamma_{ij}}$$

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$$\left.+\frac{\sqrt{\pi}}{3z\Gamma\left(\frac{D}{2}\right)}\left(\mathcal{L}_tK+K^2\right)\right)+\frac{1}{(4\pi)^{\frac{1}{2}}}\int dt\;\; a_{D+z}(\tilde{\mathcal{D}}_x)\Big|_{\gamma'_{ij}=N^{-\frac{2}{z}}\gamma_{ij}}$$

And by Gilkey's lemma

$$a_{D+z}(\Delta^z) = \frac{1}{z}\lim_{\epsilon\to 0}\left[\Gamma\left(\frac{-z+\epsilon}{2}\right)^{-1}\Gamma\left(\frac{-z+\epsilon}{2z}\right)\right]a_{D+z}(\Delta)$$

which vanishes unless $z \equiv$odd.

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$$\left. + \frac{\sqrt{\pi}}{3z\Gamma\left(\frac{D}{2}\right)}\left(\mathcal{L}_t K + K^2\right)\right)$$

For $z > D$ they are *finite*.

So now what?

- We can address Anisotropic Weyl anomalies
- Let us do it for $D = z = 2$
  Some works argued wether there was presence or not of Ricci curvatures in the anomaly
  - Some said there were not

    M. Baggio, J. de Boer and K. Holsheimer, 2011
  - But other authors showed that it appears for some particular choice of the action

    T. Griffin, P. Horava and C. M. Melby-Thompson, 2012
- With our technique we can address the question in a general setting

$$S = \int dtd^D x\, N\sqrt{\gamma}\, \phi \left(-\mathcal{L}_t^2 - \mathcal{L}_t K + \frac{1}{N^2}(N\Delta)^2 + \alpha_1 R^{(W)}\Delta + \right.$$
$$\left. + \alpha_2 \left(R^{(W)}\right)^2 + \eta \left(K_{ij}K^{ij} - \frac{1}{D}K^2 \right) \right) \phi$$

where $R^{(W)} = R + \frac{2(D-1)}{z}\nabla_i a^i - \frac{(D-2)(D-1)}{z^2} a_i a^i$.

So now what?

- We can address Anisotropic Weyl anomalies

Anomalies will be also given by the Heat Kernel critical coefficient

$$\delta W = -z\, a_{D+z}(\omega, \mathcal{D})$$

So now what?

$$S = \int dt d^D x \, N\sqrt{\gamma}\, \phi \left(-\mathcal{L}_t^2 - \mathcal{L}_t K + \frac{1}{N^2}(N\Delta)^2 + \alpha_1 R^{(W)}\Delta + \right.$$
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$$a_4 = \frac{1}{8\pi}\int dt \; d^2 x N\sqrt{\gamma}\, \omega \left[\frac{1}{2}\left(\frac{\alpha_1^2}{4} - \alpha_2\right)\left(R^{(W)}\right)^2 - \frac{1}{2}\left(\eta + \frac{1}{4}\right)\left(K_{ij}K^{ij} - \frac{1}{2}K^2\right) + \right.$$
$$\left. - \frac{1}{6}\left(\mathcal{L}_t K + K^2\right) \right]$$

This reproduces all the preovious results and gives an insight about the presence of Ricci curvature.

So now what?

$$a_4 = \frac{1}{8\pi}\int dt\, d^2x N\sqrt{\gamma}\,\omega\,\left[\frac{1}{2}\left(\frac{\alpha_1^2}{4}-\alpha_2\right)\left(R^{(W)}\right)^2 - \frac{1}{2}\left(\eta+\frac{1}{4}\right)\left(K_{ij}K^{ij}-\frac{1}{2}K^2\right)+ -\frac{1}{6}\left(\mathcal{L}_t K + K^2\right)\right]$$

This has dramatic consequences for holography

- Massive vector in Einstein gravity $\longrightarrow$ No R^2 term
- There are tow possible solutions
  - Add matter to bulk theory
  - Use Horava-Lifshitz gravity
  - Are both options the same???

So now what?

In arbitrary D and z

$$S=\int dt\,d^Dx\,N\sqrt{\gamma}\left\{\mathcal{L}_t\phi\mathcal{L}_t\phi+\frac{1}{4}\left(1-\frac{z^2}{D^2}\right)\phi^2K^2+\frac{1}{2}\left(1-\frac{z}{D}\right)\phi^2\mathcal{L}_tK+\right.$$
$$\left.+\eta\phi^2\left(K_{\mu\nu}K^{\mu\nu}-\frac{1}{D}K^2\right)+\phi\prod_{i=0}^{z-1}\left(\alpha_i\hat{\Delta}+\beta_iR^{(W)}\right)\phi\right\}$$

For either $\beta_i=0$ or $\alpha_i=\lambda\beta_i$, we have that the spatial part is always of the form $\mathcal{O}^z$ so we conclude

- For even D and $z>D$ there is no anomaly.
- For even D and $D=z$ the anomaly is proportional to $K_{ij}K^{ij}-\frac{1}{D}K^2$

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For the future

- Entanglement entropy?

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For the future

- Entanglement entropy?
- Application to non-scalar matter. Horava-Lifshitz gravity.