Phenomenology of Semileptonic *B*-meson Decays with Form Factors from Lattice QCD

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IFT Seminar | Universidad Autonoma Madrid
November 23, 2015

Why Exclusive Semileptonic Decays?

- Rare processes are sensitive to non-Standard physics: leptoquarks, Z', 4th generation, non-Standard Higgs bosons, supersymmetry.
- Several "tensions":
  - CKM from inclusive *vs*. exclusive decays;
  - excess in $B \to D^{(*)}\tau\nu$;
  - deficits in $B \to K^{(*)}\mu\mu$.
- Experimental results available; more on the way.
- Nonperturbative hadronic matrix elements available (with full error budgets).

Energy Scales

high energy

TeV New Physics — $\mathscr{L} = ?$

Electroweak (Higgs, top, W, Z) — $\mathscr{L} = \mathscr{L}_{\mathrm{SM}}$

Bottom quark — $\mathscr{L} = -\mathscr{H}_{\mathrm{eff}}$

Hadronic physics: Λ_{QCD} — Lattice QCD

low energy

Processes

$\ell = \mu, e$

- CKM Determination

$$B \to \pi \ell \nu$$

$$B \to D \ell \nu$$

$$B \to D^* \ell \nu$$

(in pedagogical order)

- New Physics Search

$$B \to \pi \tau \nu$$

$$B \to D \tau \nu$$

$$B \to K \nu \bar{\nu}$$

$$B \to \pi \nu \bar{\nu}$$

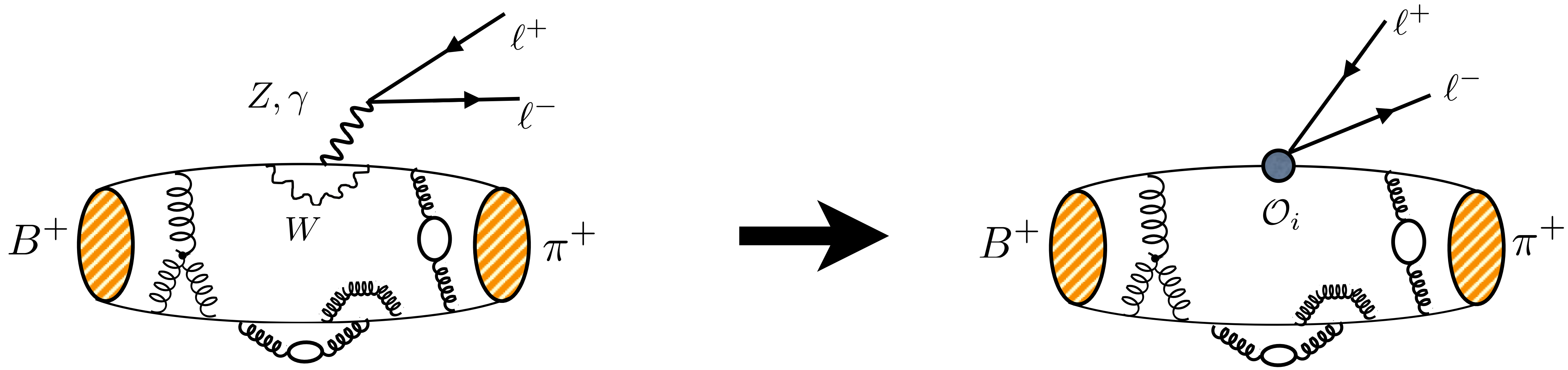
$$B \to K \ell^+ \ell^-$$

$$B \to \pi \ell^+ \ell^-$$

Effective Hamiltonian

- Masses of W, Z, top, and Higgs are much greater than m_b:

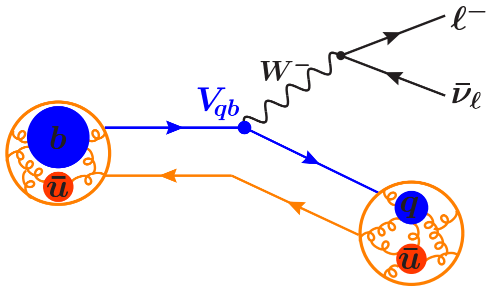
$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \mathbf{NP}) \mathscr{L}_i[\ell, q, \gamma, g]$$



- Contributions of unknown particles lumped into Wilson coefficients $\mathscr{C}_i$.

- We use SM Wilson coeff's of Huber, Lunghi, Misiak, Wyler [hep-ph/0512066].

Matrix Elements and Form Factors



- Decompose amplitudes in form factors ($q = p - k = \ell + \nu$):

$$\langle \pi(k)|\bar{u}\gamma^\mu b|B(p)\rangle = \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right) f_+(q^2) + \frac{M_B^2 - M_\pi^2}{q^2}q^\mu f_0(q^2),$$
$$= \sqrt{2M_B}\left[p^\mu f_\parallel(q^2)/M_B + k_\perp^\mu f_\perp(q^2)\right]$$

$$\langle \pi(k)|\bar{u}\sigma^{\mu\nu} b|B(p)\rangle = -2i\frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2),$$

$$\langle \pi(k)|\bar{u}b|B(p)\rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u} f_0(q^2),$$

PCVC: same

- The kinematic variable $q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$.

Computing Hadronic Matrix Elements

- Use correlation functions ($x_4 = it$ is "Euclidean time"):

$$\langle \pi(x_4) J(x_4') B^\dagger(0) \rangle = \sum_{m,n} \langle 0|\hat{\pi}|\pi_n\rangle \langle \pi_n|\hat{J}|B_m\rangle \langle B_m|\hat{B}^\dagger|0\rangle e^{-M_{\pi_n}(x_4 - x_4') - M_{B_m} x_4'}$$

  in which desired matrix element lies in the middle (of leading term).

- Find other information from two-point correlation functions:

$$\langle \pi(x_4) \pi^\dagger(0) \rangle = \sum_n |\langle 0|\hat{\pi}|\pi_n\rangle|^2 e^{-M_{\pi_n} x_4}$$

$$\langle B(x_4) B^\dagger(0) \rangle = \sum_n |\langle 0|\hat{B}|B_n\rangle|^2 e^{-M_{B_n} x_4}$$

- Compute LHS with lattice QCD; use humans to analyze RHS.

Outline

- Motivation
- Lattice QCD Calculations
- $|V_{ub}|$ and $|V_{cb}|$
- Penguin Processes
- Outlook

arXiv:1510.02349

arXiv:1509.06235 arXiv:1507.01618 arXiv:1503.07839

arXiv:1503.07237 arXiv:1403.0635

Fermilab Lattice and MILC Collaborations

Jon Bailey, Alexei Bazavov, Claude Bernard, Chris Bouchard,
Carleton DeTar, Daping Du, Aida El-Khadra, Elizabeth Freeland,
Elvira Gámiz, Steve Gottlieb, Urs Heller, A.S.K., Jack Laiho,
Ludmila Levkova, Yuzhi Liu, Paul Mackenzie, Yannick Meurice,
Ethan Neil, Si-Wei Qiu, Jim Simone, Bob Sugar,
Doug Toussaint, Ruth Van de Water, Ran Zhou

with special guest

Enrico Lunghi

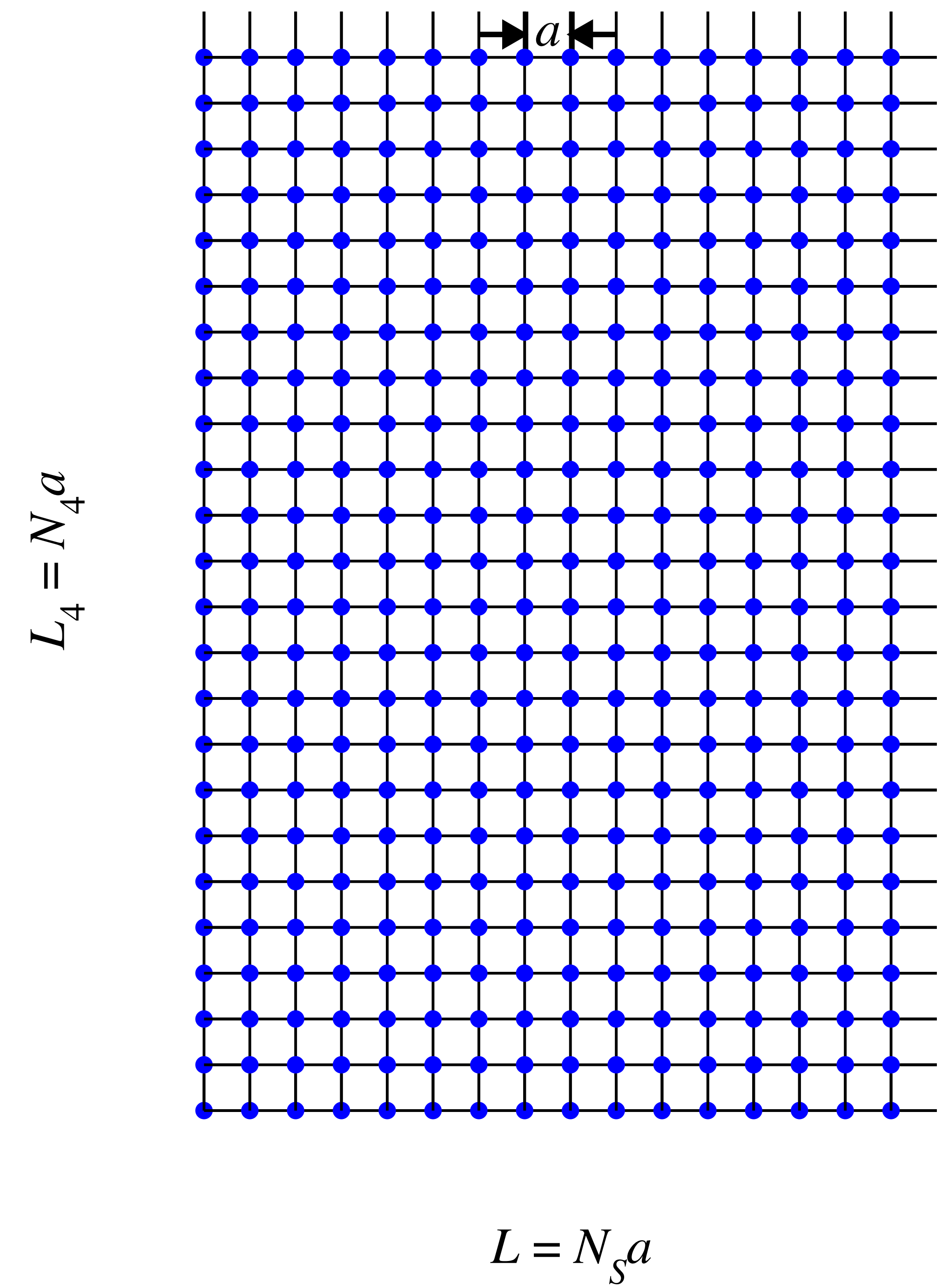
Numerical Lattice QCD

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

$$= \frac{1}{Z} \int \mathcal{D}U \ \mathrm{Det}(\not{D} + m) \exp\left(-S_{\mathrm{gauge}}\right) [\bullet']$$

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^9, so compute integrals numerically.



Lattice Gauge Theory

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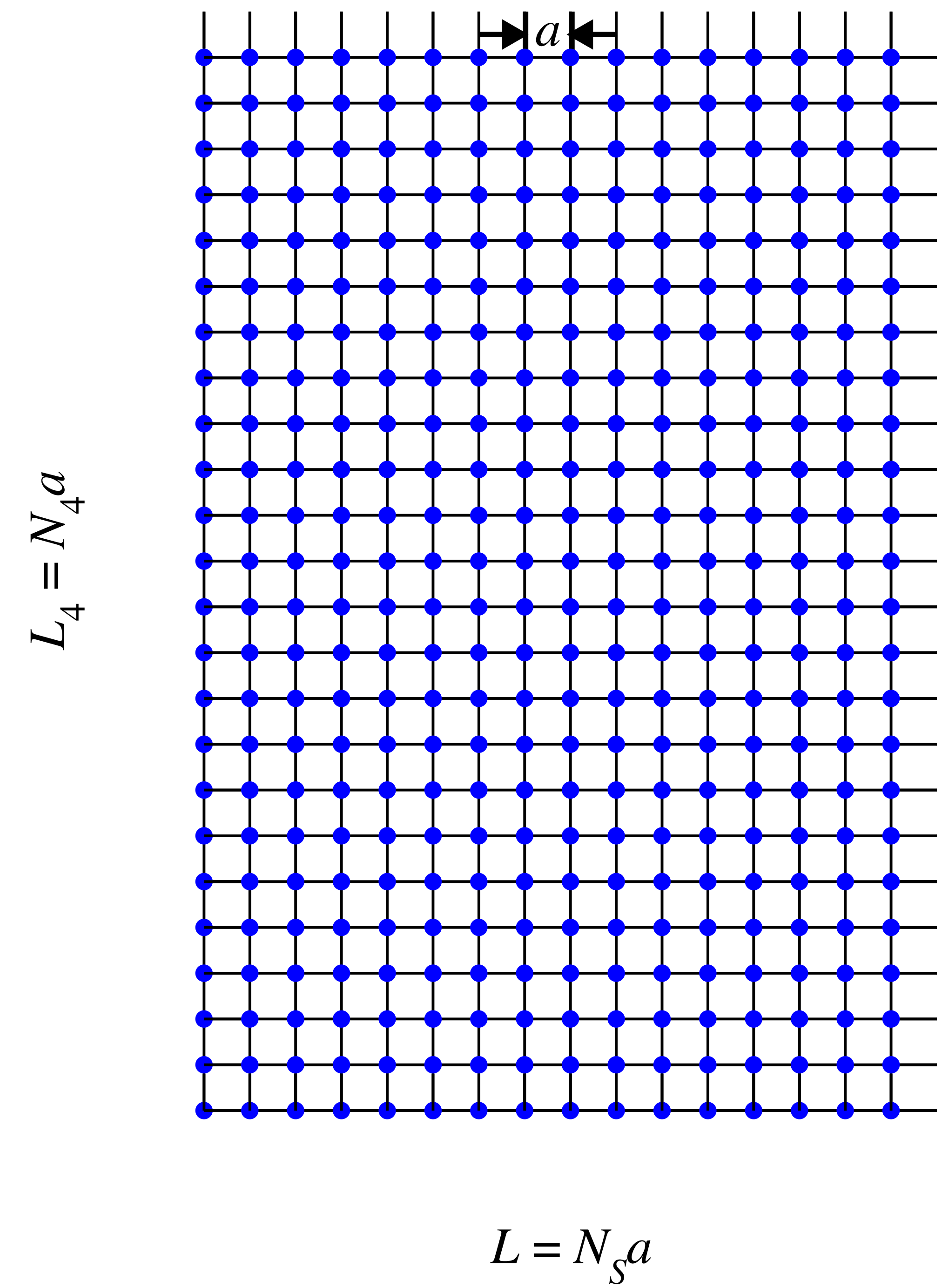

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \underbrace{\boxed{\mathcal{D}U}}_{\text{MC}} \underbrace{\boxed{\mathcal{D}\psi \mathcal{D}\bar{\psi}}}_{\text{hand}} \exp\left(-S\right) [\bullet]$$

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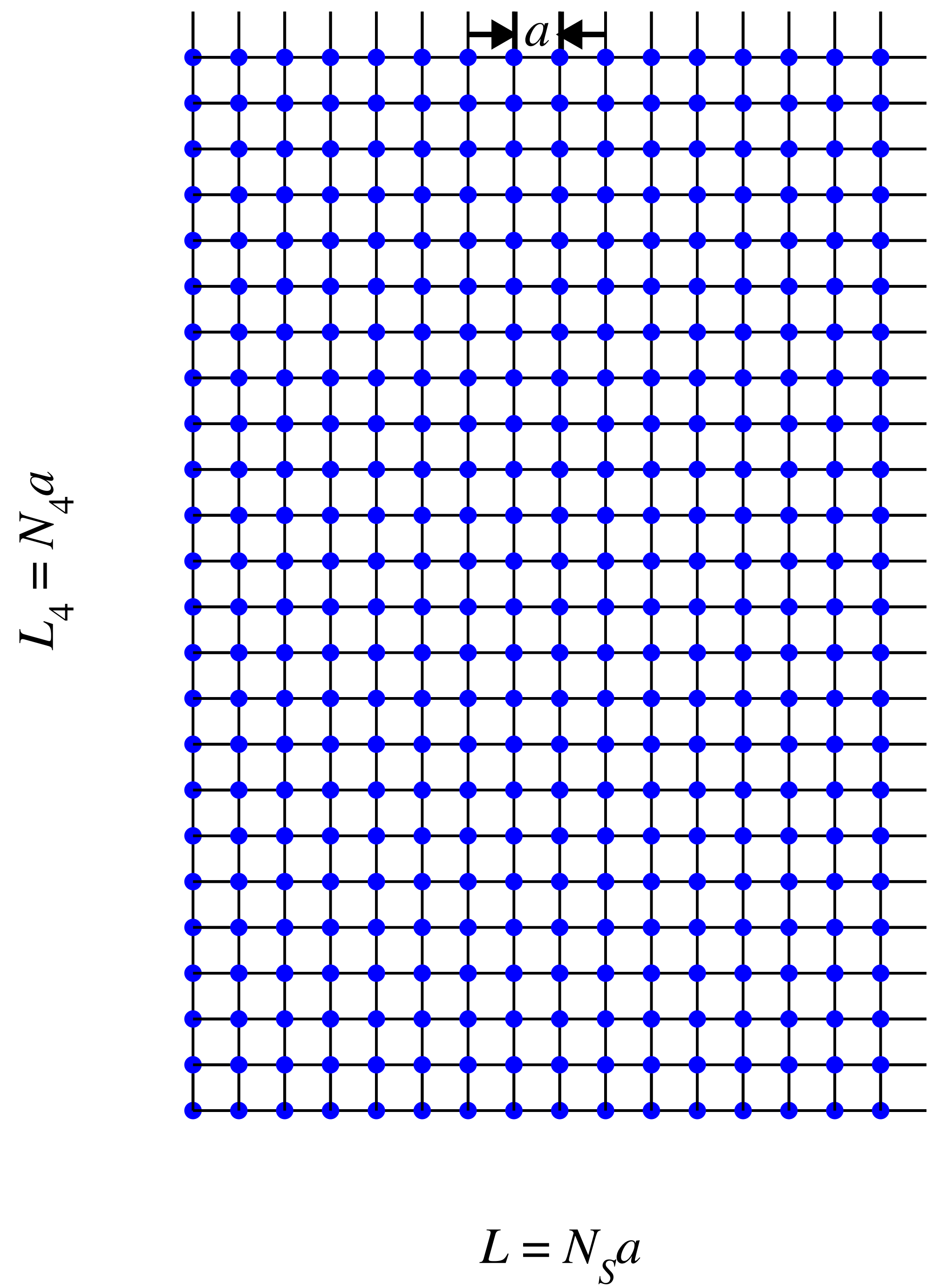


Some algorithmic issues

e.g., ASK, hep-lat/0205021

- lattice $N_S^3 \times N_4$, spacing a
- memory $\propto N_S^3 N_4 = L_S^3 L_4 / a^4$
- $\tau_\mathrm{g} \propto a^{-(4+z)}$, $z = 1$ or 2.
- $\tau_\mathrm{q} \propto (m_\mathrm{q} a)^{-p}$, $p = 1$ or 2.
- Imaginary time:
  - static quantities
- size $L_S = N_S a$, $L_4 = N_4 a$;
- dimension of spacetime = 4
- critical slowing down
- especially dire with sea quarks
- thermodynamics: $T = 1/N_4 a$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U\, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S)\, [\bullet]$$
$$= \mathrm{Tr}\{\bullet\, e^{-\hat{H}/T}\} / \mathrm{Tr}\{e^{-\hat{H}/T}\}$$

Numerical Lattice Gauge Theory

- The lattice provides a UV cutoff; a finite volume provides an IR cutoff; a finite Euclidean time leads to a nonzero temperature.
- Write a random number generator to create lattice gauge fields distributed with the weight e^{-S}.
- Fit correlations functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.

Effective Field Theory

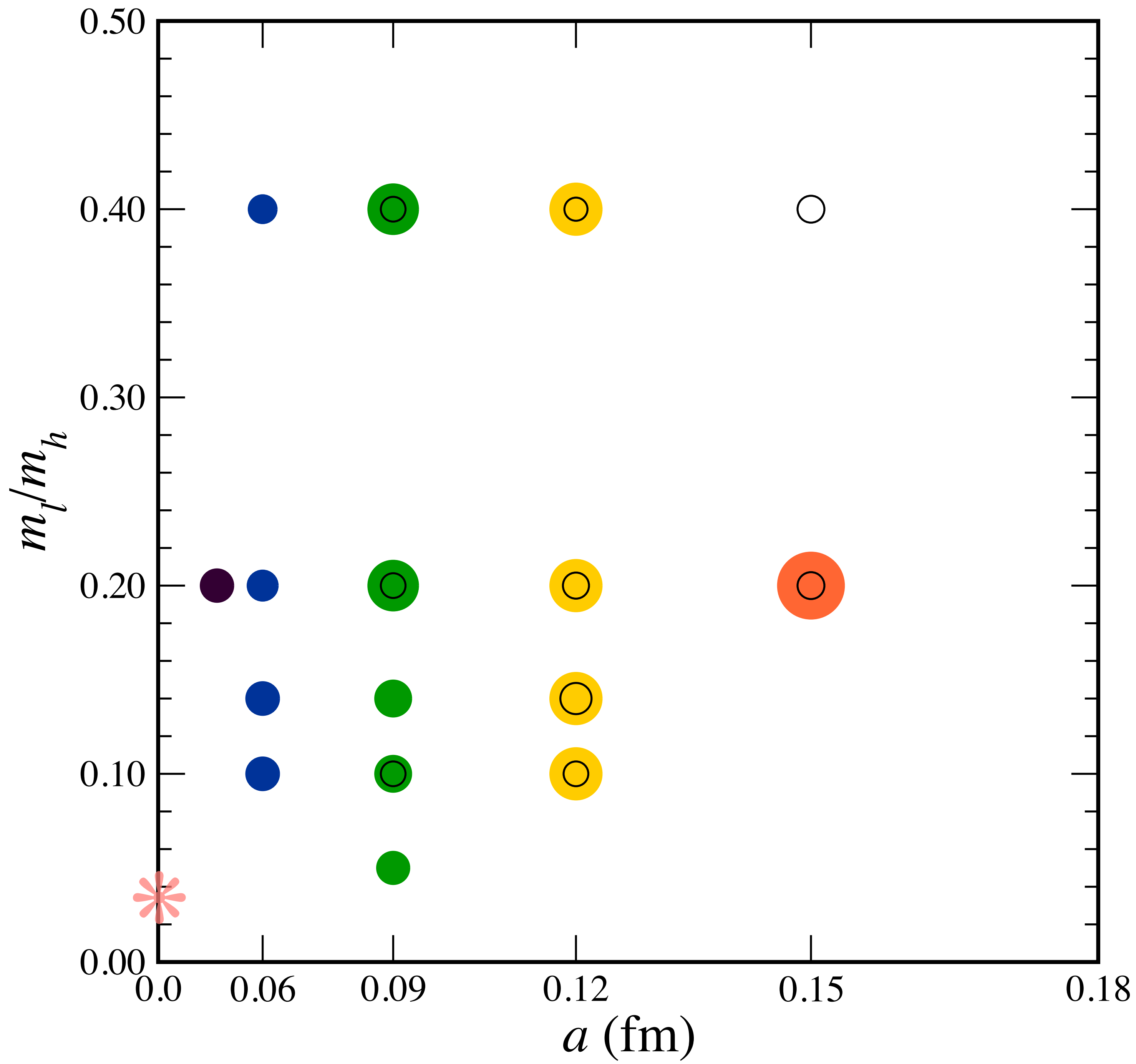
- The lattice provides a UV cutoff: Symanzik effective field theory.
- The finite volume provides an IR cutoff: effective field theory in a box:
    - loop integrals become finite sums;
    - these effects are either very small or very useful (absorptive parts).
- Sometimes the light quarks aren't light enough: chiral perturbation theory:
    - replace the computer's pion cloud with Nature's.
- Sometimes heavy-quark masses have $m_Q a \approx 1$: HQET or NRQCD.

asqtad Ensembles: 2+1

MILC, arXiv:0903.3598

a (fm)	size	am'_l/am'_s	# confs	# sources
≈ 0.15	$16^3\times$ 48	0.0097/0.0484	628	24
≈ 0.12	$20^3\times$ 64	0.02/0.05	2052	4
≈ 0.12	$20^3\times$ 64	0.01/0.05	2256	4
≈ 0.12	$20^3\times$ 64	0.007/0.05	2108	4
≈ 0.12	$24^3\times$ 64	0.005/0.05	2096	4
≈ 0.09	$28^3\times$ 96	0.0124/0.031	1992	4
≈ 0.09	$28^3\times$ 96	0.0062/0.031	1928	4
≈ 0.09	$32^3\times$ 96	0.00465/0.031	984	4
≈ 0.09	$40^3\times$ 96	0.0031/0.031	1012	4
≈ 0.09	$64^3\times$ 96	0.00155/0.031	788	4
≈ 0.06	$48^3\times$144	0.0072/0.018	576	4
≈ 0.06	$48^3\times$144	0.0036/0.018	672	4
≈ 0.06	$56^3\times$144	0.0025/0.018	800	4
≈ 0.06	$64^3\times$144	0.0018/0.018	824	4
≈ 0.045	$64^3\times$192	0.0028/0.014	800	4

asqtad Ensembles: 2+1



Blue-Gene/Q at ALCF

Cluster at Fermilab

CKM: $|V_{ub}|$ and $|V_{cb}|$

Basic Formulas for $B \to \pi l \nu$

- Relevant term in effective Hamiltonian: $\mathscr{L}_i = \bar{b}\gamma^\mu(1-\gamma^5)u\,\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell$
- Differential decay rate:

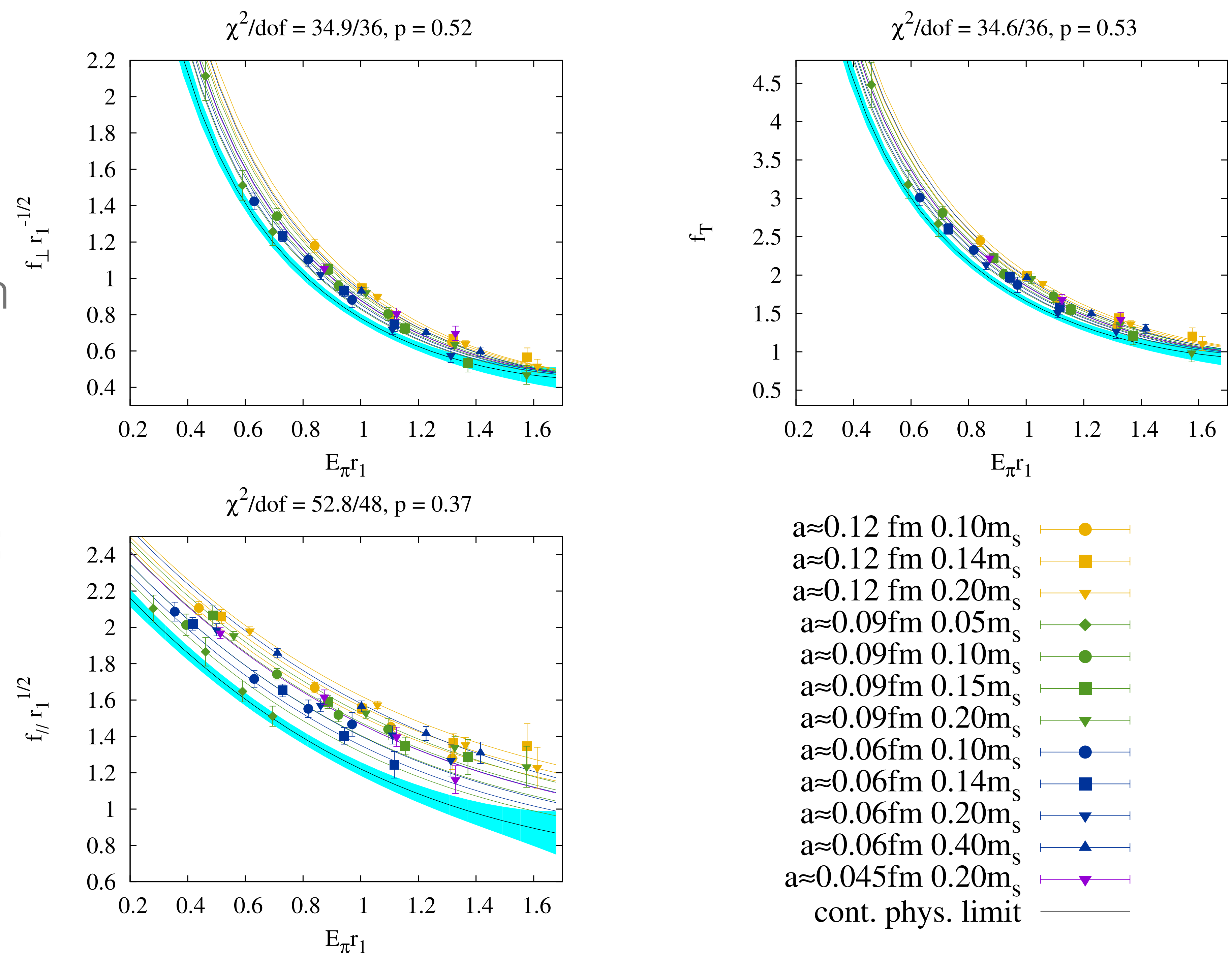
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{ub}|^2|\boldsymbol{k}|^3|f_+(q^2)|^2 + \mathrm{O}(m_\ell^2),$$

- Steps:
  - generate numerical data at several $\boldsymbol{k}$, m_l, a;
  - chiral continuum extrapolation;
  - extend to full kinematic range with z expansion.

Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

arXiv:1503.07839

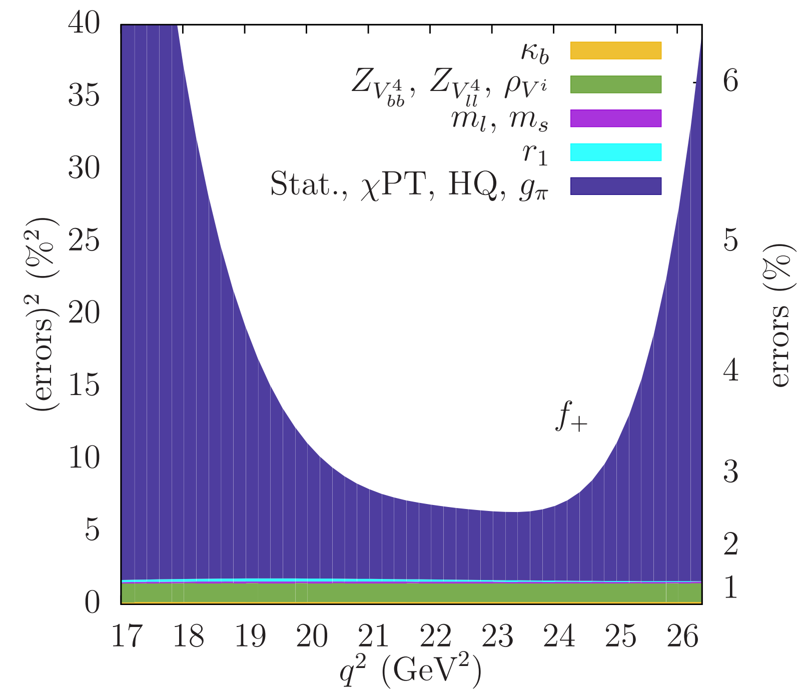
- Compute $f(\boldsymbol{k}, m_s, m_l, a)$
- Combine data with Symanzik EFT & chiral PT:
  - $m_l \to \frac{1}{2}(m_u + m_d)$;
  - $a \to 0$.
- Limited range: $|\boldsymbol{k}|a \ll 1$.

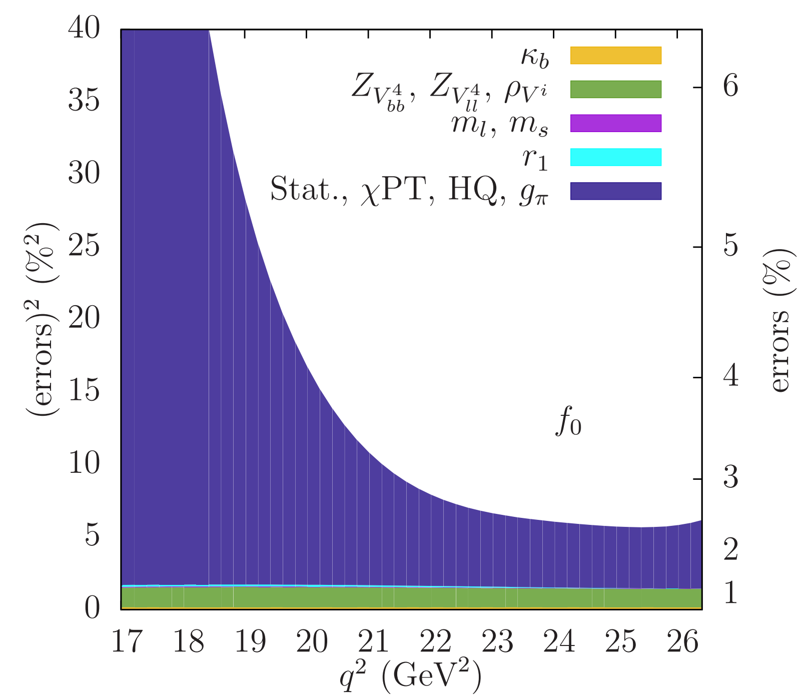


Error Budgets

in region with lattice data

- The largest uncertainty, by far, comes from MC statistics, as **amplified** via the chiral-continuum extrapolation.
- Next (and independent of q^2) is matching from LGT to continuum QCD.
- Input parameters (m_l, m_s, $\varkappa_b$) & relative scale (r_1) disappear in quadrature sum.
- Challenge: extend reach to lower q^2, without being killed by the (amplified) statistical error.





Analyticity and Unitarity

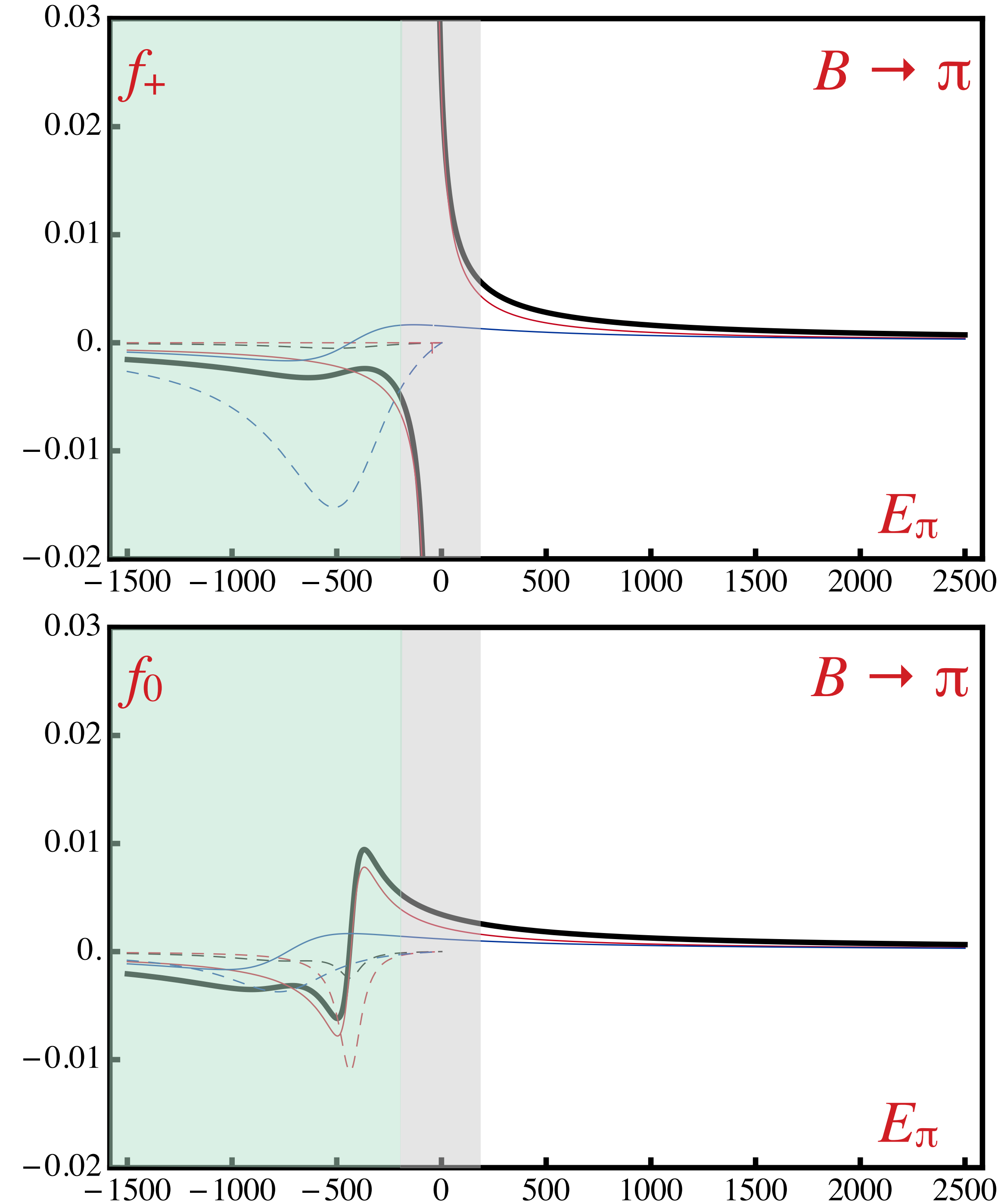
νl decay

- The form factor is analytic in q^2 except a cut for $q^2 \geq (M_B + M_\pi)^2$ and (possibly) subthreshold poles $(M_B - M_\pi)^2 \leq q^2 < (M_B + M_\pi)^2$.

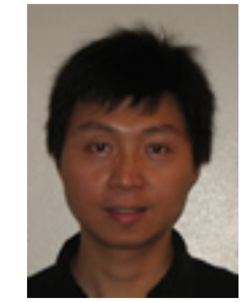
- With $t_+ = (M_B + M_\pi)^2$, set

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

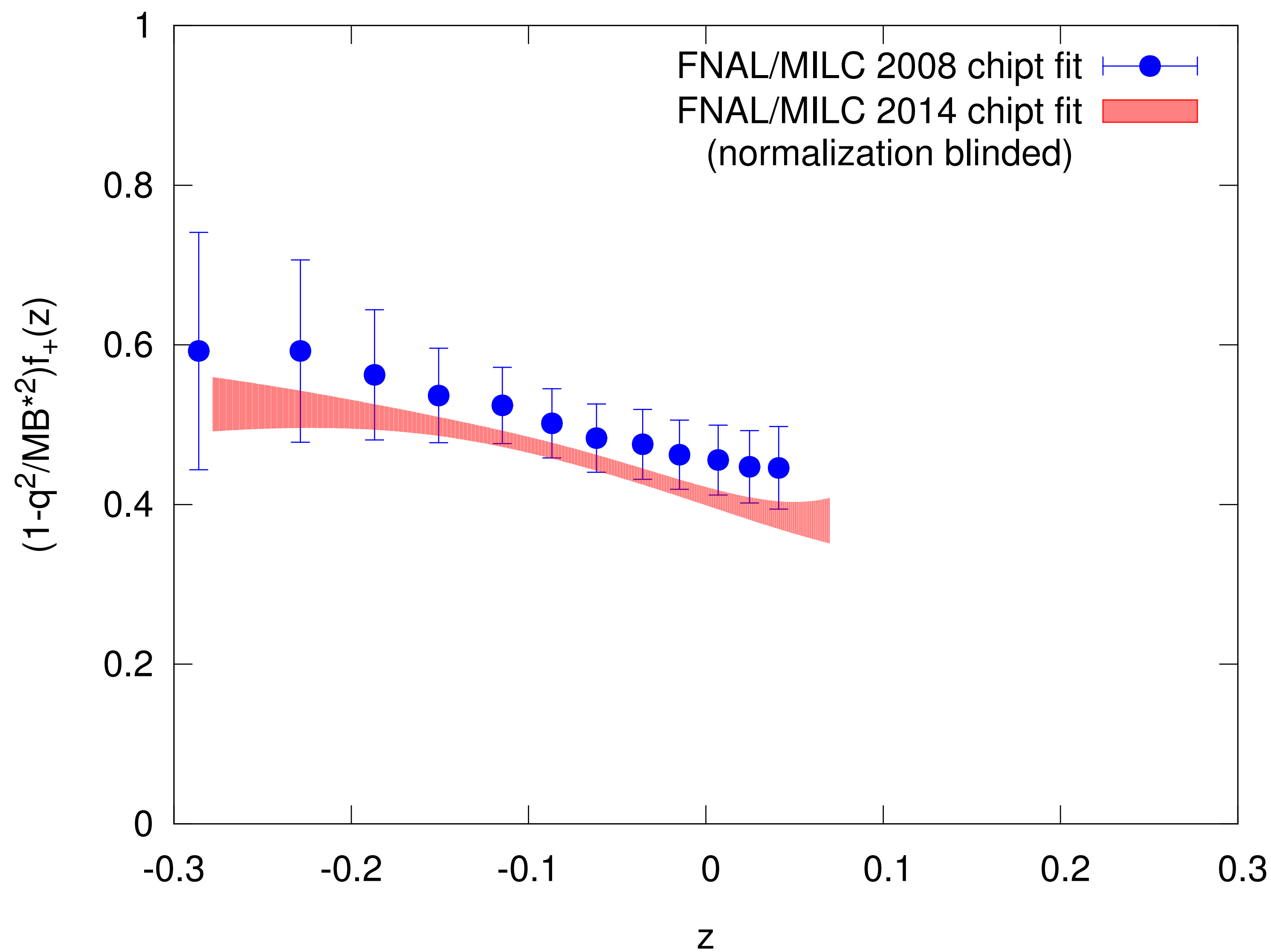
which maps cut to unit circle and semileptonic decay to real $|z| \leq 0.28$ for optimal t_0.



Determination of $|V_{ub}|$



- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



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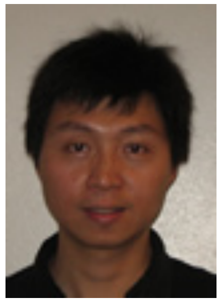

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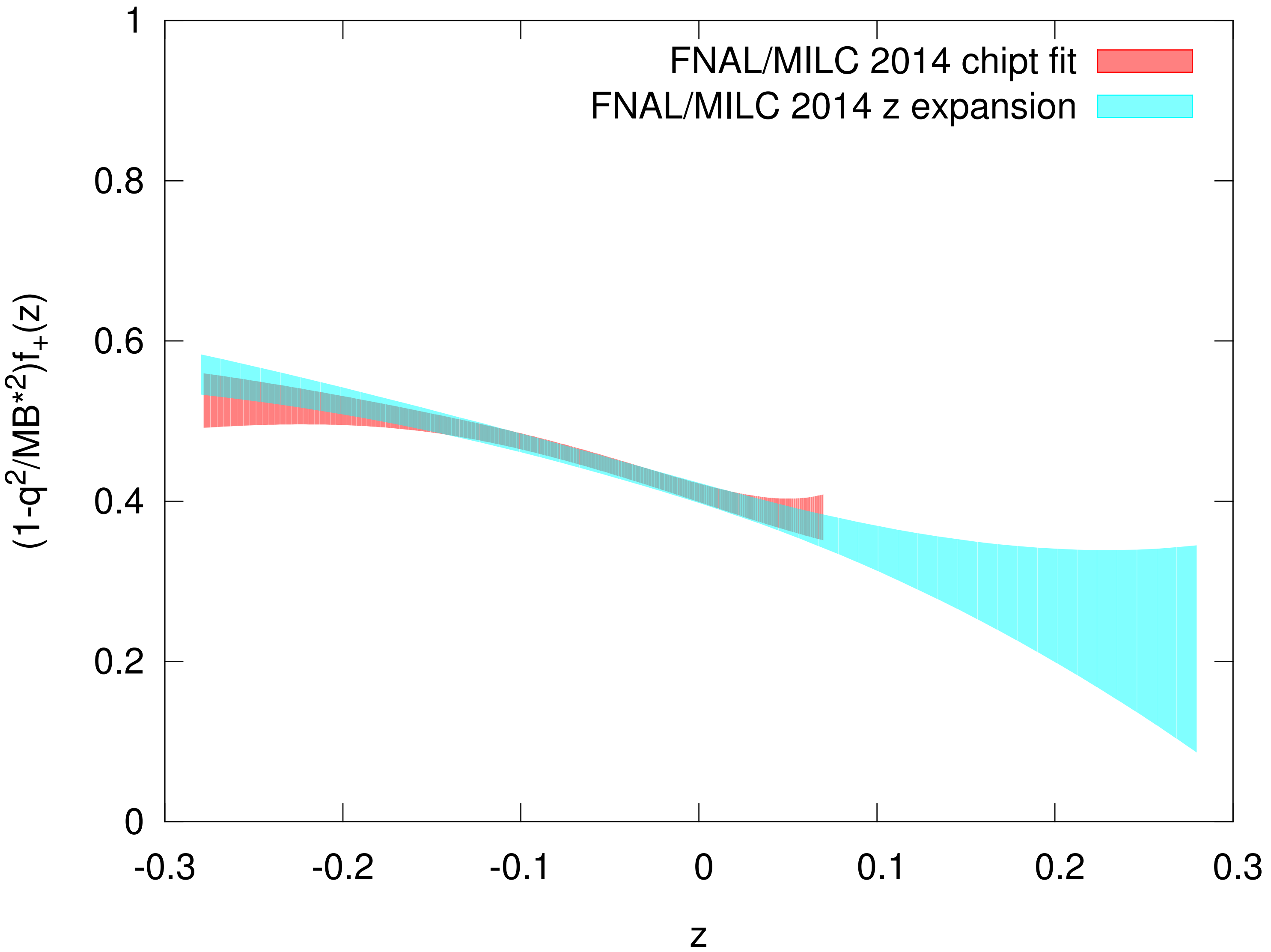

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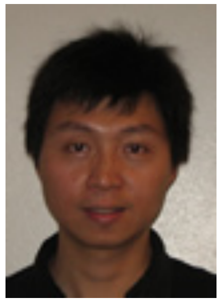

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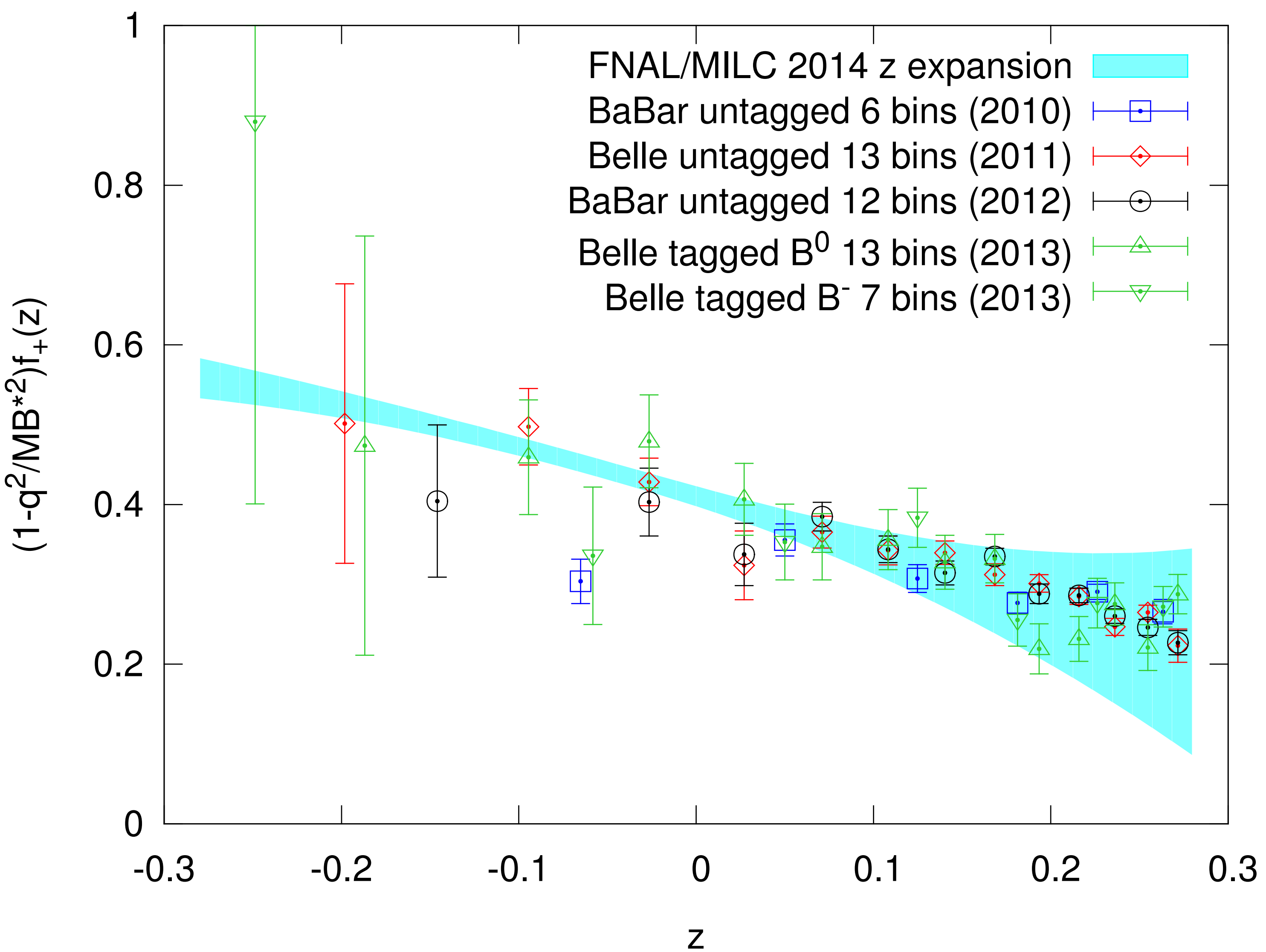

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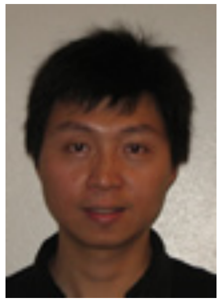

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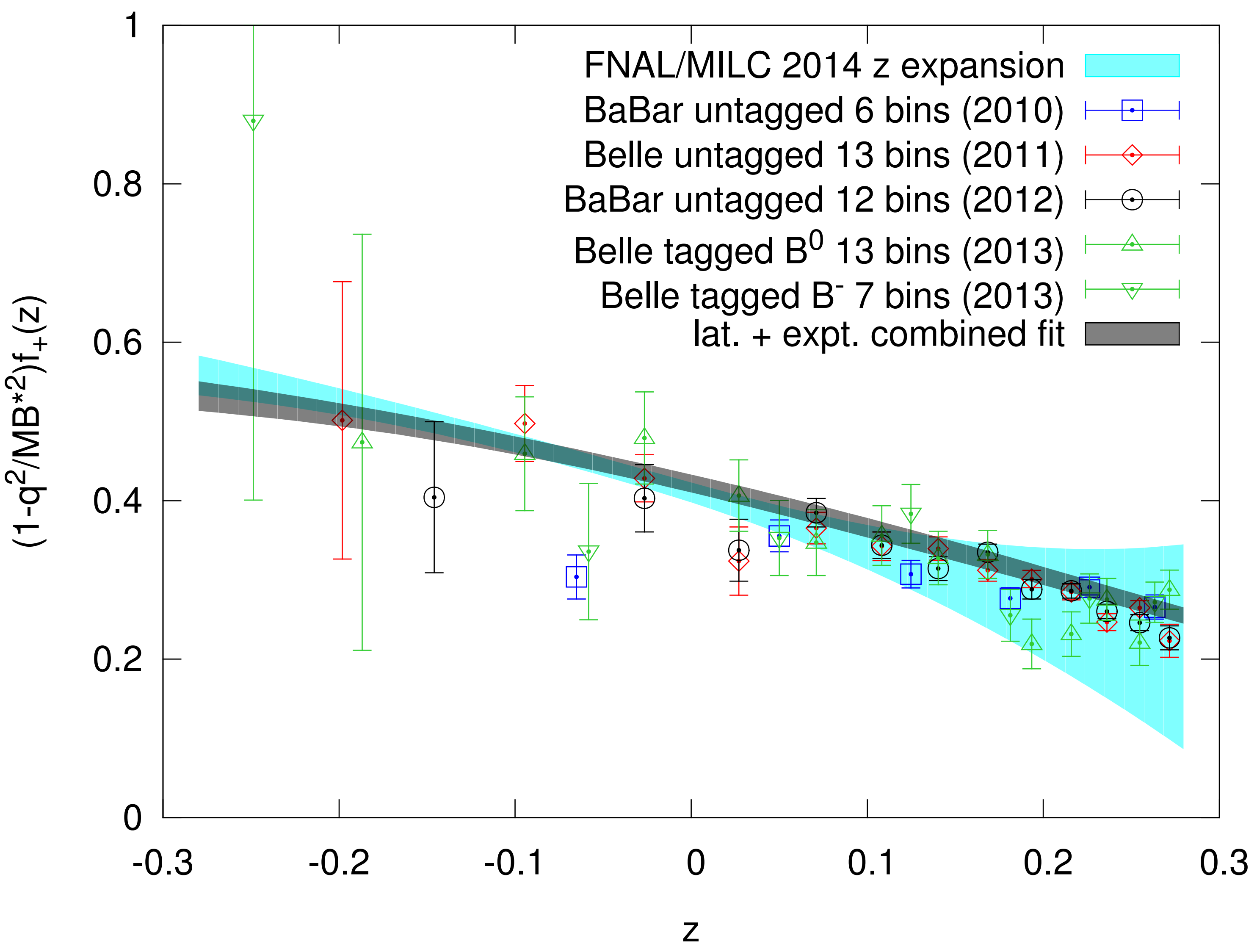
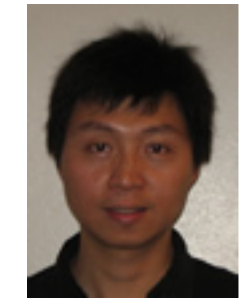


Determination of $|V_{ub}|$



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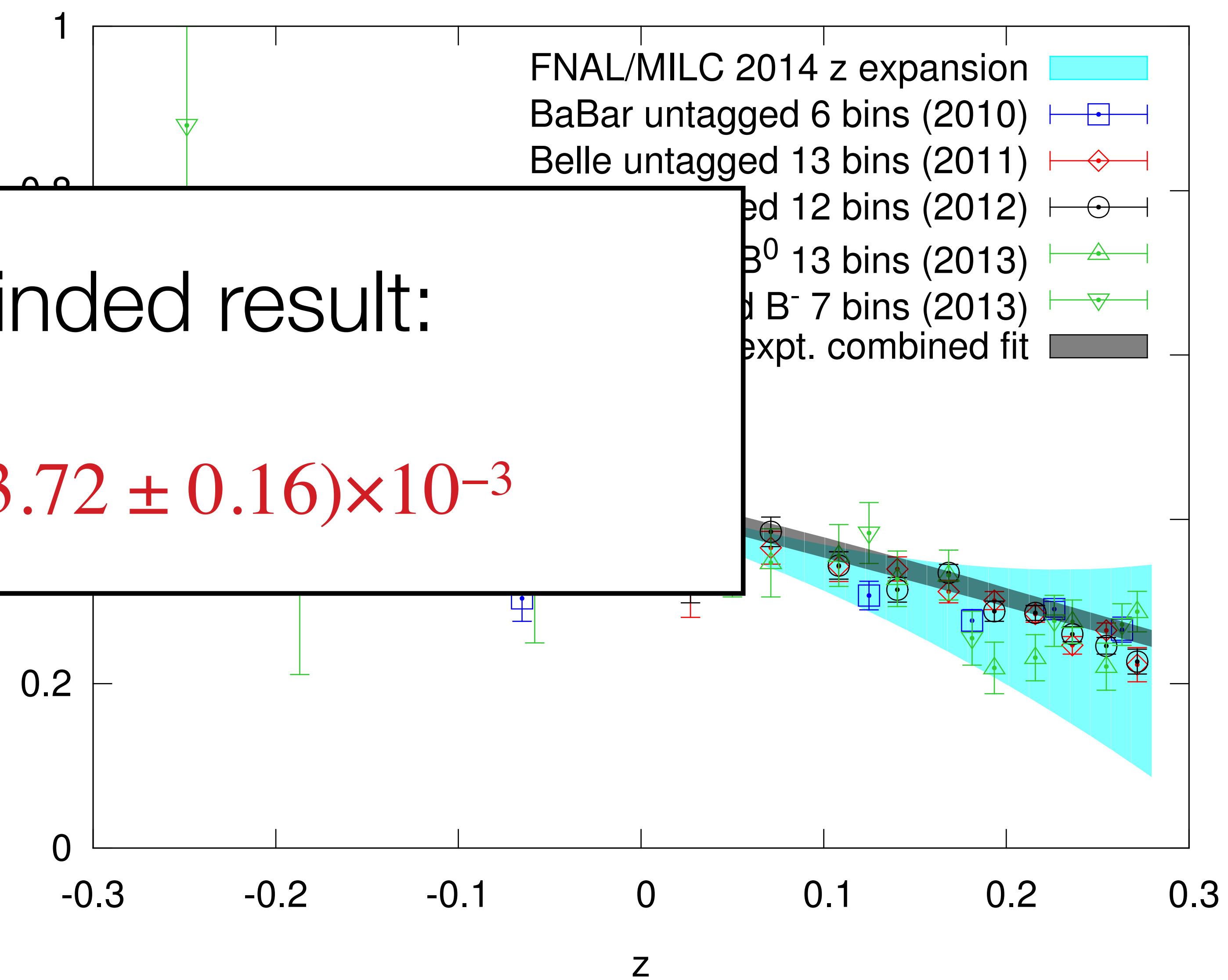


Unblinded result:

$|V_{ub}| = (3.72 \pm 0.16)\times 10^{-3}$

Reconstructing Form Factors

- For additional applications, the z expansion provides a useful summary.
- Formulas (Bourrely, Caprini, Lellouch, arXiv:0807.2722):

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z-1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]$$

$$f_0(z) = \sum_{n=0}^{N_z} b_n^0 z^n$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Subthreshold 1^- pole in f_+; first 0^+ excitation (for f_0) is unstable.

- Coefficients and correlations:

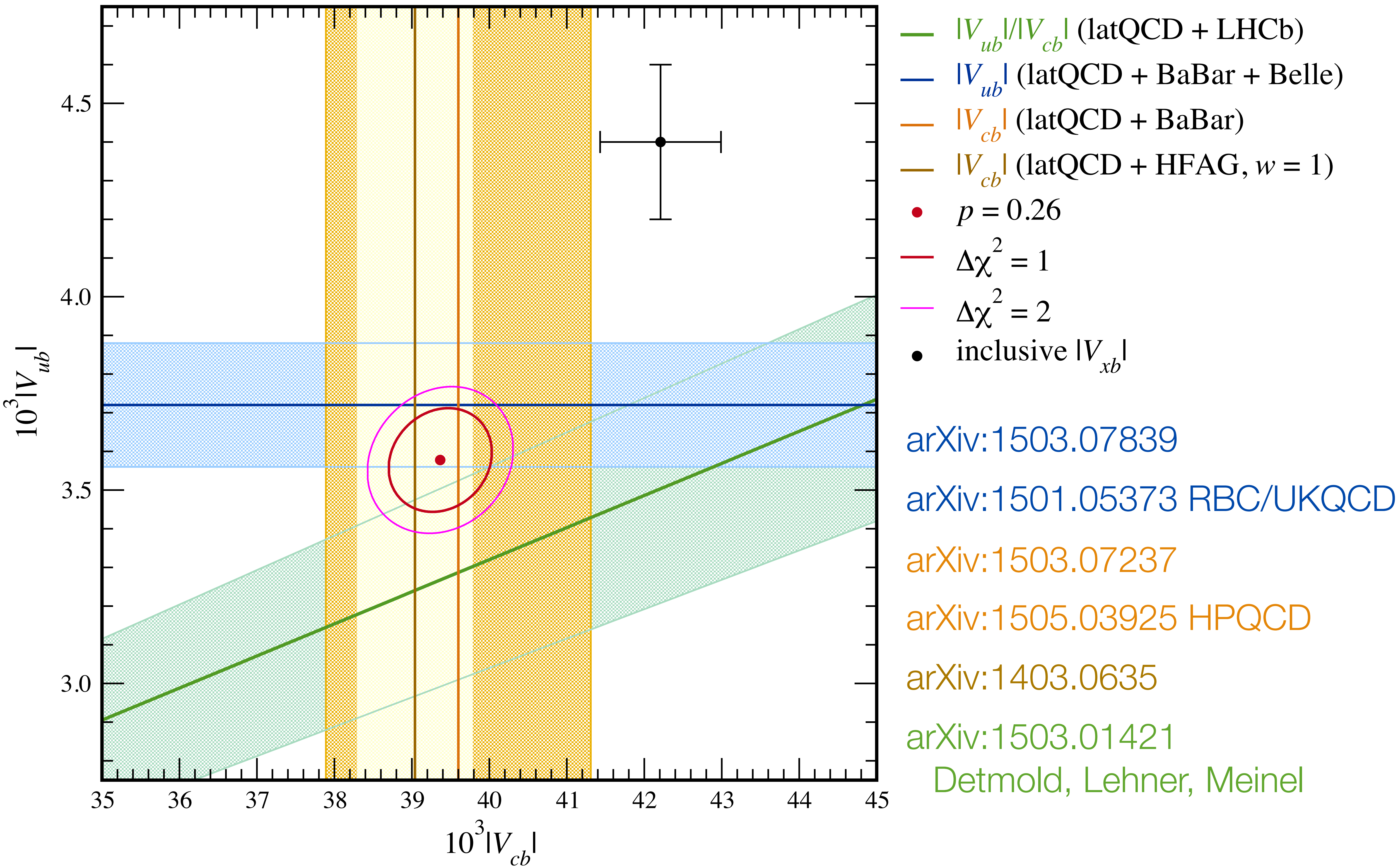
	b_0^+	b_1^+	b_2^+	b_3^+	b_0^0	b_1^0	b_2^0	b_3^0
	0.407(15)	−0.65(16)	−0.46(88)	0.4(1.3)	0.507(22)	−1.77(18)	1.27(81)	4.2(1.4)
b_0^+	1	0.451	0.161	0.102	0.331	0.346	0.292	0.216
b_1^+		1	0.757	0.665	0.430	0.817	0.854	0.699
b_2^+			1	0.988	0.482	0.847	0.951	0.795
b_3^+				1	0.484	0.833	0.913	0.714
b_0^0					1	0.447	0.359	0.189
b_1^0						1	0.827	0.500
b_2^0							1	0.838
b_3^0								1

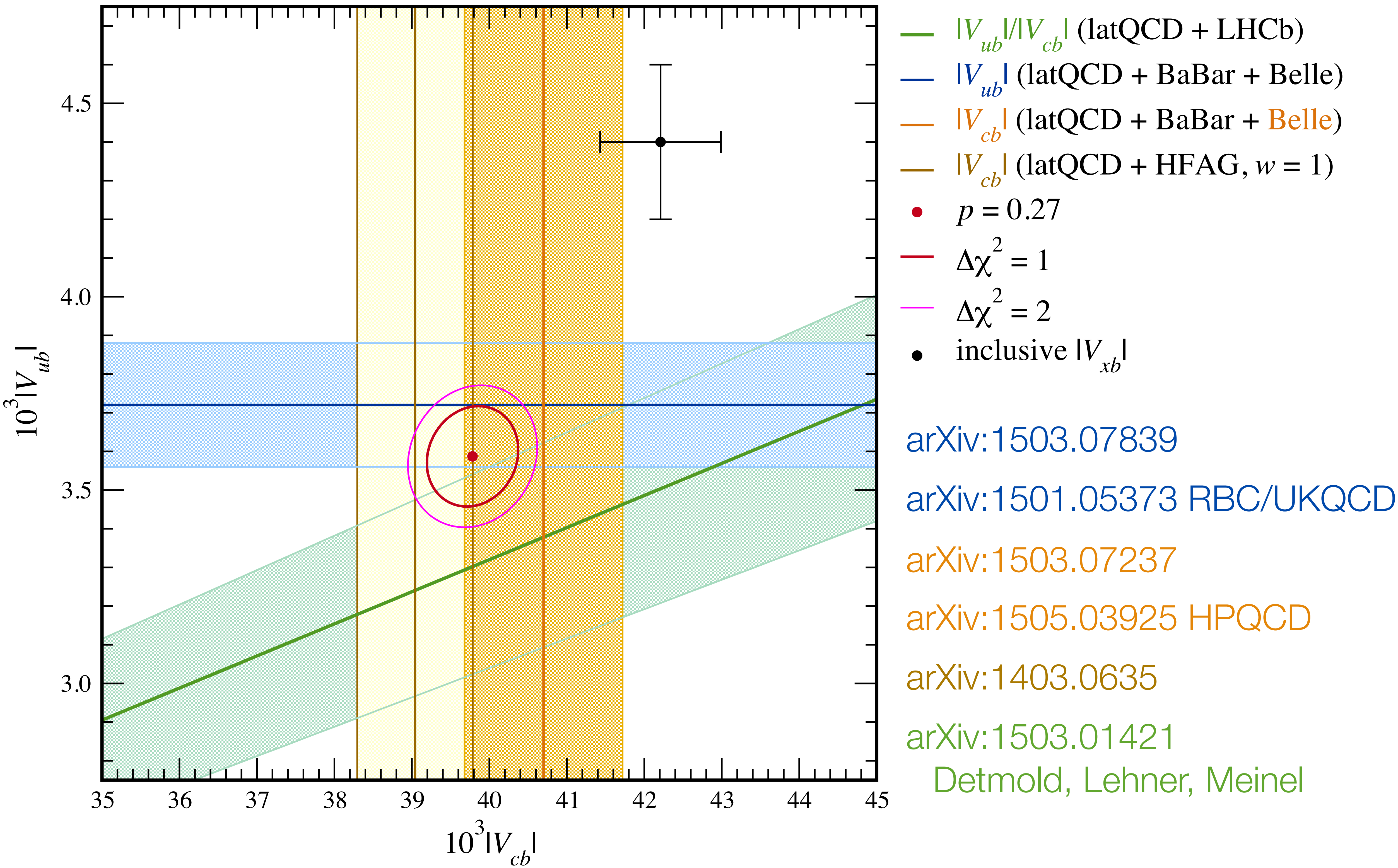
- If you are interesting in semileptonic B decays, just take this table and the formulas from the last slide and use the resulting form factors.

Semileptonic $B \to Dl\nu$ for $|V_{cb}|$

arXiv:1503.07237

- Similar strategy as above:
  - compute sequence of form-factor values;
  - chiral continuum extrapolation;
  - combined z-expansion fit to obtain $|V_{cb}|$.
- Differences:
  - HQET control of cutoff effects more central [hep-lat/0002008, hep-lat/0112044, hep-lat/0112045];
  - use Boyd, Grinstein, Lebed form of z expansion [hep-ph/9508211].


|V_ub|/|V_cb| (latQCD + LHCb)
|V_ub| (latQCD + BaBar + Belle)
|V_cb| (latQCD + BaBar)
|V_cb| (latQCD + HFAG, w = 1)
p = 0.26
Δχ^2 = 1
Δχ^2 = 2
inclusive |V_xb|
10^3|V_ub|
10^3|V_cb|
4.5
4.0
3.5
3.0
35
36
37
38
39
40
41
42
43
44
45
arXiv:1503.07839
arXiv:1501.05373 RBC/UKQCD
arXiv:1503.07237
arXiv:1505.03925 HPQCD
arXiv:1403.0635
arXiv:1503.01421
Detmold, Lehner, Meinel



|V_ub|/|V_cb| (latQCD + LHCb)
|V_ub| (latQCD + BaBar + Belle)
|V_cb| (latQCD + BaBar + Belle)
|V_cb| (latQCD + HFAG, w = 1)
p = 0.27
Δχ² = 1
Δχ² = 2
inclusive |V_xb|
10³|V_ub|
10³|V_cb|
4.5
4.0
3.5
3.0
35
36
37
38
39
40
41
42
43
44
45
arXiv:1503.07839
arXiv:1501.05373 RBC/UKQCD
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arXiv:1505.03925 HPQCD
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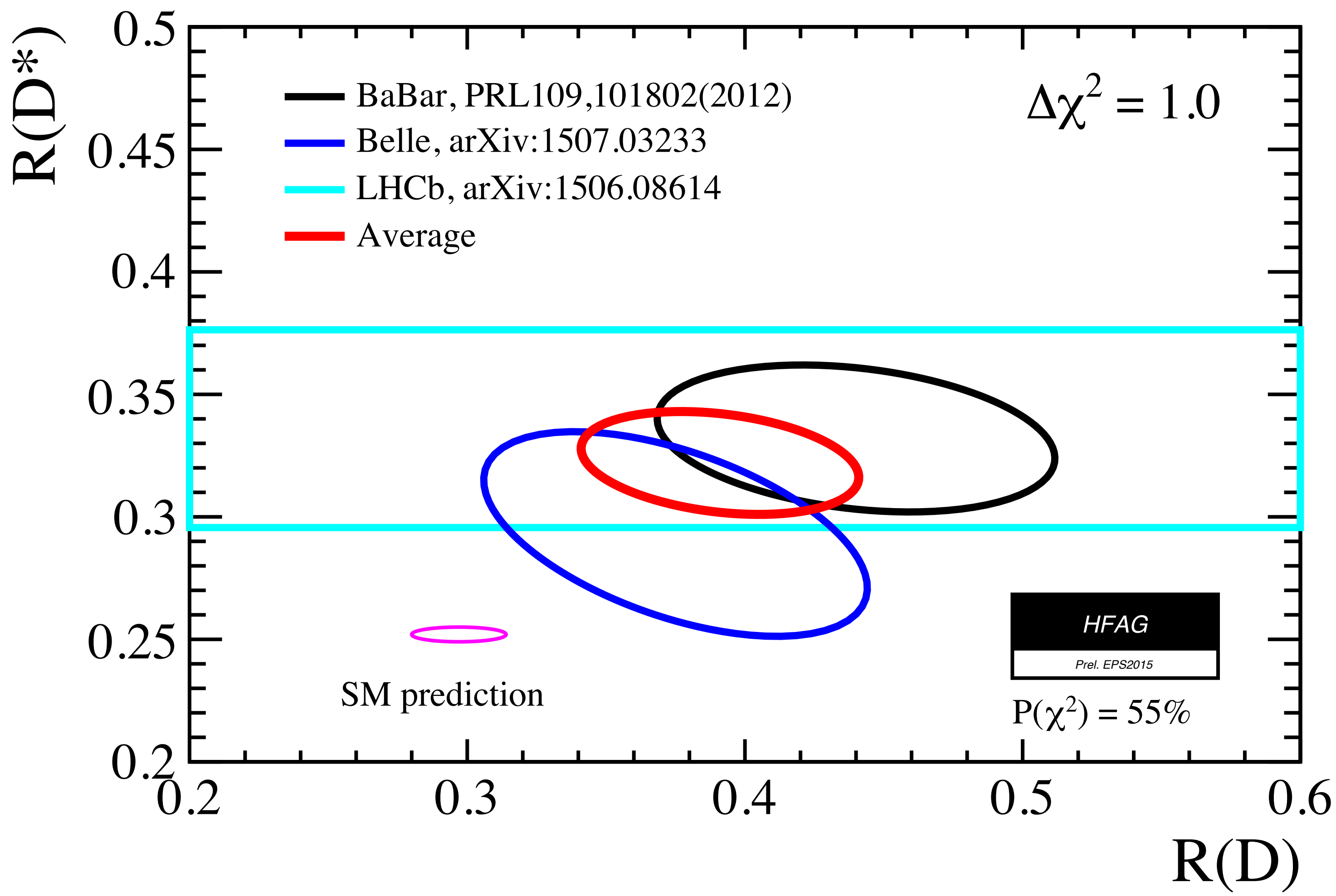
arXiv:1503.01421
Detmold, Lehner, Meinel

New Physics in $B \to D^{(*)}\tau\nu$?

BaBar, arXiv:1205.5442; Belle, arXiv:1507.03233; LHCb, arXiv:1506.08614

- BaBar presented evidence for an excess in both channels:
  - 2.0σ for $R(D)$; 2.7σ for $R(D^*)$; 3.4σ combined.
- With Belle & LHCb:
  - 3.9σ combined.
- Estimated form factors w/
  - HQET;
  - quenched QCD.

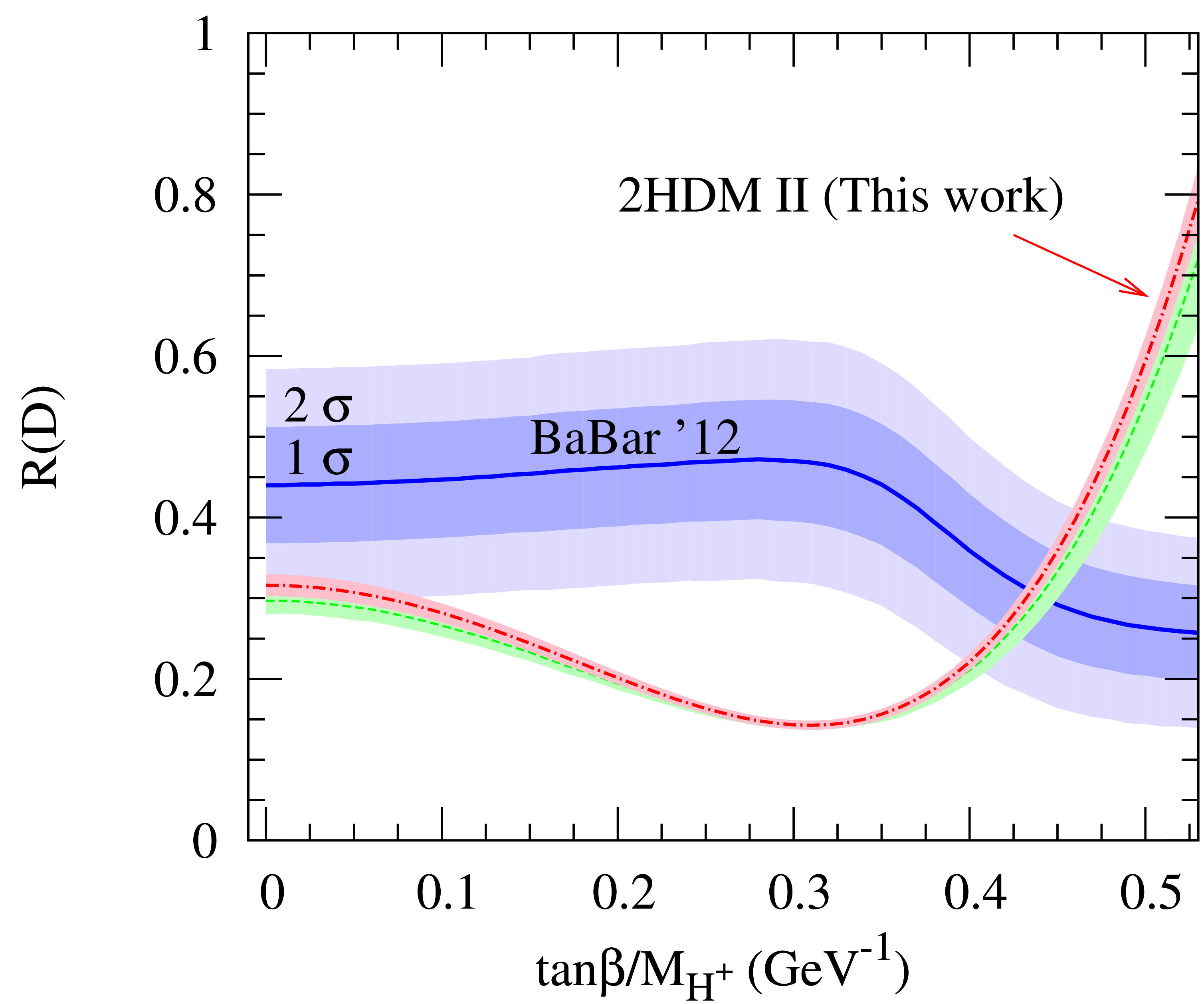


Form Factors for $B \to D^{(*)}\tau\nu$

Fermilab/MILC, arXiv:1206.4992, arXiv:1503.07237; HPQCD, arXiv:1505.03925

see also arXiv:1206.4977.

- $R(D)$ values:
  - 0.297 ± 0.017 (est.);
  - 0.316 ± 0.014 (F/M ’12);
  - 0.299 ± 0.011 (F/M ’15);
  - 0.300 ± 0.008 (HPQCD).
- Lattice QCD work for $R(D^*)$ yet to come.

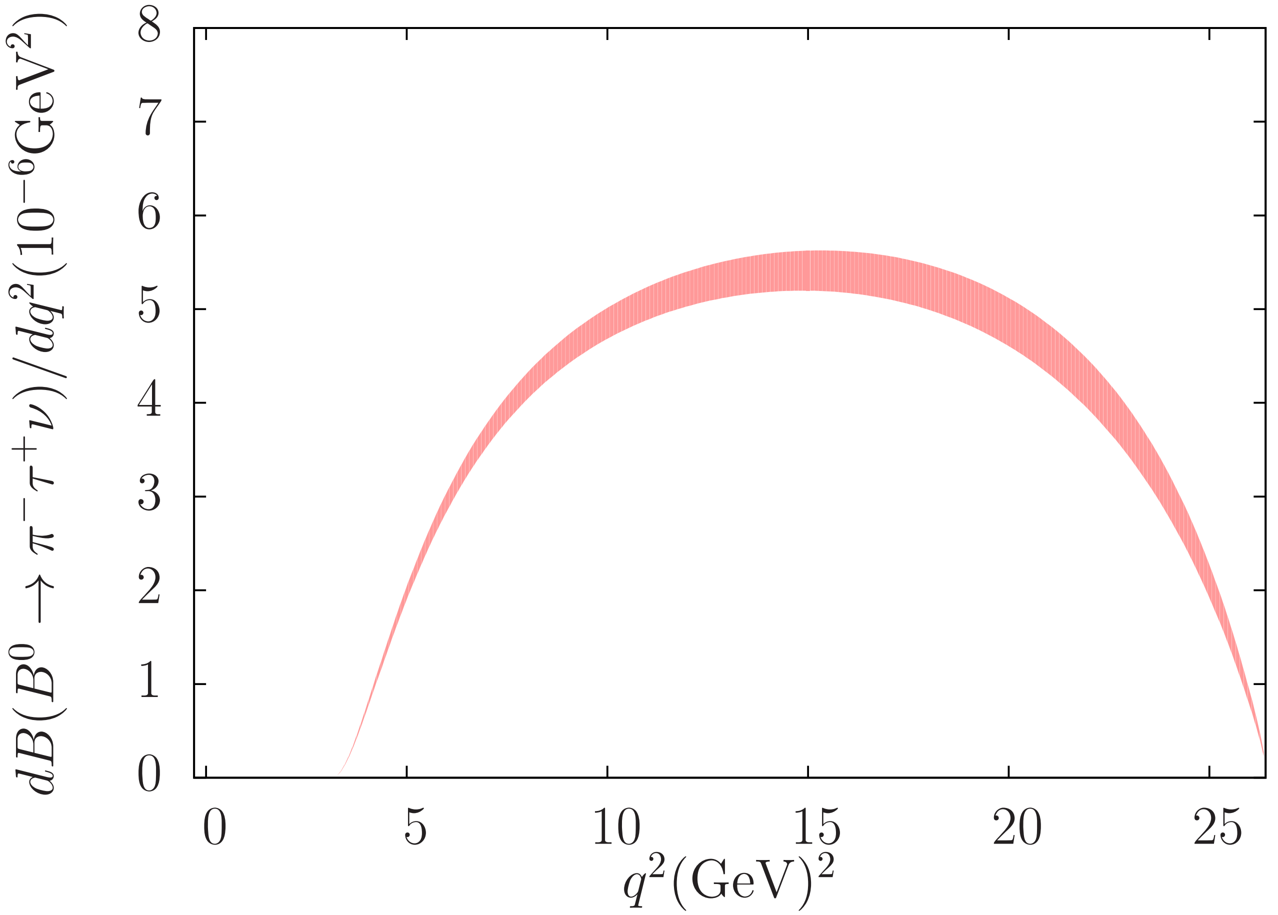


New Physics in $B \to \pi\tau\nu$?

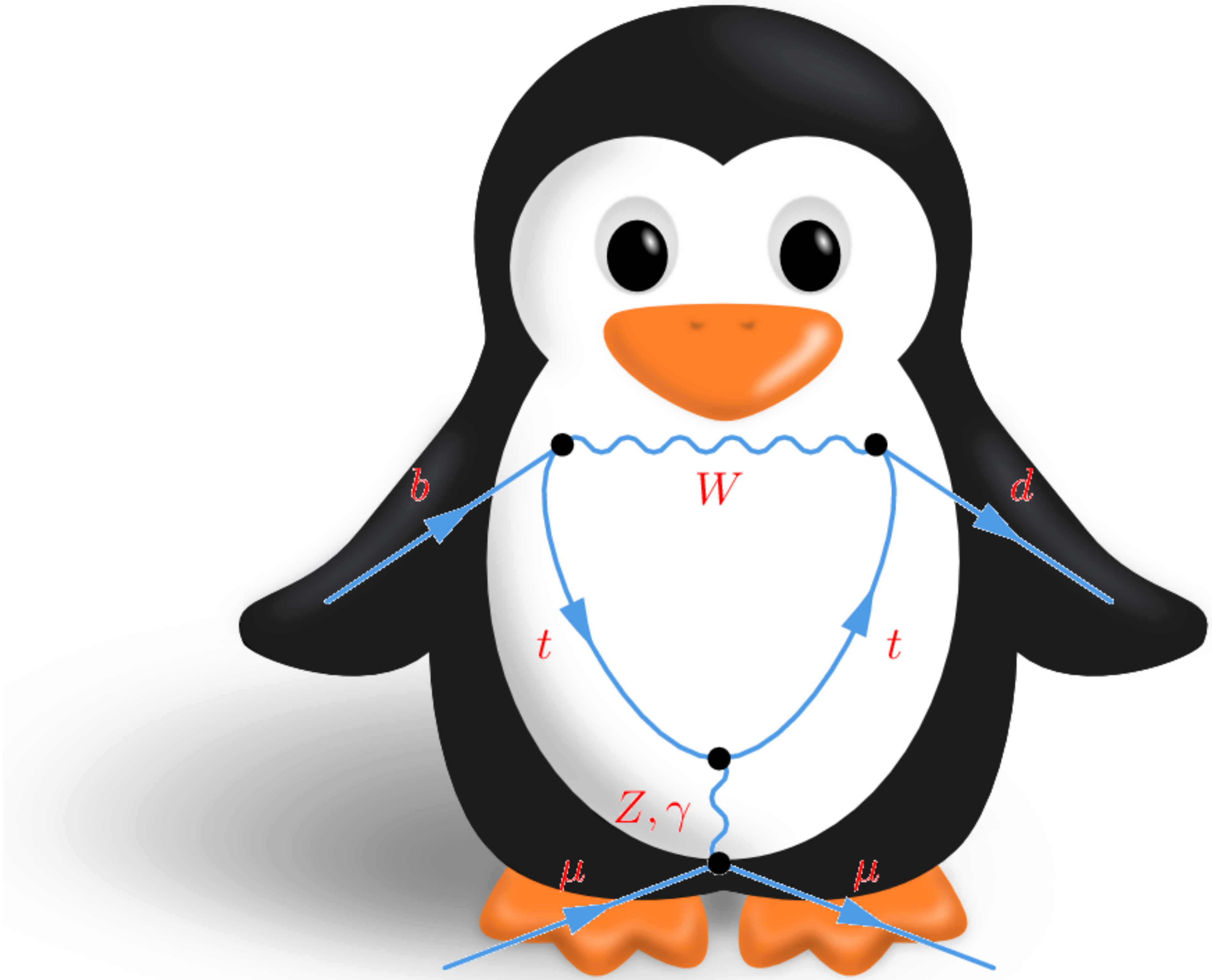
arXiv:1510.02349

- A charged Higgs mediating $b \to c$ could also mediate $b \to u$.
- SM prediction, including term $\sim m_\tau^2 |f_0|^2$.
- With the Fermilab/MILC form factors, we find

$$R(\pi) \equiv \frac{\mathscr{B}(B \to \pi\tau\nu_\tau)}{\mathscr{B}(B \to \pi\ell\nu_\ell)} = 0.641(17)$$



Penguins



graphic by Daping Du

Basic Formulas for $B^0 \to \pi^0\nu\nu$, $K^0\nu\nu$

- Relevant term in effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \frac{e^2}{16\pi^2} \frac{X_t}{\sin^2\theta_W}\, \bar{q}_L \gamma_\mu b_L \sum_\nu \bar{\nu}_L \gamma^\mu \nu_L$$

- Differential decay rate:

$$\frac{d\mathscr{B}(B \to P\nu\bar{\nu})}{dq^2} = C_P \tau_B \left|V_{tb} V_{tq}^*\right|^2 \frac{G_F^2 \alpha^2}{32\pi^5} \frac{X_t^2}{\sin^4\theta_W} |\boldsymbol{k}|^3 |f_+(q^2)|^2$$

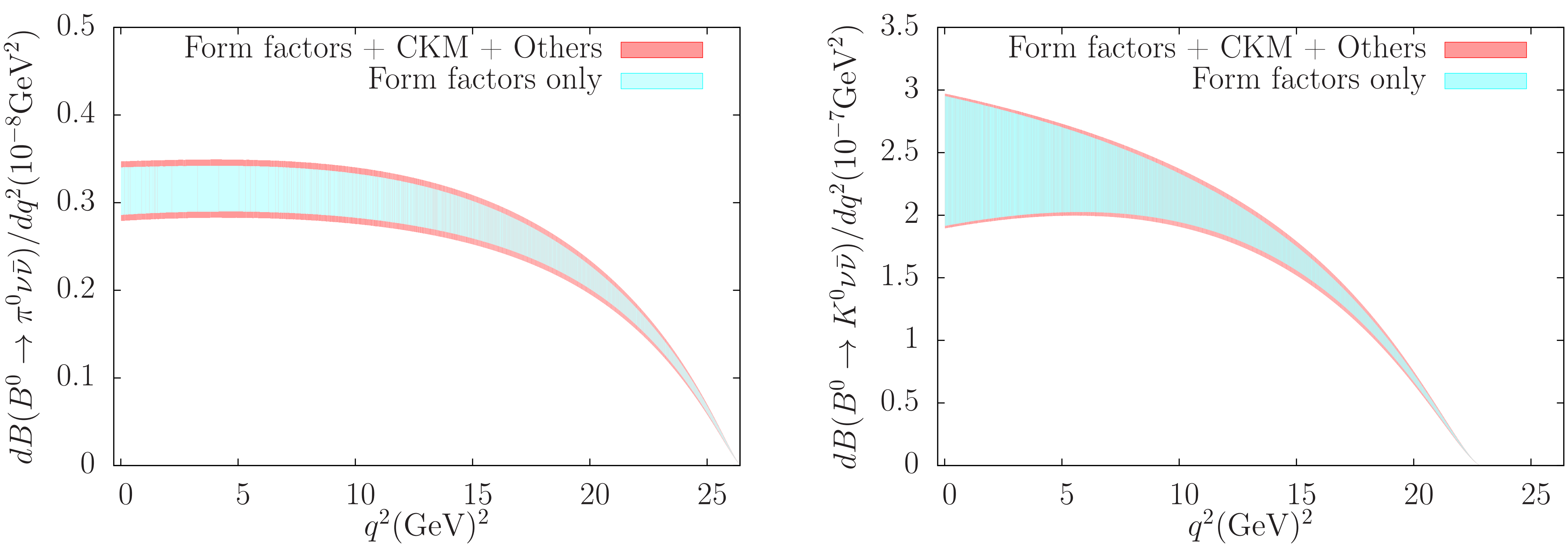
$$q = d \Rightarrow P = \pi^0, \quad C_{\pi^0} = \tfrac{1}{2}$$

$$q = s \Rightarrow P = K^0, \quad C_{K^0} = 1$$

- Thus, can use same form factor as above (and kaon counterpart).

SM Predictions for the Differential Rate

arXiv:1510.02349



- NB: charged counterparts receive contribution from the cascade $B \to \tau\nu$, $\tau \to \pi\nu$ (or $K\nu$), with amplitude proportional to $|V_{ub}|f_B\, |V_{ud}|f_\pi$ (or $|V_{us}|f_K$) [Kamenik & Smith, arXiv:0908.1174].

Basic Formulas for $B \to \pi l^+ l^-$, $K l^+ l^-$

cf., arXiv:1510.02349, Sec. 2 & Appendix B

- One-loop effective Hamiltonian contains many operators ($q = d, s$):

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \qquad Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \qquad Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q') \qquad Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q')$$

$$Q_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q') \qquad Q_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q')$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \ell) \qquad Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Matrix elements of Q_7, Q_9, Q_{10} yield form factors, including tensor f_T.

Other Matrix Elements

- Schematically,

$$\langle P\ell\ell|Q_i(y)|\bar{B}\rangle \sim (\bar{u}_\ell \gamma_\mu v_\ell) \int d^4x\, e^{iq\cdot(x-y)} \langle P|T\, J^\mu_{\mathrm{em}}(x) Q_i(y)|\bar{B}\rangle$$

- Four regions:
  - when q^2 is close to M^2 of ρ, ω, ϕ: very challenging [arXiv:1506.07760];
  - when q^2 is near charmonium resonances: also very challenging;
  - low q^2, between these two: SCET (aka QCD factorization);
  - high $q^2 \sim M_B{}^2$: OPE [Grinstein, Pirjol, hep-ph/0404250], leading back to the three form factors, f_+, f_0, & f_T.

Soft Collinear Effective Theory

hep-ph/9905312, hep-ph/0006124, hep-ph/0011336, hep-ph/0109045

- The RHS of the equation on the last slide takes the form

$$C_i\langle P\ell\ell|Q_i|\bar{B}\rangle \sim C_i\ \left[\boxed{(1+\alpha_s)f_T} + \boxed{(1+\alpha_s)f_+} + \phi_B \star T \star \phi_P\right], \quad i=1,\ldots,6,$$
$$C_8\langle P\ell\ell|Q_8|\bar{B}\rangle \sim C_8\ \left[\boxed{\alpha_s\ f_T} + \boxed{\alpha_s\ f_+} + \phi_B \star T \star \phi_P\right],$$

light-cone distribution amplitudes

$$C_7\langle P\ell\ell|Q_7|\bar{B}\rangle \sim C_7 f_T \rightsquigarrow C_7^{\text{eff}} f_T,$$
$$C_9\langle P\ell\ell|Q_9|\bar{B}\rangle \sim C_9 f_+ \rightsquigarrow C_9^{\text{eff}} f_+,$$
$$C_{10}\langle P\ell\ell|Q_{10}|\bar{B}\rangle \sim C_{10} f_+,$$

hep-ph/0008255, hep-ph/0106067

- We add the nonfactorizable terms to the amplitude using the known hard-scattering kernel T and distribution amplitudes from the literature.

- SM C_7, C_9, C_{10} from Huber, Lunghi, Misiak, Wyler [hep-ph/0512066].

Coefficients and Correlations: $B \to K l^+ l^-$

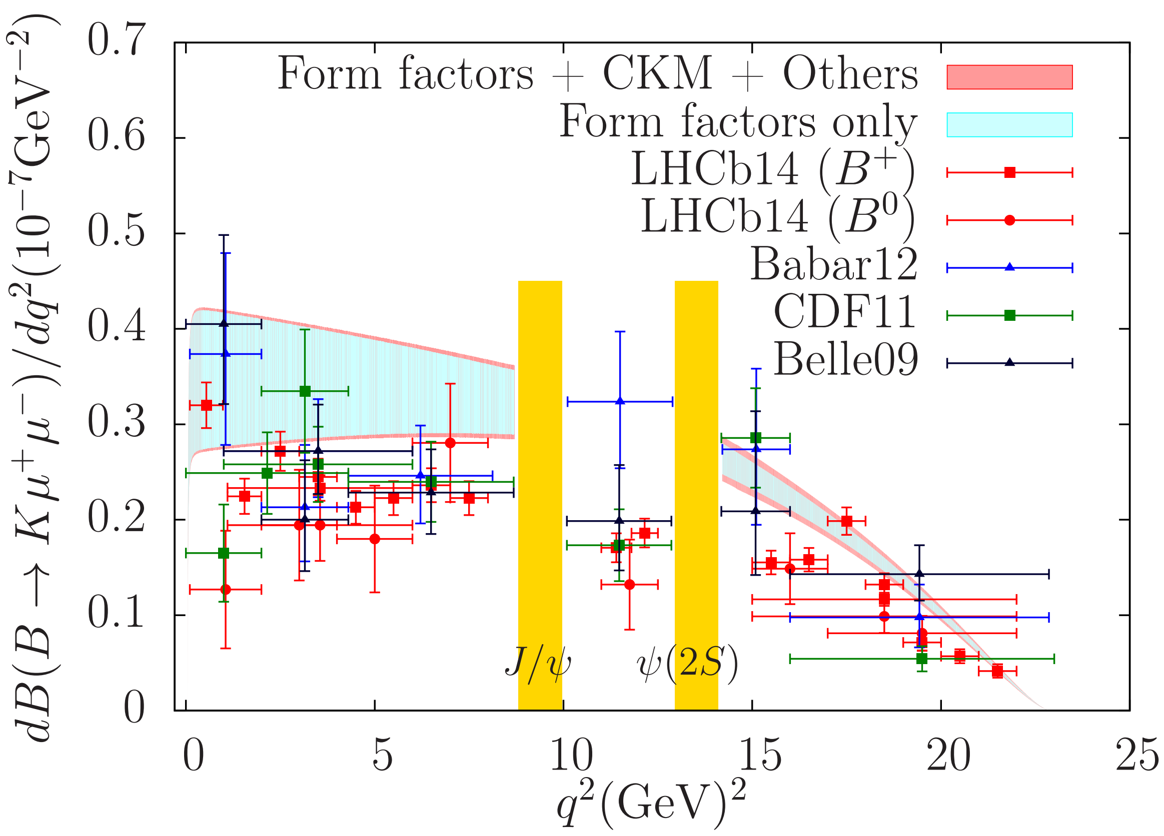
arXiv:1509.06235

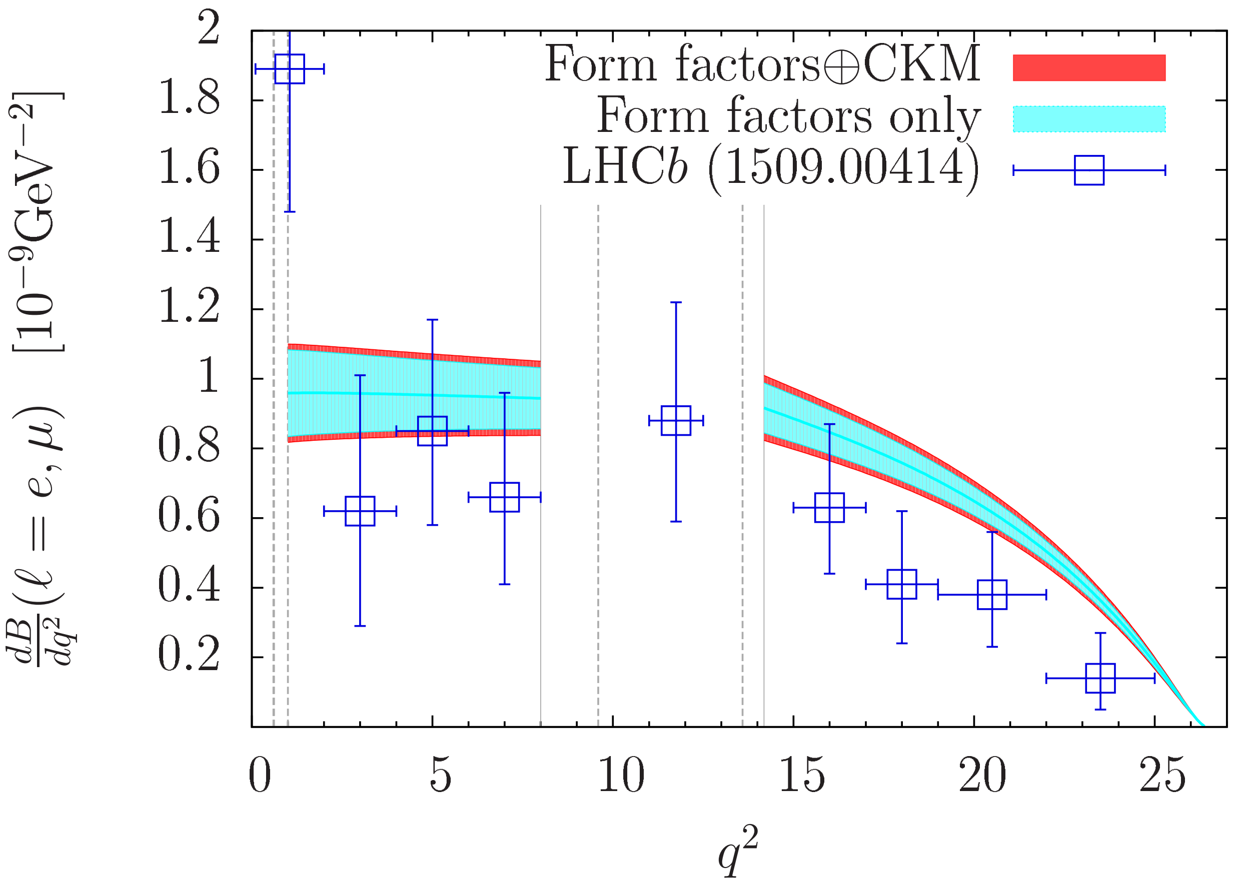
	b_0^+	b_1^+	b_2^+	b_0^0	b_1^0	b_2^0	b_0^T	b_1^T	b_2^T
mean	0.466	−0.885	−0.213	0.292	0.281	0.150	0.460	−1.089	−1.114
error	0.014	0.128	0.548	0.010	0.125	0.441	0.019	0.236	0.971
b_0^+	1	0.450	0.190	0.857	0.598	0.531	0.752	0.229	0.117
b_1^+		1	0.677	0.708	0.958	0.927	0.227	0.443	0.287
b_2^+			1	0.595	0.770	0.819	−0.023	0.070	0.196
b_0^0				1	0.830	0.766	0.582	0.237	0.192
b_1^0					1	0.973	0.324	0.372	0.272
b_2^0						1	0.268	0.332	0.269
b_0^T							1	0.590	0.515
b_1^T								1	0.897
b_2^T									1

- Corresponding information for $B \to \pi l^+ l^-$ in arXiv:1503.07839 and arXiv:1507.01618.

Kinematic Distributions

- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot's theory **preceded** measurement:





- arXiv:1510.02349 also contains predictions for the ratio, the flat terms, etc.

Other Results in arXiv:1510.02349

- Tests of heavy-quark and SU(3)-flavor symmetries.

- Comparisons with LHCb over wide bins, $q^2 \in [1\ \mathrm{GeV}^2, 6\ \mathrm{GeV}^2]$, and $q^2 \in [15\ \mathrm{GeV}^2, 22\ \mathrm{GeV}^2]$).

- SM predictions for $B \to \pi\tau^+\tau^-$, $B \to K\tau^+\tau^-$

- SM contribution to lepton-universality violation ($q^2 \in [1\ \mathrm{GeV}^2, 6\ \mathrm{GeV}^2]$):

$$R^{\mu e}_{K^+}\big|_{\mathrm{SM}} = 1.00050(43), \text{ vs. } R^{\mu e}_{K^+}\big|_{\mathrm{LHCb}} = 0.745^{+97}_{-82}$$

$$R^{\mu e}_{K^+} = \int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} d\mathscr{B}(B \to K\mu\mu) \left[\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} d\mathscr{B}(B \to Kee)\right]^{-1}$$

- Neutrino modes and $B \to \pi\tau\nu$, discussed earlier.

CKM: $|V_{td}|$ and $|V_{ts}|$

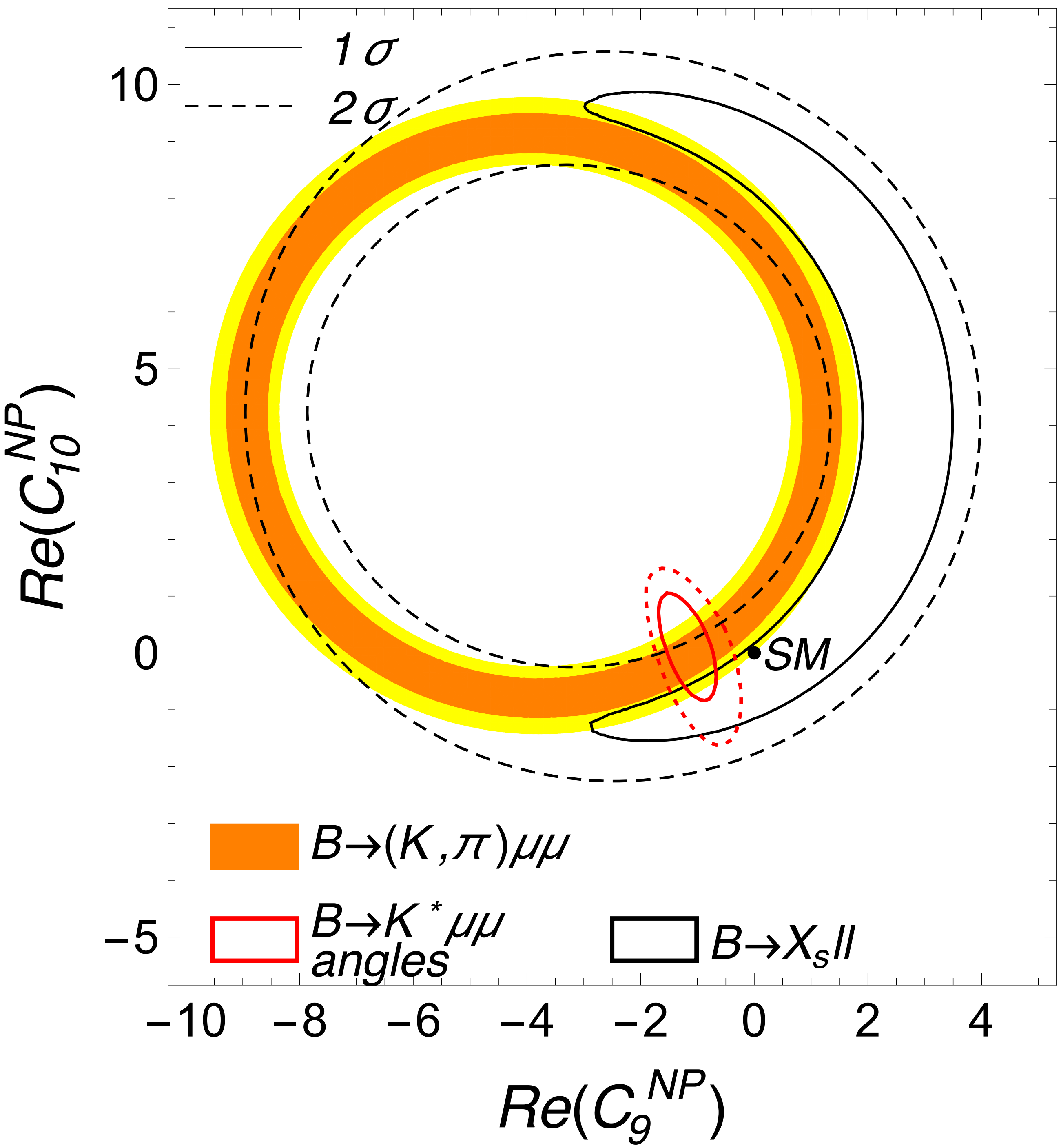
- Assume that there is no new physics buried in the Wilson coefficients.
- Then the combination of our calculations with experimental measurements yield the third row of the CKM matrix.
- We find

$$|V_{td}/V_{ts}| = 0.201(20)$$
$$|V_{tb}^* V_{td}| \times 10^3 = 7.45(69)$$
$$|V_{tb}^* V_{ts}| \times 10^3 = 35.7(1.5)$$

- The uncertainty here is commensurate from the neutral *B*-meson mixing.
- Result for $|V_{ts}|$ is 1.4σ lower than that from mixing.

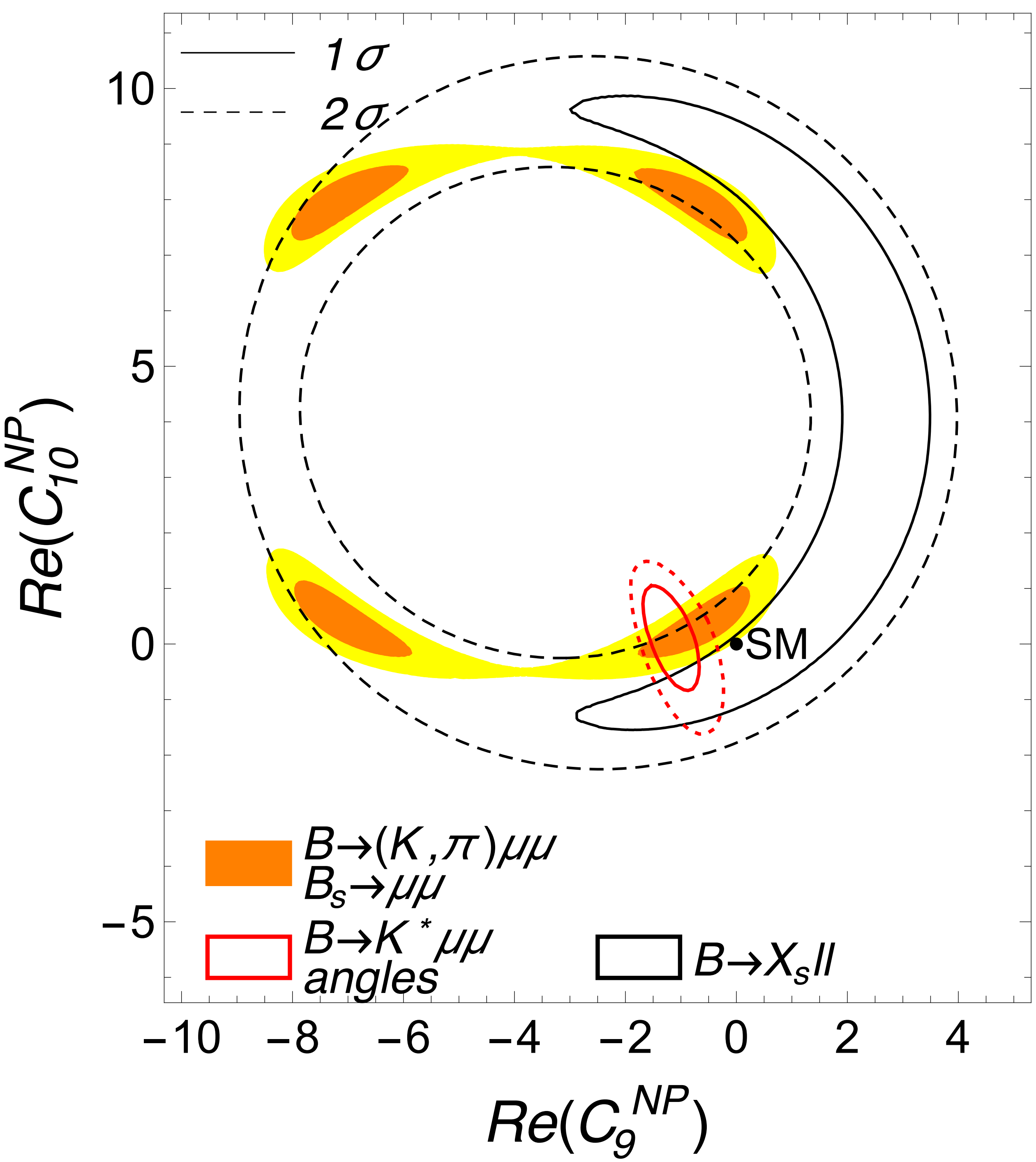
Wilson Coefficients

- Assuming no new physics is sad:
  - take the CKM matrix from a global fit;
  - determine best fit to Wilson coefficients C_9 and C_{10}.
- From the observables considered here, the SM is 2σ away from the best fit.
- Comparable but complementary to angular observables in $B \to K^* \mu\mu$.



Wilson Coefficients 2

- Add $B_s \to \mu\mu$, which also relies on lattice QCD—f_{B_s}.
- Favored region shrinks but only away from SM point.
- NB: assuming no new CPV and avoiding $b \to s\gamma$ constraints.



Outlook

- Perhaps too much information to summarize.
- The overarching take-home message:
  - we provide a convenient useful parametrization of the form factors, including correlations needed for joint fits, ratios, *etc*.;
  - the scope of application is not limited to what we've done;
  - just like collider physicists use CTEQ or MRSW parton densities, flavor physicists can use our (or other group's) form factors.
- Future work, e.g., on MILC HISQ ensembles, will improve the precision (over the coming few years).

¡Muchas gracias!