Phenomenology of Semileptonic $B$-meson Decays with Form Factors from Lattice QCD

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Why Exclusive Semileptonic Decays?

• Rare processes are sensitive to non-Standard physics: leptoquarks, Z\', 4th generation, non-Standard Higgs bosons, supersymmetry.

• Several “tensions”:
  • CKM from inclusive vs. exclusive decays;
  • excess in $B \rightarrow D(\ast)\tau\nu$;
  • deficits in $B \rightarrow K(\ast)\mu\mu$.

• Experimental results available; more on the way.

• Nonperturbative hadronic matrix elements available (with full error budgets).
Energy Scales

TeV New Physics

Electroweak (Higgs, top, $W$, $Z$)

Bottom quark

Hadronic physics: $\Lambda_{QCD}$

Lattice QCD
### Processes

<table>
<thead>
<tr>
<th>CKM Determination</th>
<th>New Physics Search</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow \pi \ell \nu$</td>
<td>$B \rightarrow \pi \tau \nu$</td>
</tr>
<tr>
<td>$B \rightarrow D \ell \nu$</td>
<td>$B \rightarrow D \tau \nu$</td>
</tr>
<tr>
<td>$B \rightarrow D^* \ell \nu$</td>
<td>$B \rightarrow K \nu \bar{\nu}$</td>
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(in pedagogical order)

$l = \mu, e$
Effective Hamiltonian

- Masses of $W$, $Z$, top, and Higgs are much greater than $m_b$:

$$\mathcal{L} = \mathcal{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i C_i(\alpha, \alpha_s, G_F, \sin^2 \theta, m_\ell, m_q, V; \text{NP}) \mathcal{L}_i[\ell, q, \gamma, g]$$

- Contributions of unknown particles lumped into Wilson coefficients $C_i$.

- We use SM Wilson coeff’s of Huber, Lunghi, Misiak, Wyler [hep-ph/0512066].
Matrix Elements and Form Factors

- Decompose amplitudes in form factors ($q = p - k = \ell + \nu$):

\[
\langle \pi(k)|\bar{u}\gamma^\mu b|B(p)\rangle = \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) f_+(q^2) + \frac{M_B^2 - M_\pi^2}{q^2} q^\mu f_0(q^2),
\]

\[
= \sqrt{2M_B} \left[ p^\mu f_+ (q^2) / M_B + k^\mu_\perp f_-(q^2) \right]
\]

\[
\langle \pi(k)|\bar{u}\sigma^{\mu\nu} b|B(p)\rangle = -2i \frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2),
\]

\[
\langle \pi(k)|\bar{u}b|B(p)\rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u} f_0(q^2),
\]

- The kinematic variable $q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$. 

PCVC: same
Computing Hadronic Matrix Elements

• Use correlation functions \( x_4 = it \) is “Euclidean time”:

\[
\langle \pi(x_4) J(x_4') B^\dagger(0) \rangle = \sum_{m,n} \langle 0 | \hat{\pi} | \pi_n \rangle \langle \pi_n | \hat{J} | B_m \rangle \langle B_m | \hat{B}^\dagger | 0 \rangle e^{-M_{\pi n} (x_4 - x_4') - M_{B m} x_4'}
\]

in which desired matrix element lies in the middle (of leading term).

• Find other information from two-point correlation functions:

\[
\langle \pi(x_4) \pi^\dagger(0) \rangle = \sum_n \langle 0 | \hat{\pi} | \pi_n \rangle |^2 e^{-M_{\pi n} x_4}
\]

\[
\langle B(x_4) B^\dagger(0) \rangle = \sum_n \langle 0 | \hat{B} | B_n \rangle |^2 e^{-M_{B n} x_4}
\]

• Compute LHS with lattice QCD; use humans to analyze RHS.
Outline

• Motivation

• Lattice QCD Calculations

• $|V_{ub}|$ and $|V_{cb}|$

• Penguin Processes

• Outlook

arXiv:1510.02349
Fermilab Lattice and MILC Collaborations


with special guest

Enrico Lunghi
Numerical Lattice QCD
Lattice Gauge Theory

\[ \langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp (-S) [\bullet] \]

\[ = \frac{1}{Z} \int \mathcal{D}U \ Det(\mathcal{D} + m) \exp (-S_{\text{gauge}}) [\bullet'] \]

- Infinite continuum: uncountably many d.o.f. \((\Rightarrow\) UV divergences);

- Infinite lattice: countably many; used to define QFT;

- Finite lattice: finite dimension \(\sim 10^9\), so compute integrals numerically.
Lattice Gauge Theory

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\[ L = N_s a \]

\[ L_4 = N_s a \]
Some algorithmic issues

e.g., ASK, hep-lat/0205021

• lattice $N^3_S \times N_4$, spacing $a$

• memory $\propto N^3_S N_4 = L_S^3 L_4 / a^4 $

• $\tau_g \propto a^{-\left(4+z\right)}$, $z = 1$ or 2.

• $\tau_q \propto (m_q a)^{-p}$, $p = 1$ or 2.

• Imaginary time:

• static quantities

• size $L_S = N_S a$, $L_4 = N_4 a$;

• dimension of spacetime = 4

• critical slowing down

• especially dire with sea quarks

• thermodynamics: $T = 1 / N_4 a$

$$
\langle \bullet \rangle = \frac{1}{Z} \int\mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet] \\
= \text{Tr}\{\bullet e^{-\hat{H}/T}\} / \text{Tr}\{e^{-\hat{H}/T}\}
$$
Numerical Lattice Gauge Theory

- The lattice provides a UV cutoff; a finite volume provides an IR cutoff; a finite Euclidean time leads to a nonzero temperature.

- Write a random number generator to create lattice gauge fields distributed with the weight $e^{-S}$.

- Fit correlations functions to get masses and matrix elements.

- Repeat several times while varying bare gauge coupling and bare masses.

- Find a trajectory with constant pion, kaon, $D_s, B_s$, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.

- Convert units to MeV.
Effective Field Theory

- The lattice provides a UV cutoff: Symanzik effective field theory.
- The finite volume provides an IR cutoff: effective field theory in a box:
  - loop integrals become finite sums;
  - these effects are either very small or very useful (absorptive parts).
- Sometimes the light quarks aren’t light enough: chiral perturbation theory:
  - replace the computer’s pion cloud with Nature’s.
- Sometimes heavy-quark masses have $m_q a \approx 1$: HQET or NRQCD.
### asqtad Ensembles: 2+1

MILC, arXiv:0903.3598

<table>
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<th>$a$ (fm)</th>
<th>size</th>
<th>$am'/am'_s$</th>
<th># confs</th>
<th># sources</th>
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<td>0.0036/0.018</td>
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<tr>
<td>≈ 0.06</td>
<td>$56^3 \times 144$</td>
<td>0.0025/0.018</td>
<td>800</td>
<td>4</td>
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<td>≈ 0.06</td>
<td>$64^3 \times 144$</td>
<td>0.0018/0.018</td>
<td>824</td>
<td>4</td>
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<td>≈ 0.045</td>
<td>$64^3 \times 192$</td>
<td>0.0028/0.014</td>
<td>800</td>
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</table>
asqtad Ensembles: 2+1
Blue-Gene/Q at ALCF

Cluster at Fermilab
CKM: |V_{ub}| and |V_{cb}|
Basic Formulas for $B \rightarrow \pi l \nu$

- Relevant term in effective Hamiltonian: $\mathcal{L}_i = \bar{b} \gamma^\mu (1 - \gamma^5) u \bar{\nu}_\ell \gamma_\mu (1 - \gamma^5) \ell$

- Differential decay rate:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 |k|^3 |f_+(q^2)|^2 + O(m_\ell^2),$$

- Steps:
  - generate numerical data at several $k, m_l, a$;
  - chiral continuum extrapolation;
  - extend to full kinematic range with $z$ expansion.
Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

- Compute $f(k, m_s, m_l, a)$
- Combine data with Symanzik EFT & chiral PT:
  - $m_l \to \frac{1}{2}(m_u+m_d)$;
  - $a \to 0$.
- Limited range: $|k|a \ll 1$. 

\[ \chi^2/\text{dof} = 34.9/36, \ p = 0.52 \]
\[ \chi^2/\text{dof} = 52.8/48, \ p = 0.37 \]
\[ \chi^2/\text{dof} = 34.6/36, \ p = 0.53 \]
Error Budgets

• The largest uncertainty, by far, comes from MC statistics, as **amplified** via the chiral-continuum extrapolation.

• Next (and independent of $q^2$) is matching from LGT to continuum QCD.

• Input parameters ($m_l, m_s, \kappa_b$) & relative scale ($r_1$) disappear in quadrature sum.

• Challenge: extend reach to lower $q^2$, without being killed by the (amplified) statistical error.
The form factor is analytic in $q^2$ except a cut for $q^2 \geq (M_B + M_\pi)^2$ and (possibly) subthreshold poles $(M_B - M_\pi)^2 \leq q^2 < (M_B + M_\pi)^2$.

With $t_+ = (M_B + M_\pi)^2$, set

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

which maps cut to unit circle and semileptonic decay to real $|z| \leq 0.28$ for optimal $t_0$. 
Determination of $|V_{ub}|$

- Much more precise than 2008.
- **BLINDED PLOTS!!**
- $z$ variable extends range.
- Functional fitting method.
- Relative norm’n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.
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Unblinded result:

$$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$$
Reconstructing Form Factors

• For additional applications, the $z$ expansion provides a useful summary.

• Formulas (Bourrely, Caprini, Lellouch, arXiv:0807.2722):

\[
  f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z-1} b_n^+ \left[ z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]
\]

\[
f_0(z) = \sum_{n=0}^{N_z} b_n^0 z^n
\]

\[
t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2
\]

• Subthreshold $1^-$ pole in $f_+$; first $0^+$ excitation (for $f_0$) is unstable.
- Coefficients and correlations:

<table>
<thead>
<tr>
<th></th>
<th>$b_0^+$</th>
<th>$b_1^+$</th>
<th>$b_2^+$</th>
<th>$b_3^+$</th>
<th>$b_0^0$</th>
<th>$b_1^0$</th>
<th>$b_2^0$</th>
<th>$b_3^0$</th>
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<tbody>
<tr>
<td>$b_0^+$</td>
<td>1</td>
<td>0.451</td>
<td>0.161</td>
<td>0.102</td>
<td>0.331</td>
<td>0.346</td>
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<tr>
<td>$b_1^+$</td>
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<td>0.757</td>
<td>0.665</td>
<td>0.430</td>
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<td>$b_2^+$</td>
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<td>0.988</td>
<td>0.482</td>
<td>0.847</td>
<td>0.951</td>
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<tr>
<td>$b_3^+$</td>
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<td>0.484</td>
<td>0.833</td>
<td>0.913</td>
<td>0.714</td>
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<tr>
<td>$b_0^0$</td>
<td>1</td>
<td>0.447</td>
<td>0.359</td>
<td>0.189</td>
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<tr>
<td>$b_3^0$</td>
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</tbody>
</table>

- If you are interested in semileptonic B decays, just take this table and the formulas from the last slide and use the resulting form factors.
Semileptonic $B \rightarrow Dl\nu$ for $|V_{cb}|$

- Similar strategy as above:
  - compute sequence of form-factor values;
  - chiral continuum extrapolation;
  - combined $z$-expansion fit to obtain $|V_{cb}|$.

- Differences:
  - HQET control of cutoff effects more central [hep-lat/0002008, hep-lat/0112044, hep-lat/0112045];
  - use Boyd, Grinstein, Lebed form of $z$ expansion [hep-ph/9508211].
\[ 10^3 |V_{cb}| \]

- Green line: \(|V_{ub}| / |V_{cb}| \) (latQCD + LHCb)
- Blue line: \(|V_{ub}| \) (latQCD + BaBar + Belle)
- Orange line: \(|V_{cb}| \) (latQCD + BaBar)
- Brown line: \(|V_{cb}| \) (latQCD + HFAG, \( w = 1 \))
- Red dot: \( p = 0.26 \)
- Red line: \( \Delta \chi^2 = 1 \)
- Purple line: \( \Delta \chi^2 = 2 \)
- Black dot: inclusive \(|V_{xb}|\)

References:
- arXiv:1503.07839
- arXiv:1501.05373 RBC/UKQCD
- arXiv:1503.07237
- arXiv:1505.03925 HPQCD
- arXiv:1403.0635
- arXiv:1503.01421

Detmold, Lehner, Meinel
New Physics in $B \rightarrow D^{(*)}\tau\nu$?


• BaBar presented evidence for an excess in both channels:
  • $2.0\sigma$ for $R(D);$ $2.7\sigma$ for $R(D^*)$; $3.4\sigma$ combined.

• With Belle & LHCb:
  • $3.9\sigma$ combined.

• Estimated form factors w/
  • HQET;
  • quenched QCD.

![Diagram showing correlation between R(D) and R(D*) with data points and error ellipses from BaBar, Belle, and LHCb.]

$\Delta \chi^2 = 1.0$

$P(\chi^2) = 55\%$
Form Factors for $B \to D^{(*)}\tau\nu$


- $R(D)$ values:
  - $0.297 \pm 0.017$ (est.);
  - $0.316 \pm 0.014$ (F/M ’12);
  - $0.299 \pm 0.011$ (F/M ’15);
  - $0.300 \pm 0.008$ (HPQCD).

- Lattice QCD work for $R(D^*)$ yet to come.

see also arXiv:1206.4977.
New Physics in $B \to \pi \tau \nu$?

- A charged Higgs mediating $b \to c$ could also mediate $b \to u$.

- SM prediction, including term $\sim m_\tau^2 |f_0|^2$.

- With the Fermilab/MILC form factors, we find

$$R(\pi) \equiv \frac{\mathcal{B}(B \to \pi \tau \nu_\tau)}{\mathcal{B}(B \to \pi \ell \nu_\ell)} = 0.641(17)$$

![Graph showing $d\mathcal{B}(B^0 \to \pi^+ \nu)/dq^2(10^{-6} \text{GeV}^2)$ vs $q^2(\text{GeV}^2)$]
Penguins
Basic Formulas for $B^0 \rightarrow \pi^0 \nu \nu, K^0 \nu \nu$

- Relevant term in effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \frac{e^2}{16\pi^2} \frac{X_t}{\sin^2 \theta_W} \bar{q} L \gamma_\mu b_L \sum_v \bar{v}_L \gamma_\mu v_L$$

- Differential decay rate:

$$\frac{d\mathcal{B}(B \rightarrow P \nu \bar{\nu})}{dq^2} = C_P \tau_B \left| V_{tb} V_{tq}^* \right|^2 \frac{G_F^2 \alpha^2}{32\pi^5} \frac{X_t^2}{\sin^4 \theta_W} |\mathbf{k}|^3 |f_+(q^2)|^2$$

\[ q = d \Rightarrow P = \pi^0, \quad C_{\pi^0} = \frac{1}{2} \]

\[ q = s \Rightarrow P = K^0, \quad C_{K^0} = 1 \]

- Thus, can use same form factor as above (and kaon counterpart).
SM Predictions for the Differential Rate

arXiv:1510.02349

- NB: charged counterparts receive contribution from the cascade $B \to \tau \nu$, $\tau \to \pi \nu$ (or $K \nu$), with amplitude proportional to $|V_{ub}| f_B |V_{ud}| f_\pi$ (or $|V_{us}| f_K$) [Kamenik & Smith, arXiv:0908.1174].
Basic Formulas for $B \rightarrow \pi l^+l^-, Kl^+l^-$

cf., arXiv:1510.02349, Sec. 2 & Appendix B

- One-loop effective Hamiltonian contains many operators ($q = d, s$):

  \[ Q^u_1 = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \]
  \[ Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \]
  \[ Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q') \]
  \[ Q_5 = (\bar{q}_L \gamma_\mu \gamma_\nu \gamma_\rho L)(\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho L) \]
  \[ Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \gamma^\mu b_R) F_{\mu \nu} \]
  \[ Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \ell) \]
  \[ Q_2 = (\bar{q}_L \gamma_\mu b_L) \]
  \[ Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q') \]
  \[ Q_6 = (\bar{q}_L \gamma_\mu \gamma_\nu \gamma_\rho L)(\bar{q} \gamma^\mu \gamma^\nu \gamma^\rho L) \]
  \[ Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \gamma^\mu T^a b_R) G_{\mu \nu}^a \]
  \[ Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \gamma_5 \ell) \]

- Matrix elements of $Q_7, Q_9, Q_{10}$ yield form factors, including tensor $f_T$. 

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Other Matrix Elements

- Schematically,

\[ \langle P \ell \ell | Q_i(y) | \bar{B} \rangle \sim (\bar{u}_{\ell} \gamma_{\mu} v_{\ell}) \int d^4 x e^{i q \cdot (x-y)} \langle P | T J_{\text{em}}^\mu(x) Q_i(y) | \bar{B} \rangle \]

- Four regions:
  - when \( q^2 \) is close to \( M^2 \) of \( \rho, \omega, \phi \): very challenging \( \text{[arXiv:1506.07760]} \);
  - when \( q^2 \) is near charmonium resonances: also very challenging;
  - low \( q^2 \), between these two: SCET (aka QCD factorization);
  - high \( q^2 \sim M_B^2 \): OPE [Grinstein, Pirjol, \text{hep-ph/0404250}], leading back to the three form factors, \( f_+, f_0, \& f_T \).
Soft Collinear Effective Theory  

- The RHS of the equation on the last slide takes the form

\[ \begin{align*}
C_i \langle P \ell \ell | Q_i | \bar{B} \rangle & \sim C_i \left[ (1 + \alpha_s) f_T + (1 + \alpha_s) f_+ + \phi_B \ast T \ast \phi_P \right], \quad i = 1, \ldots, 6, \\
C_8 \langle P \ell \ell | Q_8 | \bar{B} \rangle & \sim C_8 \left[ \alpha_s f_T + \alpha_s f_+ + \phi_B \ast T \ast \phi_P \right], \\
C_7 \langle P \ell \ell | Q_7 | \bar{B} \rangle & \sim C_7 f_T \sim C_{7\text{eff}} f_T, \\
C_9 \langle P \ell \ell | Q_9 | \bar{B} \rangle & \sim C_9 f_+ \sim C_{9\text{eff}} f_+, \\
C_{10} \langle P \ell \ell | Q_{10} | \bar{B} \rangle & \sim C_{10} f_+. 
\end{align*} \]

- We add the nonfactorizable terms to the amplitude using the known hard-scattering kernel \( T \) and distribution amplitudes from the literature.

- SM \( C_7, C_9, C_{10} \) from Huber, Lunghi, Misiak, Wyler [hep-ph/0512066].
Coefficients and Correlations: $B \rightarrow Kl^+l^-$

![Coefficients and Correlations](arXiv:1509.06235)

<table>
<thead>
<tr>
<th></th>
<th>$b_0^+$</th>
<th>$b_1^+$</th>
<th>$b_2^+$</th>
<th>$b_0^0$</th>
<th>$b_1^0$</th>
<th>$b_2^0$</th>
<th>$b_0^T$</th>
<th>$b_1^T$</th>
<th>$b_2^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.466</td>
<td>-0.885</td>
<td>-0.213</td>
<td>0.292</td>
<td>0.281</td>
<td>0.150</td>
<td>0.460</td>
<td>-1.089</td>
<td>-1.114</td>
</tr>
<tr>
<td>error</td>
<td>0.014</td>
<td>0.128</td>
<td>0.548</td>
<td>0.010</td>
<td>0.125</td>
<td>0.441</td>
<td>0.019</td>
<td>0.236</td>
<td>0.971</td>
</tr>
</tbody>
</table>

Kinematic Distributions

- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot’s theory preceded measurement:

- arXiv:1510.02349 also contains predictions for the ratio, the flat terms, etc.
Other Results in arXiv:1510.02349

- Tests of heavy-quark and SU(3)-flavor symmetries.
- Comparisons with LHCb over wide bins, \( q^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2] \), and \( q^2 \in [15 \text{ GeV}^2, 22 \text{ GeV}^2] \).
- SM predictions for \( B \to \pi \tau^+ \tau^- \), \( B \to K \tau^+ \tau^- \)
- SM contribution to lepton-universality violation (\( q^2 \in [1 \text{ GeV}^2, 6 \text{ GeV}^2] \)):

\[
R_{K^+}^{\mu e} \bigg|_{\text{SM}} = 1.00050(43), \ \text{vs.} \ \ R_{K^+}^{\mu e} \bigg|_{\text{LHCb}} = 0.745^{+97}_{-82}
\]

\[
R_{K^+}^{\mu e} = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} d^2 B(B \to K \mu \mu) \left[ \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} d^2 B(B \to K e e) \right]^{-1}
\]

- Neutrino modes and \( B \to \pi \nu \nu \), discussed earlier.
CKM: $|V_{td}|$ and $|V_{ts}|$

- Assume that there is no new physics buried in the Wilson coefficients.

- Then the combination of our calculations with experimental measurements yield the third row of the CKM matrix.

- We find

\[
|V_{td}/V_{ts}| = 0.201(20) \\
|V_{tb}^*V_{td}| \times 10^3 = 7.45(69) \\
|V_{tb}^*V_{ts}| \times 10^3 = 35.7(1.5)
\]

- The uncertainty here is commensurate from the neutral $B$-meson mixing.

- Result for $|V_{ts}|$ is $1.4\sigma$ lower than that from mixing.
Wilson Coefficients

• Assuming no new physics is sad:
  • take the CKM matrix from a global fit;
  • determine best fit to Wilson coefficients $C_9$ and $C_{10}$.

• From the observables considered here, the SM is $2\sigma$ away from the best fit.

• Comparable but complementary to angular observables in $B \rightarrow K^* \mu \mu$. 
Wilson Coefficients 2

- Add $B_s \rightarrow \mu \mu$, which also relies on lattice QCD — $f_{B_s}$.

- Favored region shrinks but only away from SM point.

- NB: assuming no new CPV and avoiding $b \rightarrow s \gamma$ constraints.
Outlook

• Perhaps too much information to summarize.

• The overarching take-home message:
  
  • we provide a convenient useful parametrization of the form factors, including correlations needed for joint fits, ratios, etc.;
  
  • the scope of application is not limited to what we’ve done;
  
  • just like collider physicists use CTEQ or MRSW parton densities, flavor physicists can use our (or other group’s) form factors.

• Future work, e.g., on MILC HISQ ensembles, will improve the precision (over the coming few years).
¡Muchas gracias!