New approaches in the analysis of Dark Matter direct detection data

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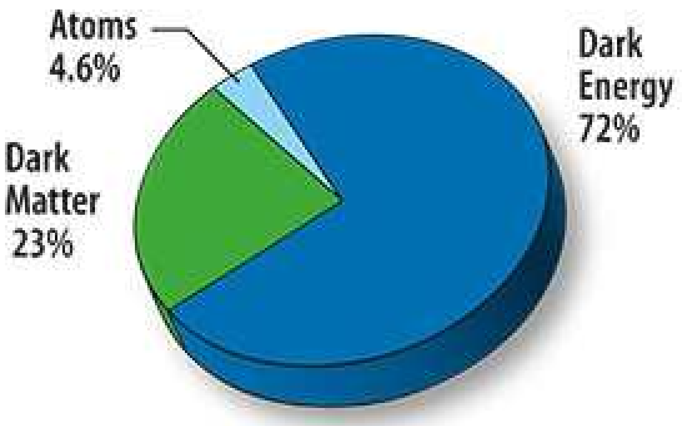
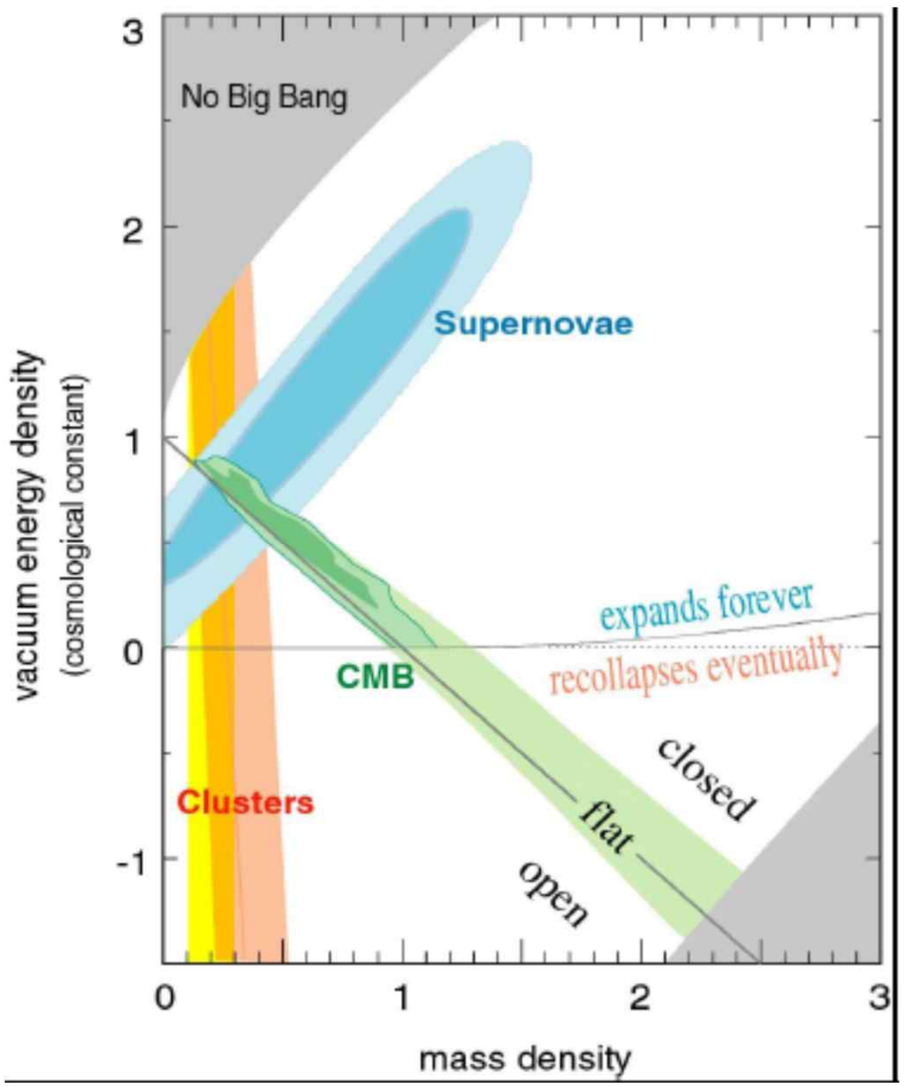
Plan of the talk:

- Introduction
- Sources of uncertainty in direct detection and generalizations
- Are constraints robust? A few counter-examples (DAMA and more)
- Some late developments (work in progress)

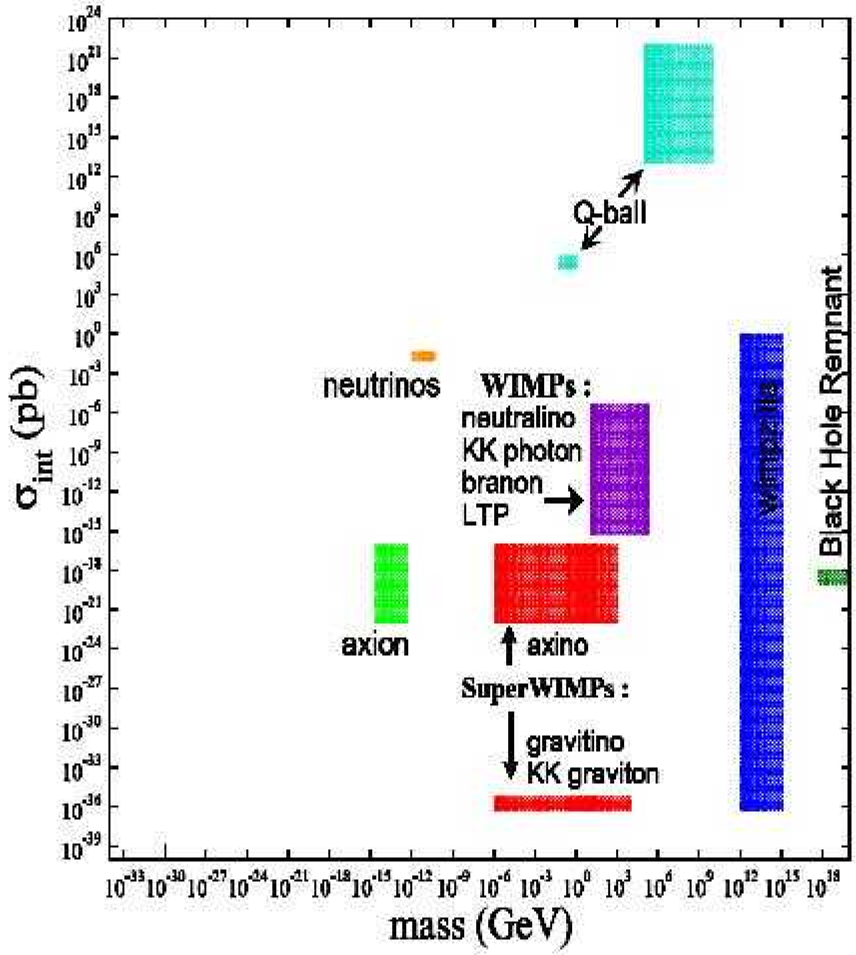
The concordance model



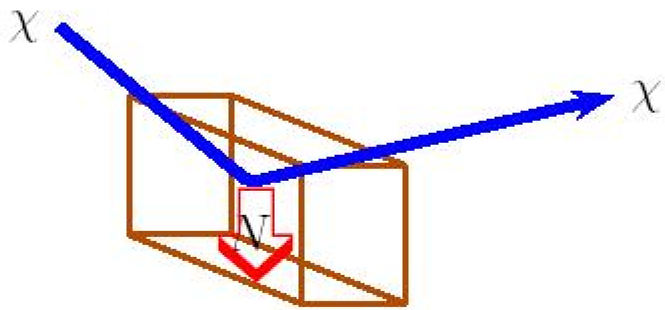


(Incomplete) List of DM candidates

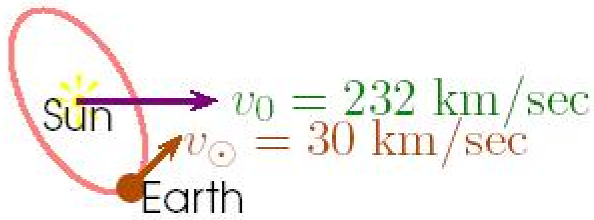
- Neutrinos
- Axions
- WIMPS (including Lightest Supersymmetric particle LSP such a neutralino or sneutrino)
- SuperWIMPS (gravitino)
- Lighest Kaluza-Klein Particle (LKP)
- Heavy photon in Little Higgs Models
- Solitons (Q-balls, B-balls)
- Black Hole remnants
- …



WIMP direct detection



- Elastic recoil of non relativistic halo WIMPs off the nuclei of an underground detector
- Recoil energy of the nucleus in the keV range
- Yearly modulation effect due to the rotation of the Earth around the Sun (the relative velocity between the halo, usually assumed at rest in the Galactic system, and the detector changes during the year)



WIMP differential detection rate

$$\frac{dR}{dE_R} = N_T \frac{\rho_\chi}{m_\chi} \int_{vmin}^{v_{max}} d\vec{v} f(\vec{v}) |\vec{v}| \frac{d\sigma(\vec{v}, E_R)}{dE_R}$$

E_R=nuclear energy
N_T=# of nuclear targets
v=WIMP velocity in the Earth's rest frame

Astrophysics

- ρ_χ=WIMP local density
- f(v)= WIMP velocity distribution function

Particle and nuclear physics

- $\frac{d\sigma(\vec{v}, E_R)}{dE_R}$ =WIMP-nucleus elastic cross section

$$\frac{d\sigma(\vec{v}, E_R)}{dE_R} = \left(\frac{d\sigma(\vec{v}, E_R)}{dE_R}\right)_{\text{coherent}} + \left(\frac{d\sigma(\vec{v}, E_R)}{dE_R}\right)_{\text{spin-dependent}}$$

usually dominates, α (atomic number)^2

N.B.: dependence on galactic model contained in function:

$$\boxed{\mathcal{I}(v_{min}) \equiv \int_{v_{min}} \frac{f(v)}{v}\, d^3\vec{v}}$$

f(v) usually assumed to be at Maxwellian at rest in the Galactic system (possibility of *corotation* can be also considered):

$$f_G(\vec{v}_G) = \left(\frac{3}{2\pi v_{rms}^2}\right)^{\frac{3}{2}} e^{-\frac{3v_G^2}{2v_{rms}^2}}\, d^3\vec{v}_G$$

$$\vec{v}_G = \vec{w} + \vec{v}$$

WIMP velocity in Galactic reference frame

Earth velocity in Galactic reference frame

WIMP velocity in Earth reference frame

N.B.: theoretical predictions for the WIMP direct detection rate depend on two main ingredients:

1) a scaling law for the cross section, in order to compare experiments using different targets

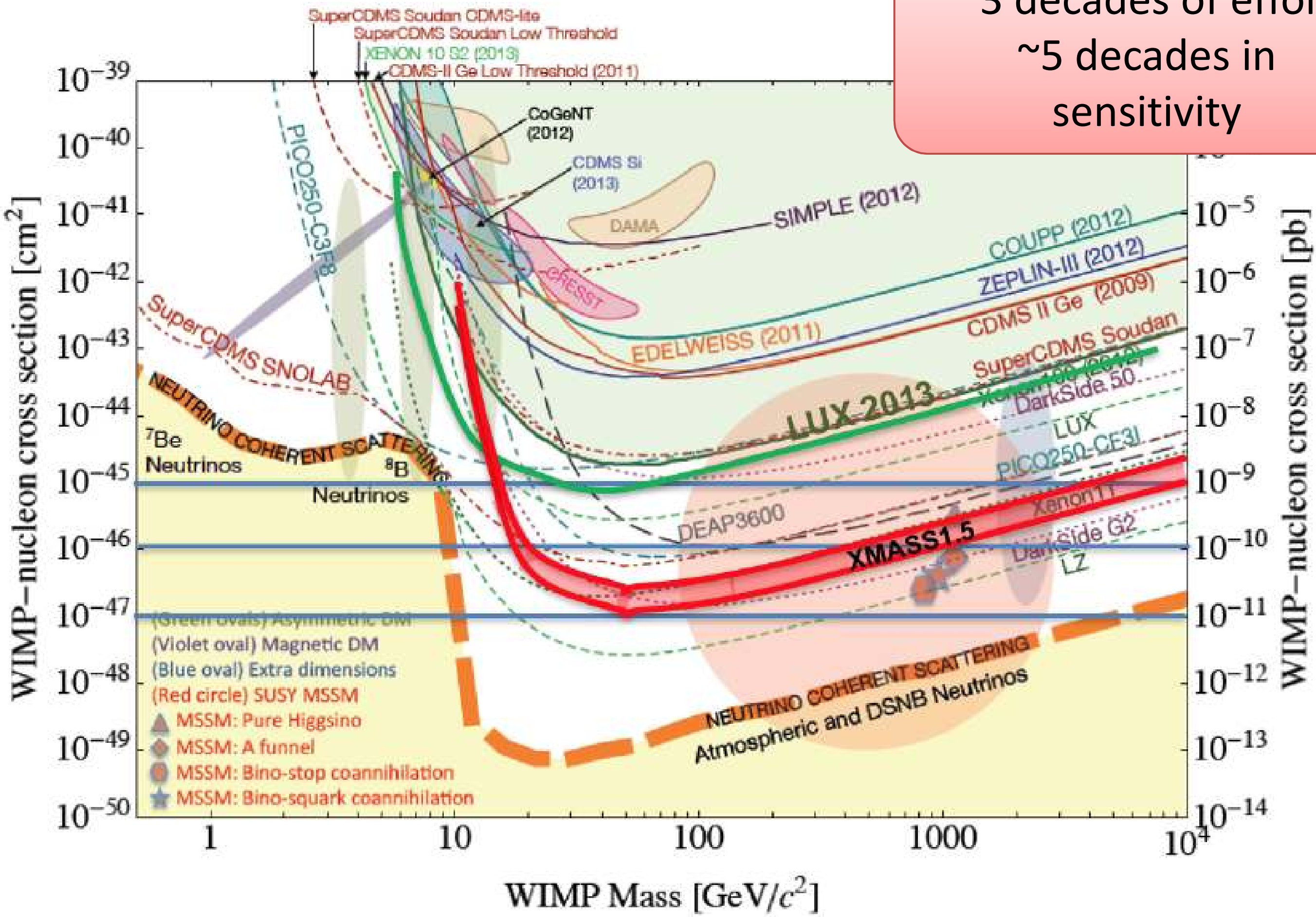
   Traditionally spin-independent cross section (proportional to (atomic mass number)^2) or spin-dependent cross section (proportional to the product $\boldsymbol{S}_{WIMP} \cdot \boldsymbol{S}_{nucleus}$) is assumed

2) a model for the velocity distribution of WIMPs

   Traditionally a Maxwellian distribution is assumed

WIMP direct searches: spin-independent interaction+Maxwellian distribution

~3 decades of effort
~5 decades in sensitivity



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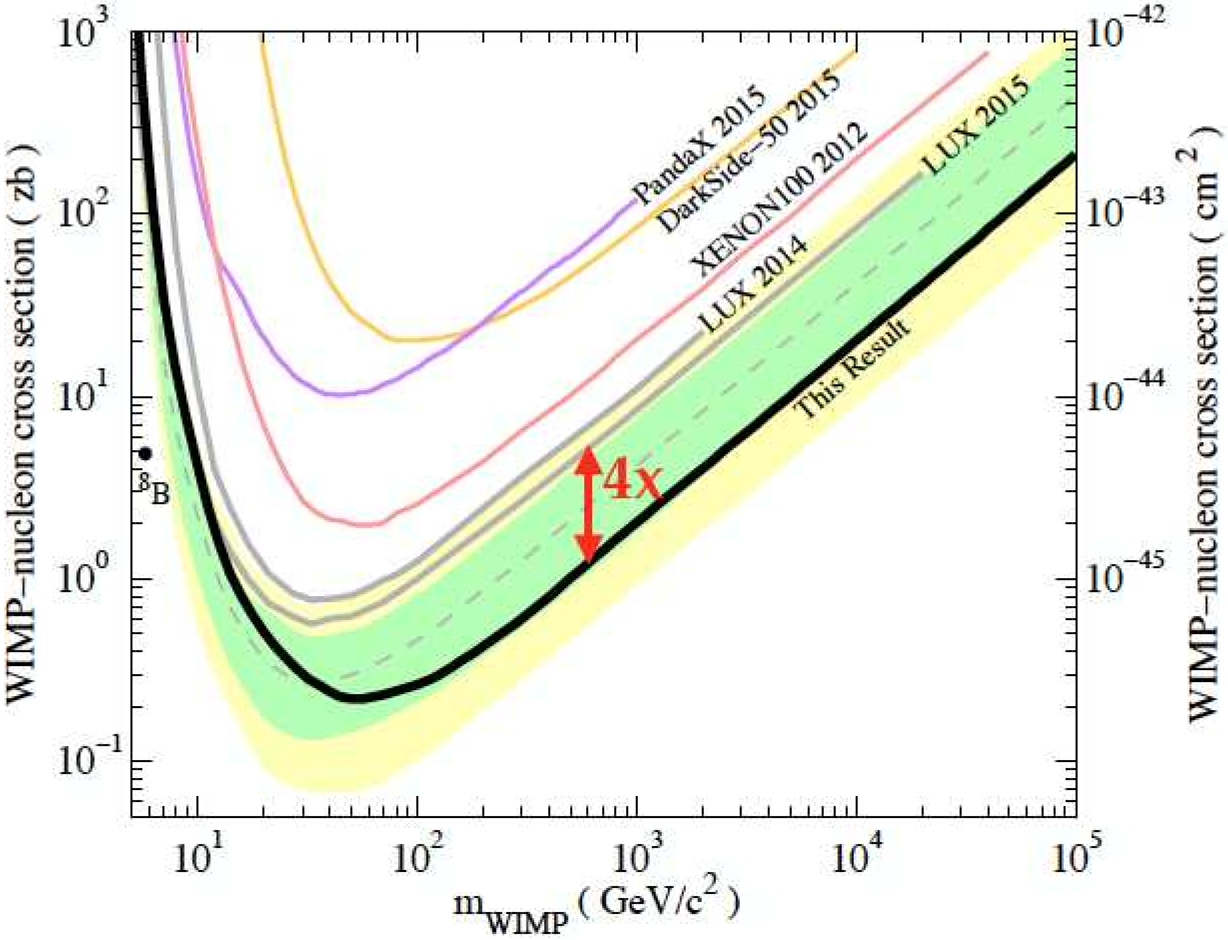
Will the race discover DM before eventually reaching the irreducible background of solar and atmospheric neutrinos???

(from Y. Suzuki talk @IDM 2016, July 2016)

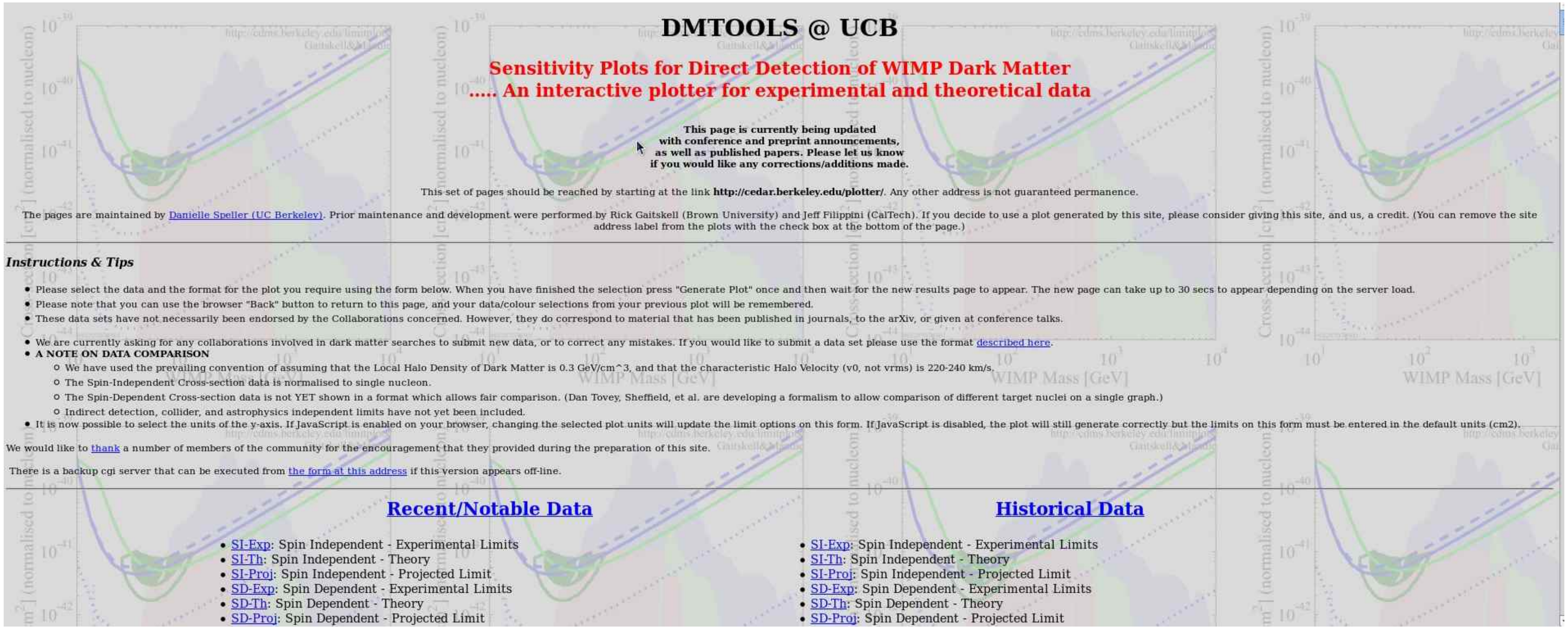
LUX 2016 (332 live days)



(A. Manalaysay, IDM 2016)


Is WIMP direct detection alive and well?
IFT, 2020something
© Warner Bros.

Getting an updated mass-cross section plot has never been easier!



DMTOOLS @ UCB

Sensitivity Plots for Direct Detection of WIMP Dark Matter
..... An interactive plotter for experimental and theoretical data

This page is currently being updated with conference and preprint announcements, as well as published papers. Please let us know if you would like any corrections/additions made.

This set of pages should be reached by starting at the link **http://cedar.berkeley.edu/plotter/**. Any other address is not guaranteed permanence.

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Instructions & Tips

- Please select the data and the format for the plot you require using the form below. When you have finished the selection press "Generate Plot" once and then wait for the new results page to appear. The new page can take up to 30 secs to appear depending on the server load.
- Please note that you can use the browser "Back" button to return to this page, and your data/colour selections from your previous plot will be remembered.
- These data sets have not necessarily been endorsed by the Collaborations concerned. However, they do correspond to material that has been published in journals, to the arXiv, or given at conference talks.
- We are currently asking for any collaborations involved in dark matter searches to submit new data, or to correct any mistakes. If you would like to submit a data set please use the format described here.
- **A NOTE ON DATA COMPARISON**
  - We have used the prevailing convention of assuming that the Local Halo Density of Dark Matter is 0.3 GeV/cm^3, and that the characteristic Halo Velocity (v0, not vrms) is 220-240 km/s.
  - The Spin-Independent Cross-section data is normalised to single nucleon.
  - The Spin-Dependent Cross-section data is not YET shown in a format which allows fair comparison. (Dan Tovey, Sheffield, et al. are developing a formalism to allow comparison of different target nuclei on a single graph.)
  - Indirect detection, collider, and astrophysics independent limits have not yet been included.
- It is now possible to select the units of the y-axis. If JavaScript is enabled on your browser, changing the selected plot units will update the limit options on this form. If JavaScript is disabled, the plot will still generate correctly but the limits on this form must be entered in the default units (cm2).

We would like to thank a number of members of the community for the encouragement that they provided during the preparation of this site.

There is a backup cgi server that can be executed from the form at this address if this version appears off-line.

Recent/Notable Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

Historical Data

- SI-Exp: Spin Independent - Experimental Limits
- SI-Th: Spin Independent - Theory
- SI-Proj: Spin Independent - Projected Limit
- SD-Exp: Spin Dependent - Experimental Limits
- SD-Th: Spin Dependent - Theory
- SD-Proj: Spin Dependent - Projected Limit

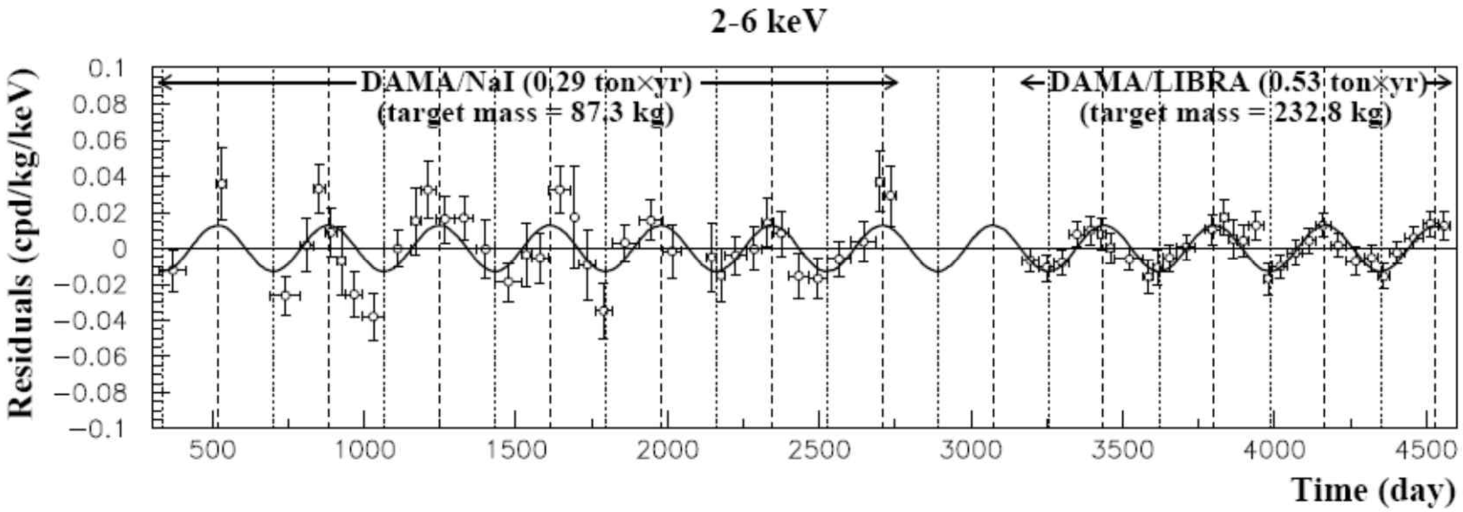
(http://cedar.berkeley.edu/plotter/)

…at least for the most common assumptions: spin-independent, spin-dependent interaction+ Maxwellian

“Excesses”

DAMA/Libra result (Bernabei et al., Eur.Phys.J.C56:333-355,2008, arXiv:0804.2741)

0.53 ton x year (0.82 ton x year combining previous data)
8.2 σ C.L. effect

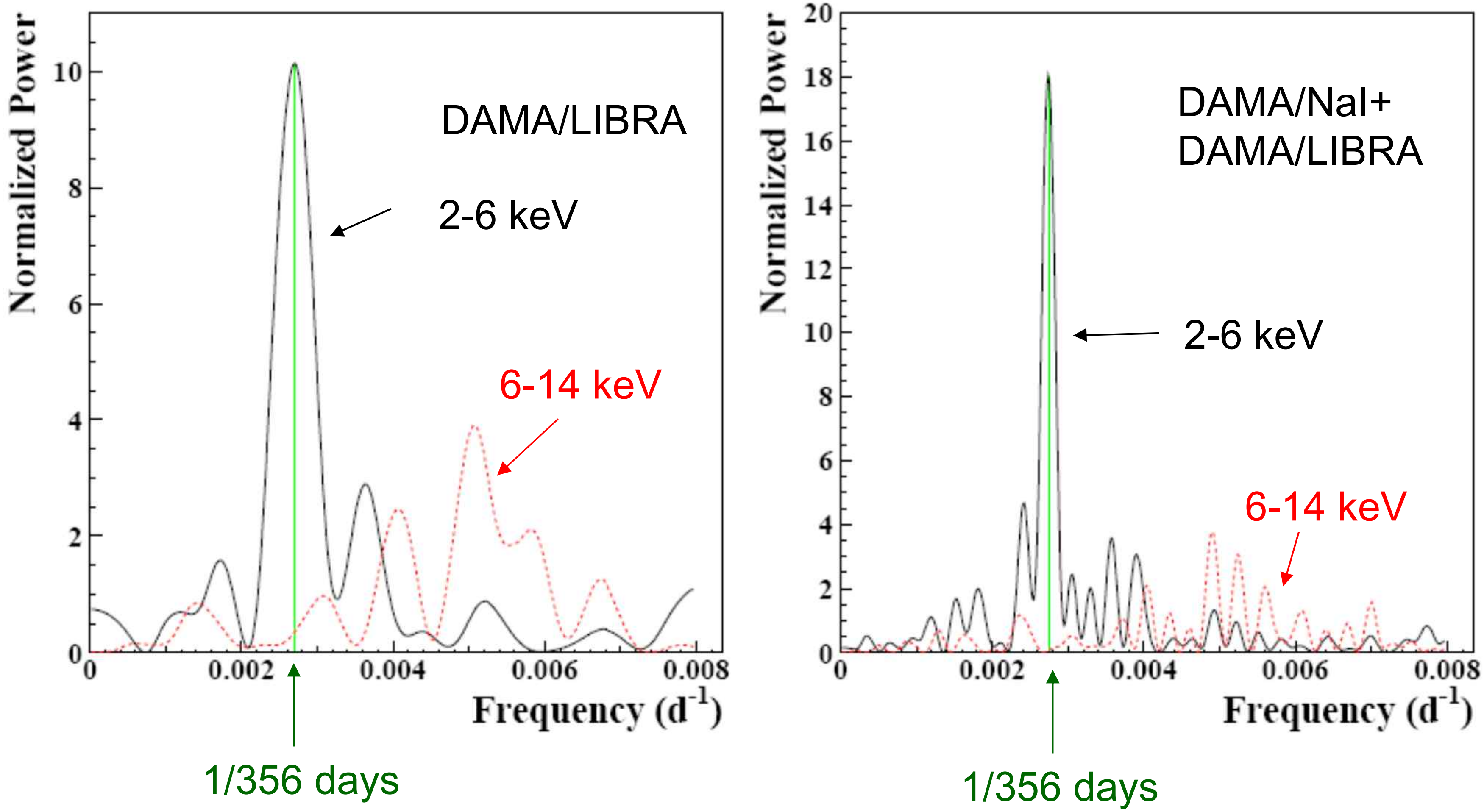


	A (cpd/kg/keV)	$T = \frac{2\pi}{\omega}$ (yr)	t_0 (day)	C.L.
DAMA/NaI				
(2–4) keV	0.0252 ± 0.0050	1.01 ± 0.02	125 ± 30	5.0σ
(2–5) keV	0.0215 ± 0.0039	1.01 ± 0.02	140 ± 30	5.5σ
(2–6) keV	0.0200 ± 0.0032	1.00 ± 0.01	140 ± 22	6.3σ
DAMA/LIBRA				
(2–4) keV	0.0213 ± 0.0032	0.997 ± 0.002	139 ± 10	6.7σ
(2–5) keV	0.0165 ± 0.0024	0.998 ± 0.002	143 ± 9	6.9σ
(2–6) keV	0.0107 ± 0.0019	0.998 ± 0.003	144 ± 11	5.6σ
DAMA/NaI+ DAMA/LIBRA				
(2–4) keV	0.0223 ± 0.0027	0.996 ± 0.002	138 ± 7	8.3σ
(2–5) keV	0.0178 ± 0.0020	0.998 ± 0.002	145 ± 7	8.9σ
(2–6) keV	0.0131 ± 0.0016	0.998 ± 0.003	144 ± 8	8.2σ

$A \cos[\omega\,(t-t_0)]$

$\omega = 2\pi/T_0$

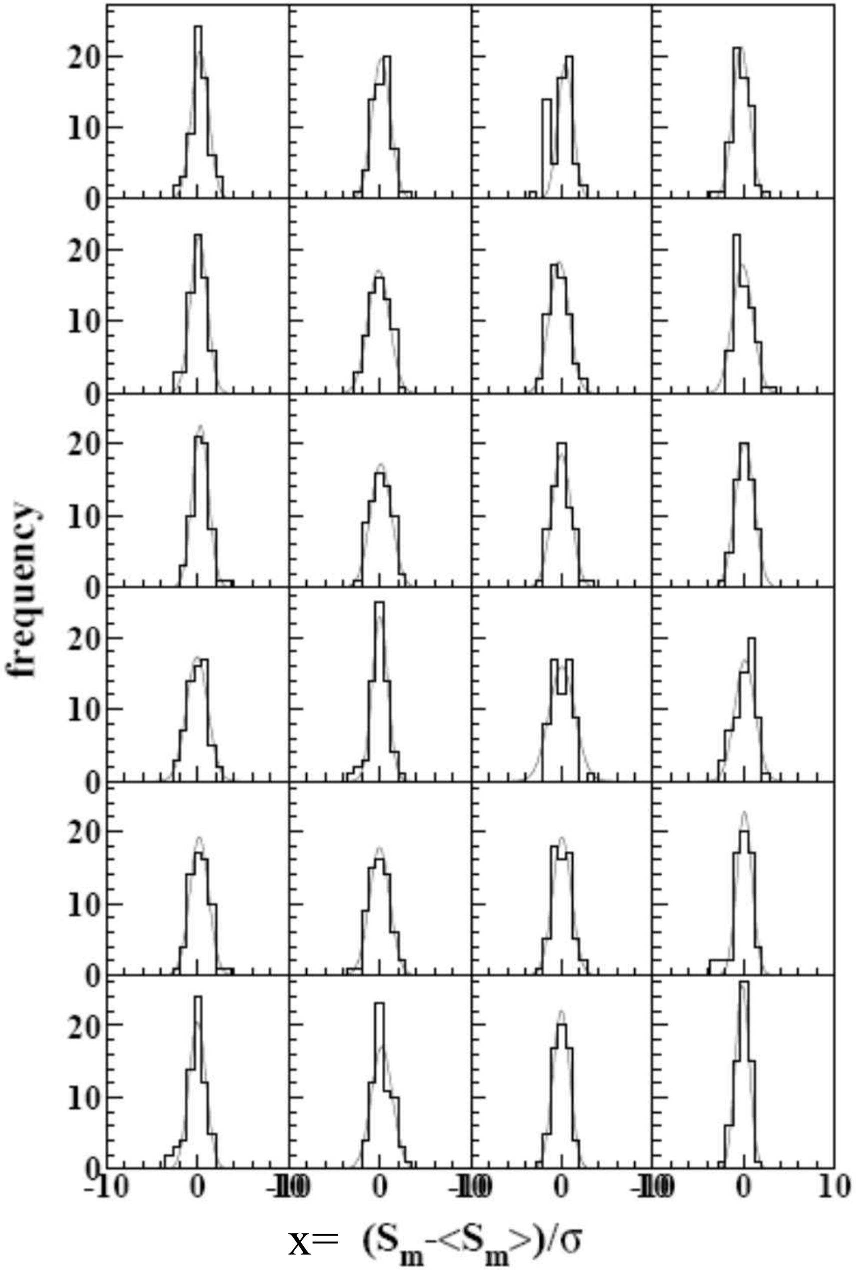
Power spectrum



the peak is only in the 2-6 keV energy interval
absent in the 6-14 keV interval just above

The WIMP signal decays exponentially with energy and is expected near threshold

$x=(S_m-<S_m>)/\sigma$ distribution

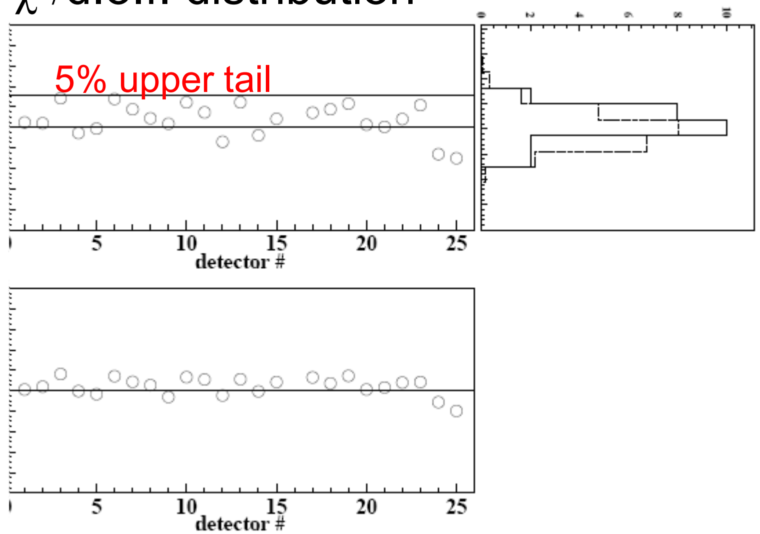


Effect is “spread out” on all 24 detectors (and affects only “single hits”)

each panel: distribution of $x=(S_m-<S_m>)/\sigma$ in one DAMA/LIBRA detector over 4 years

$\chi^2=\sum x^2$ (64 d.o.f:16 x 0.5 keV energy bins x 4 years)

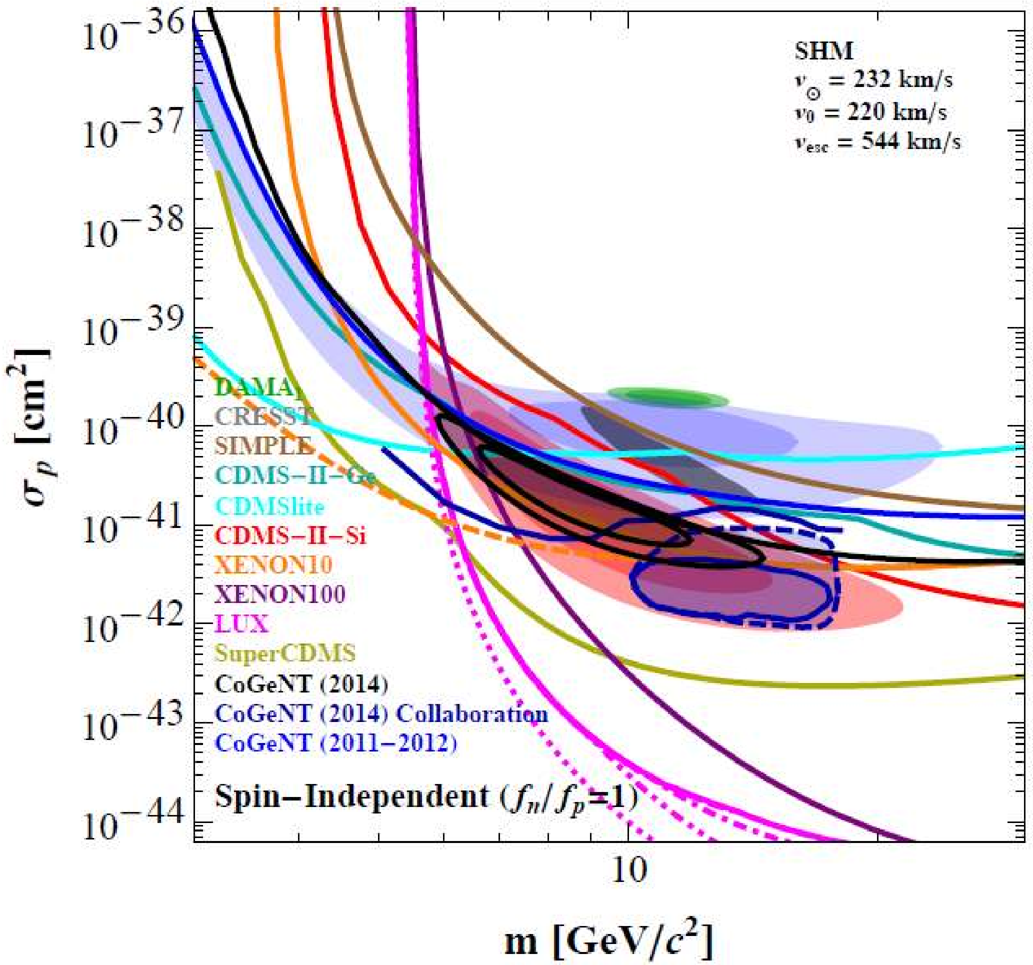
χ^2/d.o.f. distribution



$<\chi^2/d.o.f.>=1.072$

Situation @ low WIMP mass

Spin-independent interaction, isothermal sphere



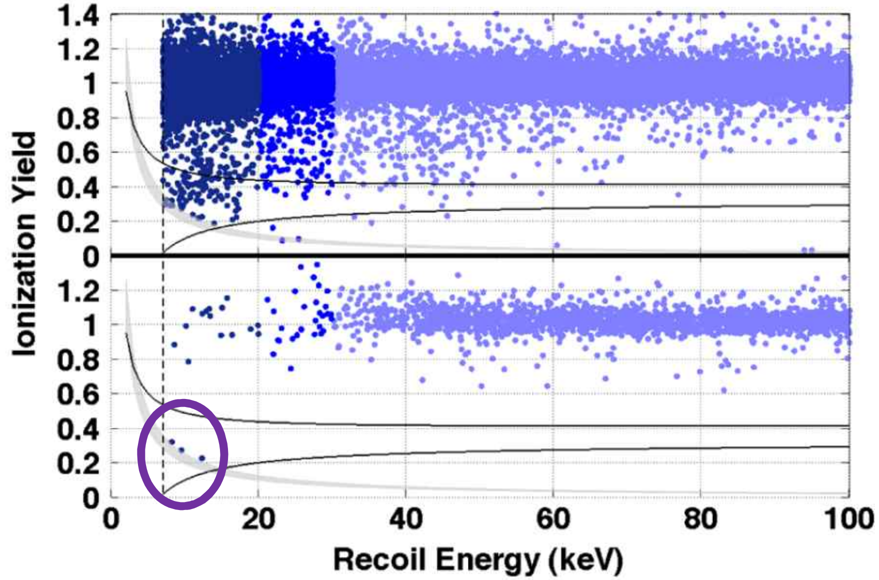
- qualitatively LUX is similar to XENON100
- stronger constraints at lowest masses from CDMSlite + Xenon10

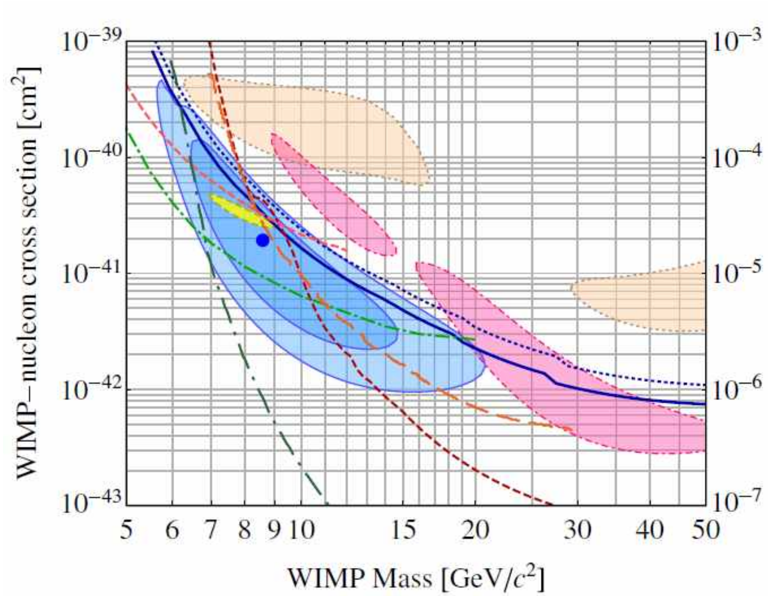
An explanation of DAMA in terms of a WIMP signal seems doomed

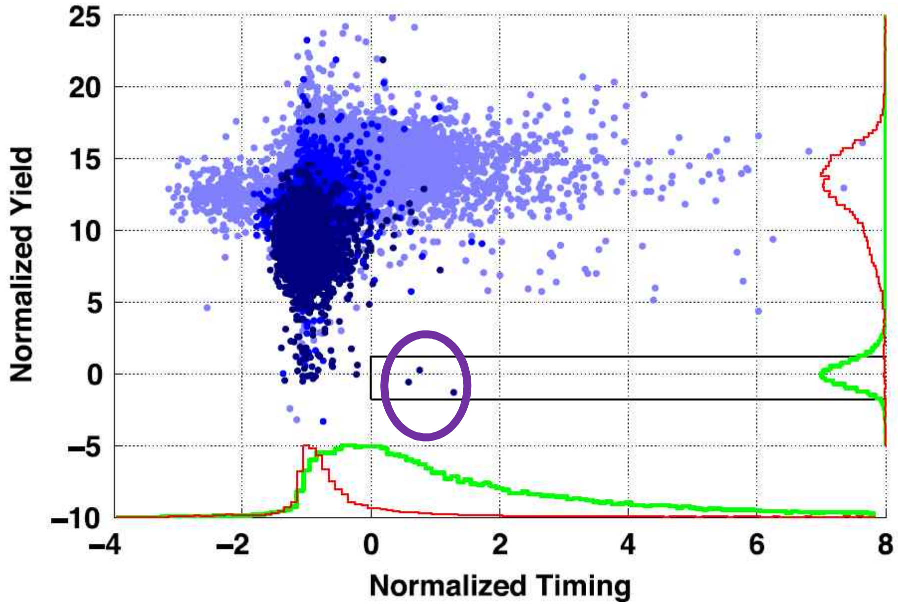
(E. Del Nobile, G. Gelmini, P. Gondolo, J.H. Huh , 1405.5582)

The CDMS II Silicon excess

- dual signal (phonons+ionization) used to discriminate background
- total exposure of 140.2 kg days with eight Silicon detectors of ~106 g each in the energy range 7-100 keV
- ~23.4 kg day equivalent exposure after selection cuts for 10 GeV WIMP
- 3 WIMP-candidate events survive with expected background <0.6 events (~5% probability of bck fluctuation)







R.Agnese et al. (CDMS Collaboration), Phys.Rev.Lett.111, 251301 (2013),1304.4279

The CRESST excess (btw: is it gone)?

CRESST 2012:

G. Angloher et al (CRESST Coll.) Eur. Phys. J.C72, 1971 (2012), 1109.0702

•730 kg day with $CaWO_4$ (light+phonons)

•"excess" (total of 34 events in Tungsten recoil band for 12 keVnr<E_R<24 keVnr vs. 7.4 expected due to lead recoil background from ^{210}Po decay)

• sizeable surface background from non-scintillating clamps holding the crystals.

•**CRESST 2014**:

G. Angloher et al(CRESST-II Collaboration),1407.3146

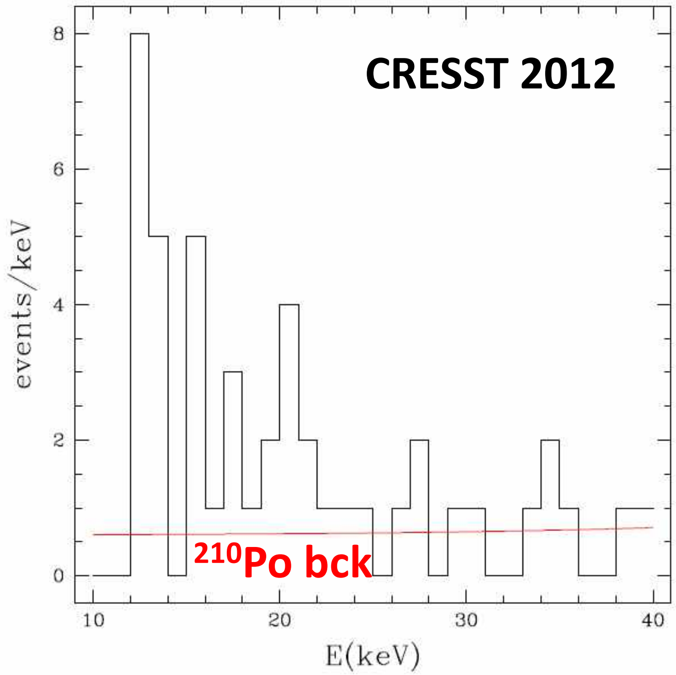
•Improved radiopurity and fully-scintillating design for one 250 g detector module (TUM-40)

•total exposure: 29 kg days

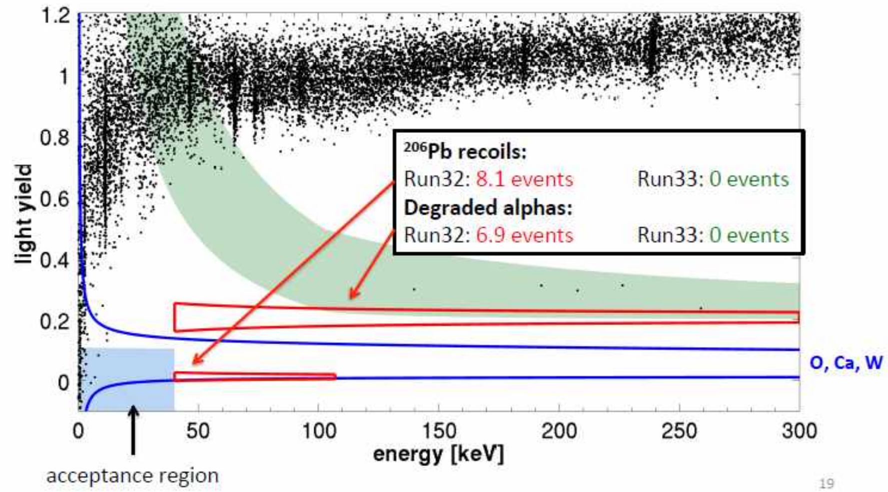
• additional light from surface events allows efficient veto of surface background

• no longer events in previous excess region and **lower threshold**: low-mass WIMP solution ruled out **while high-mass WIMP solution survives**

• back-of-the-envelope estimation: 30*29/730~1.2 events. 90% CL upper bound of 0 is 2.3, simply exposure is too low to rule out previous effect → need more statistics

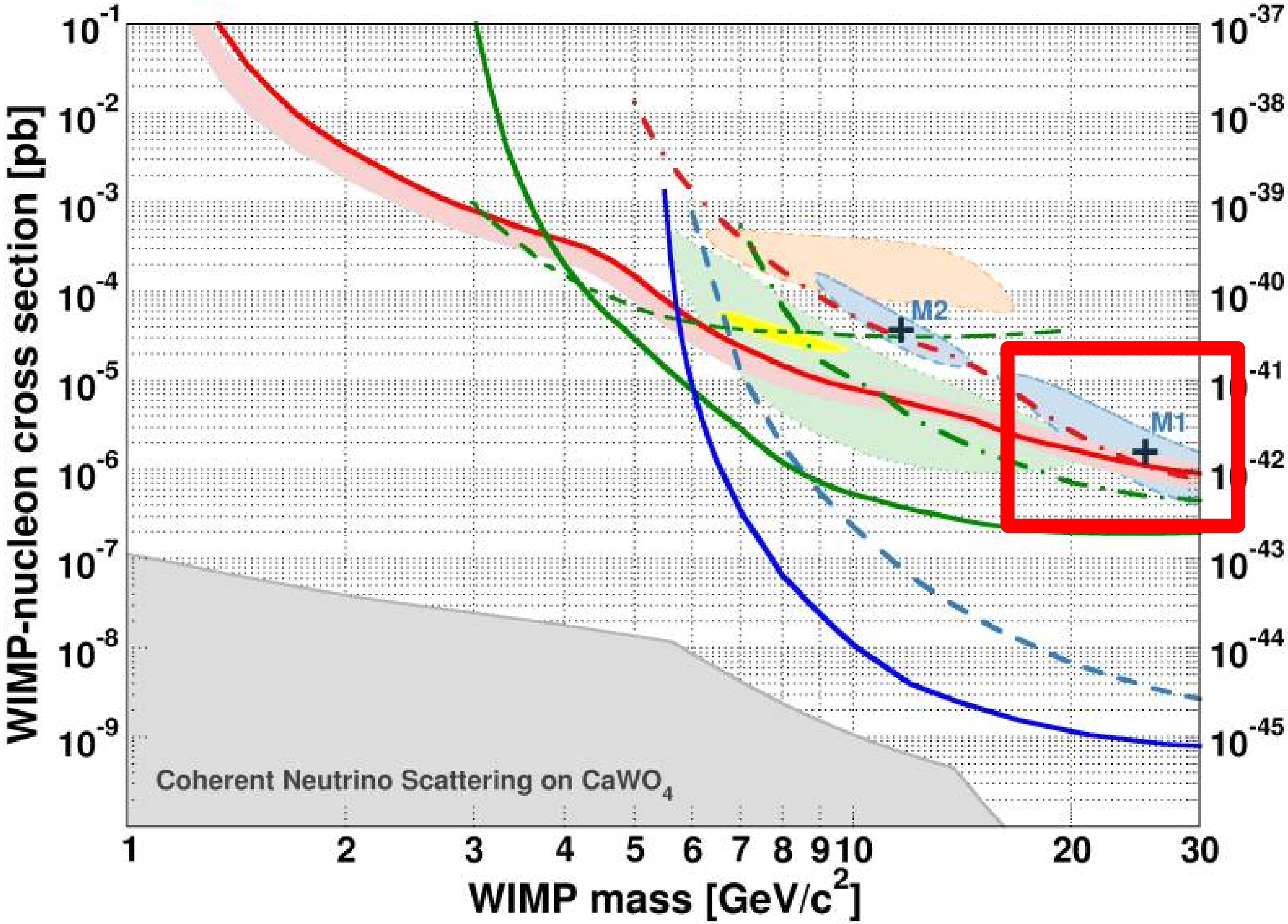


CRESST 2014



The CRESST excess

G. Angloher et al(CRESST-II Collaboration),1407.3146



- still marginal compatibility for high-mass solution assuming isothermal sphere
- full compatibility relaxing assumptions on velocity distribution

thresholdinos?

Indeed, spin-independent and spin-dependent cross sections are predicted for the neutralino in supersymmetry and numerical simulations of galaxy formation support the choice of a Maxwellian for the velocity distributions.

However a bottom-up approach would also be desireable,especially if no hints come from high-energy physics about the fundamental properties of the WIMP particle. Indeed two questions arise:

- what is the most general class of scaling laws for a WIMP-nucleus cross section?
- the detailed merger history of the Milky Way is not known, allowing for the possibility of the presence of sizeable non–thermal components for which the density, direction and speed of WIMPs are hard to predict, *especially in the high velocity tail of the distribution*: do we need to assume a Maxwellian velocity distribution?

Recently both aspects have been addressed

Compatibility among different experiments (ex. DAMA/Libra vs. CoGeNT) can be verified without assuming any model for the halo

Write expected WIMP rate as:

$$\frac{dR}{dE_R} = \frac{\rho_\chi \sigma_n}{2m_\chi \mu_{n\chi}^2} \frac{C_T}{f_n^2} F^2(E_R) \epsilon(E_R) g(v_{\min}, t)$$

$F^2(E_R)$ is the form factor, and the function:

$$g(v_{\min}, t) = \int_{v_{\min}}^{\infty} \frac{f_{\text{local}}(\vec{v}, t)}{v} d^3 v$$

contains all the dependence on the halo model with:

$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu^2}}$$

So there is a one-to-one correspondence between the recoil energy E_R and v_{min}

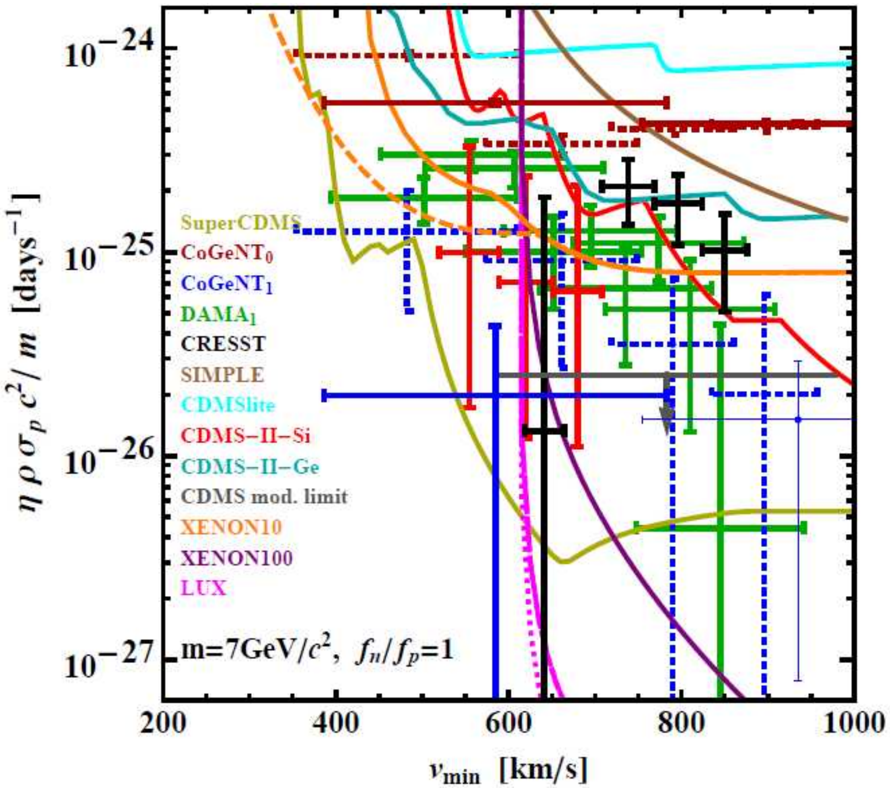
→ map the event rate expected in different experiments into the same intervals in v_{min} (P.J. Fox, J. Liu, N. Weiner, PRD83,103514 (2011))

In this way the dependence on the galactic model cancels out in the ratio of the expected count rates of the two experiments because <u>they depend on the same integrals of $f_{local}(v)$</u>

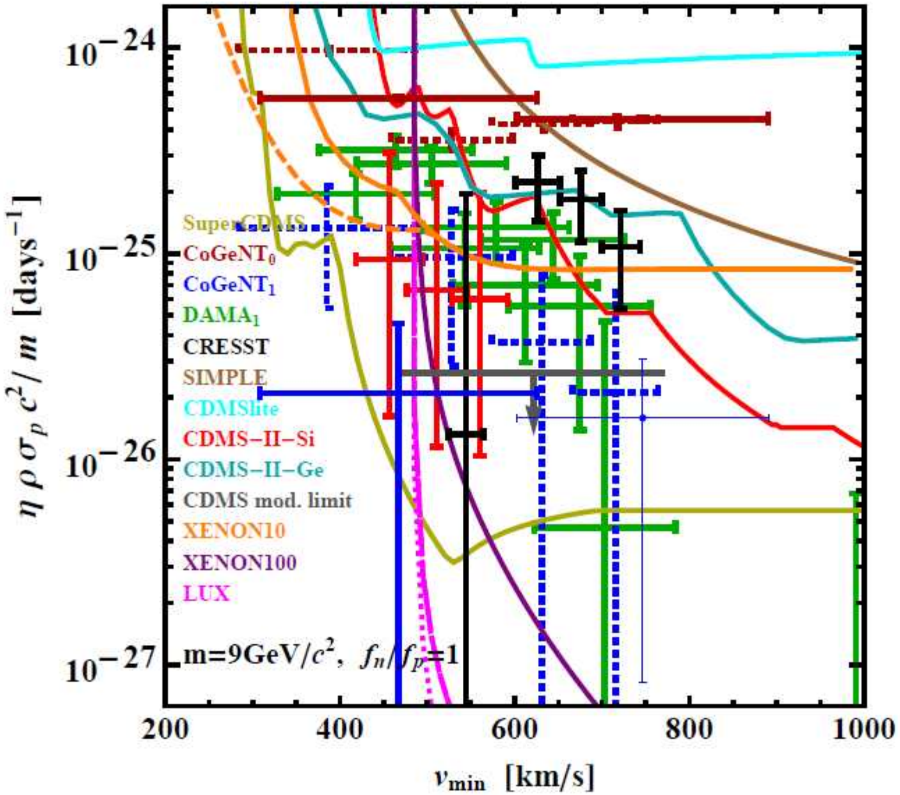
halo-independent analysis for elastic scattering

Del Nobile, Gelmini, Gondolo, Huh, arXiv:1405.5582

m_{WIMP}=7 GeV



m_{WIMP}=9 GeV



$$R^{\rm SI}_{[E'_1,E'_2]}(t) = \int_0^\infty {\rm d}v_{\rm min}\, \tilde\eta(v_{\rm min},t)\, \mathcal{R}^{\rm SI}_{[E'_1,E'_2]}(v_{\rm min}) \qquad \tilde\eta(v_{\rm min},t) \equiv \frac{\rho\sigma_p}{m}\int_{v\geqslant v_{\rm min}} {\rm d}^3v\, \frac{f(\mathbf{v},t)}{v}$$

$$\tilde\eta(v_{\rm min},t) \simeq \tilde\eta^0(v_{\rm min}) + \tilde\eta^1(v_{\rm min})\cos[\omega(t-t_0)]$$

N.B. : only halo dependence factorized. Results depend on assumptions on other quantities such as quenching factors, L_{eff}, Q_y etc.

$$\tilde{\eta}(v_{min}, t) \equiv \frac{\rho}{m_{WIMP}} \sigma_0 \eta(v_{min}, t)$$

•Annual modulation

Experimental data fits (DAMA, CoGeNT, KIMS) assume a sinusoidal behaviour:

$$\tilde{\eta}(v_{\min}, t) \simeq \tilde{\eta}^0(v_{\min}) + \tilde{\eta}^1(v_{\min}) \cos[\omega(t - t_0)]$$

The usual "halo-independent" approach to analyze yearly modulation data: factorize a modulated halo function $\tilde{\eta}_1$ with the only constraint $\tilde{\eta}_1 < \tilde{\eta}_0$.

(In the case of a Maxwellian typically $\tilde{\eta}_1 / \tilde{\eta}_0 \leq 0.07$)

Standard lore: cannot predict $\tilde{\eta}_1 / \tilde{\eta}_0$ without a model for the velocity distribution. Is it really so? More on that later

Summarizing, the minimal requirements for halo functions $\eta_{0,1}$ are:

$$\tilde{\eta}_0(v_{\min,2}) \leq \tilde{\eta}_0(v_{\min,1}) \quad \text{if } v_{\min,2} > v_{\min,1}$$

(decreasing function)

$$\tilde{\eta}_1 \leq \tilde{\eta}_0 \quad \text{at the same } v_{\min}$$

(modulated part<100%)

$$\tilde{\eta}_0(v_{\min} \geq v_{\rm esc}) = 0.$$

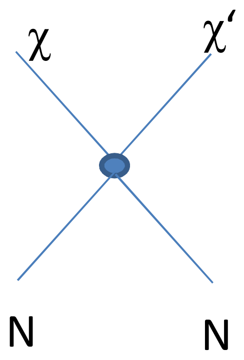
(no bound WIMPs<escape velocity)

Inelastic Dark Matter

D. Tucker-Smith and N.Weiner, Phys.Rev.D 64, 043502 (2001), hep-ph/0101138

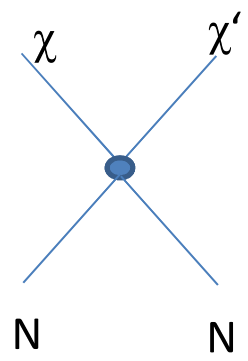
Two mass eigenstates χ and χ' very close in mass: $m_\chi - m_{\chi'} \equiv \delta$ with $\chi + N \rightarrow \chi + N$ forbidden

"Endothermic "scattering ($\delta>0$)



Kinetic energy needed to "overcome" step → rate no longer exponentially decaying with energy, maximum at finite energy E_*

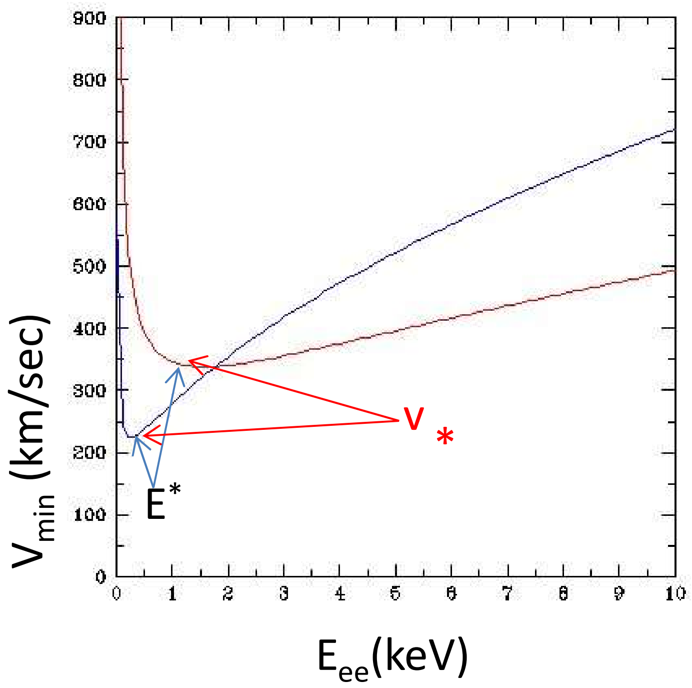
"Exothermic" scattering ($\delta<0$)



χ is metastable, δ energy deposited independently on initial kinetic energy (even for WIMPs at rest)

Inelastic DM and the halo-independent approach: recoil energy E_{ee} is no longer monotonically growing with v_{min} (energy E^* corresponds to minimal v_{min})

$$v_{min} = \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N E_R}{\mu} + \delta \right) = a\sqrt{E_r} + \frac{b}{\sqrt{E_R}}$$



N.B. for δ>0 WIMPs need a minimal absolute incoming speed v_* to upscatter to the heavier state → vanishing rate if $v_* > v_{esc}$ (escape velocity)

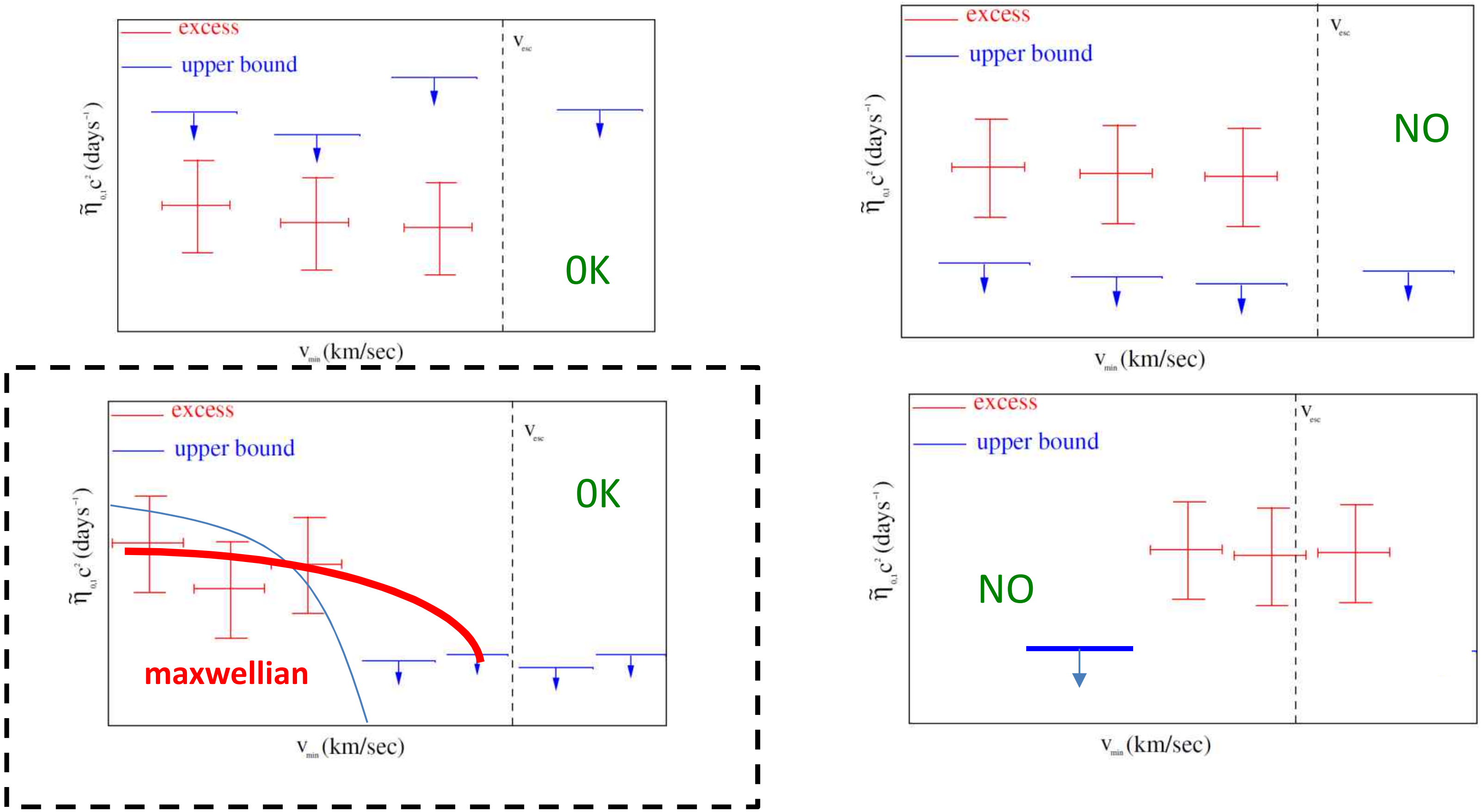
Need to rebin the data in such a way that the relation between v_{min} and E_R is invertible in each bin (easy: just ensure that for all target nuclei E^* corresponds to one of the bin boundaries)

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

comparison among different experiments for Inelastic DM

if conflicting experimental results can be mapped into non-overlapping ranges of v_{min} and if the v_{min} range of the constraint is at higher values compared to the excess (while that of the signal remains below v_{esc}) the tension between the two results can be eliminated by an appropriate choice of the $\eta_{0,1}$ functions

Four cases:



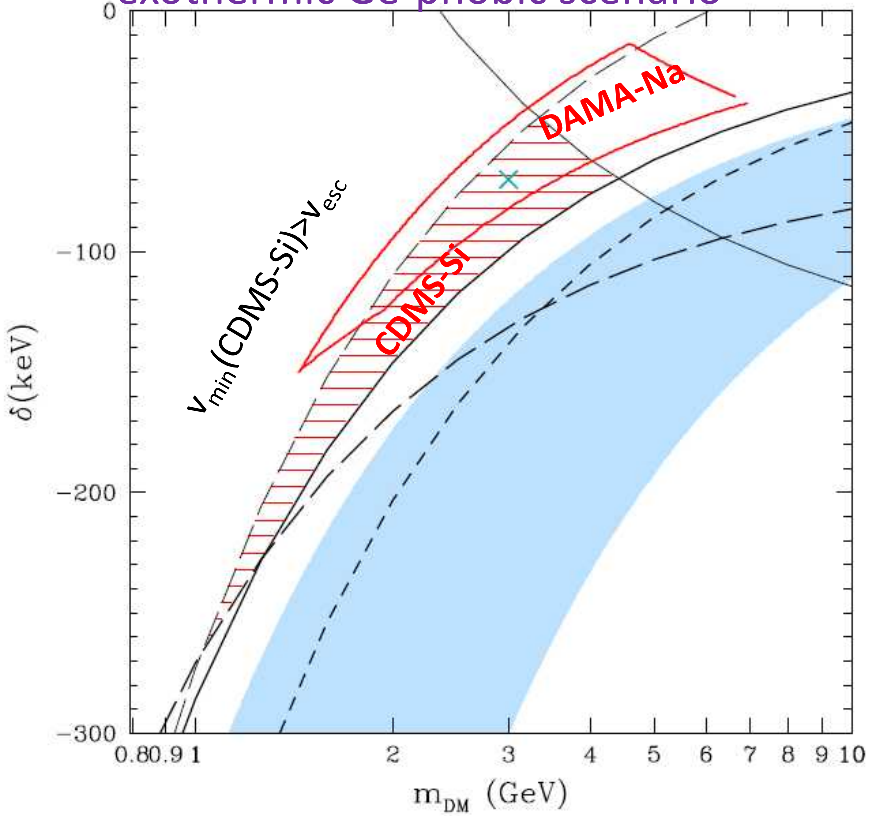
N.B: the effect of inelastic scattering ($\delta \neq 0$) only implies a “horizontal shift” of η estimations (up to negligible effects) → pick appropriate m_{DM}, δ combination to shift-away the bounds without shifting away the signal!

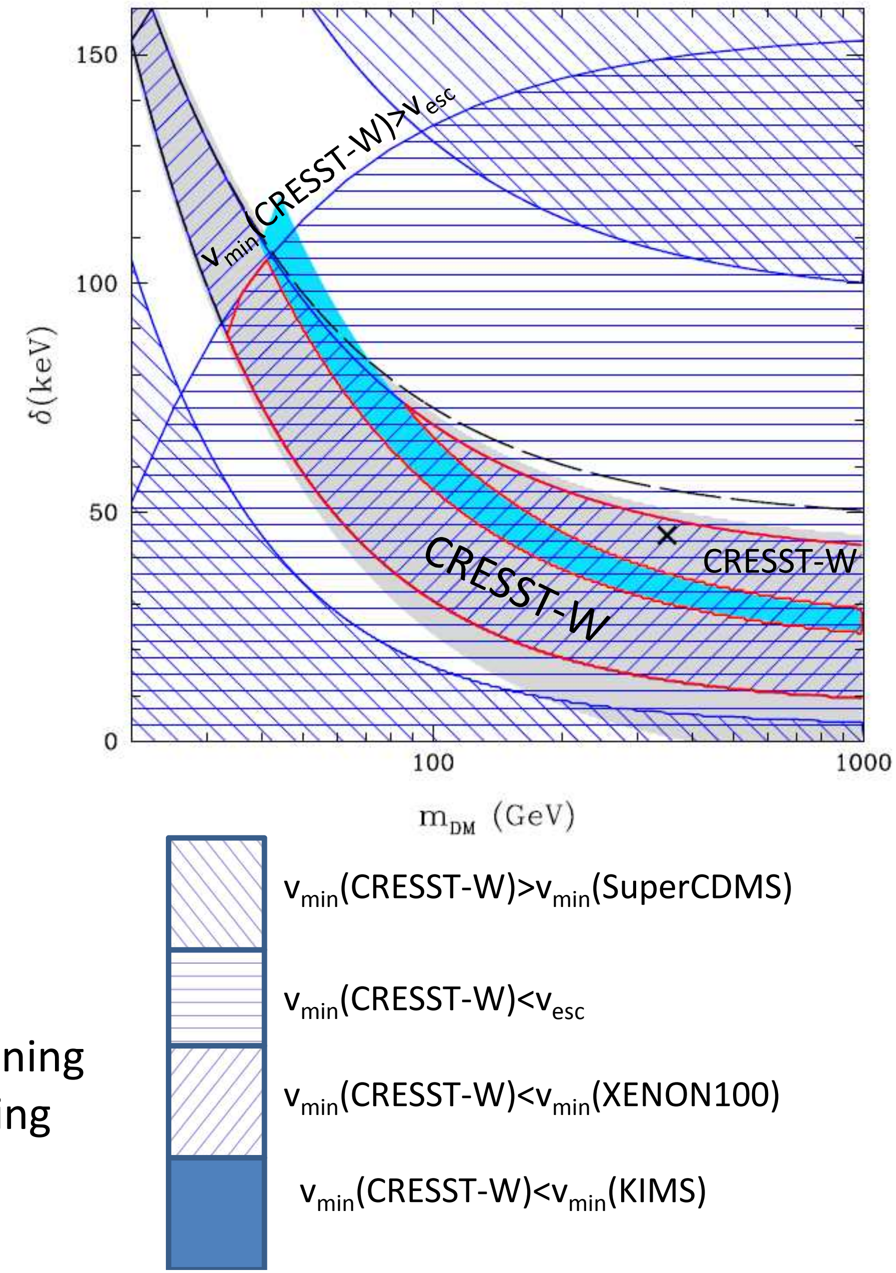
S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

Halo-independent analysis of inelastic Dark Matter

Kinematic conditions for v_{min}(bounds)>v_{min}(signals) and v_{min}(signals)<v_{esc}

"exothermic Ge-phobic scenario"





N.B. only kinematics involved (valid for different scaling laws)

At higher masses upper bound of ROI is constraining

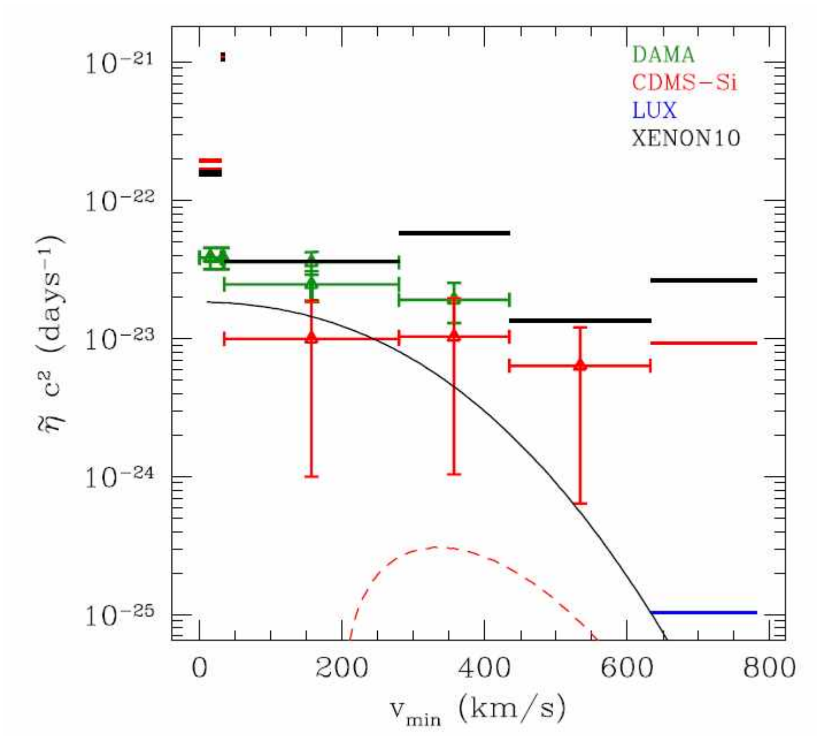
In LUX, XENON100→XENON100 more constraining than LUX due to lower light yield

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

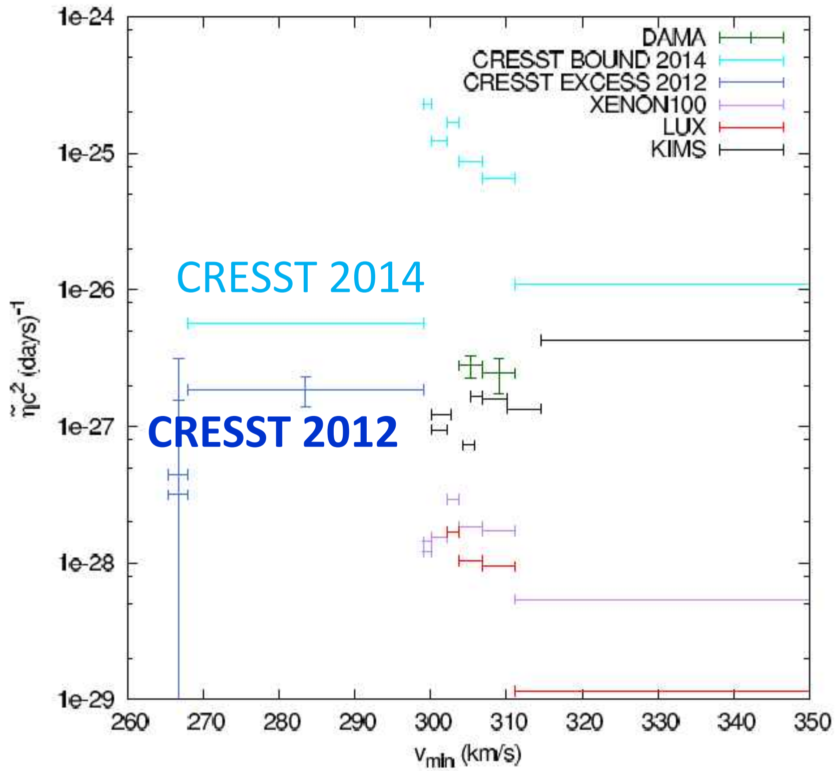
Halo-independent analysis of inelastic Dark Matter

"Agnostic" approach about velocity integral: a constraint does not affect values of v_{min} below its covered range, i.e. if v_{min}(bound)>v_{min}(signal)

m_{DM} =3 GeV, δ=-70 keV, f_n/f_p =-0.79



m_{DM} =350 GeV, δ=45 keV, f_n/f_p =1



- DAMA and CDMS-Si can be separately OK with bounds, but are always in tension between themselves
- Assuming standard Maxwellian more tension arises
- high-mass CRESST solution not affected by recent reanalysis due to low statistics

S. Scopel and K.H. Yoon, JCAP1408, 060 (2014)

isospin violation (more properly: isovector interaction)

$$R = \sigma_p \sum_i \eta_i \frac{\mu_{A_i}^2}{\mu_p^2} I_{A_i} \left[Z + (A_i - Z) f_n / f_p \right]^2$$

(spin-independent cross section, same for other interactions)

sum over isotopes

Cancellation between f_p (WIMP-proton coupling) and f_n (WIMP-nucleon coupling) when $f_n/f_p \sim -Z/(A-Z) \rightarrow$ can suppress the scatt ering cross section ona specific target (i.e. $f_n/f_p \sim -0.79$ for Germanium)

Minimal "degrading factors", i.e. maximal factors by which the reciprocal scaling law between two elements can be reduced (limited by multiple isotopes, one choice of f_n/f_p ratio cannot fit all)

Element	Xe	Ge	Si	Ca	W	Ne	C
Xe (54, ∗)	1.00	8.79	149.55	138.21	10.91	34.31	387.66
Ge (32, ∗)	22.43	1.00	68.35	63.14	130.45	15.53	176.47
Si (14, ∗)	172.27	30.77	1.00	1.06	757.44	1.06	2.67
Ca (20, ∗)	173.60	31.53	1.17	1.00	782.49	1.10	2.81
W (74, ∗)	2.98	13.88	177.46	166.15	1.00	41.64	466.75
Ne (10, ∗)	163.65	28.91	4.39	4.09	726.09	1.00	11.52
C (6, ∗)	176.35	32.13	1.07	1.02	789.59	1.12	1.00
I (53, 127)	1.94	5.51	127.04	118.35	20.68	28.92	326.95
Cs (55, 133)	1.16	7.15	139.65	127.61	12.32	31.88	355.27
O (8, 16)	178.49	32.13	1.08	1.03	789.90	1.13	1.01
Na (11, 23)	101.68	13.77	8.45	8.33	481.03	2.27	22.68
Ar (18, 36)	178.49	32.13	1.08	1.03	789.90	1.13	1.01
F (9, 19)	89.39	10.88	12.44	11.90	425.93	3.05	33.47

(J.L.Feng, J.Kumar, D.Marfatia and D.Sanford, Phys.Lett.B703, 124 (2011), 1102.4331)

On the most general WIMP-nucleus cross section
(i.e. beyond “spin-dependent” and “spin”independent”)

Most general approach: consider ALL possible NR couplings, including those depending on velocity and momentum

$$\mathcal{H} = \sum_i \left(c_i^0 + c_i^1 \tau_3\right) \mathcal{O}_i$$

τ_3=nuclear isospin operator, i.e.

$$c_i^{\rm p} = (c_i^0 + c_i^1)/2 \quad \text{(proton)}$$
$$c_i^{\rm n} = (c_i^0 - c_i^1)/2 \quad \text{(neutron)}$$

(if $c_i^p = c_i^n \rightarrow c_i^1=0$)

N.R. operators O_i guaranteed to be Hermitian if built out of the following four 3-vectors:

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^\perp, \quad \vec{S}_\chi, \quad \vec{S}_N$$

with:

$$\left.\begin{array}{l} \vec{v}^\perp = \vec{v} + \dfrac{\vec{q}}{2\mu_N} \\ \vec{v} \equiv \vec{v}_{\chi,\rm in} - \vec{v}_{N,\rm in} \end{array}\right\} \Rightarrow \vec{v}^\perp \cdot \vec{q} = 0$$

$$\mathcal{O}_1 = 1_\chi 1_N,$$
$$\mathcal{O}_2 = (v^\perp)^2,$$
$$\mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N,$$
$$\mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right),$$
$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$
$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp,$$
$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp,$$
$$\mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right),$$
$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N},$$
$$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}.$$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

Additional operators <u>that do not </u>arise for traditional spin-0 or spin-1 mediators:

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp),$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$

$$\mathcal{O}_{14} = i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)(\vec{S}_N \cdot \vec{v}^\perp),$$

$$\mathcal{O}_{15} = -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left[(\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]$$

$$\mathcal{O}_{16} = -\left[(\vec{S}_\chi \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right]\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right)$$

Connection to relativistic effective theory:

j	$\mathcal{L}^j_{\rm int}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-\frac{\vec q}{m_\chi}\cdot\vec S_\chi\frac{\vec q}{m_N}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q^2}{2m_N m_M}1_\chi 1_N + 2(\frac{\vec q}{m_\chi}\times\vec S_\chi + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$\frac{\vec q^2}{2m_N m_M}\mathcal{O}_1 - 2\frac{m_N}{m_M}\mathcal{O}_3 + 2\frac{m_N^2}{m_M m_\chi}(\frac{\vec q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp + \frac{2}{m_\chi}i\vec S_\chi\cdot(\vec S_N\times\vec q)$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_M}\cdot\vec S_N$	$2\frac{m_N}{m_M}\mathcal{O}_{10}$	O/O
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^2}{2m_\chi m_M}1_\chi 1_N - 2(\frac{\vec q}{m_N}\times\vec S_N + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)$	$-\frac{\vec q^2}{2m_\chi m_M}\mathcal{O}_1 + \frac{2m_N}{m_M}\mathcal{O}_5 - 2\frac{m_N}{m_M}(\frac{\vec q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$4(\frac{\vec q^2}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6)$	E/E
11	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma^\mu\gamma^5 N$	$4i(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
12	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec q^2}{m_\chi m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)]\frac{\vec q}{m_M}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\frac{\vec q^2}{m_M^2}\mathcal{O}_{10} - 4\frac{\vec q^2}{m_M^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
13	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma_\mu N$	$2\vec v^\perp\cdot\vec S_\chi + 2i\vec S_\chi\cdot(\vec S_N\times\frac{\vec q}{m_N})$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec S_\chi\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
15	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma^\mu\gamma^5 N$	$-4\vec S_\chi\cdot\vec S_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i\vec v^\perp\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$	E/O
17	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu N$	$2i\frac{\vec q}{m_M}\cdot\vec S_\chi$	$2\frac{m_N}{m_M}\mathcal{O}_{11}$	O/O
18	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q}{m_M}\cdot\vec S_\chi[i\frac{\vec q^2}{m_N m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_N)]$	$\frac{\vec q^2}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
19	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu\gamma^5 N$	$-4i\frac{\vec q}{m_M}\cdot\vec S_\chi\vec v_\perp\cdot\vec S_N$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	E/O
20	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4\frac{\vec q}{m_M}\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	E/E

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In the expected rate WIMP physics (encoded in the R functions that depend on the c_i couplings) and the nuclear physics (contained in 8 (6+2) response functions W factorize in a simple way:

$$\frac{d\mathcal{R}}{dE_R} = \sum_T \frac{d\mathcal{R}_T}{dE_R} \equiv \sum_T \xi_T \frac{\rho_\chi}{2\pi m_\chi} \int_{v>v_{\min}(q)} \frac{f(\vec{v}+\vec{v}_e(t))}{v} P_{\rm tot}(v^2,q^2)\, d^3v$$

$$\begin{aligned}
P_{\rm tot}(v^2,q^2) &= \frac{4\pi}{2j_N+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \Bigg[R_M^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_M^{\tau\tau'}(y) \\
&+ R_{\Sigma''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma''}^{\tau\tau'}(y) + R_{\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Sigma'}^{\tau\tau'}(y) \Bigg] \\
&+ \frac{q^2}{m_N^2} \Bigg[R_{\Phi''}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi''}^{\tau\tau'}(y) + R_{\Phi'' M}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Phi'' M}^{\tau\tau'}(y) \\
&+ R_{\tilde{\Phi}'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\tilde{\Phi}'}^{\tau\tau'}(y) + R_{\Delta}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta}^{\tau\tau'}(y) \\
&+ R_{\Delta\Sigma'}^{\tau\tau'}(v_{\chi T}^{\perp 2}, \frac{q^2}{m_N^2})\, W_{\Delta\Sigma'}^{\tau\tau'}(y) \Bigg] \Bigg\},
\end{aligned}$$

N.B.: besides usual spin-independent and spin-dependent terms new contributions arise, with explicit dependences on the transferred momentum q and the WIMP incoming velocity

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
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WIMPs response funtions

$$R_M^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=c_1^\tau c_1^{\tau'}+\frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2}v_T^{\perp 2}c_5^\tau c_5^{\tau'}+v_T^{\perp 2}c_8^\tau c_8^{\tau'}+\frac{q^2}{m_N^2}c_{11}^\tau c_{11}^{\tau'}\right]$$

$$R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\left[\frac{q^2}{4m_N^2}c_3^\tau c_3^{\tau'}+\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau-\frac{q^2}{m_N^2}c_{15}^\tau\right)\left(c_{12}^{\tau'}-\frac{q^2}{m_N^2}c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Phi''M}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\left[c_3^\tau c_1^{\tau'}+\frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau-\frac{q^2}{m_N^2}c_{15}^\tau\right)c_{11}^{\tau'}\right]\frac{q^2}{m_N^2}$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'}+\frac{q^2}{m_N^2}c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2}$$

$$R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\frac{q^2}{4m_N^2}c_{10}^\tau c_{10}^{\tau'}+\frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'}+\right.$$
$$\left.\frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'}+c_6^\tau c_4^{\tau'})+\frac{q^4}{m_N^4}c_6^\tau c_6^{\tau'}+v_T^{\perp 2}c_{12}^\tau c_{12}^{\tau'}+\frac{q^2}{m_N^2}v_T^{\perp 2}c_{13}^\tau c_{13}^{\tau'}\right]$$

$$R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\frac{1}{8}\left[\frac{q^2}{m_N^2}v_T^{\perp 2}c_3^\tau c_3^{\tau'}+v_T^{\perp 2}c_7^\tau c_7^{\tau'}\right]+\frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'}+\right.$$
$$\left.\frac{q^2}{m_N^2}c_9^\tau c_9^{\tau'}+\frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau-\frac{q^2}{m_N^2}c_{15}^\tau\right)\left(c_{12}^{\tau'}-\frac{q^2}{m_N^2}c_{15}^{\tau'}\right)+\frac{q^2}{2m_N^2}v_T^{\perp 2}c_{14}^\tau c_{14}^{\tau'}\right]$$

$$R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2}c_5^\tau c_5^{\tau'}+c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2}$$

$$R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2},\frac{q^2}{m_N^2}\right)=\frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'}-c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.$$

general form:

$$R_k^{\tau\tau'}=R_{0k}^{\tau\tau'}+R_{1k}^{\tau\tau'}\frac{(v_T^\perp)^2}{c^2}=R_{0k}^{\tau\tau'}+R_{1k}^{\tau\tau'}\frac{v_T^2-v_{min}^2}{c^2}$$

Nuclear response functions

Assuming one-body dark matter-nucleon interactions, the Hamiltonian density for dark matter-nucleus interactions is:

$$\begin{aligned}
\mathcal{H}_{ET}(\vec{x}) &= \sum_{i=1}^{A} l_0(i)\ \delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} l_0^A(i)\ \frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i \cdot \vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\cdot\frac{1}{i}\overrightarrow{\nabla}_i\right] \\
&+ \sum_{i=1}^{A} \vec{l}_5(i)\cdot\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \sum_{i=1}^{A} \vec{l}_M(i)\cdot\frac{1}{2M}\left[-\frac{1}{i}\overleftarrow{\nabla}_i\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\frac{1}{i}\overrightarrow{\nabla}_i\right] \\
&+ \sum_{i=1}^{A} \vec{l}_E(i)\cdot\frac{1}{2M}\left[\overleftarrow{\nabla}_i \times \vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i) + \delta(\vec{x}-\vec{x}_i)\vec{\sigma}(i)\times\overrightarrow{\nabla}_i\right]
\end{aligned}$$

So the WIMP-nucleus Hamiltonian has the general form:

$$\int d\vec{x}\ e^{-i\vec{q}\cdot\vec{x}}\ \left[l_0\langle J_i M_i|\hat{\rho}(\vec{x})|J_i M_i\rangle - \vec{l}\cdot\langle J_i M_i|\hat{\vec{j}}(\vec{x})|J_i M_i\rangle\right]$$

With:

$$\begin{aligned}
e^{i\vec{q}\cdot\vec{x}_i} &= \sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^J j_J(qx_i)Y_{J0}(\Omega_{x_i}) \\
\hat{e}_\lambda e^{i\vec{q}\cdot\vec{x}_i} &= \begin{cases} \displaystyle\sum_{J=0}^{\infty}\sqrt{4\pi}\ [J]\ i^{J-1}\frac{\vec{\nabla}_i}{q} j_J(qx_i)Y_{J0}(\Omega_{x_i}), & \lambda = 0 \\ \displaystyle\sum_{J\geq 1}^{\infty}\sqrt{2\pi}\ [J]\ i^{J-2}\left[\lambda j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i}) + \frac{\vec{\nabla}_i}{q}\times j_J(qx_i)\vec{Y}_{JJ1}^{\lambda}(\Omega_{x_i})\right], & \lambda = \pm 1 \end{cases}
\end{aligned}$$

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which depends on the expectations of six distinct nuclear response functions, defined as:

$$M_{JM}(q\vec{x})$$

$$\Delta_{JM}(q\vec{x}) \equiv \vec{M}_{JJ}^{M}(q\vec{x}) \cdot \frac{1}{q}\vec{\nabla}$$

$$\Sigma'_{JM}(q\vec{x}) \equiv -i\left\{\frac{1}{q}\vec{\nabla} \times \vec{M}_{JJ}^{M}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{-\sqrt{J}\ \vec{M}_{JJ+1}^{M}(q\vec{x}) + \sqrt{J+1}\ \vec{M}_{JJ-1}^{M}(q\vec{x})\right\} \cdot \vec{\sigma}$$

$$\Sigma''_{JM}(q\vec{x}) \equiv \left\{\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{\sqrt{J+1}\ \vec{M}_{JJ+1}^{M}(q\vec{x}) + \sqrt{J}\ \vec{M}_{JJ-1}^{M}(q\vec{x})\right\} \cdot \vec{\sigma}$$

$$\tilde{\Phi}'_{JM}(q\vec{x}) \equiv \left(\frac{1}{q}\vec{\nabla} \times \vec{M}_{JJ}^{M}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right) + \frac{1}{2}\vec{M}_{JJ}^{M}(q\vec{x}) \cdot \vec{\sigma}$$

$$\Phi''_{JM}(q\vec{x}) \equiv i\left(\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right)$$

with $M_{JM} = j_J Y_{JM}$ Bessel spherical harmonics and $\vec{M}^{M}_{JL} = j_J \vec{Y}_{JM}$ vector spherical harmonics.

- **M**= vector-charge (scalar, **usual spin-independent part**, non-vanishing for all nuclei)
- **Φ''**=vector-longitudinal, related to spin-orpit coupling σ·l (also spin-independent, non-vanishing for all nuclei)
- **Σ'** and **Σ''** = associated to longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require nuclear spin j>0
- **Δ**=associated to the orbital angular momentum operator l, also requires j>0
- **$\tilde{\Phi}$'**= related to a vector-longitudinal operator that transforms as a tensor under rotations, requires j>1/2

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542;
N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

Squaring the ampitude get the following nuclear response functions:

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=0,2,\ldots}^{\infty} \langle j_N || O_{J;\tau}(q) || j_N \rangle \langle j_N || O_{J;\tau'}(q) || j_N \rangle \text{ for } O = M, \Phi'',$$

$$W_O^{\tau\tau'}(y) \equiv \sum_{J=1,3,\ldots}^{\infty} \langle j_N || O_{J;\tau}(q) || j_N \rangle \langle j_N || O_{J;\tau'}(q) || j_N \rangle \text{ for } O = \Sigma'', \Sigma', \Delta,$$

$$W_{\tilde{\Phi}'}^{\tau\tau'}(y) = \sum_{J=2,4,\ldots}^{\infty} \langle j_N || \tilde{\Phi}'_{J;\tau}(q) || j_N \rangle \langle j_N || \tilde{\Phi}'_{J;\tau'}(q) || j_N \rangle,$$

$$W_{\Phi'' M}^{\tau\tau'}(y) = \sum_{J=0,2,\ldots}^{\infty} \langle j_N || \Phi''_{J;\tau}(q) || j_N \rangle \langle j_N || M_{J;\tau'}(q) || j_N \rangle,$$

(interference terms)

$$W_{\Delta\Sigma'}^{\tau\tau'}(y) = \sum_{J=1,3,\ldots}^{\infty} \langle j_N || \Delta_{J;\tau}(q) || j_N \rangle \langle j_N || \Sigma'_{J;\tau'}(q) || j_N \rangle.$$

These 8 (6+2 interferences) W nuclear response functions have been calculated for most nuclei using a numerical (truncated) harmonic potential shell model (Fitzpatrick et al., JCAP 1302 1302(2013), Catena and Schwabe, JCAP 1504 no. 04, 042 (2015)) with oscillator parameter:

$$b[\text{fm}] = \sqrt{41.467/(45A^{-1/3} - 25A^{-2/3})} \qquad y = (qb/2)^2$$

One of the most popular scenarios for WIMP-nucleus scattering is a spin-dependent interaction where the WIMP particle is a χ fermion (either Dirac or Majorana) that recoils through its coupling to the spin of nucleons N=p,n:

$$\mathcal{L}_{int} \propto \vec{S}_\chi \cdot \vec{S}_N = c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n$$

(for instance, predicted by supersymmetry when the WIMP is a neutralino that couples to quarks via Z-boson or squark exchange)

A few facts of life:

Nuclear spin is mostly carried by odd-numbered nucleons. Even-even isotopes carry no spin.

• the DAMA effect is measured with Sodium Iodide. Both Na and I have spin **carried by an unpaired proton**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{23}Na	3/2	11	12	100 %
^{127}I	5/2	53	74	100 %

Germanium experiments carry only a very small amount of ^{73}Ge, the only isotope with spin, **carried by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{73}Ge	9/2	32	41	7.7 %

Xenon experiment contain two isotopes with spin, **both carried mostly by an unpaired neutron**

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{129}Xe	½	54	75	26%
^{131}Xe	3/2	54	77	21%

→several authors have considered the possibility that $c_n \ll c_p$: in this case the WIMP particle is seen by DAMA but does not scatter on xenon and germanium detectors

However another class of Dark Matter experiments (superheated droplet detector and bubble chambers) **all use nuclear targets with an unpaired proton**:

Experiment	Target	Type	Energy thresholds (keV)	Exposition (kg day)
SIMPLE	$C_2Cl\,F_5$	superheated droplets	7.8	6.71
COUPP	$C\,F_3\,I$	bubble chamber	7.8, 11, 15.5	55.8, 70, 311.7
PICASSO	$C_3\,F_8$	bubble chamber	1.7, 2.9, 4.1, 5.8, 6.9, 16.3, 39, 55	114
PICO-2L	$C_3\,F_8$	bubble chamber	3.2, 4.4, 6.1, 8.1	74.8, 16.8, 82.2, 37.8

Isotope	Spin	Z (# of protons)	A-Z (# of neutrons)	Abundance
^{19}F	1/2	9	10	100
^{35}Cl	3/2	17	18	75.77 %
^{37}Cl	3/2	17	20	24.23 %
^{127}I	5/2	53	74	100

These experiments are sensitive to c_p , so for $c_n << c_p$ spin-dependent scatterings on Fluorine have been shown to lead to tension with the DAMA (C. Amole et al., (PICO Coll.) PLB711, 153(2012), E. Del Nobile, G.B. Gelmini, A. Georgescu and J.H. Huh, 1502.07682)

N.B. All only sensitive to the energy threshold, which for bubble and droplets nucleation is controlled by the pressure of the liquid

Correspondence between WIMP and non-relativistic EFT nuclear response function

coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$
	velocity-independent	velocity-dependent		velocity-independent	velocity-dependent

(in parenthesis the explicit dependence on q)

$$\mathcal{H} = \sum_i \left(c_i^0 + c_i^1 \tau_3\right) \mathcal{O}_i$$

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{(v_T^{\perp})^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{v_T^2 - v_{min}^2}{c^2}$$

Relativistic couplings leading in their non-relativistic limits **to the most general spin-dependent** terms:

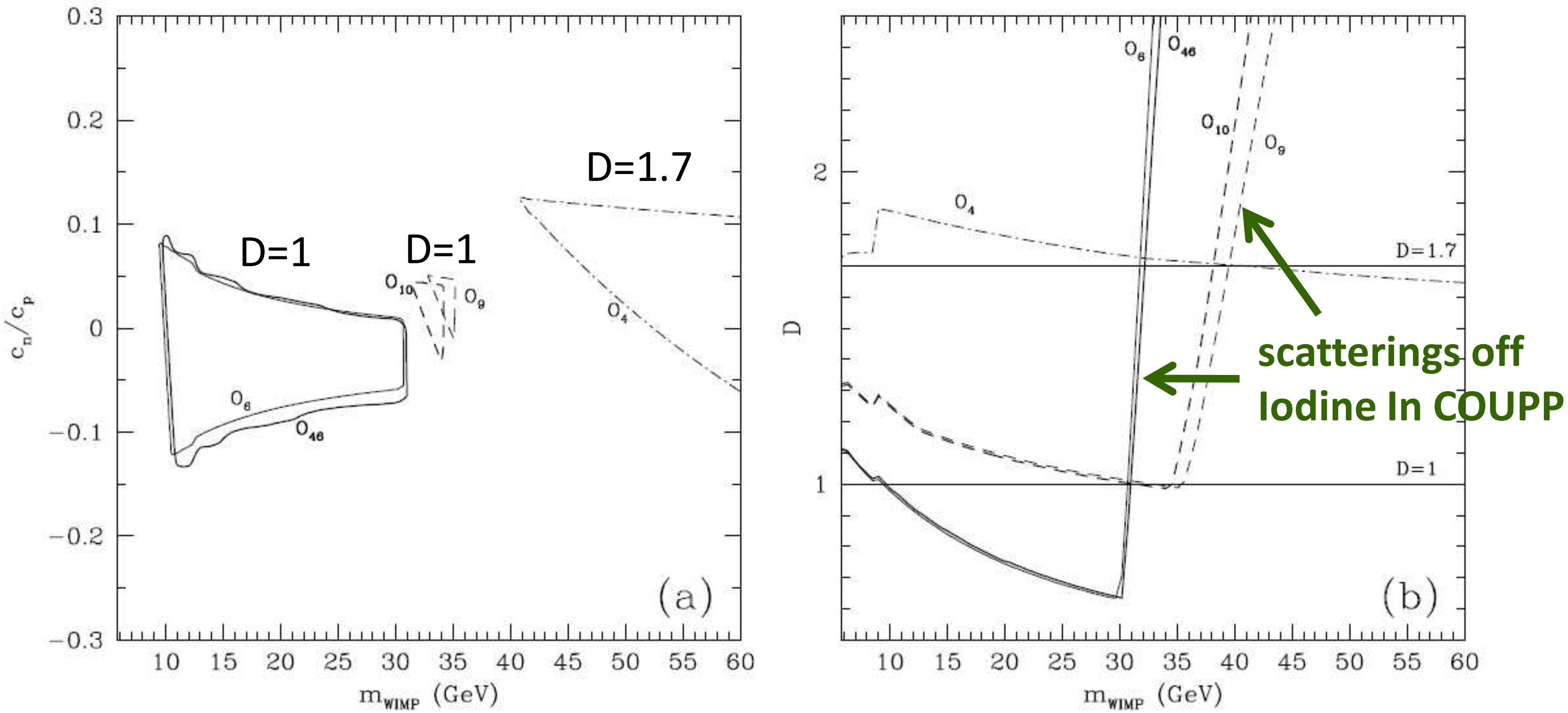
	Relativistic EFT	Nonrelativistic limit	$\sum_i \mathcal{O}_i$	cross section scaling
1	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4\vec{S}_\chi\cdot\vec{S}_N$	$-4\mathcal{O}_4$	$W_{\Sigma''}^{\tau\tau'}(q^2)+W_{\Sigma'}^{\tau\tau'}(q^2)$
2	$2\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N+$ $+\bar{\chi}\gamma^\mu\gamma_5\chi\bar{N}i\sigma_{\mu\nu}\frac{q^\nu}{m_{WIMP}}N$	$-4\vec{S}_N\cdot\vec{v}_T^\perp$	$-4\mathcal{O}_7$	$(v_T^\perp)^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
3	$2\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N+$ $-\bar{\chi}i\sigma_{\mu\nu}\frac{q^\nu}{m_{WIMP}}\chi\bar{N}\gamma^\mu\gamma_5 N$	$-4\vec{S}_N\cdot\vec{v}_T^\perp$	$-4\mathcal{O}_7$	$(v_T^\perp)^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
4	$\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma^5 N$	$-2\vec{S}_N\cdot\vec{v}_T^\perp+$ $+\frac{2}{m_{WIMP}}i\vec{S}_\chi\cdot\left(\vec{S}_N\times\vec{q}\right)$	$-2\mathcal{O}_7+2\frac{m_N}{m_{WIMP}}\mathcal{O}_9$ $\simeq 2\frac{m_N}{m_{WIMP}}\mathcal{O}_9$	$\simeq q^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
5	$\bar{\chi}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\chi\bar{N}\gamma^\mu\gamma_5 N$	$4i(\frac{\vec{q}}{m_M}\times\vec{S}_\chi)\cdot\vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	$q^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
6	$\bar{\chi}\gamma^\mu\gamma_5\chi\bar{N}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}N$	$4i\vec{S}_\chi\cdot(\frac{\vec{q}}{m_M}\times\vec{S}_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	$q^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
7	$i\bar{\chi}\chi\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$	$q^2 W_{\Sigma''}^{\tau\tau'}(q^2)$
8	$i\bar{\chi}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}\gamma_5\chi\bar{N}\gamma_\mu\gamma_5 N$	$-4i(\frac{\vec{q}}{m_N}\cdot\vec{S}_\chi)(\vec{v}_T^\perp\cdot\vec{S}_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	$(v_T^\perp)^2 q^2 W_{\Sigma'}^{\tau\tau'}(q^2)$
9	$\bar{\chi}\gamma_5\chi\bar{N}\gamma^5 N$	$-\frac{\vec{q}}{m_{WIMP}}\cdot\vec{S}_\chi\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$-\frac{m_N}{m_{WIMP}}\mathcal{O}_6$	$q^4 W_{\Sigma''}^{\tau\tau'}(q^2)$
10	$\bar{\chi}i\sigma^{\mu\alpha}\frac{q_\alpha}{m_M}\gamma_5\chi\bar{N}i\sigma_{\mu\beta}\frac{q^\beta}{m_M}\gamma_5 N$	$4\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	$q^4 W_{\Sigma''}^{\tau\tau'}(q^2)$
11	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4\left(\frac{\vec{q}}{m_M}\times\vec{S}_\chi\right)\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$4\left(\frac{q^2}{m_M^2}\mathcal{O}_4-\frac{m_N^2}{m_M^2}\mathcal{O}_6\right)$	$q^4 W_{\Sigma'}^{\tau\tau'}(q^2)$

- the resulting scaling laws include the most general velocity and momentum dependences allowed by Galilean invariance through the product $(v_T^\perp)^{2n}$ $(q^2)^m$ (n=0,1; m=0,1,2)

S.Scopel, J.H.Yoon and K.Yoon, JCAP 1507(2015)041

Numerical results

S.Scopel, J.H.Yoon and K.Yoon, JCAP 1507(2015)041



- If D<1 all constraints are verified
- Possible for O_6,O_{46} (q^4 momentum dependence) and to a lesser extent for O_9,O_{10} (q^2 momentum dependence), no compatibility for O_4 (usual spin-dependent interaction, no q dependence)
- as long as scatterings off Fluorine (and/or Chlorine) dominate in bubble chambers and droplets detectors momentum transfers q=sqrt($m_{nucleus}$ E) have a smaller values compared to Sodium , due to the lighter target mass and to the lower energy threshold of the former→ reduced sensitivity to DAMA for $(q^2)^n$, n=1,2
- for m_{WIMP}>30 GeV scatterings off Iodine in COUPP are kinematically accessible with much larger values of momentum transfer q →steep rise in compatibility factor when n=1,2

S.Scopel and K.Yoon, JCAP 1602(2016)050

An alternative way to evade Fluorine constraints for a WIMP with spin-dependent coupling to protons: inelastic scattering

$$v_{\min} = \frac{1}{\sqrt{2 m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|$$

$v_{min} > v^*_{min}$ $\qquad$ $$v^*_{\min} = \sqrt{\frac{2\delta}{\mu_{\chi N}}}$$

$A_{sodium}=23$ $\qquad$ $A_{Fluorine}=19$

$m_{Sodium} > m_{Fluorine} \rightarrow \mu_{\chi N}^{Sodium} > \mu_{\chi N}^{Sodium}$

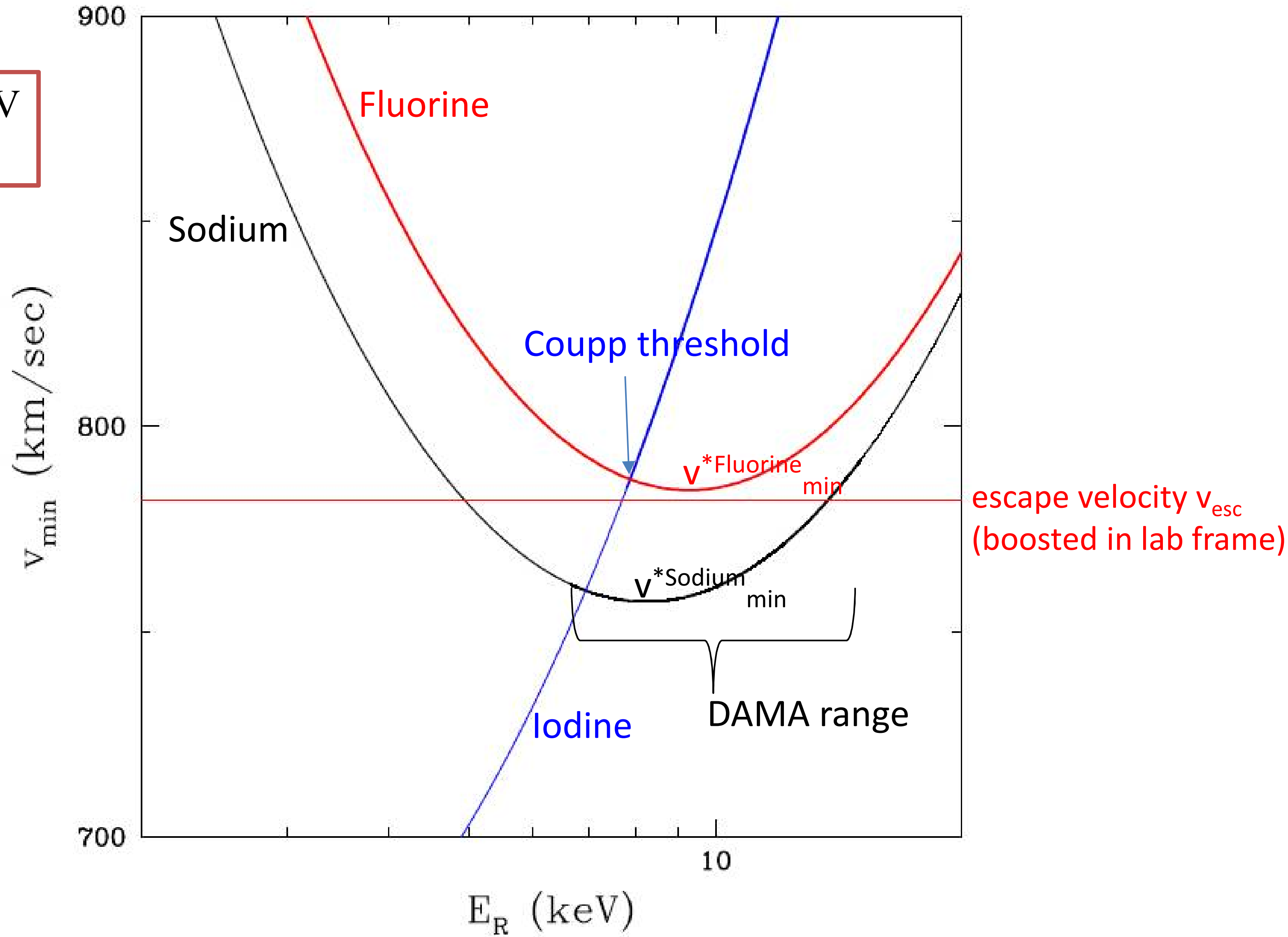
$\rightarrow v^{*Sodium}_{min} < v^{*Fluorine}_{min}$

$\boxed{\text{what if } v^{*Sodium}_{min} < v_{esc} < v^{*Fluorine}_{min}\text{?}}$

(N.B. v_{esc} in lab frame)

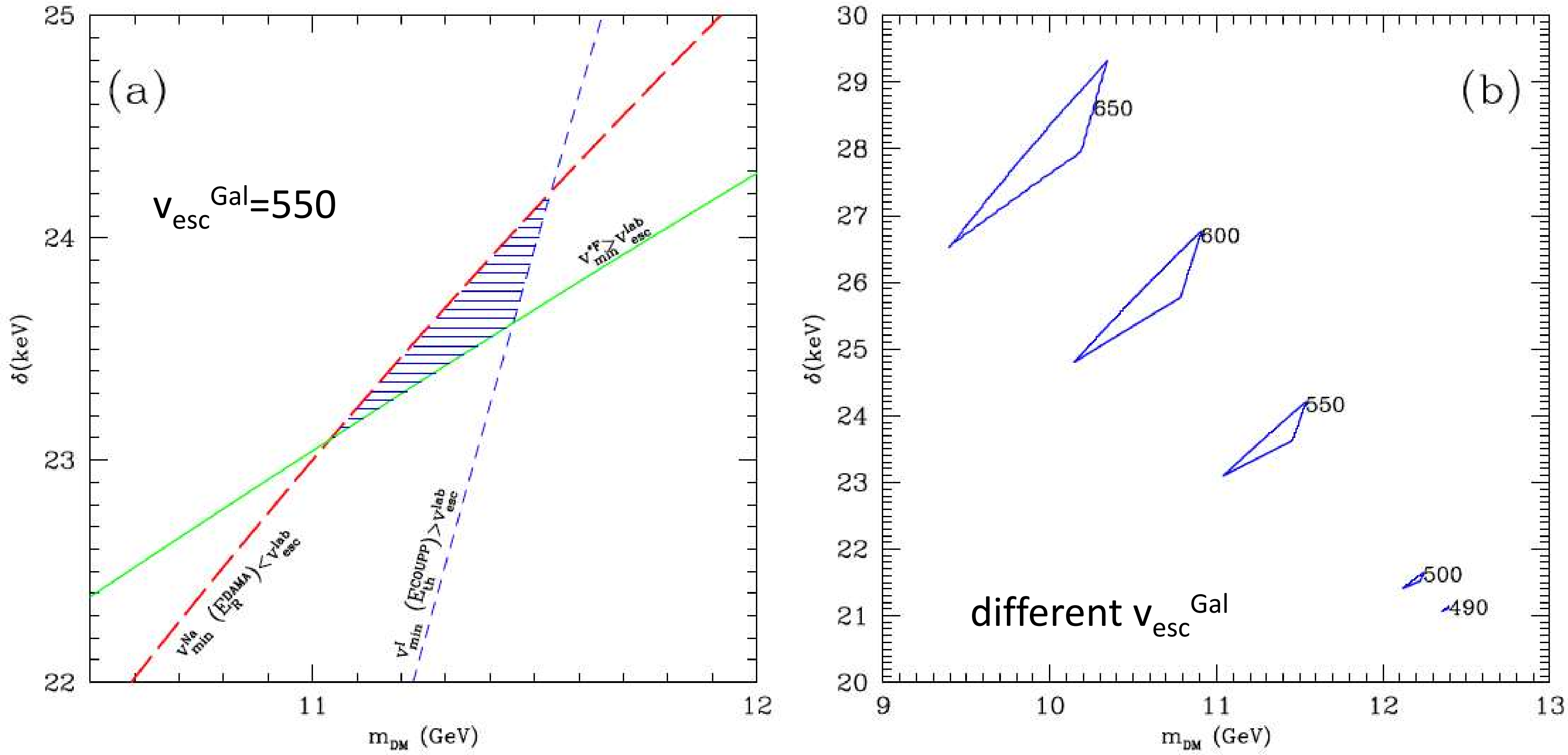
S.Scopel and K.Yoon, JCAP 1602(2016)050

m_χ = 11.4 GeV
δ=23.7 keV



depending on m_χ and δ, can drive Fluorine (and Iodine in COUPP) beyond v_{esc} while Sodium remains below → no constraint on DAMA from droplet detectors and bubble chambers

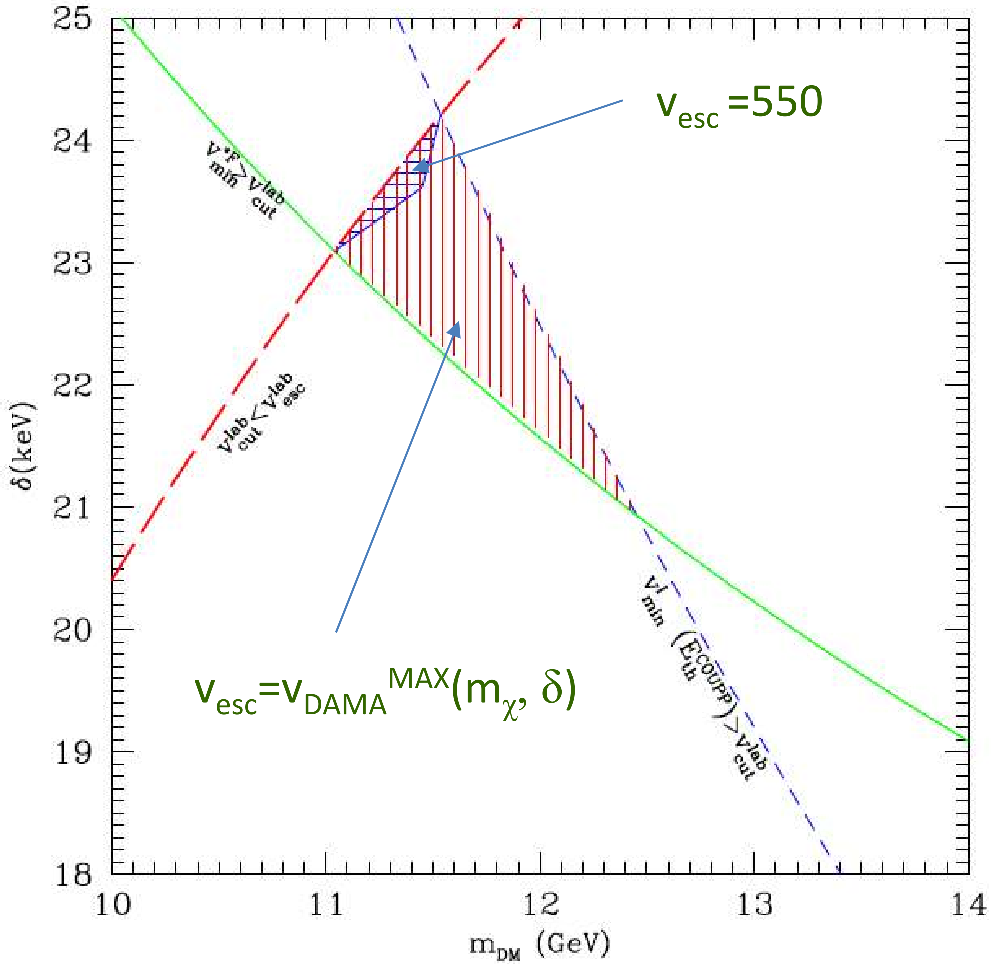
S.Scopel and K.Yoon, JCAP 1602(2016)050



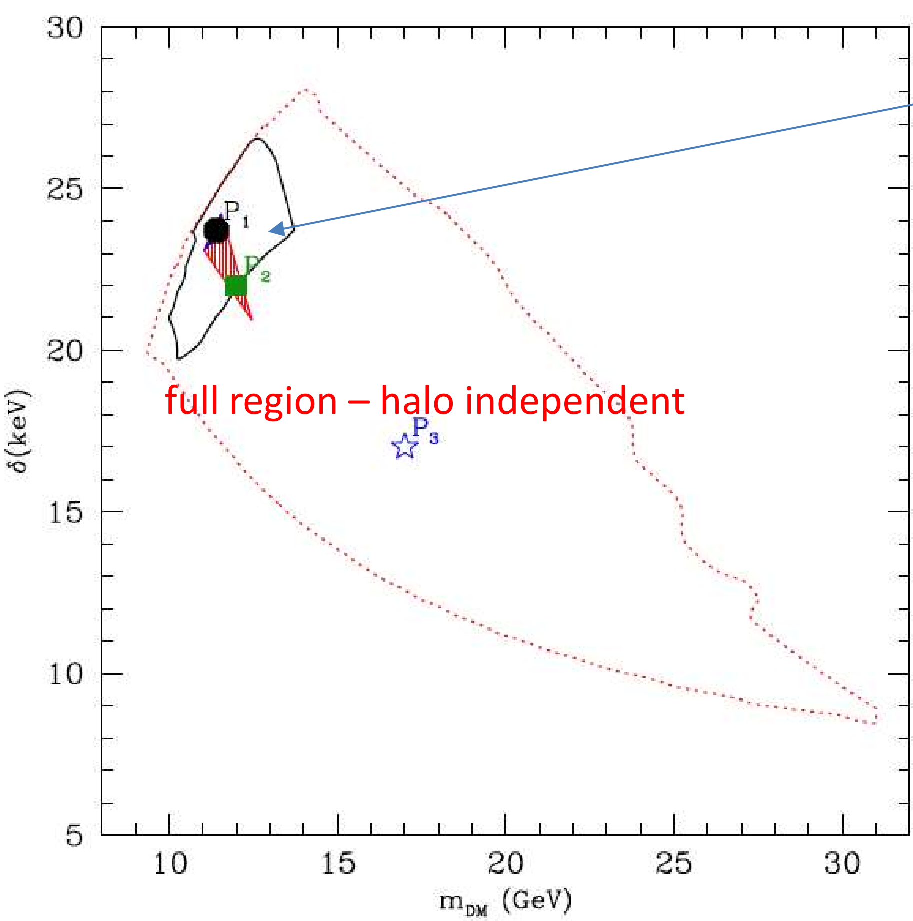
very tuned region. but this is just kinematics

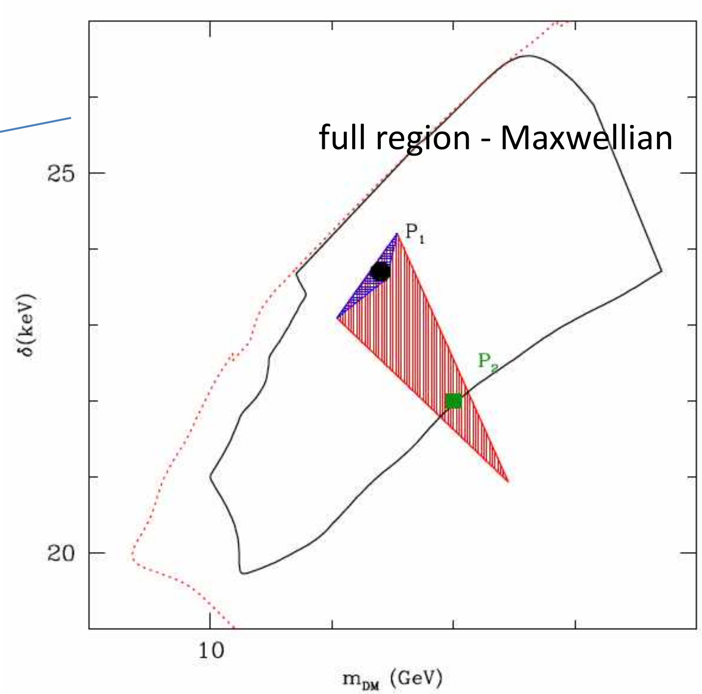
S.Scopel and K.Yoon, JCAP 1602(2016)050

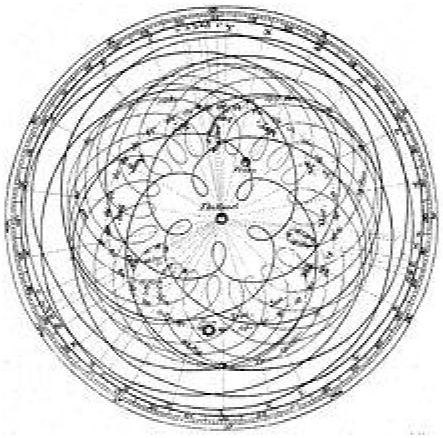
taking v_{esc}=v_{DAMA}^{MAX}(m_χ, δ) the kinematic region enlarges considerably



when including also the dynamics (through a full calculation of the compatibility factor) the two regions (Maxwellian and halo-independent) enlarge even more





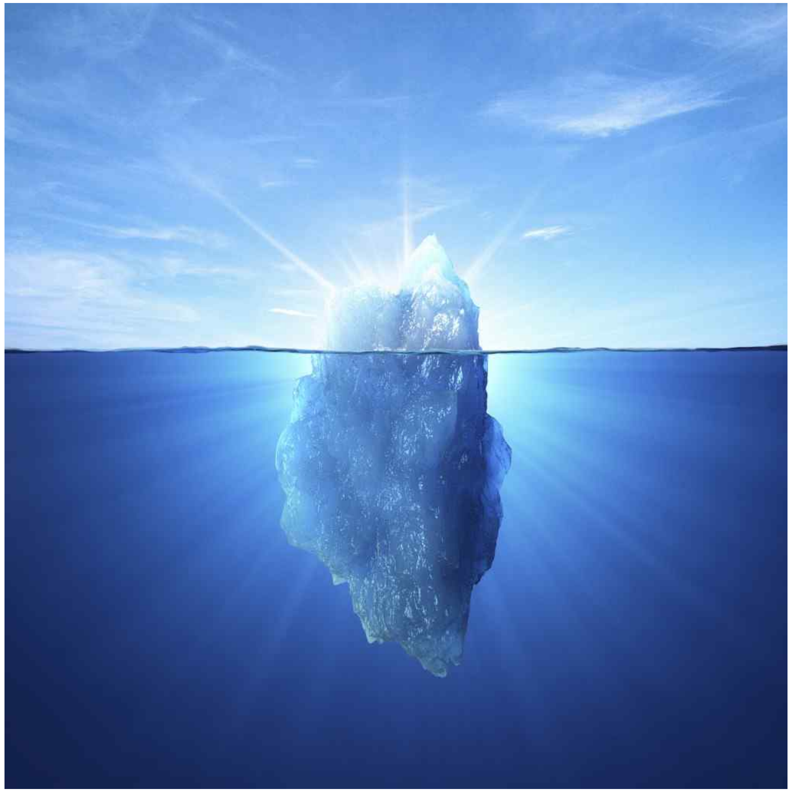


Several epicycles added to the usual scenario:

- Halo-independent
- Non-standard coupling
- Inelastic scattering
- Isospin violation
- ….

- Indeed, combining a halo-independent approach and/or a non-standard coupling (other than SI or SD) and/or inelastic scattering (different kinematics) and/or isospin violation compatibility among any of the “excesses” and constraints from null experiments can be achieved (S.S. and K.H. Yoon, JCAP 1602 (2016) no.02, 050; S.S.,K.H. Yoon and J.H. Yoon, JCAP 1507 (2015) no.07, 041; S.S. and J. H. Yoon, Phys.Rev. D91 (2015) no.1, 015019; S.S. and K.H. Yoon, JCAP 1408 (2014) 060)
- “Proofs of concept”

The bottom line:
Based on very well motivated theoretical assumptions we got used to a very simple WIMP direct detection parameter space (i.e. mass vs. SI sigma exclusion plots for isothermal sphere). However in principle it may be much larger: are we just starting now to scratch its surface?



Using data to study the model-independent halo functions η_0 and η_1 and extract the cross section σ: the stream approach

S.S. P. Gondolo, *work in progress*

For a standard velocity-independent WIMP nucleus cross section the definition of the halo function η_0 is given by:

$$\eta_0(v_{min}, t) = \int_{v_{min}}^{\infty} \frac{f(\vec{v}, t)}{v}\, d^3v \qquad v \equiv |\vec{v}|$$

with f the velocity distribution in the lab rest frame. Present-day detectors are not sensitive to directionality, so if the cross section is isotropical the signal depends on the angular average of f:

$$\bar{f}(v, t) = v^2 \int f(\vec{v}, t)\, d\Omega_v$$

In the following: will drop the bar and indicate with f the angular average:

$$\bar{f}(v, t) \rightarrow f(v, t)$$

The distribution f in the lab rest frame is the result of a Galilean boost of the galactic velocity distribution f_{gal} . Assuming that f_{gal} is time-independent on the time-scale of the experiment the only time dependence of f comes from the boost:

$$f(\vec{v},t) = f_{gal}\left(\vec{v} + \underbrace{\vec{v}_{\odot}}_{\text{velocity of the solar system In the Galactic frame}} + \underbrace{\vec{v}_{\oplus}}_{\text{velocity of the Earth in the solar system frame}}(t)\right)$$

In the following we will assume for simplicity that also f_{gal} is isotropical. This is not strictly necessary for the method to work, although simplifies the numerical procedure:

$$\vec{u} \equiv \vec{v} + \vec{v}_{\odot} + \vec{v}_{\oplus}(t) \qquad u \equiv |\vec{u}|$$

$$\boxed{f_{gal}(\vec{u}) = f_{gal}(u)}$$

The modulated halo function η_1 has several definitions, which depend on how the modulated part of the signal is extracted from the exp data:

$$\eta_1(v_{min}) = \frac{1}{2}\left[\eta_0(v_{min}, t_0) - \eta_0(v_{min}, t_0 + \frac{T}{2})\right]$$

$$\eta_1(v_{min}) = \frac{1}{T}\int_0^T \eta_0(v_{min}, t)\cos[\omega(t-t_0)]\, dt$$

etc. (all definitions coincide for a sinusoidal modulation)

T=365 days, t_0=2 June
$\omega=2\pi/365$

The expected rate in bin of *observed* energy E' is given by:

$$N(t)_{[E_1',E_2']} = \int_{E_1'}^{E_2'} \frac{dR}{dE'}(E',t)\, dE'$$

$$v_{min}(E_R) = \left(\frac{m_T E_R}{2\mu_T^2}\right)^{\frac{1}{2}}$$ (elastic scattering)

$$\frac{dR}{dE'}(E',t) = \epsilon(E') \int_0^{\infty} \frac{dR}{dE_{ee}}(E_{ee},t)\mathcal{G}_T(E',E_{ee})\, dE_{ee} \qquad E_{ee} = Q(E_R)E_R$$

$$\frac{dR}{dE_R}(E_R,t) = \sum_T N_T \frac{\rho}{m_{WIMP}} \int_{v_{min}[E_R]} d^3v f(\vec{v},t) v \frac{d\sigma}{dE_R}$$

$$\frac{d\sigma}{dE_R} = \frac{\sigma_0 F(E_R)}{E_R^{max}} = \frac{\sigma_0 F(E_R)}{2\mu_T/m_N v^2}$$

m_T=nuclear mass of target T

μ_T=WIMP-nucleus reduced mass

$F(E_R)$=nuclear form factor (for finite size nucleus)

$Q(E_R)$=quenching factor=fraction of energy deposited in detected channel such as ionization or scintillation

$\mathcal{G}_T(E',E_{ee})$ =energy resolution of the detector

ε(E')=detector acceptance

N_T= number of targets of isotope T

Combining everything together :

contains all uncertainties

$$\boxed{N_{[E_1',E_2']}(t) = \int_0^{\infty} \mathcal{R}_{[E_1',E_2']}(v_{min})\tilde{\eta}(v_{min},t)d\,v_{min}}$$

$$\tilde{\eta}(v_{min},t) \equiv \frac{\rho}{m_{WIMP}}\sigma_0\eta(v_{min},t)$$

(σ_0=point-like WIMP-nucleus cross section)

E. Del Nobile, G. Gelmini, P. Gondolo and J.H. Huh, JCAP 1310 (2013) 048

N.B. the previous derivation requires no explicit velocity dependence in the cross section. Can do better!

An apparently innocuous trick: take out the velocity integral, and write the expected number of events as:

$$N(t)_{[E'_1,E'_2]} = \int_0^\infty \mathcal{H}_{[E'_1,E'_2]}(v)\, f(v,t)\, dv$$

where the response function contains all the dependences on the cross section and the experimental quantities. By setting:

$$f(v,t) \equiv -v\frac{\partial \eta(v,t)}{\partial v}$$

integrating by parts and incorporating as usual the point-like cross section and the local density in the definition of the halo function leads to:

$$N_{[E'_1,E'_2]}(v)(t) = \int_0^\infty \mathcal{R}_{[E'_1,E'_2]}(v)\, \tilde{\eta}(v,t)\, dv$$

This expression looks pretty much the same as the previous one (with $v_{min} \rightarrow v$) but *is valid in principle for any velocity dependence in the cross section*.

The two different response functions are related by:

$$\mathcal{R}_{[E'_1,E'_2]}(v) = \frac{\partial}{\partial v}\left[v\mathcal{H}_{[E'_1,E'_2]}(v)\right], \qquad \mathcal{H}_{[E'_1,E'_2]}(v) = \frac{1}{v}\int_0^v \mathcal{R}_{[E'_1,E'_2]}(v')\, dv'$$

P. Gondolo, S. Scopel, work in progress

A mathematical theorem:

- given the $N+1$ known functions $g^i(x)$ $(i = 1, ..., N)$ and $h(x)$ and the unknown function $f(x)$, all defined in the same domain, the N constraints:

$$I_g^i = \int_0^\infty g^i(x)\, f(x)\, dx,\ i = 1, ...N,$$

imply that the extreme values of the integral:

$$I_h = \int_0^\infty h(x)\, f(x)\, dx,$$

are obtained by expressing the unknown function $f(x)$ in terms of the N parametrizations:

$$f_n(x) = \sum_{j=1}^{n} \lambda_j\, \delta(x - x_j),\ n = 1, ..., N,$$

with $\sum_{i=1}^{n} \lambda_i$=1 and n=1,..,N.

I. Pinelis, *"On the extreme points of moments sets"*, arXiv:1205.0134; H. P. Mulholland and P. Rogers, *"Representation theorems for distribution functions"*, Proc. of London Math. Society s3-8(2) (1958) 177–223.

P. Gondolo, S. Scopel, in preparation

In practice, this means that, at fixed n, the maximal range of the I_g integral is swept by the λ_j , x_j parameters that satisfy the n constraints with $f_n(x)$ given by the superposition of n streams, i.e. the system of n + 1 linear equations:

$$\sum_{j=1}^{n} \lambda_j g^i(x - x_j) = I_f^i, \quad i = 1, n$$

$$\sum_{k=1}^{n} \lambda_k = 1, \ \lambda_k > 0.$$

The full range of I_g is then obtained by combining the N intervals at fixed n.

Direct application to the analysis of direct detection data: given n experimental measurements any other quantity of the form

$$A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$$

can be bracketed for *any* f(v).

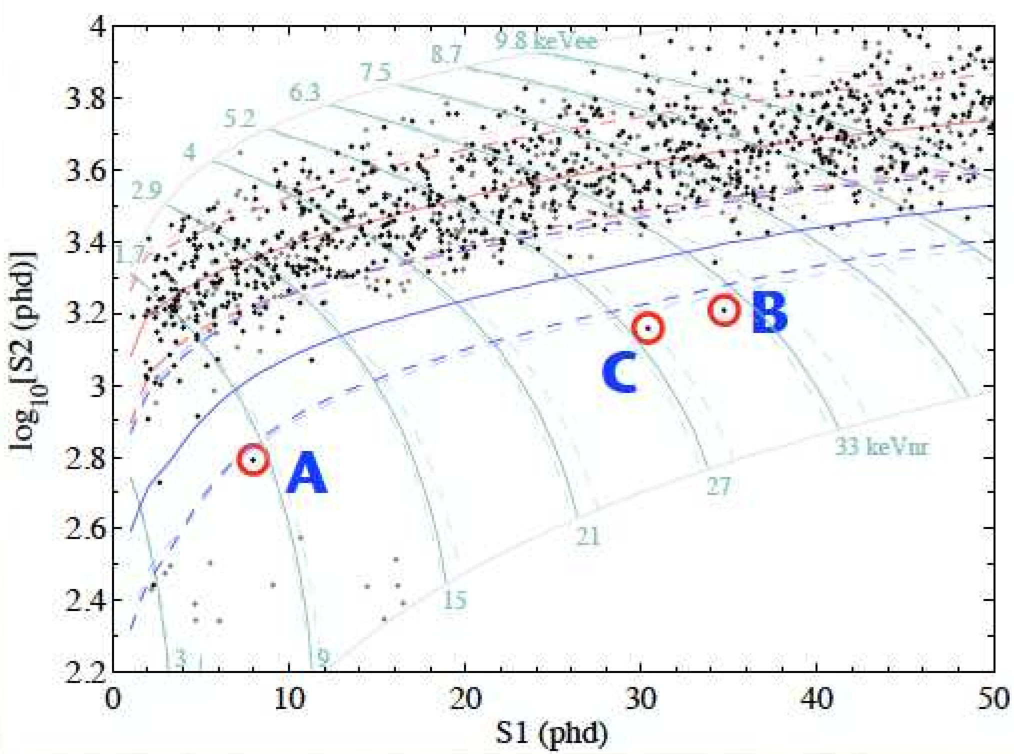
Recap:

- Given n independent direct detection measurements a parameterization of the velocity distribution in terms of n streams, combined with analogous parameterizations for n-1,n-2,…1 brackets *any* observable of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ where only A(v) is known.

Where do we get from here?

WIMP-search data

- After salting, events outside the ER band were scrutinized again.
- Two populations of rare pathological events were identified, that had contributed three particularly dangerous events.



From A. Manalaysay talk at IDM2016

Three events surviving unblinding in LUX, excluded by modified pos-unblinding cuts.
Let's just assume they were not (**CAVEAT: not for real, just playing with them!**)

Problem #1: only tree events in the $0<S_1<50$ phe window, which binning should we use? (N.B. S_1 in phe=photo-electons can be converted to the recoil energy).

Solution: can use the data to construct the *extended* likelihood function, which does not need binning:

$$\frac{L}{2} = N_{tot} - \sum_{k=1}^{N} \ln\left(\frac{dR}{dS_1}\right)_k$$

with N=3 events and where where both N_{TOT} and dR/dS_1 are given by the sum of a background + a signal contributions:

$$\left(\frac{dR}{dS_1}\right)_k = \left(\frac{dR_s}{dS_1}\right)_k + \left(\frac{dR_b}{dS_1}\right)_k$$

$$N_{tot} = N_{tot,s} + N_{tot,b}$$

$$N_{tot,s} = \int_{S_1^{min}}^{S_1^{max}} \frac{dR_s}{dS_1}\, dS_1, \quad N_{tot,b} = \int_{S_1^{min}}^{S_1^{max}} \frac{dR_b}{dS_1}\, dS_1$$

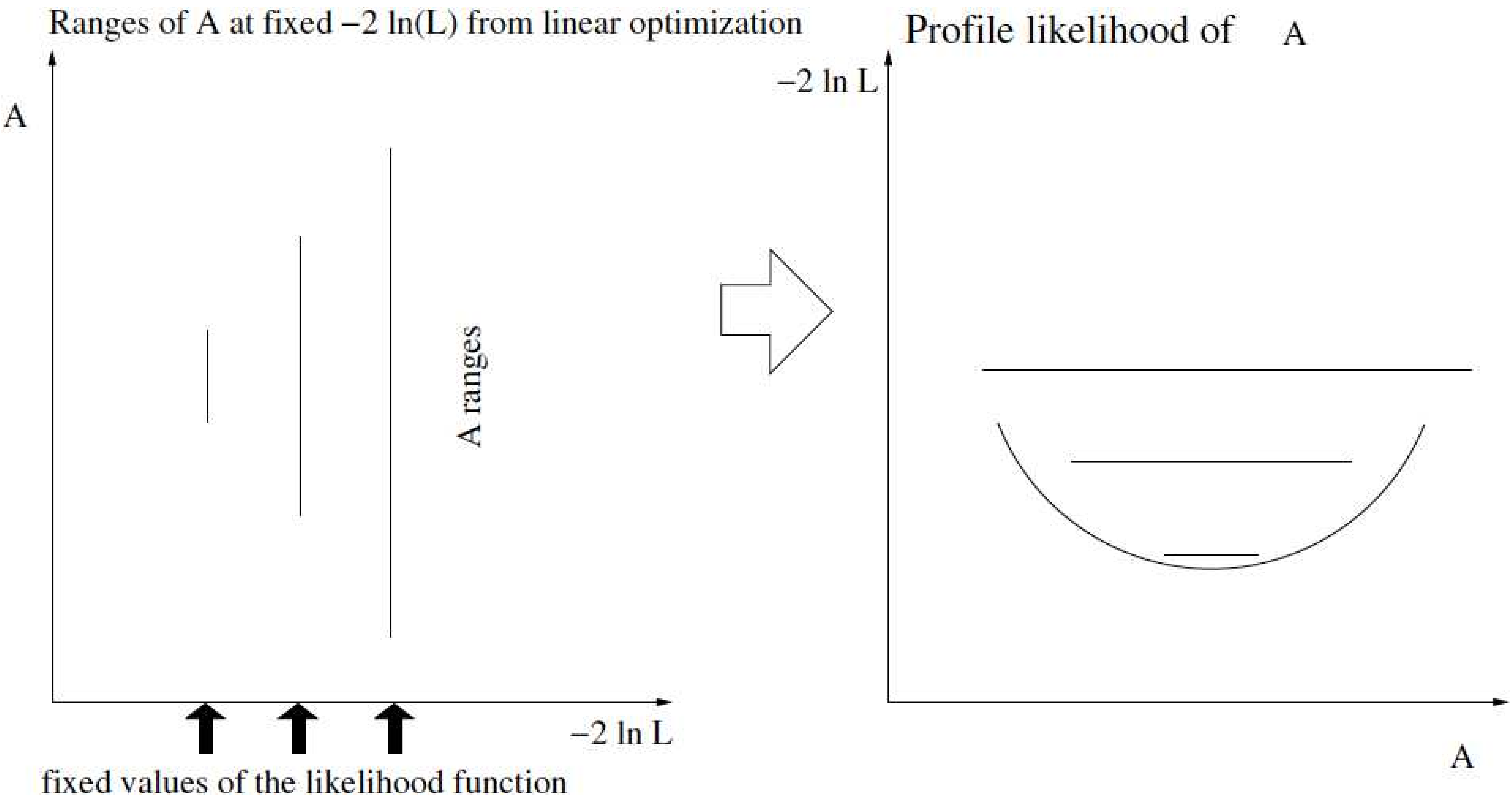
N.B. need an estimation of the background. In LUX the background estimation is 1.5 events in the full range 0.5 PE$<S_1<50$ PE

The signal part can be expressed in terms of integrals of f(v) times some response function only dependent on experimental quantities:

$$
\begin{aligned}
N_{tot,s} &= \int_0^\infty \mathcal{H}_{[S_1^{min}, S_1^{max}]}(v) f(v)\, dv, \\
\left(\frac{dR_s}{dS_1}\right)_k &= \int_0^\infty \mathcal{H}_{S_1^k}(v) f(v)\, dv,
\end{aligned}
$$

According to the previous theorem for any choice of the four quantities N_{TOT} and $(dR/dS_1)_k$ any other quantity of the form $A = \int_0^\infty \mathcal{A}(v)\, f(v)\, dv$ can be bracketed for any f(v). Actually, fixing N_{TOT} and $(dR/dS_1)_k$ fixes L, *so this is also true for a fixed value of L*

If we can bracket the full range of A at fixed L by turning the plot 90 degrees we can get the profile-likelihood of A



The n-sigma range of A is obtained by taking the points with $L-L_{min}<n^2$

Can choose A as any quantity which can be expressed as an integral of f(v) times a response function. For instance, take an average of the halo function η in some range of v:

$$\tilde{\eta}(v_{min}) = \frac{\rho}{m_\chi}\sigma \int_{v_{min}}^{\infty} f(v)\, dv$$

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1}, v_{min,2}]} = \frac{1}{v_{min,2} - v_{min,1}} \int_{v_{min,1}}^{v_{min,2}} \tilde{\eta}(v_{min})\, dv_{min}$$

Indeed, this average can be expressed as:

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1}, v_{min,2}]} = \boxed{\int_0^{\infty} \mathcal{H}_\eta^{[v_{min,1}, v_{min,2}]}(v) f(v)\, dv}$$

with:

$$\mathcal{H}_\eta^{[v_{min,1}, v_{min,2}]}(v) = \frac{\rho}{m_\chi}\sigma \begin{cases} 0 & \text{if } v < v_{min,1} \\ \frac{1}{v}\frac{v - v_{min,1}}{v_{min,2} - v_{min,1}} & \text{if } v_{min,1} \leq v \leq v_{min,2} \\ \frac{1}{v} & \text{if } v > v_{min,2}. \end{cases}$$

Problem #2: how do we sample the parameter space with N=1,2,3,4 streams?
According to the theorem the full range of $< \tilde{\eta}(v_{min}) >_{[v_{min,1},v_{min,2}]}$ is spanned by using:

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1},v_{min,2}]} = \sum_{k=1}^{m} \lambda_k \mathcal{H}_{\eta}^{[v_{min,1},v_{min,2}]}(v_k), \quad m = 1, 2, ..., N+1$$

Need to do that numerically.

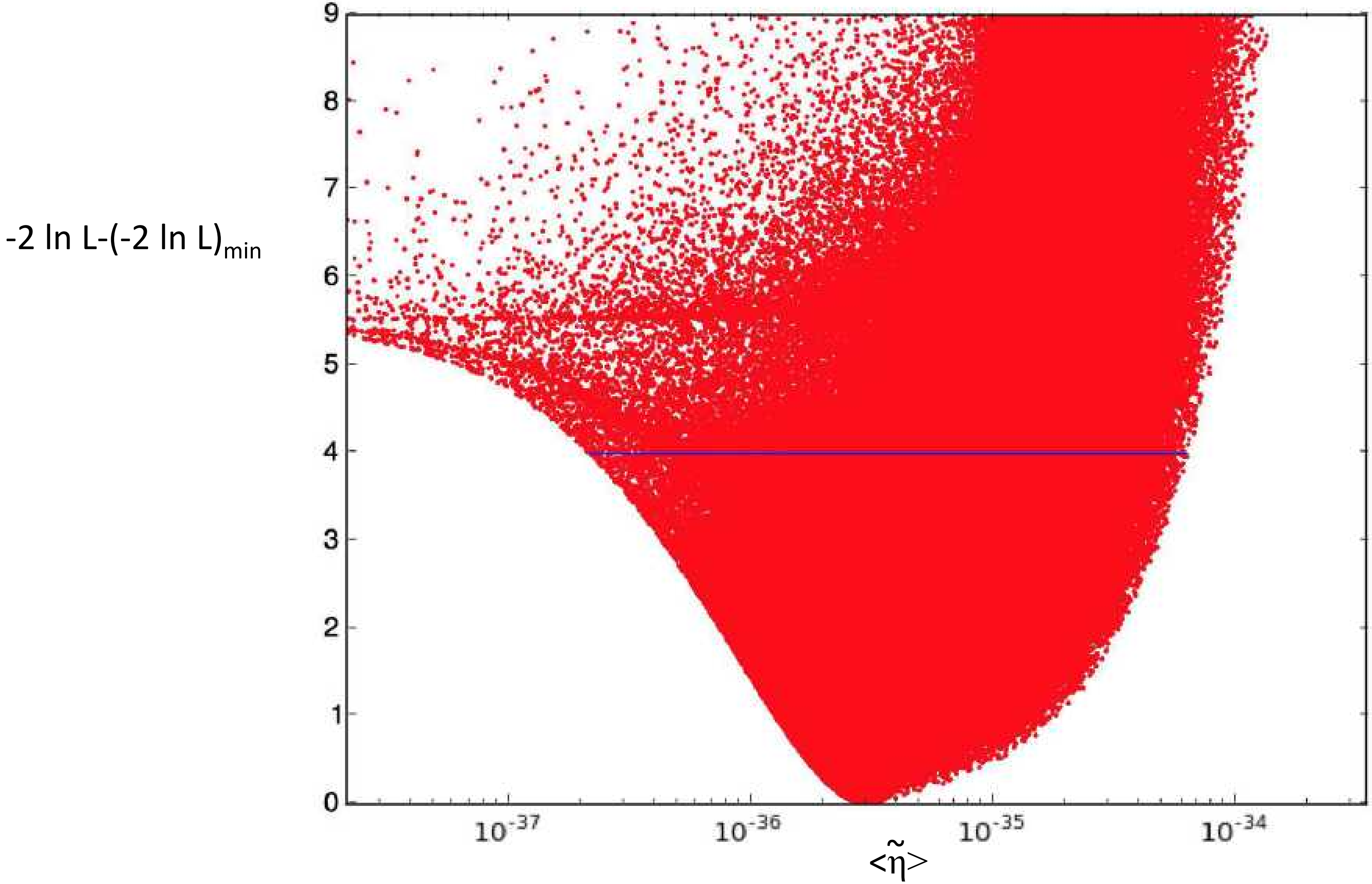
Suitable for a Markov Chain sampling. Two advantages:

- the sampling is driven by the Likelihood itself, don't waste time in low-probability regions
- Perfect for profiling, highest density of points where -2 ln L is minimal

Can use a Markov–Chain Montecarlo code* to generate large sets {v} of v_k velocities and {λ} of λ_k coefficients for $1 \leq k \leq m$ and $1 \leq m \leq N+1=4$ to calculate both -2 ln L and $<\tilde{\eta}(v_{min})>$

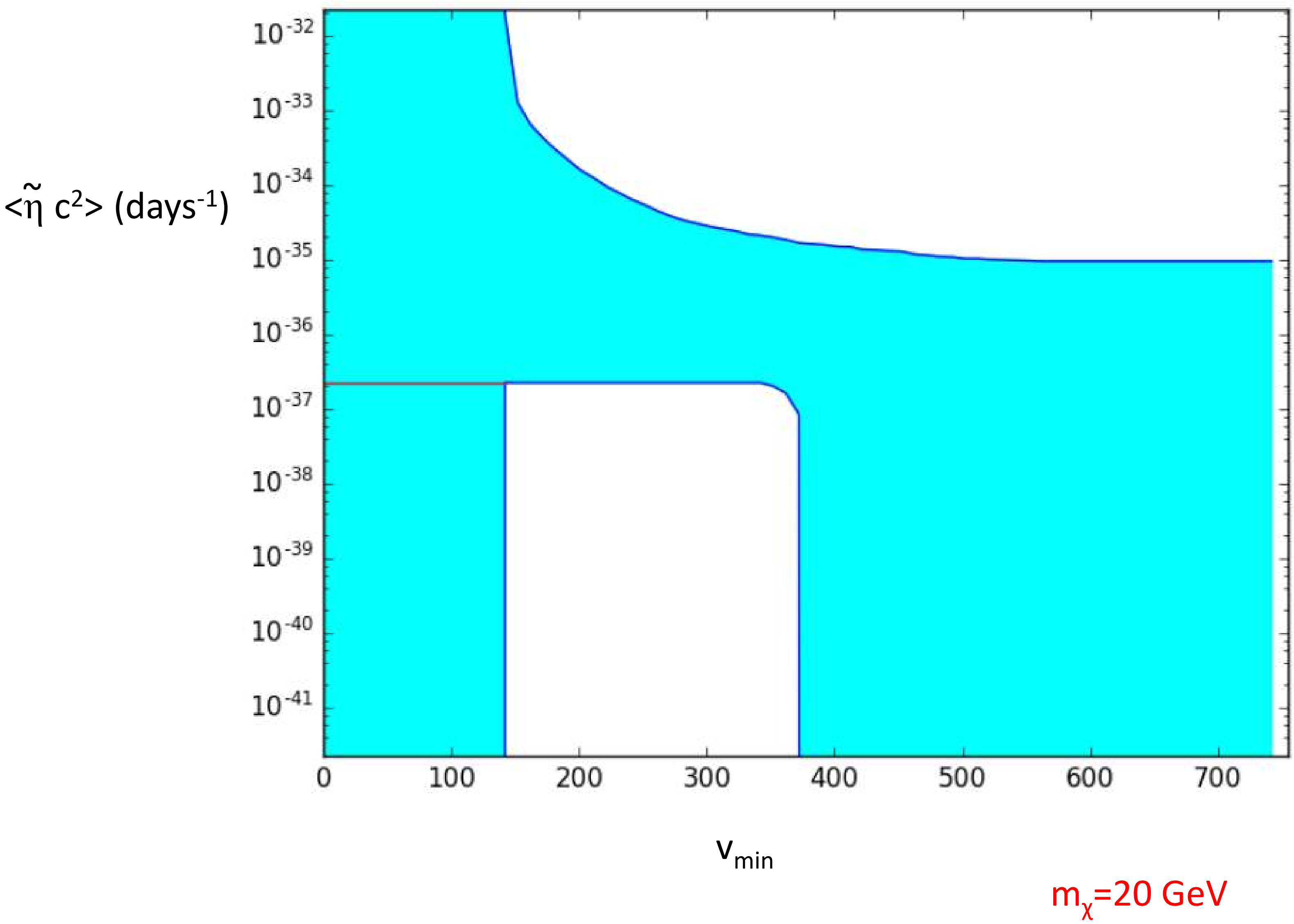
*emcee, D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, emcee: The mcmc hammer, Publications of the Astronomical Society of the Pacific 125 (2013), no. 925 306.

Example: 2-sigma interval for <η> in the range 200 km/s<v_{min}<250 km/s <u>and zero background</u>

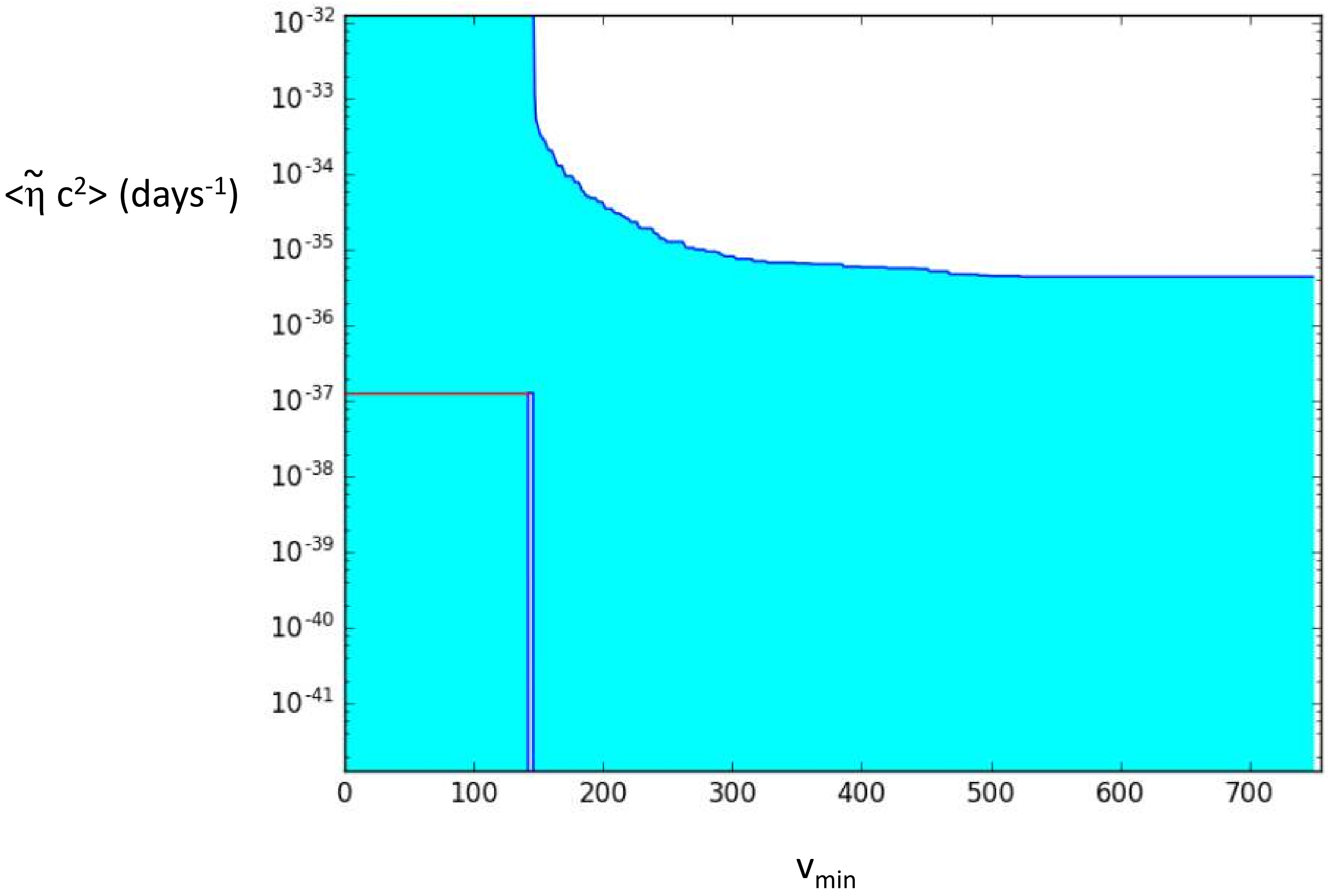


5×10^6 points using 250 independent walkers and a Metropolis-Hastings sampler

Repeat the same exercise ad different velocity ranges:



On the other hand, assuming a flat background of 1.5 events, 3 observed events have a much smaller significance (less than 1 sigma, only upper bound for the halo function):



m_χ=20 GeV

N.B. the new physics is contained in the cross section σ, which is a normalizing factor in the response function:

$$\tilde{\eta}(v_{min}) = \frac{\rho}{m_\chi}\sigma \int_{v_{min}}^{\infty} f(v)\, dv$$

Since:

$$< \tilde{\eta}(v_{min}) >_{[v_{min,1},v_{min,2}]} = \sum_{k=1}^{m} \lambda_k \mathcal{H}_\eta^{[v_{min,1},v_{min,2}]}(v_k)$$

a convenient way is to normalize the streams to σ:

$$\sigma = \sum_k \lambda_k$$

What kind of info can we get on σ? It depends…

From direct detection data to suppression scale (simple halo-independent recipe)

Once $\tilde{\eta}$ is fixed by experiment need f(v) to get info on the cross section and the suppression scale Λ

$$\tilde{\eta}(v_{min}) \equiv \frac{\rho_\chi}{m_\chi}\sigma_0 \eta(v_{min})$$

M*aximize η* and *minimize* cross section taking:

$$f(\vec{v}) = \delta(v_s - v_{min})$$

(v_s = maximal value of the v_{min} range corresponding to the signal)

$$\Rightarrow \quad \tilde{\eta}^{max}(v_{min}) = \tilde{\eta}^{fit}\theta(v_s - v_{min})$$

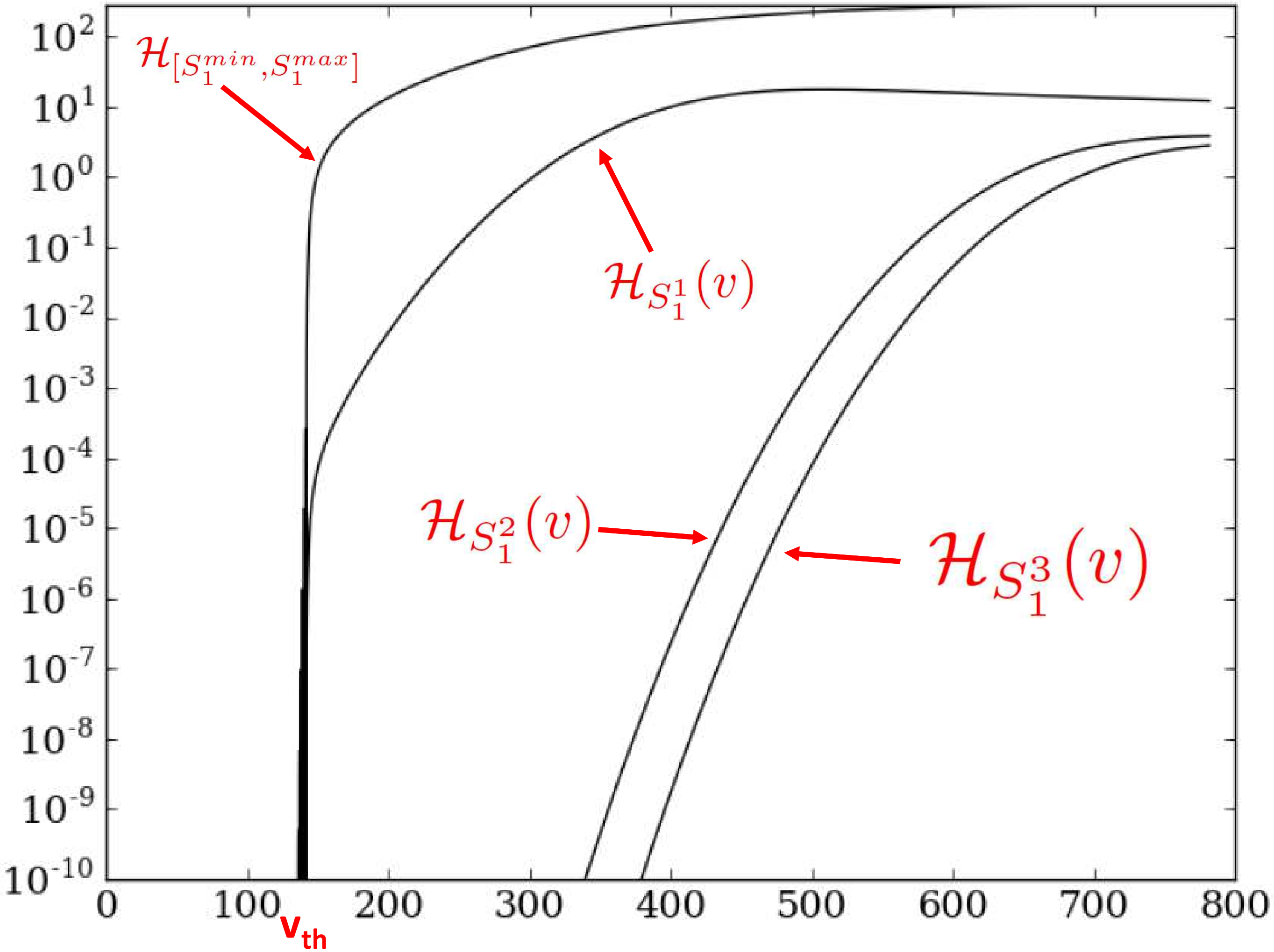
N.B. corresponds to fitting the experimental etas to a constant value, works only if this is compatible to data

Then use:

$$\tilde{\eta}^{fit} = \frac{\rho}{m_\chi}\sigma\frac{1}{v_s}$$

Profile likelihood of σ?

The four response functions in -2 ln L, corresponding to $N_{tot,s}$ and $(dR/dS_1)_k$ with S_k=(7.9, 30.40, 34.7) PE



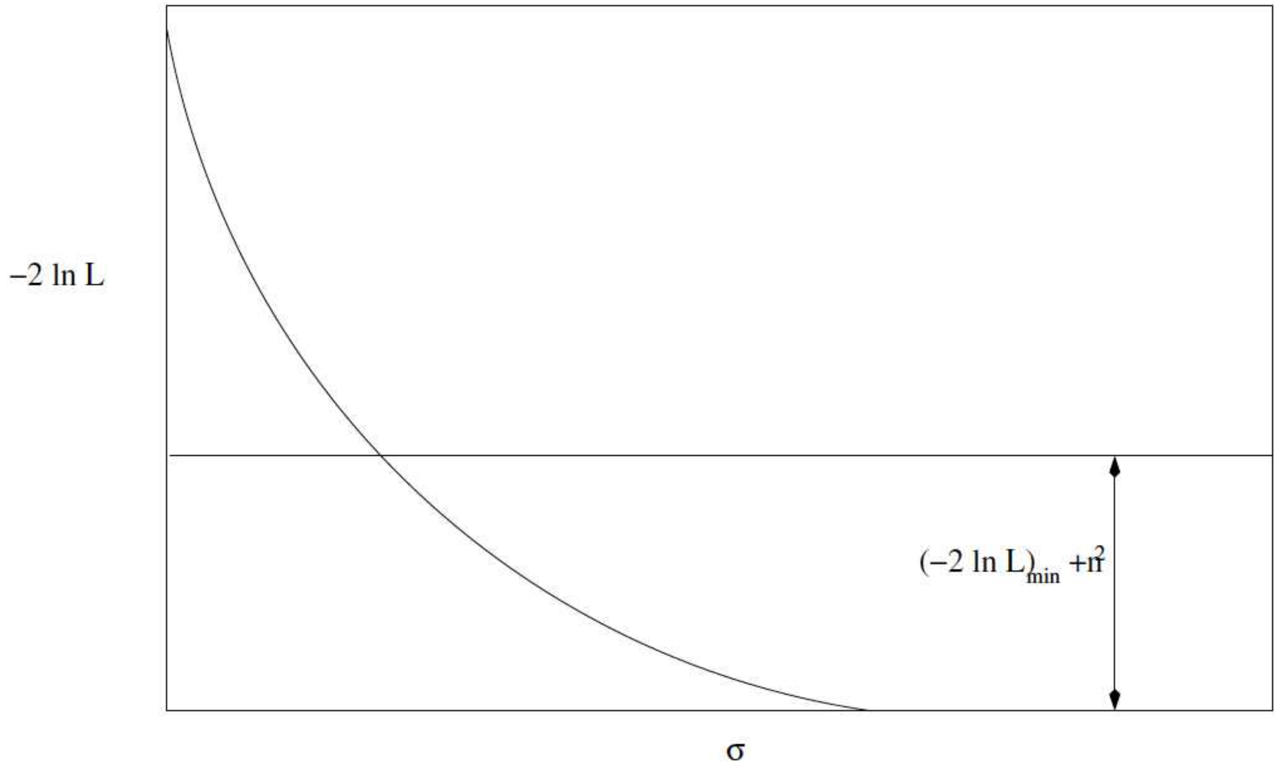
All response functions vanish for $v \rightarrow v_{th}$ (low-energy threshold - extended by energy resolution)

m_χ=20 GeV

This means that the likelihood is degenerate when:

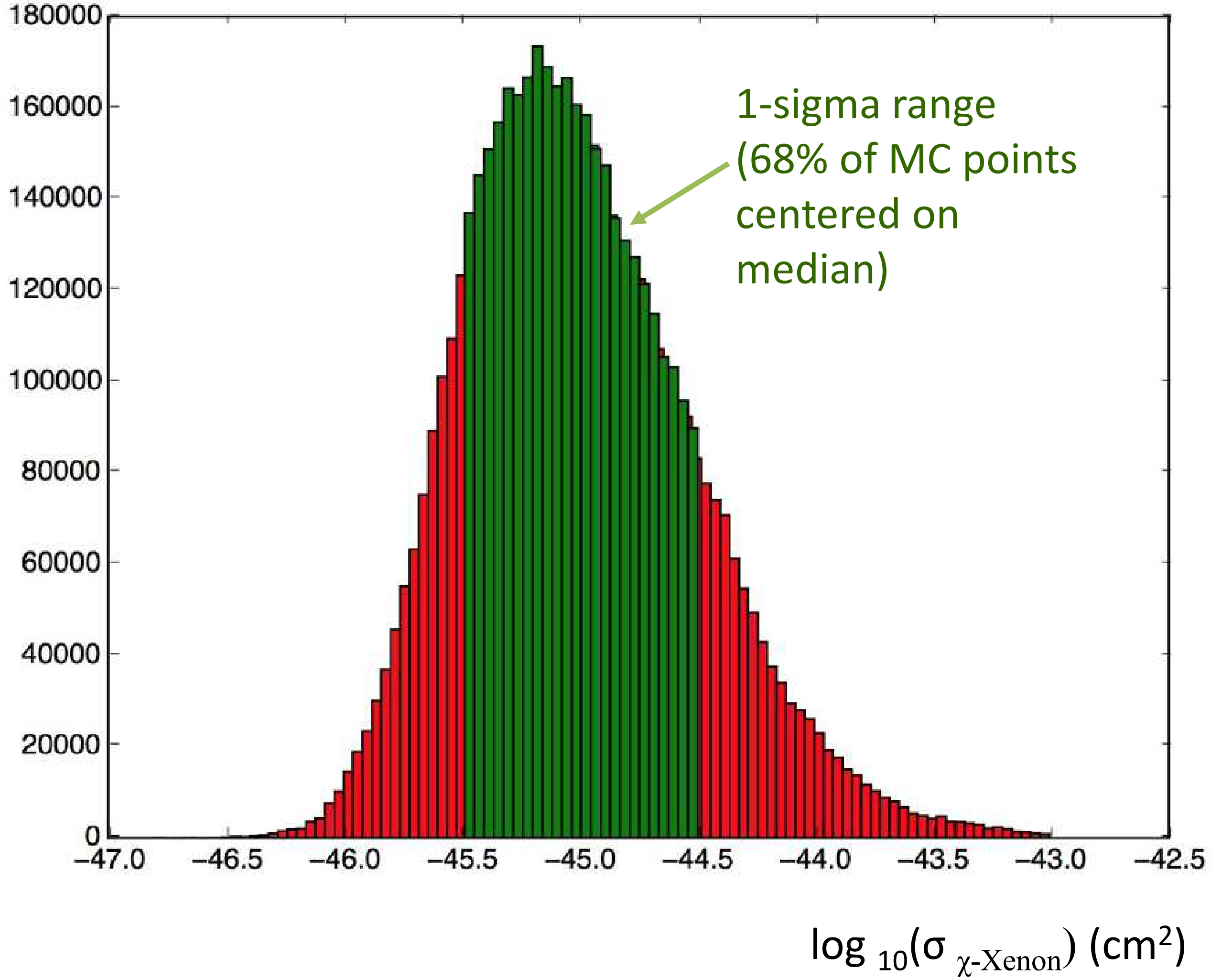
$$\lim_{v_k \to v_{th}} \mathcal{H}_i(v_k) = 0, \; \sigma \to \infty \text{ at fixed } \lambda_k \mathcal{H}_i(v_k)$$

→ profiling of σ at n sigma can only yields a lower bound



Lower boun on σ → upper bound on NP scale

Actually can also get an *interval* on σ through Bayes theorem, but need to assume a *prior distribution* on the parameters (namely, on the v_k's):



(flat prior in range $0<v_k<v_{esc}$=782 km/sec)

N.B. -2 ln L flat directions with $\sigma \to \infty$ for $v_k \to v_{th}$, small volume diluted in flat prior range

Halo-independent yearly-modulated fractions

Due to the rotation of the Earth around the Sun the signal in a direct detection experiment depends on time. Assuming that the only time dependence is due to the boost from the Galactic to the Lab rest frame:

$$S(t)_{[E_1',E_2']} = \int \mathcal{H}_{[E_1',E_2']}(v)\, f(v,t)\, dv =$$

$$S(t)_{0,[E_1',E_2']} + S_{m,[E_1',E_2']} \cos\left[\frac{2\pi}{365\,\text{days}}(t-t_0)\right] =$$

$$\int \mathcal{R}_{[E_1',E_2']}(v)\tilde{\eta}(v,t)\, dv =$$

$$\int \mathcal{R}_{[E_1',E_2']}(v)\left\{\tilde{\eta}_0(v) + \tilde{\eta}_1(v)\cos\left[\frac{2\pi}{365\,\text{days}}(t-t_0)\right]\right\} dv$$

Standard lore: need to know explicitly f(v) to get the modulated fraction $\tilde{\eta}_1(v)/\tilde{\eta}_0(v)$ (ex: <10% for a Maxwellian)

$$f(\vec{v},t) = f_{gal}\left(\vec{u} = \vec{v} + \vec{v}_{\odot} + \vec{v}_{\oplus}(t)\right)$$

Lab rest frame

Galaxy rest frame

velocity of the Sun

velocity of the Earth

Change integration variable from v (lab frame) to u (Galactic frame):

$$S_{[E_1',E_2']}(t) = \int \mathcal{H}_{[E_1',E_2']}\left(\vec{u} - \vec{v}_{\odot} - \vec{v}_{\oplus}(t)\right)\, f_{gal}(\vec{u})\, d^3u$$

N.B. : the time dependence is now is only in the response function

The unmodulated and modulated parts are obtained via a Fourier time analysis:

$$S_{0,[E_1',E_2']} = \frac{1}{T}\int_0^T dt,\, S_{[E_1',E_2']}(t)$$

$$S_{m,[E_1',E_2']} = \frac{1}{T}\int_0^T dt,\cos\left[\frac{2\pi}{365}(t-t_0)\right] S_{[E_1',E_2']}(t)$$

Assuming isotropic response (this is usually the case) this implies:

$$S_{0,[E'_1,E'_2]} = \int \mathcal{H}_{0,[E'_1,E'_2]}(u)\, f_{gal}(u)\, du$$
$$S_{m,[E'_1,E'_2]} = \int \mathcal{H}_{m,[E'_1,E'_2]}(u)\, f_{gal}(u)\, du$$

$$u \equiv |\vec{u}|$$

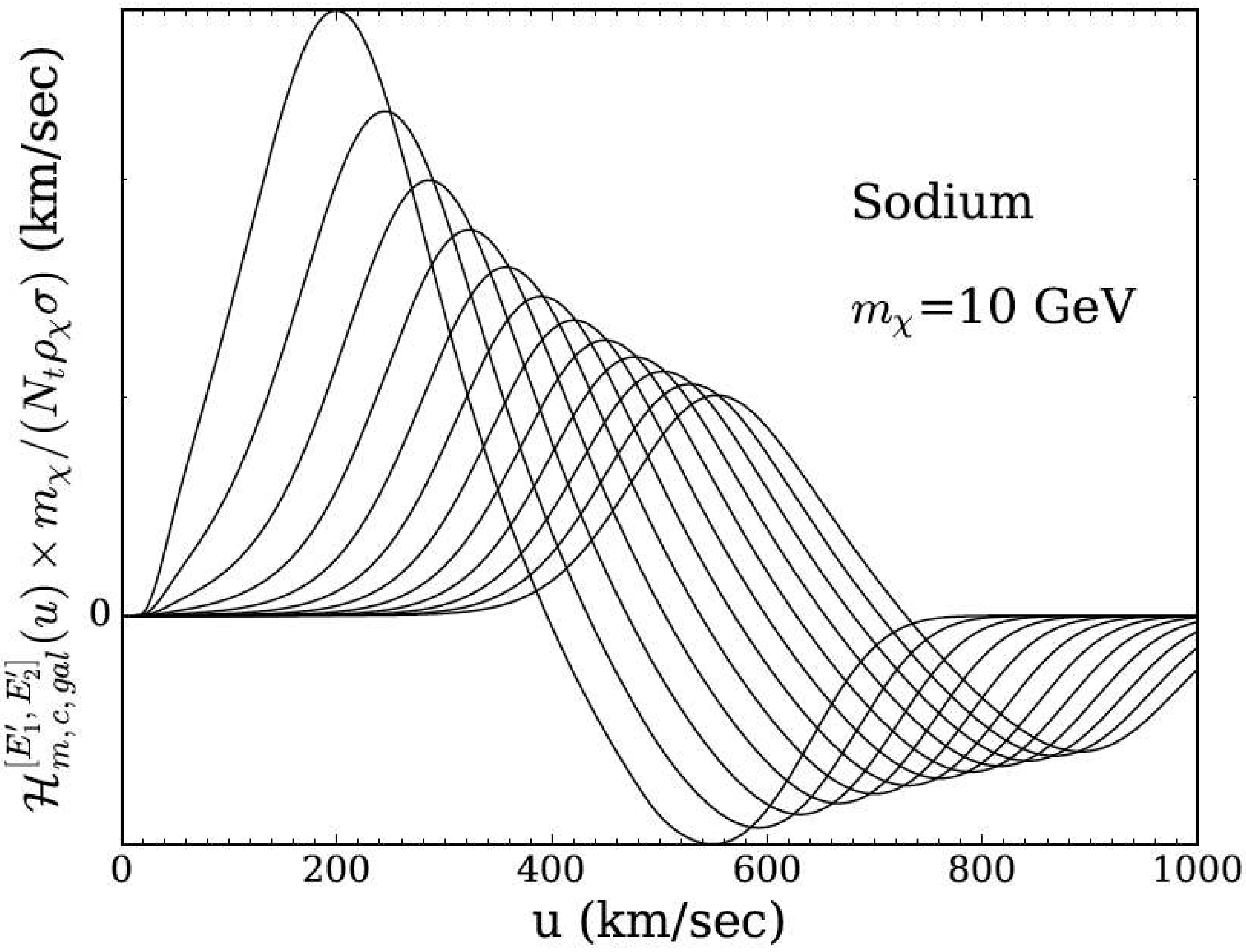
with:

$$\mathcal{H}_{0,\mathrm{gal}}(u) = \frac{1}{4\pi} \int d\Omega_u \, \frac{1}{T} \int_0^T dt\, \mathcal{H}\left(|\vec{u} - \vec{v}|\right)$$

$$\mathcal{H}_{m,\mathrm{gal}}(u) = \frac{1}{4\pi} \int d\Omega_u \, \frac{1}{T} \int_0^T dt \cos\left[\frac{2\pi}{365}(t - t_0)\right] \mathcal{H}\left(|\vec{u} - \vec{v}|\right)$$

N.B. The modulated amplitude depends on the cosine transform of the response function, which is completely known→ modulation as a property of the detector

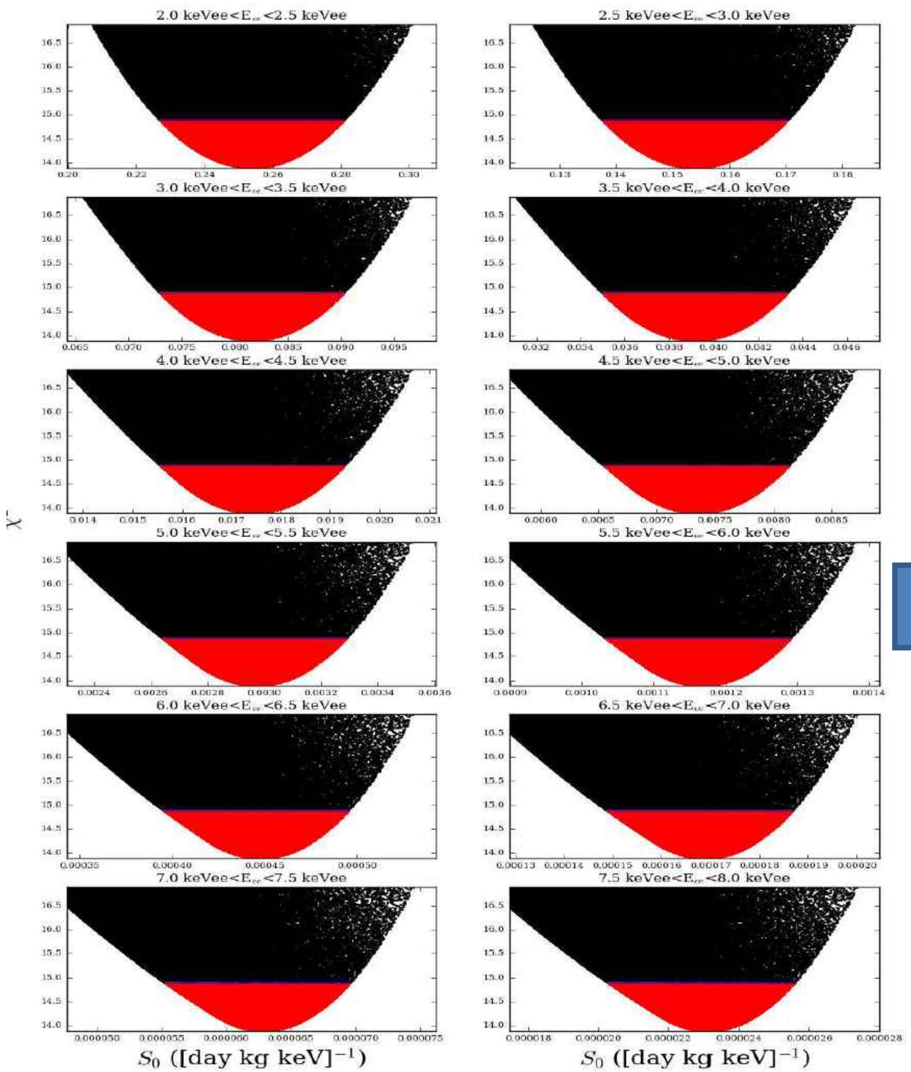
Modulated response functions in DAMA (galactic rest frame)



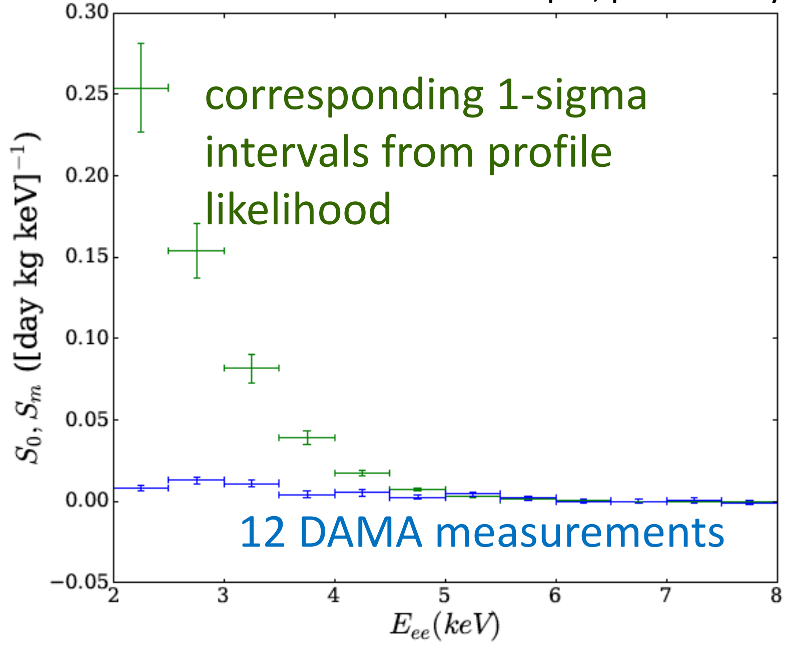
S_0 and S_m are both given by the integral of a know response function times *the same* unknown f(u) → use theorem on extreme distributions to profile out the unmodulated amplitudes in DAMA starting from measured modulated amplitudes

$$-2\ln\mathcal{L} = \sum_{k=1}^{12}\left(\frac{S_m^k - S_{m,exp}^k}{\sigma_k}\right)^2$$

$$f_{gal}(u) = \sum_{k=1}^{N}\lambda_k\delta(u-u_k), \quad N=1,12$$

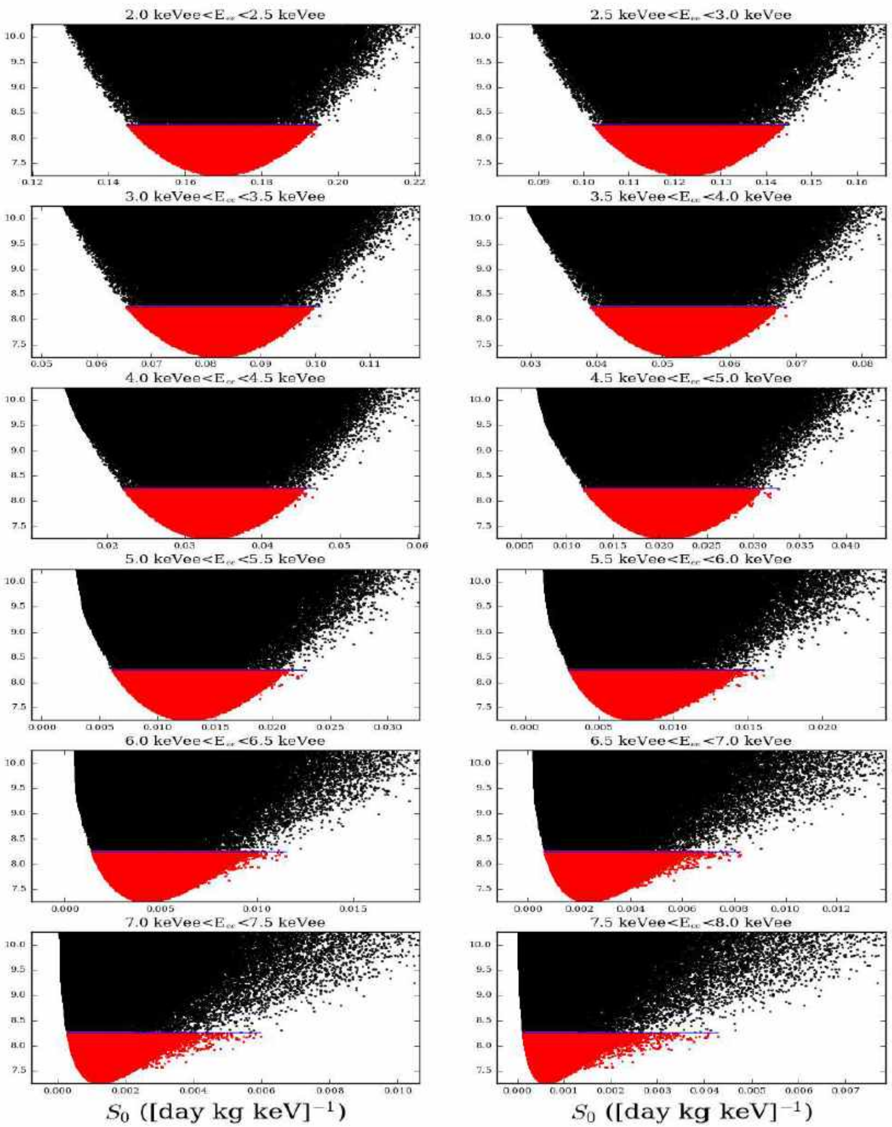


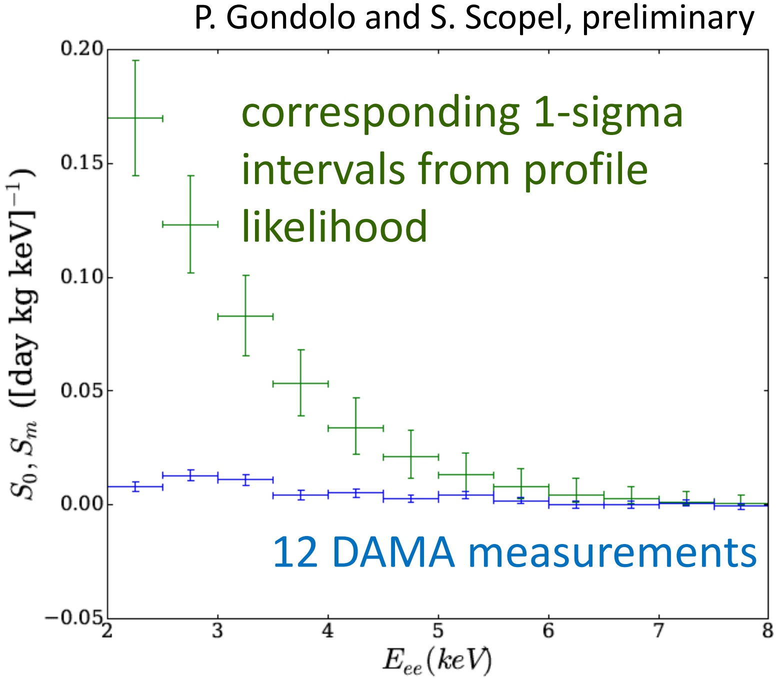
P. Gondolo and S. Scopel, preliminary



m_χ=5 GeV

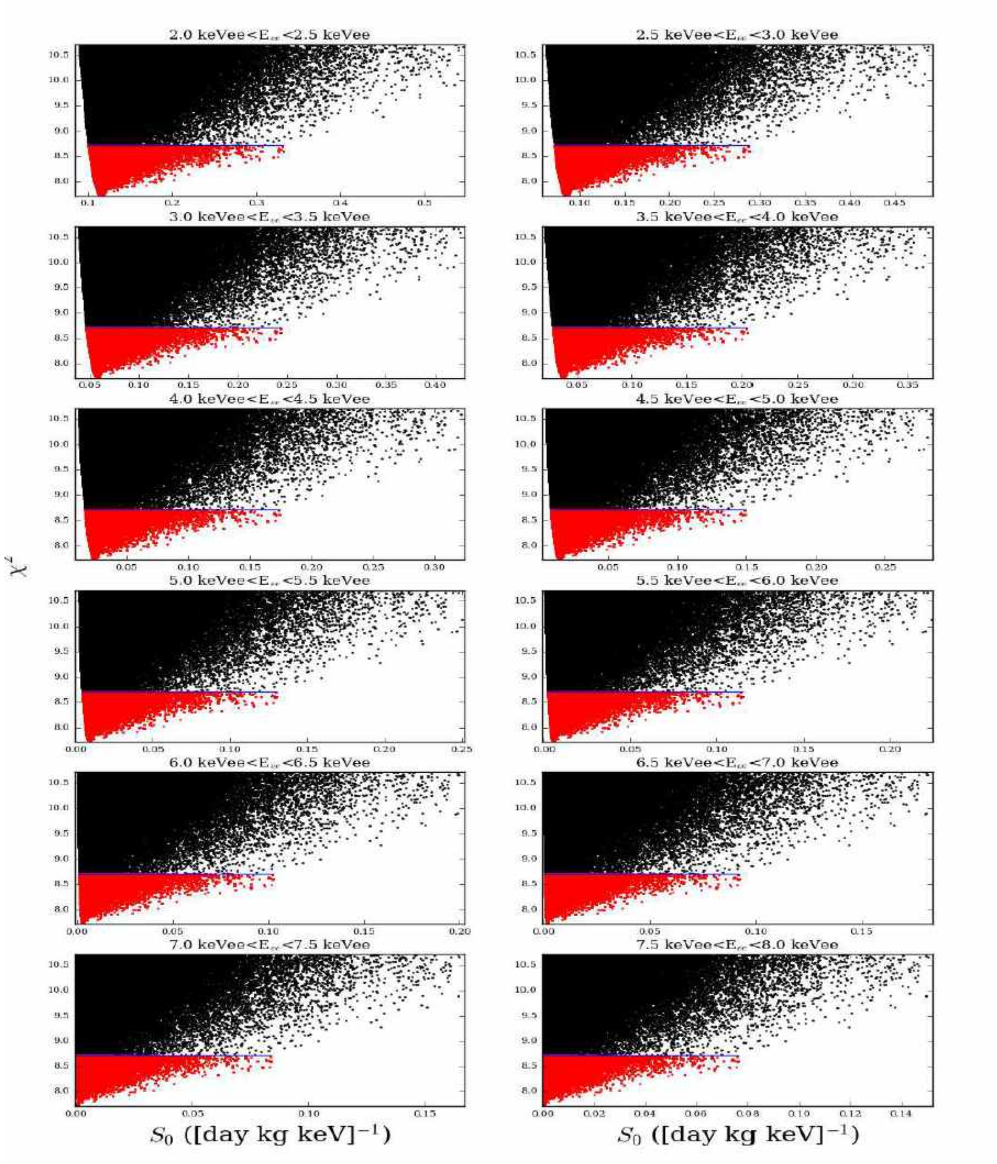
5x10[6] points Markov chain, 250 independent walkers Metropolis-Hastings sampler



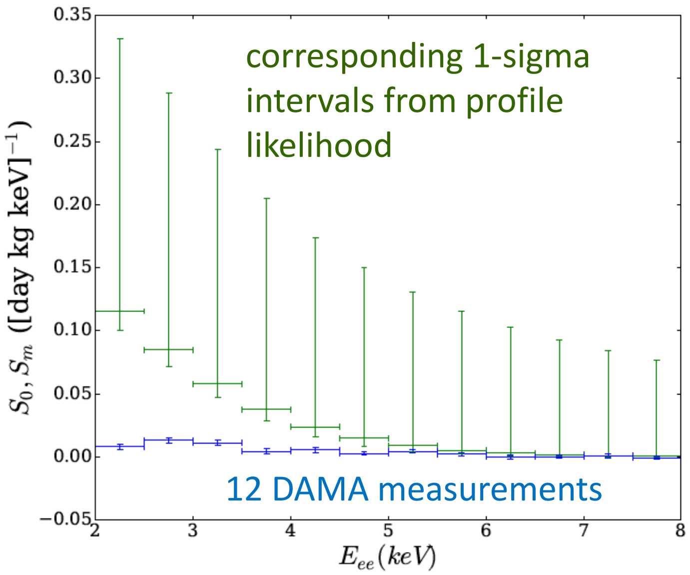


m_χ=10 GeV

$5x10^6$ points Markov chain, 250 independent walkers Metropolis-Hastings sampler



P. Gondolo and S. Scopel, preliminary

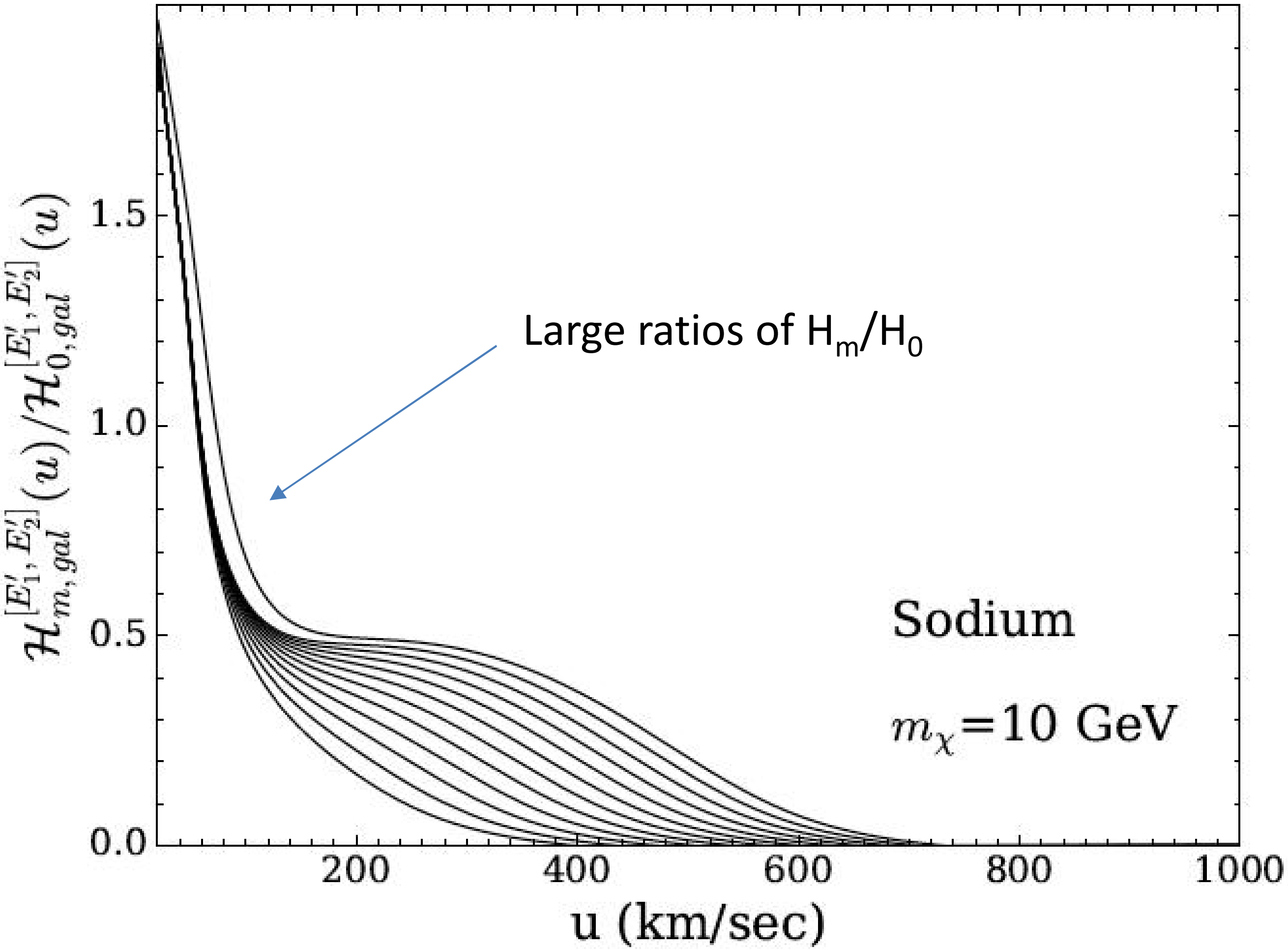


m_χ=15 GeV

5x10^6 points Markov chain, 250 independent walkers Metropolis-Hastings sampler

1-sigma ranges for modulation fractions:
m_χ=5: 0.03<S_m/S_0<0.13; m_χ=10: 0.05<S_m/S_0<0.13; m_χ=20: 0.07<S_m/S_0<0.19

N.B. Non-isotropic distributions can easily predict larger modulation fractions (up to 100 %).
However, also the space of isotropic f(u) contains large modulation solutions, which however are disfavored by the data



Conclusions

• an explanation of the DAMA modulation result (or of other, less statistically significant "excesses") in terms of a WIMP signal is incompatible with the constraints published by other Dark Matter direct detection experiments only if direct-detection data are analyzed with ALL the following assumptions:

1) **spin-dependent** or **isoscalar spin-independent** cross section
2) **Maxwellian** velocity distribution in our Galaxy
3) WIMP **elastic** scattering

All these assumptions are reasonable if for instance the WIMP is a susy neutralino and if the DM particles in our Galaxy are fully thermalized.

•However, without any hint from the LHC about the underlying fundamental physics and without a detailed knowledge of the merger history of our Galaxy it appears safer to adopt a bottom-up layman approach. This includes:

1) using non-relativistic effective theory which introduces new response functions with explicit dependence on the transferred momentum and the WIMP incoming velocity
2) factorizing the halo-function dependence
3) allowing for inelastic scattering
4) allowing for isovector couplings

• In this way a much wider parameter space opens up.

• First explorations show that indeed compatibility between excesses and constraints can be achieved → **full correlation with indirect signals and relic abundance needs still to be worked out**

• **"Proofs of concept" (but if by chance you have a nice model for spin-dependent Inelastic Dark Matter that couples only to protons it works just fine for DAMA)**

- New methods using *representation theorems for distribution functions* allow to get intervals on unknown quantities for the most general halo function → for instance, in this way it is possible to get info on the average rate knowing only the modulated fractions
- Given a signal, strictly speaking halo-independent methods without *any* assumption on the velocity distribution f(v) can only yield a lower bound on the interaction cross section → an upper bound on σ requires some prior assumptions on the f(v)