Non-Relativistic limits

Joaquim Gomis

Universitat Barcelona

IFT

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Movitation

- Non-Relativistic gravities
- Non-Relativistic holography

Points to be addressed

- Non-Relativistic limit (NR) is not unique
- NR limit of particles, string in a Newton-Cartan (NC) background
- NR limit of 3d Chern-Simons gravities

NR limit of a massive relativistic particle

$$L = -m\sqrt{-\dot{x}^2}$$

NR limit $(c=1)$

$$\begin{cases} x^0 = \lambda t \\ m = \lambda M \end{cases} \qquad \lambda \to \infty \qquad p_0 = -E/\lambda$$

$$L = -\underset{\uparrow}{M\lambda^2 \dot{t}} + \frac{1}{2} M \frac{\dot{\vec{x}}^2}{\dot{t}} + O\left(\frac{1}{\lambda^2}\right)$$

divergent term, total derivative

How to eliminate this term?

Jaume Gomis
H. Ooguri (2000)

Add a coupling to EM field

$$L_\tau = L + L^{extra} = -m\sqrt{-\dot{x}^2} + e A_\mu \dot{x}^\mu$$

if we choose $A_\mu = (M\omega, \vec{0})$, $F_{\mu\nu} = 0$

$$L_\tau = -M\lambda^2 \dot{t} + \frac{1}{2} M \frac{\dot{\vec{x}}^2}{\dot{t}} + M\lambda^2 \dot{t} + O\left(\frac{1}{\omega^2}\right)$$

$$\xrightarrow[\lambda \to \infty]{} \frac{1}{2} M \frac{\dot{\vec{x}}^2}{\dot{t}}$$

The canonical action becomes

$$S = \int d\tau \left[p_\mu \dot{x}^\mu - \frac{e}{2}\left[(p_\mu - e A_\mu)^2 + m^2 \right] \right] \to$$

$$= \int d\tau \left[-E\dot{t} + \vec{p}\,\dot{\vec{x}} - \frac{e}{2}\left(\vec{p}^{\,2} - 2mE\right) \right]$$

Related facts

- Existence of divergent terms
- $U(1)$ gauge field
- Non-trivial EC cohomology of the Galilei group. Bargmann algebra

$$\begin{cases} \delta \vec{x} = \vec{\beta} t \\ \delta \vec{p} = m \vec{\beta} \\ \delta E = \vec{p} \vec{\beta} \end{cases}$$

noether charges

$$\vec{G} = \vec{p} t - m \vec{x}$$

$$\vec{P} = \vec{p}$$

$$\{ G_i, P_j \} = \delta_{ij} \, m \, \vec{1}$$

Z new generator

Massless non-relativistic particle

Souriau (1970)

Consider a relativistic tachyon.

Batlle, Gomis, Hezincescu, Townsend (2017)

Define the NR limit

$$\begin{cases} x^0 = \lambda t \,, \quad p_0 = -\dfrac{E}{\lambda} \\ m = M \end{cases}$$

$$S = \int d\tau \left[p_\mu \dot{x}^\mu - \frac{e}{2} (p^2 - m^2) \right] =$$

Tachyon

$$= \int d\tau \left[-E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2} \left(-\frac{E^2}{c^2} + \vec{p}^2 - M^2 \right) \right]$$

$$\to \int d\tau \left[-E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2} (\vec{p}^2 - M^2) \right]$$

$$\delta \vec{p} = 0 \qquad \vec{G} = \vec{p}\, t$$

$$\delta E = \vec{p}\vec{\beta} \qquad \vec{P} = p$$

$$\{ \vec{G}, \vec{P} \} = 0$$

massless

Equations of motion

$$\begin{cases} \dot{t} = 0 & \text{instantaneous interaction} \\ \dot{\vec{p}} = 0 \\ \dot{\vec{x}} = e\vec{p} \\ \dot{E} = 0 \\ \dot{\pi}_e = -\frac{1}{2}(p^2 - m^2) \approx 0 \end{cases}$$

gauge fixing

$$\begin{cases} x^1 = \tau, \\ p^1 = \sqrt{m^2 - \vec{p}_\perp^{\,2}} \end{cases}$$

physical subspace

$(x_\perp, p_\perp)$ only transverse modes

General properties of NR limit

Batlle, Gomis et al (2017)

Consider a relativistic lagrangian $\mathcal{L}$ invariant under the relativistic transformation δ_R

$$\delta_R \mathcal{L} = d F_R$$

We scale fields and parameters with λ

$$\begin{cases} \mathcal{L} = \lambda^2 \mathcal{L}_2 + \mathcal{L}_0 + \omega^{-2} \mathcal{L}_{-2} + O\left(\frac{1}{\omega^4}\right) \\ \delta_R = \delta_0 + \lambda^{-2} \delta_{-2} + \cdots \\ F_R = \lambda^2 F_2 + F_0 + \lambda^{-2} F_{-2} \end{cases}$$

NR Transformation

The condition of the relativistic symmetry imply the infinite equations

$\lambda^2 \qquad \delta_0 \mathcal{L}_2 = dF_2$

$\lambda^0 \qquad \delta_0 \mathcal{L}_0 + \delta_2 \mathcal{L}_2 = dF_0$

$\lambda^{-2} \qquad \delta_0 \mathcal{L}_2 + \delta_{-2} \mathcal{L}_0$

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Notice that always $\mathcal{L}_2$ is NR invariant. A sufficient condition for $\mathcal{L}_0$ be NR invariant is that $\mathcal{L}_2$ is a total derivative.

If $\mathcal{L}_2$ is not a total derivative we could ask to add to $\mathcal{L}$ + $\mathcal{L}_{extra}$ such the divergent terms calcel. If this is the case the NR lagrangian is

$$\mathcal{L}_{NR} = \mathcal{L}_0 + \mathcal{L}_0^{extra}$$

The non-relativistic limit is not unique

Jaume Gomis Ooguri (2000)

Batlle, Gomis, Not (2017)

Barducci, Casalbuoni, Gomis (2018)

Let us construct

k - NR limits or p-brane NR limits

Consider partitioning the $D+1$ dimensional space time into a k dimensional Minkowskian part and in $(D+1-k)$ dimensional euclidean

$$\alpha, \beta = 0, 1, \ldots k-1 \qquad (-, + \cdots +)$$

$$a, b = k, \ldots D \qquad (+, +, \cdots +)$$

$ISO(1, k-1)$ $\quad M_{\alpha\beta},\ P_\alpha,\quad \alpha,\beta = 0,1,\dots,k-1$

$ISO(D+1-k)$ $\quad M_{ab},\ P_a,\quad a,b = k \dots D$

The generators of whole Poincaré group are

$ISO(1,D)$: $M_{\alpha\beta}\ M_{ab},\ P_\alpha,\ P_a,\quad M_{\alpha b} =: B_{\alpha b}$

galilei $\begin{cases} P_a = \lambda \tilde{P}_a \\ B_{\alpha a} = \frac{1}{\lambda}\tilde{B}_{\alpha a} \end{cases}$ $\quad \left(x_a = \frac{1}{\lambda}\tilde{x}_a\right)$

$\begin{cases} P_\alpha = \frac{1}{\lambda} P_\alpha \\ B_{\alpha a} = \frac{1}{\lambda}\tilde{B}_{\alpha a} \end{cases}$ $\quad \left(x^\alpha = \lambda \tilde{x}^\alpha\right)$

<u>Examples</u>

$k=1$ - particle limit of Nambu-Goto

Batlle, Gomis, Not [2017]

$$S = -wT\int d^2\sigma \left[\sqrt{(\dot{t}\vec{x}' - t'\dot{\vec{x}})^2} \right. +$$

$$+\frac{1}{2\lambda}\frac{(\dot{\vec{x}}\,\vec{x}')}{\sqrt{(\dot{t}\,\vec{x}'-t'\,\dot{\vec{x}})^2}}+\mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

If we redefine the Tension

$$T^{NR}=\omega T$$

$$S^{NR}_{NG}=-T^{NR}\int d^2\sigma\left[\sqrt{[\dot{t}\,\vec{x}'-t'\,\dot{\vec{x}}]^2}\right]$$

The canonical is

$$S^{NR}_{NG}=-T^{NR}\int d^2\sigma\Big[-E\dot{t}+\vec{p}\,\dot{\vec{x}}-\frac{\mu}{2}\left[\vec{p}^{\,2}-T^2t'^2\right]-e\left[-Et'+\vec{p}\,\vec{x}'\right]\Big]$$

Equations of motion, gauge $\mu=1$, $\lambda=0$

$$\begin{cases}\dot{E}=T^{NR}t''\\ \dot{t}=0\\ \dot{\vec{x}}=\frac{1}{T^{NR}}\vec{p}\\ \dot{\vec{p}}=0\end{cases}$$

Collection of massless particles

K = 2, string NR limit of NG

Gomis
Ooguri (2000)

$$S = -T \int d^2\sigma \sqrt{(\dot{X} X')^2 - \dot{X}^2 X'^2} =$$

Total derivative

$$\simeq -w^2 T \int d^2\sigma \sqrt{-\det \hat{G}_{\alpha\beta}}$$
$$- \frac{T}{2} \int d^2\sigma \sqrt{-\det \hat{G}} \, (\hat{G}^{-1})^{\gamma\delta} \tilde{G}_{\gamma\delta}$$
$$+ O\left(\frac{1}{w^2}\right)$$

where $X = (X^\mu, Y^i)$

$$\hat{G}_{\alpha\beta} = \partial_\alpha X^\mu \, \partial_\beta X_\mu$$

$$\tilde{G}_{\alpha\beta} = \partial_\alpha Y^i \partial_\beta Y_i$$

If we introduce the coupling to a B field $B = w^2 \epsilon_{\mu\nu} dx^\mu \wedge dx^\nu$

$$S^{NR} = -\frac{T}{2}\int d^2\sigma \sqrt{-\det \hat{G}}\,(\hat{G}^{-1})^{\gamma\delta}\,\hat{G}_{\gamma\delta}$$

Kamimura,
Gomis, Townsend
2004
(Gomis)^2 Kamimura

Stringy Galilei Transformations

$$\begin{cases} \delta x^\mu = \epsilon^\mu + \omega^\mu{}_\nu x^\nu \\ \delta y^i = \epsilon^i + \omega^i{}_j y^j + \omega^i{}_\mu x^\mu \end{cases}$$

shift symmetries
(Galileon for p=3)

Noether charges

$$[P_a, B_{b\mu}] = \delta_{ab} Z_\mu$$

$$[B_{a\mu}, B_{b\nu}] = \delta_{ab} Z_{\mu\nu}$$

Extended Stringy Galilei algebra

Gauging Bargmann algebra

Consider the gauge field

de Pietri, Lusanna, Pauri (1994)

$$A_\mu = \tau_\mu H + e_\mu{}^i P_i + \omega_\mu{}^i G_i + \tfrac{1}{2}\omega_\mu{}^{ij} M_{ij}$$

$$F_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda]$$

Andringa, Bergshoeff, Panda, de Roo (2011)

H	τ_μ	$R_{\mu\nu}[H]$
P_i	$\omega_\mu{}^i$	$R_{\mu\nu}{}^i[P]$
M^{ij}	$\omega_\mu{}^{ij}$	$R_{\mu\nu}{}^{ij}[M]$
G^i	$\omega_\mu{}^i$	$R_{\mu\nu}{}^i(G)$
M	m_μ	$R_{\mu\nu}[Z]$

$$R_{\mu\nu}(H) = 2\,\partial_{[\mu}\tau_{\nu]} \longrightarrow \begin{array}{c}\text{Torsionless}\\ \tau = df\end{array}$$

$$R_{\mu\nu}{}^{i}(P) = 2\left(D_{[\mu} e_{\nu]}{}^{i} + \omega_{[\mu}{}^{i}\tau_{\nu]}\right)$$

$$R_{\mu\nu}{}^{ij}(M) = 2\left(\partial_{[\mu}\omega_{\nu]}{}^{ij} - \omega_{[\mu}{}^{ki}\omega_{\nu]}{}^{j}{}_{k}\right)$$

$$R_{\mu\nu}{}^{i}(G) = 2\,D_{[\mu}\omega_{\nu]}{}^{i}$$

$$R_{\mu\nu}(M) = 2\left(\partial_{[\mu} m_{\nu]} - e_{[\mu}{}^{j}\omega_{\nu]}{}^{j}\right)$$

Properties of τ_μ, $e_\mu{}^i$

$$\begin{cases} e_\mu{}^{i} e^{\mu}{}_{j} = \delta^{i}_{j} \\ e_\mu{}^{i} e^{\nu}{}_{i} + \tau_\mu \tau^\nu = \delta^{\nu}_{\mu} \\ \tau_\mu \tau^\mu = 1 \\ \tau_\mu e^{\mu}{}_{i} = \tau^\mu e_\mu{}^{i} = 0 \end{cases}$$

A Newton-Cartan structure is characterized by

$$(\tau_\mu, h^{\mu\nu}) \qquad h^{\mu\nu} = e^{\mu}{}_{i}\, e^{\nu}{}_{j}\, \delta^{ij}$$

such ∇ connection

$$\nabla_\mu \tau_\nu = 0$$
$$\nabla_\mu h^{e\sigma} = 0$$

If we impose

$$R_{\mu\nu}(H) = R_{\mu\nu}(P^i) = R_{\mu\nu}[Z]$$

conventional constraints

$$w_{\mu[0i]} = \frac{1}{2}\Big[\tau^\mu(\partial_\mu e_{i\nu} - \partial_\nu e_{i\mu}) -$$
$$- \tau^\nu e_i^e e_{j\mu}(\partial_\nu e_e^j - \partial_\nu e_\nu^j) -$$
$$- \frac{1}{2} e_i^\nu (\partial_\mu m_\nu - \partial_\nu m_\mu)$$

$$w_{\mu[ij]} = \frac{1}{2}\Big[(\partial_\mu e_{i\nu} - \partial_\nu e_{i\mu}) e_j^\nu$$
$$- e_i^\nu e_j^e \Big[e_{k\mu}(\partial_\nu e_e^k - \partial_e e_\nu^k) -$$
$$- \tfrac{1}{2}\tau_\mu (\partial_\nu \tilde{m}_e - \partial_e m_\nu) -$$
$$- (\partial_\mu e_{j\nu} - \partial_\nu e_{j\mu}) e_i^\nu$$

Particle in a NC background as a non-relativistic limit

Starting point a relativistic in a generic curved background

Bergshoeff, Rosseel, Zojer (2015)

$$S = -m \int d\tau \left[-\eta_{AB} \dot{x}_\mu E_\mu^A \dot{x}_\nu E_\nu^B - \dot{x}^\mu M_\mu \right] \qquad dM = 0$$

$$\begin{cases} P_0 = \frac{1}{2\omega} H + \omega Z \\ Y = \frac{1}{2\omega} H - \omega Z \\ M_{i0} = \omega G_{i0} \end{cases}$$

Poincaré $\otimes U(1)$

$\downarrow$

Bargmann

$[P_i, G_j] = \delta_{ij} Z$

$$A_\mu = E_\mu^A P_A + \cdots = \tau_\mu H + e_\mu^i P_i + \cdots$$

$$\Rightarrow \begin{cases} E_\mu^e = \lambda \tau_\mu + \dfrac{1}{2\lambda} m_\mu \\ M_\mu = \lambda \tau_\mu - \dfrac{1}{2\lambda} m_\mu \\ E_\mu^i = e_\mu^i - \dfrac{1}{2\lambda^2} \tau^\mu e_i^e m_e + \cdots \end{cases}$$

The NR limit we do not touch the coordinates but the gauge fields together with

$$m = \lambda M$$

we get

$$L = m \left[\frac{1}{2} \frac{h_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{\tau_\mu \dot{x}^\mu} - m_\mu \dot{x}^\mu \right]$$

Kuchar [1980]

The divergent cancel with the contribution coming from M

3d NR relativistic CS gravities

Aviles, Gomis
Hidalgo, Zanelli
(2018)

Maxwell algebra in 3d Schrader (1973)

$$[J_A, J_B] = \epsilon_{ABC} J^C, \quad [J_A, P_B] = \epsilon_{ABC} P^C$$

$$[J_A, Z_B] = \epsilon_{ABC} Z^C, \quad [P_A, P_B] = \epsilon_{ABC} Z^C$$

Invariant non-degenerate pairing

$$\langle J_A, Z_B \rangle = \langle P_A, P_B \rangle = \alpha_1 h_{AB}$$

$$\langle J_A, J_B \rangle = \alpha_2 h_{AB}$$

$$\langle P_A, J_B \rangle = \alpha_3 h_{AB}$$

$$A = E^B P_B + W^B J_B + K^B Z_B$$

CS Action

$$S_R = \int \langle A \wedge dA + \frac{1}{3} A \wedge [A, A] \rangle$$

$$= \int \{ \alpha_1 \, 2K^A R_A(w) + E^A T_A) +$$

$$+ \alpha_2 (w^A d w_A + \frac{1}{3} \epsilon_{ABC} w^A w^B w^C)$$

$$+ \alpha_3 \; E^A R_A(w) \}$$

the curvatures are

$$R^A(w) = dw^A - \frac{1}{2} \epsilon^{ABC} w_B w_C$$

$$R^A(E) = D_w E^A = T^A$$

$$R^A(K) = D_w K^A - \frac{1}{2} \epsilon^{ABC} E_B E_C$$

where the covariante derivative

$$D_w \Phi^A =: d\Phi^A - \epsilon^{ABC} w_B \Phi_C$$

U(1) Enlargements

$$A = E^B P_B + W^B J_B + K^B Z_B + h Y_1 + s Y_2$$

$$\langle Y_2, Y_1 \rangle = \alpha_4, \quad \langle Y_1, Y_2 \rangle = \alpha_5$$

$$\langle Y_2, Y_2 \rangle = 0$$

$$S_\Lambda = \int \{ \alpha_1 (2 K^A R_A(W) + E^A T_A) +$$

$$+ \alpha_2 (W^A dW_A + \frac{1}{3} \epsilon_{ABC} W^A W^B W^C)$$

$$+ \alpha_3 E^A R_A(W) + \alpha_4 h dh + 2\alpha_5 h dS$$

<u>Particle Non-Relativistic limit</u>

$$\begin{cases} P_0 = \dfrac{\tilde{H}}{2\xi} + \xi \tilde{M} \\ P_a = \tilde{P}_a \end{cases} \qquad \begin{cases} J_0 = \dfrac{\tilde{J}}{2} + \xi^2 S \\ J_a = \xi \tilde{G}_a \end{cases}$$

$$\xi \to \infty$$

$$\begin{cases} Z_a = \frac{1}{\xi}\tilde{Z}_a \\ Z_0 = \tilde{Z} \end{cases} \qquad \begin{cases} Y_1 = \frac{H}{2\xi} - \xi\hat{M} \\ Y_2 = \frac{J}{2} - \xi^2 S \end{cases}$$

Magnetic limit $\quad$ Le Bellac
Levy-Leblond
(1973)

$$Z_0 \gg Z_a$$

$$[\tilde{G}_a, \tilde{P}_b] = -\epsilon_{ab}\tilde{M} \qquad [J, Z_a] = \epsilon_{ab} Z_b$$

$$[\tilde{H}, \tilde{G}_a] = \epsilon_{ab}\tilde{P}_b \qquad [\tilde{G}_a, Z_b] = \epsilon_{ab}\tilde{Z}_b$$

$$[\tilde{J}, \tilde{P}_a] = \epsilon_{ab}\tilde{P}_b \qquad [\tilde{H}, \tilde{P}_a] = \epsilon_{ab}\tilde{Z}_b$$

$$[\tilde{G}_a, \tilde{G}_b] = -\epsilon_{ab}\tilde{S} \qquad [\tilde{P}_a, \tilde{P}_b] = -\epsilon_{ab}\tilde{Z}$$

$$[J, G_a] = \epsilon_{ab}\tilde{G}_b$$

invariant degenerate pairing

$$\langle \tilde{M}, \tilde{H} \rangle = \langle \tilde{J}, \tilde{Z} \rangle = -\tilde{\alpha}_1$$

$$\langle \tilde{P}_a, \tilde{P}_b \rangle = \langle \tilde{G}_a, \tilde{Z}_b \rangle = \tilde{\alpha}_1 \delta_{ab}$$

$$\langle \tilde{H}, \tilde{S} \rangle = \langle \tilde{M}, \tilde{J} \rangle = -\tilde{\alpha}_3$$

$$\langle \tilde{G}_a, \tilde{P}_b \rangle = \tilde{\alpha}_3 \delta_{ab}$$

$$S_{NR} = \int \tilde{\alpha}_1 \Big[-2k\,R(w) + 2k^a R_a w^b$$
$$+ e^a R_a(e^b) - \tau R(m) - m R(\tau) \Big]$$
$$- \tilde{\alpha}_2\, w\, R(w) +$$
$$+ \alpha_3 \left[e^a R_a(w^b) - \tau R(s) - m R(w) \right]$$

$$0 = R(\tau) = R(w) = R_a(w^b) = R_a(e^b) = R_a(k^a)$$
$$R(k) = \left(\frac{\tilde{\alpha}_3}{\tilde{\alpha}_1}\right)^2 R(s) = -\frac{\tilde{\alpha}_3}{\tilde{\alpha}_1} R(m)$$

If we put $S=0$, we can get a non-degenerate invariant pairing

"Electric" NR Maxwell gravity

Consider Maxwell $\times U(1)^3$

$$A = E^B P_B + W^B J_B + K^B Z_B + $$
$$+ M Y_1 + S Y_2 + T Y_3$$

$$\begin{cases} \langle Y_1, Y_1 \rangle = \langle Y_2, Y_3 \rangle = \alpha_1 \\ \langle Y_2, Y_2 \rangle = \alpha_2 \\ \langle Y_1, Y_2 \rangle = \alpha_3 \end{cases}$$

NR limit

$$P_0 = \frac{\tilde{H}}{2\varepsilon} + \varepsilon M, \quad P_a = \tilde{P}_a, \quad Y_1 = \frac{\tilde{H}}{2\varepsilon} - \varepsilon \tilde{M}$$

$$J_0 = \frac{\tilde{J}}{2} + \varepsilon^2 \tilde{S}, \quad J_a = \varepsilon \tilde{G}_a, \quad Y_2 = \frac{\tilde{J}}{2} - \varepsilon^2 \tilde{S}$$

$$Z_0 = \frac{\tilde{Z}}{2\varepsilon} + \tilde{T}, \quad Z_a = \frac{\tilde{Z}_a}{\varepsilon}, \quad Y_3 = \frac{\tilde{Z}}{2\varepsilon^2} - \tilde{T}$$

$$Z_a \gg Z_0$$

"Electric" Maxwell algebra

$$[\tilde{G}_a, \tilde{P}_b] = -\epsilon_{ab} \tilde{M}, \qquad [\tilde{J}, \tilde{Z}_a] = \epsilon_{ab} \tilde{Z}_b$$

$$[\tilde{H}, \tilde{G}_a] = \epsilon_{ab} \tilde{P}_b \qquad [G_a, Z_b] = -\epsilon_{ab} \tilde{T}$$

$$[\tilde{J}, \tilde{P}_a] = \epsilon_{ab} \tilde{P}_b \qquad [G_a, \tilde{Z}] = -\epsilon_{ab} \tilde{Z}_b$$

$$[\tilde{G}_a, \tilde{G}_b] = -\epsilon_{ab} \tilde{S} \qquad [H, P_a] = \epsilon_{ab} \tilde{Z}_b$$

$$[\tilde{J}, \tilde{G}_a] = \epsilon_{ab} \tilde{G}_b \qquad [\tilde{P}_a, \tilde{P}_b] = -\epsilon_{ab} \tilde{T}$$

Non-degenerate invariant pairing

$$\langle \tilde{H}, \tilde{M} \rangle = \langle \tilde{J}, \tilde{T} \rangle = -\tilde{\alpha}_1$$

$$\langle P_a, P_b \rangle = \langle \tilde{G}_a, \tilde{Z}_b \rangle = \tilde{\alpha}_1 \delta_{ab}$$

$$\langle \tilde{J}, \tilde{S} \rangle = -\tilde{\alpha}_2$$

$$\langle \tilde{G}_a, \tilde{G}_b \rangle = \tilde{\alpha}_2 \delta_{ab}$$

$$\langle \tilde{H}, \tilde{S} \rangle = \langle \tilde{J}, \tilde{M} \rangle = -\tilde{\alpha}_3$$

$$\langle \tilde{G}_a, \tilde{P}_b \rangle = \tilde{\alpha}_3 \delta_{ab}$$

$$S_{NR} = \int \tilde{\alpha}_1 \left[2 k_a R^a(w^b) + e_a R^a(e^b) - 2 s R(k) - 2 t R(t) \right] +$$

$$-\alpha_2 \left[-2 s R(w) + w_a R^a(w^b) \right]$$

$$+ \tilde{\alpha}_3 \left[e^a R_a(w^b) - \tau R(s) - m R(w) \right]$$

Conclusions

- The non-relativistic limit is not unique
- In order to get NR theories from relativistic ones we should cancel infinities
- Construction of NR gravities in 4d or more dimensions is difficult
- CS gravities theories can be constructed. Not unique answer
- Symmetry of NR theories is involved. Role of Free differential algebras

Kleinschmidt
Palmkvist
JG (2018) (2018)
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