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Neutrino Oscillations in Dark Matter

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Outline

- Neutrino oscillations in vacuum

    Flavors and masses; mixing and flavor change

- Neutrino oscillations in matter:

    Wolfenstein potential; Mikheyev-Smirnov adiabatic conversion

- Neutrino oscillations in dark matter

    General formulation; Implications; non-oscillations?

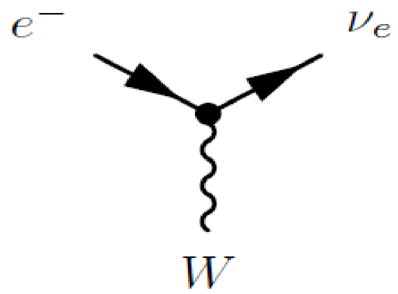
- Discussion and conclusion

Neutrino Oscillations in Vacuum

Flavors and Masses

- Flavored neutrinos: Weak interaction eigenstates

$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$



$$\frac{g}{\sqrt{2}}\, \bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha L}\, W_\mu^- + h.c.$$

- Massive neutrinos: Majorana Dirac

$\nu_1 \quad \nu_2 \quad \nu_3$

$m_1 \quad m_2 \quad m_3$

Majorana: $\nu^c = \nu \; (\nu_R \sim \nu_L^*)$

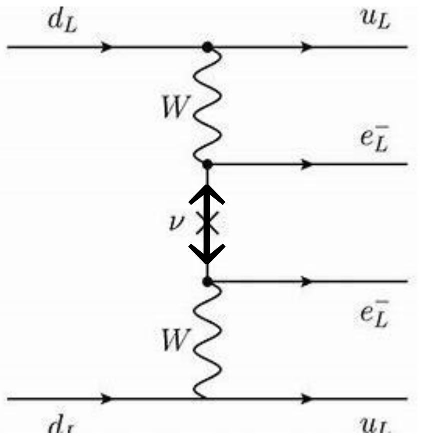
Dirac: $\nu^c \neq \nu \; (\nu^c \sim N)$

$\nu^c = C\, \bar{\nu}^T, \qquad C = i\gamma_0\gamma_2$

$\frac{1}{2} m_i \left(\overline{\nu_{iR}^c}\, \nu_{iL} + \overline{\nu_{iL}} \nu_{iR}^c \right) \qquad m_i \left(\overline{\nu_{iR}}\, \nu_{iL} + \overline{\nu_{iL}} \nu_{iR} \right)$

$= \frac{1}{2} m_i \left(\nu_{iL}^T C \nu_{iL} + {\nu_{iL}^*}^T C^+ \nu_{iL}^* \right)$

Majorana → $0\nu\beta\beta$



Mixing and flavor change

- Weak eigenstates ≠ Mass eigenstates

$$\nu_\alpha = U_{\alpha i}\, \nu_i$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P_M$$

$$P_M = \mathrm{Diag}[1, e^{i\varphi_2}, e^{i\varphi_2}]$$

- Two-flavor neutrino propagation in vacuum

$$\nu_e \to \nu_\mu$$

$$U = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}$$

$$|\nu_e(0)\rangle = c_\theta|\nu_1\rangle + s_\theta|\nu_2\rangle$$

$$|\nu_e(t)\rangle = c_\theta\, e^{i\phi_1}|\nu_1\rangle + s_\theta e^{i\phi_2}|\nu_2\rangle$$

$$\phi_i = E_i t - \boldsymbol{p}_i L$$

$$P_{e\mu} = \left|\langle\nu_\mu|\nu_e(t)\rangle\right|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Ultra-relativistic limit: $t \approx L$

$$E_i \approx \boldsymbol{p} + \frac{m_i^2}{2\boldsymbol{p}} \approx E + \frac{m_i^2}{2E}$$

$$\Delta\phi = \phi_2 - \phi_1 \approx \frac{\Delta m^2 L}{2E}$$

Neutrino evolution equation

- Propagation Hamiltonian:

$$i\frac{d}{dt}\psi = H\,\psi$$

$$\psi = (\nu_1, \nu_2, \nu_3)^T$$

$$H = \frac{m^2}{2E}$$

$$\psi = (\nu_e, \nu_\mu, \nu_\tau)^T$$

$$H_\nu = \frac{M^+M}{2E} = \frac{U^+m^2U}{2E}$$

$$H_{\bar{\nu}} = \frac{MM^+}{2E} = \frac{Um^2U^+}{2E} \left(\frac{Vm^2V^+}{2E}\right)$$

$M = U^*\,m\,U^+$ (Majorana)

$M = V\,m\,U^+$ (Dirac)

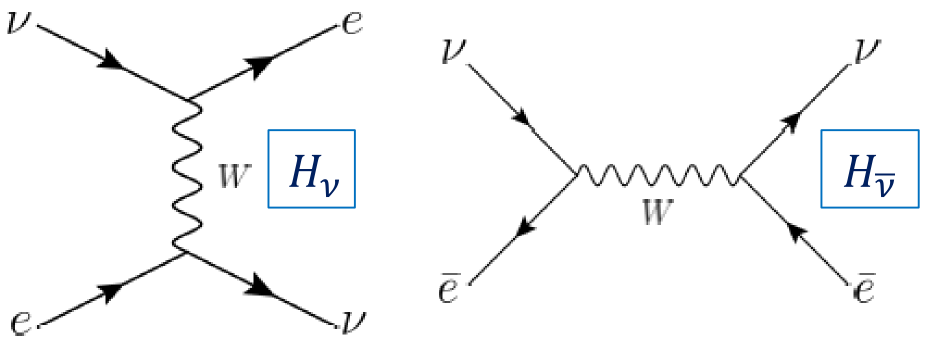
- Two-flavor evolution:

$$H = \frac{\Delta m^2}{4E}\begin{bmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{bmatrix} \rightarrow P_{e\mu} = \left|\langle\nu_\mu|e^{iHt}|\nu_e\rangle\right|^2$$

Neutrino Oscillations in Matter

Wolfenstein Potential

- Wolfenstein 1978: "Coherent forward scattering of neutrinos leaving the medium unchanged must be taken into account."



$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F\, \overline{\nu_{eL}}\gamma^\mu e_L\, \overline{e_L}\gamma_\mu \nu_{eL} \Rightarrow V_W\, \overline{\nu_{eL}}\gamma^0 \nu_{eL}$$

$$V_W = \sqrt{2}\, G_F N_e$$

- Neutrino evolution in matter: $H_{\nu,\bar{\nu}} = \frac{M^2}{2E} \pm V_W$

Neutrino propagation in matter

- Two-flavor evolution: $\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}$

$$H_\nu = \frac{\Delta m^2}{4E}\begin{bmatrix} -c_{2\theta} + x & s_{2\theta} \\ s_{2\theta} & c_{2\theta} - x \end{bmatrix} \qquad x \equiv \frac{2EV_W}{\Delta m^2}$$

- Diagonalizing H_ν: $\sin^2 2\theta_m = \dfrac{\sin^2 2\theta}{(\cos 2\theta - x)^2 + \sin^2 2\theta}$

$$H_{2m} - H_{1m} = \Delta m^2 \sqrt{(\cos 2\theta - x)^2 + \sin^2 2\theta}$$

- For $\Delta m^2 \approx m_2^2 \gg m_1^2$:

$x \gg 1$	$x \approx \cos 2\theta$	$x \approx 0$
$\theta_m \approx 0$	$\theta_m \approx 45^0$	$\theta_m \approx \theta\ (\sim 0)$
$\nu_{2m} \approx \nu_e$	$H_{2m} \approx H_{1m}$	$\nu_{2m} \approx \nu_\mu$
	$\nu_{2m} \approx \nu_e + \nu_\mu$	

Adiabatic conversion

- In a density-varying medium

$$i\frac{d}{dt}\begin{bmatrix}\nu_{1m}\\ \nu_{2m}\end{bmatrix}=\begin{bmatrix}H_{1m} & i\frac{d\theta_m}{dt}\\ -i\frac{d\theta_m}{dt} & H_{2m}\end{bmatrix}\begin{bmatrix}\nu_{1m}\\ \nu_{2m}\end{bmatrix}$$

- Adiabatic condition: $\left|\frac{d\theta_m}{dt}\right| \ll H_{2m}-H_{1m} \;\;\Rightarrow\;\; \nu_{1m} \not\leftrightarrow \nu_{2m}$
- Solar neutrino deficit:

"Oscillation or non-oscillation?"
Smirnov 1609.02386

$$\nu(t_i)=\nu_e=\cos\theta_m^i\,\nu_{1m}+\sin\theta_m^i\,\nu_{2m}$$
$$\nu(t_f)=\cos\theta_m^i\,\nu_1+\sin\theta_m^i\,e^{i\delta\phi}\,\nu_2$$

$$P_{ee}=\left|\langle\nu_e|\nu(t_f)\rangle\right|^2=\left(\cos\theta\,\cos\theta_m^i\right)^2+\left(\sin\theta\,\sin\theta_m^i\right)^2$$
$$P_{sol}=\frac{1}{2}(1+\cos2\theta_{12}\cos2\theta_{12}^m)+P_{earth}\approx 0.34$$

Adiabatic conversion

- Solar neutrino deficit:

$$P_{obs} = \frac{1}{2}(1 + \cos 2\theta_{12} \cos 2\theta_{12}^{m}) + P_{earth} \approx 0.34$$

"Oscillation or non-oscillation?"
Smirnov 1609.02386

Repubblica,
Dec. 14 2016, E. Dusi:

"C'e un errore nel Nobel della fisica del 2015"

Flavor conversion without oscillation

- Imagine a flavorful Wolfenstein potential

$$V = \begin{bmatrix} V_{ee} & V_{e\mu} \\ V_{e\mu} & V_{\mu\mu} \end{bmatrix}$$

$$\sin 2\theta_m = \frac{2V_{e\mu}}{V_{ee} - V_{\mu\mu}}$$

$$H_{2m} - H_{1m} = \sqrt{\left(V_{ee} - V_{\mu\mu}\right)^2 + 4V_{e\mu}^2}$$

- Adiabatic conversion

$$\nu_e(t_i) = \nu_e = \cos\theta_m^i\, \nu_{1m} + \sin\theta_m^i\, \nu_{2m}$$

$$\nu_e\left(t_f\right) = \cos\theta_m^i\, \nu_{1m}^f + \sin\theta_m^i\, \nu_{2m}^f$$

$$P_{e\mu} = \left|\langle \nu_\mu | \nu_e\left(t_f\right)\rangle\right|^2 = \left|\cos\theta_m^i \sin\theta_m^f - \sin\theta_m^i \cos\theta_m^f\right|^2 \neq 0$$

$$H = \frac{M^2}{2E} \quad \text{vs.} \quad V_W$$

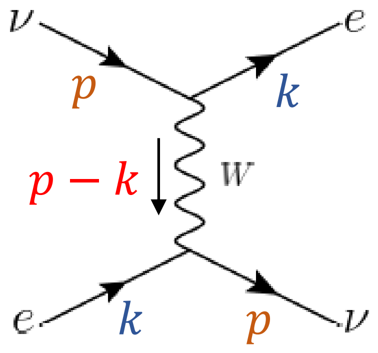
$$\Downarrow \qquad \Downarrow$$

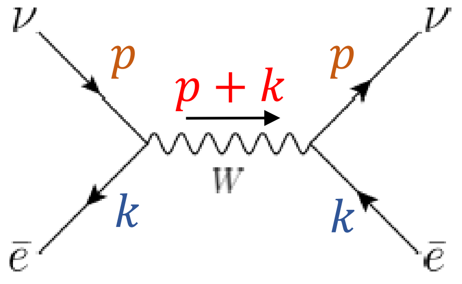
$$E^{-1} \qquad E^0$$

Neutrino Oscillations in Dark Matter

General Wolfenstein Potential

- In a medium with arbitrary N_e and $N_{\bar{e}}$





$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F m_W^2 \frac{\overline{\nu_{eL}}\gamma^\mu e_L\, \overline{e_L}\gamma_\mu \nu_{eL}}{m_W^2 - q^2}$$

$$\langle \mathcal{H}_\nu \rangle = \not{k}\, \sqrt{2} G_F m_W^2 \left[\frac{N_e/m_e}{m_W^2 - (p-k)^2} - \frac{N_{\bar{e}}/m_e}{m_W^2 - (p+k)^2} \right]$$

$$\langle \mathcal{H}_{\bar{\nu}} \rangle = \not{k}\, \sqrt{2} G_F m_W^2 \left[\frac{N_{\bar{e}}/m_e}{m_W^2 - (p-k)^2} - \frac{N_e/m_e}{m_W^2 - (p+k)^2} \right]$$

$$\langle N_e, N_{\bar{e}} | e\bar{e} | N_e, N_{\bar{e}} \rangle = -\frac{1}{2} \sum_s u_s(k)\bar{u}_s(k) \frac{N_e}{2k^0} + \frac{1}{2} \sum_s v_s(k)\bar{v}_s(k) \frac{N_{\bar{e}}}{2k^0}$$

Asymmetric medium potential

$$\Rightarrow V^m_{\nu,\bar{\nu}} \approx \sqrt{2} G_F (N_e + N_{\bar{e}}) \frac{\pm\epsilon\, m_W^4 - 2 m_e E_\nu m_W^2}{m_W^4 - 4 m_e^2 E_\nu^2}$$

$$\epsilon \equiv \frac{N_e - N_{\bar{e}}}{N_e + N_{\bar{e}}}$$

- The standard Wolfenstein potential for $m_W^2 \gg 2 m_e E_\nu$ & $\epsilon = 1$.
- Asymmetric medium distinguishes neutrinos and anti-neutrin os – opposite sign for the potential.
- For $m_W^2 \ll 2 m_e E_\nu$, $H_\nu = H_{\bar{\nu}}$ behaves like $\frac{\Delta m^2}{2E}$:

$$V^m_{\nu,\bar{\nu}} \approx \frac{\sqrt{2} G_F (N_e + N_{\bar{e}}) m_W^2 / m_e}{2 E_\nu}$$

Neutrino oscillations without mass!

Generalized medium

- Variant models of dark matter and mediator

$$\mathcal{L}' = g_{\alpha i}\,\overline{f_{iL}}\gamma^{\mu}\nu_{\alpha L}\,X_{\mu} + h.c. \quad (1)$$

$$g_{\alpha i}\,\overline{f_R}\nu_{\alpha L}\,\phi_i + h.c. \quad (2)$$

$$g_{\alpha i}\,\overline{f_{iR}}\nu_{\alpha L}\,\phi + h.c. \quad (3)$$

$$g_{\alpha\beta}\,\overline{\nu^c_{\beta R}}\nu_{\alpha L}\,\phi + h.c. \quad (4)$$

$$g_{\alpha\beta}\,\overline{\nu^c_{\beta R}}\nu_{\alpha L}\,\phi + y\,\phi\,\bar{f}_R f_L + h.c. \quad (5)$$

$$\mathcal{L}_0 = \frac{1}{2}\,\overline{\nu_L}i\partial\!\!\!/\nu_L + \frac{1}{2}\overline{\nu^c_R}i\partial\!\!\!/\nu^c_R - \frac{1}{2}\,\overline{\nu^c_R}M\nu_L - \frac{1}{2}\overline{\nu_L}M^{+}\nu^c_R$$

(u_L, v_L) or $(u_L, u_R = (v_L)^c)$

General formulation

- In a Lorenz invariant medium:

$$\mathcal{L} = \bar{u}_L(p)\,(\not{p} - \not{p}\Sigma_1 - \not{k}\Sigma_2)u_L(p) + \bar{u}_R(p)(\not{p} - \not{p}\bar{\Sigma}_1 - \not{k}\bar{\Sigma}_2)u_R(p)$$

$$- \bar{u}_R(p)(M + \Sigma_0)u_L(p) - \bar{u}_L(p)(M^+ + \bar{\Sigma}_0)u_R(p)$$

(3,4) (1,2,3,4) (5)

$\Sigma^+_{1,2} = \Sigma_{1,2}$

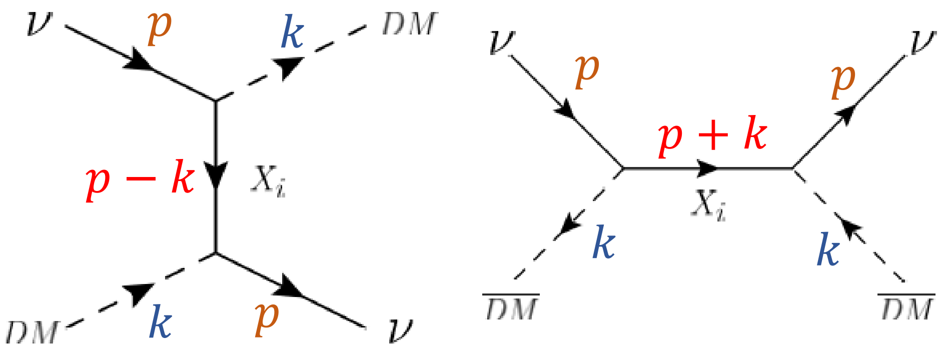
- Mass correction: $\Sigma_0 = gy\,\rho_{DM}/m_\phi^3$
- Kinetic correction → go to the canonical basis:

$$u_L \to \left(1 + \frac{\Sigma_1}{2}\right)u_L\,,\;\; u_R \to \left(1 + \frac{\bar{\Sigma}_1}{2}\right)u_R\,; \qquad M \to \widetilde{M} = \left(1 + \frac{\bar{\Sigma}_1}{2}\right)M\left(1 + \frac{\Sigma_1}{2}\right)$$

$\widetilde{M}^T = \widetilde{M}$ only if $\bar{\Sigma}_1 = \Sigma_1^T$

- DM potential: $V^{\mathrm{DM}}_{\nu(\bar{\nu})} = k^0\Sigma_2(\bar{\Sigma}_2)$

A model calculation



$$\mathcal{L}' = g_{\alpha i}\,\overline{f_{iR}}\nu_{\alpha L}\,\phi^* + h.c. = g^*_{\alpha i}\,\overline{f^c_{iL}}\nu^c_{\alpha R}\,\phi + h.c.$$

$$\mathcal{L}_{\rm eff} = \bar{u}_L(p)\left(\not p - \lambda\left[\frac{\not p - \not k}{(p-k)^2 - m_X^2}\frac{N_{\rm DM}}{2k^0} + \frac{\not p + \not k}{(p+k)^2 - m_X^2}\frac{N_{\overline{\rm DM}}}{2k^0}\right]\right)u_L(p)$$
$$+\bar{u}_R(p)\left(\not p - \lambda^*\left[\frac{\not p - \not k}{(p-k)^2 - m_X^2}\frac{N_{\overline{\rm DM}}}{2k^0} + \frac{\not p + \not k}{(p+k)^2 - m_X^2}\frac{N_{\rm DM}}{2k^0}\right]\right)u_R(p)$$
$$-\bar{u}_R(p)Mu_L(p) - \bar{u}_L(p)M^+u_R(p)$$

$$\lambda_{\alpha\beta} \equiv g^*_{\alpha i}\,g_{\beta i}\ \ (\lambda^T = \lambda^*)$$

Calculation of $\Sigma_{1,2}$ and DM potential

$$\Sigma_1(\bar{\Sigma}_1) = \frac{\lambda^{(T)}}{2} \frac{\rho_{\rm DM}}{m_{\rm DM}^2} \frac{\pm\epsilon 2 m_{\rm DM} E_\nu - m_X^2}{m_X^4 - 4 m_{\rm DM}^2 E_\nu^2}$$

$$\Sigma_2(\bar{\Sigma}_2) = \frac{\lambda^{(T)}}{2} \frac{\rho_{\rm DM}}{m_{\rm DM}^2} \frac{\pm\epsilon m_X^2 - 2 m_{\rm DM} E_\nu^2}{m_X^4 - 4 m_{\rm DM}^2 E_\nu^2}$$

$$V_{\nu\,(\bar{\nu})}^{\rm DM} = \frac{\lambda^{(T)}}{2} \frac{\rho_{\rm DM}}{m_{\rm DM}} \frac{\pm\epsilon m_X^2 - 2 m_{\rm DM} E_\nu}{m_X^4 - 4 m_{\rm DM}^2 E_\nu^2}$$

$$\widetilde{M} = \left(1 + \frac{\bar{\Sigma}_1}{2}\right) M \left(1 + \frac{\Sigma_1}{2}\right)$$

$$\bar{\Sigma}_{1,2} = \Sigma_{1,2}^* \left(\widetilde{M}^T = \widetilde{M}\right) \text{ if } \epsilon = 0$$

$$\epsilon \equiv \frac{N_{\rm DM} - N_{\overline{\rm DM}}}{N_{\rm DM} + N_{\overline{\rm DM}}},$$

$$\rho_{\rm DM} \equiv m_{\rm DM}(N_{\rm DM} + N_{\overline{\rm DM}})$$

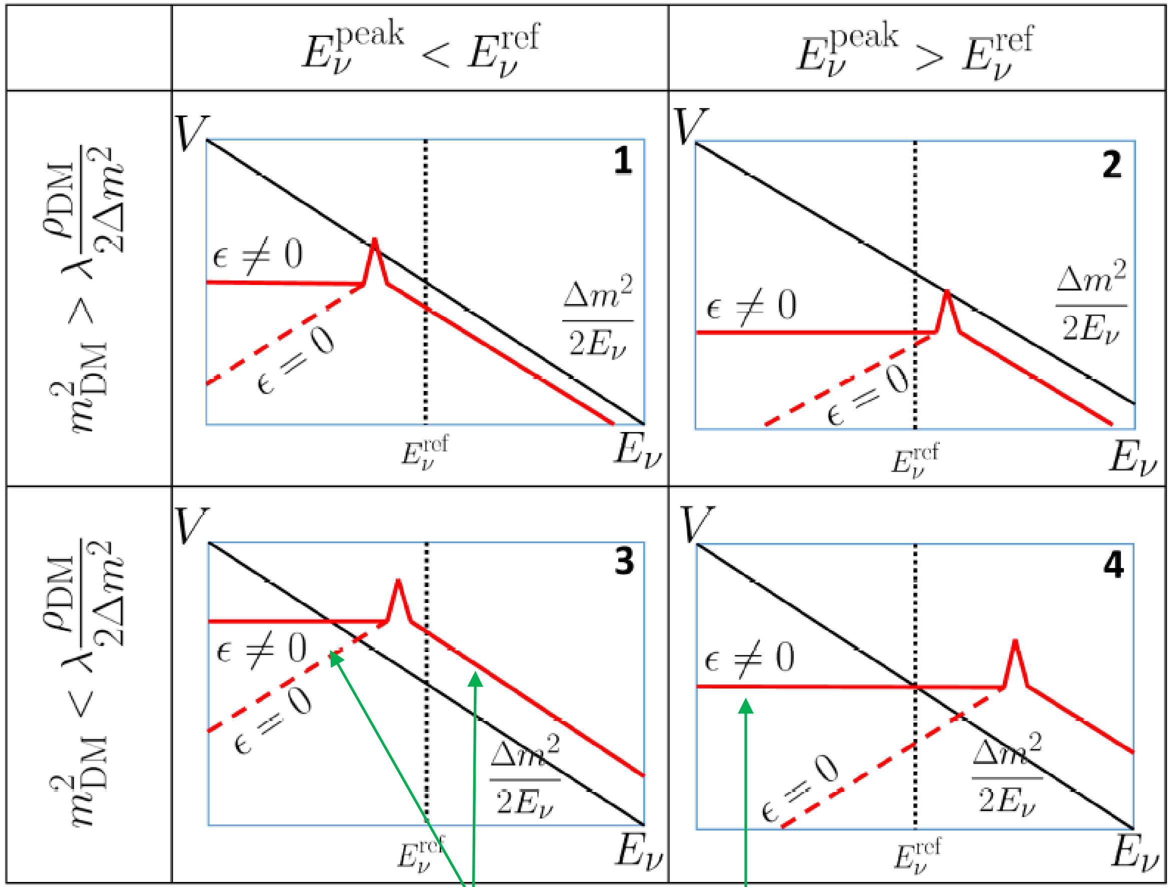
*) $m_X \to 0$ with $\epsilon = 0$ corresponds to Ge-Murayama 1904.02518 (sign mistake).

Behaviors of DM potential

$$E_\nu^{\text{peak}} \equiv \frac{m_X^2}{2m_{\text{DM}}}$$

$$E_\nu^{\text{ref}} = 1\text{MeV} \sim 100\text{GeV}$$

"Window of observed neutrino oscillations"



$$V_{\nu,\bar{\nu}}^{\text{DM}} = \frac{\lambda^{(T)}}{2} \frac{\rho_{\text{DM}}/m_{\text{DM}}^2}{2E_\nu}$$

$$V_{\nu,\bar{\nu}}^{\text{DM}} = \pm\epsilon \frac{\lambda^{(T)}}{4} \frac{\rho_{\text{DM}}}{m_{\text{DM}}^2 E_\nu^{\text{peak}}}$$

1. Constrained DM potential: $1\sim10\%$ of $\Delta m^2/2E$
2. NSI probes: $\varepsilon_{\alpha\beta} \equiv V_{\alpha\beta}^{\text{DM}}/V_W^{\text{SM}}$
3. Excluded DM potential.
4. Future probe of DM potential governing oscillations of ultra-relativistic neutrinos

Corrections to standard oscillations

- Conventional NSI parametrization for $E_\nu^{\rm peak} > 100{\rm GeV}$:

$$\varepsilon_{\alpha\beta} \equiv \frac{V_{\alpha\beta}^{\rm DM}}{V_W^{\rm SM}} = \pm 0.01\epsilon\lambda_{\alpha\beta}\left(\frac{20{\rm meV}}{m_{\rm DM}}\right)^2\left(\frac{1{\rm TeV}}{E_\nu^{\rm peak}}\right)\left(\frac{\rho_{\rm DM}}{0.3{\rm GeVcm}^{-3}}\right) \quad \text{with } N_e \approx 10^{24}\ {\rm cm}^{-3}$$

- Considering corrections of 1% for $\varepsilon, \Sigma_1/\bar{\Sigma}_1$:

$$m_{\rm DM} > {\rm Max}\left[20{\rm meV}\,|\epsilon\lambda|^{\frac{1}{2}}\left(\frac{1{\rm TeV}}{E_\nu^{\rm peak}}\right)^{\frac{1}{2}}, 10^{-4}{\rm meV}\,|\lambda|^{\frac{1}{3}}\left(\frac{1{\rm TeV}}{E_\nu^{\rm peak}}\right)^{\frac{1}{3}}\right] \quad \text{for } E_\nu^{\rm peak} > 100{\rm GeV}$$

$$m_{\rm DM} > {\rm Max}\left[200{\rm meV}\,|\lambda|^{\frac{1}{2}}, 0.3{\rm meV}\,|\epsilon\lambda|^{\frac{1}{3}}\left(\frac{1{\rm MeV}}{E_\nu}\right)^{\frac{1}{3}}\right] \quad \text{for } E_\nu^{\rm peak} < 1{\rm MeV}$$

Oscillations without mass

- If $E_\nu^{\text{peak}} < 1\text{MeV} < E_\nu$, the neutrino propagation Hamiltonian is

$$\boxed{H_{\nu(\bar{\nu})} \approx \lambda^{(T)} \frac{\rho_{\text{DM}}/m_{\text{DM}}^2}{2E_\nu}} \Rightarrow \lambda^{(T)} \frac{3\times 10^{-3}\text{eV}^2}{2E_\nu}\left(\frac{20\text{meV}}{m_{\text{DM}}}\right)^2$$

- Then the observed oscillations can be explained by

$$\begin{aligned}\lambda &= \frac{2m_{DM}^2}{\rho_{DM}} U^* \text{diag}(\Delta m^2) U^T, \\ &\simeq \begin{pmatrix} 0.026 & 0.091 & 0.085 \\ 0.091 & 0.381 & 0.408 \\ 0.085 & 0.408 & 0.478 \end{pmatrix} \left(\frac{m_{DM}}{20\text{meV}}\right)^2 \left(\frac{0.3\,\text{GeV cm}^{-3}}{\rho_{DM}}\right)\end{aligned}$$

Normal hierarchy
No CP phase

Oscillations by DM potential at High E

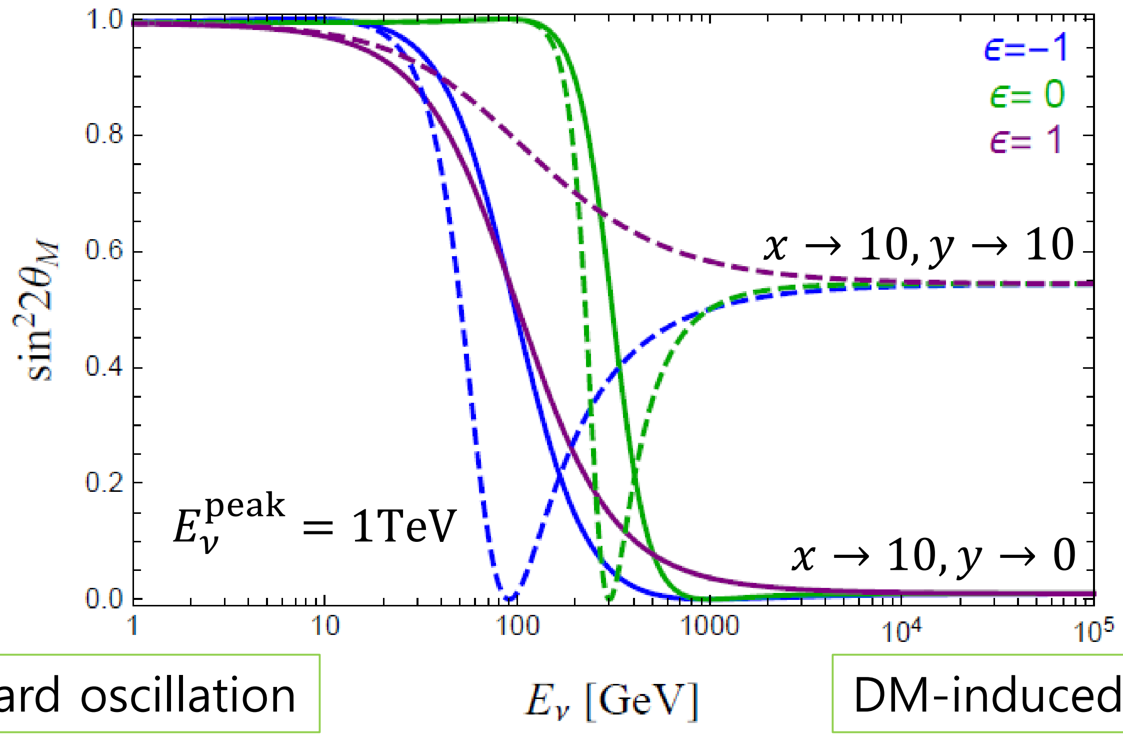
- Standard oscillation for $E_\nu < 100GeV$ if $E_\nu^{\mathrm{peak}} \gg 100\mathrm{GeV}$.
- DM-assisted oscillation for $E_\nu \gg E_\nu^{\mathrm{peak}}$.

$$\mathcal{H}_M = \mathcal{H}_{\mathrm{vac}} + \begin{pmatrix} V_{\mu\mu} & V_{\mu\tau} \\ V_{\mu\tau}^* & V_{\tau\tau} \end{pmatrix} \rightarrow \mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{pmatrix} -(\cos 2\theta - x) & \sin 2\theta + y \\ \sin 2\theta + y & \cos 2\theta - x \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} \sin^2 2\theta_M &= \frac{(\sin 2\theta + y)^2}{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2}, \\ \Delta m_M^2 &= \Delta m^2 \sqrt{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2}, \end{aligned}}$$

$$x \equiv \frac{(V_{\mu\mu} - V_{\tau\tau})/2}{\Delta m^2/4E}, \qquad y \equiv \frac{V_{\mu\tau}}{\Delta m^2/4E}.$$

Energy dependent oscillation angle



Standard oscillation

DM-induced oscillation

Realistic?

- Sensible UV-completion

    Dark sector coupling only to neutrinos…

- Observable DM effect if $m_{X,\mathrm{DM}} \ll \mathrm{meV}$ and $|\lambda| \ll 1$

    BBN, neutrino-DM scattering, star cooling, etc.

- Origin of DM density

    Ultra-light cold DM, coherent oscillation

Conclusion

- An effort to provide a more general study of neutrino propaga tion in a (Lorenz-invariant) medium.
- Asymmetric medium induces CPT violation in neutrino oscillati ons which may be tested in the future.
- Neutrino oscillations can be due to either the medium effect o r the genuine mass effect, or both.
- Observation of ultra-relativistic neutrino oscillations and the ab solute mass measurement will be crucial.
- Further studies on phenomenological implications and viability are needed.