Vortex Solution in Holographic Two-Band Superfluid/Superconductor

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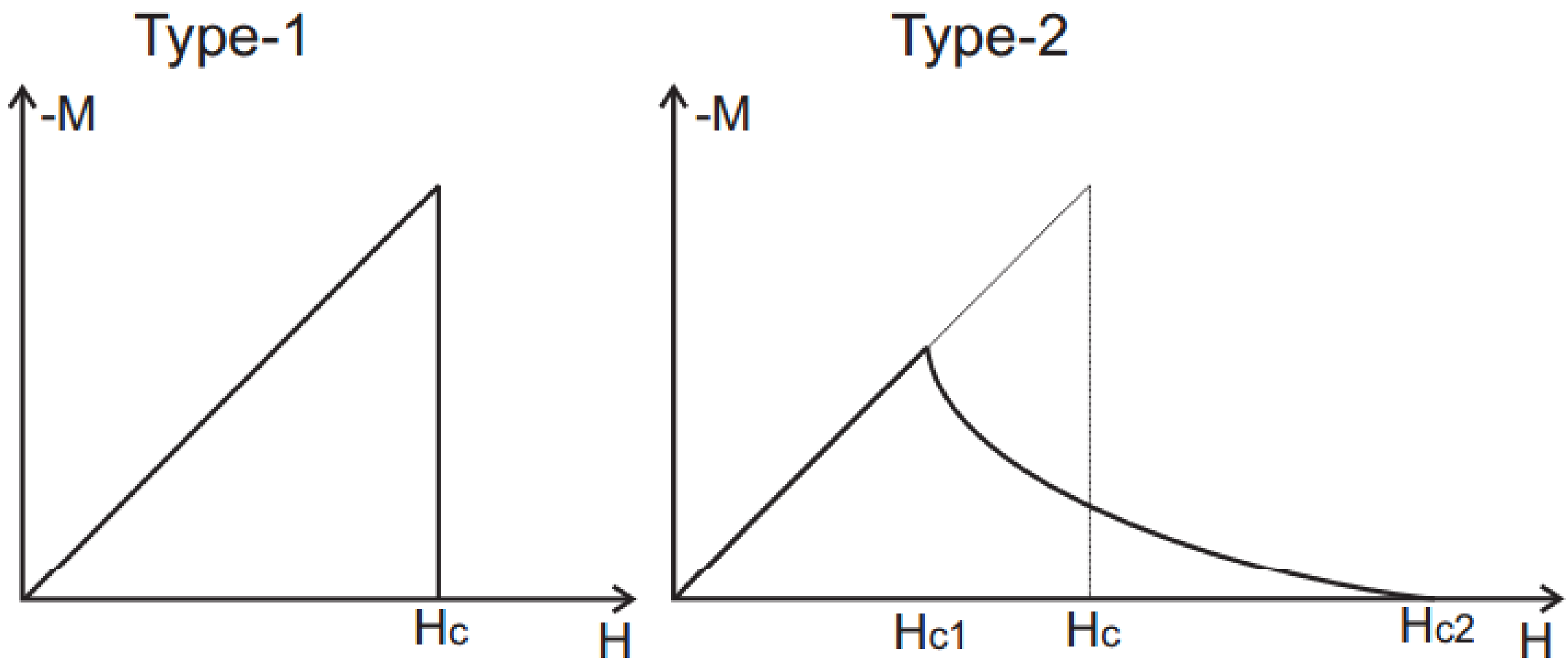
arXiv:1511.01325, M.-S. Wu, Shang-Yu Wu and HQZ

SUPERCONDUCTING VORTEX IN CONDENSED MATTER PHYSICS

* Ginzburg-Landau (GL) parameter κ

Type I	Type II
• Meissner • Normal	• Meissner • Vortex • Normal
$\kappa < 1/\sqrt{2}$	$\kappa > 1/\sqrt{2}$

$\kappa = \lambda/\xi$, λ-penetration length, ξ-coherence length



M- Magnetization

H- External Magnetic Field

* Supercurrent velocity from GL theory

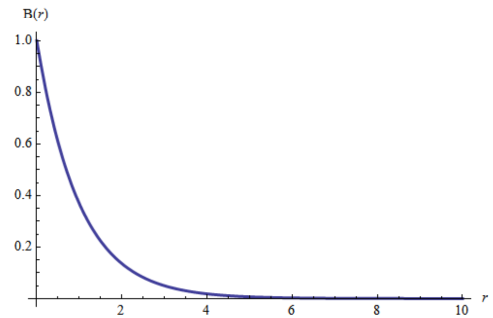
$$m^* v_s = \hbar \nabla \varphi - \frac{e^* A}{c}$$

* Coherence length ξ and penetration length λ near Tc

$$\xi(T) \propto \frac{1}{\sqrt{1 - T/T_c}} \qquad \lambda(T) \propto \frac{1}{\sqrt{1 - T/T_c}}$$

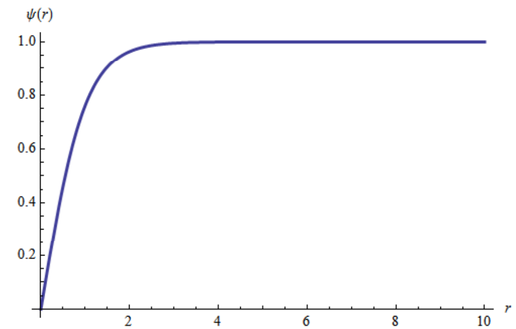
* Magnetic field and penetration length λ

$$B(r) \approx \sqrt{\frac{\lambda}{r}} e^{-r/\lambda}$$



* Condensate and coherence length

$$\psi(r) = \psi_0 \tanh\left(\frac{r}{\sqrt{2}\xi}\right)$$



PRB, **69**, 054508 (2004)

* Two component Ginzburg-Landau (TCGL)

$$F_{GL} = \int dx \Big[\alpha_1 |\Delta_1|^2 + \alpha_2 |\Delta_2|^2 - \gamma(\Delta_1^* \Delta_2 + \Delta_2^* \Delta_1)$$

$$+ \frac{1}{2}\beta_1 |\Delta_1|^4 + \frac{1}{2}\beta_2 |\Delta_2|^4 + K_{1i} |\nabla_i \Delta_1|^2 + K_{2i} |\nabla_i \Delta_2|^2 \Big],$$

$$\nabla_i = \partial_i + i \frac{2\pi}{\Phi_0} A_i$$

γ — Josephson interband coupling

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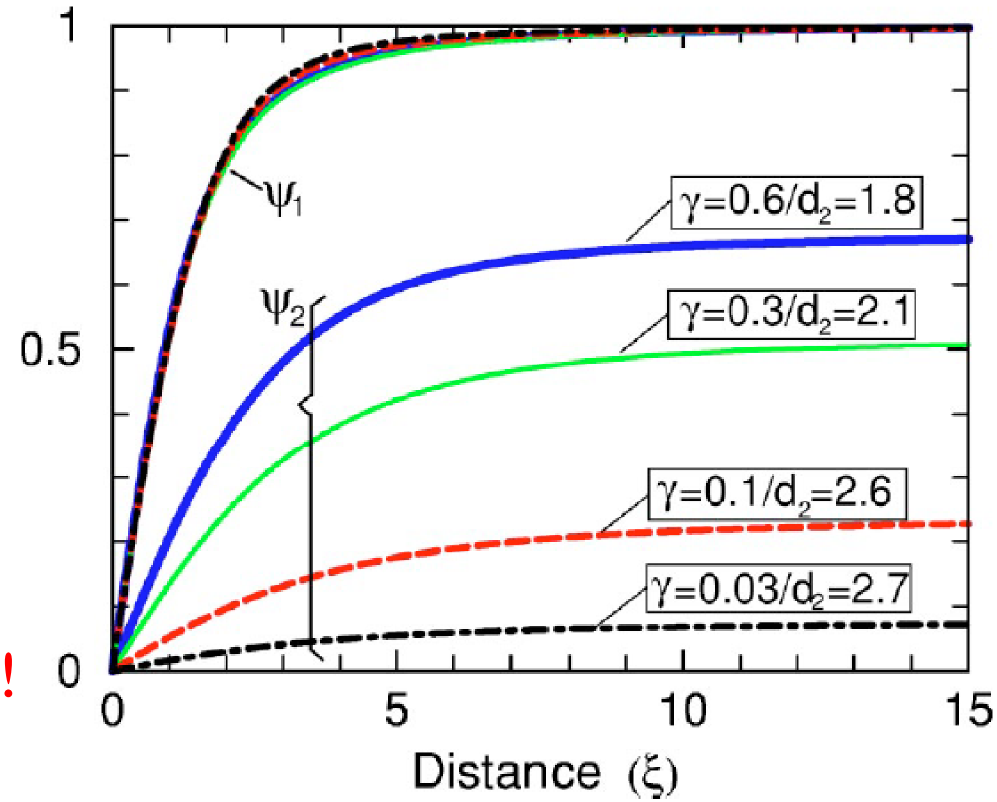

Note: one single vortex solution!

HOLOGRAPHIC STUDY

* Action of the two-band superconductor

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R + \frac{6}{L^2} - \frac{1}{4}F^2 - |\partial\psi_1 - iqA\psi_1|^2 - |\partial\psi_2 - iqA\psi_2|^2 - V(\psi_1,\psi_2)],$$

$$V(\psi_1,\psi_2) = m_1^2|\psi_1|^2 + m_2^2|\psi_2|^2 + \epsilon(\psi_1\psi_2^* + \psi_1^*\psi_2) + \eta|\psi_1|^2|\psi_2|^2$$

ϵ is the intercomponent Josephson coupling

η is a density-density coupling

* 4d AdS-Schwarzchild black hole

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + d\rho^2 + \rho^2 d\phi^2\right), \quad f(r) = 1 - \left(\frac{z}{z_h}\right)^3$$

$z \to 0$ is the infinite boundary

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* Ansatz for vortex

$$\psi_1 = \varphi_1 e^{i\theta_1},\ \psi_2 = \varphi_2 e^{i\theta_2}$$

$$\varphi_1(\rho, z),\ \varphi_2(\rho, z),\ A_t(\rho, z),\ A_\phi(\rho, z),\ \theta_1 = n_1\phi,\ \theta_2 = n_2\phi$$

* Equation of Motions

$$0 = f\partial_z^2 A_t + \frac{\partial_\rho A_t}{\rho} + \partial_\rho^2 A_t - \frac{2q^2 A_t}{z^2}\left({\varphi_1}^2 + \varphi_2^2\right) ,$$

$$0 = \partial_z f \partial_z A_\phi + f\partial_z^2 A_\phi - \frac{\partial_\rho A_\phi}{\rho} + \partial_\rho^2 A_\phi + \frac{2q}{z^2}\varphi_1^2\left(n_1 - qA_\phi\right) + \frac{2q}{z^2}\varphi_2^2\left(n_2 - qA_\phi\right)$$

$$0 = -\frac{\varphi_1}{\rho^2}\left(qA_\phi - n_1\right)^2 + \frac{q^2 {A_t}^2 \varphi_1}{f} - \frac{m_1^2\varphi_1}{z^2} - \frac{\epsilon e^{i(n_2 - n_1)\phi}\varphi_2}{z^2} - \frac{\eta\varphi_1{\varphi_2}^2}{z^2}$$
$$+ \left(\partial_z f - \frac{2f}{z}\right)\partial_z\varphi_1 + f\partial_z^2\varphi_1 + \frac{\partial_\rho\varphi_1}{\rho} + \partial_\rho^2\varphi_1 ,$$

$$0 = -\frac{\varphi_2}{\rho^2}\left(qA_\phi - n_2\right)^2 + \frac{q^2 {A_t}^2 \varphi_2}{f} - \frac{m_2^2\varphi_2}{z^2} - \frac{\epsilon e^{i(n_1 - n_2)\phi}\varphi_1}{z^2} - \frac{\eta\varphi_2{\varphi_1}^2}{z^2}$$
$$+ \left(\partial_z f - \frac{2f}{z}\right)\partial_z\varphi_2 + f\partial_z^2\varphi_2 + \frac{\partial_\rho\varphi_2}{\rho} + \partial_\rho^2\varphi_2 .$$

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$$+\left(\partial_z f - \frac{2f}{z}\right)\partial_z\varphi_1 + f\partial_z^2\varphi_1 + \frac{\partial_\rho\varphi_1}{\rho} + \partial_\rho^2\varphi_1\ ,$$

$$n_1 = n_2 = n$$

$$0 = -\frac{\varphi_2}{\rho^2}\left(qA_\phi - n_2\right)^2 + \frac{q^2{A_t}^2\varphi_2}{f} - \frac{m_2^2\varphi_2}{z^2} - \frac{\epsilon e^{i(n_1-n_2)\phi}\varphi_1}{z^2} - \frac{\eta\varphi_2{\varphi_1}^2}{z^2}$$
$$+\left(\partial_z f - \frac{2f}{z}\right)\partial_z\varphi_2 + f\partial_z^2\varphi_2 + \frac{\partial_\rho\varphi_2}{\rho} + \partial_\rho^2\varphi_2\ .$$

* Boundary condition (B.C.) near z ~ 0

$$\varphi_i(\rho, z)|_{z\to 0} = \varphi_i^{(1)}(\rho) z^{3-\Delta_i} + \varphi_i^{(2)}(\rho) z^{\Delta_i}, \quad i = 1, 2,$$

$$A_t(\rho, z)|_{z\to 0} = \mu(\rho) - \varrho(\rho) z,$$

$$A_\phi(\rho, z)|_{z\to 0} = a_\phi(\rho) + J_\phi(\rho) z.$$

* B.C. near horizon

At=0, regular b.c. for other fields

* Numerical parameters:

$$q = L = 1 \qquad m_1^2 = -2, m_2^2 = -5/4 \qquad \boxed{\eta = 0}$$

↓

$$\Delta_1 = 2, \Delta_2 = 5/2$$

Holographic Superfluid Vortex

* B.C. at vortex core ρ=0

$$\varphi|_{\rho\to 0} = 0, \quad \partial_\rho A_t|_{\rho=0} = 0, \quad A_\phi|_{\rho=0} = 0, \quad \text{for } n \neq 0$$

$$\partial_\rho \varphi_{\rho=0} = 0 \qquad \ldots \qquad \ldots \qquad \text{for } n = 0$$

* B.C. at z=0

$$\varphi_i^{(1)} \equiv 0$$

* Dirichlet B.C. at z=0

$$A_\phi|_{z=0} = a_\phi(\rho) = \frac{1}{2}\rho^2 B$$

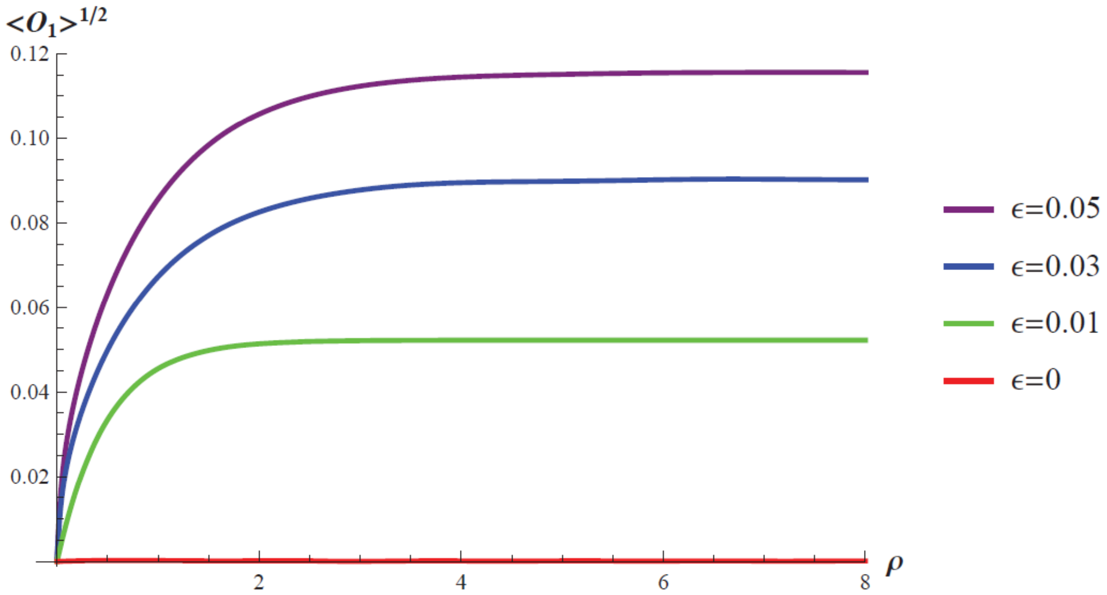
* B.C. at vortex border ρ=R

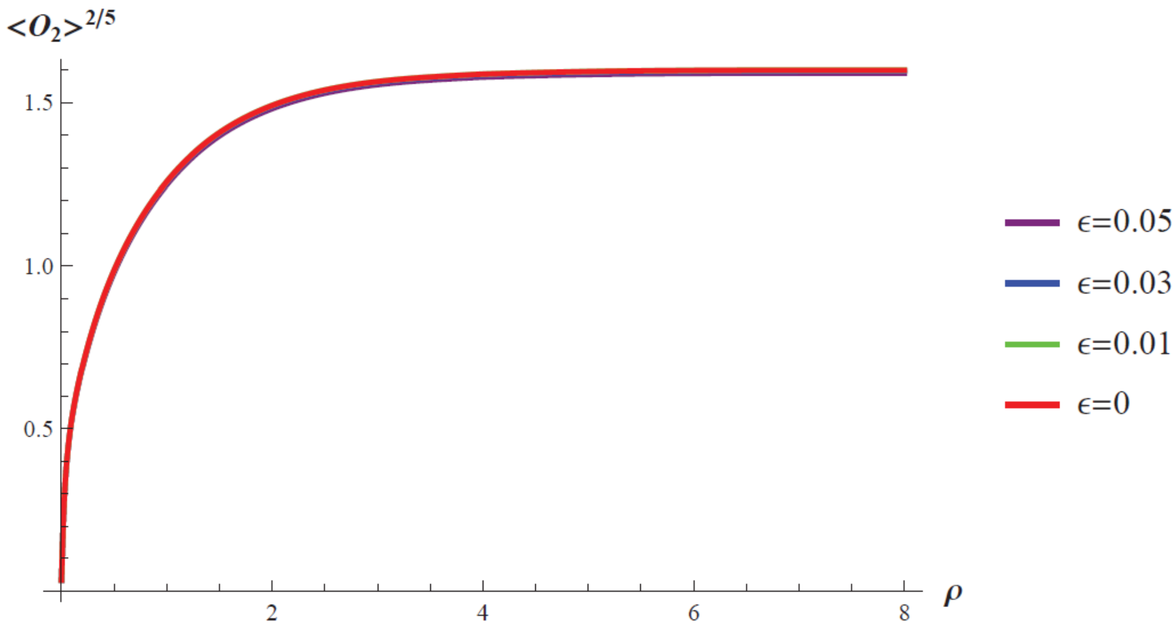
$$A_\phi|_{\rho=R} = \frac{1}{2}BR^2$$

← Angular velocity

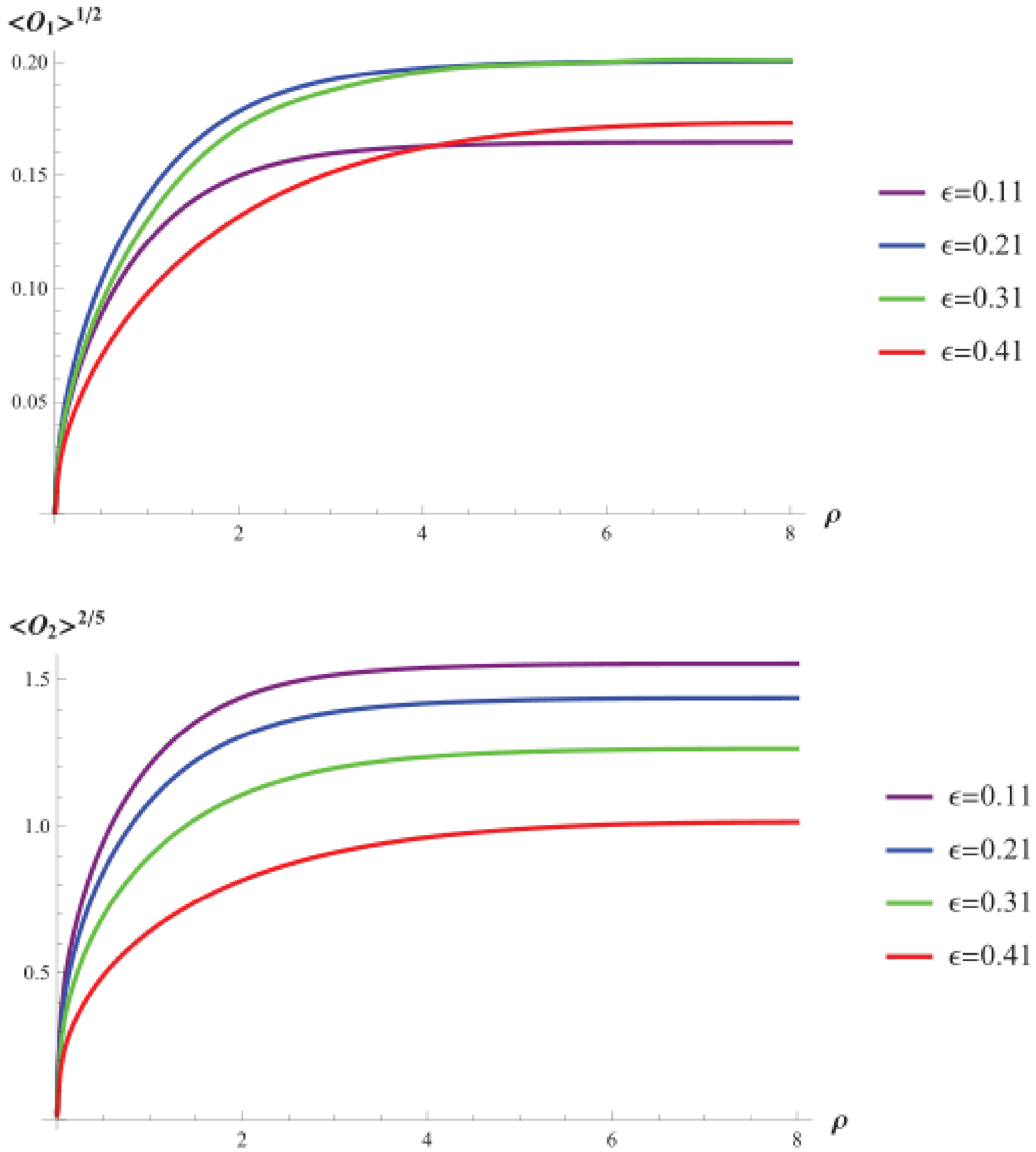
Homogenerous b.c. for At and φ_i

For small ϵ





For $\epsilon \geq 0.1$



Free Energy

* On-shell action

$$S_{os} = \frac{1}{2\kappa^2}\left(-\frac{1}{2}\right)\int d^4x \partial_a \left[\sqrt{-g}\left(A_b F^{ab} + \psi_1^* \partial^a \psi_1 + \psi_1 \partial^a \psi_1^* + \psi_2^* \partial^a \psi_2 + \psi_2 \partial^a \psi_2^*\right)\right]$$
$$+\frac{1}{2\kappa^2}\frac{iq}{2}\int d^4x \sqrt{-g} A_b \left[\psi_1^*\left(\partial^b - iqA^b\right)\psi_1 - \psi_1\left(\partial^b + iqA^b\right)\psi_1^* \right.$$
$$\left. +\psi_2^*\left(\partial^b - iqA^b\right)\psi_2 - \psi_2\left(\partial^b + iqA^b\right)\psi_2^*\right]$$
$$+\frac{\eta}{2\kappa^2}\int d^4x \sqrt{-g}|\psi_1|^2|\psi_2|^2.$$

* Counter term

$$S_{ct} = \frac{-1}{2\kappa^2}\int d^3x \sqrt{-\gamma}\left(\psi_1 \psi_1^*\right) + \frac{-1/2}{2\kappa^2}\int d^3x \sqrt{-\gamma}\left(\psi_2 \psi_2^*\right)$$

Free Energy

* On-shell action

$$S_{os} = \frac{1}{2\kappa^2}\left(-\frac{1}{2}\right)\int d^4x\partial_a\left[\sqrt{-g}\left(A_bF^{ab}+\psi_1^*\partial^a\psi_1+\psi_1\partial^a\psi_1^*+\psi_2^*\partial^a\psi_2+\psi_2\partial^a\psi_2^*\right)\right]$$
$$+\frac{1}{2\kappa^2}\frac{iq}{2}\int d^4x\sqrt{-g}A_b\left[\psi_1^*\left(\partial^b-iqA^b\right)\psi_1-\psi_1\left(\partial^b+iqA^b\right)\psi_1^*\right.$$
$$\left.+\psi_2^*\left(\partial^b-iqA^b\right)\psi_2-\psi_2\left(\partial^b+iqA^b\right)\psi_2^*\right]$$
$$+\frac{\eta}{2\kappa^2}\int d^4x\sqrt{-g}|\psi_1|^2|\psi_2|^2.$$

* Counter term

$$S_{ct} = \frac{-1}{2\kappa^2}\int d^3x\sqrt{-\gamma}\left(\psi_1\psi_1^*\right)+\frac{-1/2}{2\kappa^2}\int d^3x\sqrt{-\gamma}\left(\psi_2\psi_2^*\right)$$

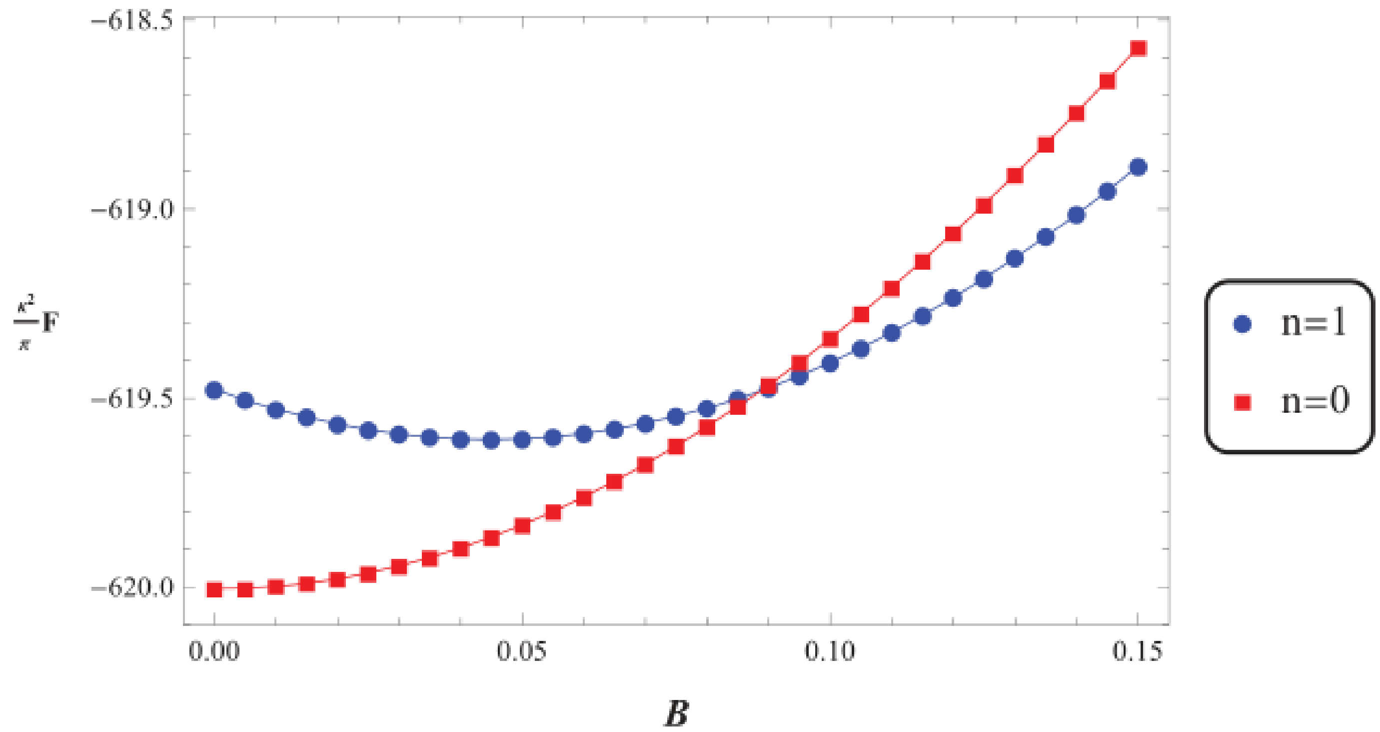
* Regularized free energy

$$F = -TS_{reg.} = -T\left(S_{os} + S_{ct}\right)$$

$$= \frac{-T}{2\kappa^2}\int dt d\phi \left[\int d\rho\rho\left(\frac{\varrho\mu}{2}+\frac{BJ_\phi}{4}\right)\Big|_{z=0} - \int dz \frac{A_\phi \partial_\rho A_\phi}{2\rho}\Big|_{\rho=R}\right.$$

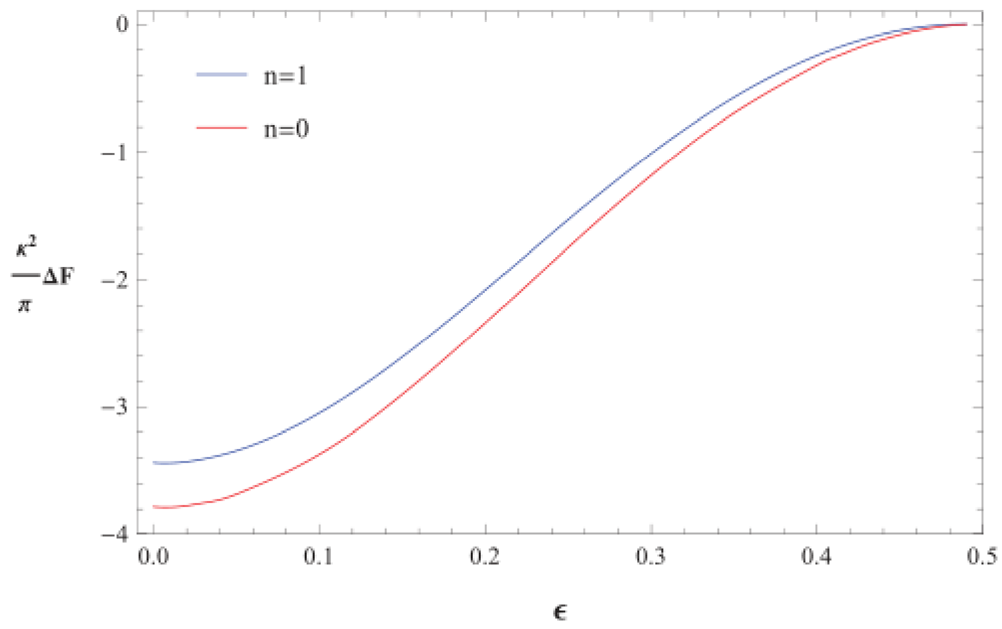
$$\left. + \int dz d\rho \frac{\rho}{z^2}\left(-\frac{q^2 A_t^2(\varphi_1^2+\varphi_2^2)}{f} - \frac{qA_\phi}{\rho^2}\left(\varphi_1^2\left(n_1 - qA_\phi\right) + \varphi_2^2(n_2 - qA_\phi)\right) + \frac{\eta\varphi_1^2\varphi_2^2}{z^2}\right)\right]$$

* Critical angular velocity B_{c1}

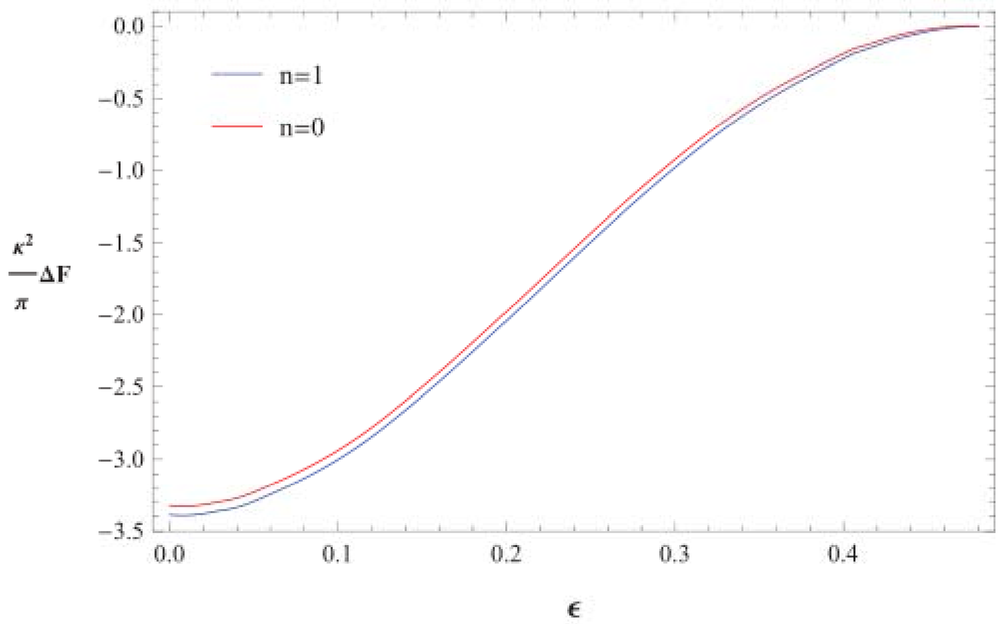


* For n=1

$$\Delta F = F(\varphi_i \neq 0) - F(\varphi_i = 0)$$



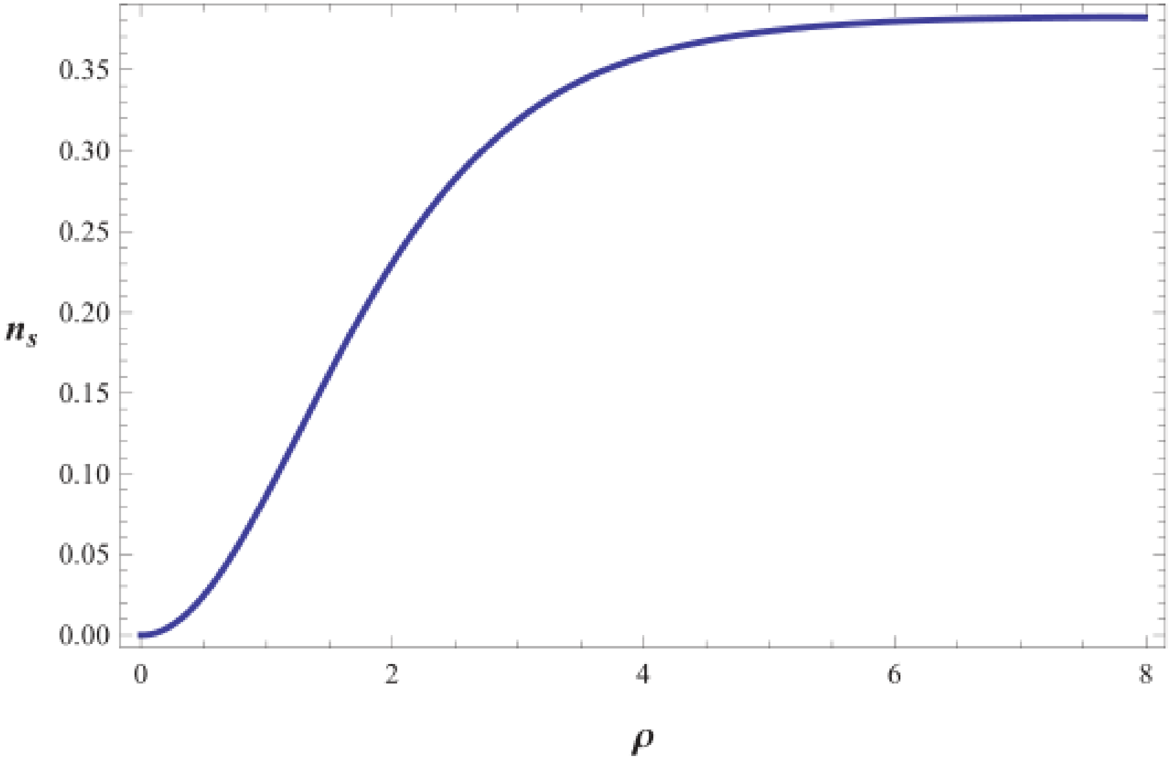
$$B = 0.03 < B_{c1}$$



$$B = 0.1 > B_{c1}$$

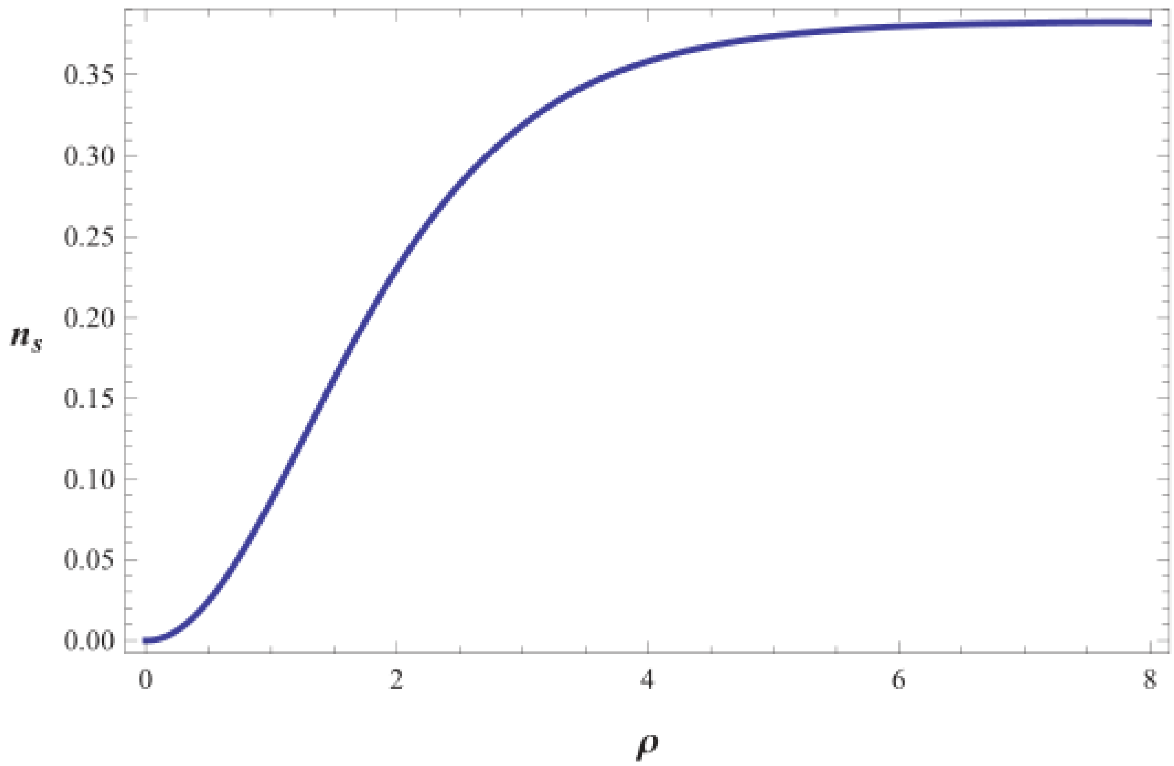
* Superfluid density and current

$$n_s = \frac{J_\phi}{n - a_\phi}$$



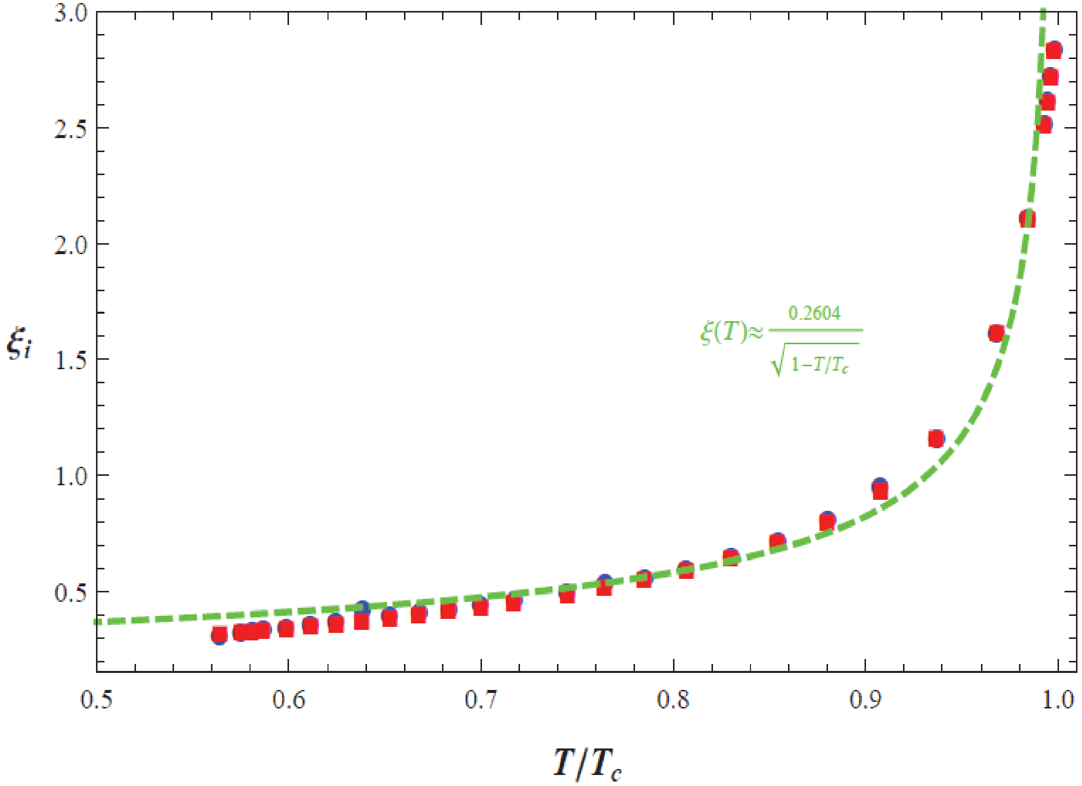
* Superfluid density and current

$$n_s = \frac{J_\phi}{n - a_\phi} \quad \Longleftarrow \quad v_\phi = \nabla_\phi \varphi_i - A_\phi$$



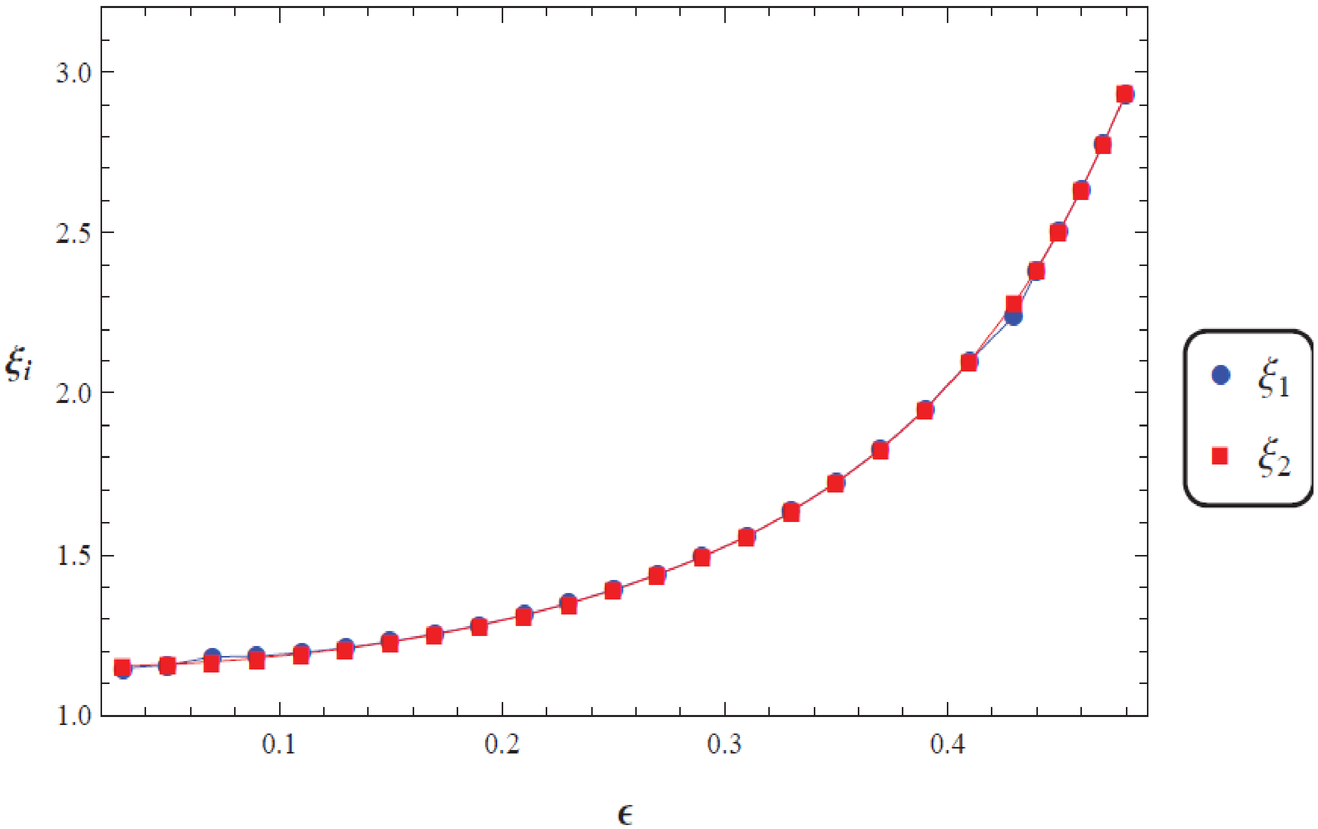
* Coherence length vs. T

$$\langle O_i(\rho)\rangle = O_i(\infty)\tanh\left(\frac{\rho}{\sqrt{2}\xi_i}\right)$$



$$\xi_i(T) = 0.2604(1 - T/T_c)^{-1/2}$$

* Dependence on Josephson coupling



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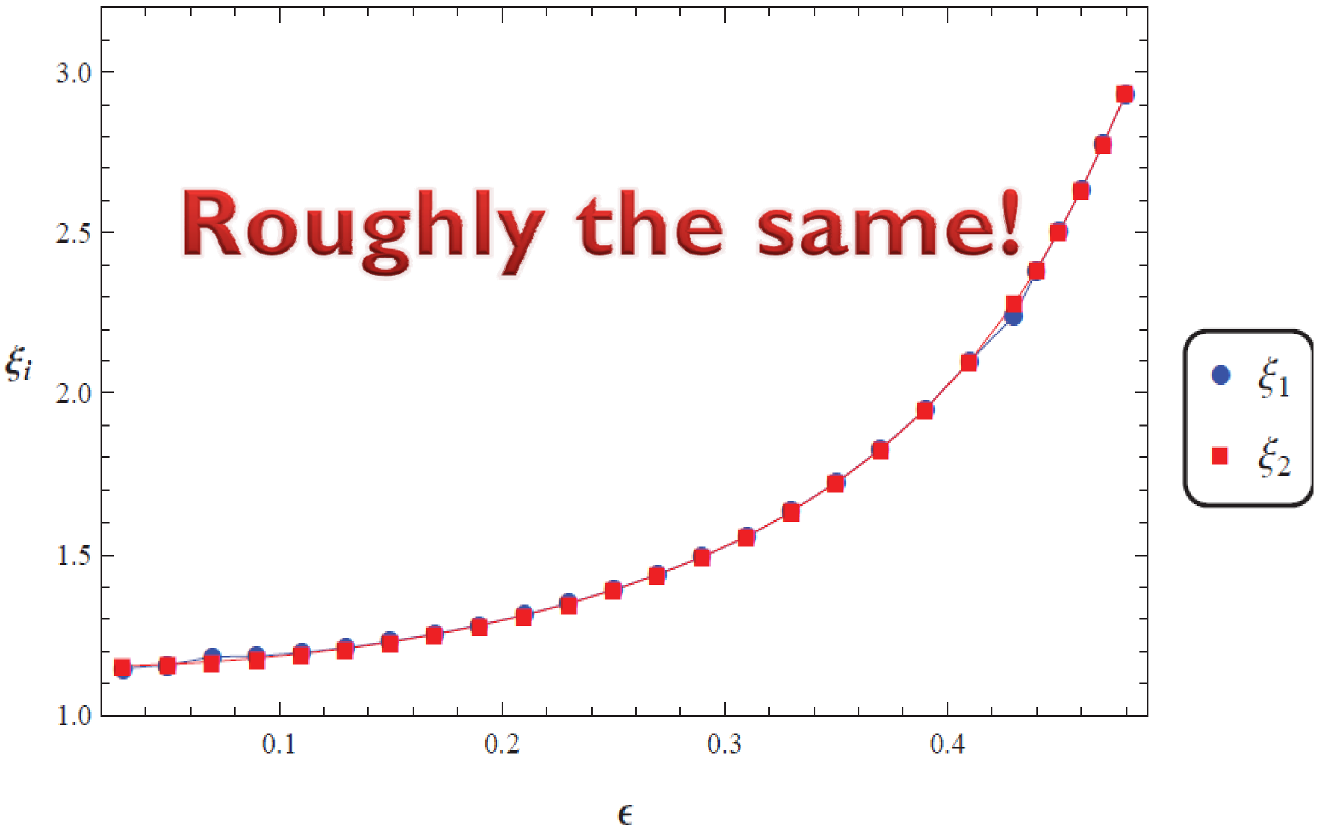
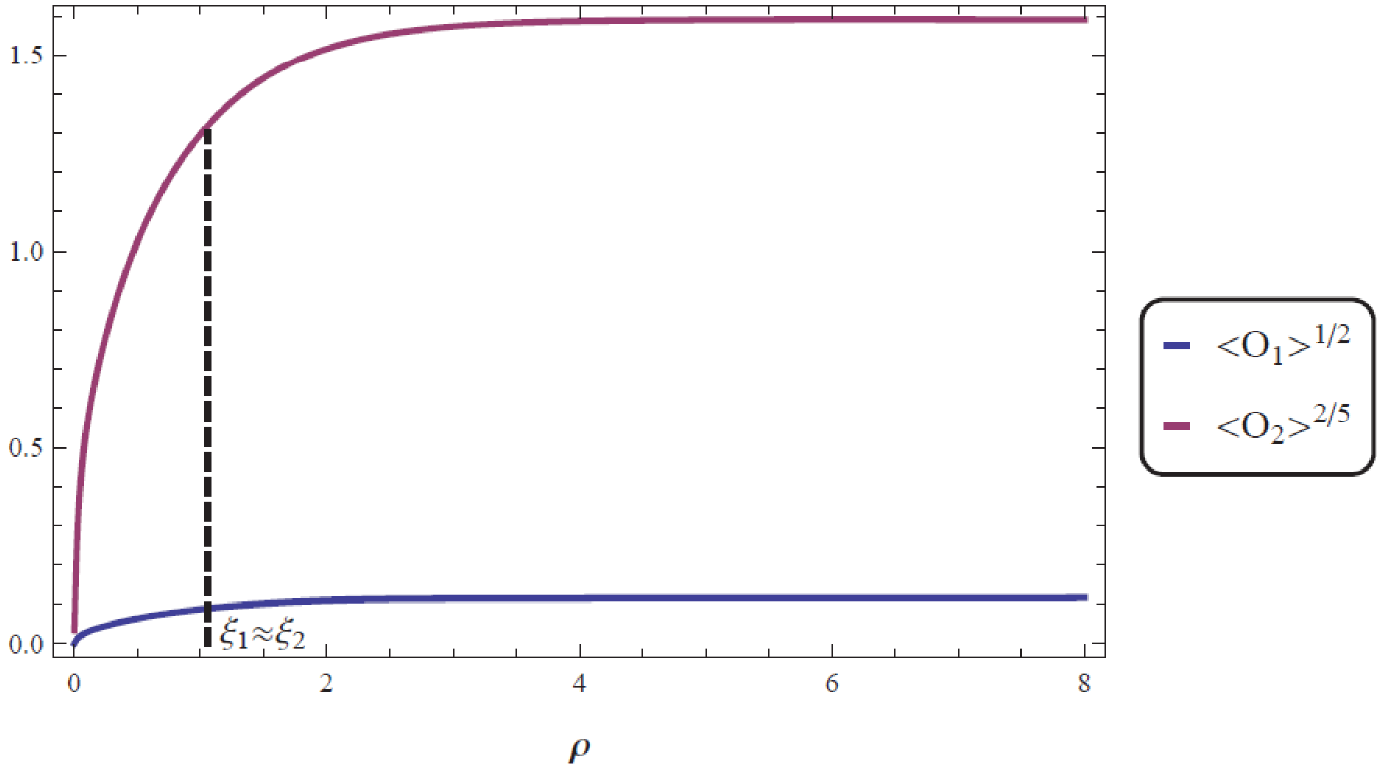


Holographic Superconductor Vortex

* Neumann B.C. at z=0

$$\partial_z A_\phi(\rho, z) = J_\phi(\rho) = 0$$



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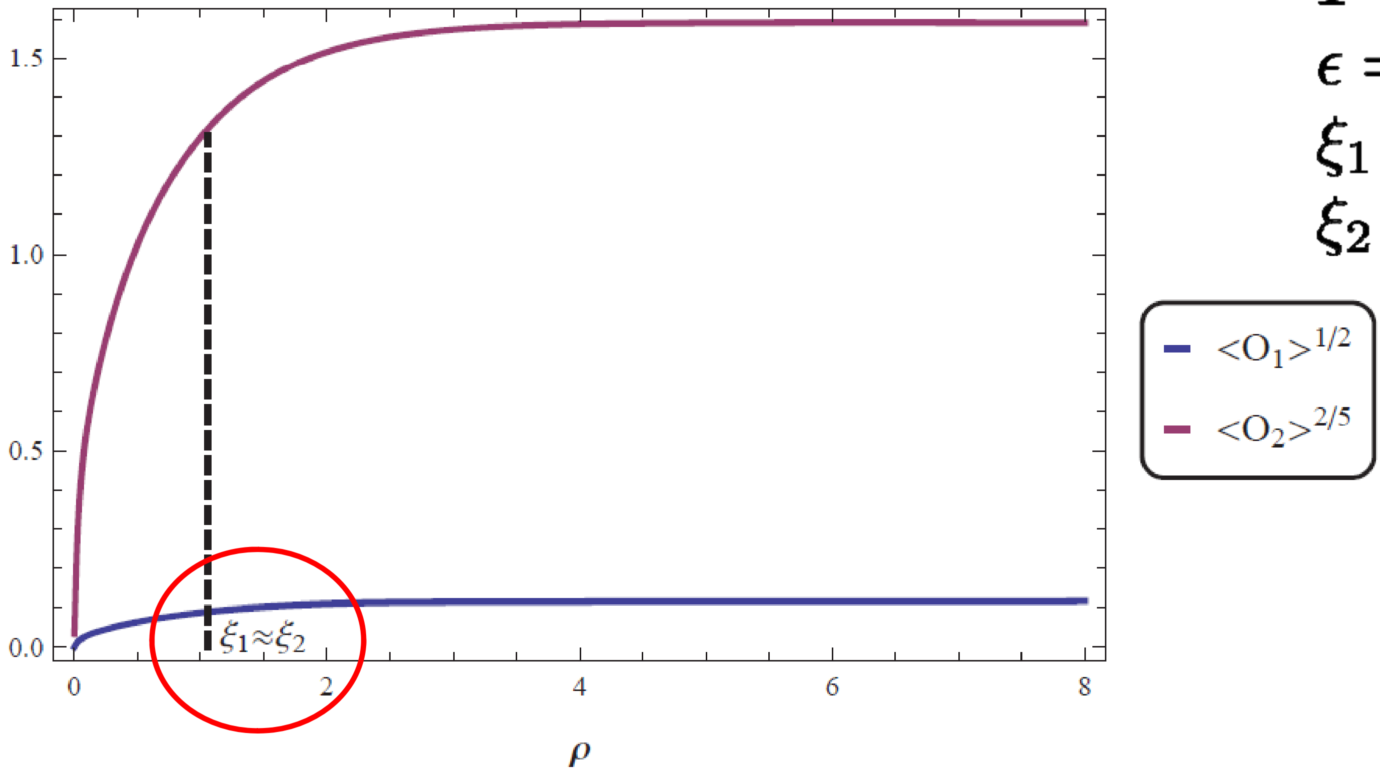
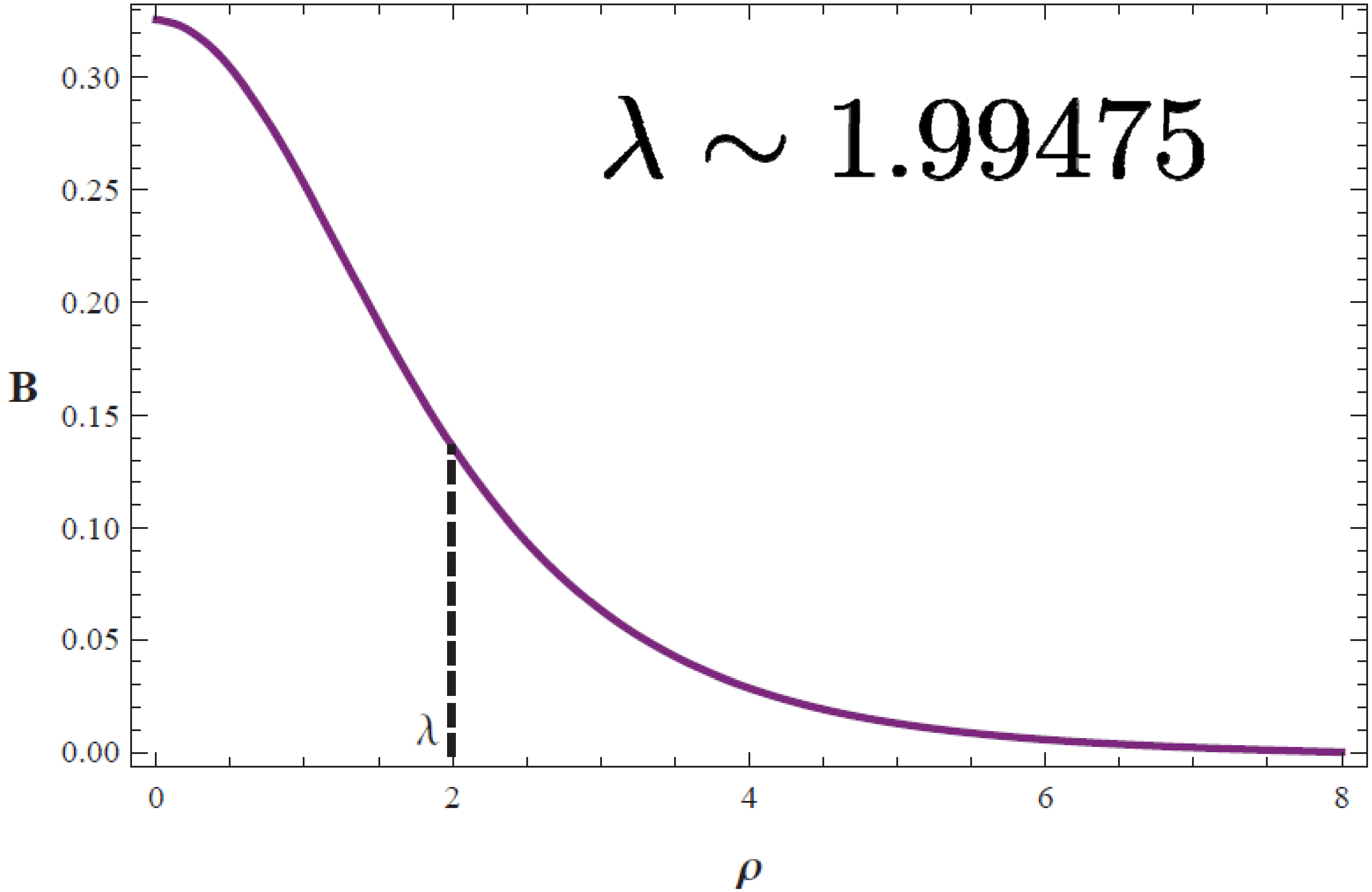


$T = 0.937T_c$
$\epsilon = 0.05$
$\xi_1 \sim 1.05687$
$\xi_2 \sim 1.01591$

* Magnetic field B $B = be^{-\rho/\lambda}$



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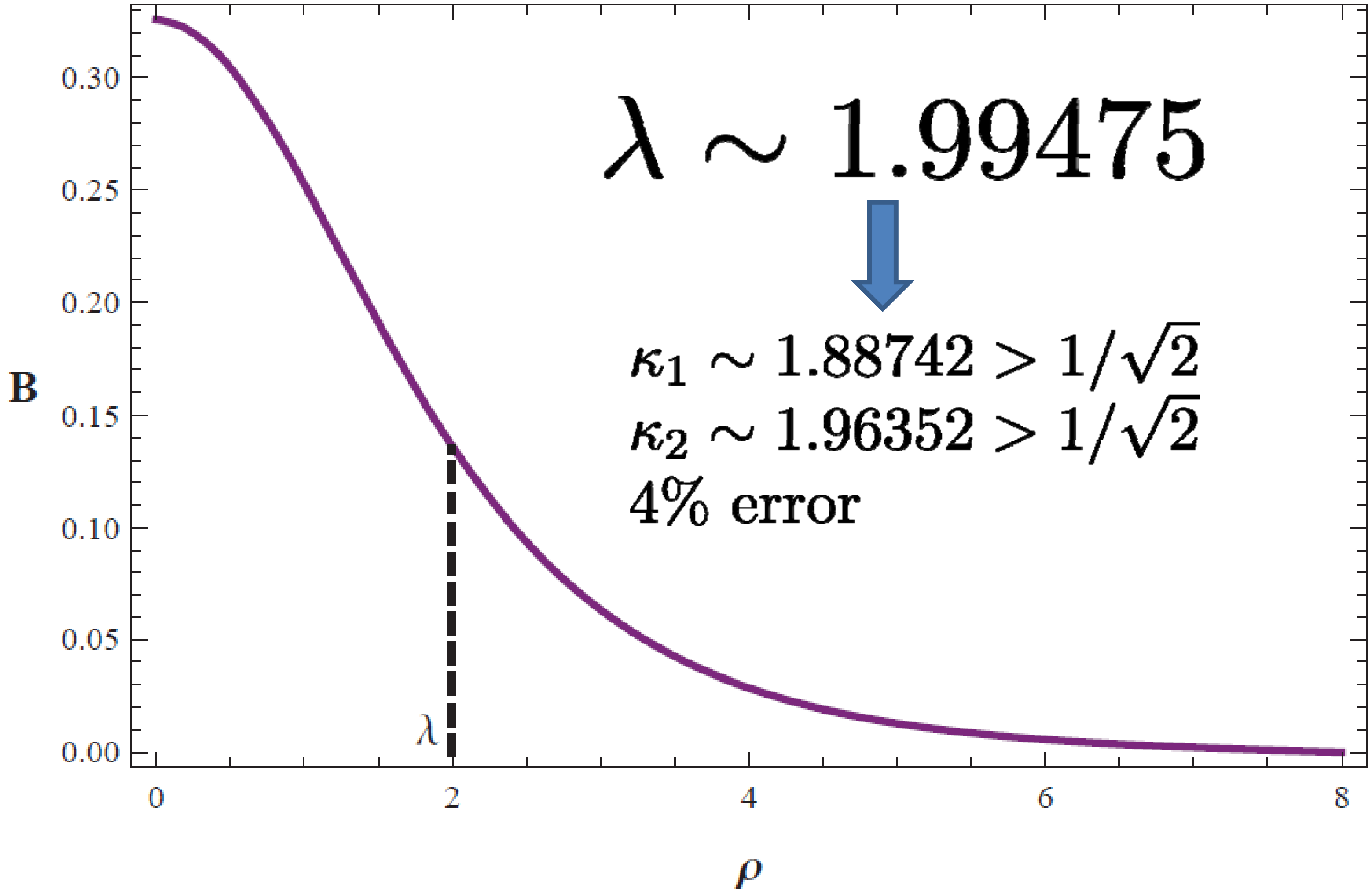

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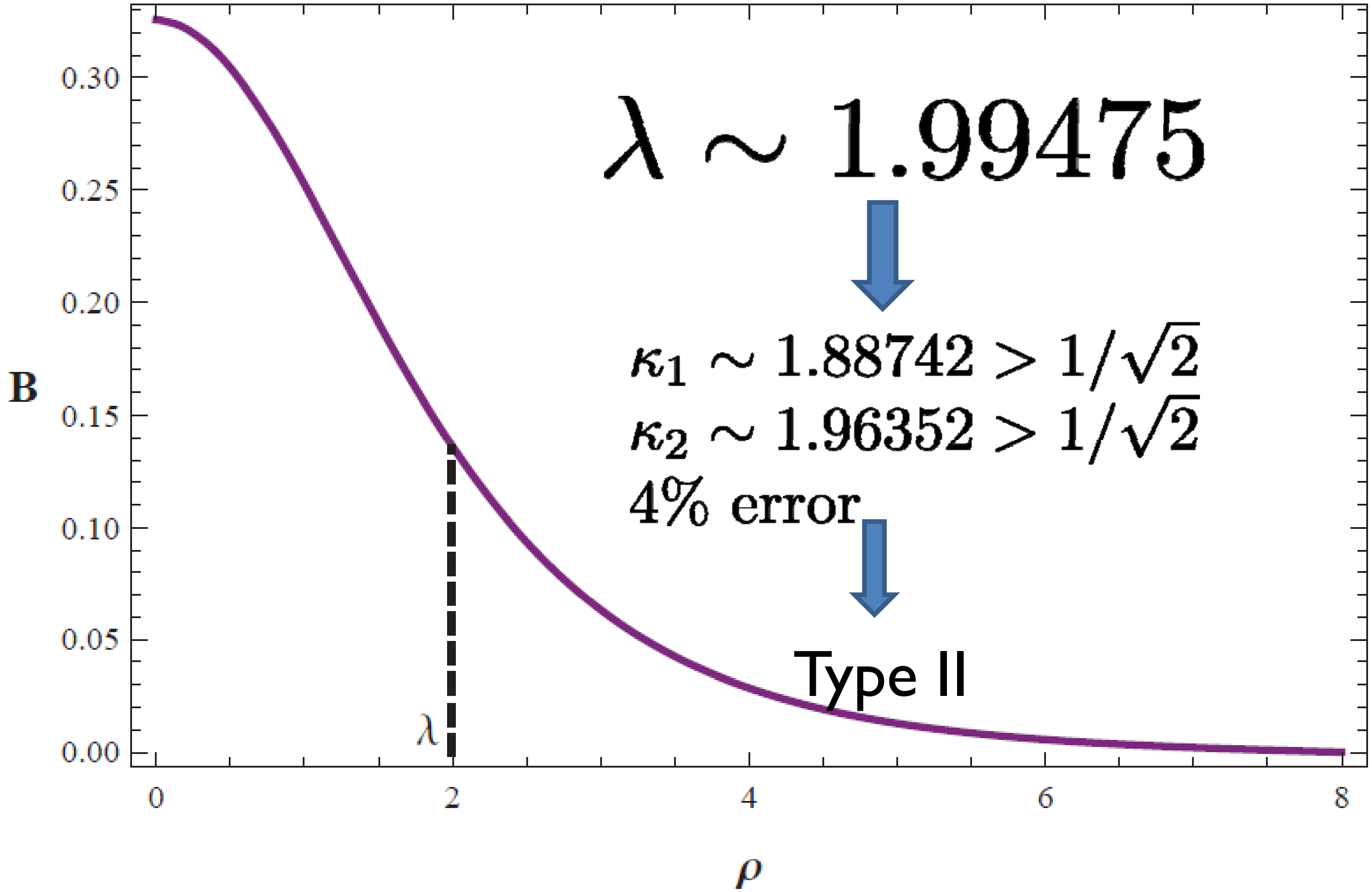
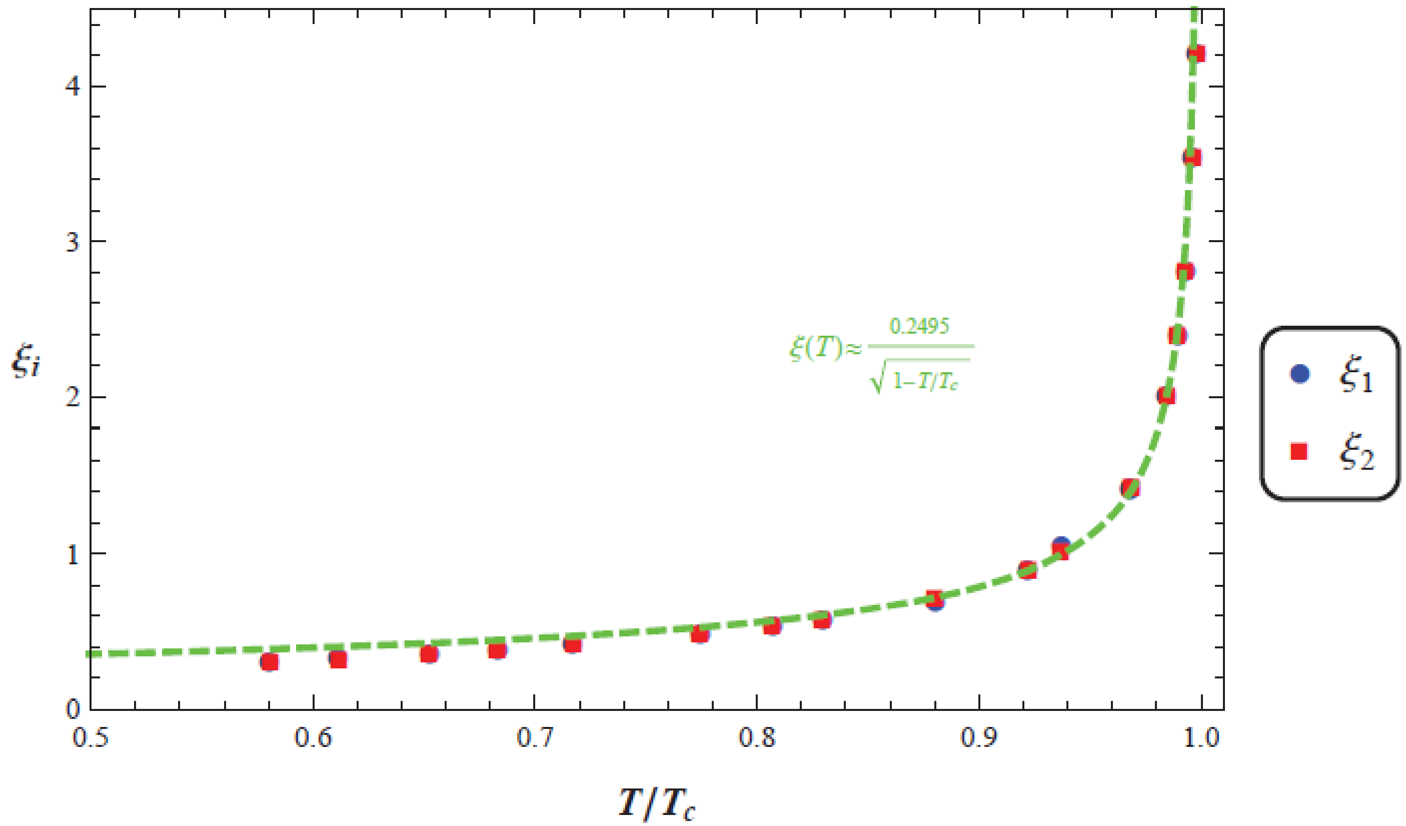
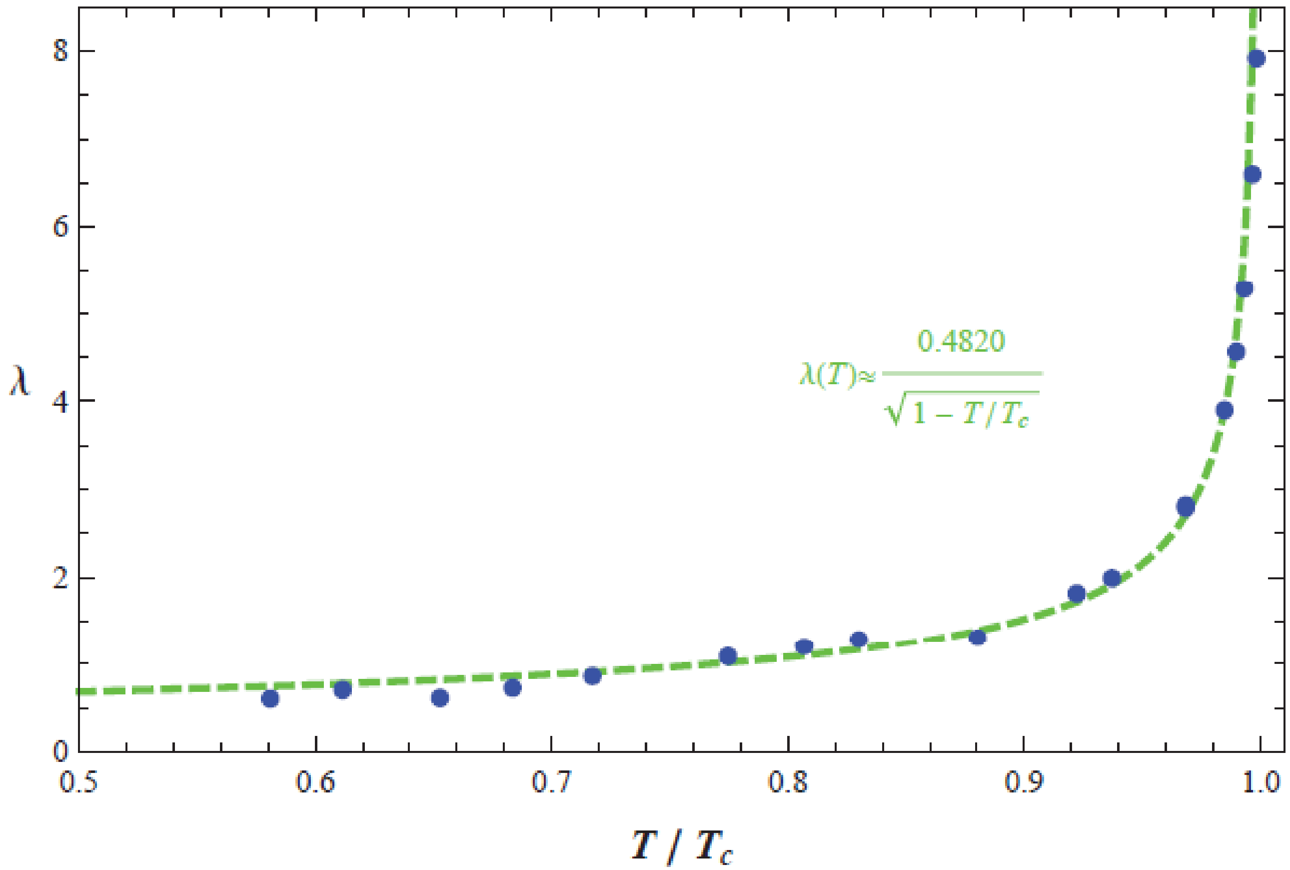


* Coherence length vs. T



* Penetration length vs. T



$$\kappa_{1,2} > 1/\sqrt{2}$$

SUMMARIES & OUTLOOK

Summaries

- Studied the angular velocity response of superfluid vortex;
- The magnetic response of sueprconductor vortex;
- Effects of Josephon coupling and temperature on vortex;
- By calculating the coherence length and penetration length, we get the type II superconductor vortex.

Outlook

Type 1.5

- Meissner
- Semi-Meissner
- Vortex
- Normal

$$\kappa_1 < 1/\sqrt{2},$$

$$\kappa_2 > 1/\sqrt{2}$$

Outlook

Type 1.5

- Meissner
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H

$$\kappa_1 < 1/\sqrt{2},$$
$$\kappa_2 > 1/\sqrt{2}$$

Selected for a Viewpoint in *Physics*

PRL **102,** 117001 (2009) PHYSICAL REVIEW LETTERS week ending 20 MARCH 2009

Type-1.5 Superconductivity

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[1]*INPAC-Institute for Nanoscale Physics and Chemistry, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*

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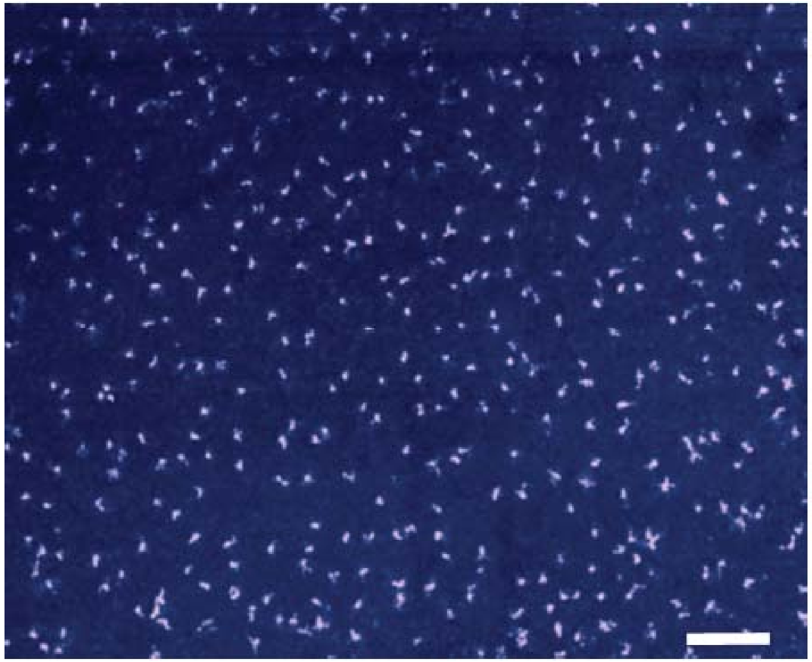
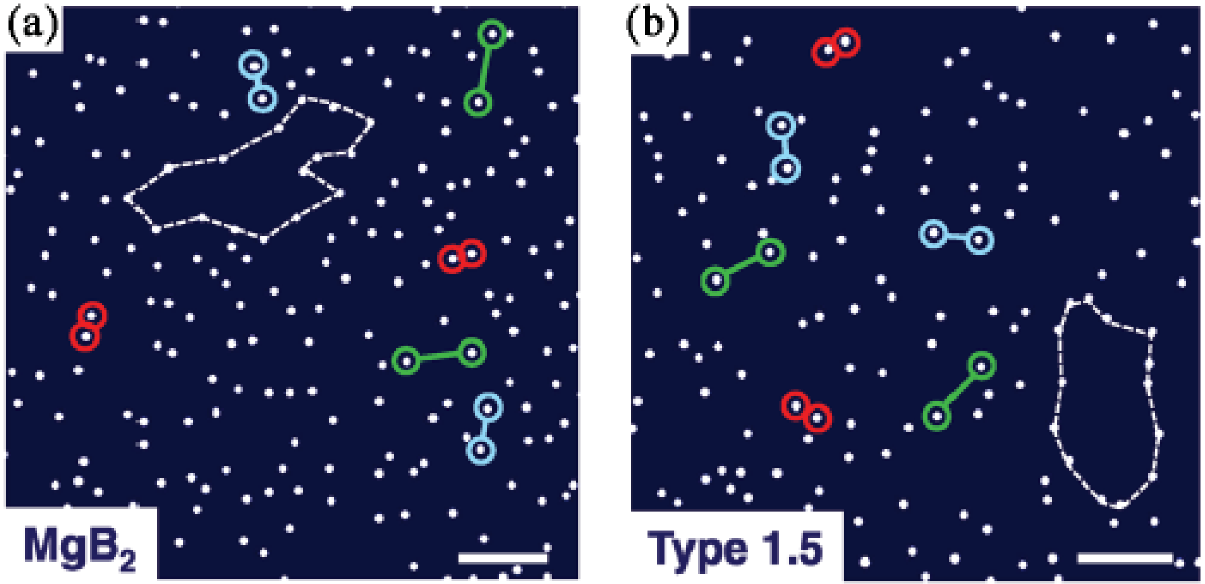
(Received 22 December 2008; published 16 March 2009)

We demonstrate the existence of a novel superconducting state in high quality two-component MgB_2 single crystalline superconductors where a unique combination of both type-1 ($\lambda_1/\xi_1 < 1/\sqrt{2}$) and type-2 ($\lambda_2/\xi_2 > 1/\sqrt{2}$) superconductor conditions is realized for the two components of the order parameter. This condition leads to a vortex-vortex interaction attractive at long distances and repulsive at short distances, which stabilizes unconventional stripe- and gossamerlike vortex patterns that we have visualized in this type-1.5 superconductor using Bitter decoration and also reproduced in numerical simulations.

$$\kappa_\pi = 0.66 < 1/\sqrt{2} \quad \text{(type I)}$$
$$\kappa_\sigma = 3.68 > 1/\sqrt{2} \quad \text{(type II)}$$



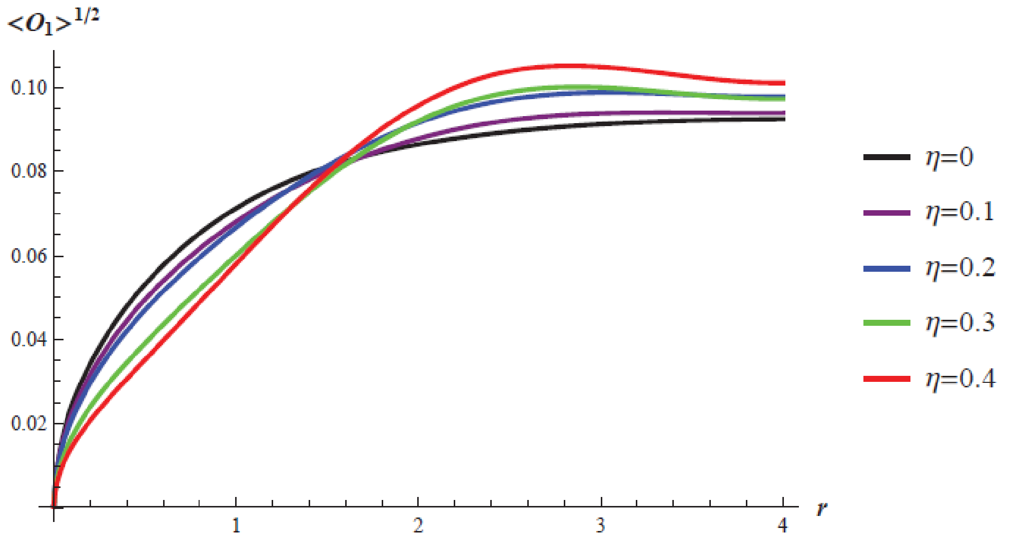
Vortex clusters

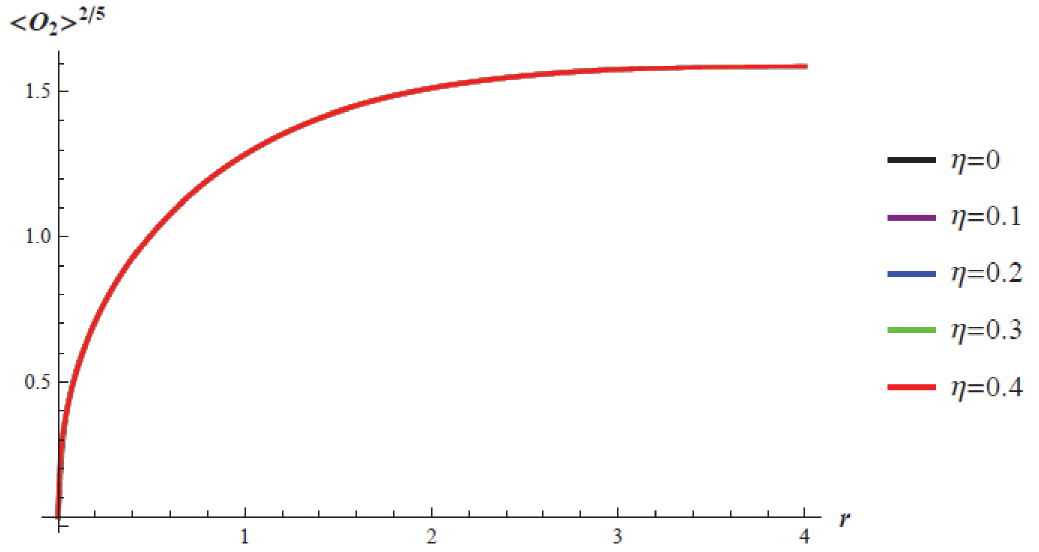


Features of type 1.5

- Long-range attractive , short-range repulsive
- Semi-Meissner: mixtures of Meissner and vortex
- Non-axialsymmetric vortex
- Vortex clusters

- The effects of η





Thank you!