Theory status of the muon g-2
and
experimental prospects

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Outline of Talk:

- ❖Introduction
- ❖The Muon $g-2$ Experiments
- ❖Standard Model Prediction for a_μ
- ❖Evaluation of $a_\mu^{\rm had}$
- ❖About the Hadronic Light-by-Light Scattering Contribution
- ❖Theory vs Experiment; do we see New Physics?
- ❖Summary and Outlook

Next Generation Muon (g-2)

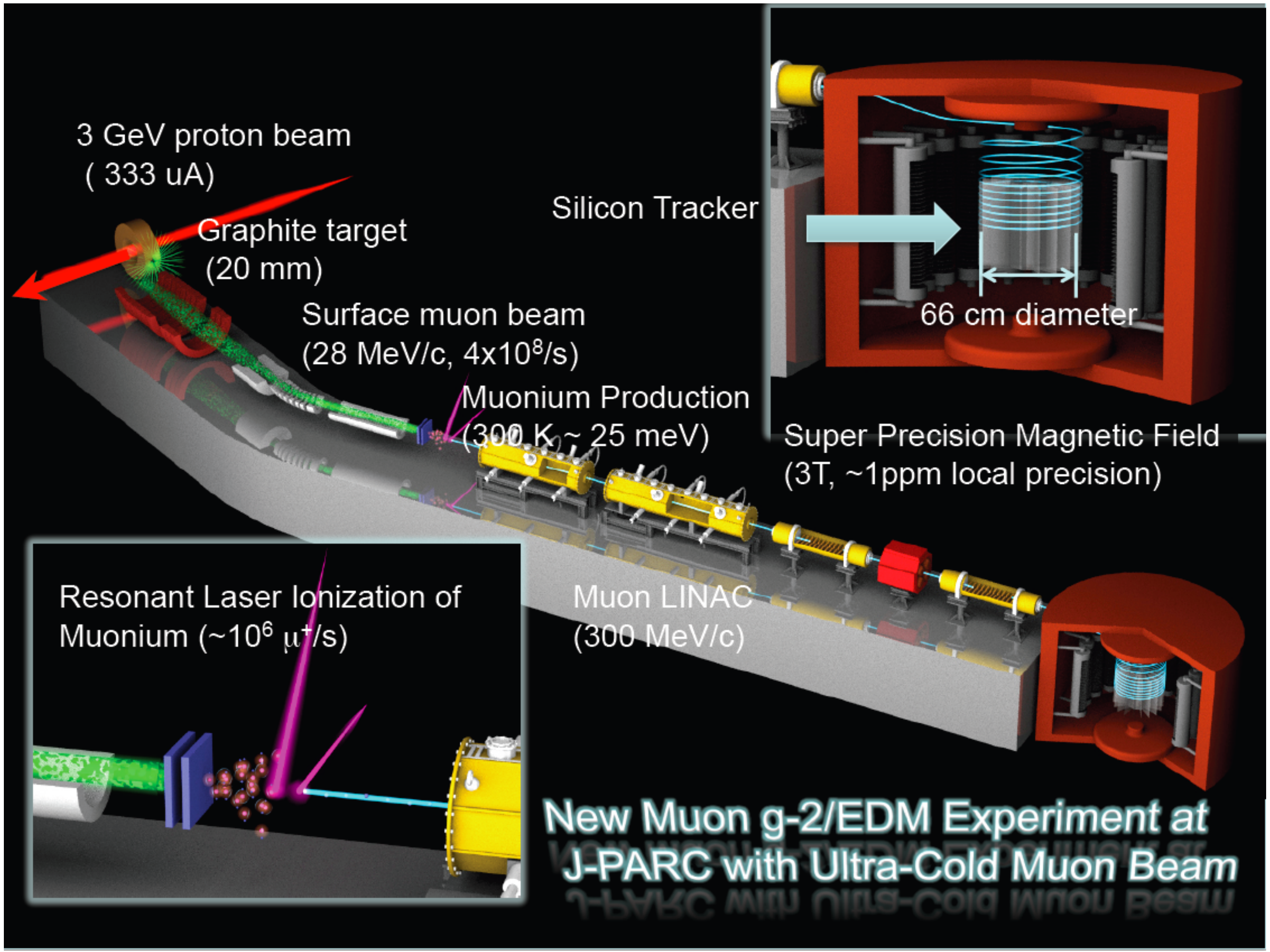
Goal $\delta a_\mu \sim 140$ ppb

David Hertzog, University of Washington
for the E989 Muon (g-2) Collaboration

A novel approach: J-PARC g-2/EDM with Ultra-Cold Muon Beam

vs. Ultra-Hot Muon Beam (CERN/BNL/FNAL)

very different systematics

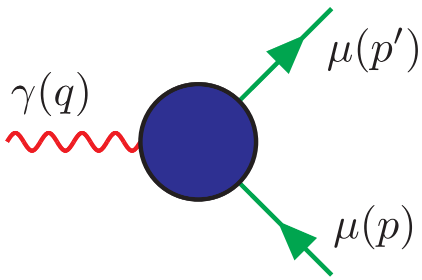


Introduction

Particle with spin $\vec{s}$ $\Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} \; ; \quad g_\mu = 2\,(1 + a_\mu)$$

Dirac: $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly



Electromagnetic Lepton Vertex

$$= (-\mathrm{i}e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu} F_2(q^2)\right] u(p)$$

$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

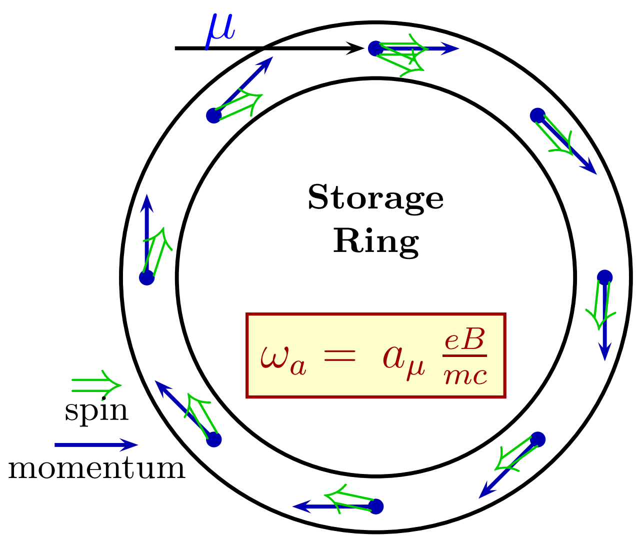
the simplest object
you can think of
in the static limit

a_μ responsible for the Larmor (spin) precession $\Rightarrow$need polarized muons orbiting

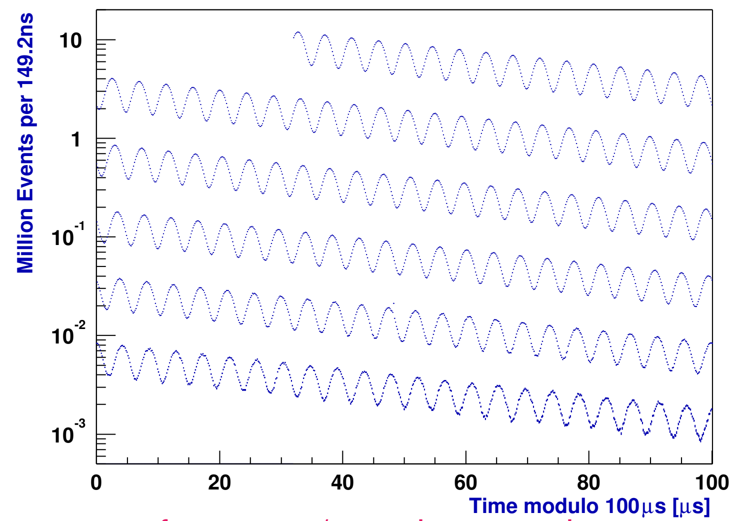
Shoot protons on target producing pions which decay by P violating weak process

$$\pi^+ \to \mu^+ \nu_\mu \; ; \quad \mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$



actual precession × 2 ~ 12′/circle



frequency/counting experiment

Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy ~ 3.1 GeV

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]^{E\sim 3.1\,\mathrm{GeV}}_{\text{at "magic } \gamma\text{"}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

CERN, BNL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

Crucial: 3.1 GeV muons live time in labe frame $\gamma\tau_\mu$ 29 times longer!

$$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10} \text{ BNL updated}$$

To come – :

New muon $g-2$ experiments at Fermilab and J-PARC: improve error by factor 4

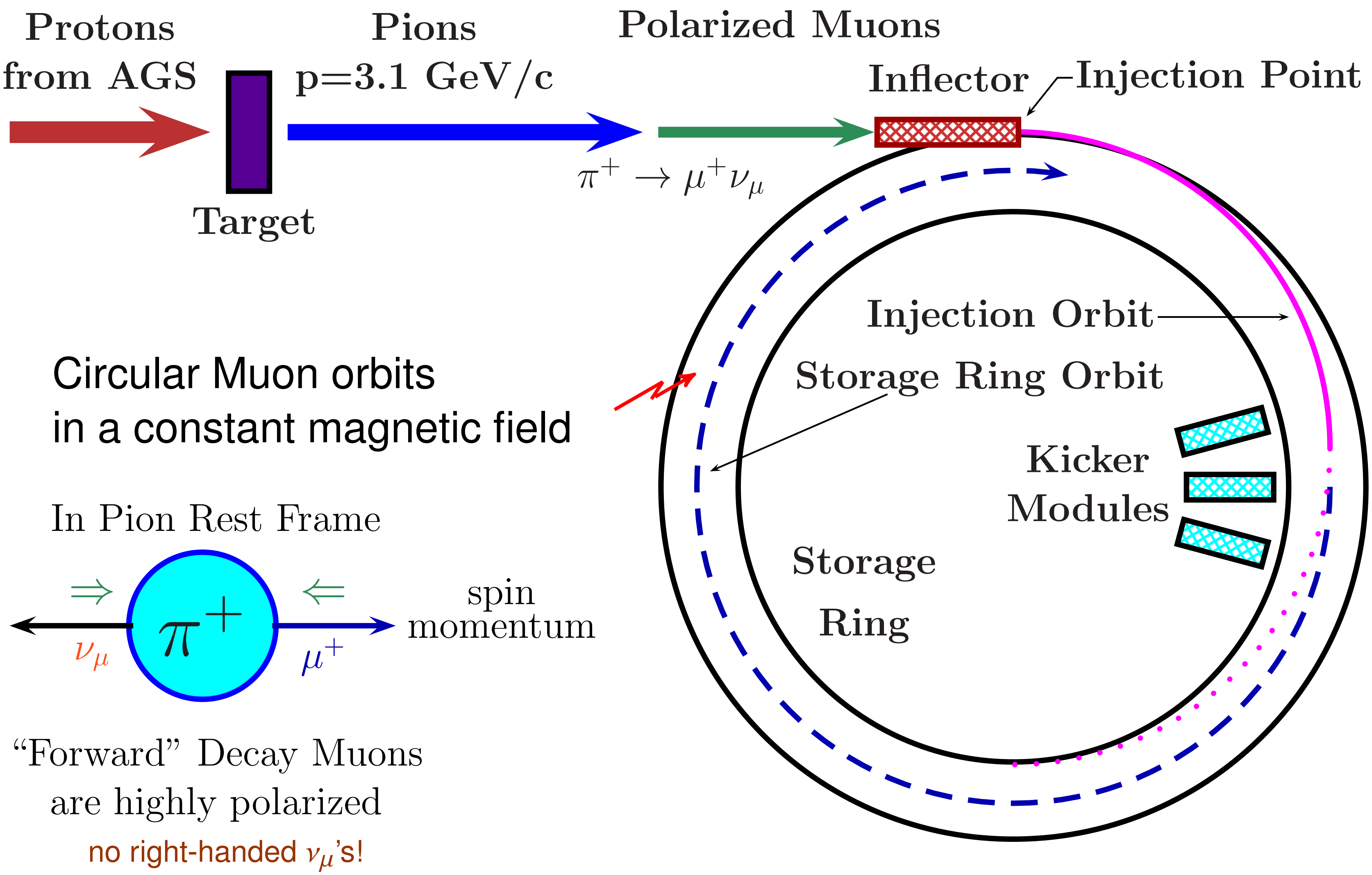
$\Rightarrow \quad \Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the} = 6.1\,\sigma$ if theory as today

Reduction of hadronic uncertainty by factor 2 $\Rightarrow \quad \Delta a_\mu = 10.7\,\sigma$

That's what we hope to achieve!

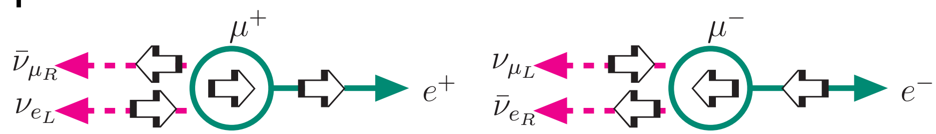
A Case for Challenging the Standard Model

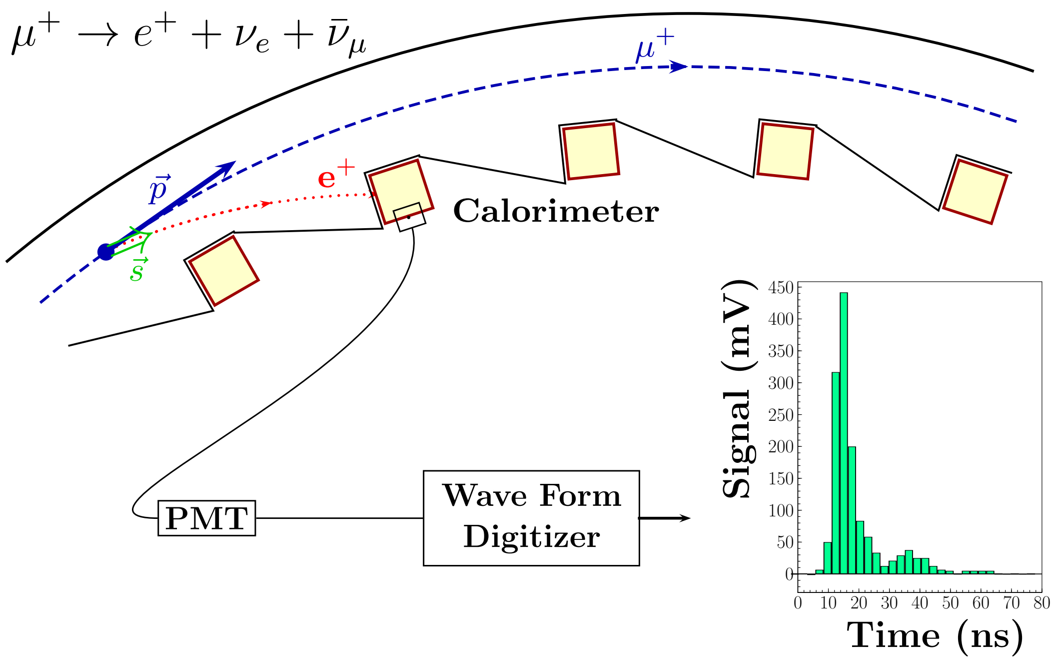
The Principle of the Muon $g-2$ Experiment:



The schematics of muon injection and storage in the $g-2$ ring

Muons are circling in the ring many times before they decay into a positron plus two neutrinos: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. Maximal parity violation implies that the positron is emitted along the spin axis of the muon:





Decay of μ^+ and detection of the emitted e^+ (PMT=Photomultiplier)

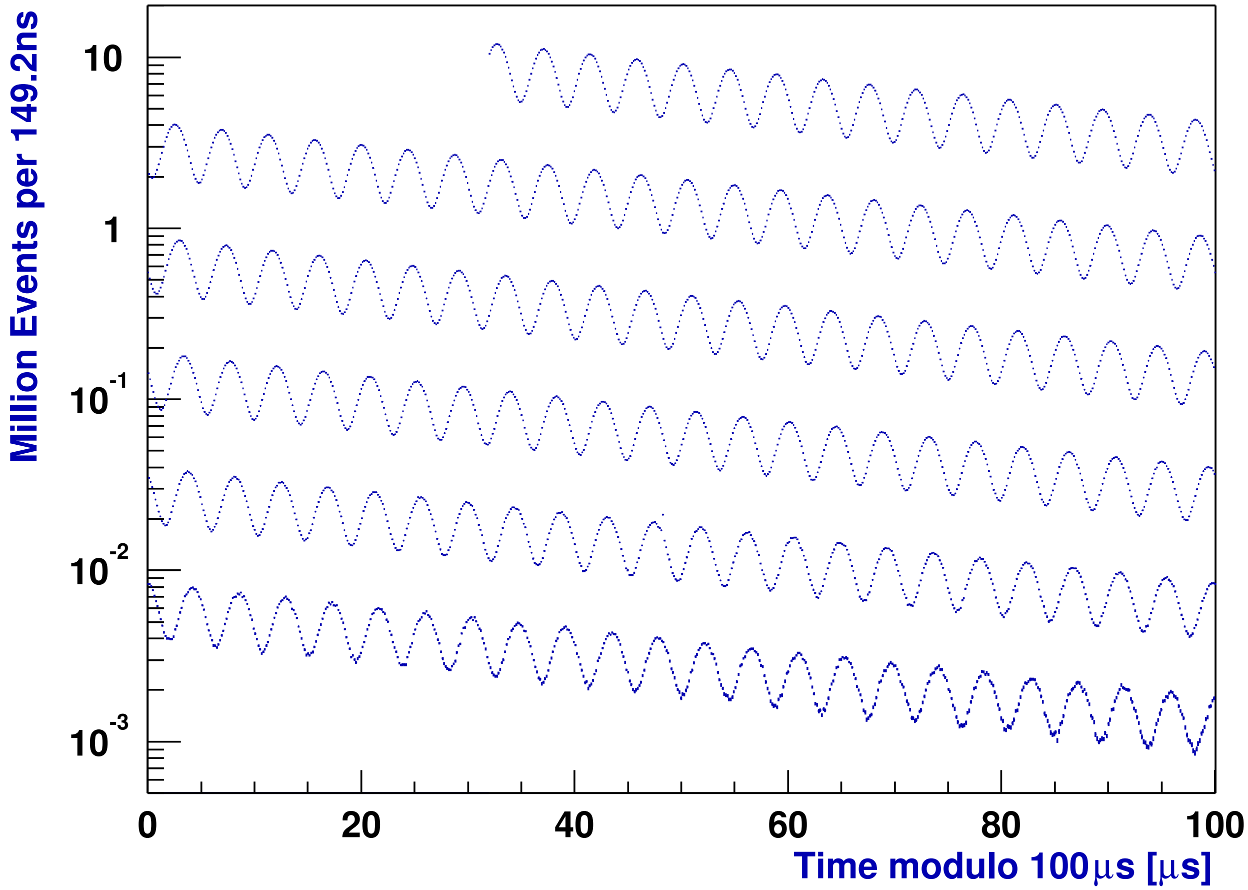
The decay positrons detected by 24 lead/scintillating fiber calorimeters inside the muon storage ring and the measured positron energy provides the direction of the muon spin.

The number of decay positrons with energy greater than E emitted at time t after muons are injected into the storage ring is

$$N(t) = N_0(E)\ \exp\left(-t/\gamma\tau_\mu\right)\ [1 + A(E)\ \sin(\omega_a t + \phi(E))]\ ,$$

– $N_0(E)$ is a normalization factor, – τ_μ the muon life time, – $A(E)$ is the asymmetry factor for positrons of energy greater than E.

- exponential decay modulated by the $g-2$ angular frequency ω_a
- angular frequency ω_a neatly determined from the time distribution of the decay positrons observed with the electromagnetic calorimeters



Distribution of counts versus time for the 3.6 billion decays in the 2001 negative muon data–taking period

5 parameter fit

The Muon $g-2$ experiments

BNL muon storage ring: r= 7.112 meters, aperture of the beam pipe 90 mm, field 1.45 Tesla, momentum of the muon $p_\mu = 3.094$ GeV/c

BNL Result and Update

a_μ measured via a ratio of frequencies (measurement of a_μ and B)

$$B = \frac{\hbar\omega_p}{2\mu_p}\,,\ \omega_a = \frac{e a_\mu}{m_\mu c} B\,,\ \mu_\mu = \left(1 + a_\mu\right) \frac{e\hbar}{2m_\mu c} \Leftrightarrow \mu_\mu = \left(1 + a_\mu\right) \frac{\hbar}{2} \frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu} + 1\right) \frac{\omega_a}{\omega_p} \mu_p \Leftrightarrow$$

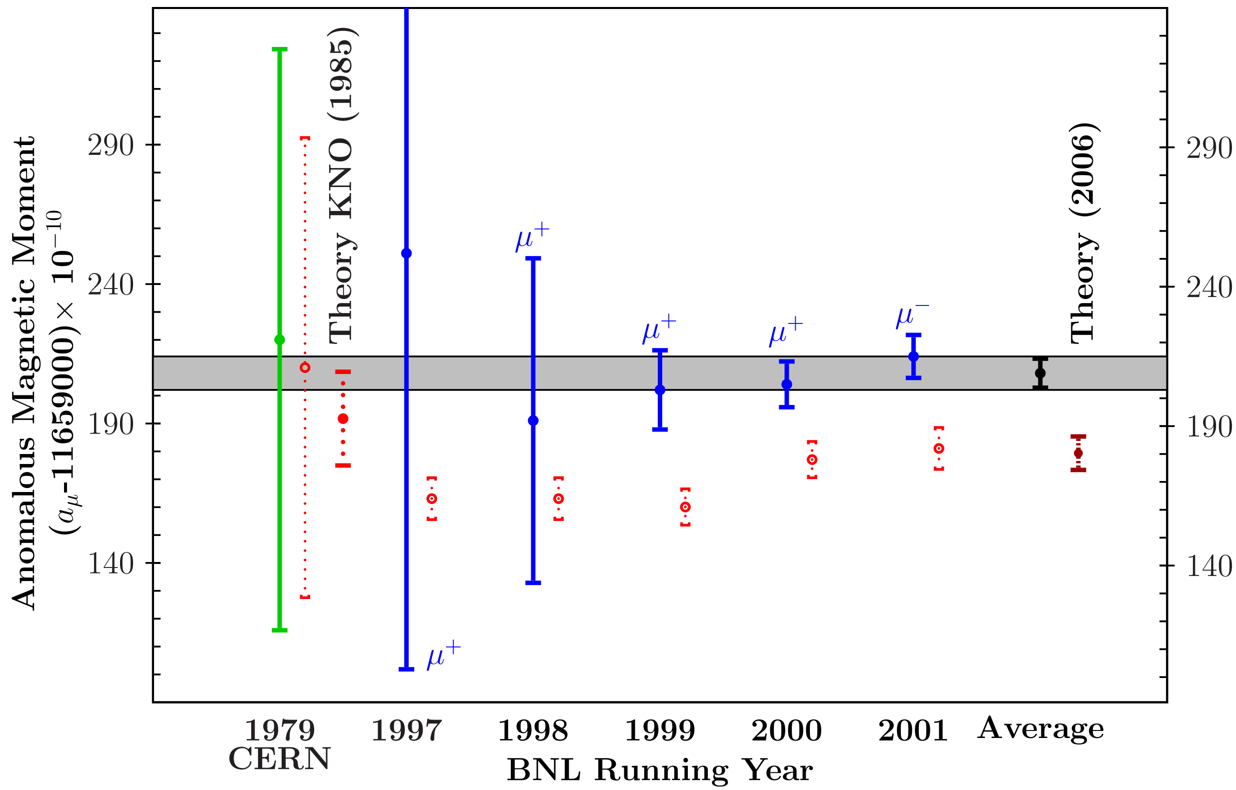
$$\boxed{a_\mu = \frac{\mathcal{R}}{\lambda - \mathcal{R}}}$$ in terms of measurables

- $\overline{\omega}_p = (e/m_\mu)\langle B\rangle$ free proton NMR frequency
- $\mathcal{R} = \omega_a/\overline{\omega}_p$ from E-821
- $\lambda = \omega_L/\overline{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

value used by E-821	$3.18334539(10)$	
new value	$3.183345137(85)$	Mohr et al. RMP 80 (2008) 633

⇒change in a_μ: $+0.92 \times 10^{-10}$ review in RPP2009

$a_\mu^{\rm exp} = (11\,659\,208.9 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$ updated



Results of individual E821 measurements, together with last CERN result and theory values quoted by the experiments

Standard Model Prediction for a_μ: what is new?

$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{Weak}) + a_\mu(\mathrm{LO\ HVP}) + a_\mu(\mathrm{HO\ HVP}) + a_\mu(\mathrm{HLbL})$

- new CODATA values for lepton mass ratios m_μ/m_e, m_μ/m_τ
- spectacular progress by Aoyama, Hayakawa, Kinoshita and Nio on 5–loop QED calculation 12672 diagrams (as well as improved 4–loop results 871 diagrams)
  - $O(\alpha^5)$ electron $g-2$, substantially more precise $\alpha(a_e)$
  - Complete $O(\alpha^5)$ muon $g-2$, settles better the QED part
- QED Contribution

The QED contribution to a_μ has been computed through 5 loops

Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Input:

$a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

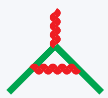
$\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25 \text{ ppb}]$ Aoyama et al 2012

$$a_\mu^{\rm QED} = 116\,584\,718.851 \underbrace{(0.029)}_{\alpha_{\rm inp}} \underbrace{(0.009)}_{m_e/m_\mu} \underbrace{(0.018)}_{\alpha^4} \underbrace{(0.007)}_{\alpha^5}[0.36] \times 10^{-11}$$

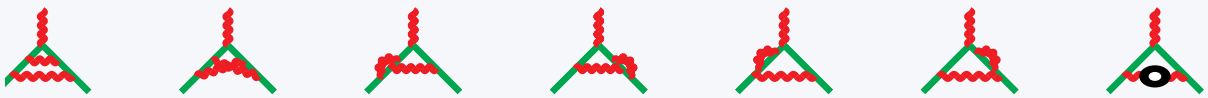
The current uncertainty is well below the $\pm 60 \times 10^{-11}$ experimental error from E821

# n of loops	C_i $[(\alpha/\pi)^n]$	$a_\mu^{\rm QED} \times 10^{11}$
1	+0.5	116140973.289 (43)
2	+0.765 857 426(16)	413217.628 (9)
3	+24.050 509 88(32)	30141.9023 (4)
4	+130.8796(63)	381.008 (18)
5	+753.290(1.04)	5.094 (7)
tot		116584718.851 (0.036)

❶ 1 diagram Schwinger 1948



❷ 7 diagrams Peterman 1957, Sommerfield 1957



❸ 72 diagrams Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

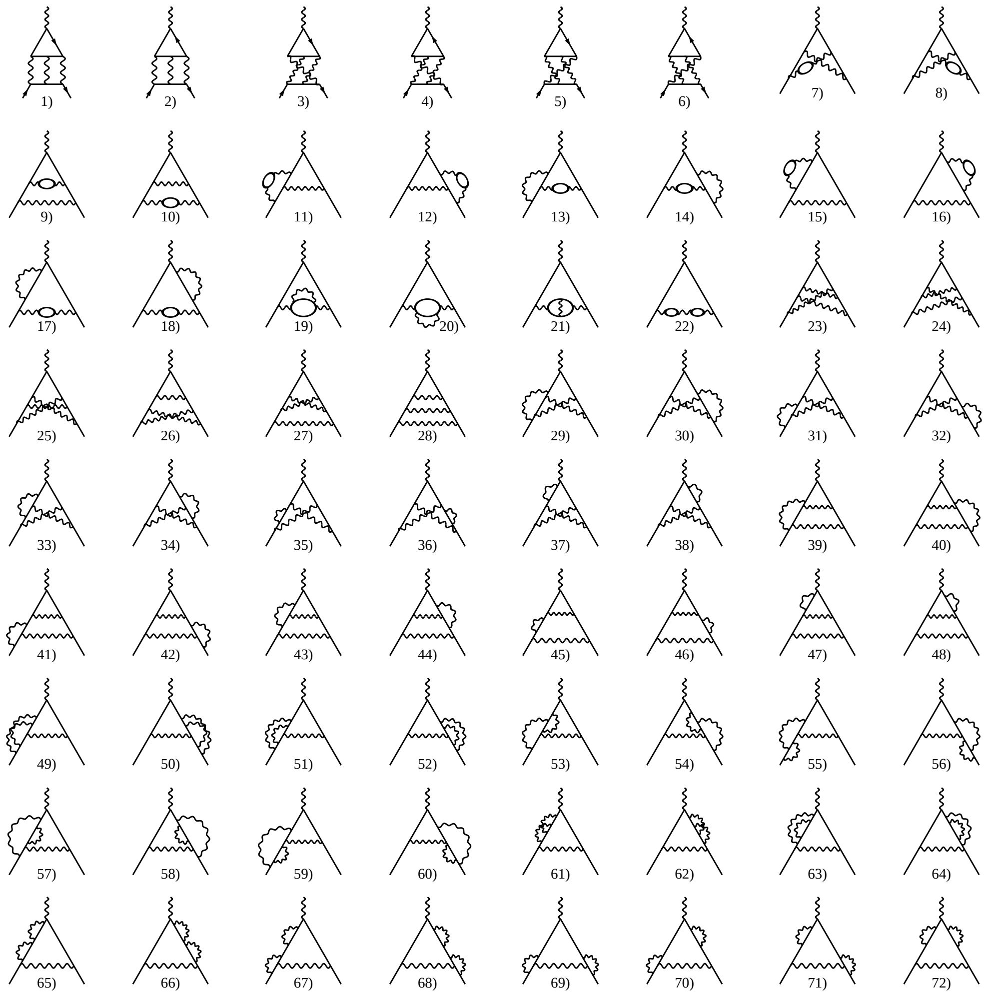
❹ 871 diagrams Kinoshita 1999, Kinoshita, Nio 2004, Ayoama et al. 2009/2012

❺ estimates of leading terms Karshenboim 93, Czarnecki, Marciano 00, Kinoshita, Nio 05

❏ all 12672 diagrams (fully automated numerical) Ayoama et al. 2012

Universal 3–loop contribution:

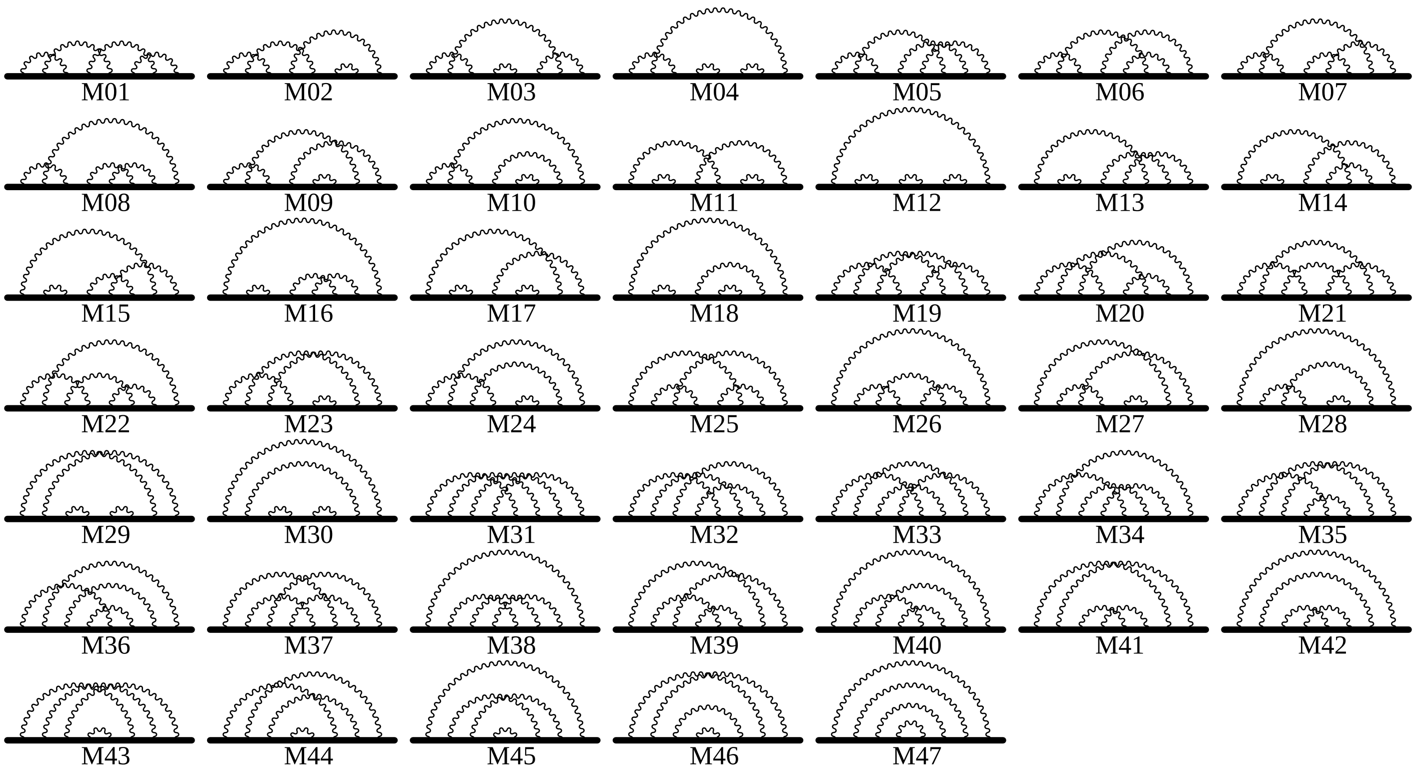
(Remiddi et al., Remiddi, Laporta 1996 [27 years for 72 diagrams])



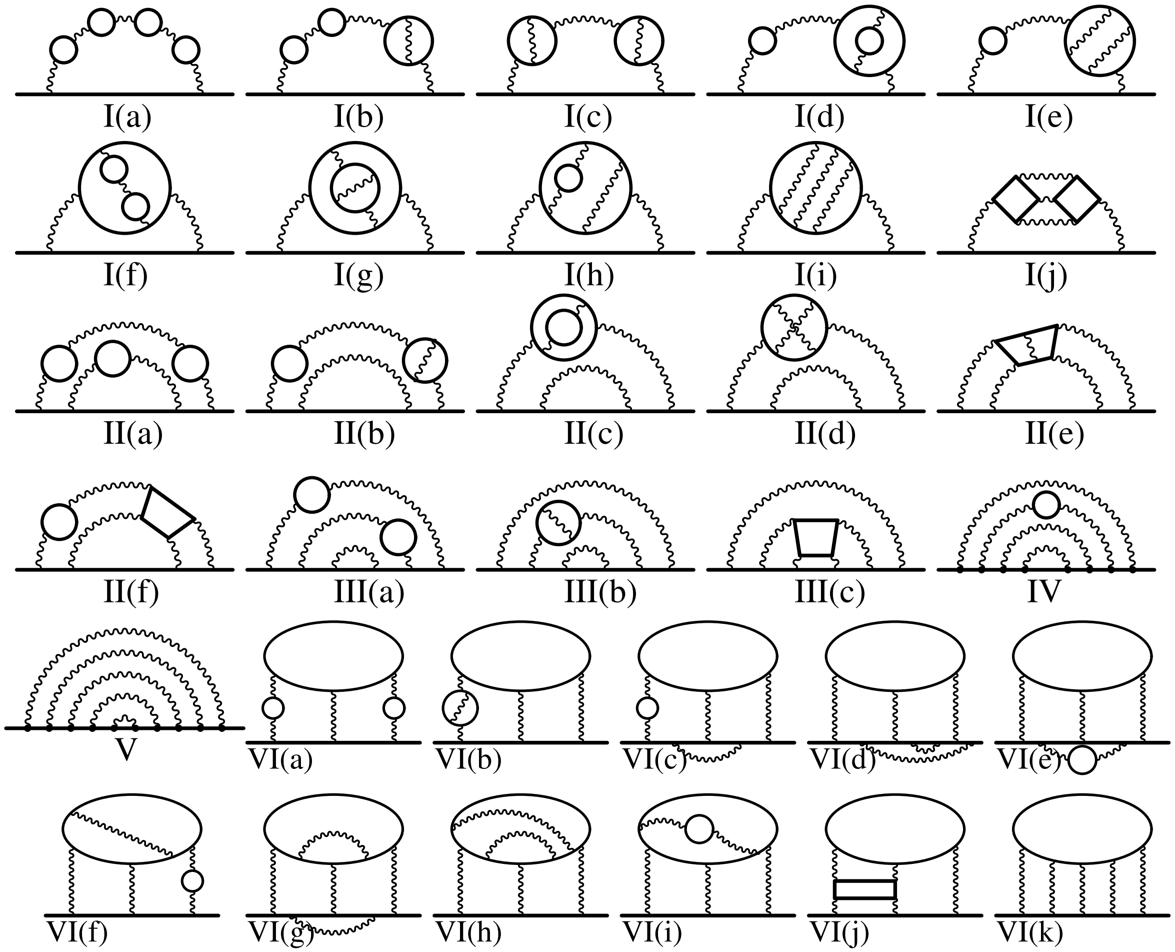
$$a_{\ell\,\mathrm{uni}}^{(6)} = \Big[\frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2\ln 2 + \frac{139}{18}\zeta(3) + \frac{100}{3}\left\{\mathrm{Li}_4(\frac{1}{2}) + \frac{1}{24}\ln^4 2 - \frac{1}{24}\pi^2\ln^2 2\right\} - \frac{239}{2160}\pi^4 + \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5)\Big]\left(\frac{\alpha}{\pi}\right)^3$$

30 years of heroic efforts: demanding 4–loop contribution:

(Kinoshita et al., Aoyama, Hayakawa, Kinoshita and Nio 2007)



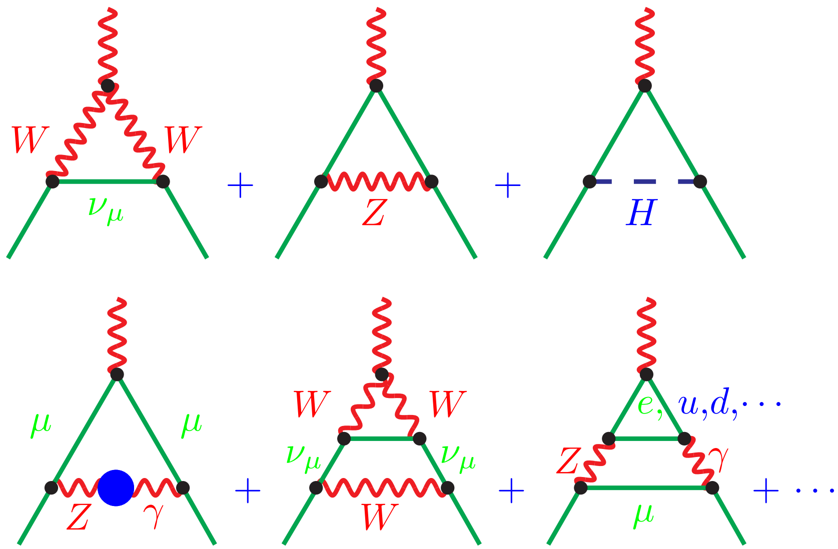
4-loop Group V diagrams. 47 self-energy-like diagrams of $M_{01} - M_{47}$ represent 518 vertex diagrams [by inserting the external photon vertex on the virtual muon lines in all possible ways].



5-loop self-energy-like diagrams. 32 gauge-invariant subsets.

Improved coefficients $a_e^{(8)} = -1.9097(20)\,(\alpha/\pi)^4$ and new $a_e^{(10)} = 9.16(58)\,(\alpha/\pi)^5$ improve $\alpha^{-1}(a_e)$ by factor 4.5 !

❒ Weak contributions



Brodsky, Sullivan 67, ...,
Bardeen, Gastmans, Lautrup 72
Higgs contribution tiny!

$$a_\mu^{\mathrm{weak}(1)} = (194.82 \pm 0.02) \times 10^{-11}$$

Kukhto et al 92
potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$
Peris, Perrottet, de Rafael 95
quark-lepton (triangle anomaly) cancellation
Czarnecki, Krause, Marciano 96

Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05 full 2–loop result

Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\mathrm{weak}} = (154.0 \pm 1.0[\mathrm{had}] \pm 0.3[\mathrm{m_H, m_t, 3-loop}]) \times 10^{-11}$$

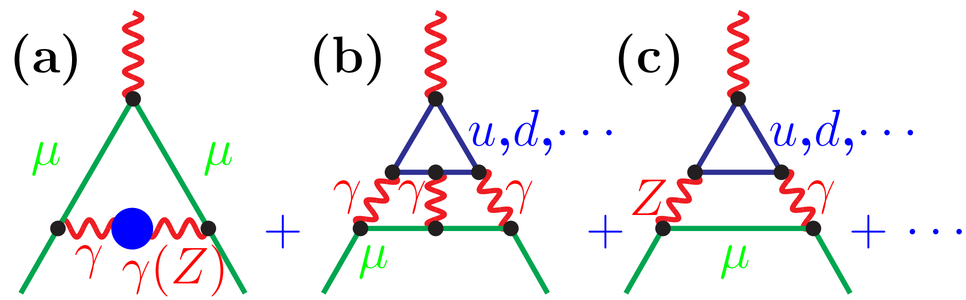
new: m_H known!

(Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano, Vainshtein 02, FJ 12, Gnendiger, Stöckinger, Stöckinger-Kim 13)

❒ Hadronic stuff: the limitation to theory

General problem in electroweak precision physics:

contributions from hadrons (quark loops) at low energy scales



(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$ Light quark loops

(b) Hadronic light-by-light scattering $O(\alpha^3)$ ↓

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$ Hadronic "blobs"

Evaluation of non-perturbative effects:

- ❒ data in conjunction with Dispersion Relations (DR),
- ❒ low energy effective modeling, RLA: HLS, ENJL
- ❒ lattice QCD

(a) HVP via dispersion integral over $e^+e^- \to \mathrm{hadrons}$-data (1 independent amplitude to be determined by one specific data set), HLS, lattice QCD

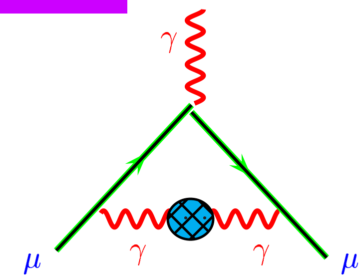
(b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM in accord with chiral structure of QCD), $\gamma\gamma \to \mathrm{hadrons}$-data dispersive approach (28 independent amplitudes to be determined by as many independent data sets), lattice QCD Blum et al

(c) quark and lepton triangle diagrams: VVV = 0 by Furry $\Rightarrow$only VVA (of $f\bar{f}Z$-vertex) contributes $\Rightarrow$ABJ anomaly is perturbative and non-perturbative simultaneously i.e. leading effects calculable (anomaly cancellation) de Rafael, Knecht, Perrottet, Melnikov, Vainshtein

❐ Evaluation of $a_\mu^{\rm had}$

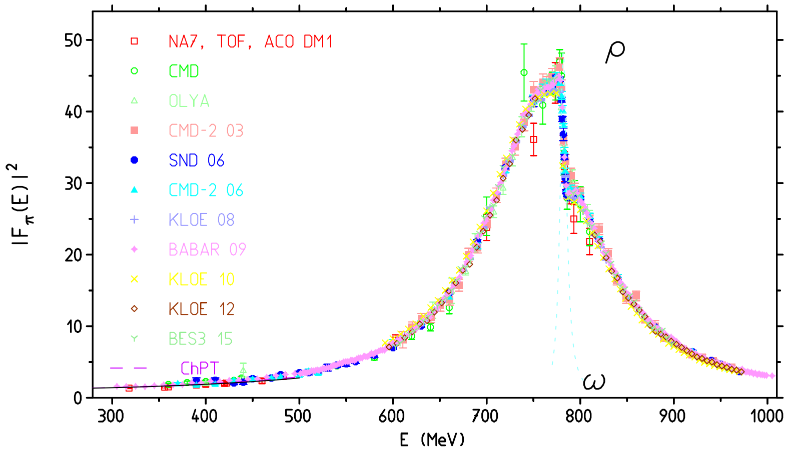
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via Dispersion Relation (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E_{\rm cut}^2} ds \, \frac{R_\gamma^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int\limits_{E_{\rm cut}^2}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right)$$



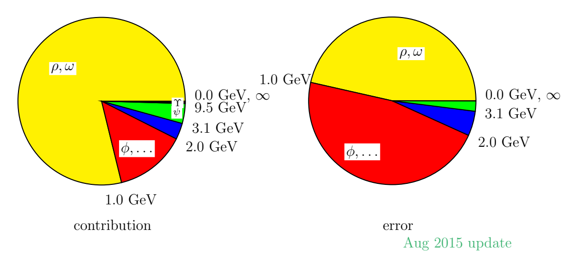
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

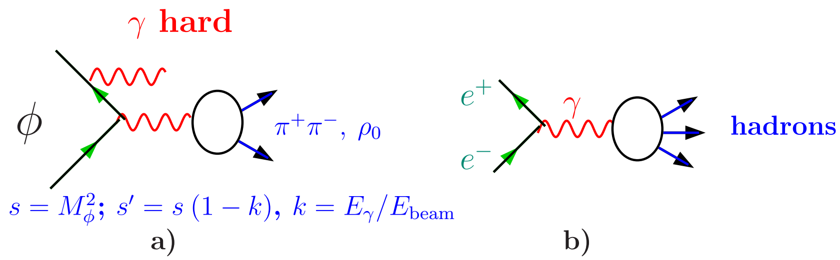
Data: NSK, KLOE, BaBar, BES3



$$a_\mu^{\rm had(1)} = (688.6 \pm 4.8)[688.9 \pm 3.5]\ 10^{-10}$$

e^+e^-–data based [incl. τ JS11]



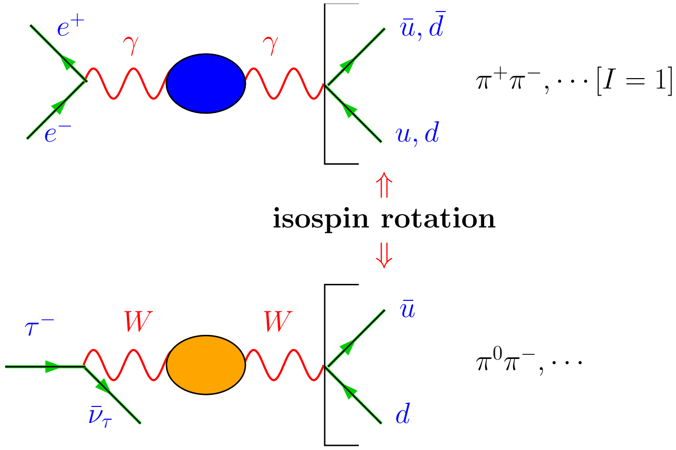


a) Initial state radiation (ISR), b) Standard energy scan.

SCAN: CMD-2, SND (NSK); ISR: KLEO, BaBar, BESIII

New experimental input for HVP: BESIII-ISR,VEPP-2000

c) τ–decay spectra + isospin breakings

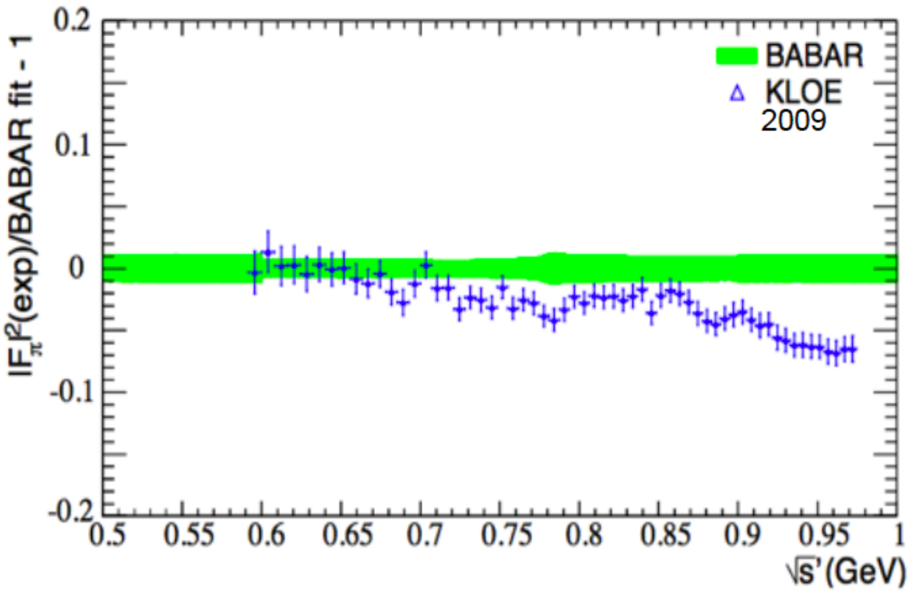


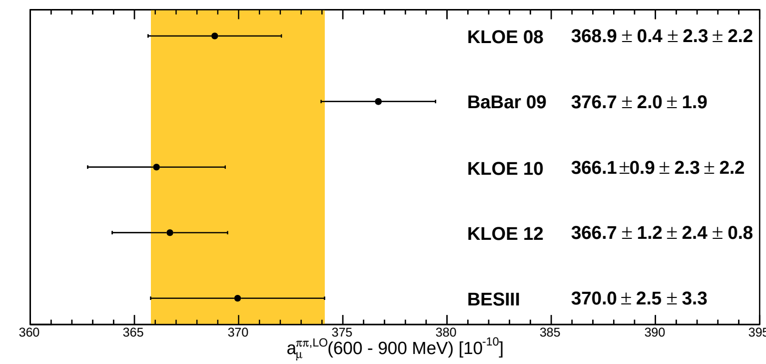
$e^+e^- \to \pi^+\pi^-, \cdots$ vs $\tau^- \to \bar{\nu}_\tau \pi^0\pi^-, \cdots$

τ–spectra: ALEPH, OPAL, CLEO, Belle

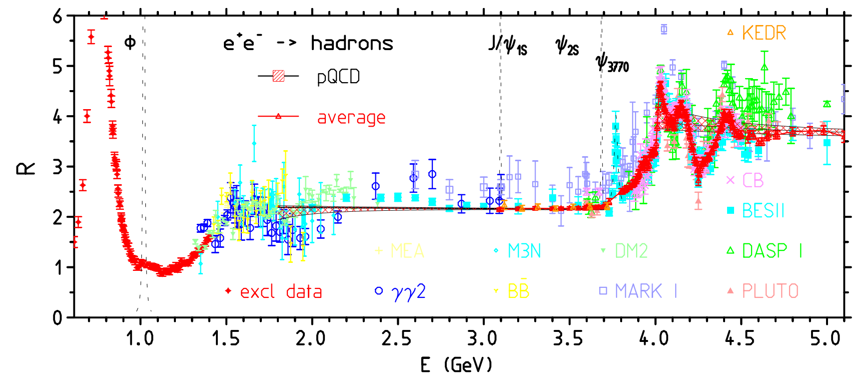
include $I = 1$ $\tau \to \pi\pi\nu_\tau$ in range [0.63-0.96] GeV:

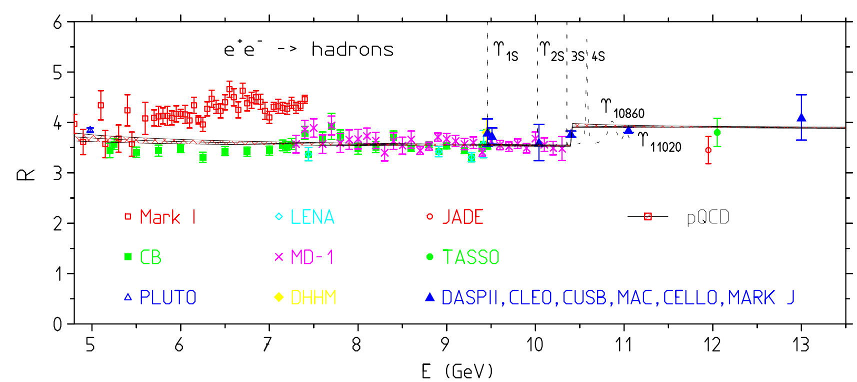
e^+e^-	:	353.82(0.88)(2.17)[2.34]
τ	:	354.25(1.24)(0.61)[1.38]
$e^+e^- + \tau$	:	354.14(0.82)(0.86)[1.19]





BES-III ~ KLOE; $1.9\,\sigma$ below BaBar





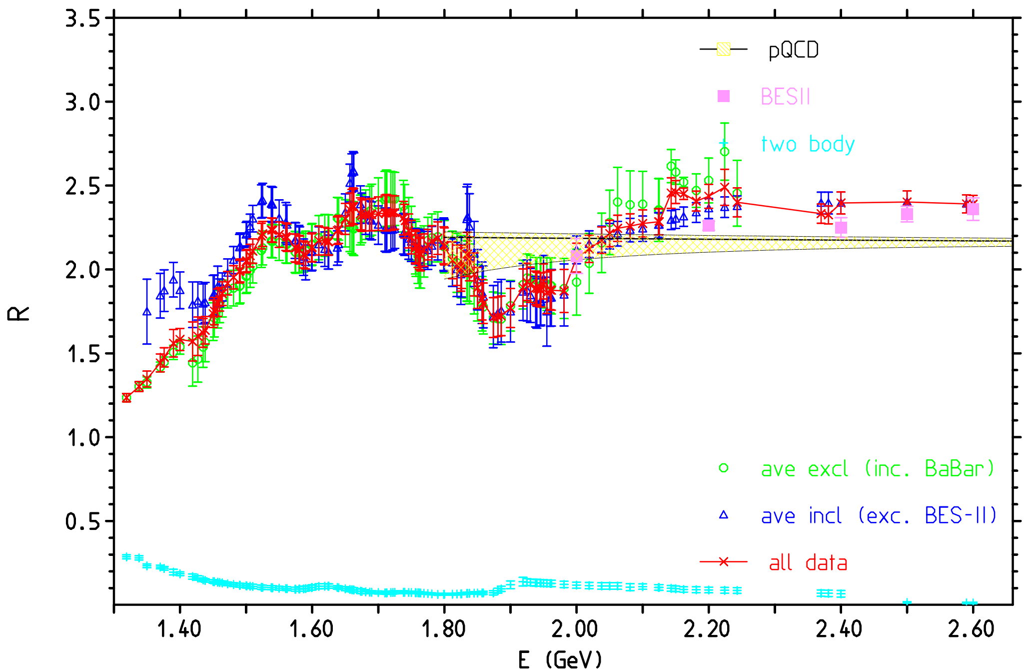
Most precise ISR measurements in conflict.
BESIII steps to resolve this

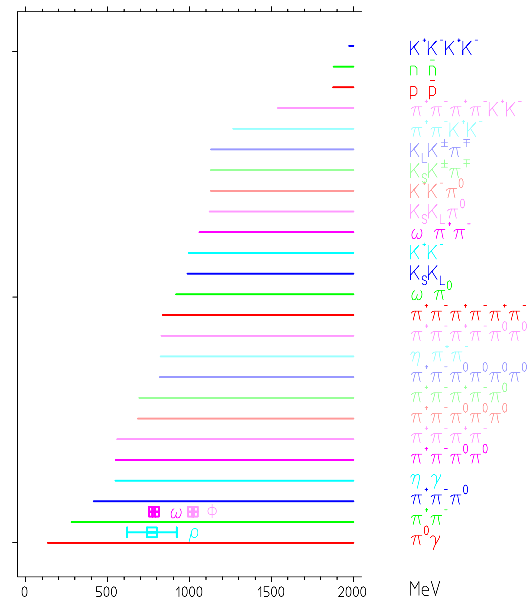
Recent/preliminary results:

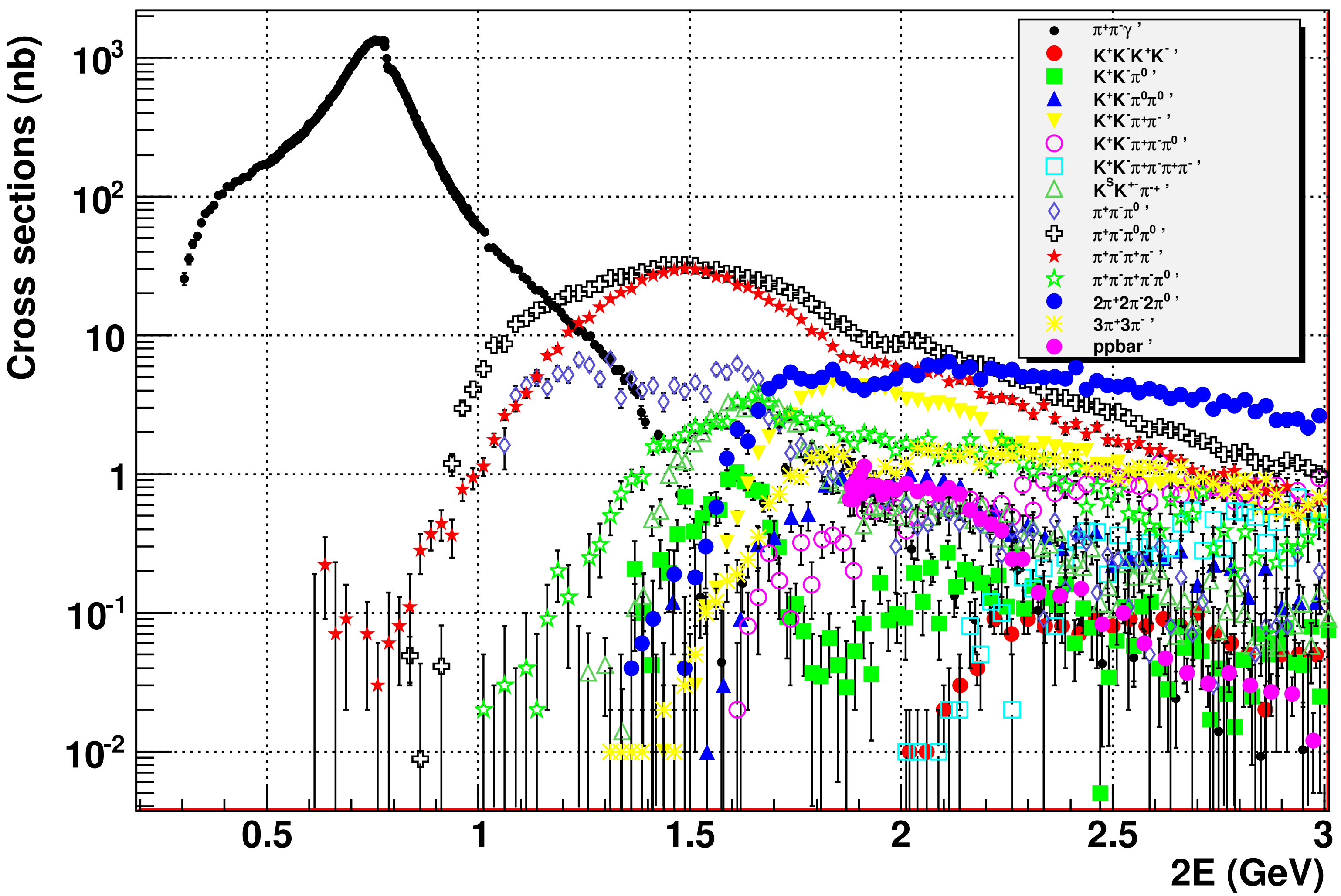
- $e^+e^- \to \pi^+\pi^-$ from CMD-3
- $e^+e^- \to \pi^+\pi^-\pi^0$ from Belle
- $e^+e^- \to K^+K^-$ from CMD-3
- $e^+e^- \to K^+K^-$ from SND
- $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $e^+e^- \to \pi^+\pi^-$ from BES-III most recent

Main issue in HVP

- region 1.2 to 2 GeV bad data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) Who will do it? BES III radiative return!

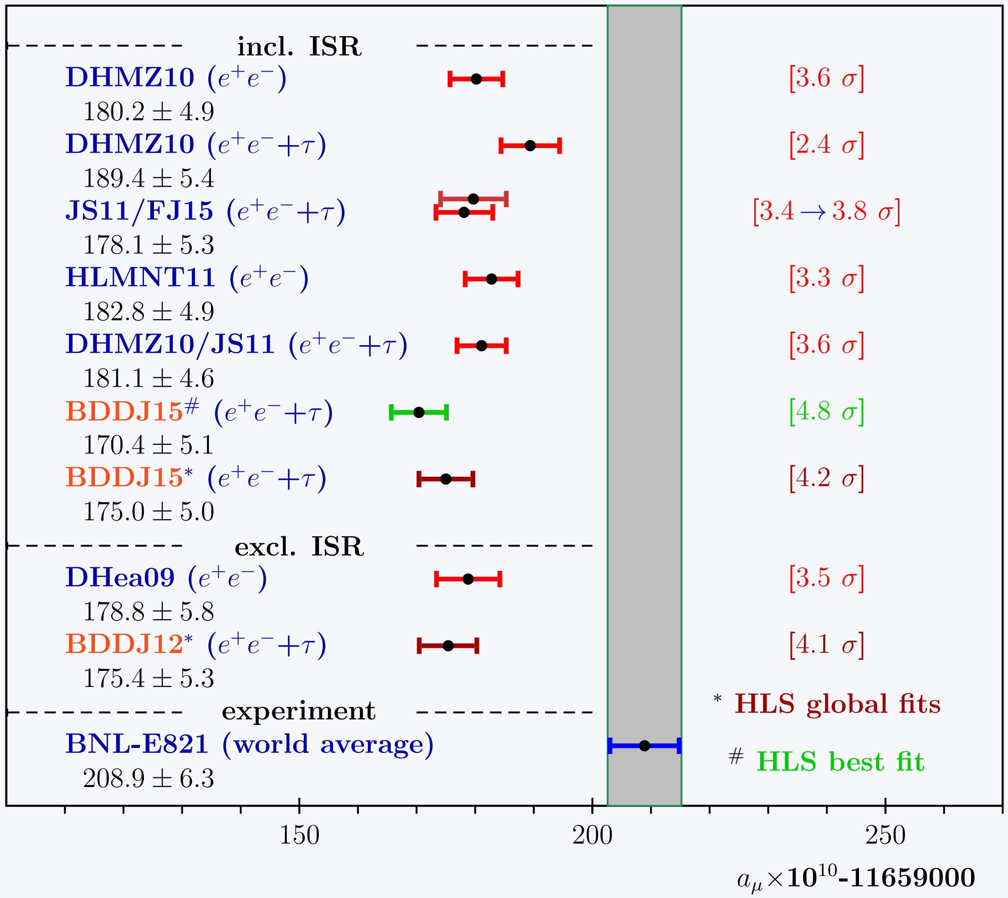






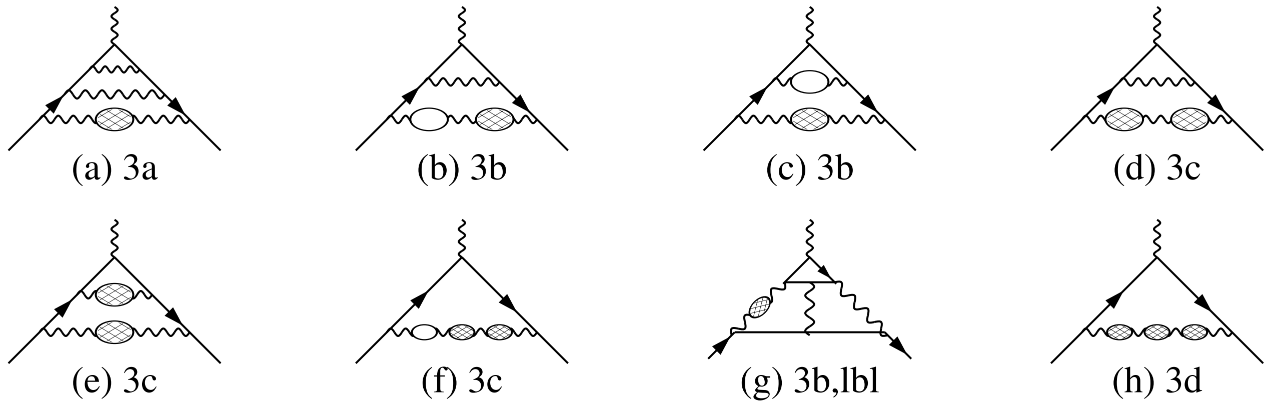
Eidelman et al 2011

Comparison of different estimates and leading uncertainties:



Note: some do not include τ data. HLS best fit (NSK+KLOE10+KLOE12) does not include BaBar data. The JS11/FJ15 is updated to include the new BES III data.

NNLO HVP effects based on 2015 update



Class		results Kurz et al	my evaluation
$a_\mu^{(3a)}$	=	0.80×10^{-10}	$0.782(77) \times 10^{-10}$
$a_\mu^{(3b)}$	=	-0.41×10^{-10}	$-0.403(37) \times 10^{-10}$
$a_\mu^{(3b,\mathrm{lbl})}$	=	0.91×10^{-10}	$0.900(77) \times 10^{-10}$
$a_\mu^{(3c)}$	=	-0.06×10^{-10}	$-0.0544(7) \times 10^{-10}$
$a_\mu^{(3d)}$	=	0.0005×10^{-10}	$5.22(15) \times 10^{-14}$
$a_\mu^{\mathrm{had,NNLO}}$	=	$12.4(1) \times 10^{-11}$	$12.25(12) \times 10^{-11}$

Kurz et al. 2014

LO, NLO and NNLO:

$$a_\mu^{\mathrm{had}(1)} = (688.91 \pm 3.52)\ 10^{-10}$$ (LO)

$$a_\mu^{\mathrm{had}(2)} = (-99.17 \pm 1.00)\ 10^{-10}$$ (NLO)

$$a_\mu^{\mathrm{had}(3)} = (1.225 \pm 0.012)\ 10^{-10}$$ (NNLO) Kurz et al 2014

all e^+e^-–data based [2015 update]

Effective field theory: the Resonance Lagrangian Approach

HVP dominated by spin 1 resonance physics! need theory of $\rho, \omega, \phi, \cdots$

❒ Principles to be included: Chiral Structure of QCD, VMD & electromagnetic gauge invariance.

❖General framework: resonance Lagrangian extension of chiral perturbation theory (CHPT), i.e. implement VMD model with Chiral structure of QCD. Specific version Hidden Local Symmetry (HLS) effective Lagrangian Bando, Kugo, Yamawaki. First applied to HLbL of muon $g-2$ Hayakawa, Kinoshita, Sanda.

Global Fit strategy: Benayoun et al.
Data below $E_0 = 1.05$ GeV (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).

❒ Effective theory predicts cross sections:

$$\pi^+\pi^-,\ \pi^0\gamma,\ \eta\gamma,\ \eta'\gamma,\ \pi^0\pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0 \quad \text{(83.4\%)},$$

- Missing part:

$$4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi \text{ and regime } E > E_0$$

evaluated using data directly and pQCD for perturbative region and tail

- Including **self-energy effects** is mandatory ($\gamma\rho$-mixing, $\rho\omega$-mixing ..., decays with proper phase space, energy dependent width etc)

- Method works in reducing uncertainties by using **indirect constraints**

- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar

Main goal of HLS global fit strategy:

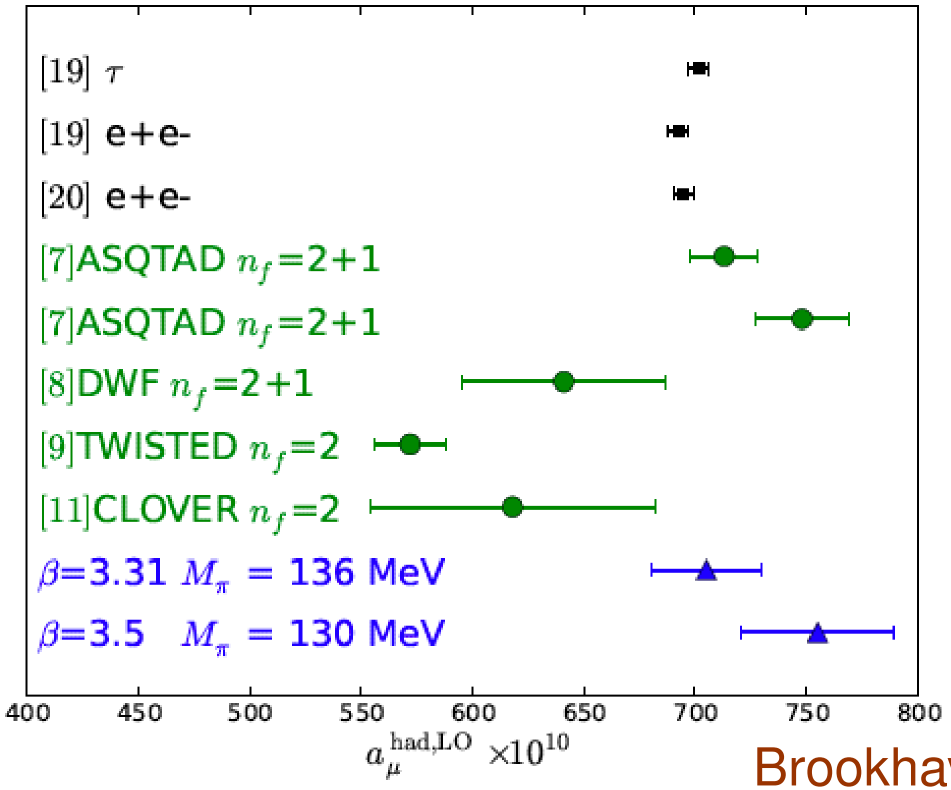
- recuction of HVP uncertainty via indirect constraints, in particular by taking into account τ data (after solving e^+e^- vs. τ data puzzle)

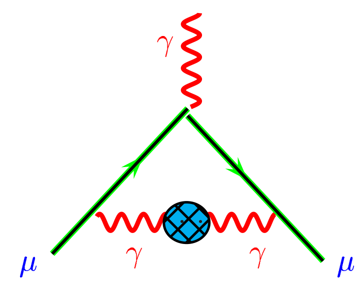
  [typically: $683.50 \pm 4.74 \to 681.77 \pm 3.14$]

- **Single out** representative effective resonance Lagrangian by global fit is expected to help in improving EFT calculations of hadronic light-by-light scattering (such concept so far missing)

HVP from lattice QCD

Lattice: $\sim$ 3-10% quoted errors, but incomplete, Experiment: 0.6% errors





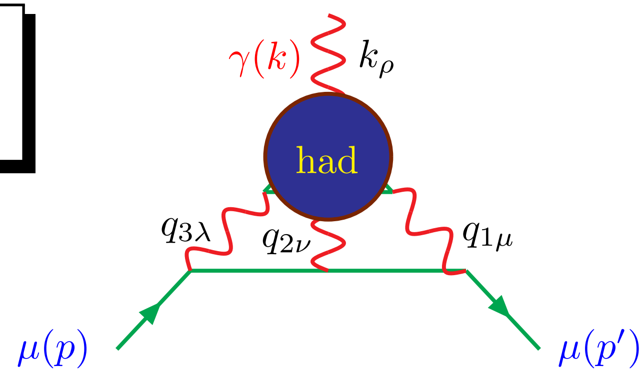
$$a_\mu^{(2)\,\rm had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty {\rm d}Q^2\, f(Q^2)\, \Pi(Q^2)$$

Brookhaven, Zeuthen, Mainz, Edinburgh, ...

Plot from Laurent Lellouch

The hadronic LbL: setup and problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

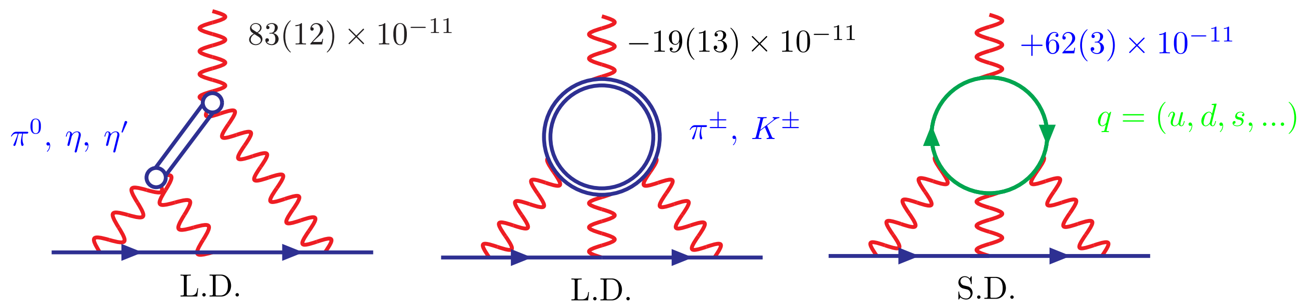
$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int d^4x_1\, d^4x_2\, d^4x_3\, e^{i\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\, 0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\,.$$

- non-perturbative physics
- covariant decomposition involves 138 Lorentz structures (43 gauge invariant)
- 28 can contribute to $g-2$
- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', ...$ described by the effective Wess-Zumino Lagrangian

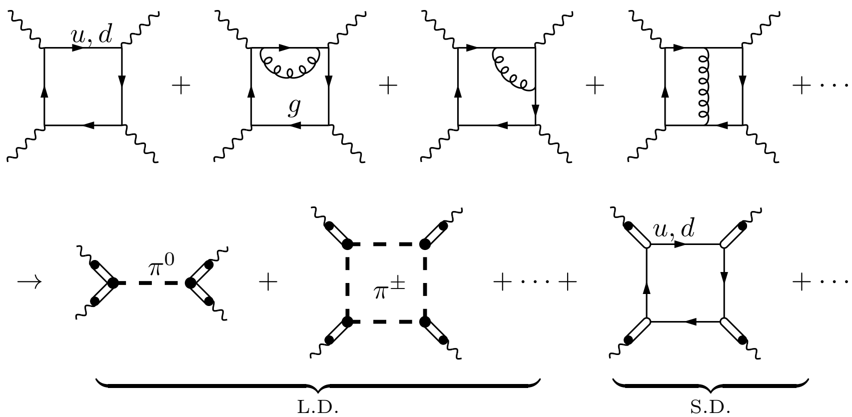
- generally, pQCD useful to evaluate the short distance (S.D.) tail

- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar pions as well as the vector mesons ($\rho, \cdots$) which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

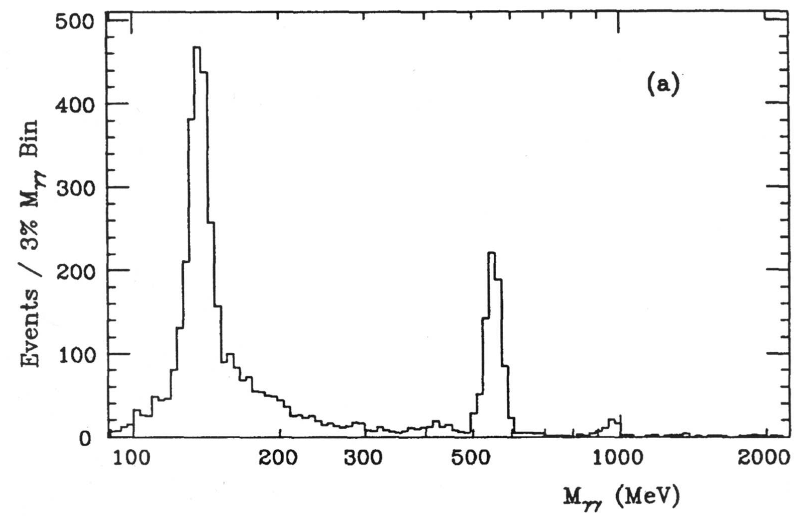
Need appropriate low energy effective theory ⇒amount to calculate the following type diagrams



LD contribution requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex

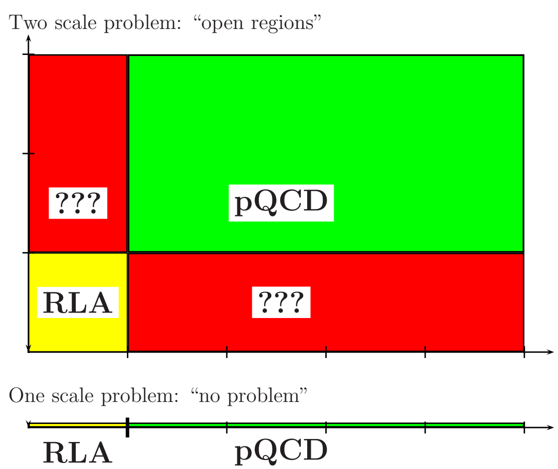


Crystal Ball 1988



Data show almost background free spikes of the PS mesons!

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



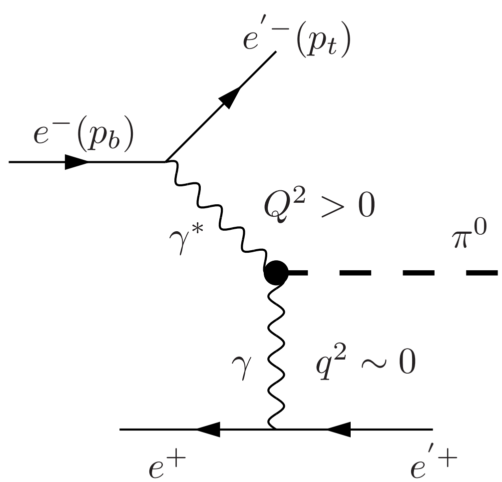
???

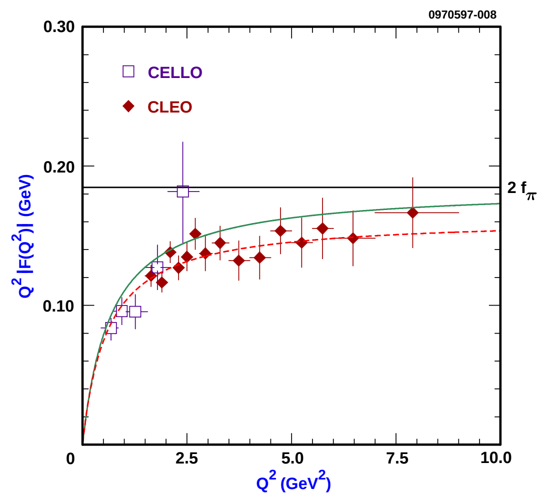
– Data + Dispersion Relation, OPE,
– QCD factorization,
– Brodsky-Lepage approach
– Models constrained by data

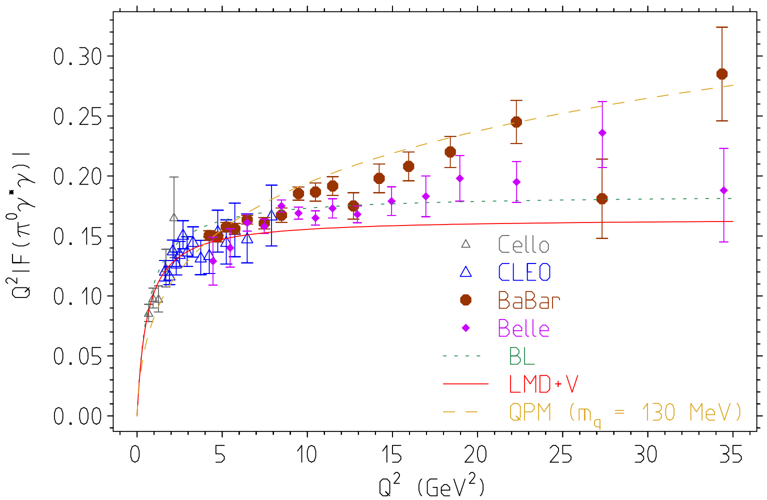
❐ Constraint I: $\Gamma(\pi^0\gamma\gamma) \leftrightarrow$ effective WZW-Lagrangian

❖ The constant $e^2 \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV^{-1} well determined by $\pi^0 \to \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

❖ Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments





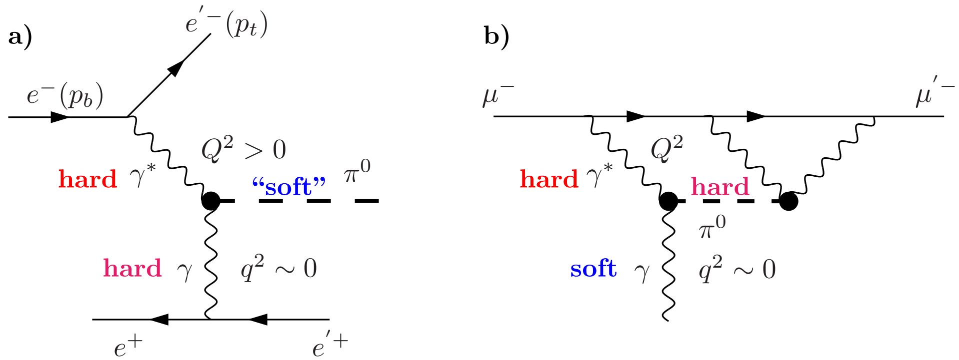


CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2. Outdated by BaBar? Belle conforms with theory expectations!

❒ Constraint II: VMD mechanism ↔ Brodsky-Lepage behavior

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$$

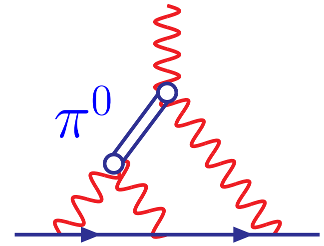
then cannot miss to get reasonable result!



Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ or $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q^2,q^2,0)$ if integral to be evaluated in Minkowski space.

Can we check such questions experimentally or in lattice QCD?

❑ π^0-exchange



model	$a_\mu^{\pi^0} \cdot 10^{10}$	group
EJLN/BPP	5.9(0.9)	Bijnens, Pallante, Prades 1995
Nonlocal quark model	6.72	Dorokhov, Broniowski 2008
Dyson-Schwinger Eq. Approach	5.75	Goecke, Fisher, Williams 2011
LMD+V/KN	(5.8 – 6.3)	Knecht, Nyffeler 2002
MV: LMD+V+OPE[WZW]	6.3(1.0)	Melnikov, Vainshtein 1997
Formfactor inspired by AdS/QCD	6.54	Cappiello, Cata, d'Ambrosio 2011
Chiral quark model	6.8	Greynat, de Rafael 2012
Magnetic susceptibility constraint	7.2	Nyffeler 2009

My estimation: Leading LbL contribution from PS mesons:

$$a_\mu[\pi^0, \eta, \eta'] \sim (93.91 = [63.14 + 14.87 + 15.90] \pm 12.40) \times 10^{-11}$$

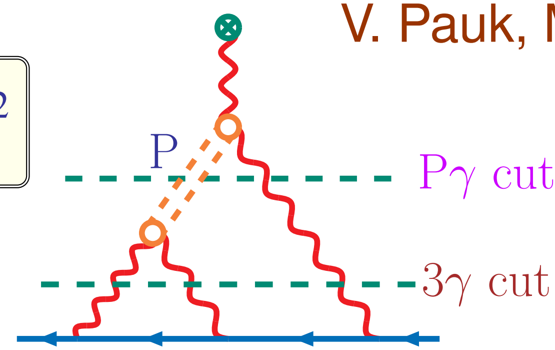
Still controversial: ❑ VMD at external vertex ?

❑ Pion-pole approximation ?

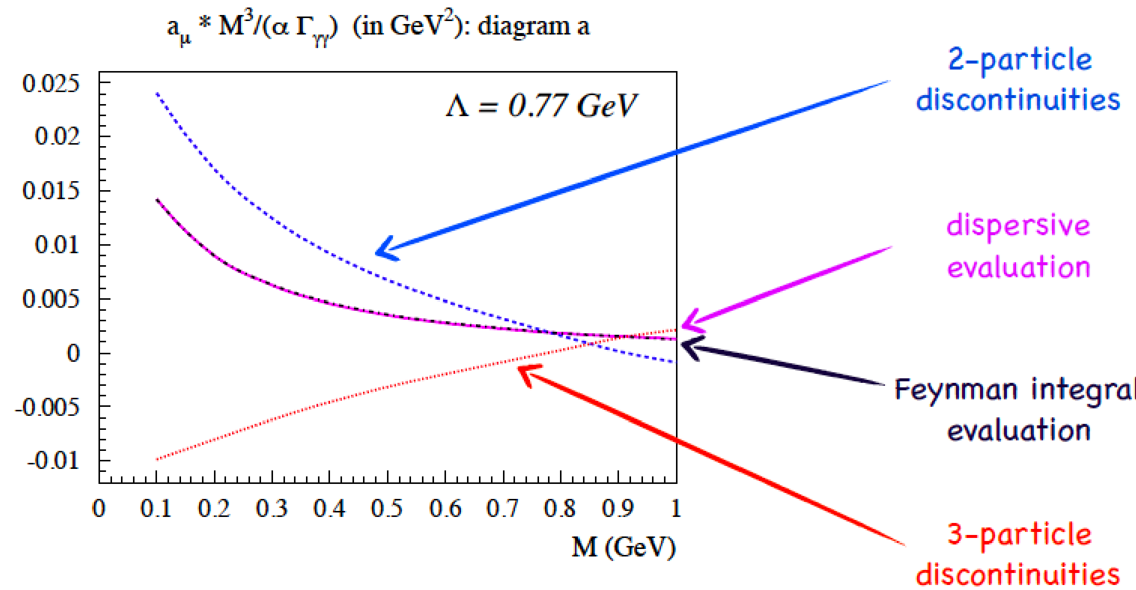
❑ a_μ Dispersion Relation Approach

V. Pauk, M. Vanderhaeghen

$$a_\mu = F_2(0)\ ; \quad F_2(0) = \frac{1}{2\pi \mathrm{i}} \int \frac{\mathrm{d}q^2}{q^2}\, \mathrm{Abs}\, F_2(q^2$$



both time-like
$e^+e^- \to P\gamma$
and space-like
$\gamma^*\gamma^* \to P$
data needed as input



Dispersive approach to $\gamma^*\gamma^* \to \gamma^*\gamma^*$: Colangelo, Hoferichter, Procura, Stoffer

- Very ambitious long term project, requires all kind of data not yet available

New:

- Axial vector meson contributions re-evaluated V. Pauk, F.J.
- Landau-Yang theorem constraint built in correctly
- Tensor meson contributions evaluated V. Pauk, M. Vanderhaeghen
- Results depending slightly on assuming nonet symmetry, ideal mixing etc

JN09 based on Nyffeler 09: the only result relaxing from pole approximation

$$a_\mu^{\mathrm{LbL;had}} = (103 \pm 34) \times 10^{-11}$$

Note old MV result for axials:

$$a_\mu[a_1, f_1', f_1] \sim (28.13 = [7.02 + 19.38 + 1.74] \pm 5.63) \times 10^{-11}$$

versus new:

$$a_\mu[a_1, f_1', f_1] \sim (7.55 = [1.89 + 5.19 + 0.47] \pm 2.71) \times 10^{-11}$$

Beyond single meson exchanges: 28 amplitudes to be determined by data

The role of models

Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-N_c $\pi^0\gamma\gamma$ form-factor

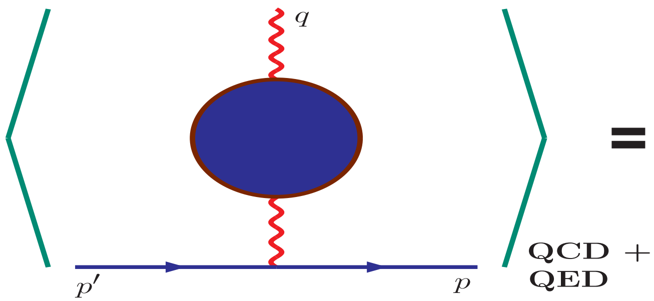
- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LMD+V form-factor

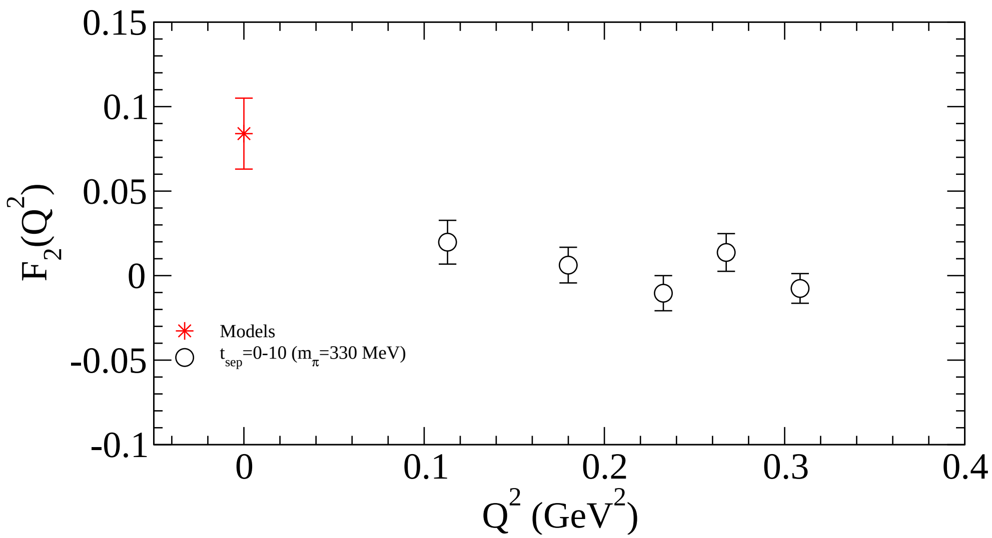
$$
\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2, q_1^2, q_2^2) &= \frac{F_\pi}{3} \frac{\mathcal{P}(q_1^2, q_2^2, p_\pi^2)}{\mathcal{Q}(q_1^2, q_2^2)} \\
\mathcal{P}(q_1^2, q_2^2, p_\pi^2) &= h_7 + h_6\, p_\pi^2 + h_5\,(q_2^2 + q_1^2) + h_4\, p_\pi^4 + h_3\,(q_2^2 + q_1^2)\, p_\pi^2 \\
&\quad + h_2\, q_1^2\, q_2^2 + h_1\,(q_2^2 + q_1^2)^2 + q_1^2\, q_2^2\,(p_\pi^2 + q_2^2 + q_1^2) \\
\mathcal{Q}(q_1^2, q_2^2) &= (q_1^2 - M_1^2)\,(q_1^2 - M_2^2)\,(q_2^2 - M_1^2)\,(q_2^2 - M_2^2)
\end{aligned}
$$

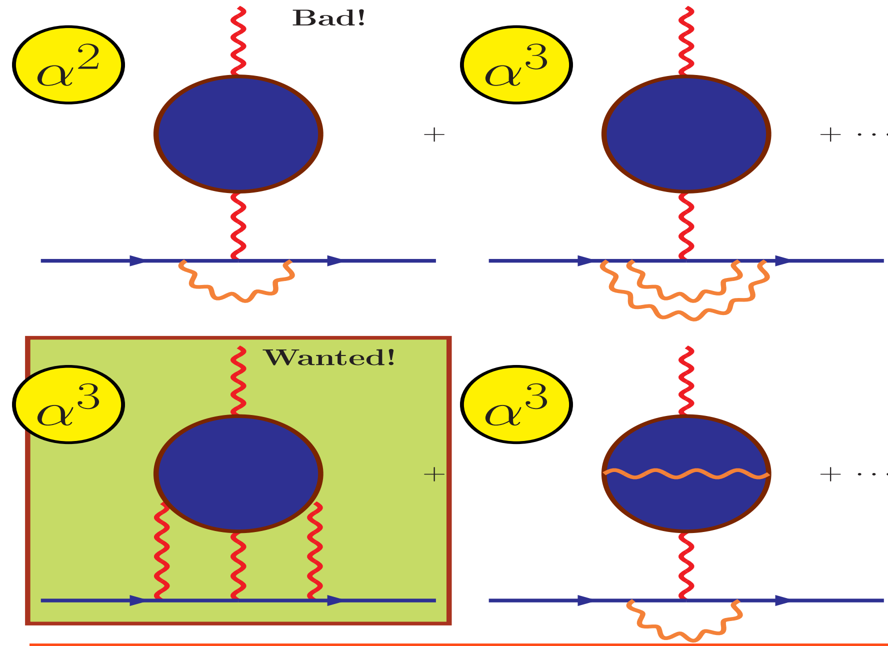
all constants are constraint by SD expansion (OPE). Again, need data to fix parameters! $(h_1, \cdots, h_7)$ Models: ENJL, HLS, LMD+V, etc

❑ HLBL in lattice QCD

Blum, Izubuchi et al.







Subtraction of unwanted terms:
works for diagram by diagram
and configuration by configuration
(important for noise reduction)

Proof of principle Blum et al. Phys.Rev.Lett. 114 (2015) 1

LbL: Present

JN09: result relaxing from pole approximation, new axial + NLO-HLbL+tensor

$$a_\mu^{\mathrm{LbL;had}} = (103 \pm 34) \times 10^{-11}$$

Summary of results

Contribution	HKS	BPP	KN	MV	PdRV	N/JN/J15
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	94±12
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13
axial vectors	1.7±1.7	2.5±1.0	–	22± 5	15±10	7.6± 2.7
scalars	–	−6.8±2.0	–	–	−7± 7	−5.0± 1.2
quark loops	9.7±11.1	21± 3	–	–	2.3	21± 3
total	89.6±15.4	83±32	80±40	136±25	105±26	103±34

Is this the final answer? How to improve? A limitation to more precise $g-2$ tests?

Looking for new ideas to get ride of model dependence: new "DR + data" approach (as HVP) but now 28 invariant amplitude (in place of 1)!

New evaluations included:

New contribution	Reference	$a_\mu \cdot 10^{11}$			
NNLO HVP	Kurz et al. 2014	12.4	$\pm$	0.1	
NLO HLBL	Colangelo et al. 2014	3	$\pm$	2	
New axial exchange HLBL	Pauk, Vanderhaeghen, F.J. 2014	7.55	$\pm$	2.71	
Old axial exchange HLBL	Melnikov, Vainshtein 2004	22	$\pm$	5	
Tensor exchange HLBL	Pauk, Vanderhaeghen 2014	1.1	$\pm$	0.1	
Total change		+2.1	$\pm$	3.4	$[\leftarrow 5]$

a_μ Theory vs Experiment: do we see New Physics?

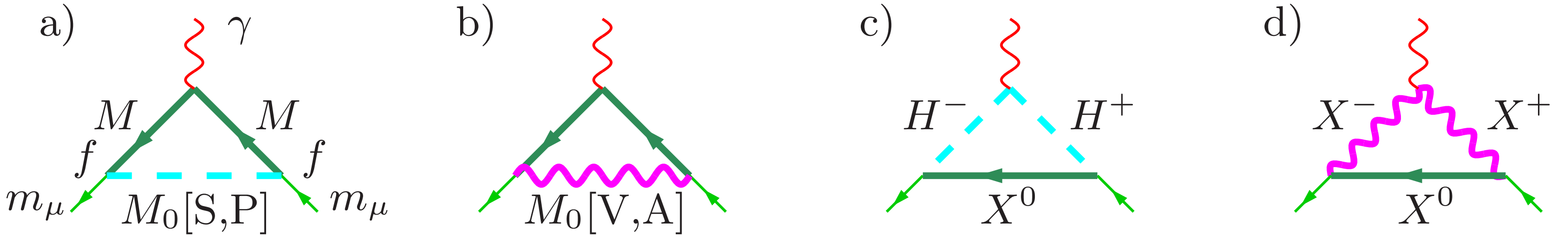
Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471.8851	0.036	Remiddi, Kinoshita ...
Leading hadronic vac. pol.	688.91	3.52	$e^+e^- + \tau$ data
Subleading hadronic vac. pol.	-9.917	0.100	2015 update
NNLO hadronic vac. pol.	1.240	0.010	2014 KLMS
Hadronic light–by–light	10.3	3.4	evaluation (J&N 09/J 14)
Weak incl. 2-loops	15.40	0.10	CMV06/FJ12/BSS13
Theory	11 659 177.82	4.90	–
Experiment	11 659 209.1	6.3	BNL Updated
Exp.- The. 3.9 standard deviations	31.28	7.98	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4 σ deviation: ❒ new physics? ❒ a statistical fluctuation? ❒ underestimating uncertainties (experimental, theoretical)?

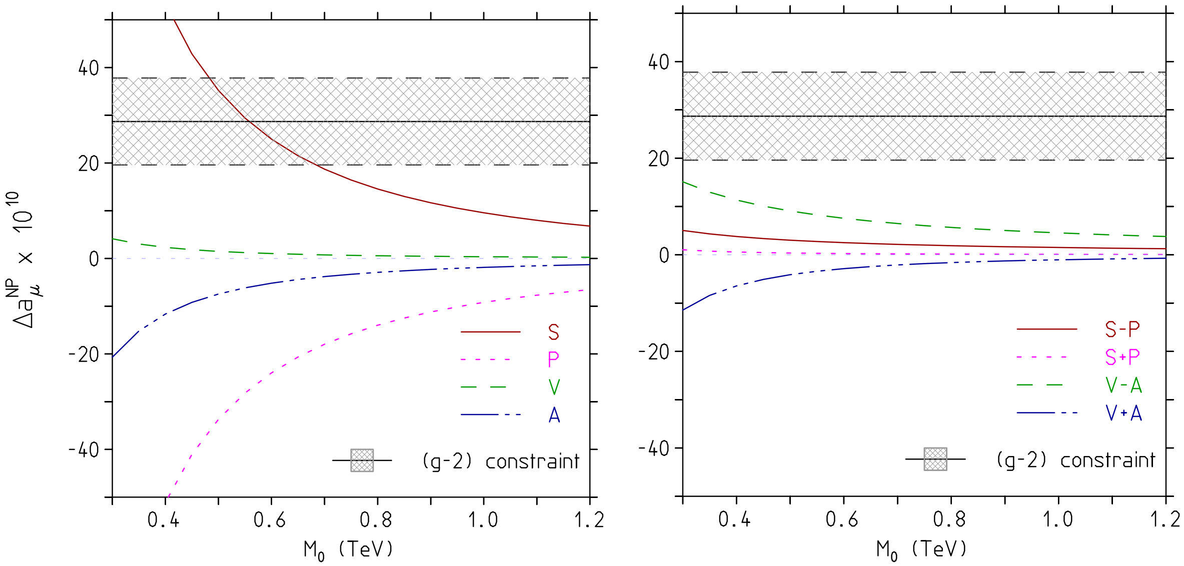
❖do experiments measure what theoreticians calculate?

What could fill the gap?

Most natural New Physics contributions: (examples)



neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector



Left: $m_\mu = M \ll M_0$ Right: $m_\mu \ll M_0 = M$

In general:

$$\Delta a_\mu^{\rm NP} = \alpha^{\rm NP} \, \frac{m_\mu^2}{M_{\rm NP}^2}$$

NP searches (LEP, Tevatron, LHC): typically $M_{\rm NP} >> M_W$, then $\Delta a_\mu^{\rm exp-the} = \Delta a_\mu^{\rm NP}$ requires $\alpha^{\rm NP} \sim 1$ spoiling perturbative arguments. Exception: 2HDM, SUSY $\tan\beta$ enhanced coupling! Note: NP sensitivity enhanced for muon by $\sim 40\,000$ relative to electron, while a_e is only 2250 times more precise than $a_\mu \Rightarrow \sim$ 18 in sensitivity!

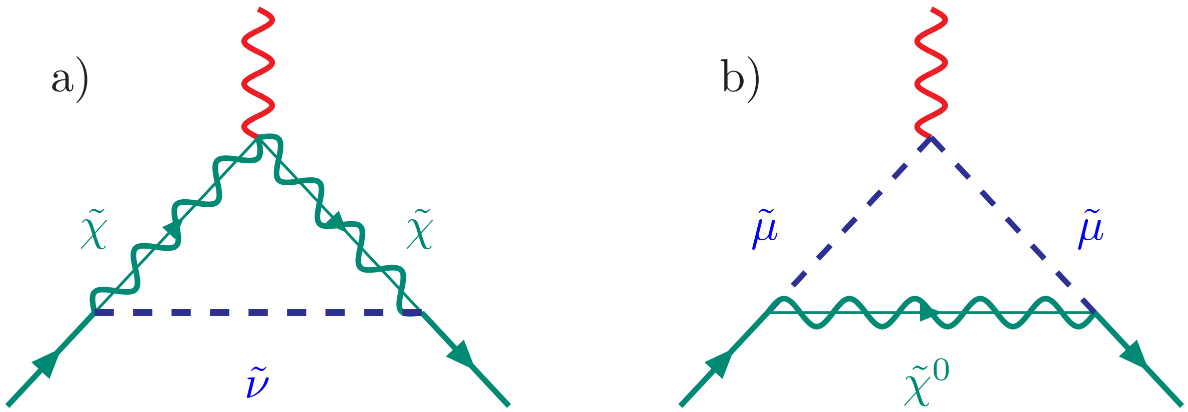
Problem: LEP, Tevatron and LHC direct bounds on masses of possible new states

[typically $M_X > 800$ GeV]

Need enhanced couplings! as in SUSY extensions of SM

- $M_{\rm SUSY}$ lightest SUSY particle; SUSY requires two Higgs doublets
- $\tan\beta = \frac{v_1}{v_2}$, $v_i =< H_i >$; $i = 1, 2$; $\tan\beta \sim m_t/m_b \sim 40 \quad [4-40]$

- muon $g-2$ in contrast requires moderately light SUSY masses and in the pre-LHC era fitted rather well with expectations from SUSY



$$a_\mu^{\mathrm{SUSY}} \simeq \frac{\mathrm{sign}(\mu M_2)\,\alpha(M_Z)}{8\pi \sin^2\Theta_W}\,\frac{(5+\tan^2\Theta_W)}{6}\,\frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{M_{\mathrm{SUSY}}}{m_\mu}\right)$$

with M_{SUSY} a typical SUSY loop mass, sign given by Higgsino mass term μ

❐ a particular role is played by the mass of the light Higgs

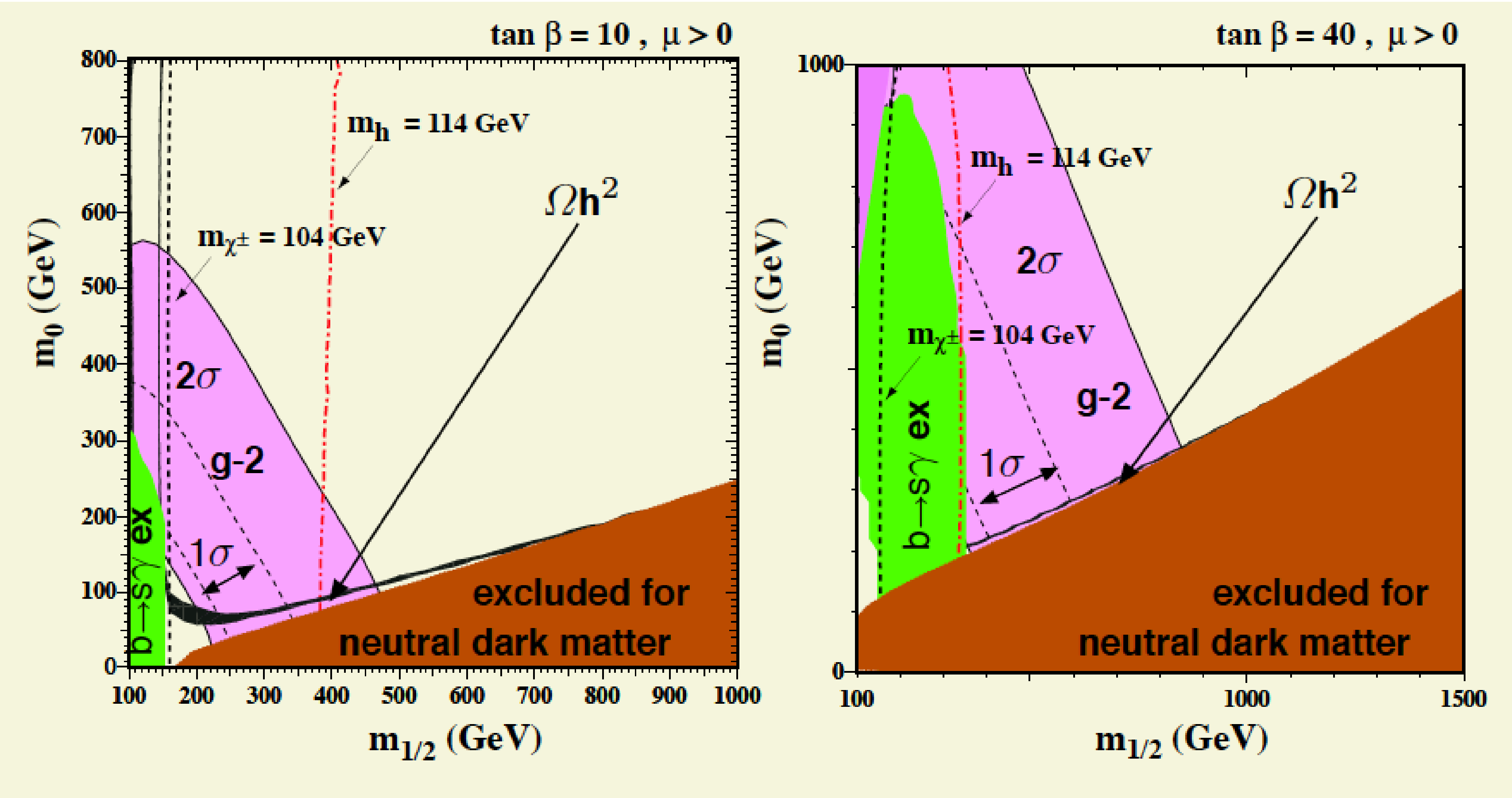
A ramarkable 2–loop calculation within MSSM exist by Heinemeyer, Stöckinger, Weiglein corrections can be large for $M_{\mathrm{LOSP}} < 500$ GeV, increasing for lower masses.

SUSY constrains the Higgs Boson Mass:

At tree level in the MSSM $m_h \leq M_Z$. This bound receives large radiative corrections from the $t/\tilde{t}$ sector, which changes the upper bound to (Haber & Hempfling 1990)

$$m_h^2 \sim M_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} G_\mu m_t^4}{2\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \cdots$$

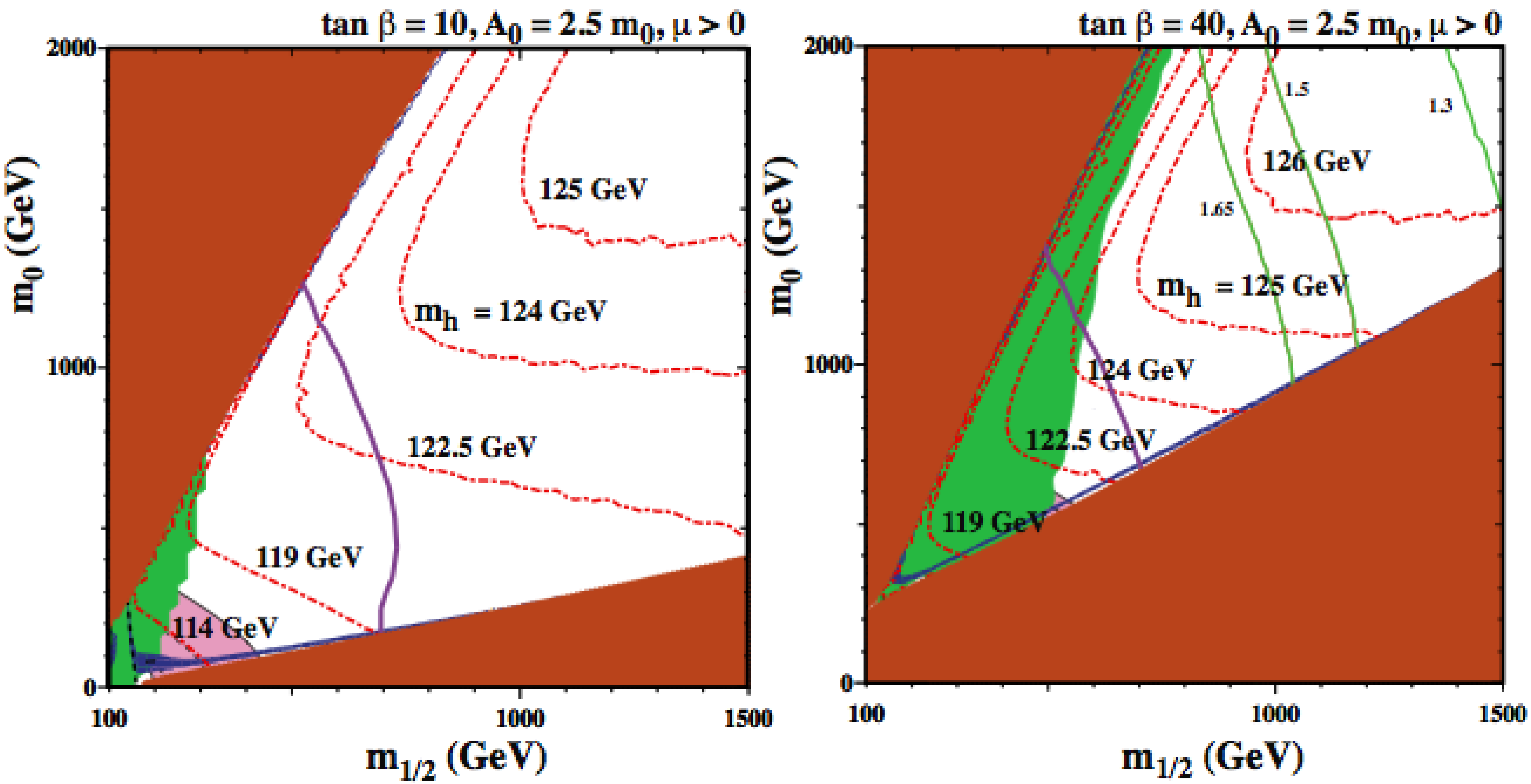
which in any case is well below 200 GeV. A given value of m_h fixes the value of $m_{1/2}$ represented by $\{m_{\tilde{t}_1}, m_{\tilde{t}_2}\}$



constraints from LEP, B-physics, g-2, cosmic relict density [plots Olive 09].

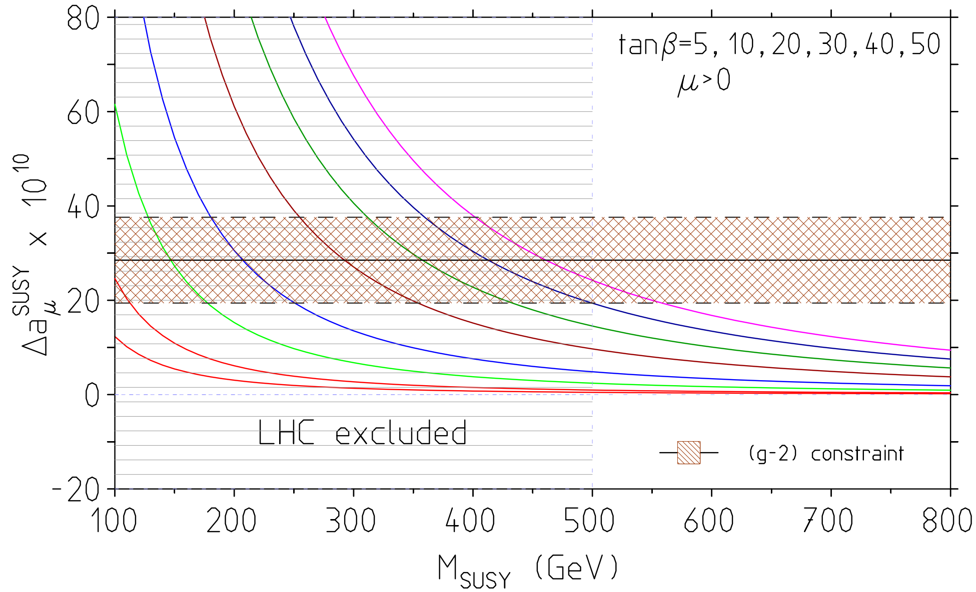
$\mathbf{m_0}$ scalar mass $\quad$ $\mathbf{m_{1/2}}$ gaugino mass

After first LHC results:



J. Ellis et al. CMSSM

❒ if Higgs is established at 125 GeV (LHC/CERN) we must have $m_{1/2} > 800 GeV$ or higher!



Constraint on large $\tan\beta$ SUSY contributions as a function of $M_{\rm SUSY}$. The horizontal band shows $\Delta a_\mu^{\rm NP} = \delta a_\mu$. The region left of $M_{\rm SUSY} \sim$ 500 GeV is excluded by LHC searches. If $m_h \sim 125 \pm 1.5$ GeV actually $M_{\rm SUSY} > 800$ GeV depending on details of the stop sector ($\{\tilde{t}_1, \tilde{t}_2\}$ mixing and mass splitting) and weakly on $\tan\beta$.

SUSY-GUT likely out, but no direct exclusion bounds on sneutrino-chargino and smuons-neutralino states!

Future

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come

Provided deviation is real $3\sigma \to 9\sigma$ possible? Provided theory and needed cross section data improves the same as the muon $g-2$ experiments!

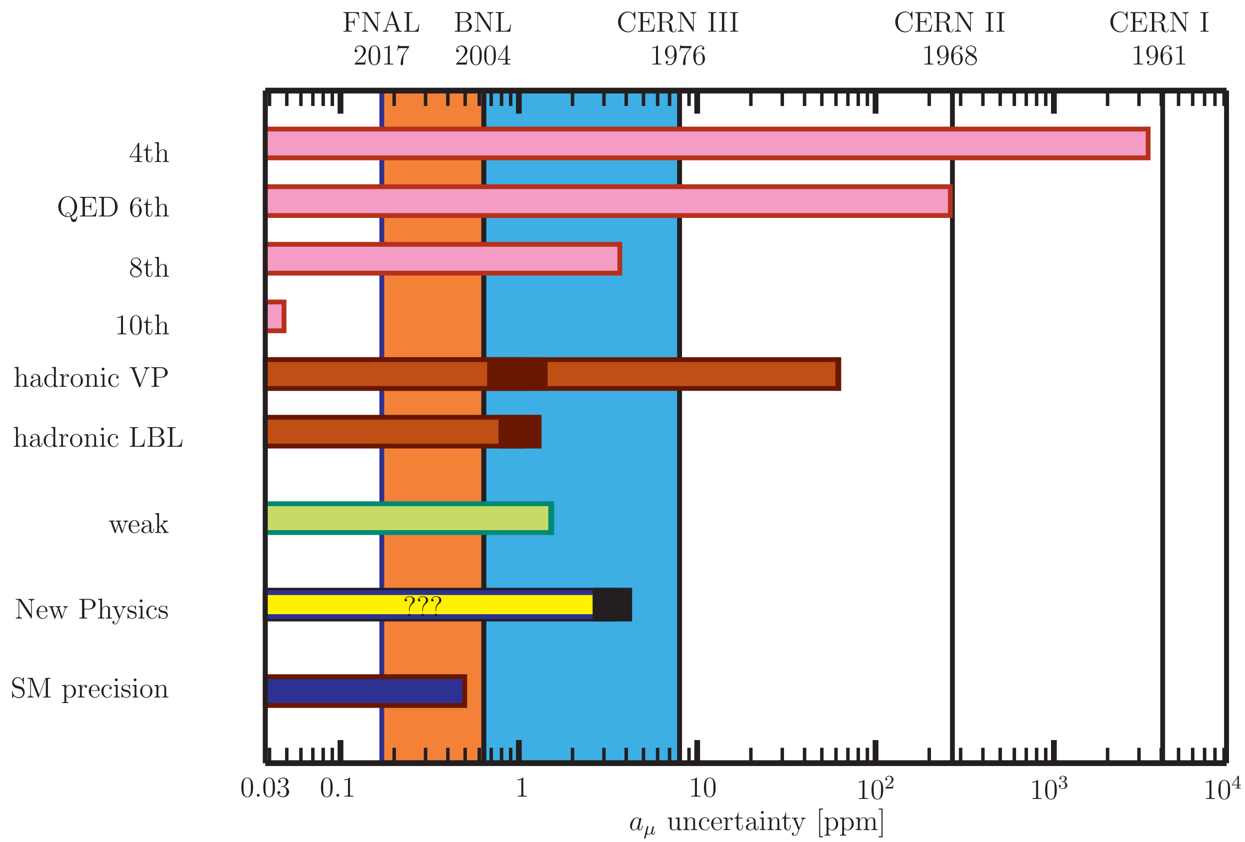
Key: need substantial progress in non-perturbative QCD

For muon $g-2$:

- main obstacle: hadronic light-by-light [data, lattice QCD, RLA]
- progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...), lattice QCD in reach (recent progress Jansen et al, Wittig et al, Blum et al)

  in both cases lattice QCD will be the answer one day,
- also low energy effective RL and DR approach need be further developed.

And here we are:



Sensitivity of $g-2$ experiments to various contributions. The increase in precision with the BNL $g-2$ experiment is shown as a cyan vertical band. New Physics is illustrated by the deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$

The challenge:

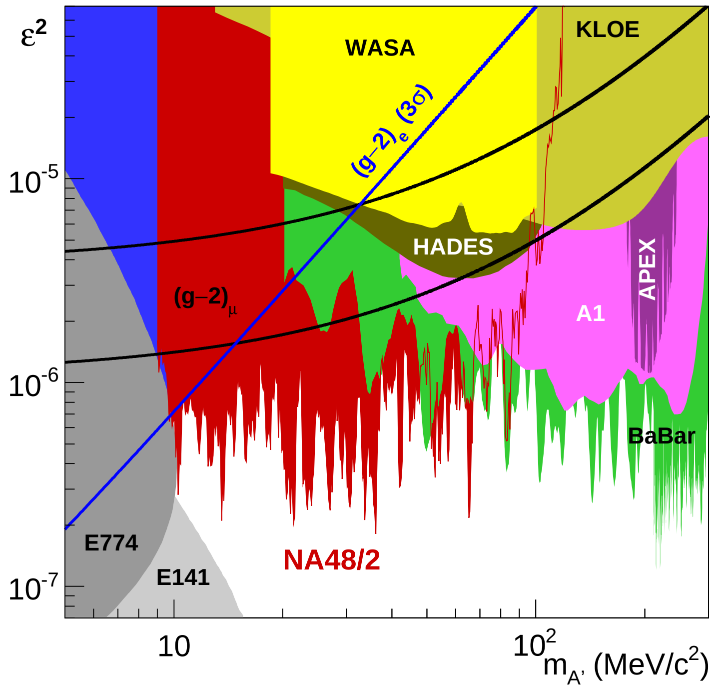
$a_\mu^{\text{had,VP}}[LO]$	$(6923 \pm 42) \times 10^{-11}$	$+58.82 \pm 0.36$ ppm
$a_\mu^{\text{had,VP}}[NLO]$	$(-98 \pm 1) \times 10^{-11}$	
a_μ^{EW}	$(154 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,LbL}}$	$[(105 \div 115) \pm (26 \div 40)] \times 10^{-11}$	$+0.90 \pm 0.22$ ppm
$\delta a_\mu^{\text{exp}}$ present	63×10^{-11}	± 0.54 ppm
$\delta a_\mu^{\text{exp}}$ future	16×10^{-11}	± 0.14 ppm

Next generation experiments require a factor 4 reduction of the uncertainty
optimistically feasible is factor 2 we hope

Summary

- Muon $g-2$ is one of the most precisely measured quantities in particle physics
- $a_\mu = (g_\mu - 2)/2$ very good monitor for physics beyond the SM:
  about 18 $\simeq$ 40000/2250 more sensitive than a_e
- At the same time it is the quantity showing a persisting discrepancy between theory and experiment $a_\mu^{\rm exp} - a_\mu^{\rm the}$ shows 3-4 σ deviation
- Could turn to about 8 σ after new Fermilab experiment improvement by factor 4, provided theory improves by factor 2 at least
- Low energy precision physics complementary to high energy as LHC
- Non-perturbative hadronic effects are limiting precision of theoretical predictions: vacuum polarization and hadronic light-by-light scattering
- Non-perturbative hadronic modeling requires experimental data to constrain models, challenge for close collaboration between theory and experiment:
  $e^+e^- \to$ hadrons and $\gamma^{(*)}\gamma^{(*)} \to \pi^0, \pi^+\pi^-,$ other hadronic states

- At the present/future level of precision it depends on all physics incorporated in the SM of particle physics: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for. Excludes light states with “normal size” couplings. Dark photon still a candidate.



see NA48/2 Coll.
arXiv:1504.00607v2

Still possible somthing from low scales ... not seen at LHC

5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

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Discussion today

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✓ ✓ ✓

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✓ ✓ ✓

Discussion today

Goals:		
	Δa_μ(Expt)	~ 140 ppb
	Δa_μ(SM)	< 220 ppb

slide from D. Hertzog

- Two new experiments on the way! more precise, and/or new concept!
- New tools and experiments to pin down more precisely the hadronic effects are on the way!

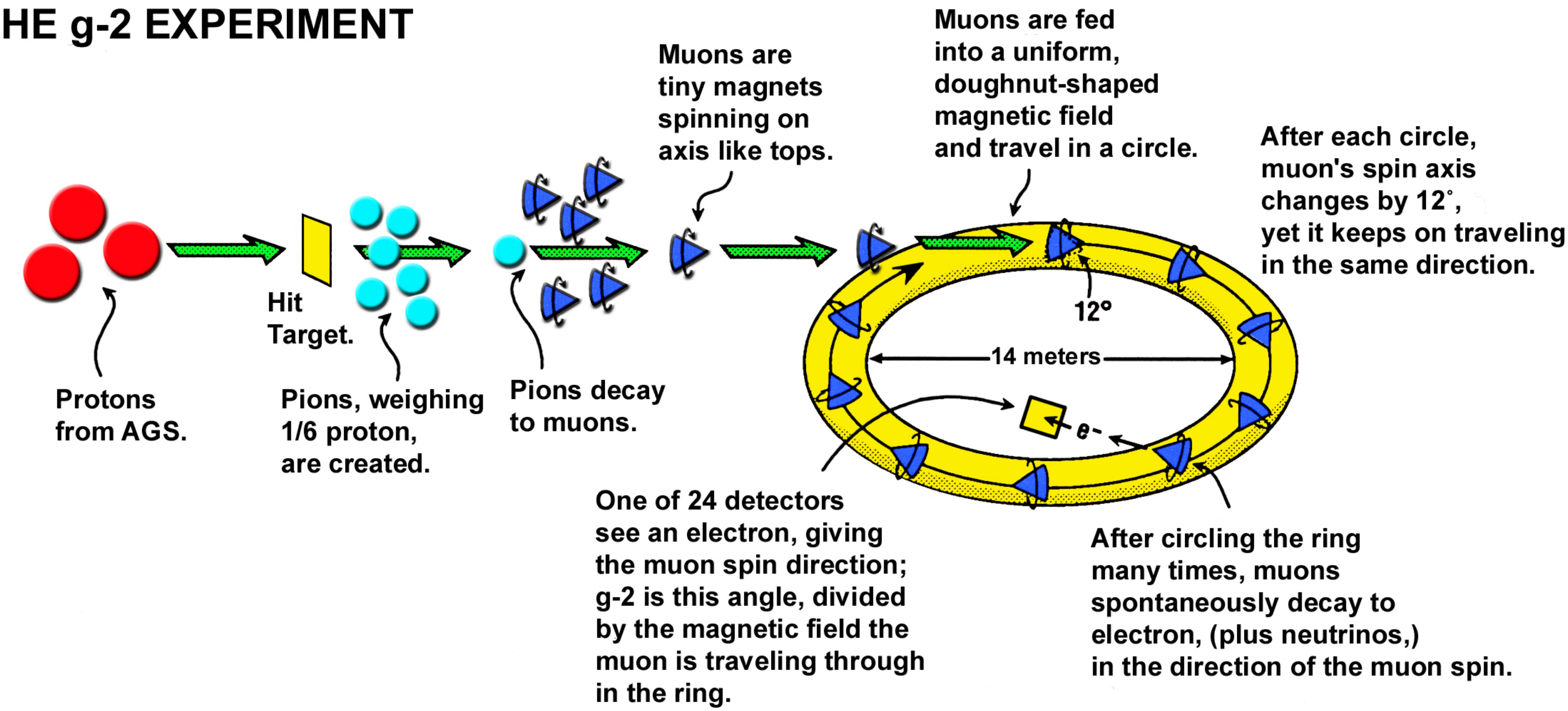
The muon $g-2$ remains a exciting and challenging topic!

After all one of the most precisely measured and precisely understood quantities showing a persistent large deviation from the SM predictions which is hard to accommodate with BSM physic not ruled out already.

Pretty much a mystery at present!

Thank you for your attention!

LIFE OF A MUON:
THE g-2 EXPERIMENT



Backup Slides

Summary HVP:

- Dominating $\pi\pi$ channel measured with $<$ 1% accuracy
  - ➠most precise ISR measurements (KLOE, BABAR) in conflict with each other
  - ➠cross check by BESIII - ISR
  - ➠VEPP-2000 aims for unprecedented accuracy 0.3%
- Higher multiplicities dominated by BABAR ISR measurements
  - ➠cross check and improvement expected by VEPP-2000, BESIII
- BELLE-II in intermediate future ?!
- Issues: - Radiative corrections
  - Precise formfactor models in MC generators
  - FSR modeling

- Lattice: Lots of interest, work on hadronic contributions, esp. HVP
  - Statistical errors (sub) 1%
  - Several groups done/doing physical m_π ($m_{\rm quark}$) simulations
  - Much effort on understanding systematics
  - 2-3% total error on connected HVP in 2 years possible
  - May be achievable for disconnected too

Summary HLBL:

- Huge experimental progress in all kinematic ranges
- KLOE-II and BESIII will measure pseudoscalar TFF in low Q2 range
- Hadronic models need to be validated by data
  - exptl. accuracy in most cases not yet precise enough

- Dispersion relations for HLbL calculation
  ➠ close interplay btw. theory and experiment

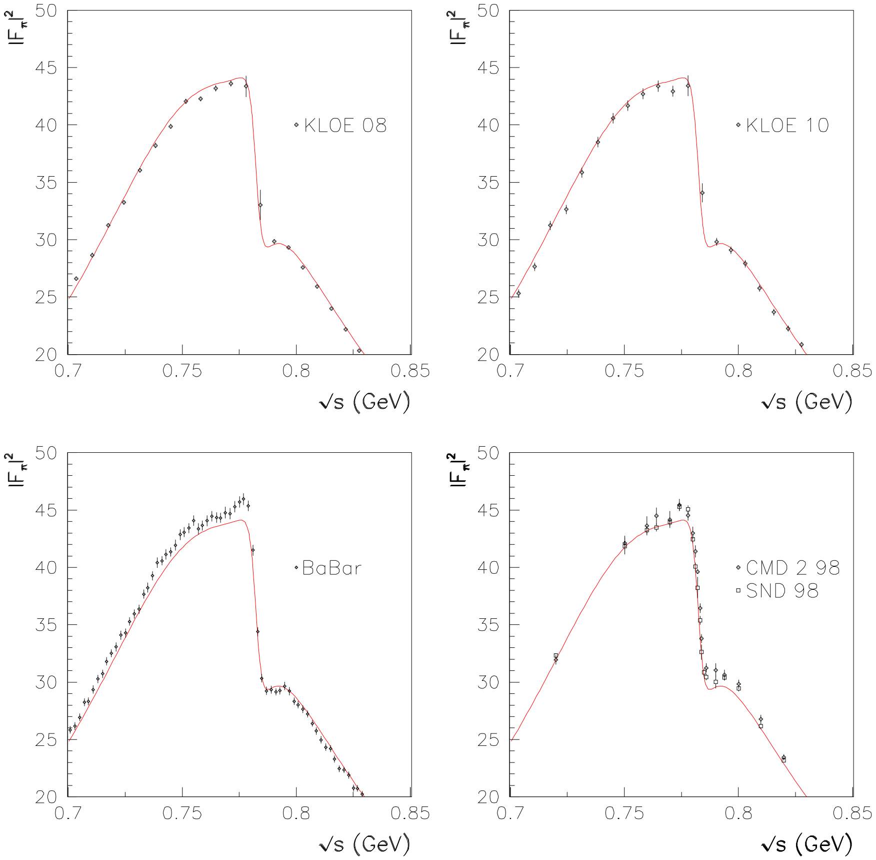
- Lattice: QCD+QED promising, but significant systematics.
  Present running with $m_\pi = 170$ MeV and
  investigating excited state contamination

- Dynamical QED+QCD is coming too

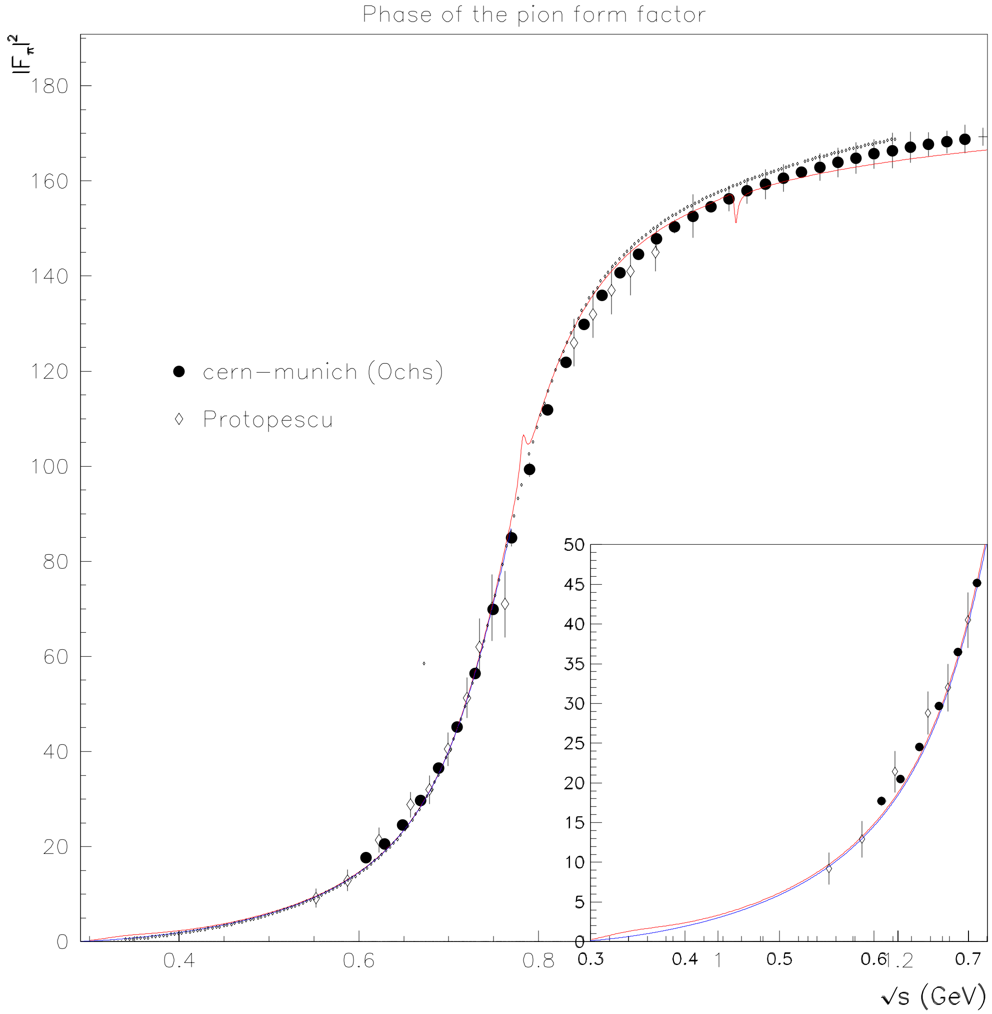
- need more groups working on it!

- Interest in 4pt function,$\pi \to \gamma^* \gamma^*$, other simpler quantities

❑ Fit of τ +PDG vs $\pi^+\pi^-$–data

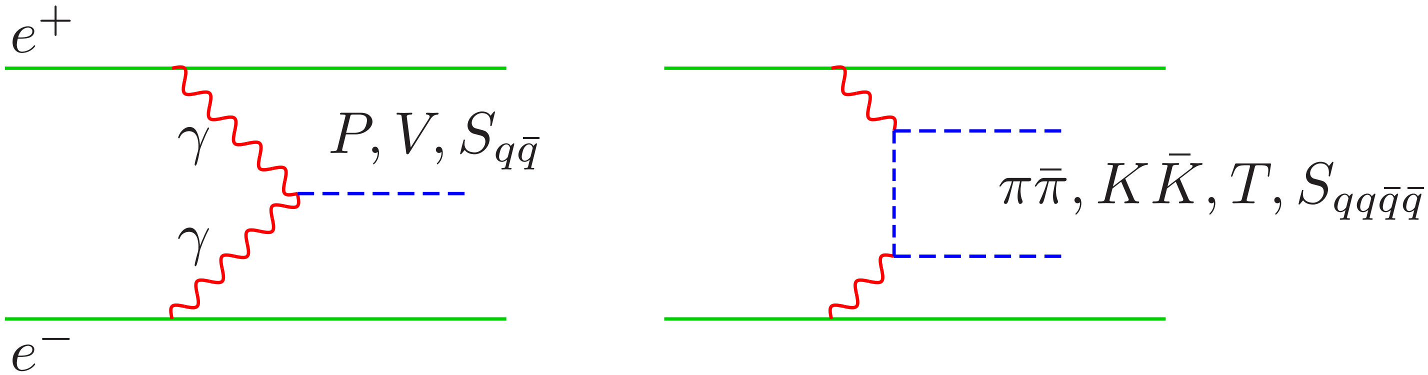


Benayoun et al 2012/13

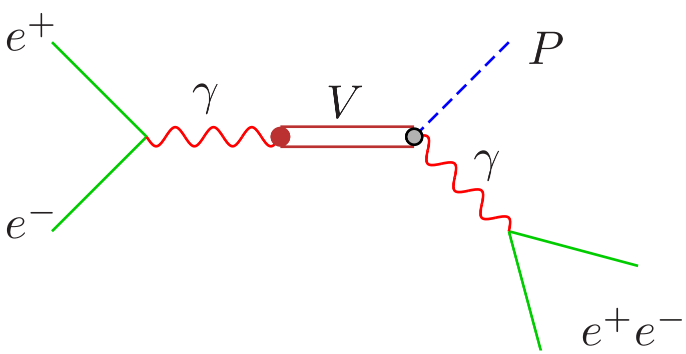


The $\pi\pi$ scattering phase of our HLS prediction

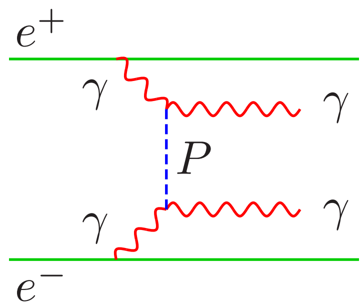
- Try exploiting possible new experimental constraints from $\gamma\gamma \to$ hadrons



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III (high luminosity)



Dalitz-decays: $\rho, \omega, \phi \to \pi^0(\eta)e^+e^-$ Novosibirsk, NA60,JLab, Mainz, Bonn, Jülich, BES



would be interesting, but is buried in the background

- all in conjunction with DR Vanderhaeghen et al 2012/14

- Still a question: do we calculate what experiments measure?

Resent: Arbuzov and Kopylowa 2013: effect of real radiation on a_μ:

$$\Delta a_f^{(1,\kappa)} = \frac{\alpha}{2\pi}\left(1+\delta d_f^{(\kappa)}\right)$$
$$\delta a_f^{(\kappa)} = \left(\frac{1}{4}+\frac{1}{2}\ \ln|\kappa|\right)\kappa + O(\kappa^2)$$

as it should smooth as $\kappa \to 0$ ("offshellness" of the muon)
Assume

$$\Delta a_\mu^{\mathrm{exp-SM}} \sim 3\times 10^{-9} \simeq \frac{\alpha}{2\pi}\,\delta a_\mu^{(\kappa)}$$

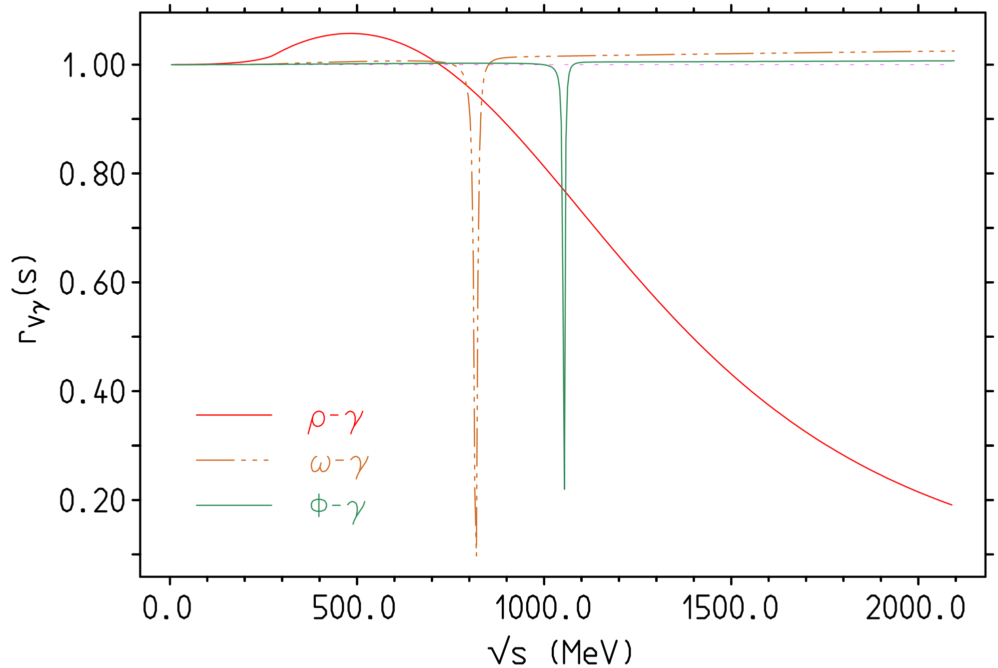
$\Rightarrow$ $\kappa \simeq -3.5\times 10^{-7}$; $\kappa m_\mu \sim 35$ eV , remember $p_\mu \simeq 3.1$ GeV

Recent: τ (charged channel) vs. e^+e^- (neutral channel) puzzle resolved

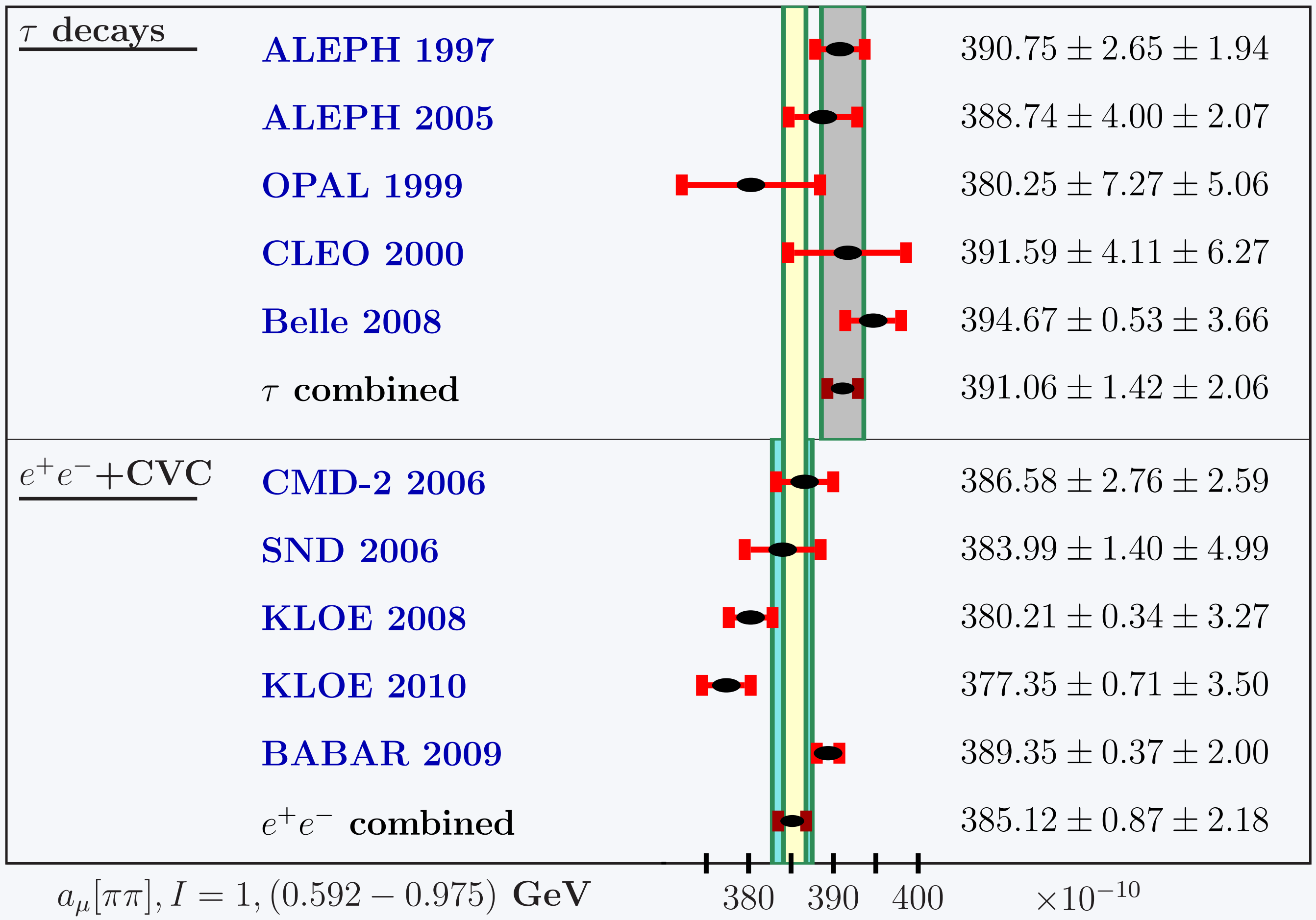
F.J.& R. Szafron, $\rho-\gamma$ interference (absent in charged channel):

$-\mathrm{i}\,\Pi^{\mu\nu\,(\pi)}_{\gamma\rho}(q) =$ 

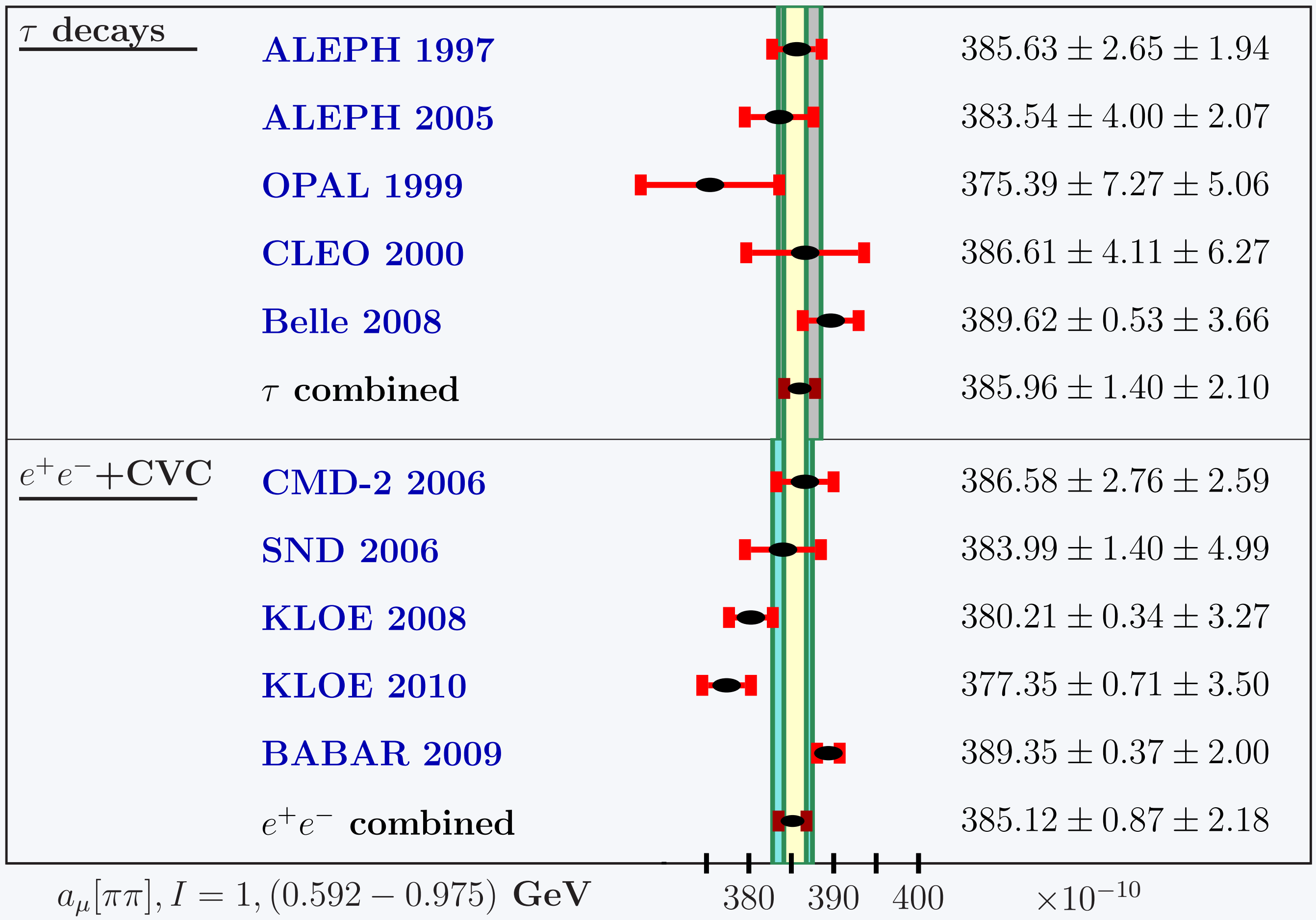
$$v_0(s) = r_{\rho\gamma}(s)\, R_{\mathrm{IB}}(s)\, v_-(s)$$



- τ require to be corrected for missing $\rho-\gamma$ mixing!
- results obtained from e^+e^- data is what goes into a_μ
- off-resonance tiny for ω, ϕ in $\pi\pi$ channel (scaled up $\Gamma_V/\Gamma(V\to\pi\pi)$)



I=1 part of $a_\mu^{\rm had}[\pi\pi]$



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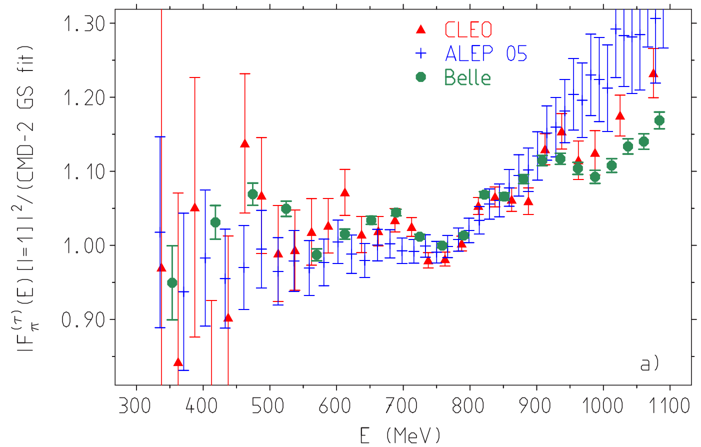
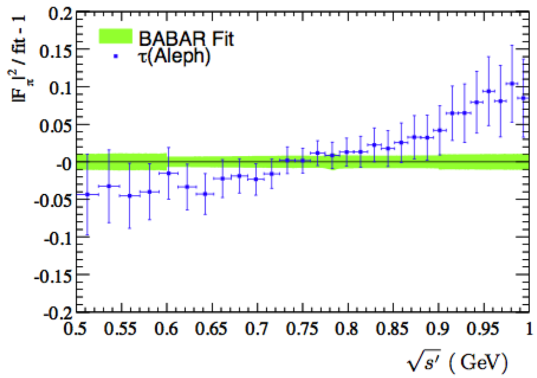
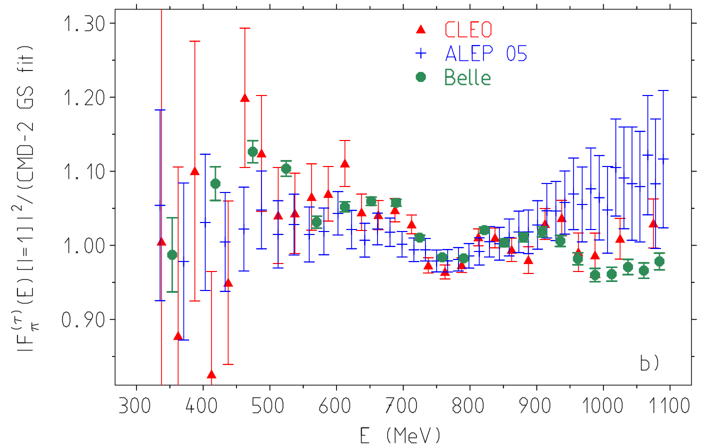


$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit)

Best "proof":





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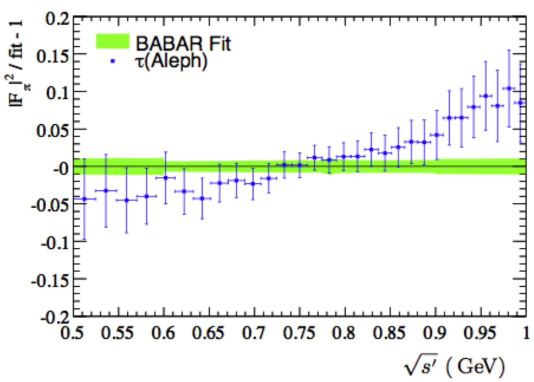
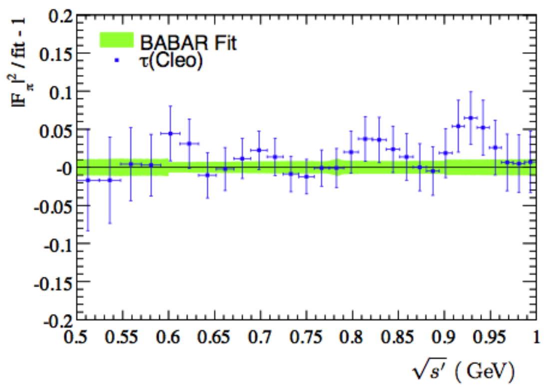
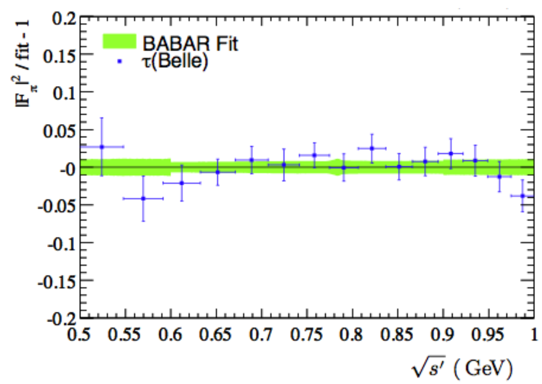
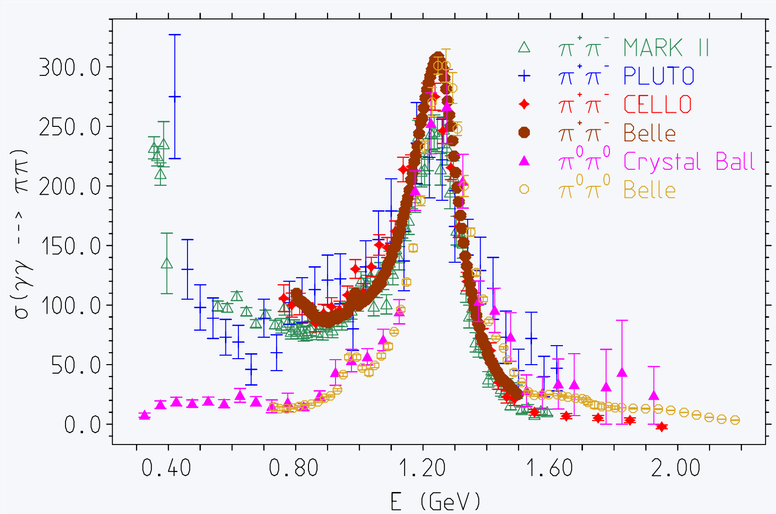






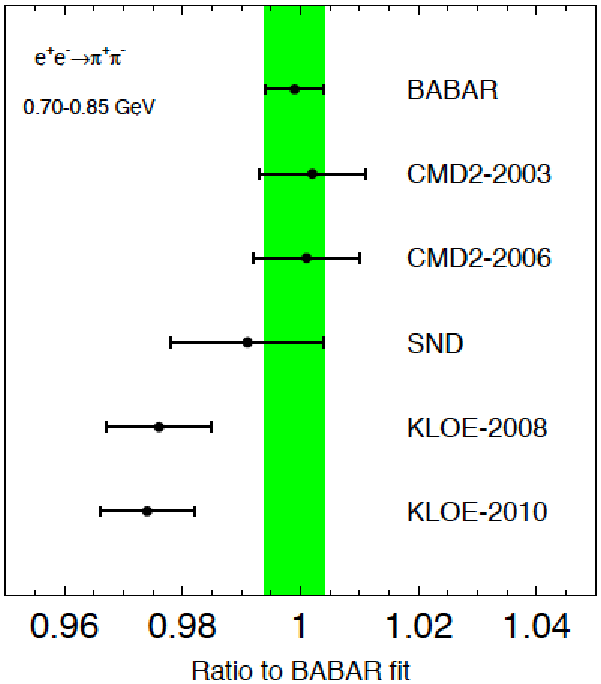
Is our model viable?



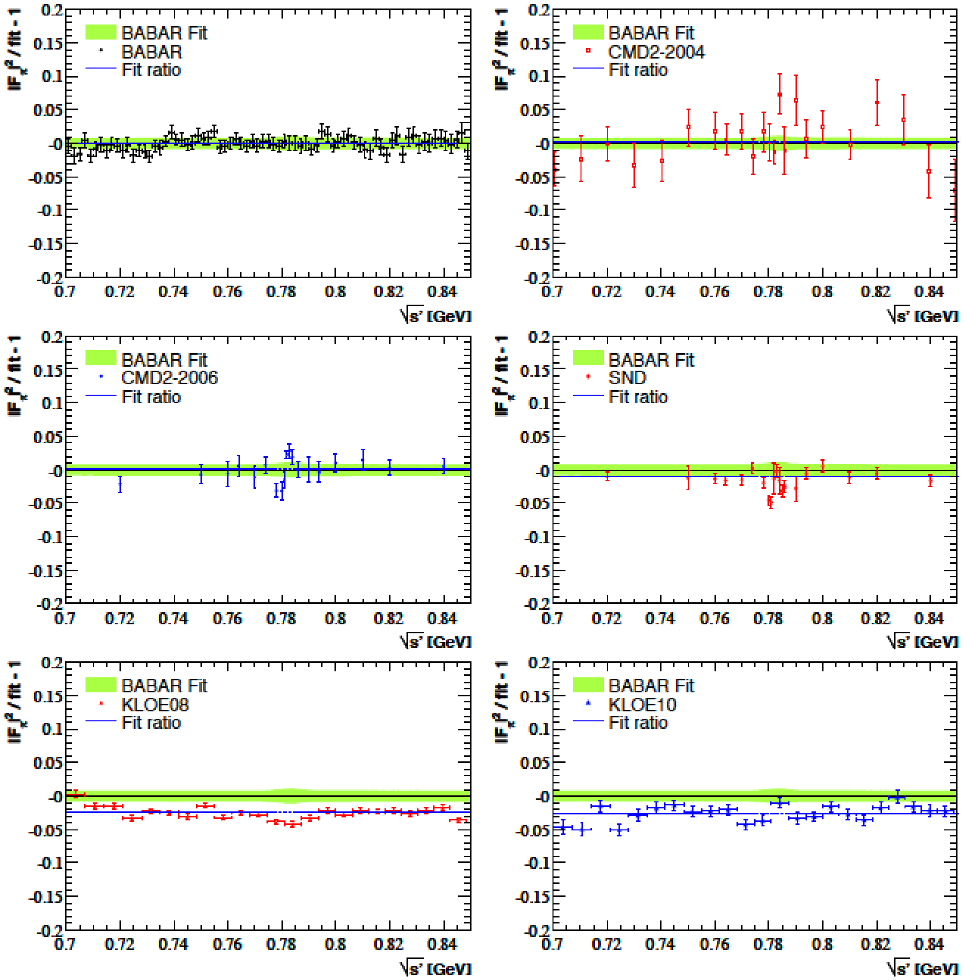
How photons couple to pions? Use $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ as a probe: what we see: 1) below about 1 GeV photons couple to pions as point-like objects (i.e. to the charged ones overwhelmingly), 2) at higher energies the photons see the quarks exclusively and form the prominent tensor resonance $f_2(1270)$. Plotted $2\,\sigma(\pi^0\pi^0)$ vs. $\sigma(\pi^+\pi^-)$

Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$ with photons directly probe the quarks! Contribution to $a_\mu^{\rm had\,LbL}$?

- discrepancy BaBar vs KLOE $\pi\pi$ data. Who can clarify it? BES III radiative return!

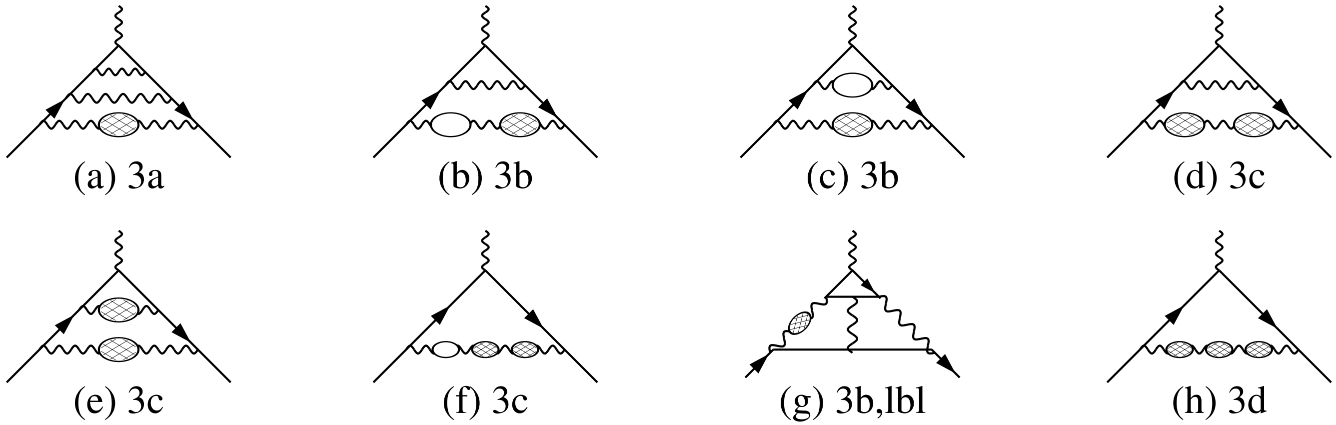


Davier&Malaescu 2013



Davier&Malaescu 2013

NNLO HVP effects

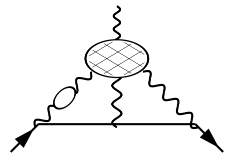


Class		results Kurz et al	my evaluation
$a_\mu^{(3a)}$	=	0.80×10^{-10}	$0.782(77) \times 10^{-10}$
$a_\mu^{(3b)}$	=	-0.41×10^{-10}	$-0.403(37) \times 10^{-10}$
$a_\mu^{(3b,\mathrm{lbl})}$	=	0.91×10^{-10}	$0.900(77) \times 10^{-10}$
$a_\mu^{(3c)}$	=	-0.06×10^{-10}	$-0.0544(7) \times 10^{-10}$
$a_\mu^{(3d)}$	=	0.0005×10^{-10}	$5.22(15) \times 10^{-14}$
$a_\mu^{\mathrm{had,NNLO}}$	=	$12.4(1) \times 10^{-11}$	$12.25(12) \times 10^{-11}$

Kurz et al. 2014

NNLO HLBL effects

New NNLO HLBL — Colangelo et al. 2014



A hadronic light-by-light next to leading order correction, which is of the same order as the NNLO corrections.

is estimated to yield

$$a_\mu^{\pi^- - \mathrm{pole,NLO}} = 1.5 \times 10^{-11}$$

using a simple VMD form-factor, which yields $a_\mu^{\pi^- - \mathrm{pole,LO}} = 57.2 \times 10^{-11}$. Including other contributions gives an estimate:

$$a_\mu^{\mathrm{HLbL,\,NLO}} = (3 \pm 2) \times 10^{-11}$$

as a correction to $a_\mu^{\mathrm{HLbL,\,LO}} = (116 \pm 39) \times 10^{-11}$.