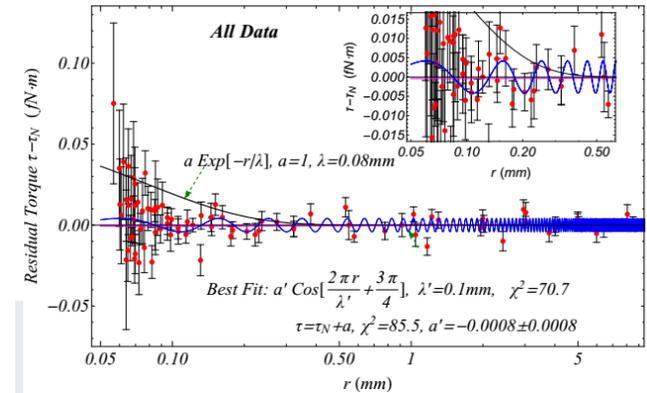
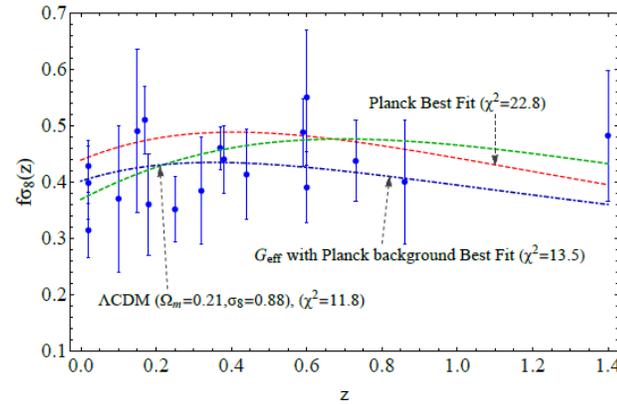
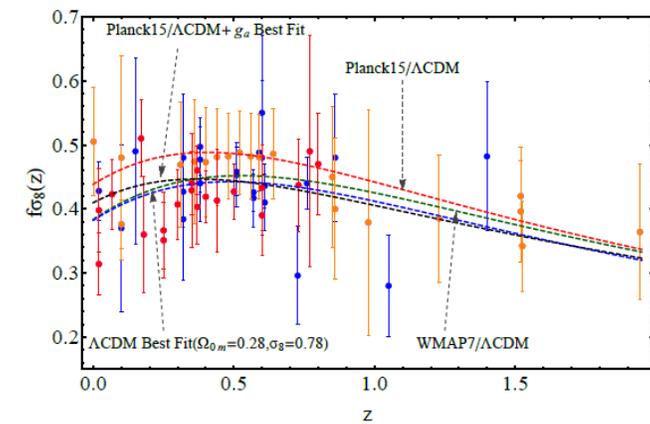


# Searching for Hints of Modified Gravity in Cosmological and Sub-mm Force Data



Leandros Perivolaropoulos

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Talk based on:

**The evolution of the  $f\sigma_8$  tension with Planck15/ $\Lambda$ CDM and implications for modified gravity theories**

Lavrentios Kazantzidis, Leandros Perivolaropoulos. Mar 4, 2018. 16 pp.

e-Print: [arXiv:1803.0133](https://arxiv.org/abs/1803.0133)

**Tension and constraints on modified gravity parametrizations of  $G_{\text{eff}}(z)$  from growth rate and Planck data**

Savvas Nesseris, George Pantazis (Ioannina U.), Leandros Perivolaropoulos

Mar 30, 2017. 19 pp.

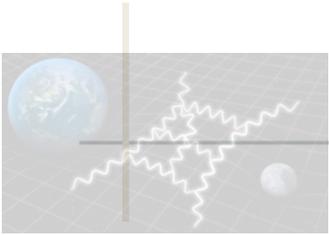
Published in Phys.Rev. D96 (2017) no.2, 023542

**Submillimeter spatial oscillations of Newton's constant: Theoretical models and laboratory tests**

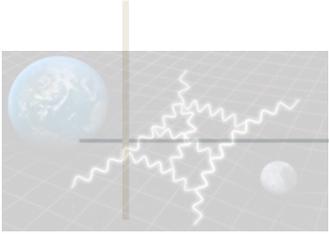
Leandros Perivolaropoulos, Nov 22, 2016. 20 pp.

Published in Phys.Rev. D95 (2017) no.8, 084050

# Structure of talk



1. Scales of tests of General Relativity:  
Common Parametrizations measuring deviations.
2. Cosmological Scales  
Growth of Density Perturbations  
Tension of Growth Data with Planck15  $\Lambda$ CDM  
Evolution of Tension with Time of Publication  
Easing the Tension with Evolution of Newton Constant  $G_{\text{eff}}(z)$   
Reconstruction of Scalar-Tensor Theory.
3. Sub-mm new forces (small scale gravity experiments)  
Oscillating Parametrizations of  $G(r)$ : Improved fit to Data  
Theoretical Models:  $f(R)$  theories, Infinite Derivative Gravity



# Scales of GR Tests I:

## Sub-mm Scales: Space Translation Invariance of G

**Yukawa Parametrization:**  $V_{eff} = -G\frac{M}{r}(1 + \alpha e^{-mr}) \Rightarrow G_{eff}(r) = G(1 + \alpha e^{-mr})$

### Constraints from

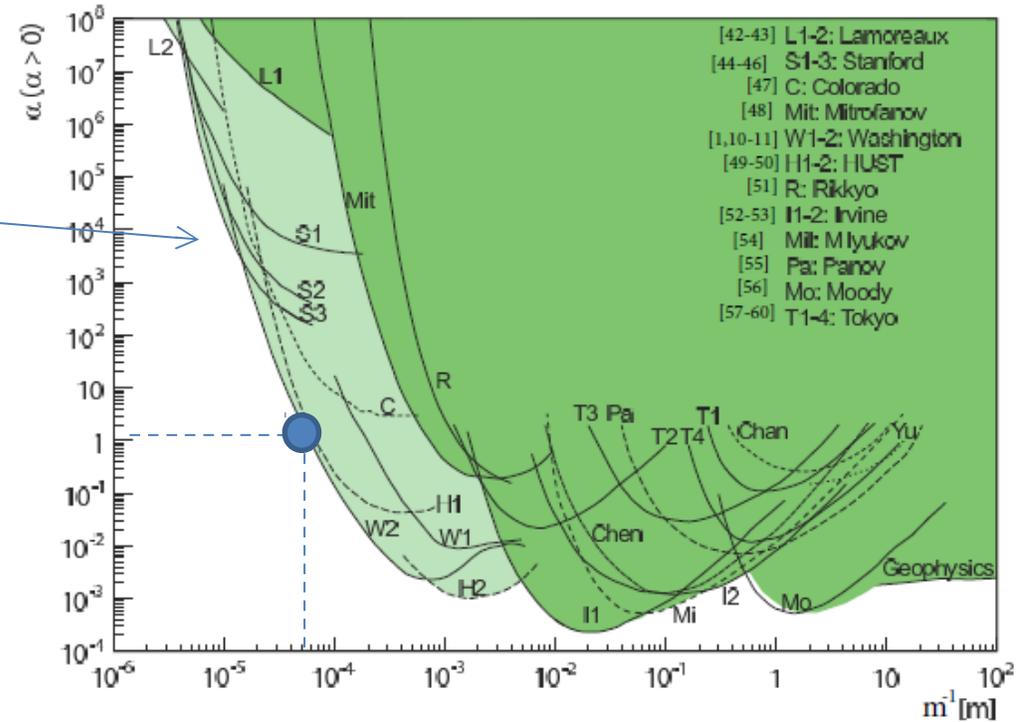
Jiro Murata and Saki Tanaka, "A review of short-range gravity experiments in the LHC era," *Class. Quant. Grav.* **32**, 033001 (2015), arXiv:1408.3588 [hep-ex].

### Theoretical Motivation:

$$f(R) = R + \frac{1}{6m^2}R^2$$

$$T_{\mu\nu} = \text{diag}(M\delta(\vec{r}), 0, 0, 0)$$

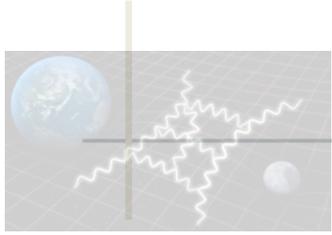
$$h_{00} = \frac{2GM}{r} \left( 1 + \frac{1}{3}e^{-mr} \right)$$



L. Perivolaropoulos, "PPN Parameter gamma and Solar System Constraints of Massive Brans-Dicke Theories," *Phys. Rev. D* **81**, 047501 (2010), arXiv:0911.3401 [gr-qc].

Similar result for massive Brans-Dicke theories

What if  $1/6m^2 < 0$ ?



# Scales of GR Tests II: Cosmological Scales- $\Lambda$ , Dark Energy or Modified Gravity

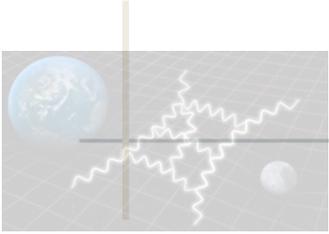
Newtonian Gauge Cosmological Perturbations:

$$ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\mathbf{x}^2$$

Modified Poisson equations:

$$\begin{aligned}\nabla^2\phi &= 4\pi G_{eff}a^2\rho\delta_m \\ \nabla^2(\phi + \psi) &= 8\pi G_L a^2\rho\delta_m\end{aligned}$$

$G_{eff}$  (matter density perturbations),  $G_L$  (lensing of light)  
parametrize deviations from GR ( $G_{eff}=G_L=G_N$  in GR)

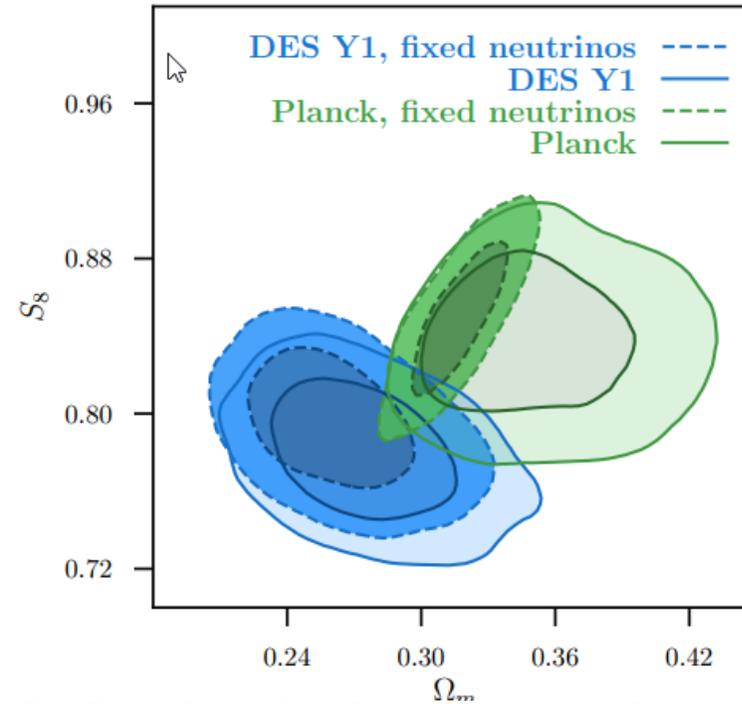
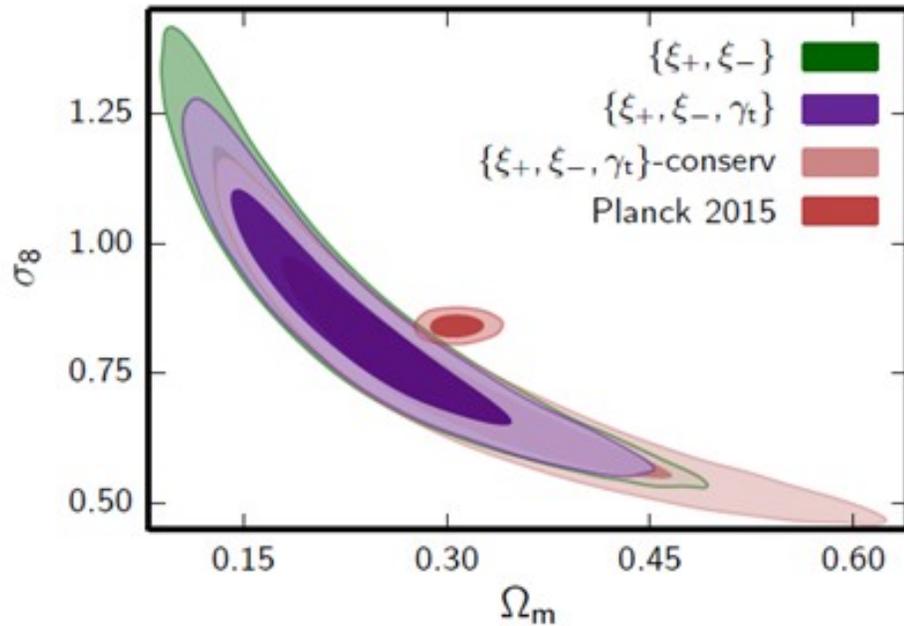
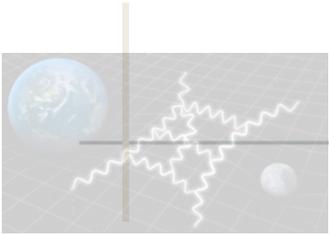


# Basic Questions

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1. Is GR consistent with data on each scale?
2. What is the optimum parametrization of  $G_{\text{eff}}$  in providing the best quality of fit to the data?
3. What are the theoretical models that support such parametrization?

# Tension from Weak Lensing



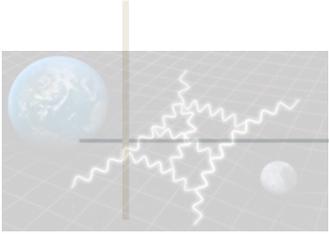
$$S_8 \equiv \sigma_8 (\Omega_m / 0.3)^{0.5}$$

KIDS-450 + 2dFLenS: Cosmological parameter constraints from weak gravitational lensing tomography and overlapping redshift-space galaxy clustering  
 Shahab Joudaki (Swinburne U., Ctr. Astrophys. Supercomput. & Oxford U.) *et al.*, Jul 20, 2017. 31 pp.  
 e-Print: [arXiv:1707.06627](https://arxiv.org/abs/1707.06627)

Dark Energy Survey Year 1 Results: Cosmological Constraints from Galaxy Clustering and Weak Lensing  
 DES Collaboration (T.M.C. Abbott (Cerro-Tololo InterAmerican Obs.) *et al.*). Aug 4, 2017. 31 pp.  
 FERMILAB-PUB-17-294-PPD  
 e-Print: [arXiv:1708.01530](https://arxiv.org/abs/1708.01530)

**Weak Lensing probes gravitational growth and indicates tension with Planck15/ $\Lambda$ CDM**

# RSD: Observational Probe of Perturbation Growth



Growth rate:  $f(a) = \frac{d \ln \delta}{d \ln a}$      $\delta(a) \equiv \frac{\delta \rho}{\rho}$

Density rms fluctuations within spheres of radius  $R = 8h^{-1}\text{Mpc}$      $\sigma(a) = \sigma_8 \frac{\delta(a)}{\delta(1)}$

Observable:  $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

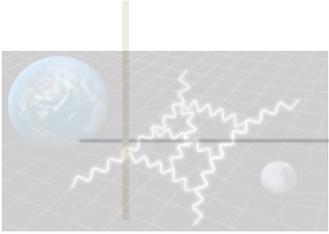
Define  $H(z)$  parametrization:  $E(a)^2 \equiv H(a)^2/H_0^2 = \Omega_{0m}a^{-3} + (1 - \Omega_{0m})a^{-3(1+w)}$

Solve the dynamical growth equation to obtain  $\delta(a, w, \Omega_{0m})$  (set  $G_{\text{eff}}/G_N=1$ ):

$$\delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \frac{\Omega_m G_{\text{eff}}(a, k)/G_N}{a^5 H(a)^2/H_0^2} \delta(a) = 0$$

Construct theoretically predicted  $f\sigma_8(a, \sigma_8, w, \Omega_{0m})$ :  $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

# Construction of Likelihood Contours for GR ( $G_{\text{eff}}/G_N=1$ )



Construct theoretically predicted  $f\sigma_8(a, \sigma_8, w, \Omega_{0m})$ :  $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$ .

Construct  $\chi^2(\sigma_8, w, \Omega_{0m})$ :  $V^i(z_i, \Omega_{0m}, \sigma_8, g_a) \equiv f\sigma_{8i} - f\sigma_8(z_i, \Omega_{0m}, \sigma_8, g_a)$   $\chi_{\text{growth}}^2 = V^i C_{ij}^{-1} V^j$ ,

$$C_{ij}^{\text{growth, total}} = \begin{pmatrix} \sigma_1^2 & 0 & 0 & \dots \\ 0 & C_{ij}^{\text{WiggleZ}} & 0 & \dots \\ 0 & 0 & \dots & \sigma_N^2 \end{pmatrix}$$

# $f\sigma_8(z)$ Growth Data (2016)

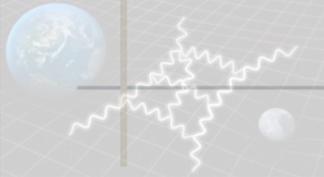
S. Nesseris, G. Pantazis and L. Perivolaropoulos,

arXiv:1703.10538 [astro-ph.CO] **Phys.Rev. D96 (2017) no.2, 023542**

Index	Dataset	$z$	$f\sigma_8(z)$	Refs.	Year	Notes
1	SDSS-LRG	0.35	$0.440 \pm 0.050$	[58]	2006	$(\Omega_m, \Omega_K) = (0.25, 0)$
2	VVDS	0.77	$0.490 \pm 0.18$	[58]	2008	$(\Omega_m, \Omega_K) = (0.25, 0)$
3	2dFGRS	0.17	$0.510 \pm 0.060$	[58]	2009	$(\Omega_m, \Omega_K) = (0.3, 0)$
4	2MASS	0.02	$0.314 \pm 0.048$	[59],[60]	2010	$(\Omega_m, \Omega_K) = (0.266, 0)$
5	SnIa+IRAS	0.02	$0.398 \pm 0.065$	[61],[60]	2011	$(\Omega_m, \Omega_K) = (0.3, 0)$
6	SDSS-LRG-200	0.25	$0.3512 \pm 0.0583$	[62]	2011	$(\Omega_m, \Omega_K) = (0.25, 0)$
7	SDSS-LRG-200	0.37	$0.4602 \pm 0.0378$	[62]	2011	
8	SDSS-LRG-60	0.25	$0.3665 \pm 0.0601$	[62]	2011	$(\Omega_m, \Omega_K) = (0.25, 0)$
9	SDSS-LRG-60	0.37	$0.4031 \pm 0.0586$	[62]	2011	
10	WiggleZ	0.44	$0.413 \pm 0.080$	[63]	2012	$(\Omega_m, h) = (0.27, 0.71)$
11	WiggleZ	0.60	$0.390 \pm 0.063$	[63]	2012	$C_{ij} \rightarrow \text{Eq. (2.8)}$ .
12	WiggleZ	0.73	$0.437 \pm 0.072$	[63]	2012	
13	SDSS-BOSS	0.30	$0.407 \pm 0.055$	[64]	2012	$(\Omega_m, \Omega_K) = (0.25, 0)$
14	SDSS-BOSS	0.40	$0.419 \pm 0.041$	[64]	2012	
15	SDSS-BOSS	0.50	$0.427 \pm 0.043$	[64]	2012	
16	SDSS-BOSS	0.60	$0.433 \pm 0.067$	[64]	2012	
17	SDSS-DR7-LRG	0.35	$0.429 \pm 0.089$	[65]	2012	$(\Omega_m, \Omega_K) = (0.25, 0)$
18	6dFGRS	0.067	$0.423 \pm 0.055$	[66]	2012	$(\Omega_m, \Omega_K) = (0.27, 0)$
19	GAMA	0.18	$0.360 \pm 0.090$	[67]	2013	$(\Omega_m, \Omega_K) = (0.27, 0)$
20	GAMA	0.38	$0.440 \pm 0.060$	[67]	2013	
21	BOSS-LOWZ	0.32	$0.384 \pm 0.095$	[68]	2013	$(\Omega_m, \Omega_K) = (0.274, 0)$
22	SDSS-CMASS	0.59	$0.488 \pm 0.060$	[69]	2013	$(\Omega_m, h, \sigma_8) = (0.307115, 0.6777, 0.8288)$
23	Vipers	0.80	$0.470 \pm 0.080$	[70]	2013	$(\Omega_m, \Omega_K) = (0.25, 0)$
24	SDSS-MGS	0.15	$0.490 \pm 0.145$	[71]	2014	$(\Omega_m, h, \sigma_8) = (0.31, 0.67, 0.83)$
25	SDSS-veloc	0.10	$0.370 \pm 0.130$	[72]	2015	$(\Omega_m, \Omega_K) = (0.3, 0)$
26	FastSound	1.40	$0.482 \pm 0.116$	[73]	2015	$(\Omega_m, \Omega_K) = (0.270, 0)$
27	6dFGS+SnIa	0.02	$0.428 \pm 0.0465$	[74]	2016	$(\Omega_m, h, \sigma_8) = (0.3, 0.683, 0.8)$
28	Vipers PDR-2	0.60	$0.550 \pm 0.120$	[75]	2016	$(\Omega_m, \Omega_b) = (0.3, 0.045)$
29	Vipers PDR-2	0.86	$0.400 \pm 0.110$	[75]	2016	
30	BOSS DR12	0.38	$0.497 \pm 0.045$	[76]	2016	$(\Omega_m, \Omega_K) = (0.31, 0)$
31	BOSS DR12	0.51	$0.458 \pm 0.038$	[76]	2016	
32	BOSS DR12	0.61	$0.436 \pm 0.034$	[76]	2016	
33	Vipers v7	0.76	$0.440 \pm 0.040$	[77]	2016	$(\Omega_m, \sigma_8) = (0.308, 0.8149)$
34	Vipers v7	1.05	$0.280 \pm 0.080$	[77]	2016	

# Robust Independent $f\sigma_8(z)$ Data

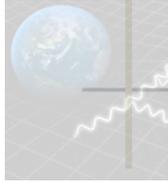
## Dataset 1



Index	Dataset	$z$	$f\sigma_8(z)$	Refs.	Year	Notes
1	6dFGS+SnIa	0.02	$0.428 \pm 0.0465$	[74]	2016	$(\Omega_m, h, \sigma_8) = (0.3, 0.683, 0.8)$
2	SnIa+IRAS	0.02	$0.398 \pm 0.065$	[61],[60]	2011	$(\Omega_m, \Omega_K) = (0.3, 0)$
3	2MASS	0.02	$0.314 \pm 0.048$	[59],[60]	2010	$(\Omega_m, \Omega_K) = (0.266, 0)$
4	SDSS-veloc	0.10	$0.370 \pm 0.130$	[72]	2015	$(\Omega_m, \Omega_K) = (0.3, 0)$
5	SDSS-MGS	0.15	$0.490 \pm 0.145$	[71]	2014	$(\Omega_m, h, \sigma_8) = (0.31, 0.67, 0.83)$
6	2dFGRS	0.17	$0.510 \pm 0.060$	[58]	2009	$(\Omega_m, \Omega_K) = (0.3, 0)$
7	GAMA	0.18	$0.360 \pm 0.090$	[67]	2013	$(\Omega_m, \Omega_K) = (0.27, 0)$
8	GAMA	0.38	$0.440 \pm 0.060$	[67]	2013	
9	SDSS-LRG-200	0.25	$0.3512 \pm 0.0583$	[62]	2011	$(\Omega_m, \Omega_K) = (0.25, 0)$
10	SDSS-LRG-200	0.37	$0.4602 \pm 0.0378$	[62]	2011	
11	BOSS-LOWZ	0.32	$0.384 \pm 0.095$	[68]	2013	$(\Omega_m, \Omega_K) = (0.274, 0)$
12	SDSS-CMASS	0.59	$0.488 \pm 0.060$	[69]	2013	$(\Omega_m, h, \sigma_8) = (0.307115, 0.6777, 0.8288)$
13	WiggleZ	0.44	$0.413 \pm 0.080$	[63]	2012	$(\Omega_m, h) = (0.27, 0.71)$
14	WiggleZ	0.60	$0.390 \pm 0.063$	[63]	2012	$C_{ij} \rightarrow \text{Eq. (2.8)}.$
15	WiggleZ	0.73	$0.437 \pm 0.072$	[63]	2012	
16	Vipers PDR-2	0.60	$0.550 \pm 0.120$	[75]	2016	$(\Omega_m, \Omega_b) = (0.3, 0.045)$
17	Vipers PDR-2	0.86	$0.400 \pm 0.110$	[75]	2016	
18	FastSound	1.40	$0.482 \pm 0.116$	[73]	2015	$(\Omega_m, \Omega_K) = (0.270, 0)$

S. Nesseris, G. Pantazis and L. Perivolaropoulos,  
arXiv:1703.10538 [astro-ph.CO] *Phys.Rev. D96* (2017) no.2, 023542

# Updated $f\sigma_8(z)$ Growth Data (2018): Dataset 2

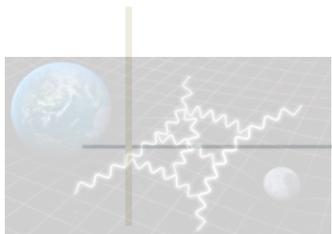


Index	Dataset	$z$	$f\sigma_8(z)$	Refs.	Year	Fiducial Cosmology
1	SDSS-LRG	0.35	$0.440 \pm 0.050$	[73]	30 October 2006	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ [74] – (0.25, 0, 0.756)
2	VVDS	0.77	$0.490 \pm 0.18$	[73]	6 October 2009	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.25, 0, 0.78)
3	2dFGRS	0.17	$0.510 \pm 0.060$	[73]	6 October 2009	$(\Omega_{\text{dm}}, \Omega_K)$ – (0.3, 0, 0.9)
4	2MRS	0.02	$0.314 \pm 0.048$	[75], [76]	13 November 2010	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.266, 0, 0.65)
5	SnIa+IRAS	0.02	$0.398 \pm 0.065$	[77], [76]	20 October 2011	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.3, 0, 0.814)
6	SDSS-LRG-200	0.25	$0.3512 \pm 0.0583$	[78]	9 December 2011	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.276, 0, 0.8)
7	SDSS-LRG-200	0.37	$0.4602 \pm 0.0378$	[78]	9 December 2011	
8	SDSS-LRG-60	0.25	$0.3665 \pm 0.0601$	[78]	9 December 2011	
9	SDSS-LRG-60	0.37	$0.4031 \pm 0.0586$	[78]	9 December 2011	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.276, 0, 0.8)
10	WiggleZ	0.44	$0.413 \pm 0.080$	[43]	12 June 2012	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.27, 0.71, 0.8)
11	WiggleZ	0.60	$0.390 \pm 0.063$	[43]	12 June 2012	$C_{ij}$ – Eq.(3.3)
12	WiggleZ	0.73	$0.437 \pm 0.072$	[43]	12 June 2012	
13	6dFCS	0.067	$0.423 \pm 0.055$	[79]	4 July 2012	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.27, 0, 0.76)
14	SDSS-BOSS	0.30	$0.407 \pm 0.055$	[80]	11 August 2012	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.25, 0, 0.804)
15	SDSS-BOSS	0.40	$0.419 \pm 0.041$	[80]	11 August 2012	
16	SDSS-BOSS	0.50	$0.427 \pm 0.043$	[80]	11 August 2012	
17	SDSS-BOSS	0.60	$0.433 \pm 0.067$	[80]	11 August 2012	
18	Vipers	0.80	$0.470 \pm 0.080$	[81]	9 July 2013	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.25, 0, 0.82)
19	SDSS-DR7-LRG	0.35	$0.429 \pm 0.089$	[82]	8 August 2013	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ [83] – (0.25, 0, 0.809)
20	GAMA	0.18	$0.360 \pm 0.090$	[84]	22 September 2013	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.27, 0, 0.8)
21	GAMA	0.38	$0.440 \pm 0.060$	[84]	22 September 2013	
22	BOSS-LOWZ	0.32	$0.384 \pm 0.095$	[85]	17 December 2013	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.274, 0, 0.8)
23	SDSS DR10 and DR11	0.32	$0.48 \pm 0.10$	[85]	17 December 2013	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ [86] – (0.274, 0, 0.8)
24	SDSS DR10 and DR11	0.57	$0.417 \pm 0.045$	[85]	17 December 2013	
25	SDSS-MGS	0.15	$0.490 \pm 0.145$	[87]	30 January 2015	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.31, 0.67, 0.83)
26	SDSS-veloc	0.10	$0.370 \pm 0.130$	[88]	16 June 2015	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ [89] – (0.3, 0, 0.89)
27	FastSound	1.40	$0.482 \pm 0.116$	[90]	25 November 2015	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ [91] – (0.27, 0, 0.82)
28	SDSS-CMASS	0.59	$0.488 \pm 0.060$	[92]	8 July 2016	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.307115, 0.6777, 0.8288)
29	BOSS DR12	0.38	$0.497 \pm 0.045$	[2]	11 July 2016	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.31, 0, 0.8)
30	BOSS DR12	0.51	$0.458 \pm 0.038$	[2]	11 July 2016	
31	BOSS DR12	0.61	$0.436 \pm 0.034$	[2]	11 July 2016	
32	BOSS DR12	0.38	$0.477 \pm 0.051$	[93]	11 July 2016	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.31, 0.676, 0.8)
33	BOSS DR12	0.51	$0.453 \pm 0.050$	[93]	11 July 2016	
34	BOSS DR12	0.61	$0.410 \pm 0.044$	[93]	11 July 2016	
35	Vipers v7	0.76	$0.440 \pm 0.040$	[53]	26 October 2016	$(\Omega_{\text{dm}}, \sigma_8)$ – (0.308, 0.8149)
36	Vipers v7	1.05	$0.280 \pm 0.080$	[53]	26 October 2016	
37	BOSS LOWZ	0.32	$0.427 \pm 0.056$	[94]	26 October 2016	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.31, 0, 0.8475)
38	BOSS CMASS	0.57	$0.426 \pm 0.029$	[94]	26 October 2016	
39	Vipers	0.727	$0.296 \pm 0.0765$	[95]	21 November 2016	$(\Omega_{\text{dm}}, \Omega_K, \sigma_8)$ – (0.31, 0, 0.7)
40	6dFCS+SnIa	0.02	$0.428 \pm 0.0465$	[96]	29 November 2016	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.3, 0.683, 0.8)
41	Vipers	0.6	$0.48 \pm 0.12$	[97]	16 December 2016	$(\Omega_{\text{dm}}, \Omega_b, n_s, \sigma_8)$ [12] – (0.3, 0.045, 0.96, 0.831)
42	Vipers	0.86	$0.48 \pm 0.10$	[97]	16 December 2016	
43	Vipers PDR-2	0.60	$0.550 \pm 0.120$	[98]	16 December 2016	$(\Omega_{\text{dm}}, \Omega_b, \sigma_8)$ – (0.3, 0.045, 0.823)
44	Vipers PDR-2	0.86	$0.400 \pm 0.110$	[98]	16 December 2016	
45	SDSS DR13	0.1	$0.48 \pm 0.16$	[99]	22 December 2016	$(\Omega_{\text{dm}}, \sigma_8)$ [89] – (0.25, 0.89)
46	2MTF	0.001	$0.505 \pm 0.085$	[100]	16 June 2017	$(\Omega_{\text{dm}}, \sigma_8)$ – (0.3121, 0.815)
47	Vipers PDR-2	0.85	$0.45 \pm 0.11$	[101]	31 July 2017	$(\Omega_b, \Omega_{\text{dm}}, h)$ – (0.045, 0.30, 0.8)
48	BOSS DR12	0.31	$0.469 \pm 0.098$	[46]	15 September 2017	$(\Omega_{\text{dm}}, h, \sigma_8)$ – (0.307, 0.6777, 0.8288)
49	BOSS DR12	0.36	$0.474 \pm 0.097$	[46]	15 September 2017	
50	BOSS DR12	0.40	$0.473 \pm 0.086$	[46]	15 September 2017	
51	BOSS DR12	0.44	$0.481 \pm 0.076$	[46]	15 September 2017	
52	BOSS DR12	0.48	$0.482 \pm 0.067$	[46]	15 September 2017	
53	BOSS DR12	0.52	$0.488 \pm 0.065$	[46]	15 September 2017	
54	BOSS DR12	0.56	$0.482 \pm 0.067$	[46]	15 September 2017	
55	BOSS DR12	0.59	$0.481 \pm 0.066$	[46]	15 September 2017	
56	BOSS DR12	0.64	$0.486 \pm 0.070$	[46]	15 September 2017	
57	SDSS DR7	0.1	$0.376 \pm 0.038$	[102]	12 December 2017	$(\Omega_{\text{dm}}, \Omega_b, \sigma_8)$ – (0.282, 0.046, 0.817)
58	SDSS-IV	1.52	$0.420 \pm 0.076$	[103]	8 January 2018	$(\Omega_{\text{dm}}, \Omega_b h^2, \sigma_8)$ – (0.26479, 0.02258, 0.8)
59	SDSS-IV	1.52	$0.396 \pm 0.079$	[104]	8 January 2018	$(\Omega_{\text{dm}}, \Omega_b h^2, \sigma_8)$ – (0.31, 0.022, 0.8225)
60	SDSS-IV	0.978	$0.379 \pm 0.176$	[105]	9 January 2018	$(\Omega_{\text{dm}}, \sigma_8)$ – (0.31, 0.8)
61	SDSS-IV	1.23	$0.385 \pm 0.099$	[105]	9 January 2018	
62	SDSS-IV	1.526	$0.342 \pm 0.070$	[105]	9 January 2018	
63	SDSS-IV	1.944	$0.364 \pm 0.106$	[105]	9 January 2018	

The evolution of the  $f\sigma_8$  tension with Planck15/ $\Lambda$ CDM and implications for modified gravity theories  
 Laurentios Kazantzidis, Leandros Perivolaropoulos. Mar 4, 2018. 16 pp.  
 e-Print: [arXiv:1803.01337](https://arxiv.org/abs/1803.01337)

$(\Omega_{\text{dm}}, \Omega_b, \sigma_8)$  – (0.282, 0.046, 0.817)  
 $(\Omega_{\text{dm}}, \Omega_b h^2, \sigma_8)$  – (0.26479, 0.02258, 0.8)  
 $(\Omega_{\text{dm}}, \Omega_b h^2, \sigma_8)$  – (0.31, 0.022, 0.8225)  
 $(\Omega_{\text{dm}}, \sigma_8)$  – (0.31, 0.8)

# Fiducial Model Correction



Alcock Paczynski effect:

True Galaxy separation:

$$dl_{\perp} = (1+z)D_A(z) d\theta$$

$$dl_{\parallel} = \frac{c dz}{H(z)}$$

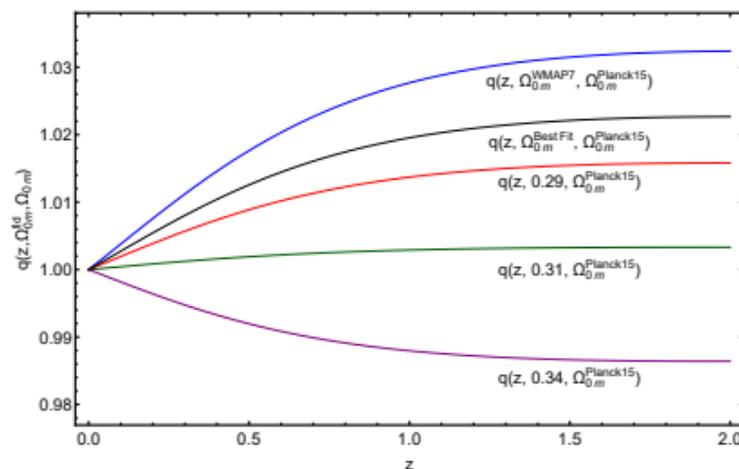
Fiducial Model Galaxy Separation:

$$dl'_{\perp} = (1+z)D'_A d\theta = \left(\frac{D'_A}{D_A}\right) dl_{\perp} = \frac{dl_{\perp}}{f_{\perp}},$$

$$dl'_{\parallel} = \frac{c dz}{H'} = \left(\frac{H}{H'}\right) dl_{\parallel} = \frac{dl_{\parallel}}{f_{\parallel}}$$

AP Induced Anisotropy:

$$F = \frac{f_{\parallel}}{f_{\perp}} = \left(\frac{H'}{H}\right) \left(\frac{D'_A}{D_A}\right)$$



Edward Macaulay, Ingunn Kathrine Wehus, and Hans Kristian Eriksen, "Lower Growth Rate from Recent Redshift Space Distortion Measurements than Expected from Planck," Phys. Rev. Lett. **111**, 161301 (2013), arXiv:1303.6583 [astro-ph.CO].

Correction Factor:

$$f\sigma_8(z) \simeq \frac{H(z)D_A(z)}{H'(z)D'_A(z)} f\sigma'_8(z) \equiv q(z, \Omega'_{0m}, \Omega_{0m}) f\sigma'_8(z)$$

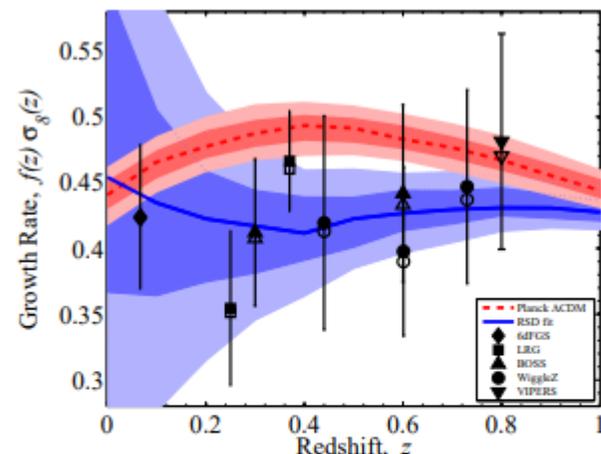
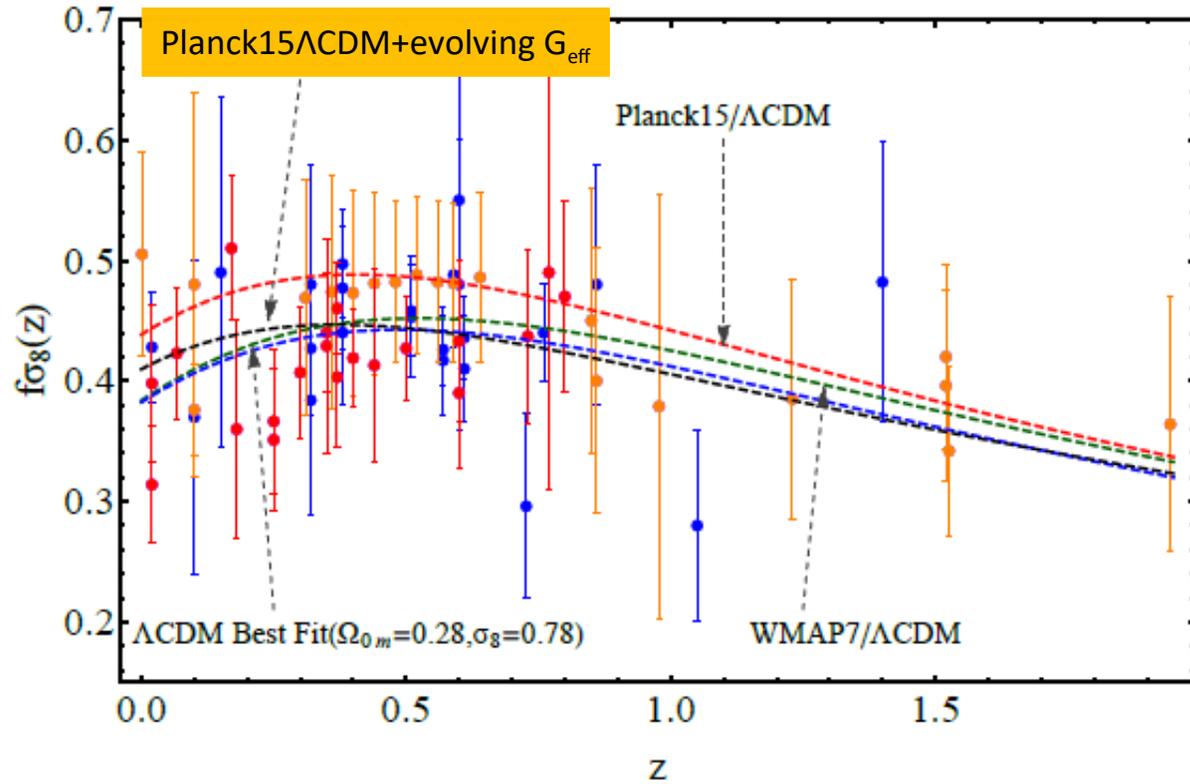


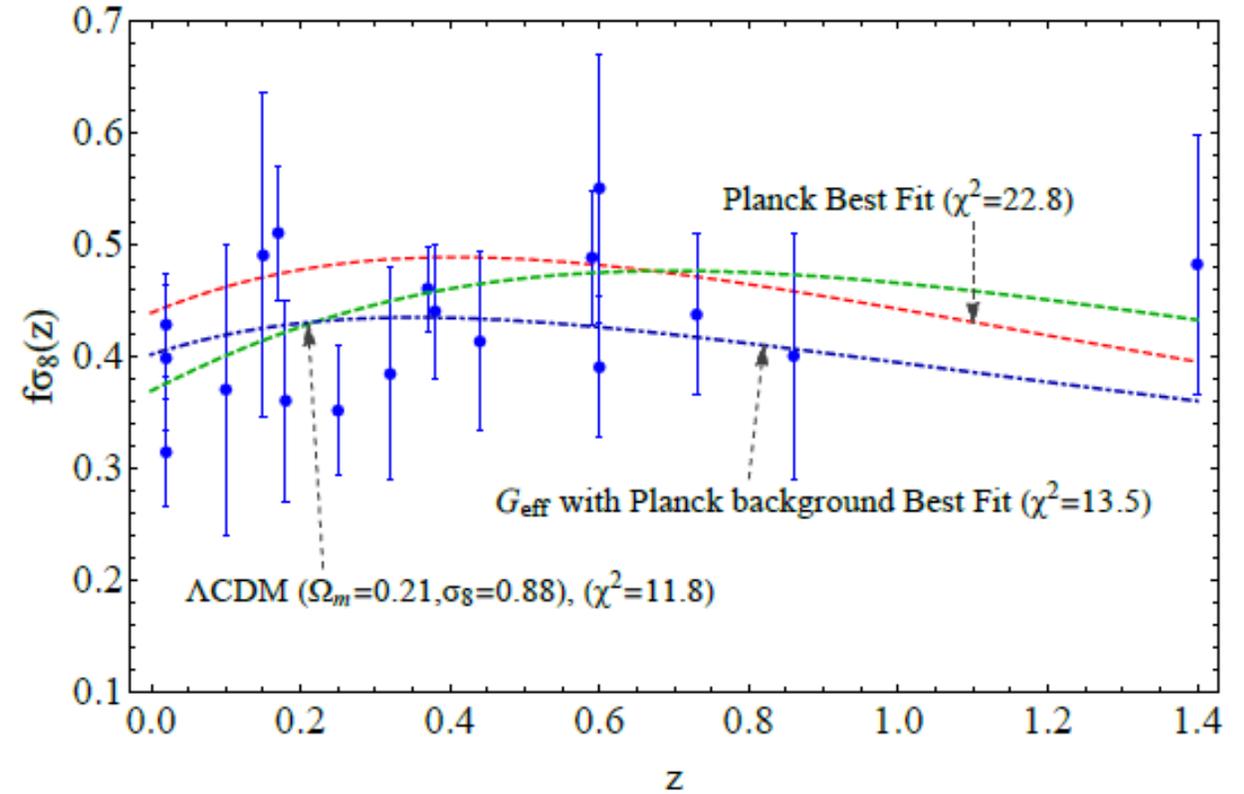
FIG. 1: Comparing models to recent measurements of  $f(z)\sigma_8(z)$ . We are plotting results for the LRG<sub>200</sub> data set. The open markers are the original published values from the RSD measurements, and the filled markers are after accounting for the Alcock-Paczynski effect in going from WMAP to Planck cosmology. The measurement error bars are at the

# Datasets and Model Predictions

Dataset 2 (63 datapoints).



Dataset 1 (18 selected datapoints)



The evolution of the  $f\sigma_8$  tension with Planck15/ΛCDM and implications for modified gravity theories

Laurentios Kazantzidis, Leandros Perivolaropoulos. Mar 4, 2018. 16 pp.

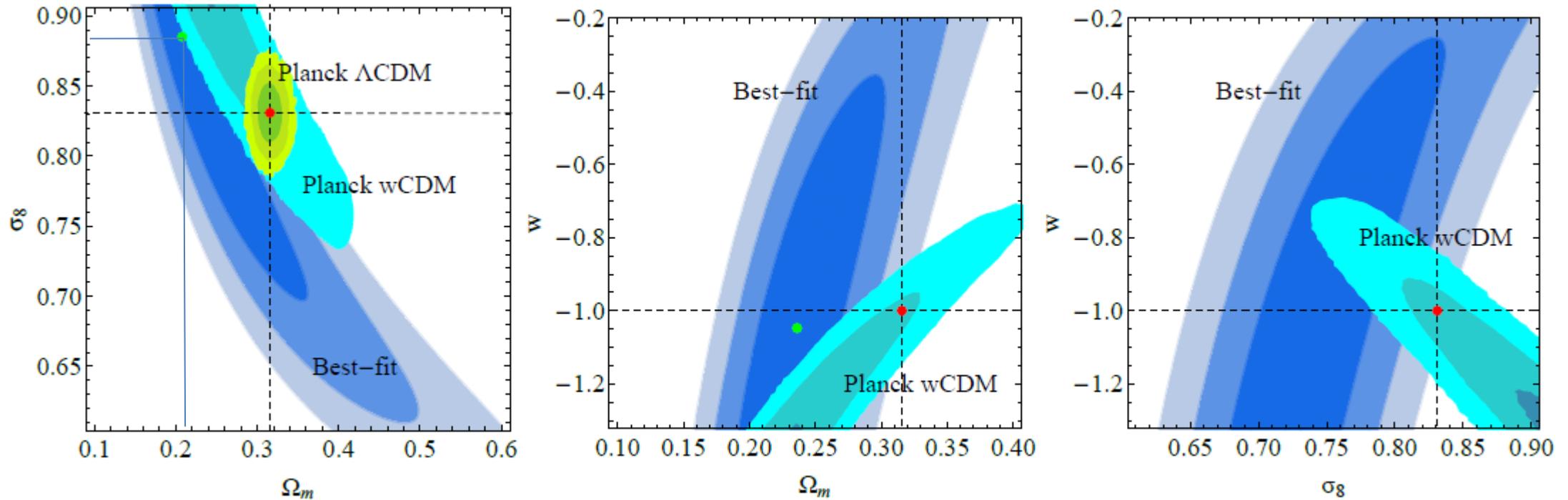
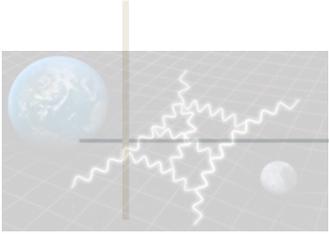
e-Print: [arXiv:1803.01337](https://arxiv.org/abs/1803.01337)

Tension and constraints on modified gravity parametrizations of  $G_{\text{eff}}(z)$  from growth rate and Planck data

Savvas Nesseris (Madrid, IFT), George Pantazis (Ioannina U.), Leandros Perivolaropoulos, Mar 30, 2017. 19 pp.

Published in Phys.Rev. D96 (2017) no.2, 023542

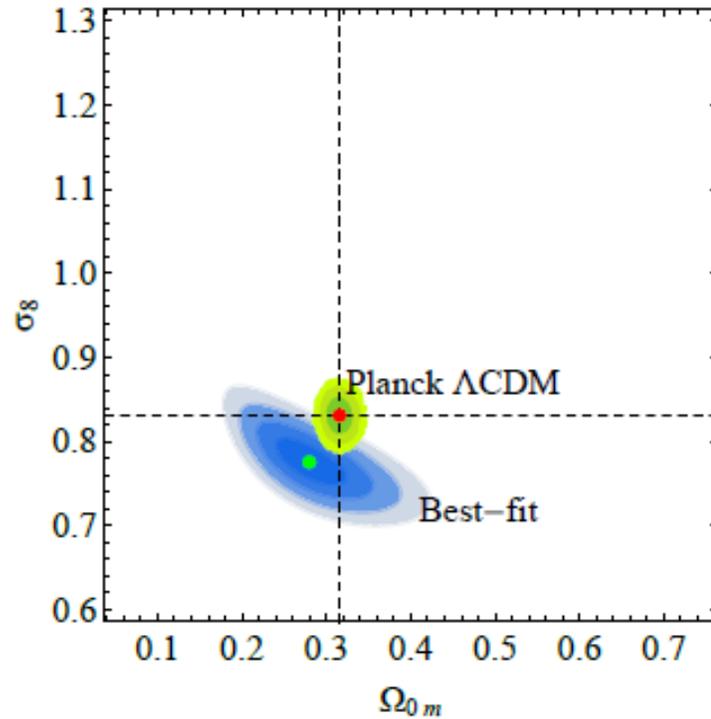
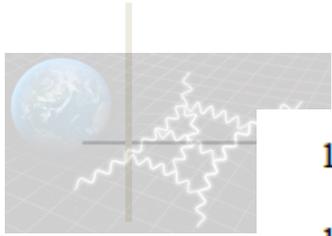
# Likelihood Contours: Dataset 1



**Tension between growth data contours and corresponding Planck15/ $\Lambda$ CDM best fit**

# Evolution of the Tension with Planck/ $\Lambda$ CDM

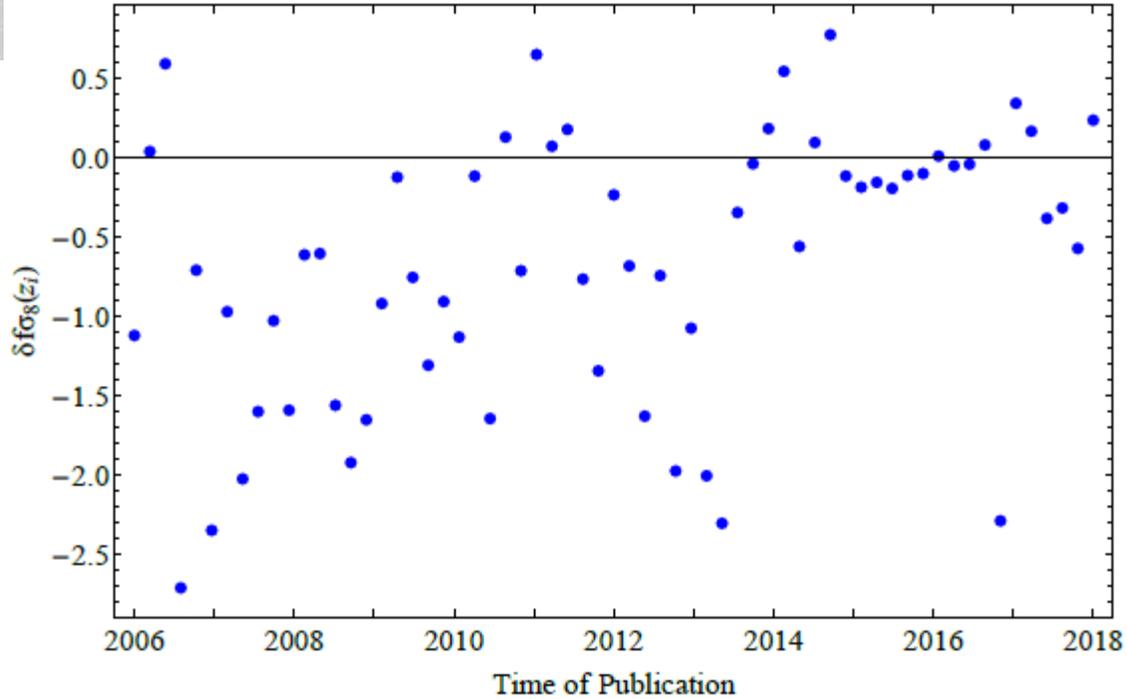
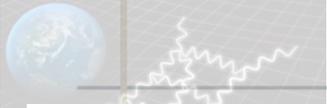
## Dataset 2



The tension is lower for more recent data.

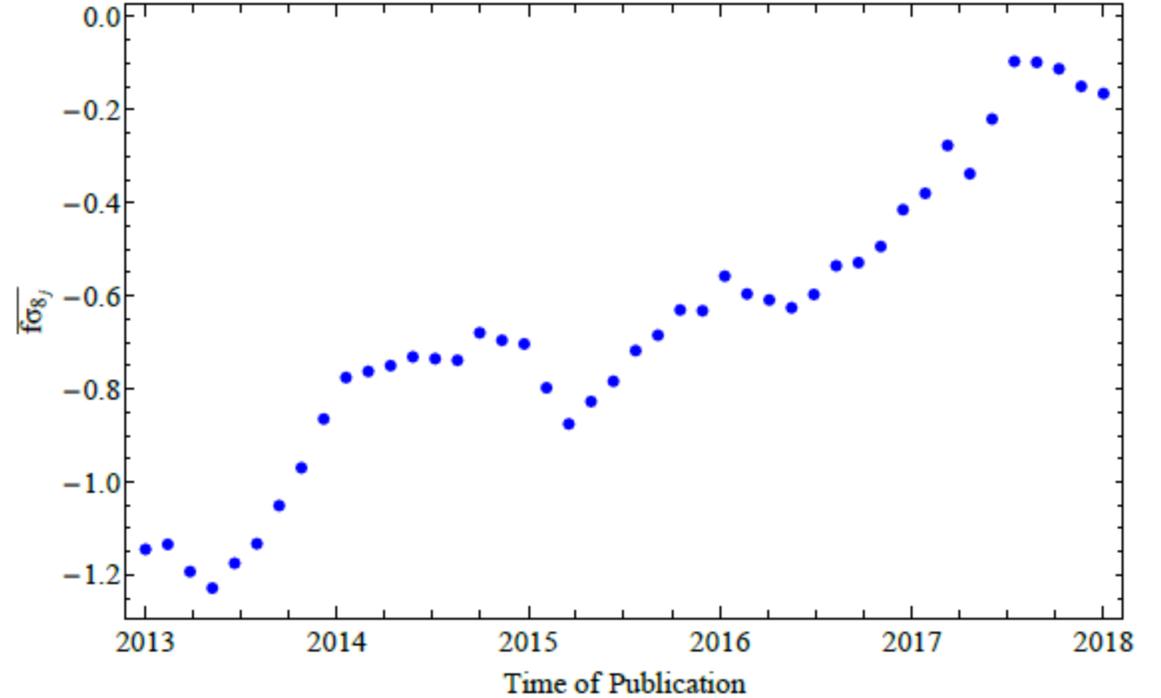
Full Dataset	Early Data	Late Data
$4.97\sigma$	$3.89\sigma$	$0.94\sigma$

# Evolution of the Tension with Planck/ $\Lambda$ CDM



Growth residuals.

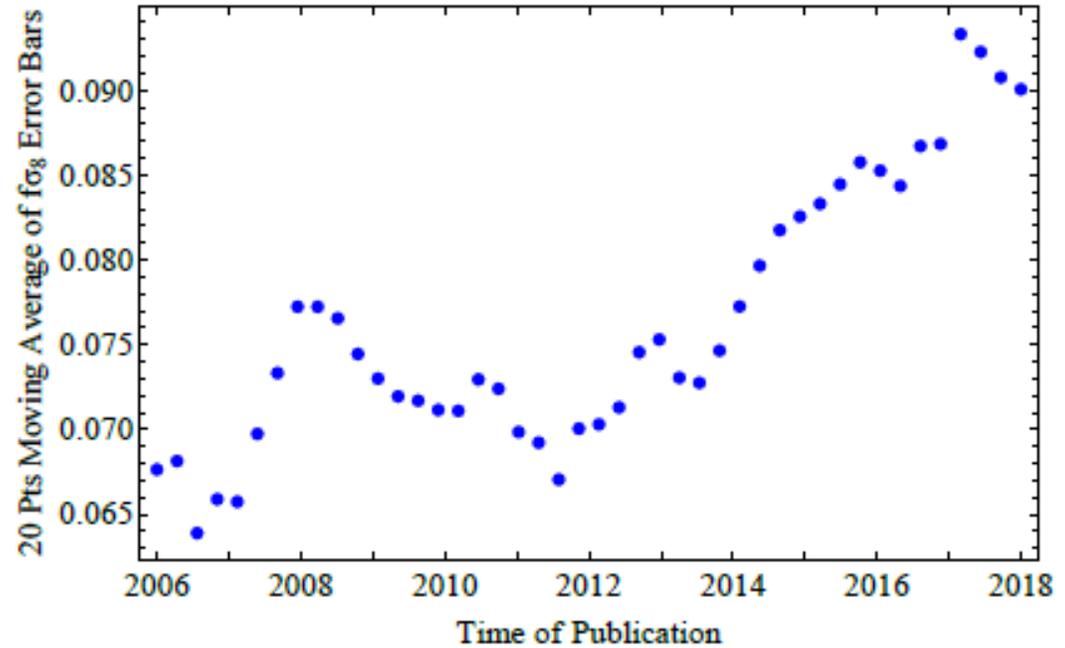
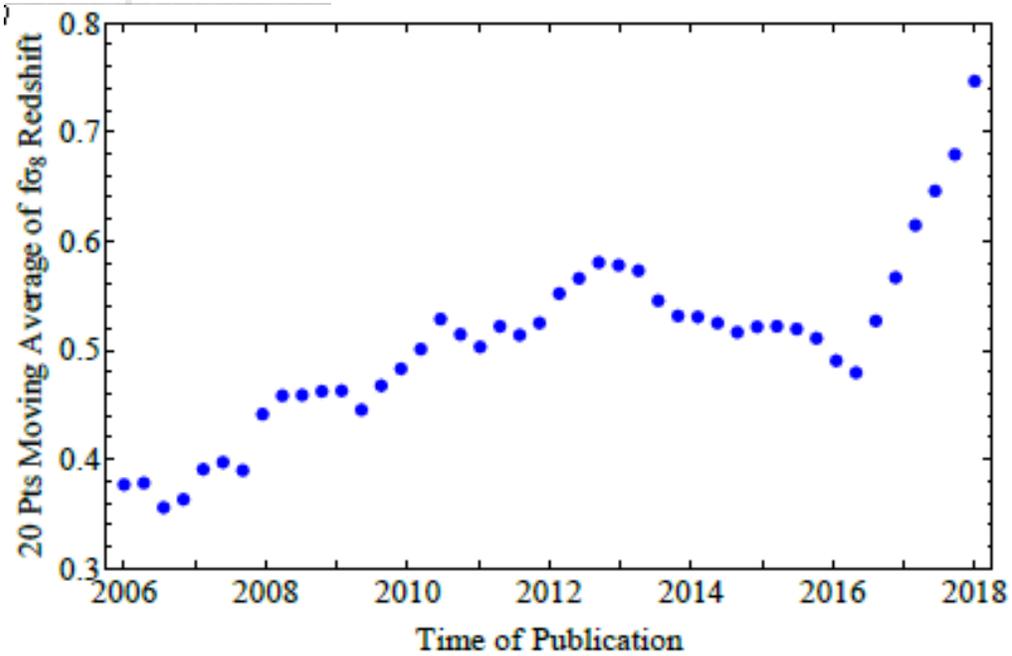
$$\delta f\sigma_8(z_i) \equiv \frac{f\sigma_8(z_i)^{data} - f\sigma_8(z_i)^{Planck15}}{\sigma_i}$$



Moving average of growth residuals  
( $N=20$ )

$$\overline{f\sigma_8_j} \equiv \sum_{i=j-N}^j \frac{\delta f\sigma_8(z_i)}{N}$$

# Moving Average of Redshifts and Errorbars



Redshifts and errorbars increase with time of publication

# Evolving $G_{\text{eff}}(z)$



Conditions to be satisfied by viable  $G_{\text{eff}}(z)$  parametrization

1.  $\lim_{z \rightarrow 0} G'_{\text{eff}}(z) \simeq 0$  (solar system tests)
2.  $\lim_{z \rightarrow \infty} G_{\text{eff}}(z) \simeq G_N$  (nucleosynthesis constraints)

$$\left| \frac{1}{G_N} \frac{dG_{\text{eff}}(z)}{dz} \Big|_{z=0} \right| < 10^{-3} h^{-1} \quad |G_{\text{eff}}/G_N - 1| \leq 0.2 \quad \text{at high } z$$

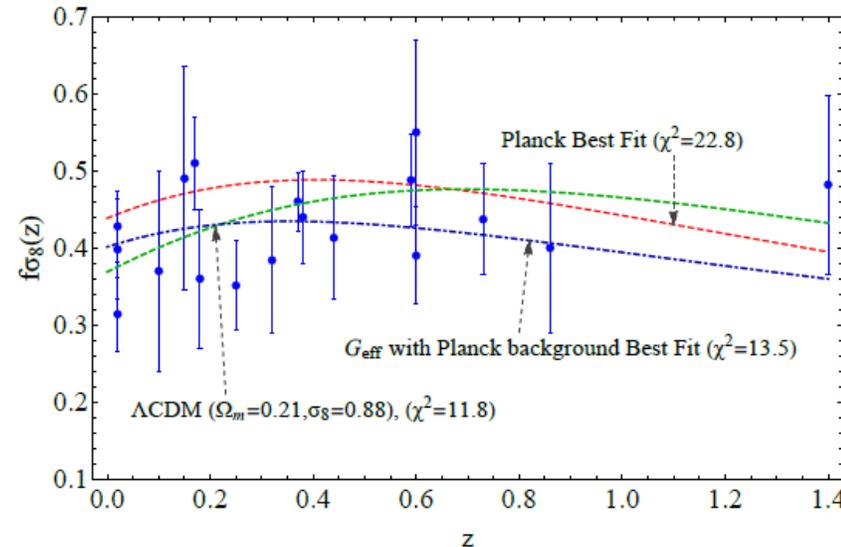
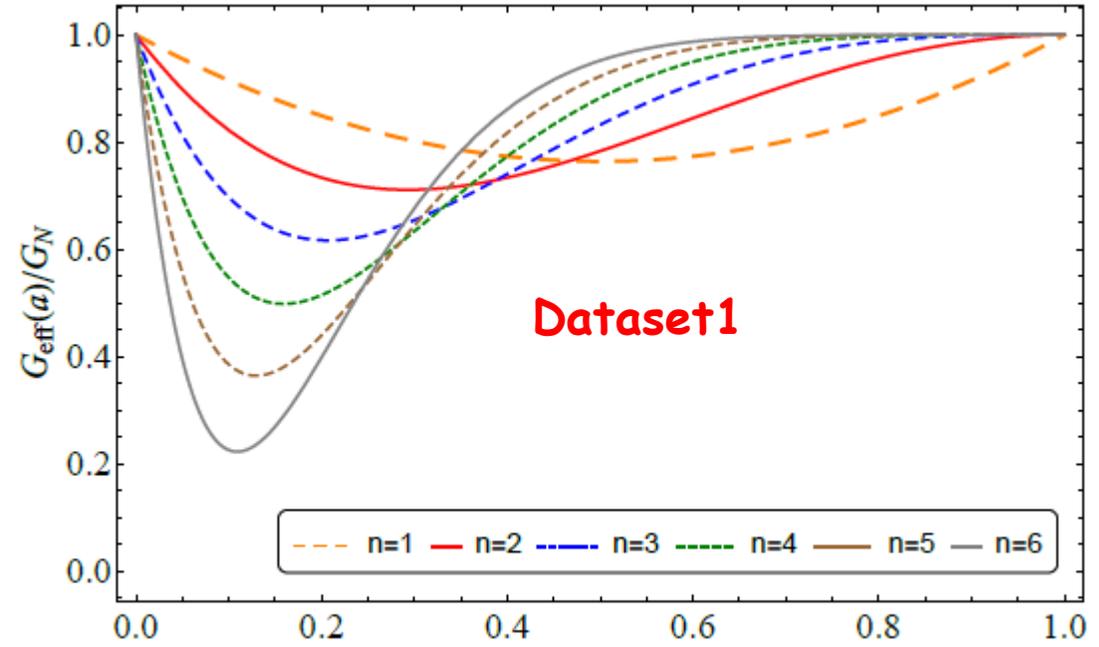
S. Nesseris and Leandros Perivolaropoulos, "The Limits of Extended Quintessence," Phys. Rev. **D75**, 023517 (2007), arXiv:astro-ph/0611238 [astro-ph].

Craig J. Copi, Ada Krauss, "A New nucleation of G," Phys. arXiv:astro-ph/03113

$$H^2 \delta''_m + \left( \frac{(H^2)'}{2} - \frac{H^2}{1+z} \right) \delta'_m \approx \frac{3}{2} (1+z) H_0^2 \frac{G_{\text{eff}}(z)}{G_{N,0}} \Omega_m$$

**Viable parametrization:**

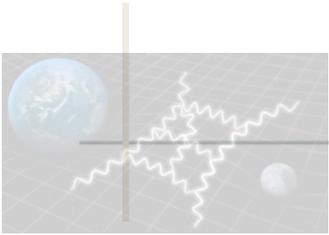
$$\frac{G_{\text{eff}}(a, n)}{G_N} = 1 + g_a (1-a)^n - g_a (1-a)^{2n}$$



$n$	$g_a$
0.343	$-1.200 \pm 1.025$
1	$-0.944 \pm 0.253$
2	$-1.156 \pm 0.341$
3	$-1.534 \pm 0.453$
4	$-2.006 \pm 0.538$
5	$-2.542 \pm 0.689$
6	$-3.110 \pm 0.771$



# Basic Questions

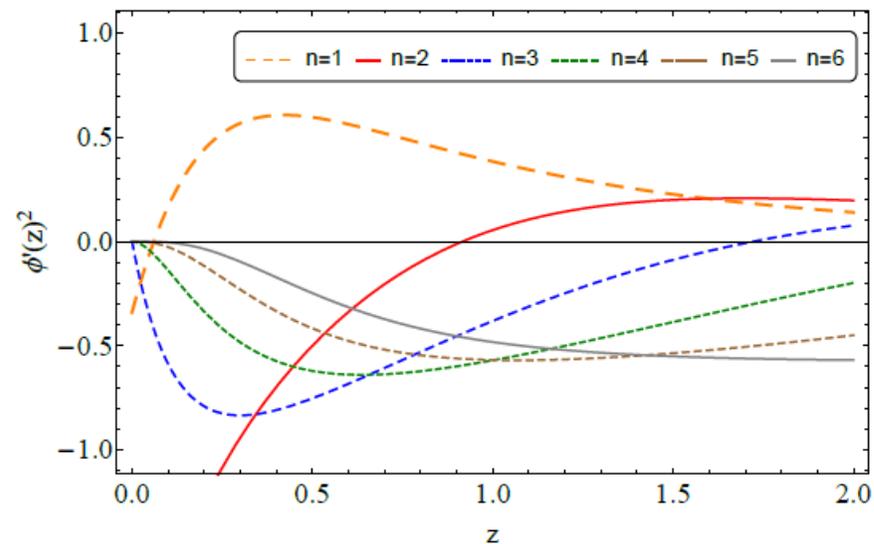
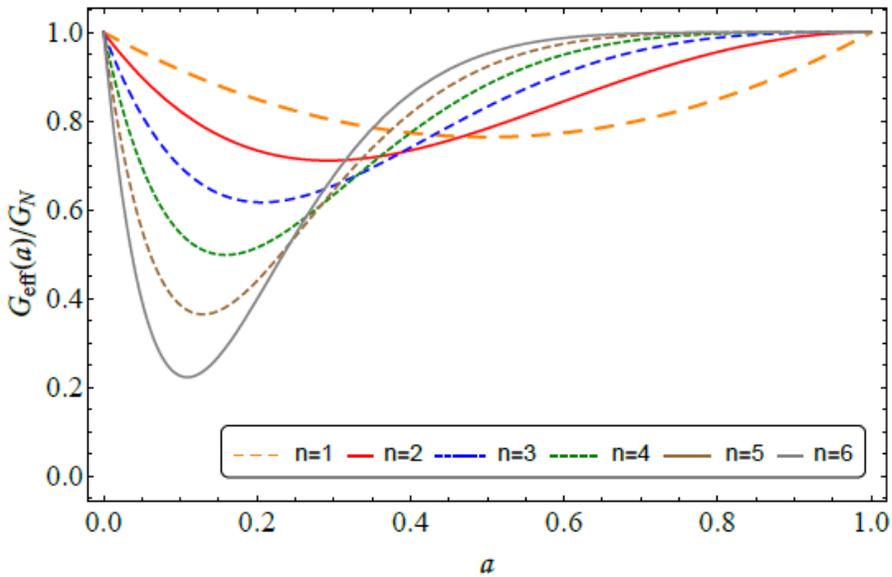


1. What Modified Gravity models are consistent with the best fit parametrizations  $G_{\text{eff}}(z)$  indicating  $g_a < 0$ ?

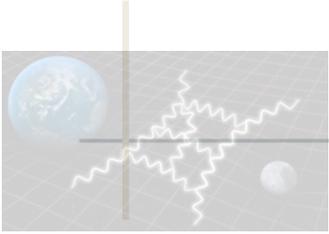
2. Can viable Scalar Tensor models get reconstructed from Planck/ $\Lambda$ CDM and the best fit  $G_{\text{eff}}(z)$ ?

No!

Reconstruction leads to negative kinetic terms in the scalar tensor action



# Reconstruction of Scalar-Tensor Quintessence



**Scalar-Tensor Action:** 
$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(\phi) R - \frac{1}{2} Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_m,$$

**FLRW Metric:** 
$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)]$$

**Dynamical Equations:**

$$3FH^2 = \rho + \frac{1}{2}\dot{\phi}^2 - 3H\dot{F} + U$$

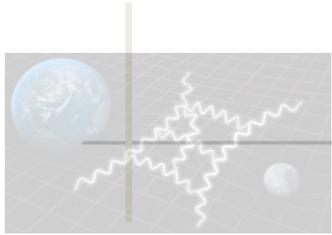
$$-2F\dot{H} = (\rho + p) + \dot{\phi}^2 + \ddot{F} - H\dot{F}$$

**Dynamical Equation wrt Redshift (eliminate potential U):**

$$F''(z) + F'(z) \left( \frac{q'(z)}{2q(z)} + \frac{2}{z+1} \right) - F(z) \frac{q'(z)}{(z+1)q(z)} + \frac{3\Omega_m(z+1)}{q(z)} = -\dot{\phi}'(z)^2 \quad q(z) \equiv E^2(z) = \frac{H^2(z)}{H_0^2}$$

$$G_{\text{eff}}(z)/G_N = \frac{1}{F(\phi)} \frac{F(\phi) + 2F_{,\phi}^2}{F(\phi) + \frac{3}{2}F_{,\phi}^2}$$

# Generic Low-z Behaviour of Scalar-Tensor Quintessence



Dynamical equation:

$$F''(z) + F'(z) \left( \frac{q'(z)}{2q(z)} + \frac{2}{z+1} \right) - F(z) \frac{q'(z)}{(z+1)q(z)} + \frac{3\Omega_m(z+1)}{q(z)} = -\phi'(z)^2$$

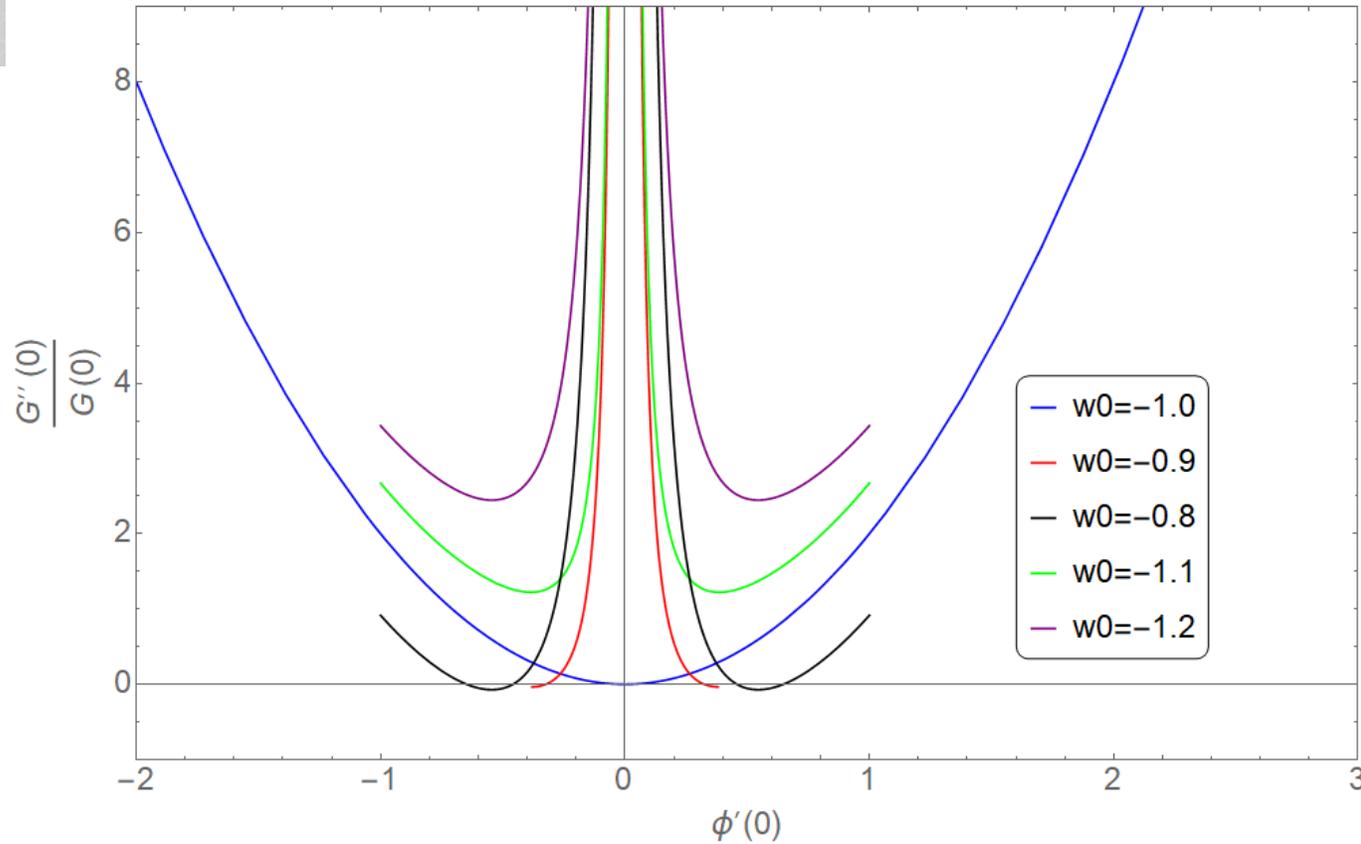
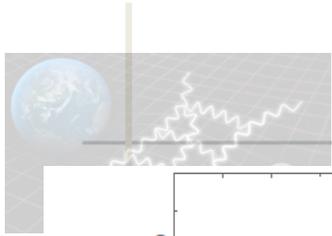
$$G_{\text{eff}}(z)/G_N = \frac{1}{F(\phi)} \frac{F(\phi) + 2F_{,\phi}^2}{F(\phi) + \frac{3}{2}F_{,\phi}^2}$$

$$q(z) = \Omega_{0m} (1+z)^3 + (1-\Omega_{0m}) (1+z)^{3(1+w_0)}$$

At  $z=0$ :  $G'(0)=0$  implies  $F'(0)=0$ ,  $G(0)=1$  implies  $F(0)=1$

$$G_{\text{eff}}(z)/G_N = \frac{1}{F(\phi)} \frac{F(\phi) + 2F_{,\phi}^2}{F(\phi) + \frac{3}{2}F_{,\phi}^2} \longrightarrow G''(z=0) = 2\phi'^2(z=0) - 9(1-\Omega_{0m})(1+w_0) + \frac{9(1-\Omega_{0m})^2(1+w_0)^2}{\phi'^2(z=0)}$$

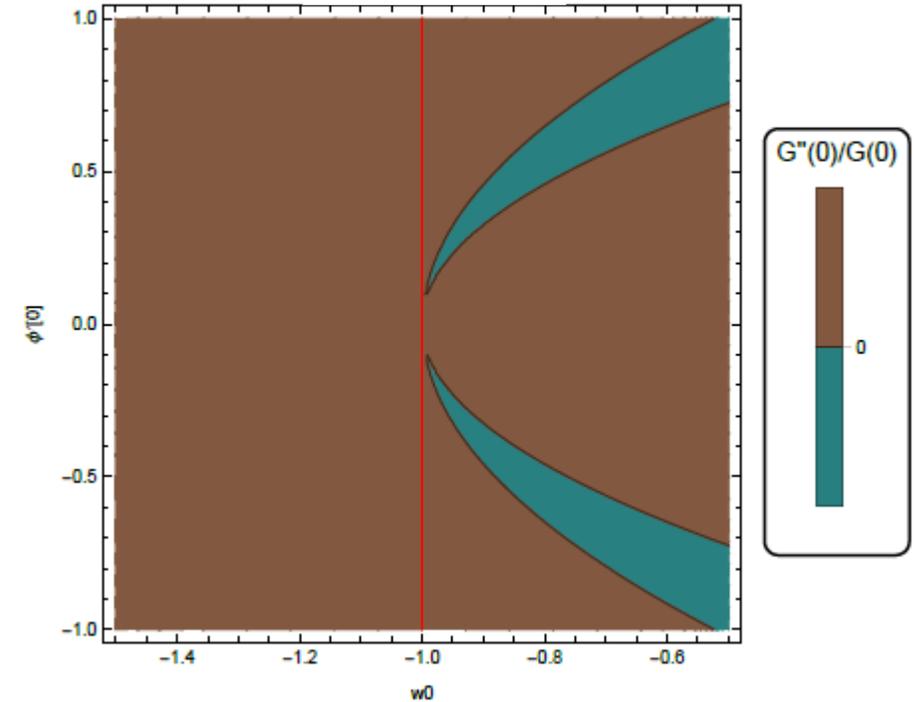
# Increasing $G_{\text{eff}}(z)$ for $w_0 \leq -1$



$$w_0 \approx -1$$

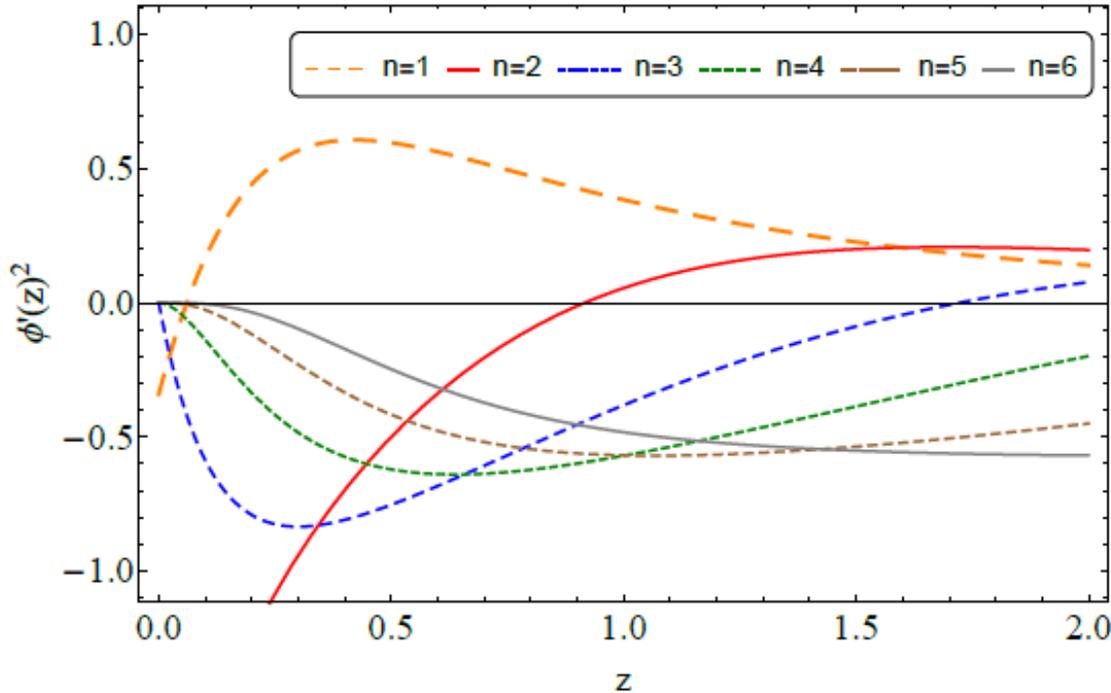
$$G(z) \approx G(0) + \frac{11}{2} G''(0) z^2 = G(0) + \phi^2(0) z^2$$

$$G''(z) = 2\phi^2(z) - 9(1 - \Omega_{\text{eff}}) \frac{9(1 - \Omega_{\text{eff}})^2 (1 + w_0)^2}{\phi^2(z)}$$



# Beyond low z: The Reconstructed Kinetic Term from best fit

$\phi'(z)$  is negative!



$$F''(z) + F'(z) \left( \frac{q'(z)}{2q(z)} + \frac{2}{z+1} \right) - F(z) \frac{q'(z)}{(z+1)q(z)} + \frac{3\Omega_m(z+1)}{q(z)} = -\phi'(z)^2$$

$$G_{\text{eff}}(z)/G_{\text{N}} = \frac{1}{F(\phi)} \frac{F(\phi) + 2F_{,\phi}^2}{F(\phi) + \frac{3}{2}F_{,\phi}^2}$$

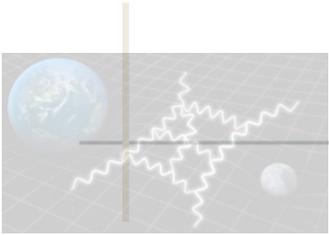
$$\frac{G_{\text{eff}}(a, n)}{G_{\text{N}}} = 1 + g_a(1-a)^n - g_a(1-a)^{2n} = 1 + g_a \left( \frac{z}{1+z} \right)^n - g_a \left( \frac{z}{1+z} \right)^{2n}$$

S. Nesseris, G. Pantazis and L. Perivolaropoulos,  
arXiv:1703.10538 [astro-ph.CO] Phys.Rev. D96 (2017) no.2. 023542

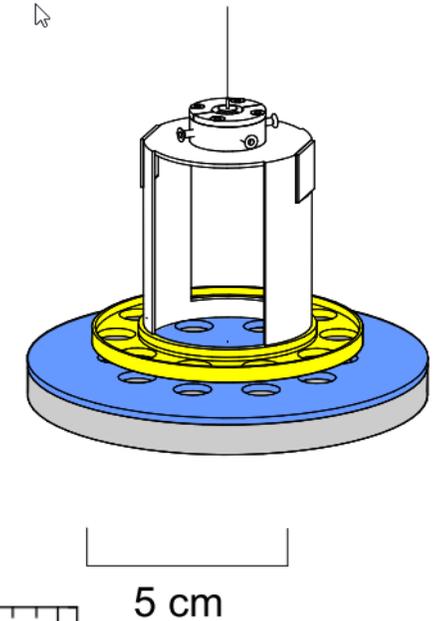
**General Result:** In a  $\Lambda$ CDM background, any  $G_{\text{eff}}(z)$  initially decreasing with  $z$  leads to a reconstructed scalar-tensor negative kinetic term for some range of low  $z$ .

If the tension is physical and the background is Planck/ $\Lambda$  CDM, then a more general modified gravity theory than scalar-tensor is required

# Testing homogeneity of Newton's constant with small scale gravity experiments



The Washington Experiment apparatus:

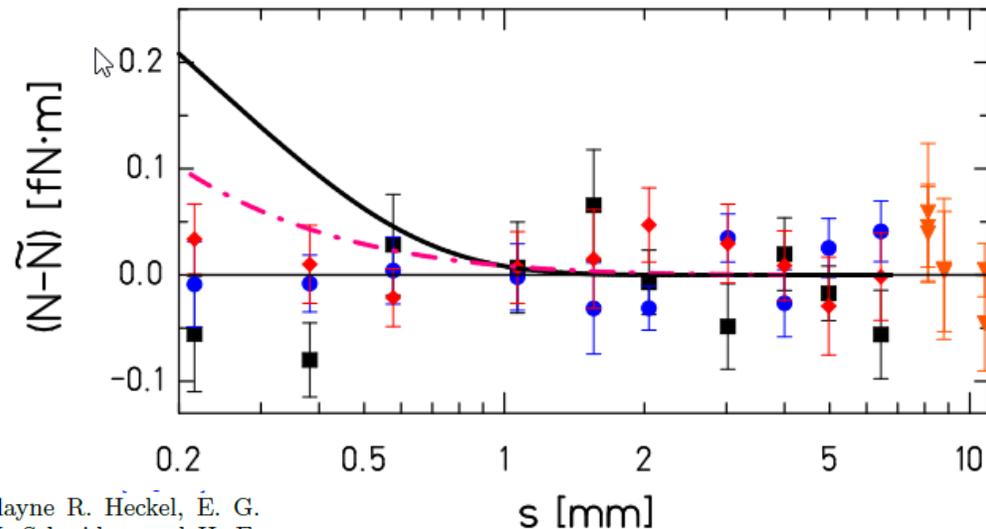


The torque from the holes of the rotating lower ring (attractor) on the holes of the upper ring (torsion pendulum) is measured by monitoring the pendulum twist for various ring separations and subtracted from the expected Newtonian torque.

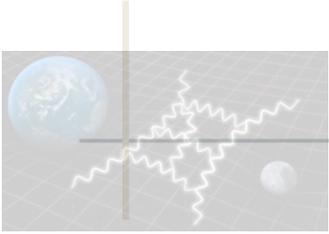
Torque residuals are measured and fit to Yukawa and power law parametrizations

D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, Blayne R. Heckel, C. D. Hoyle, and H. E. Swanson, "Tests of the gravitational inverse-square law below the dark-energy length scale," *Phys. Rev. Lett.* **98**, 021101 (2007), [arXiv:hep-ph/0611184](https://arxiv.org/abs/hep-ph/0611184) [hep-ph].

C. D. Hoyle, D. J. Kapner, Blayne R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt, and H. E. Swanson, "Sub-millimeter tests of the gravitational inverse-square law," *Phys. Rev. D* **70**, 042004 (2004), [arXiv:hep-ph/0405262](https://arxiv.org/abs/hep-ph/0405262) [hep-ph].



# Parametrizing Newton's constant on sub-mm scales



**Dark Energy Scale:**  $\lambda_{de} \equiv \sqrt[4]{\hbar c / \rho_{de}} \approx 0.085 \text{mm}$

**Yukawa parametrization:**  $V_{eff} = -G \frac{M}{r} (1 + \alpha e^{-mr}) \longrightarrow f(R) = R + \frac{1}{6m^2} R^2 + \dots \quad m^2 > 0$

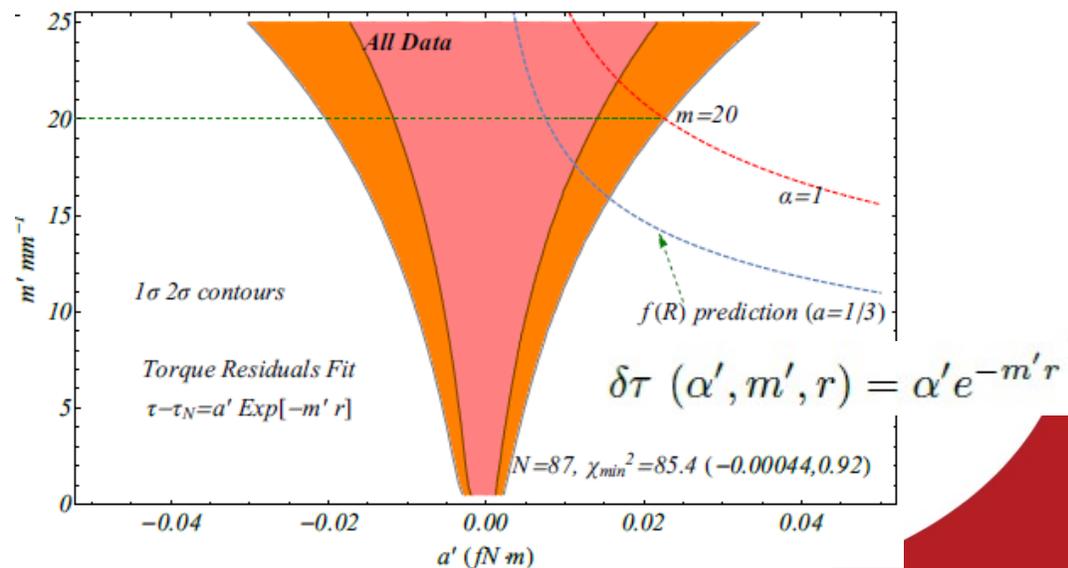
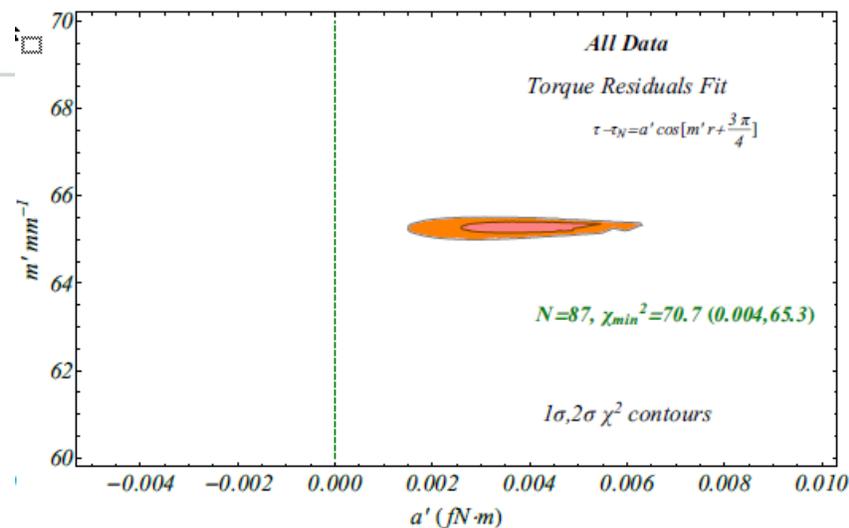
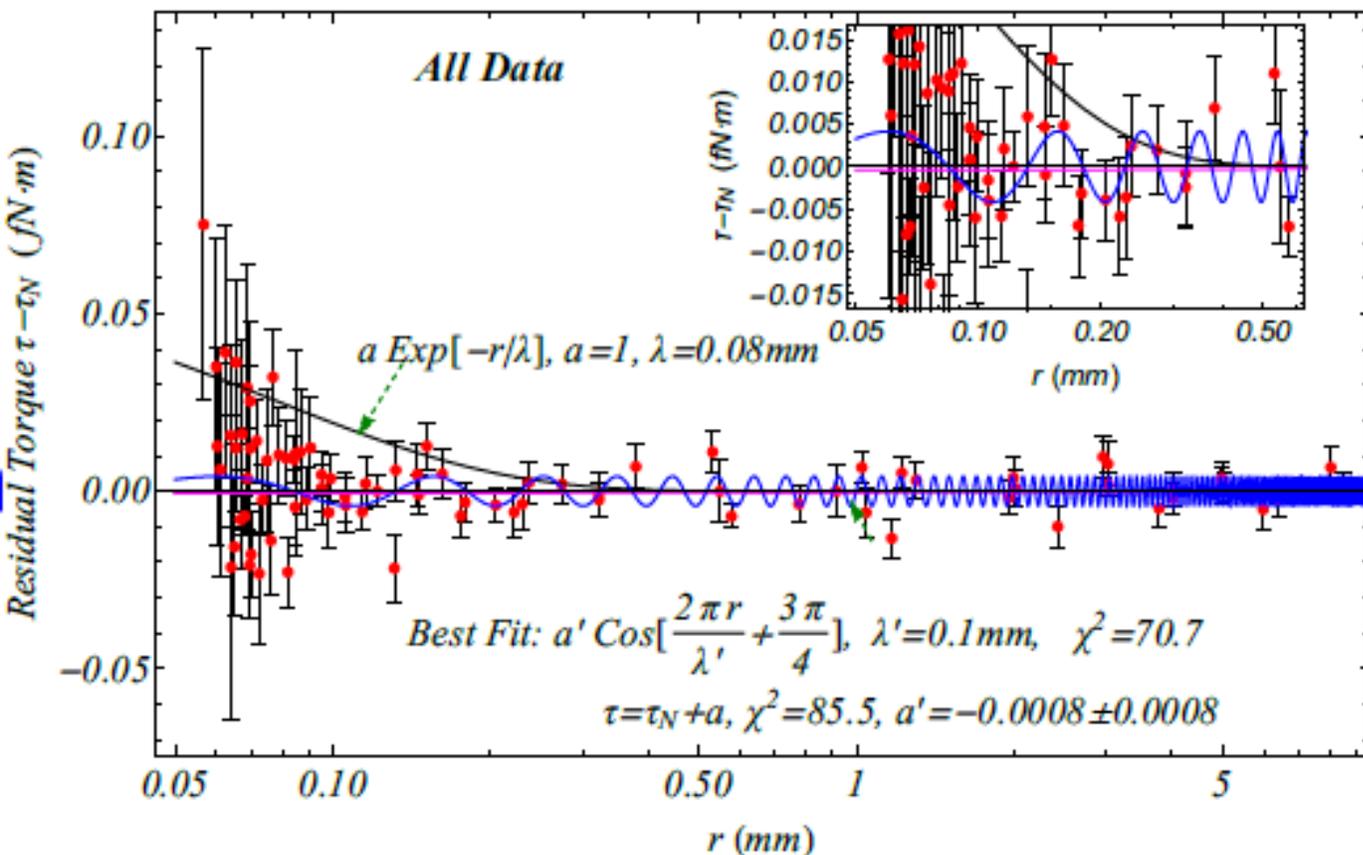
**Power law parametrization:**  $V_{eff} = -G \frac{M}{r} (1 + \beta^k (\frac{1}{mr})^{k-1})$  (brane world models)

**Oscillating parametrization:**  $V_{eff} = -G \frac{M}{r} (1 + \alpha_O \cos(\frac{2\pi}{\lambda} r + \theta)) \longrightarrow f(R) = R + \frac{1}{6m^2} R^2 + \dots \quad m^2 < 0$   
(f(R) theories (instabilities), Infinite Derivative Gravity)

# Fits to the Torque Residual Data

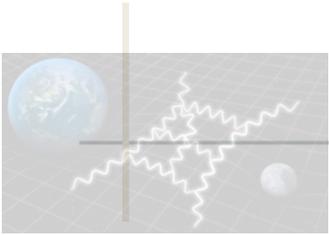
$$\delta\tau(\alpha', m', r) = \alpha' \cos(m'r + \frac{3\pi}{4})$$

$$\chi^2(\alpha', m') = \sum_{j=1}^N \frac{(\delta\tau(j) - \delta\tau_i(\alpha', m', r_j))^2}{\sigma_j^2}$$

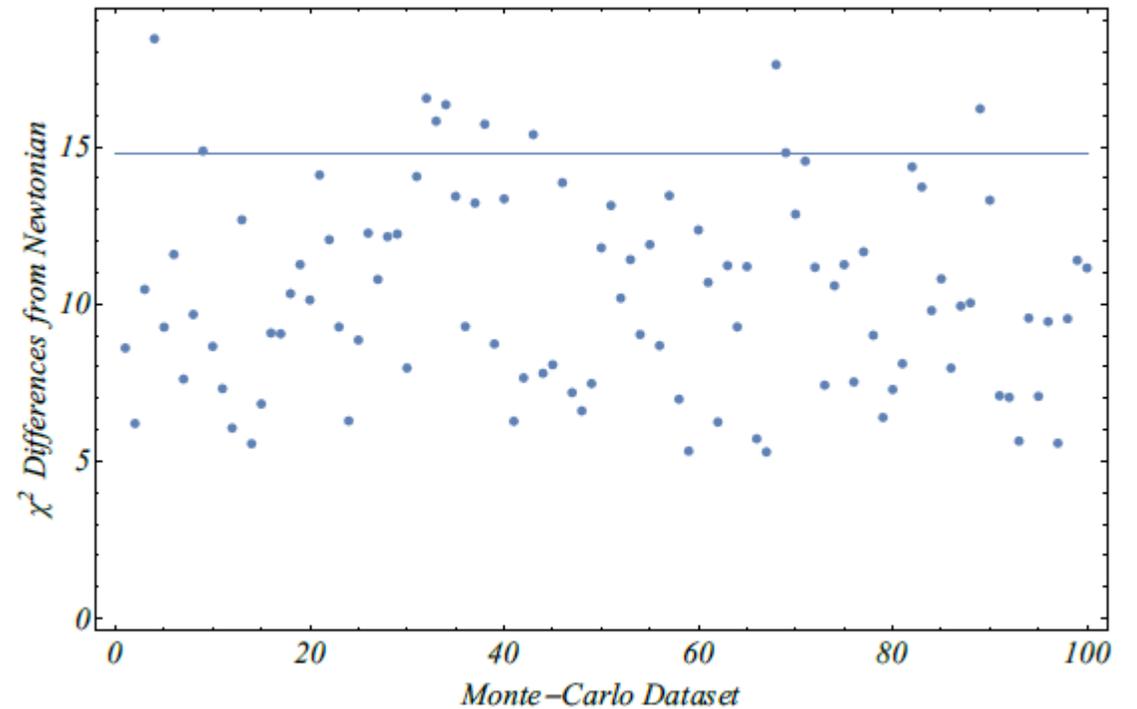
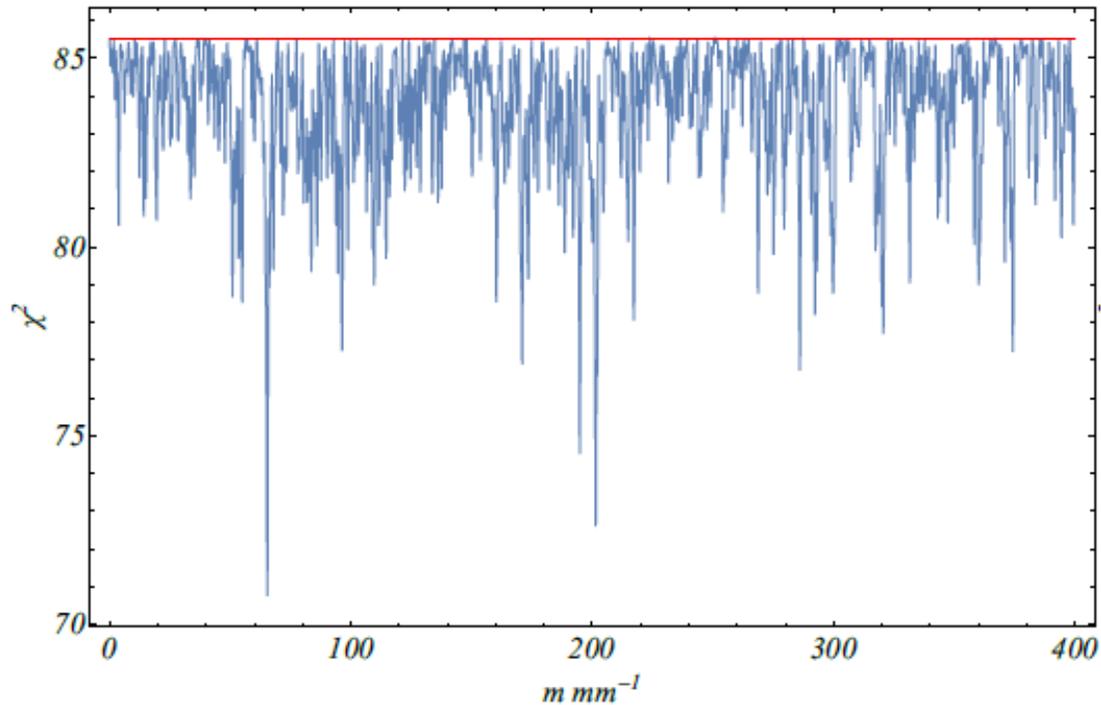


**Q: Are we seeing statistics, systematics or physics?**

# Statistical Significance

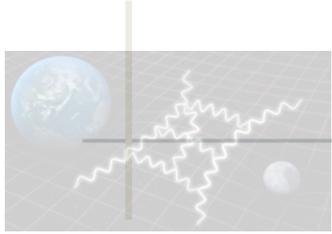


About 10% of Newtonian Monte Carlo Datasets have deeper oscillating  $\chi^2$  minima than the actual Washington experiment dataset



**There is about 10% probability that the signal is a statistical fluctuation.  
It could also be a systematic effect.**

# Theoretical Models I: $f(R)$ theories



## Weak field gravity:

$$f(R) = R + \frac{1}{6m^2} R^2$$

$$T_{\mu\nu} = \text{diag}(M\delta(\vec{r}), 0, 0, 0)$$

$$h_{00} = \frac{2GM}{r} \left( 1 + \frac{1}{3} e^{-mr} \right) \quad m^2 > 0$$

$m^2 > 0$ : Stability

$m^2 < 0$ : Instabilities

Valerio Faradani, "Matter instability in modified gravity," *Phys. Rev.* **D74**, 104017 (2006), [arXiv:astro-ph/0610734 \[astro-ph\]](#).

A. D. Dolgov and Masahiro Kawasaki, "Can modified gravity explain accelerated cosmic expansion?" *Phys. Lett.* **B573**, 1–4 (2003), [arXiv:astro-ph/0307285 \[astro-ph\]](#).

$$V_{eff} = -\frac{h_{00}}{2} = -\frac{GM}{r} \left( 1 + \frac{1}{3} \cos(|m|r + \theta) \right) \quad m^2 < 0$$

Leandros Perivolaropoulos, "Sub-millimeter Spatial Oscillations of Newton's Constant: Theoretical Models and Laboratory Tests," (2016), [arXiv:1611.07293 \[gr-qc\]](#).  
*Phys.Rev.* **D95** (2017) no.8, 084050

# Theoretical Models II: Infinite Derivative Gravity

$$\mathcal{L}_{\text{IDG}} = \frac{1}{8\pi G} \sqrt{-g} [R + \alpha (R F_1(\square) R + R^{\mu\nu} F_2(\square) R_{\mu\nu} + R^{\mu\nu\rho\sigma} F_3(\square) R_{\mu\nu\rho\sigma})]$$

$$F_i(\square) = \sum_{n=0}^{\infty} f_{i_n} \left(\frac{\square}{M^2}\right)^n \quad \square = g^{\mu\nu} \nabla_\mu \nabla_\nu$$

T. Biswas, E. Ġerwick, T. Koivisto and A. Mazumdar,  
"Towards singularity and ghost free theories of gravity,"  
Phys. Rev. Lett. **108**, 031101 (2012)

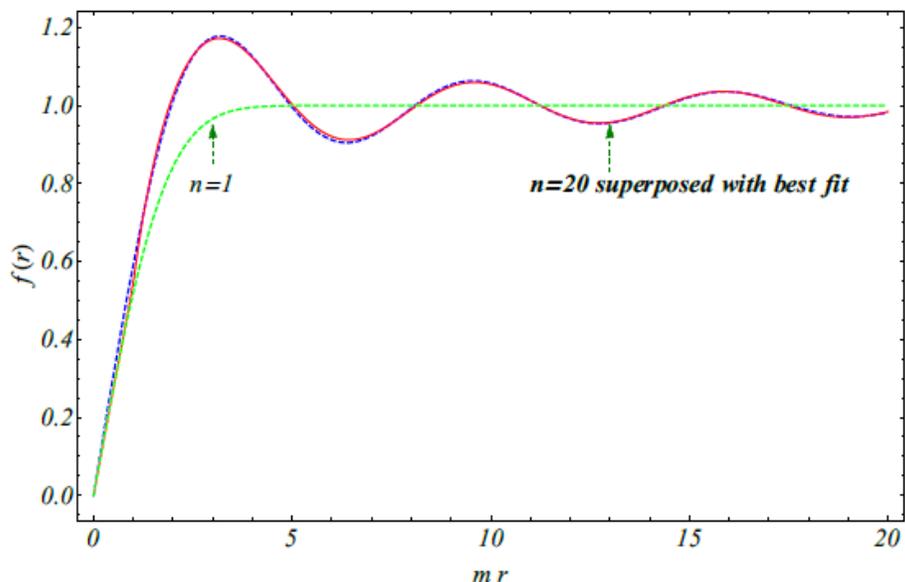
**No instabilities for proper choice of  $F_i$  (eg exponential).**

**Predicted gravitational potential:**

$$V_{\text{eff}}(r) = -\frac{GM}{r} f(r, m)$$

$$f(r, m) = \frac{1}{\pi} \int_{-\infty}^{+\infty} dk \frac{\sin(kr) e^{-\tau(k, m)}}{k}$$

$$\tau = \frac{k^{2n}}{m^{2n}}$$



**$n > 10$  is well fit as:**

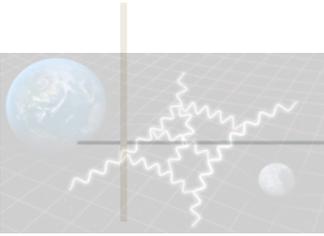
$$\begin{aligned} f(r) &= \alpha_1 \bar{r} \quad 0 < \bar{r} < 1 \\ f(r) &= 1 + \alpha_2 \frac{\cos(\bar{r} + \theta)}{\bar{r}} \quad 1 < \bar{r} \end{aligned}$$

$$\alpha_1 = 0.544, \alpha_2 = 0.572, \theta = 0.885\pi$$

**No singularities!**

Leandros Perivolaropoulos, "Sub-millimeter Spatial Oscillations of Newton's Constant: Theoretical Models and Laboratory Tests," (2016), [arXiv:1611.07293 \[gr-qc\]](https://arxiv.org/abs/1611.07293).  
Phys.Rev. D95 (2017) no.8, 084050

# Conclusions



**Tension within  $\Lambda$ CDM:** The best fit Planck15/ $\Lambda$ CDM  $\sigma_8$ - $\Omega_{0m}$  parameter values are more than  $3\sigma$  away from the corresponding best fit parameter values obtained using the latest RSD growth rate data assuming a Planck15/ $\Lambda$ CDM background cosmology. This tension is significantly weaker for  $f\sigma_8$  data obtained after 2015.

**Reduced Tension with  $G_{\text{eff}}(z)$ :** The tension can be reduced if an evolving Newton's constant is allowed leading to weaker gravity at  $z \approx 1$ . This type of evolution can not be reproduced in scalar-tensor theories with a  $\Lambda$ CDM background.

**Sub-mm Spatially Oscillating Newton Constant:** Higher derivative gravity models generically predict sub-mm spatial oscillations of Newton's constant. Hints for such oscillations have been demonstrated to exist in the Washington torsion-balance experiment.

# Fiducial cosmology Correction is not Important

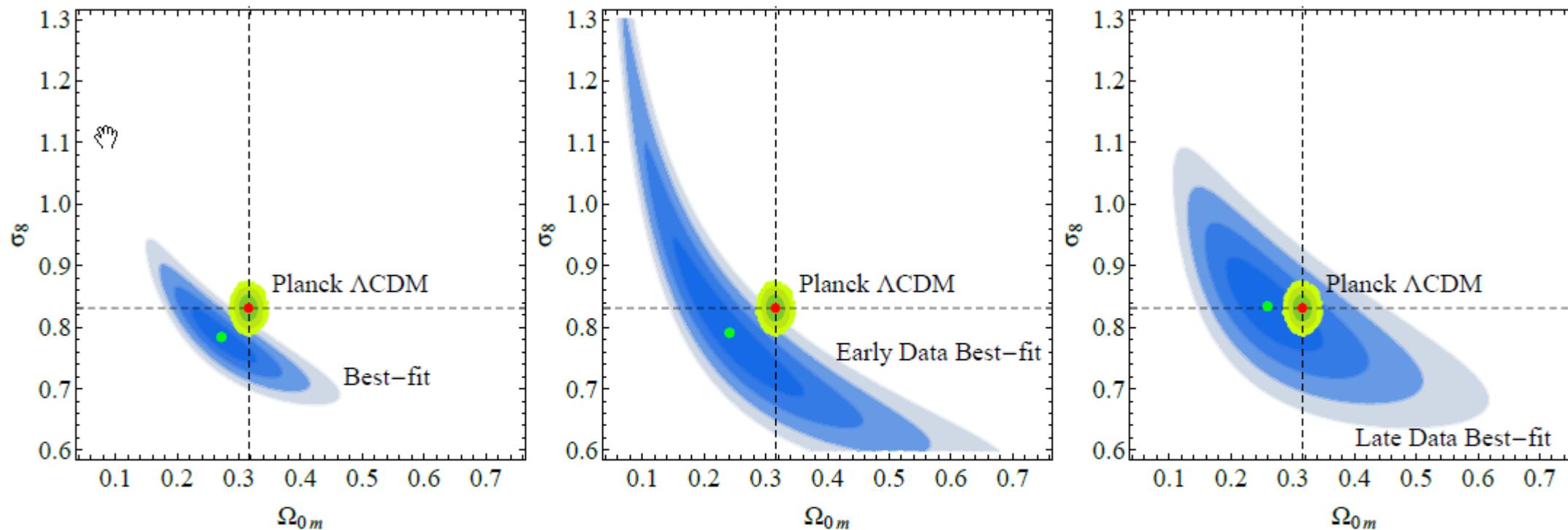


FIG. 5: Same as Fig. 4 but with no fiducial cosmology correction. The tension level in all three panels remains approximately the same.

# Correlating the Datapoints does not change the basic results

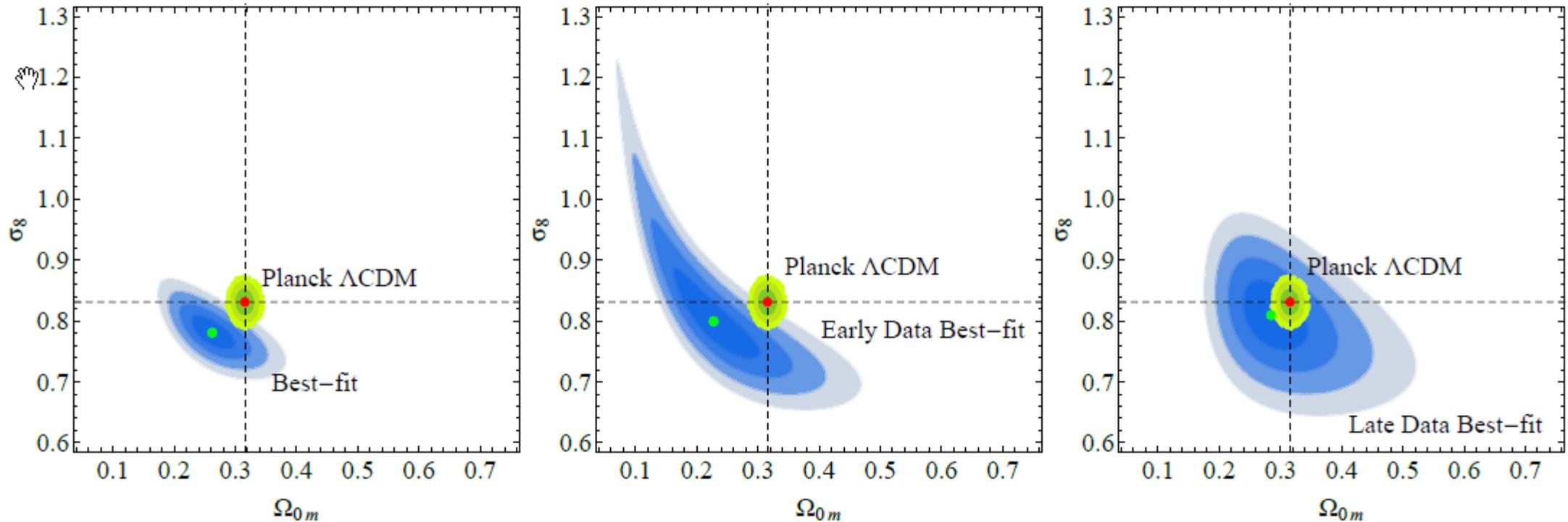


FIG. 7: Same as Fig. 4 but with a random covariance among 25% of the datapoints (assumed to be correlated in pairs). The tension level in all three panels remains approximately the same.

# Correlating the Datapoints does not change the basic results

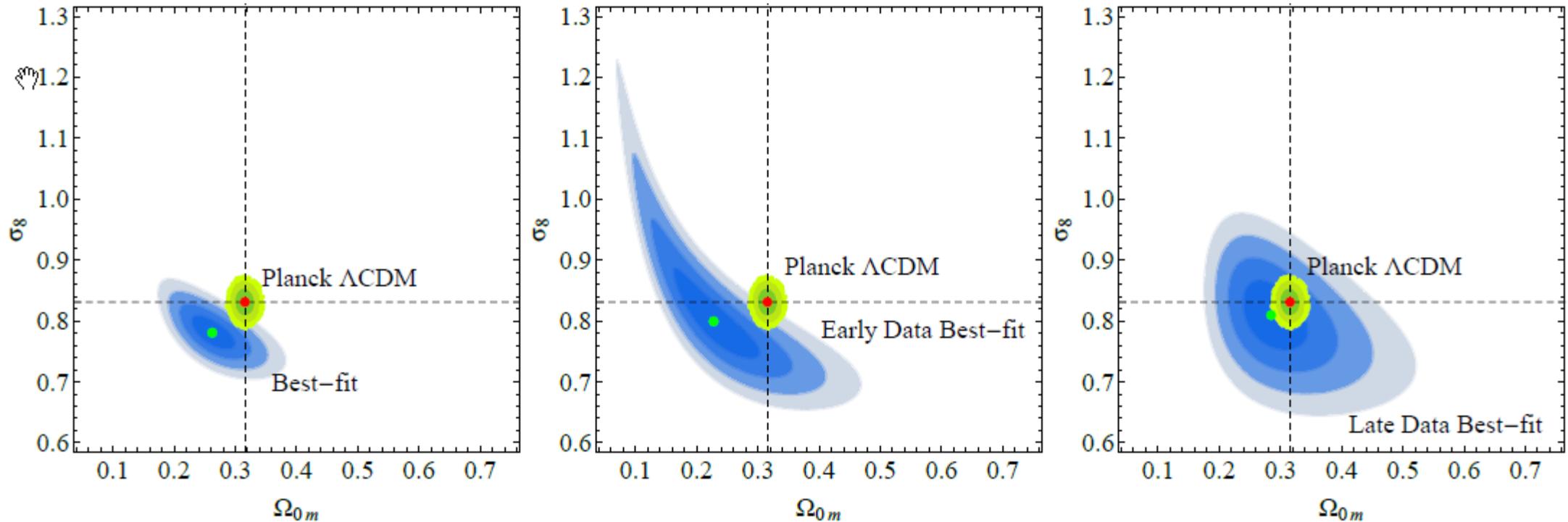
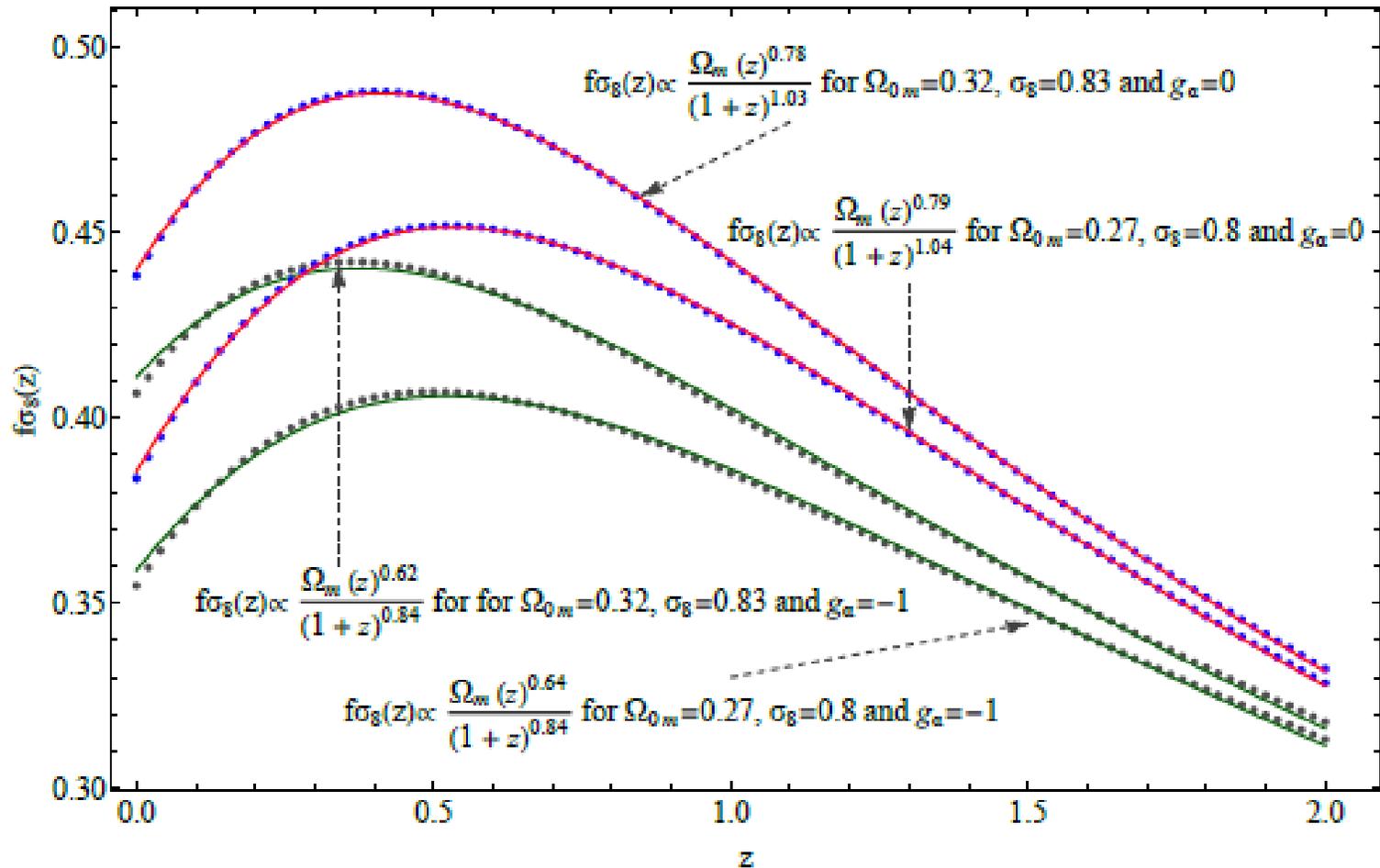
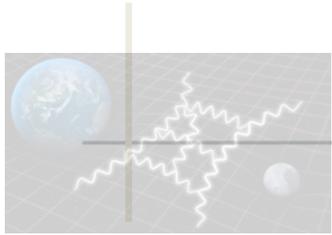


FIG. 7: Same as Fig. 4 but with a random covariance among 25% of the datapoints (assumed to be correlated in pairs). The tension level in all three panels remains approximately the same.

# The $f\sigma_8(z)$ solution is well fit by a simple parametrization



# Cosmic Growth of Density Perturbations



Perturbed metric Newtonian gauge:

$$ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\mathbf{x}^2$$

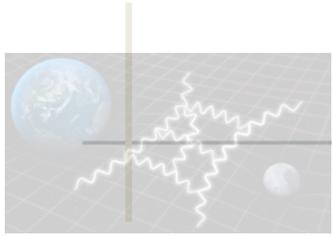
Define gauge invariant:

$$\left. \begin{aligned} \delta_m &\equiv \frac{\delta\rho}{\rho + p} + 3Hv \\ \delta u_\mu &= -\partial_\mu v \end{aligned} \right\} \begin{array}{l} \nabla_\mu T_\nu^\mu = 0 \\ T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \end{array} \left\{ \begin{array}{l} \dot{\delta}_m = -\frac{k^2}{a^2}v + 3\frac{d(\psi + Hv)}{dt} \\ \phi = \dot{v} + \frac{p}{\rho}(2Hv - \delta_m) \end{array} \right. \begin{array}{l} p = 0 \\ k^2/a^2 \gg H^2 \end{array} \longrightarrow \ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\phi \approx 0$$

$\downarrow \frac{k^2}{a^2}\phi \approx -4\pi G_{\text{eff}}\rho\delta_m$

$$\delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \frac{\Omega_m G_{\text{eff}}(a, k)/G_N}{a^5 H(a)^2/H_0^2} \delta(a) = 0$$

# Theoretical Models I: f(R) theories



Weak field gravity:

$$f(R) = R + \frac{1}{6m^2} R^2$$



$$S_{BD} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ \Phi R - \frac{3}{2} m^2 (\Phi - 1)^2 \right] + S_{matter}$$

$$T_{\mu\nu} = \text{diag}(M\delta(\vec{r}), 0, 0, 0)$$

$$\Phi = 1 + \varphi$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\left. \begin{array}{l} \varphi = \frac{2GM}{3r} e^{-mr} \\ h_{00} = \frac{2GM}{r} \left( 1 + \frac{1}{3} e^{-mr} \right) \\ h_{ij} = \frac{2GM}{r} \delta_{ij} \left( 1 - \frac{1}{3} e^{-mr} \right) \end{array} \right\}$$

$$V_{eff} = -\frac{h_{00}}{2} = -\frac{GM}{r} \left( 1 + \frac{1}{3} e^{-mr} \right) \quad m^2 > 0$$

$$V_{eff} = -\frac{h_{00}}{2} = -\frac{GM}{r} \left( 1 + \frac{1}{3} \cos(|m|r + \theta) \right) \quad m^2 < 0$$

Perturbations:

$$\bar{\varphi} = \varphi_0(r) + \delta\varphi(r, t)$$

$$(\square - m^2)\varphi = -\frac{8\pi G}{3} M\delta(\vec{r})$$



$$-\ddot{\delta\varphi} + \nabla^2 \delta\varphi - m^2 \delta\varphi = 0$$

**$m^2 > 0$ : Stability**

**$m^2 < 0$ : Instabilities**

Valerio Faraldi, "Matter instability in modified gravity," *Phys. Rev. D* **74**, 104017 (2006), [arXiv:astro-ph/0610734](https://arxiv.org/abs/astro-ph/0610734) [astro-ph].

A. D. Dolgov and Masahiro Kawasaki, "Can modified gravity explain accelerated cosmic expansion?" *Phys. Lett. B* **573**, 1-4 (2003), [arXiv:astro-ph/0307285](https://arxiv.org/abs/astro-ph/0307285) [astro-ph].

# Consistency of Evolving $G_{\text{eff}}(z)$ with CMB Power Spectrum

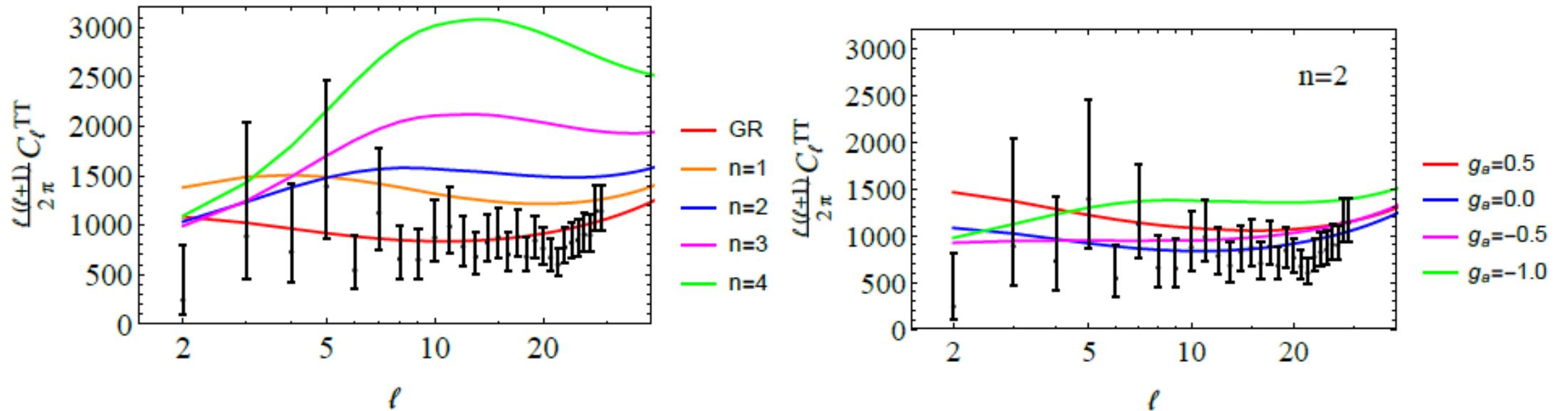


FIG. 6. The ISW effect for the  $G_{\text{eff}}$  model used in our analysis for various values of  $n$  evaluated at the minima for  $g_a$  given in Table IV (left) and for  $n = 2$  but for various values of  $g_a$  as indicated by the label. We also show the Planck15 low- $l$  binned  $C_l^{TT}$  data.