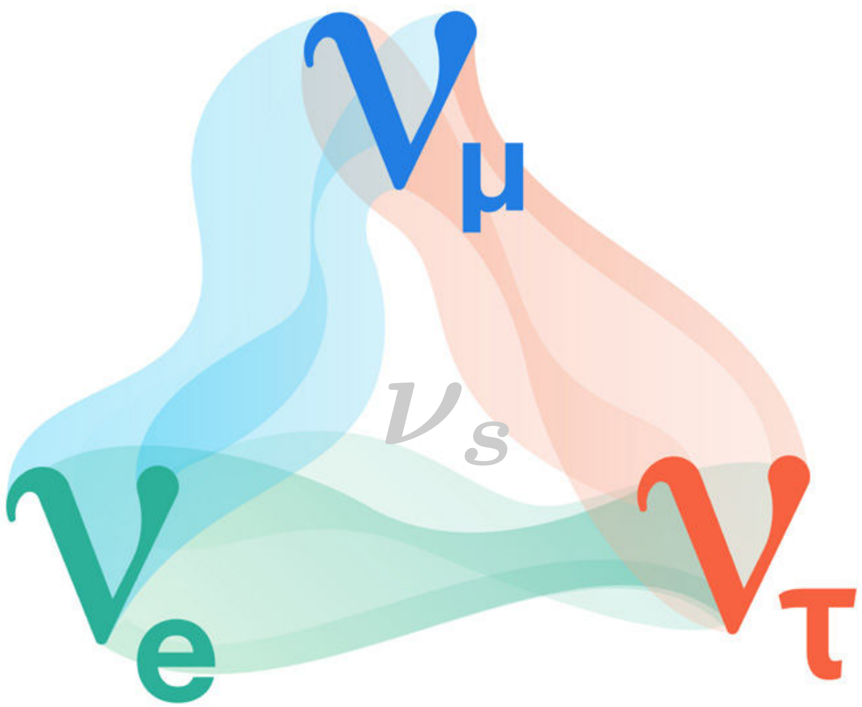
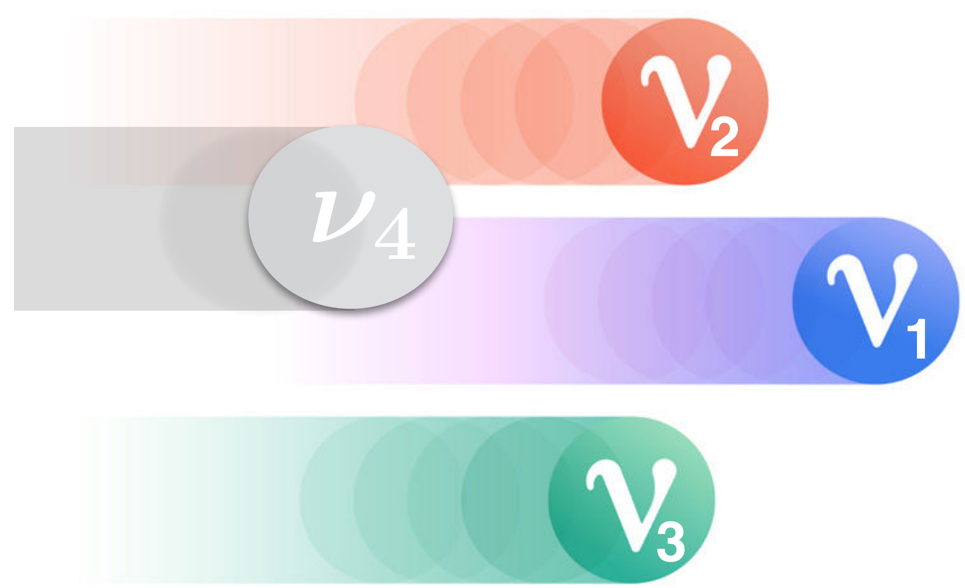
Lessons and prospects from the past, present and future of \nu_e disappearance experiments

Stephen Parke
Fermilab





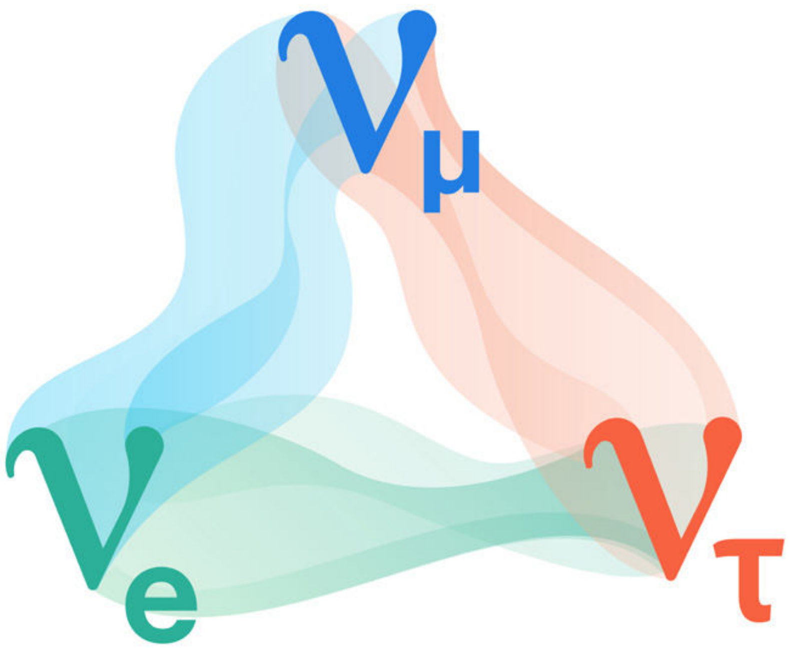
some members of the Fermilab Theory Group 1974:

Ben Lee, Mary K Gaillard, Shirley Jackson
& Tong Pagnamenta

"can live and work with pride and dignity without regard to such differences as race, religion, sex or national origin. In conflict between technical expediency and human rights, we will stand on the side of human rights." Wilson 1970

Interactions:

simple complicated



= U

unitary matrix ?

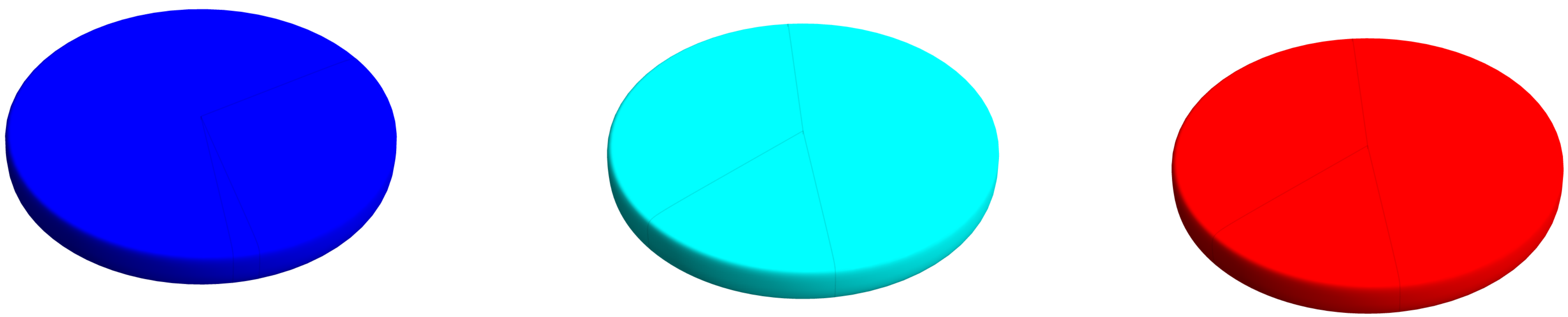
complicated simple

masses ?

Propagation:

Neutrino Flavor or Interaction States:

$W^+ \to e^+ \nu_e$ $W^+ \to \mu^+ \nu_\mu$ $W^+ \to \tau^+ \nu_\tau$



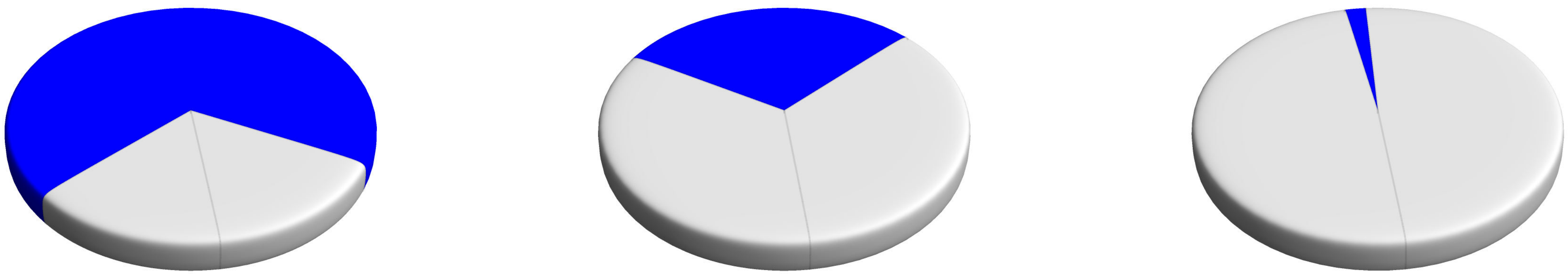
ν_e ν_μ ν_τ

provided $L/E \ll \mathbf{0.5}$ km/MeV $= \mathbf{500}$ km/GeV !!!

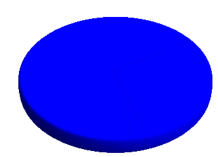
$\sim$ 1 picosecond in Neutrino rest frame !!!

Neutrino Mass EigenStates or Propagation States:

$$\text{Propagator } \nu_j \to \nu_k \;=\; \delta_{jk}\, e^{-i\left(\frac{m_j^2 L}{2E_\nu}\right)}$$



ν_1 ν_2 ν_3

$\nu_e =$ 

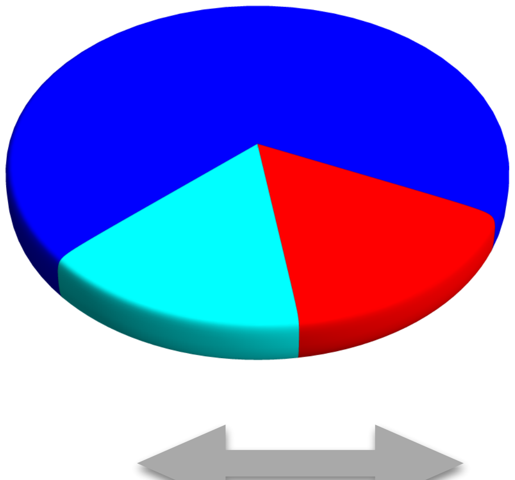
$\Delta m^2_{21} \sim 7.5 \times 10^{-5}$ eV^2

$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}$ eV^2

Neutrino Mass EigenStates or Propagation States:

ν_1

most ν_e



δ, θ_{23}

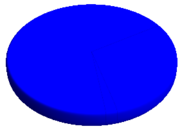
ν_2

δ, θ_{23}

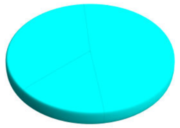
ν_3

least ν_e

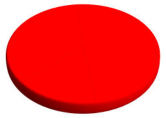
θ_{23}

$\nu_e =$ 

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, …

$\nu_\mu =$ 

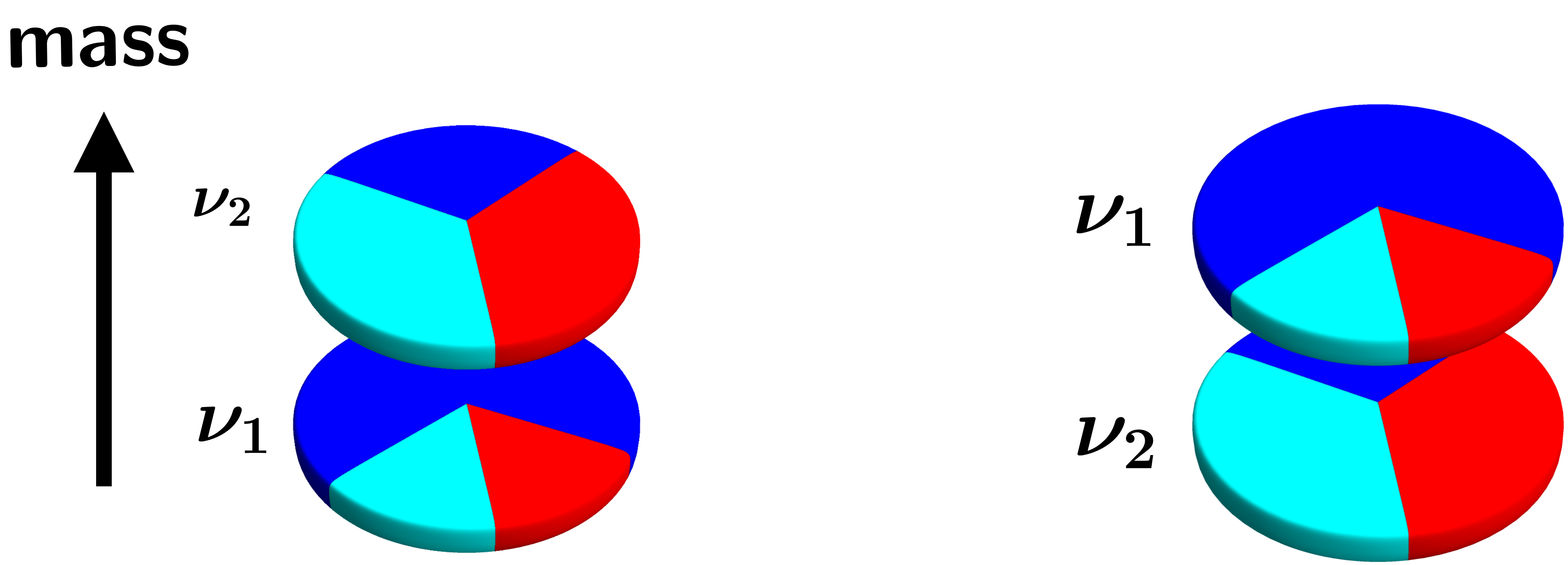
SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

$\nu_\tau =$ 

Unitarity
SK, Opera
ICECUBE ?

$\nu_1, \ \nu_2$ Mass Ordering:

–solar mass ordering



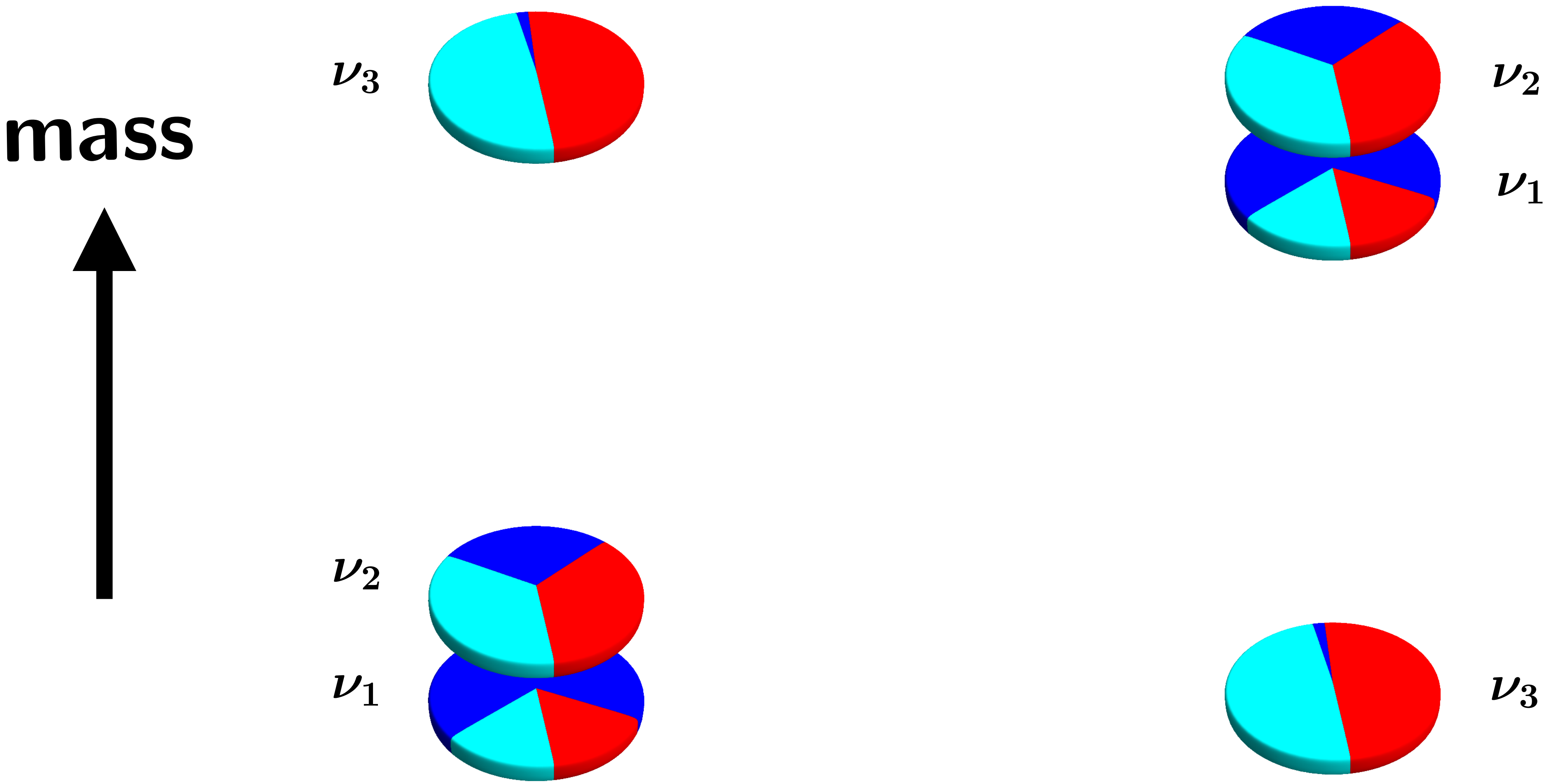
$|\Delta m^2_{21}| = |m_2^2 - m_1^2| = \mathbf{7.5} \times \mathbf{10^{-5}} \ \mathrm{eV}^2$ $\quad$ $\boldsymbol{L/E = 15} \ \mathrm{km/MeV} = \mathbf{15,000} \ \mathrm{km/GeV}$

SNO $\quad m_2 > m_1$

$\nu_e =$ ● $\quad \nu_\mu =$ ● $\quad \nu_\tau =$ ●

ν_3, ν_1/ν_2 Mass Ordering:

–atmospheric mass ordering



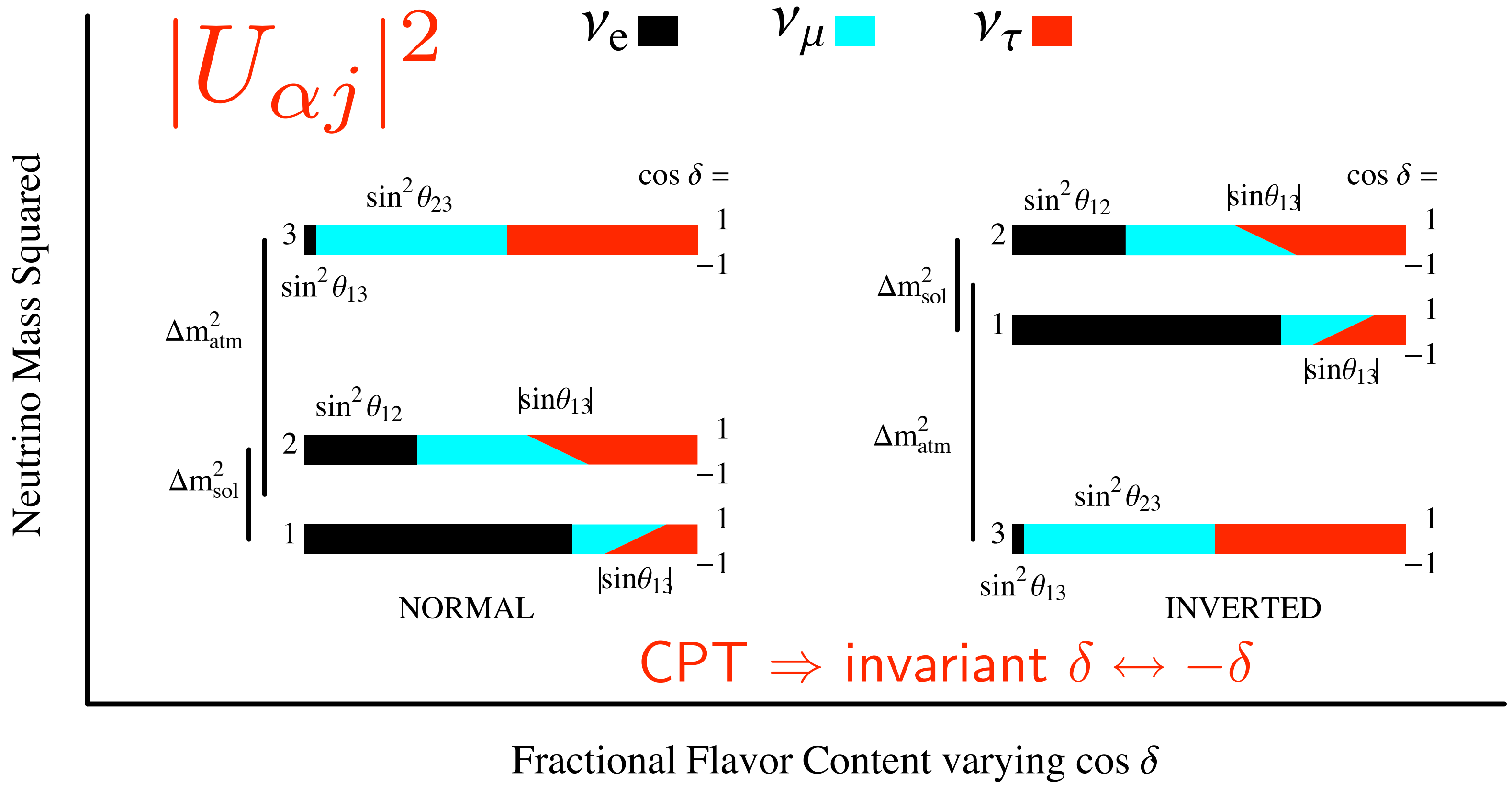
$$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3}\ \mathrm{eV}^2 \qquad L/E = 0.5\ \mathrm{km/MeV} = 500\ \mathrm{km/GeV}$$

Unknown: NOνA, JUNO, ICECUBE, DUNE, T2HKK....

$\nu_e =$ $\nu_\mu =$ $\nu_\tau =$

Summary:

- Labeling massive neutrinos: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$



CPT $\Rightarrow$ invariant $\delta \leftrightarrow -\delta$

$\sin^2\theta_{12} \sim \frac{1}{3}$

$\sin^2\theta_{23} \sim \frac{1}{2}$

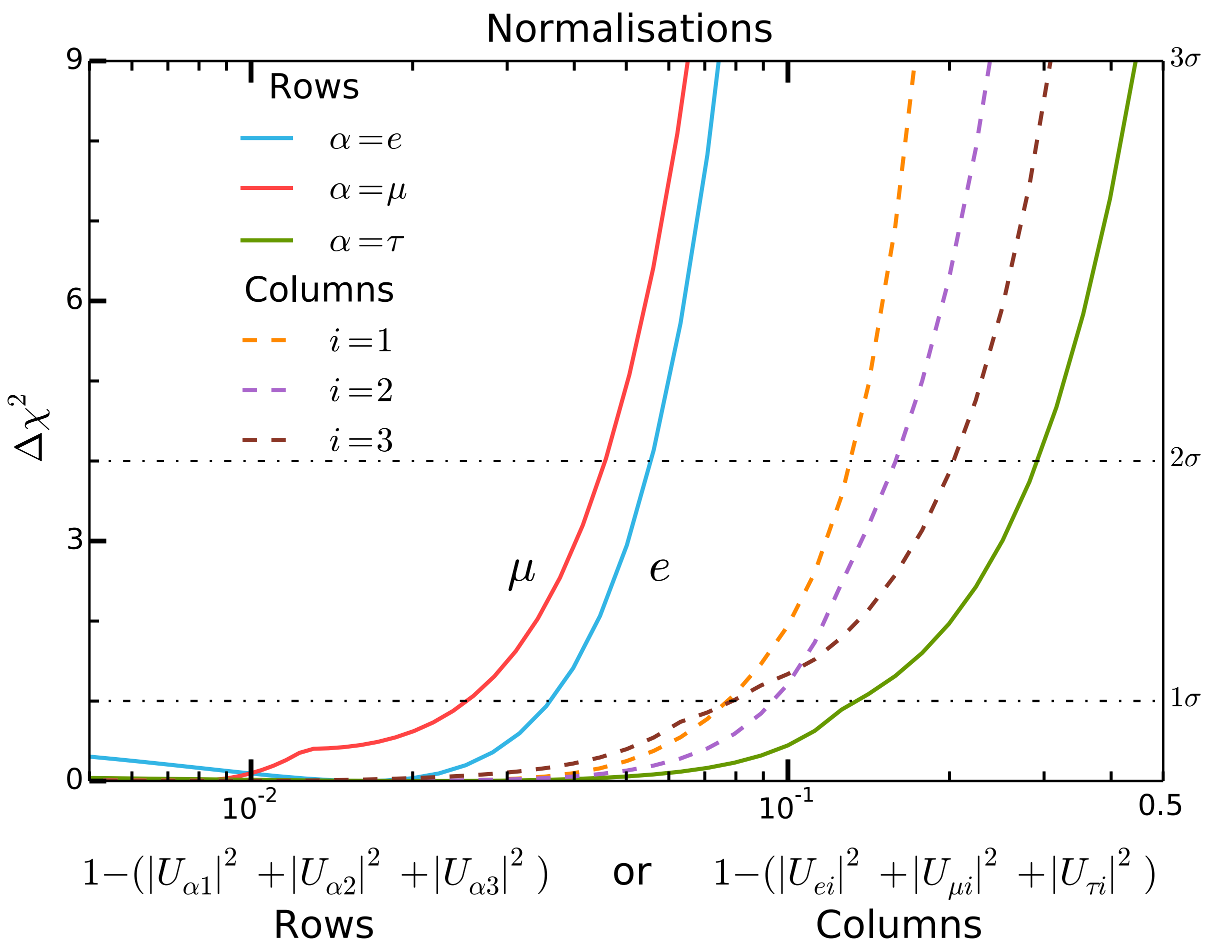
$\sin^2\theta_{13} \sim 0.02$

$0 \leq \delta < 2\pi$

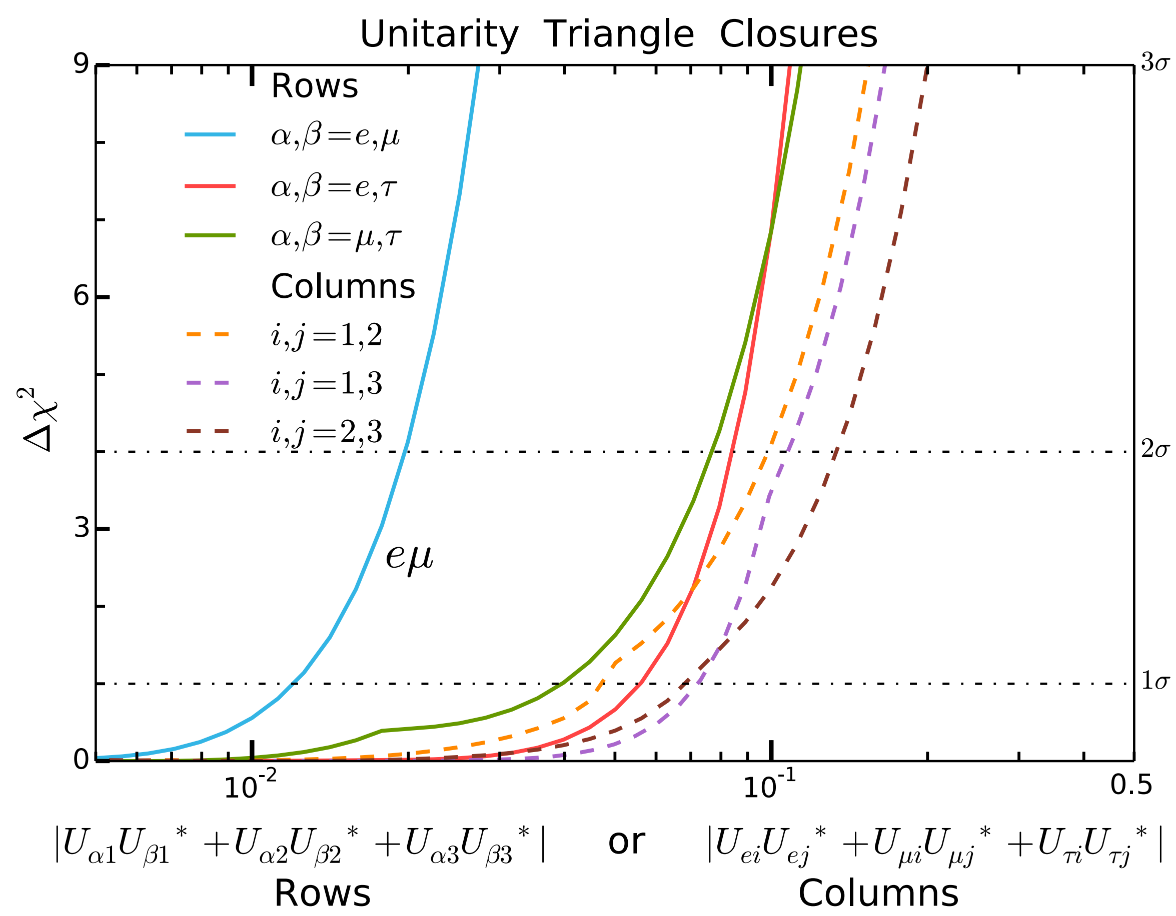
$|\Delta m^2_{21}| = |m^2_2 - m^2_1| = 7.5 \times 10^{-5}\ \mathrm{eV}^2$ $\quad L/E = 15\ \mathrm{km/MeV} = 15,000\ \mathrm{km/GeV}$

$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3}\ \mathrm{eV}^2$ $\quad L/E = 0.5\ \mathrm{km/MeV} = 500\ \mathrm{km/GeV}$

Unitarity



Ross-Lonergan+ SP 1508.05095



6 row/column plus 6 triangle conditions

2 row and 1 triangle, independent of ν_τ

$\nu_e \to \nu_e$ and $\bar{\nu}_e \to \bar{\nu}_e$

kinematic phase:

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

REACTOR NEUTRINOS:

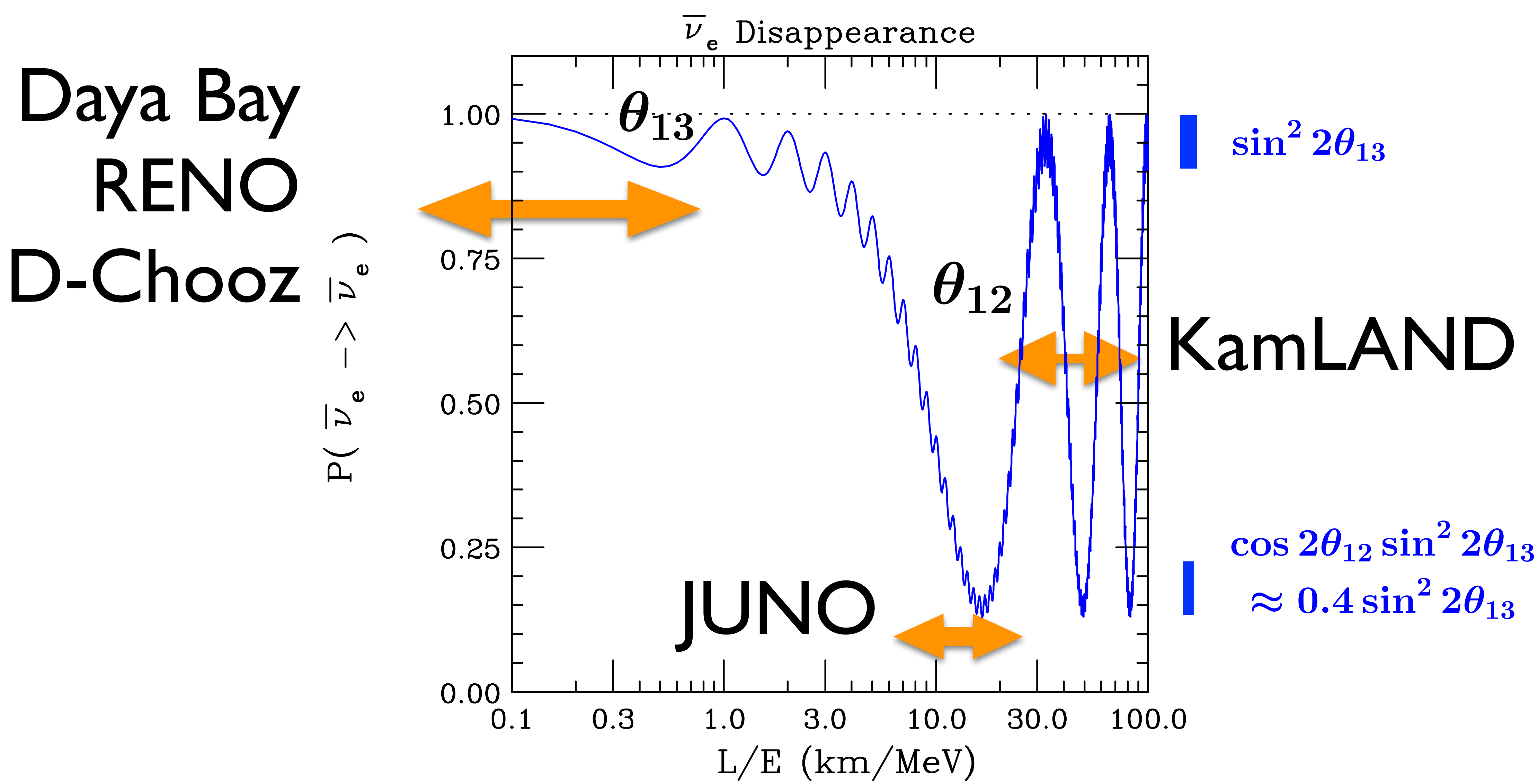
$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$P(\bar{\nu}_e \to \bar{\nu}_e) \;=\; 1 - P_{13} - P_{12} \quad \text{with}$$

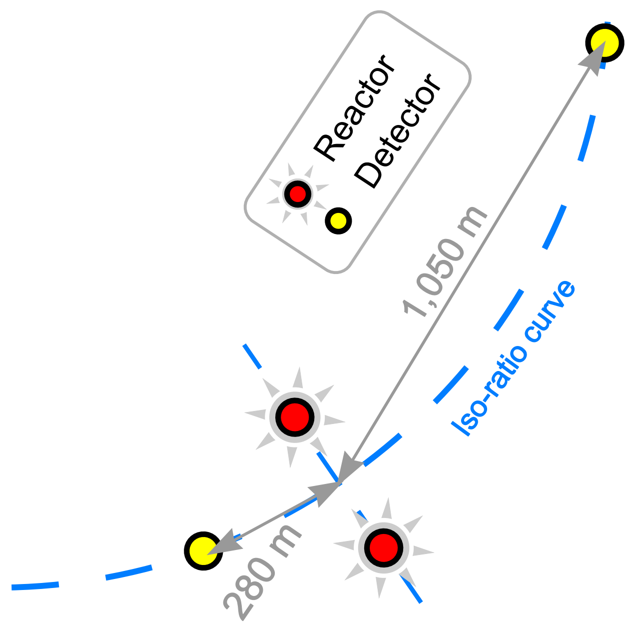
$$\boldsymbol{\theta_{13}} \quad P_{13} = 4|U_{e3}|^2(|U_{e1}|^2 \sin^2\Delta_{31} + |U_{e2}|^2 \sin^2\Delta_{32})$$

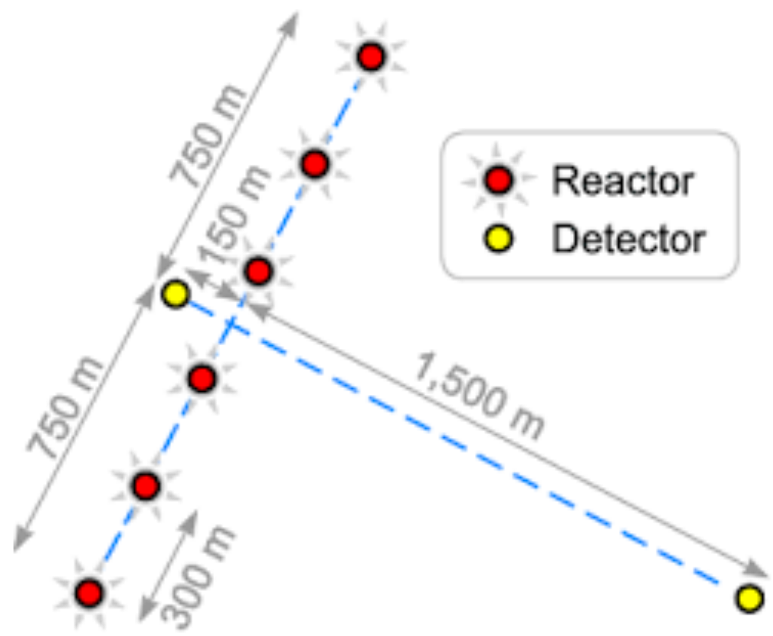
$$= \sin^2 2\theta_{13}\,(\cos^2\theta_{12} \sin^2\Delta_{31} + \sin^2\theta_{12} \sin^2\Delta_{32}),$$

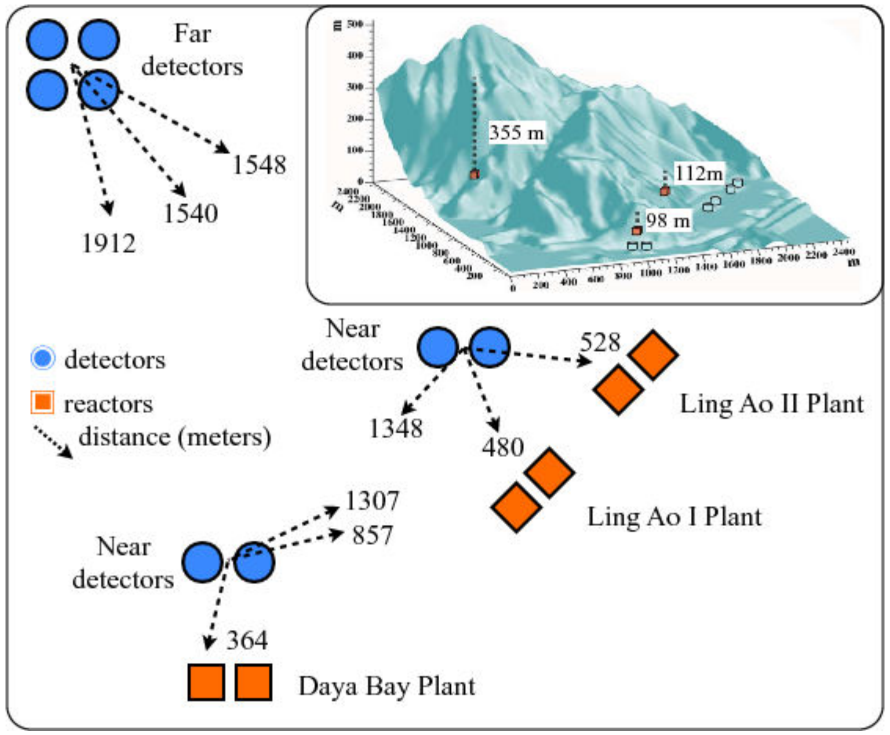
$$\boldsymbol{\theta_{12}} \quad P_{12} = 4|U_{e2}|^2|U_{e1}|^2 \sin^2\Delta_{21} = \sin^2 2\theta_{12} \cos^4\theta_{13} \sin^2\Delta_{21}$$



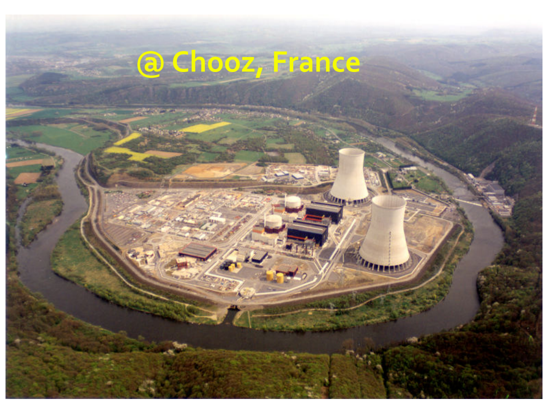
Reactor θ_{13} Experiments



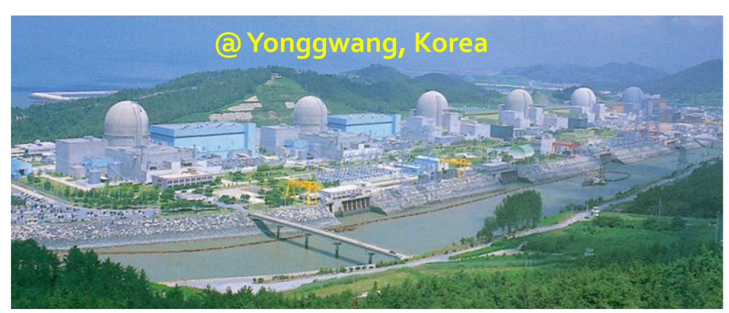




Double Chooz

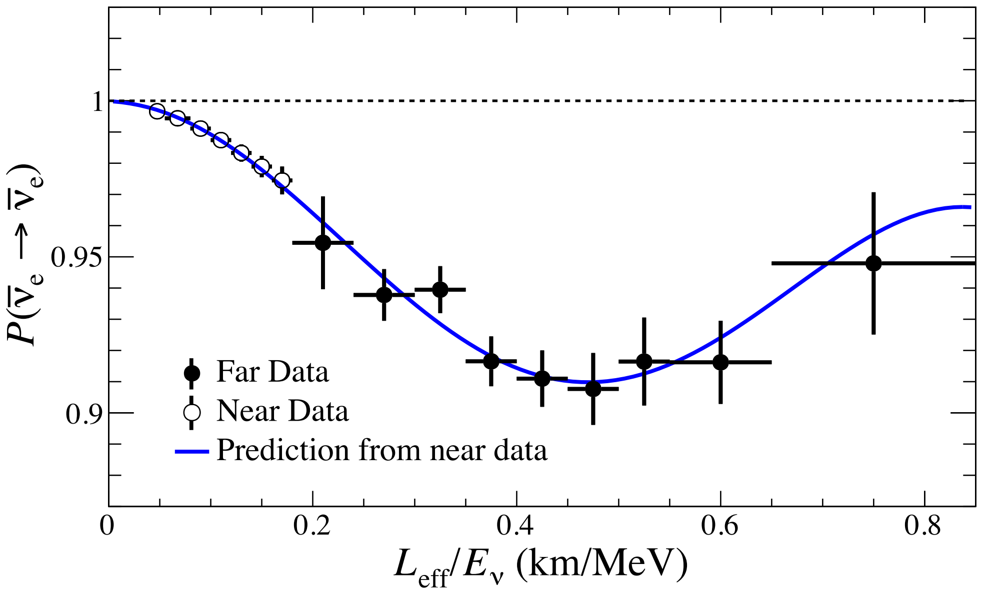


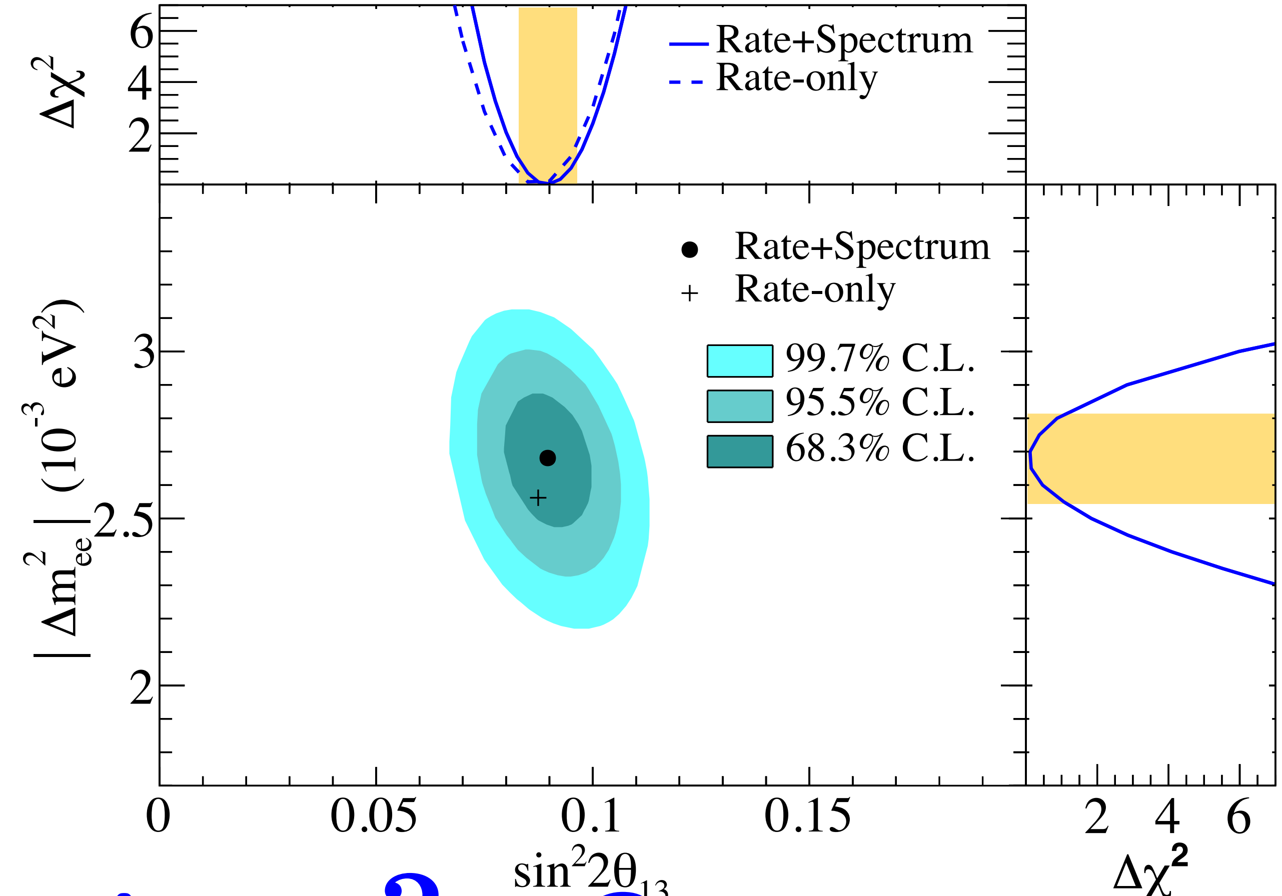
RENO



Daya Bay

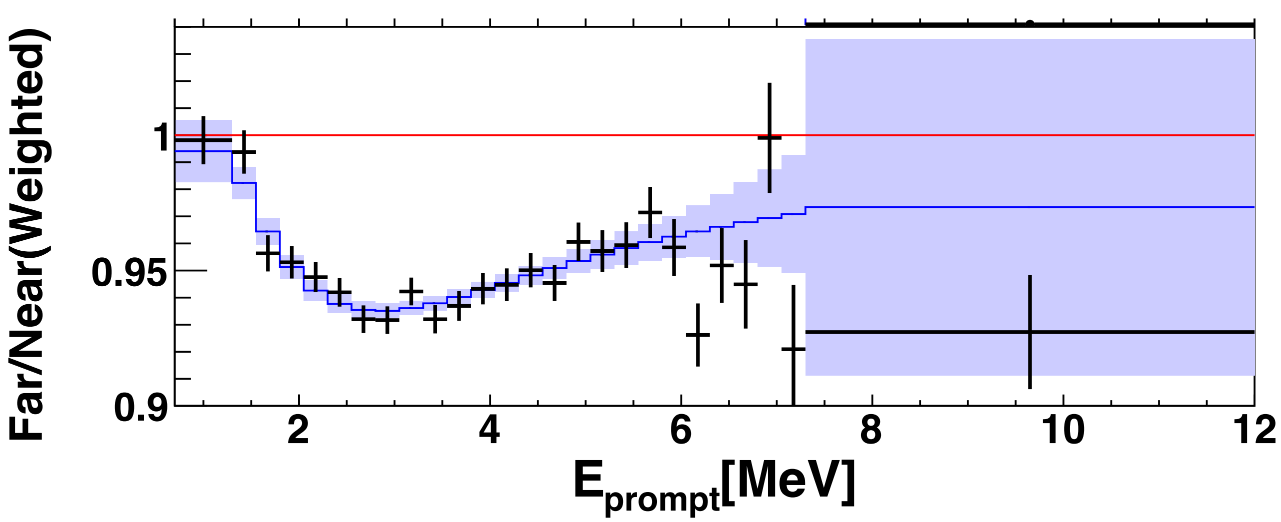
RENO 2200 days

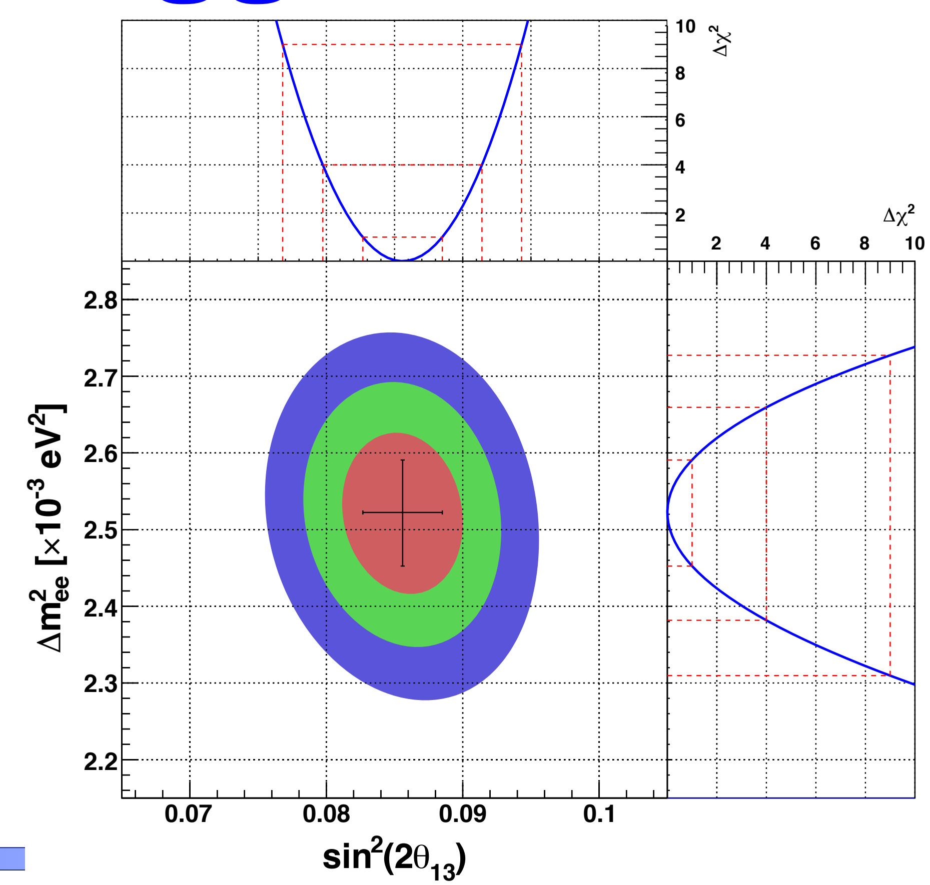




What is Δm^2_{ee} ?

Daya Bay 1958 days





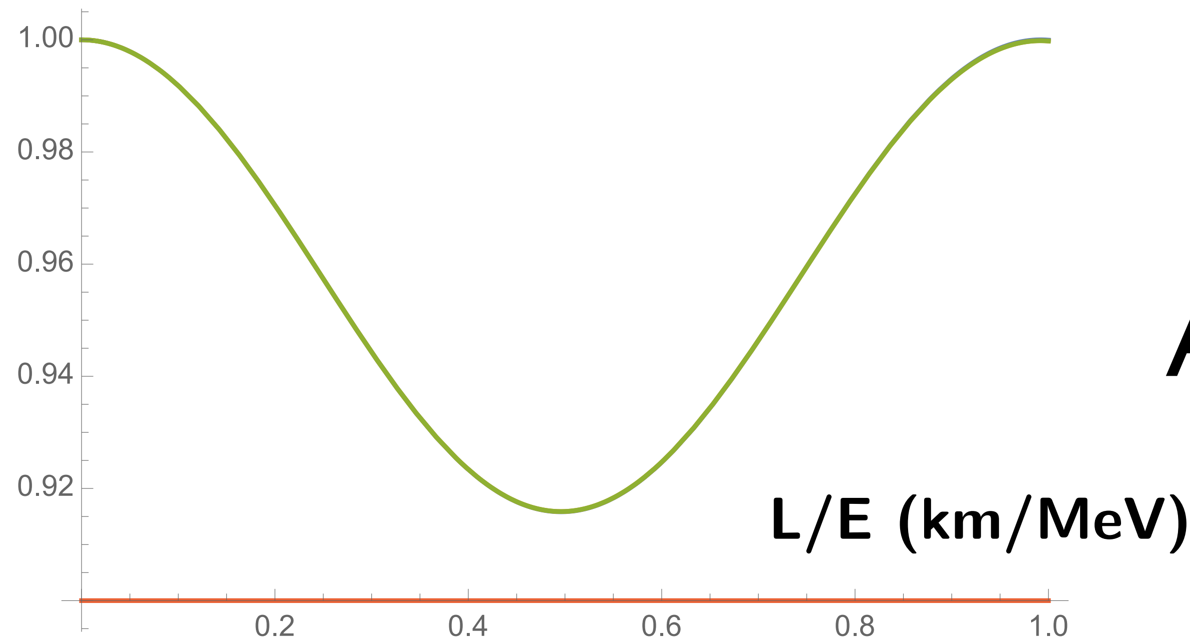
~3%

$$P_{13} = \sin^2 2\theta_{13} \left(\cos^2\theta_{12} \sin^2\Delta_{31} + \sin^2\theta_{12} \sin^2\Delta_{32}\right),$$

$$P_{13} \approx \sin^2 2\theta_{13} \, \sin^2\Delta_{ee}$$

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12} \, \Delta m^2_{31} + \sin^2\theta_{12} \, \Delta m^2_{32}$$

$1 - P_{13}$



3 curves:

NO

IO

Approx

Δm^2_{ee} is ν_e average of Δm^2_{31} and Δm^2_{32}

RENO uses this defn:

NPZ= Nunokawa, SP, Zukanovich Funchal hep-ph/0503283

WHY ?

$$\Delta_{31} = \Delta_{ee} + s^2_{12}\Delta_{21}$$
$$\Delta_{32} = \Delta_{ee} - c^2_{12}\Delta_{21}$$

$$\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32} - \sin^2\Delta_{ee}$$

$$= \quad 0 \times \Delta_{21}\sin 2\Delta_{ee} + (s^2_{12}c^2_{12})\,\Delta^2_{21}\cos 2\Delta_{ee} \pm \mathcal{O}(\Delta^3_{21})$$

$\boxed{<0.002}$

NO/IO

minimum

zero

$$\Delta_{21} = 0.1\left(\frac{L/E}{1\,\mathrm{km/MeV}}\right)$$

Daya Bay Saga:

Invisibles 2015 —> present

DB 1505.03456v1

DB 1505.03456v2 (PRL)

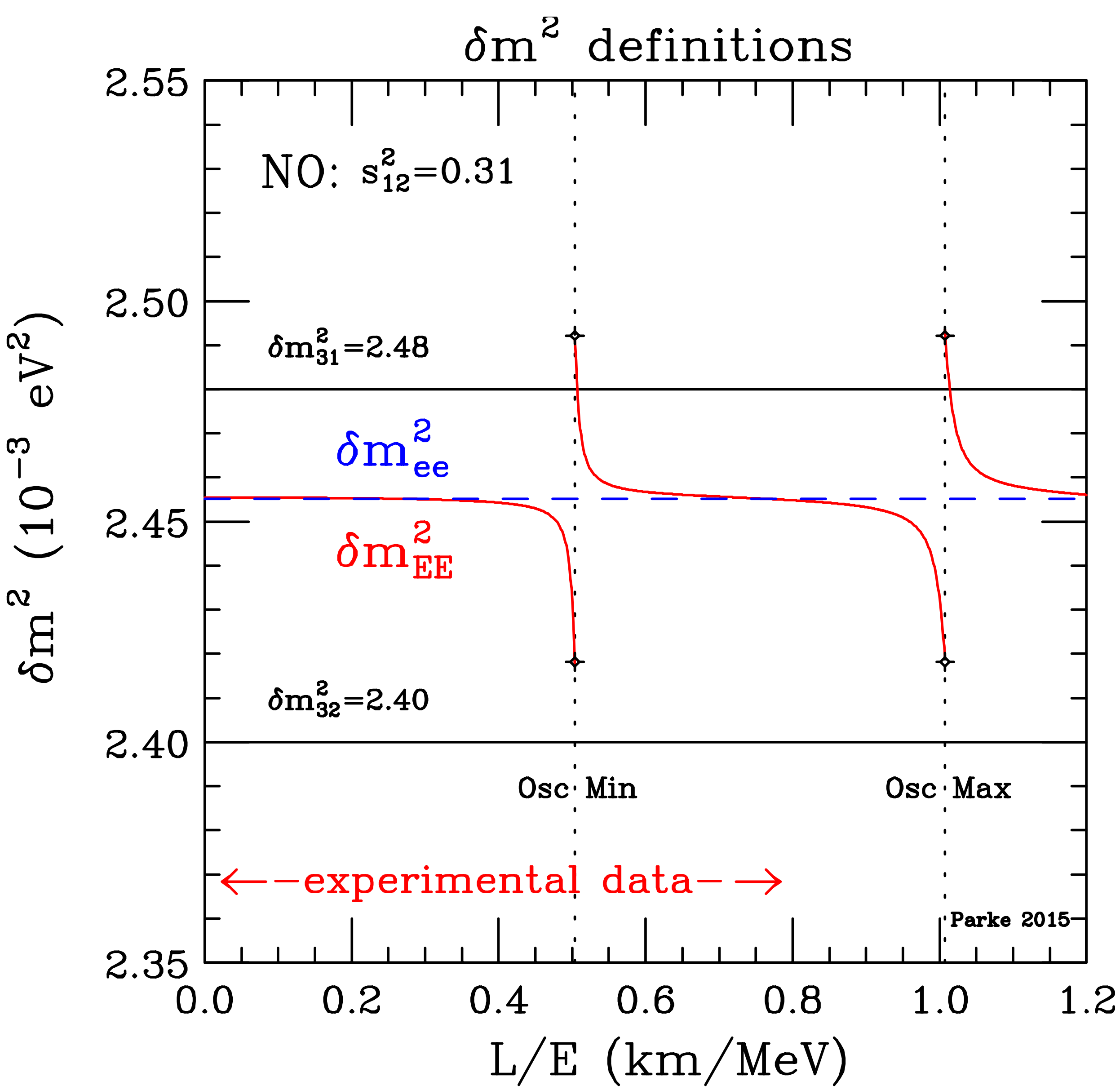
=> SP 1601.07464

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c^2_{12}\sin^2\Delta_{31} + s^2_{12}\sin^2\Delta_{32} \qquad (1)$$

$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c^2_{12}\sin^2\Delta_{31} + s^2_{12}\sin^2\Delta_{32})}\right]$$



Maladies: $\left[\frac{E}{L}\right] = M^2$

- L/E dependent
- Discontinuous at Osc. Max./Min. (L/E ≈ 0.5, 1.0, .. km/MeV) 3% jump
- no simple physical meaning !

Why???

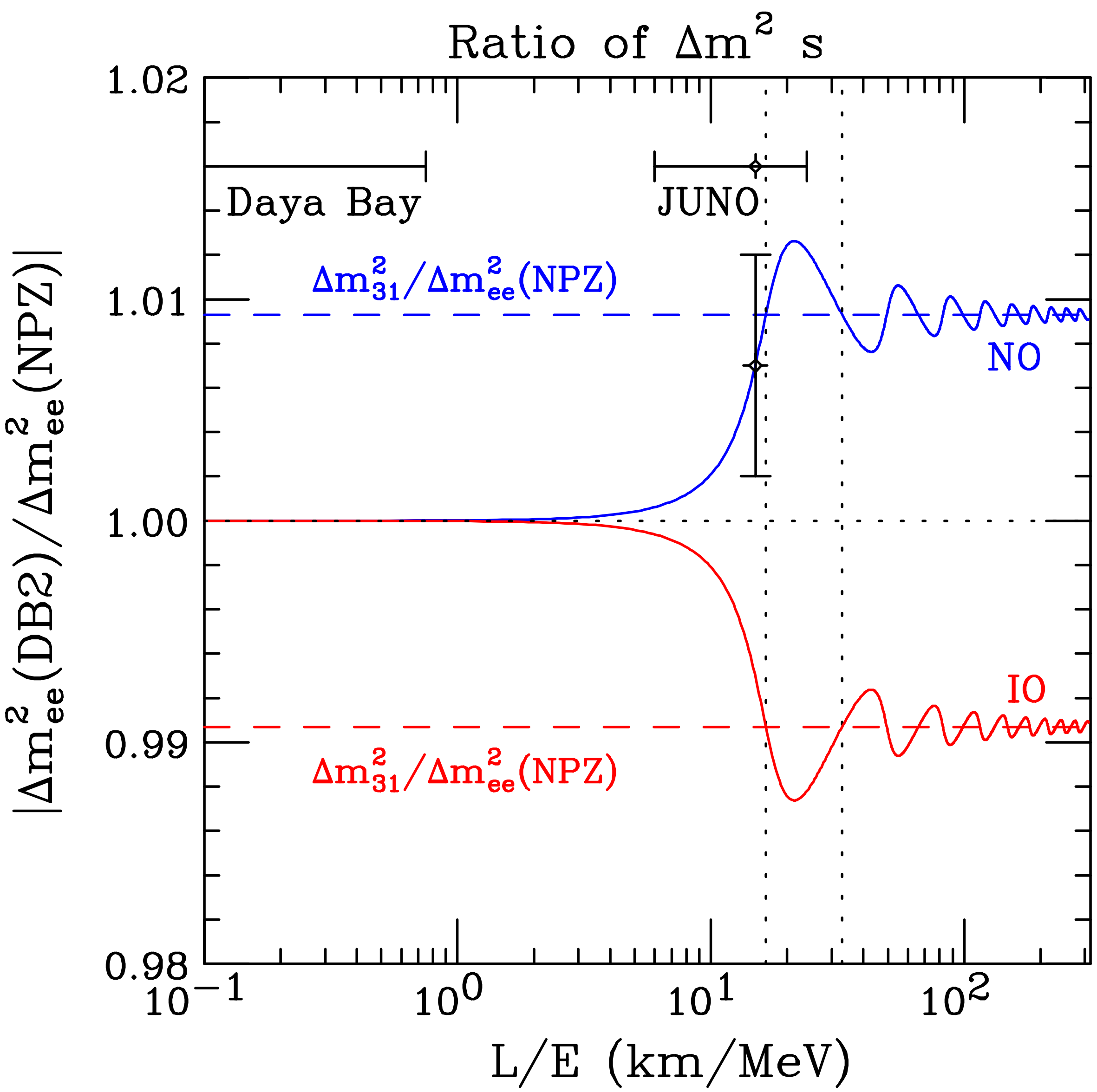
RHS (1) never gets exactly to 1,
or back to 0
whereas LHS does !

eg $\sin^2(\frac{\pi}{2} \mp \epsilon) = 1 - \epsilon^2 + \mathcal{O}(\epsilon^4)$
with $\epsilon = s_{12}c_{12}\Delta_{21}$

Daya Bay's New Defn. Δm^2_{EE} ?

(arXiv:1505.03456v2 and 1809.02261)

$$\Delta m^2_{EE} \equiv \Delta m^2_{32} + \left(\frac{2E}{L}\right) \arctan\left[\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2\theta_{12}}\right] \quad \begin{aligned} &= \Delta m^2_{ee}(NPZ) \quad L/E \ll 2\pi/\Delta m^2_{21} \\ &\approx \Delta m^2_{31} \quad L/E \geq 2\pi/\Delta m^2_{21} \end{aligned}$$



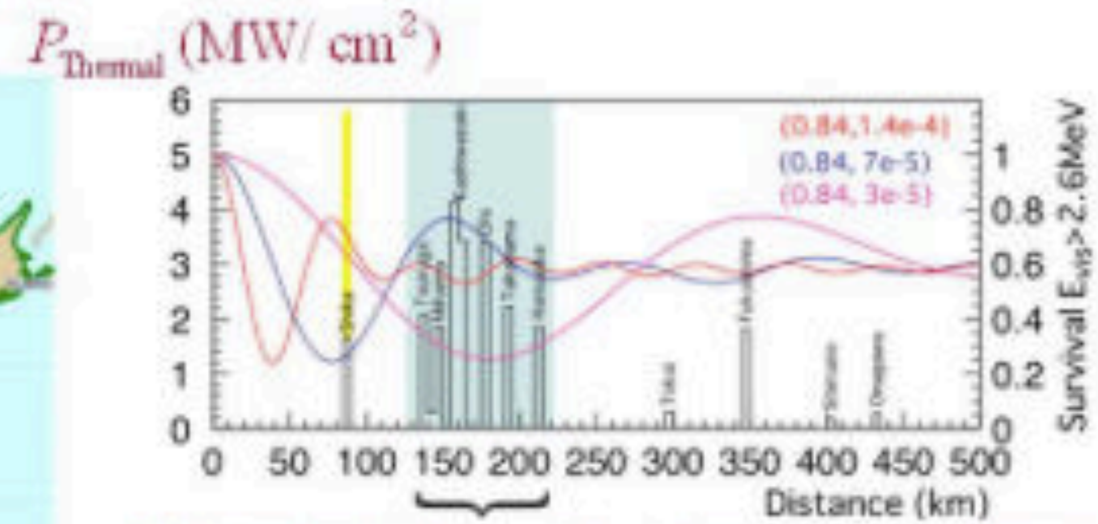
Maladies:

- L/E dependent for > 10 km/MeV
- Jump in JUNO $L/E \sim 1\%$
- physical meaning ???

$$\Delta m^2_{21}$$

KamLAND:

Reactors near the KamLAND



80% of total contribution comes from 130~220km distance

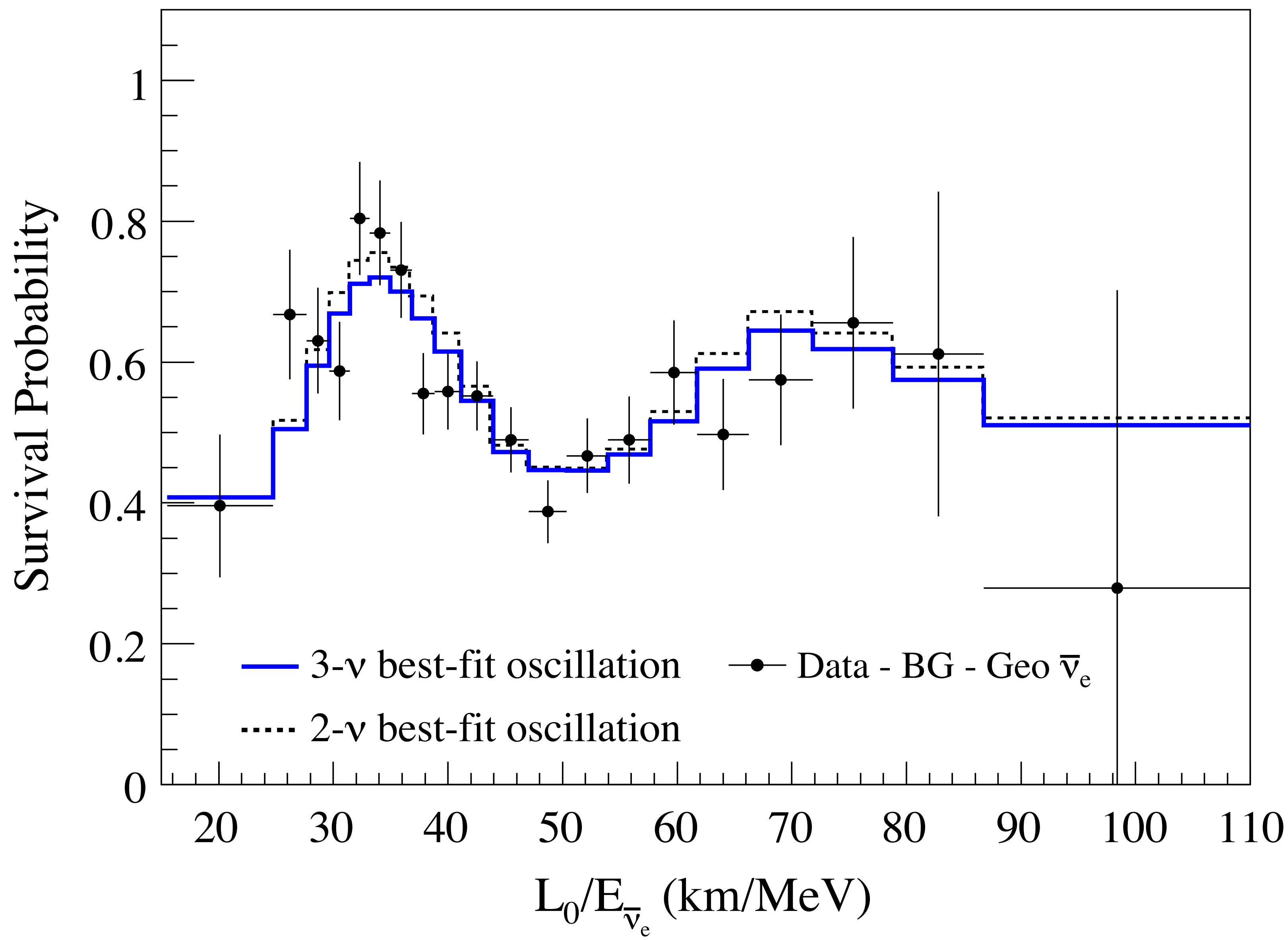
→ effective distance ~180km

Reactor neutrino flux, $\sim 6 \times 10^6/\text{cm}^2/\text{sec}$

~95.5% from Japan
~3% from Korea (2nd result period)

Reactors in Taiwan have ~0.1% contribution.

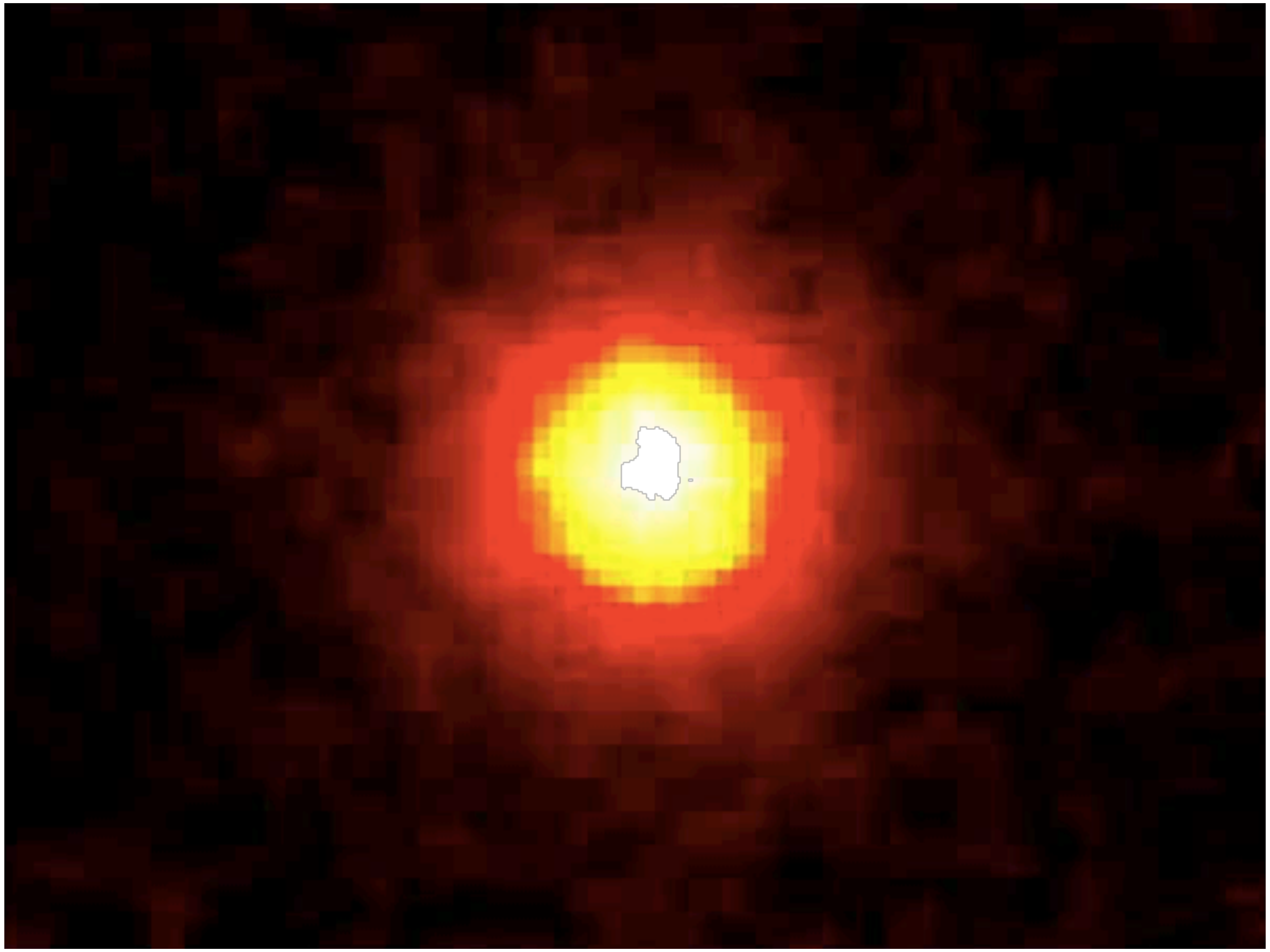
KamLAND:



Vacuum:
averaged osc
$\sim 68\%\ \nu_1$
$\sim 30\%\ \nu_2$
$\sim\ \ 2\%\ \nu_3$

$$\Delta m^2_{21} = 7.50\,^{+0.20}_{-0.20} \times 10^{-5}\ \mathrm{eV}^2,$$

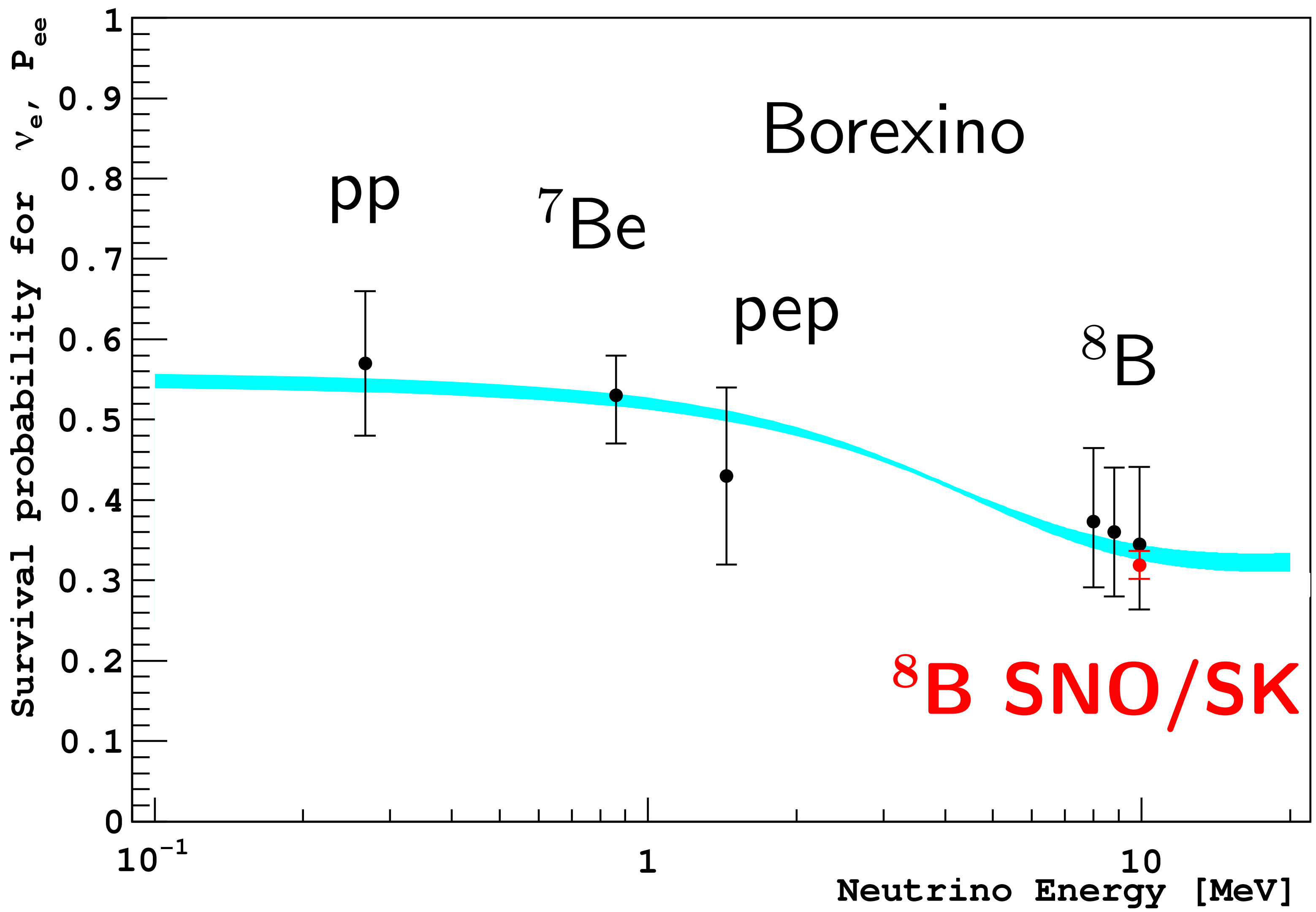
SuperK



$$\nu_? + e \rightarrow \nu + e$$

Which type of Neutrino dominates this image ?

Solar Neutrinos:



Vacuum: averaged osc

$\sim 68\% \ \nu_1$

$\sim 30\% \ \nu_2$

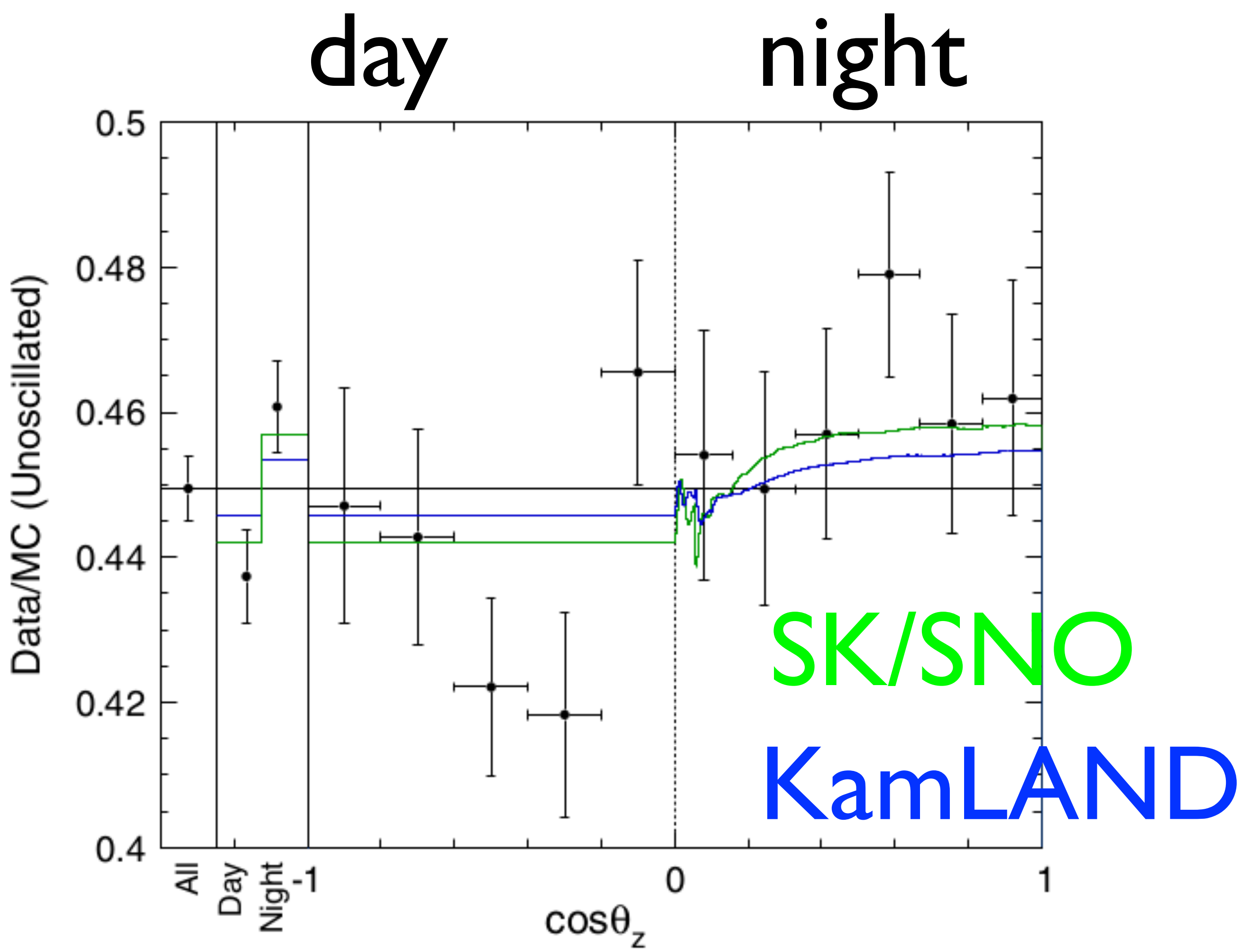
$\sim \ 2\% \ \nu_3$

MSW:

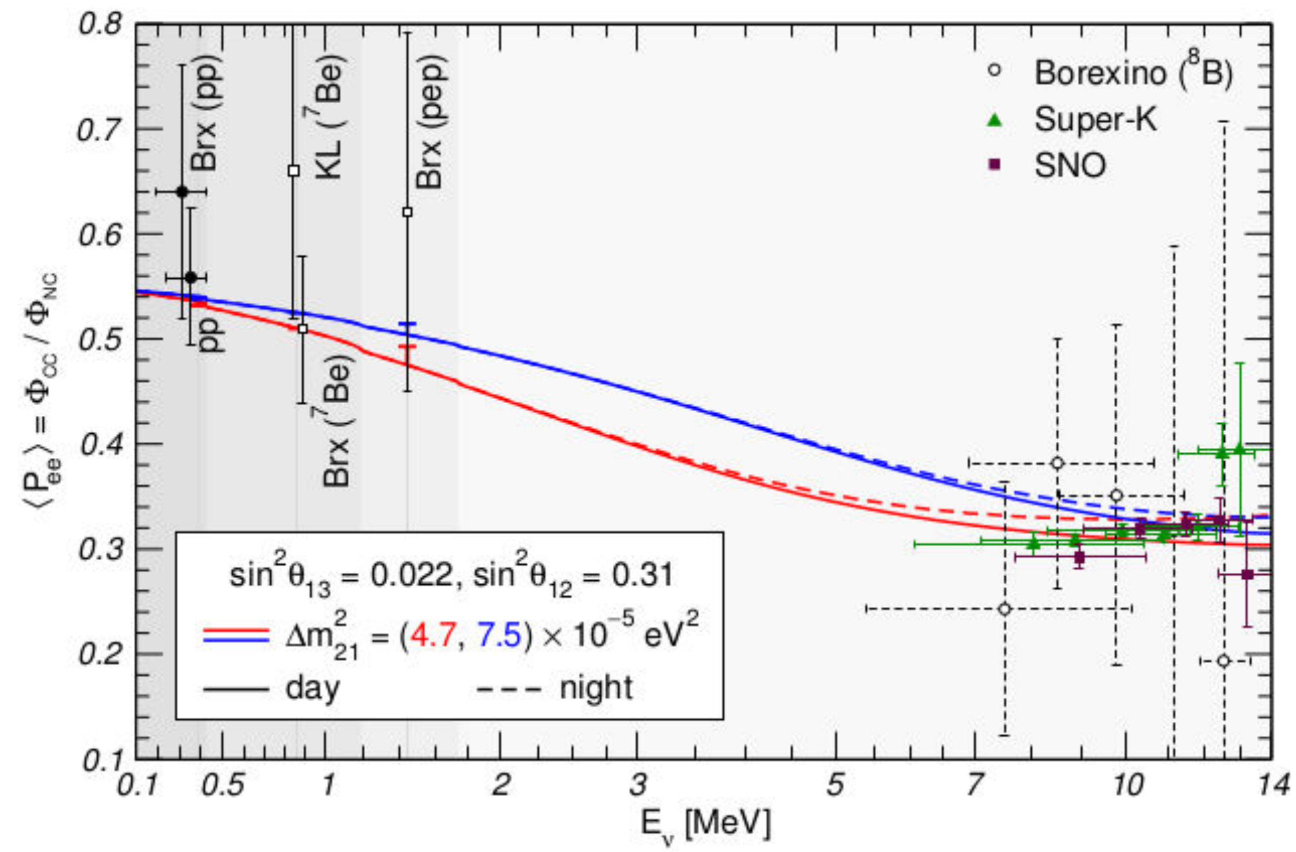
$> 90\% \ \nu_2$

matter effect

D/N asymmetry + lack of observation of upturn.



Phys. Rev. D94, 052010 (2016)



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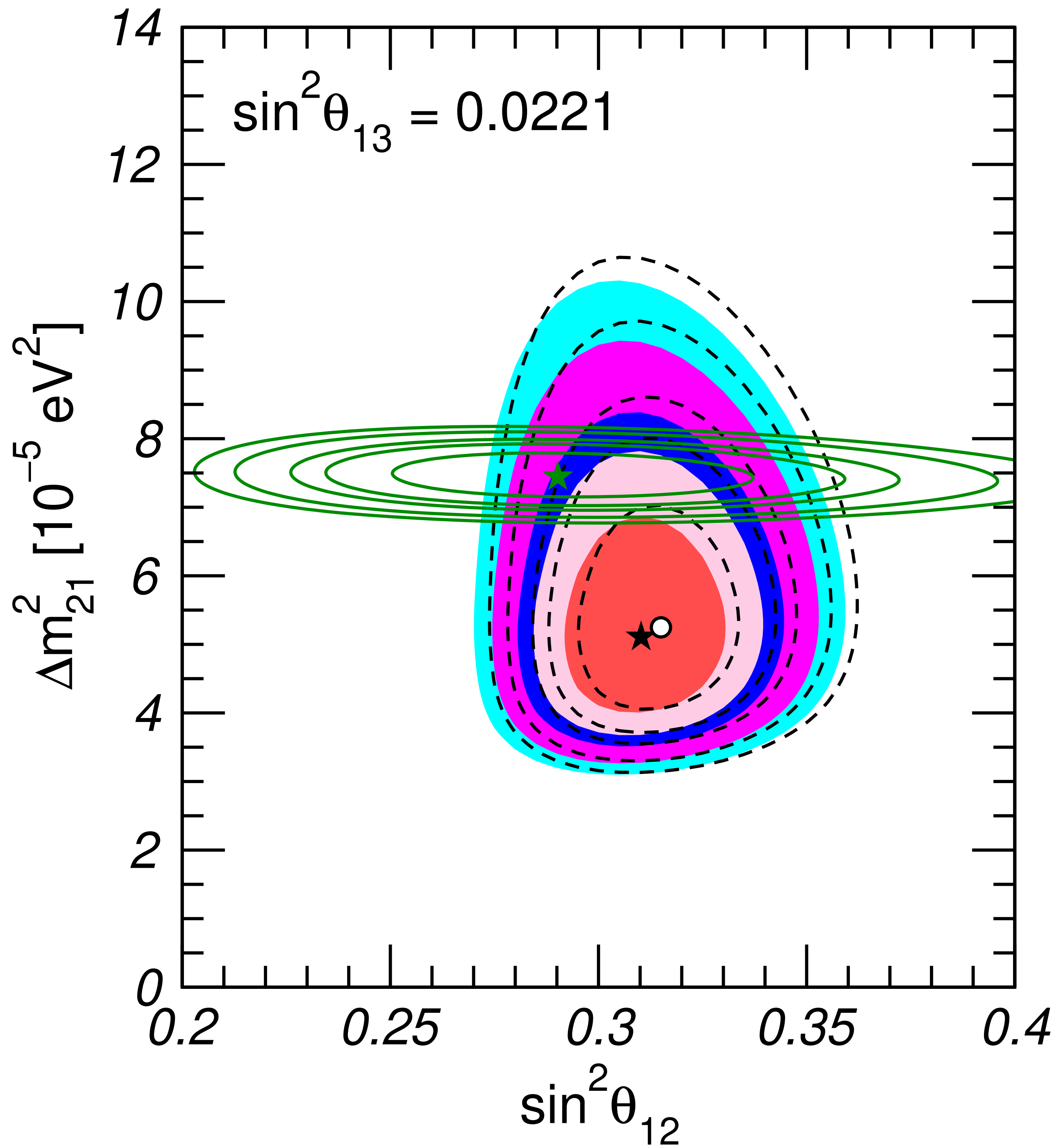
$$\Delta m^2_{21} = 5.1\,^{+1.3}_{-1.0} \times 10^{-5}\ \mathrm{eV}^2,$$

Tension between KamLAND and SNO/SK



1σ, 90%, 2σ, 99%, 3σ CL for 2 dof)

Nu-fit

Why do we care about $$\Delta m^2_{21}$$

CP Violation:

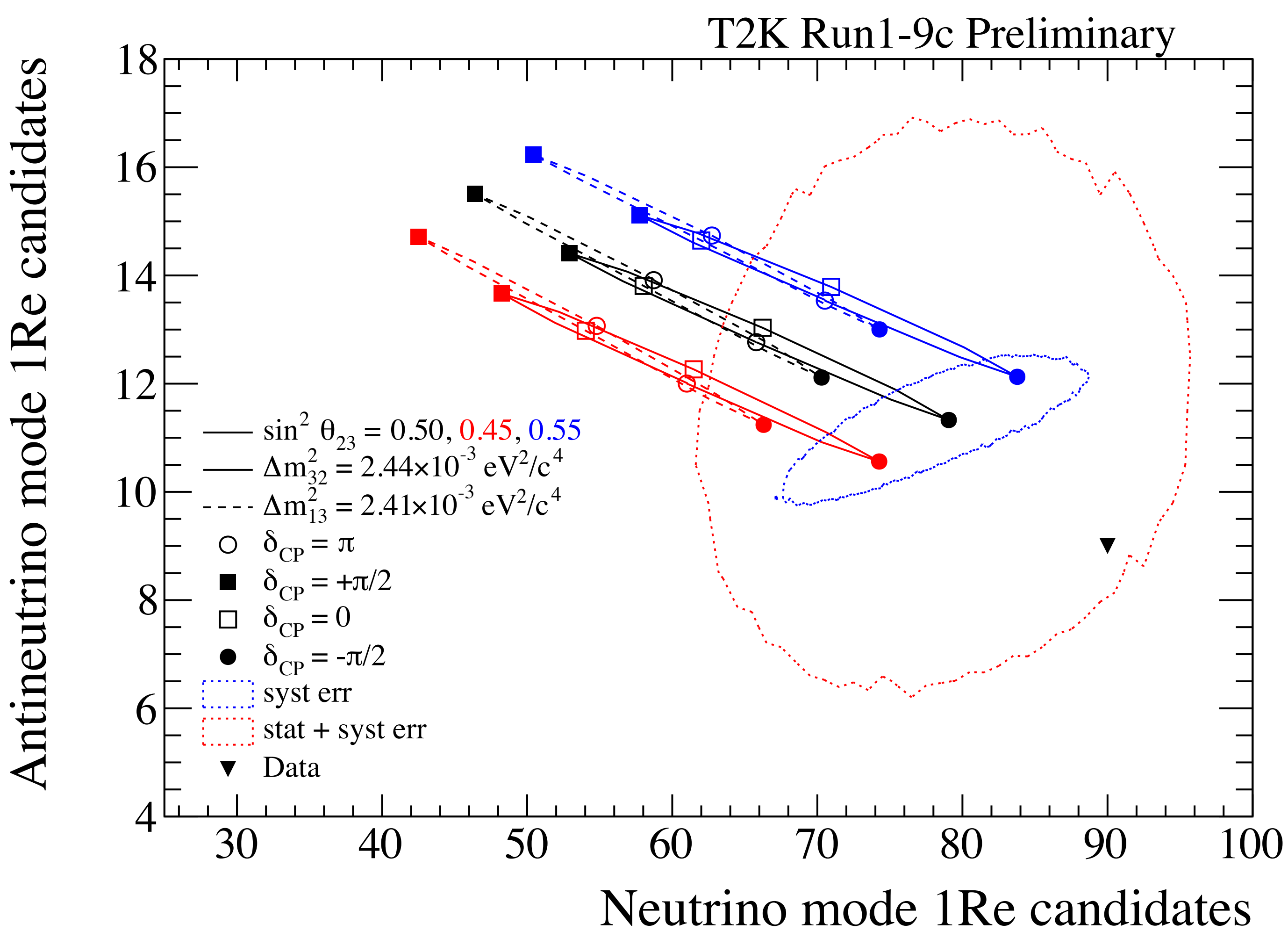
At oscillation maximum in vacuum:

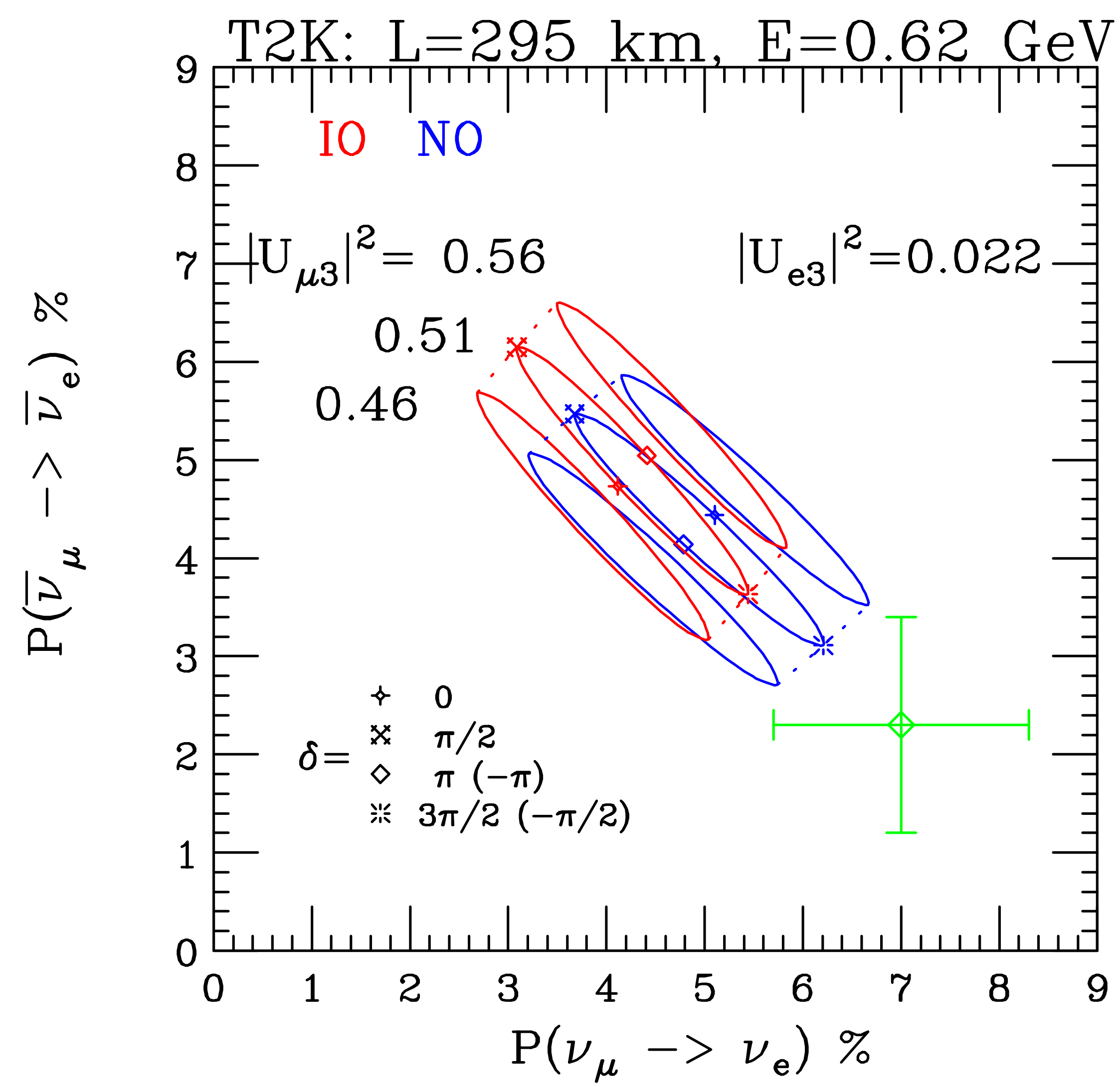
$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) \approx \pi J \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}} \right)$$

where J is Jarlskog Invariant (1986):

$$J = \sin 2\theta_{12} \sin 2\theta_{13} \cos\theta_{13} \sin 2\theta_{23} \sin\delta \approx 0.3 \sin\delta$$

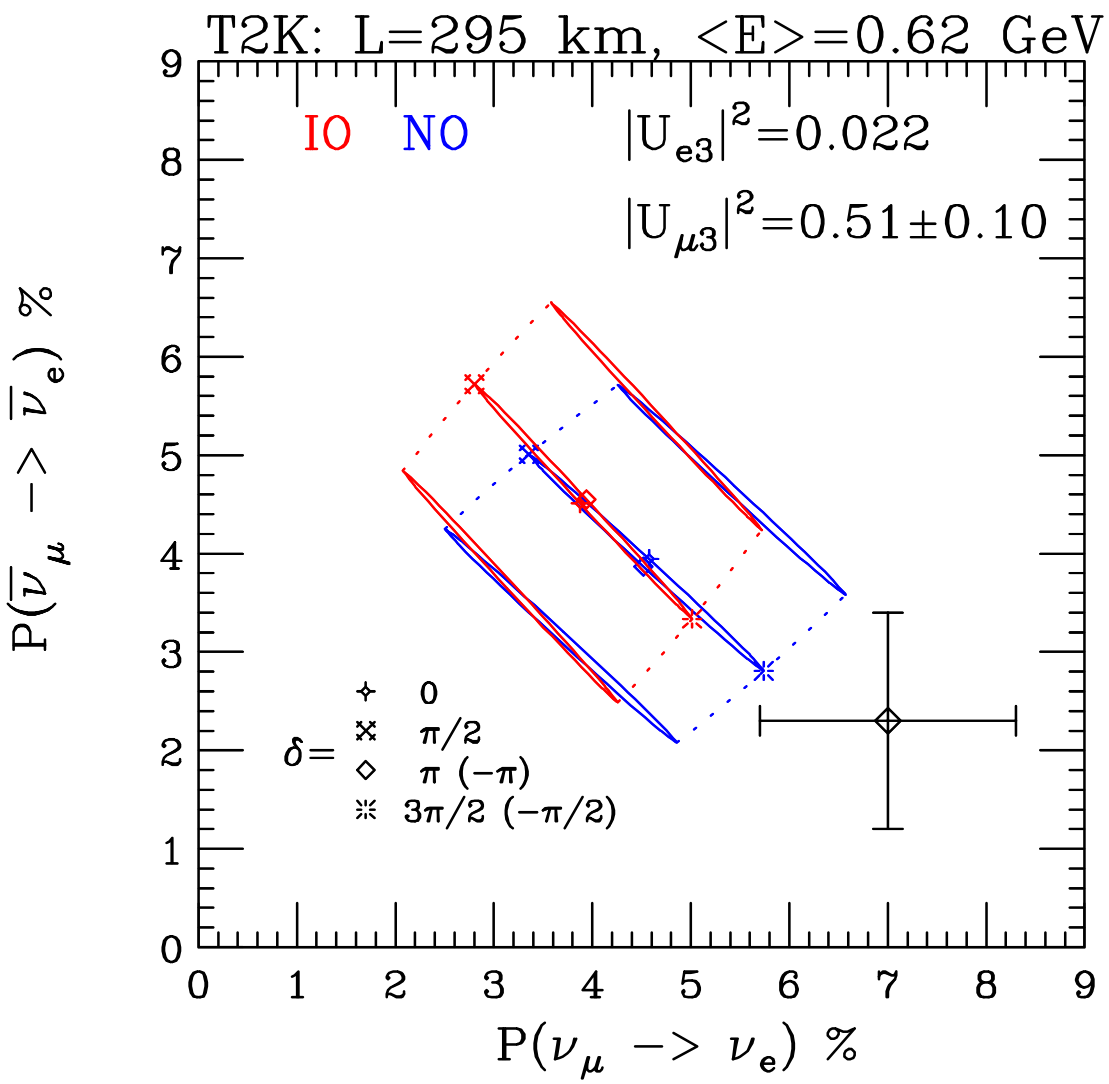
T2K



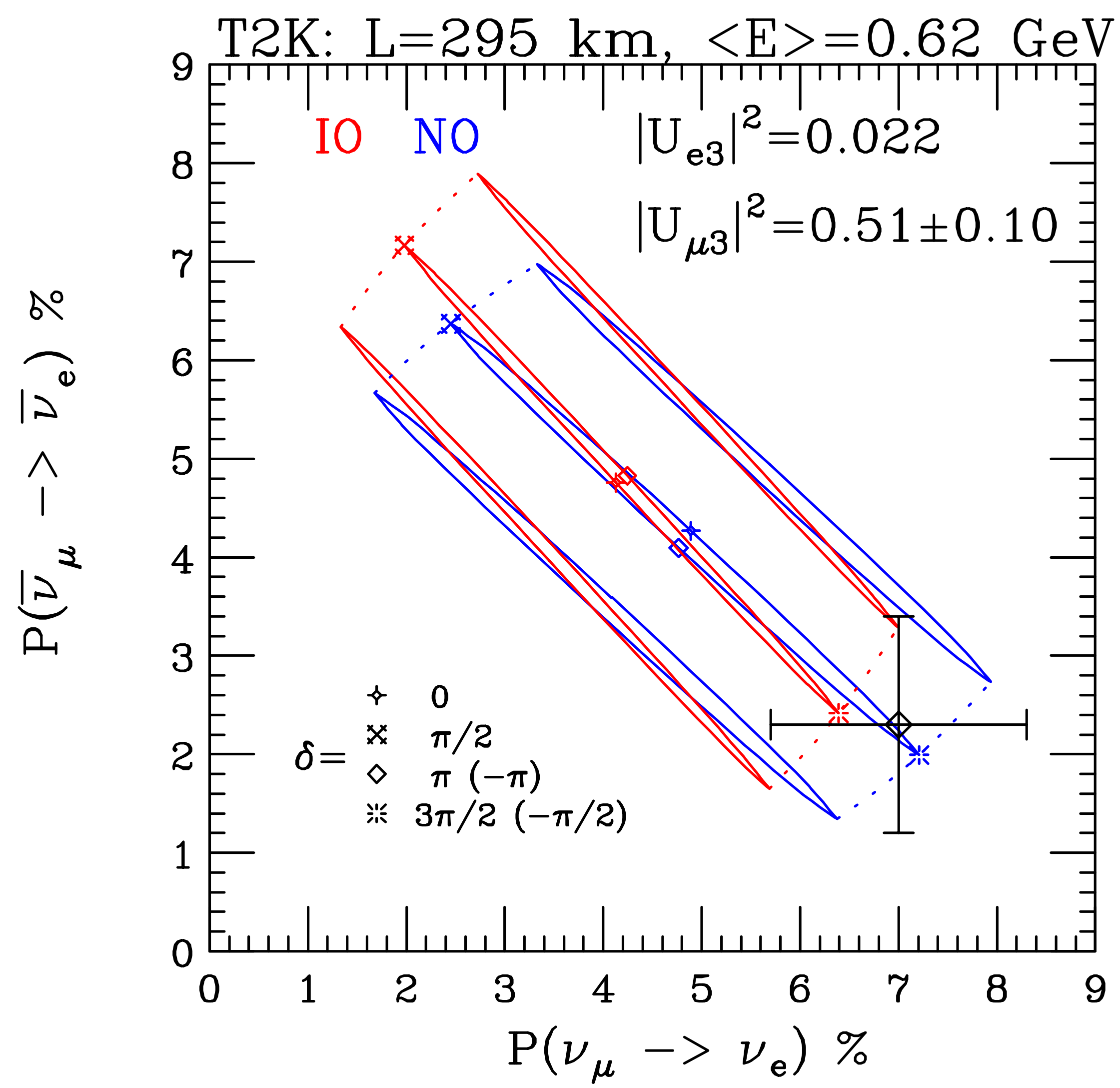


bi-event:

bi-probability:



KamLAND
Δm^2_{21}



2 x KamLAND
Δm^2_{21}

Can Short Baseline Reactor Neutrinos
say anything about

$$\Delta m^2_{21}$$

$$P(\bar\nu_e \to \bar\nu_e) = 1 - P_{13} - P_{12} \quad \text{with} \qquad \Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

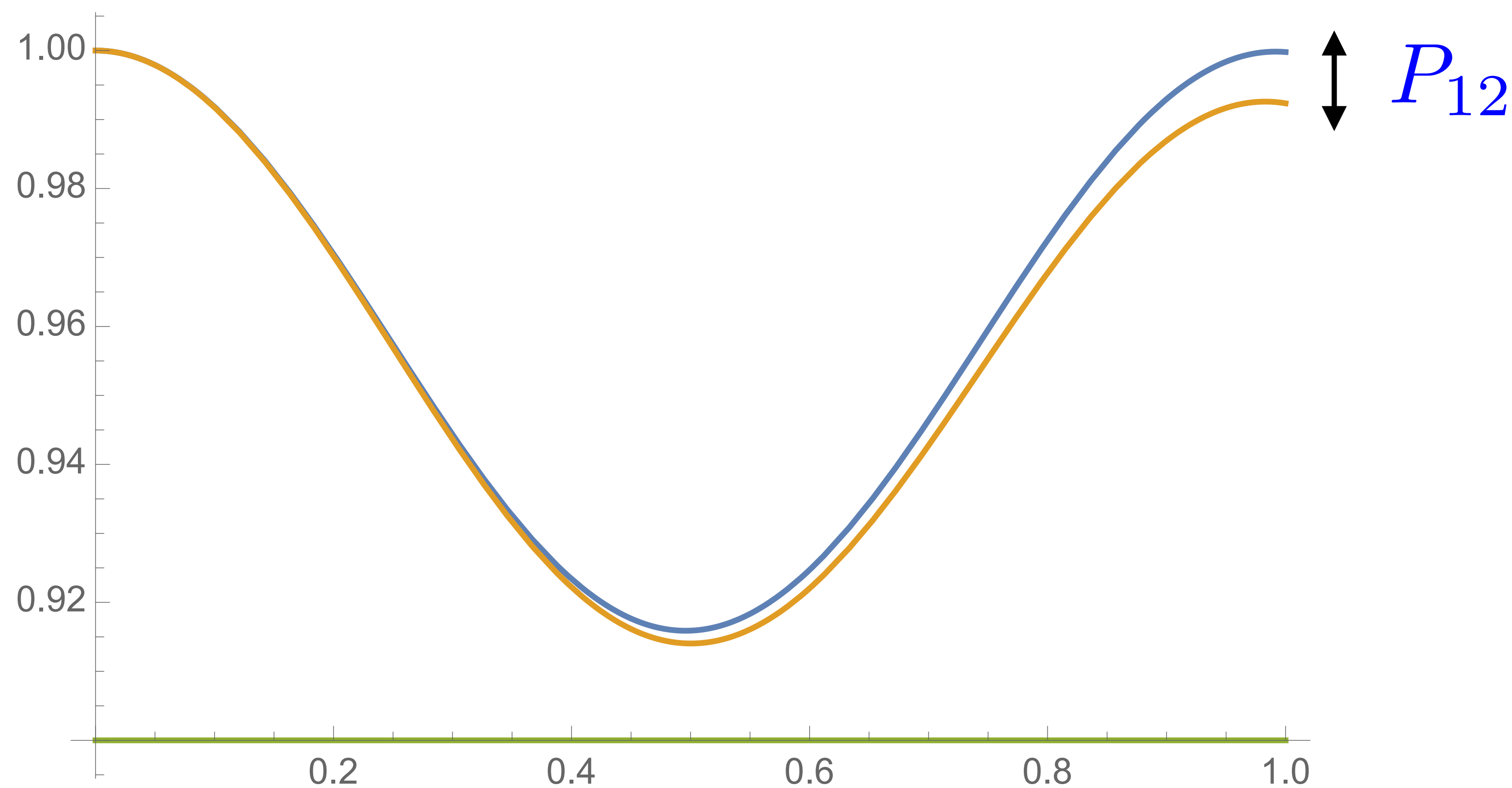
$$P_{13} = 4|U_{e3}|^2(|U_{e1}|^2 \sin^2\Delta_{31} + |U_{e2}|^2 \sin^2\Delta_{32}) \quad (< 0.1)$$

$$= \sin^2 2\theta_{13}\,(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}),$$

$$P_{12} = 4|U_{e2}|^2|U_{e1}|^2 \sin^2\Delta_{21} = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

(< 0.01)

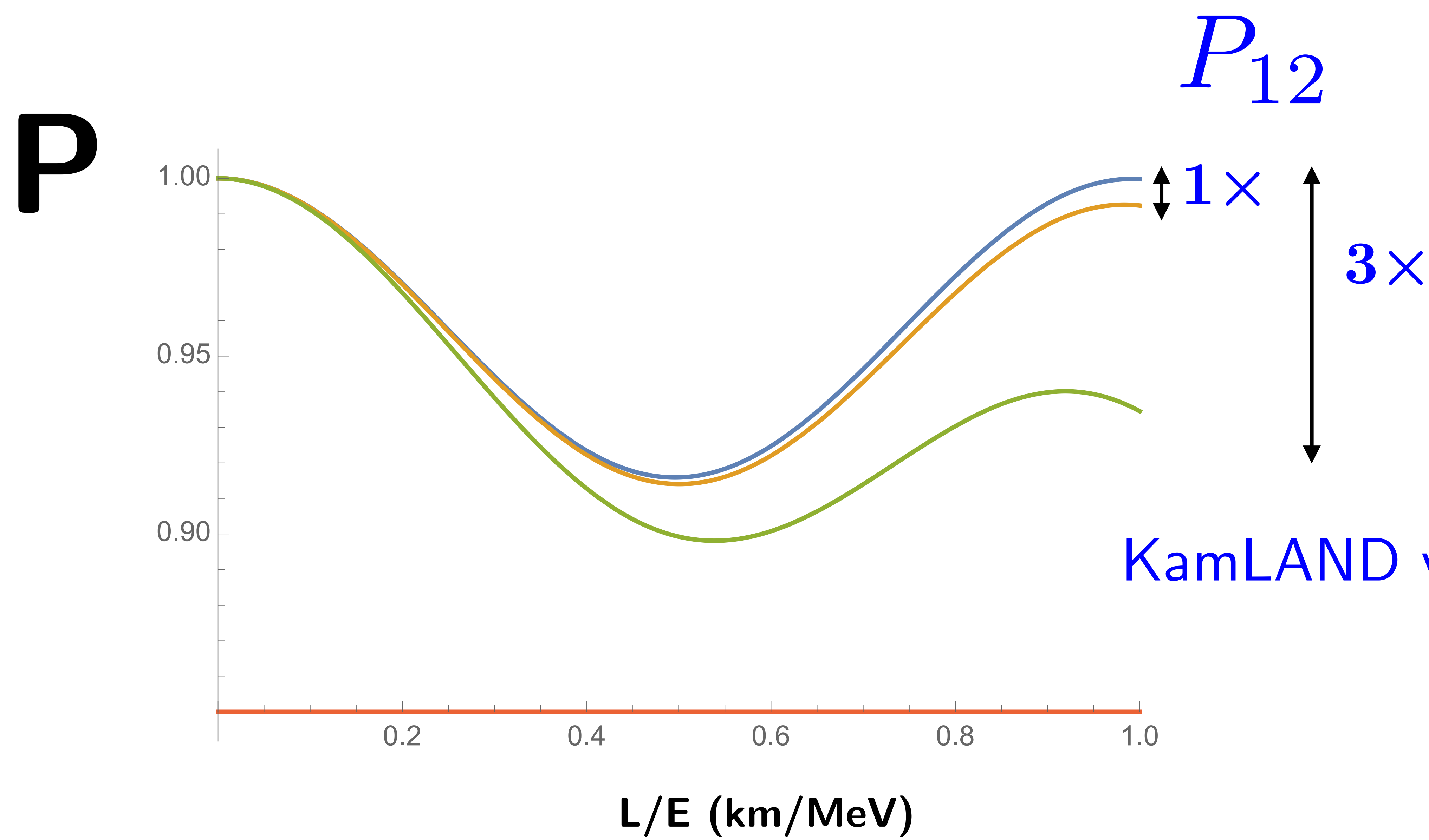
P



L/E (km/MeV)

Dependence on Solar Parameters:

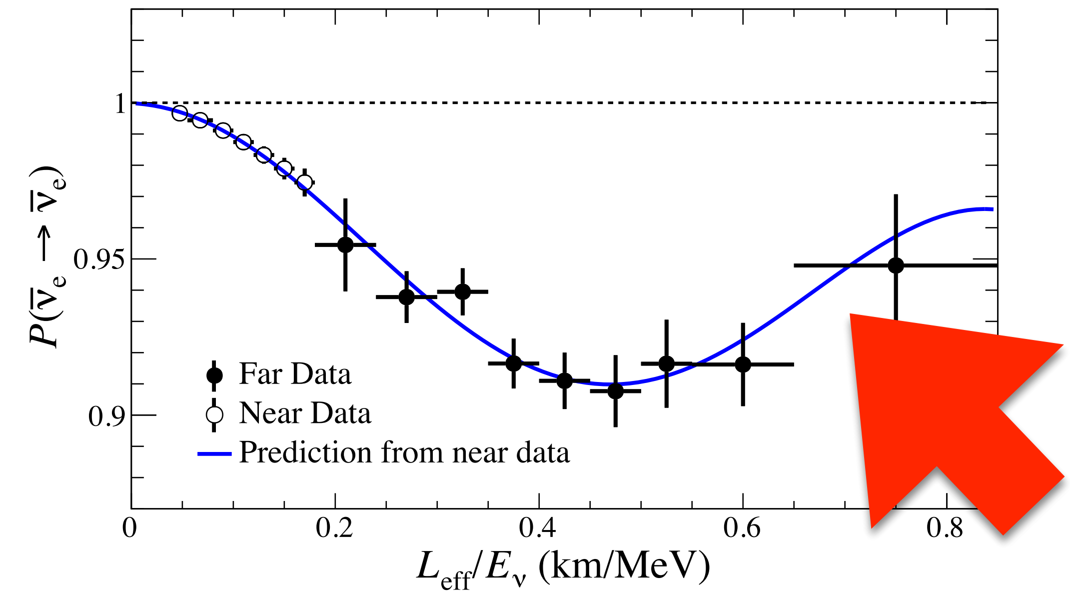
S.H. Seo and SP arXiv:1808.09150

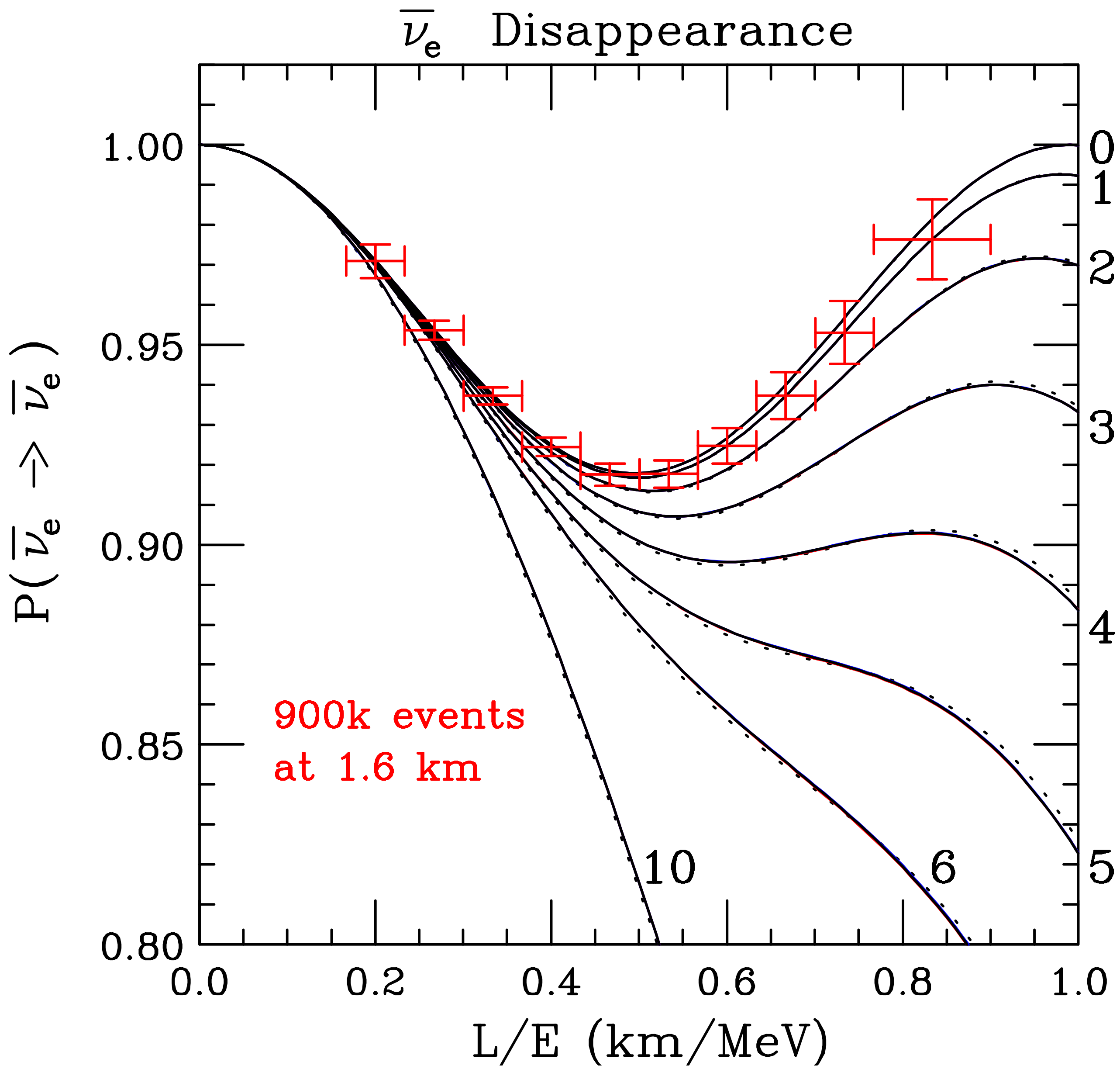


If Δm^2_{21} is 3 times bigger, P_{12} is 9 times larger !

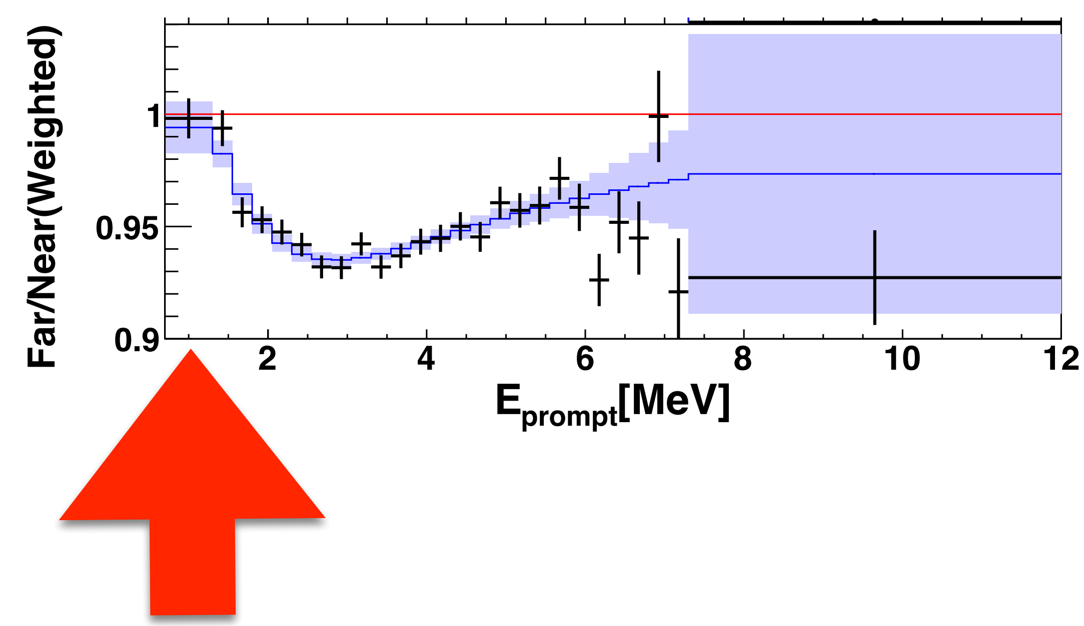
dependence is on $\sin 2\theta_{12}\Delta m^2_{21}$

RENO





Daya Bay

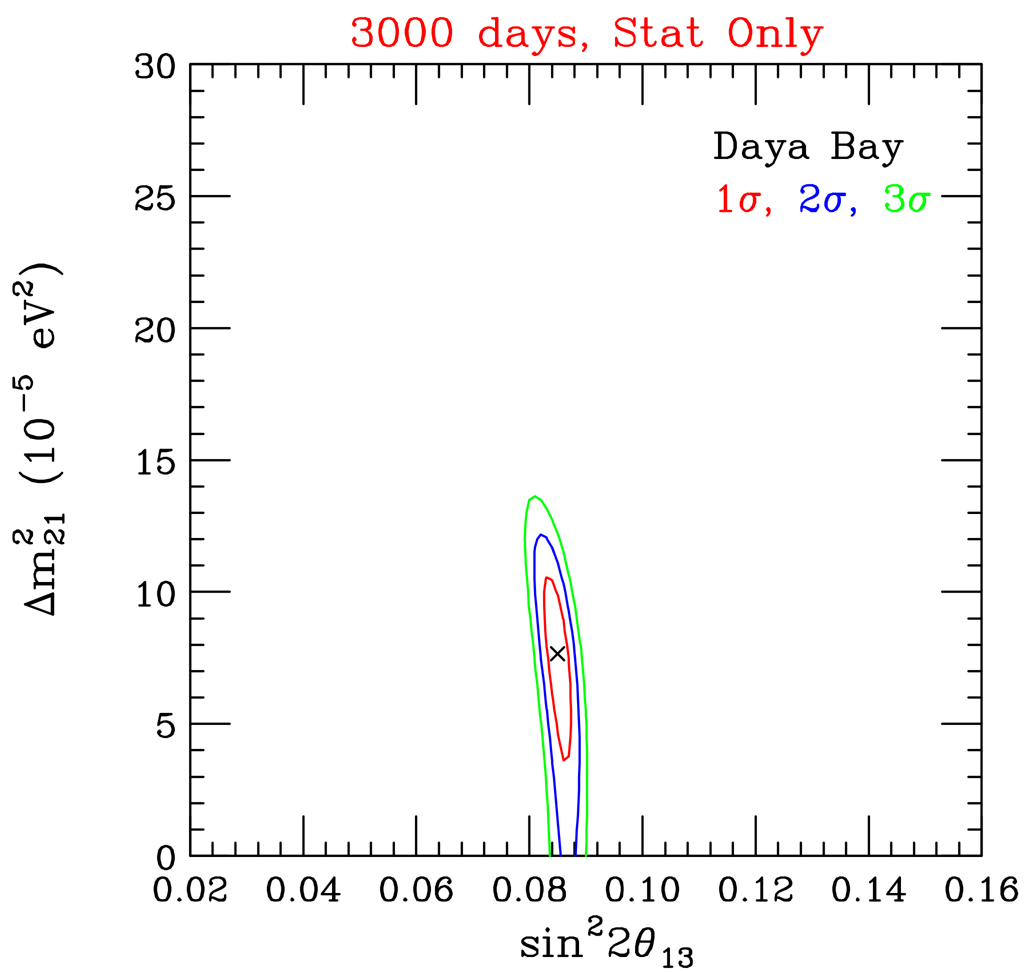


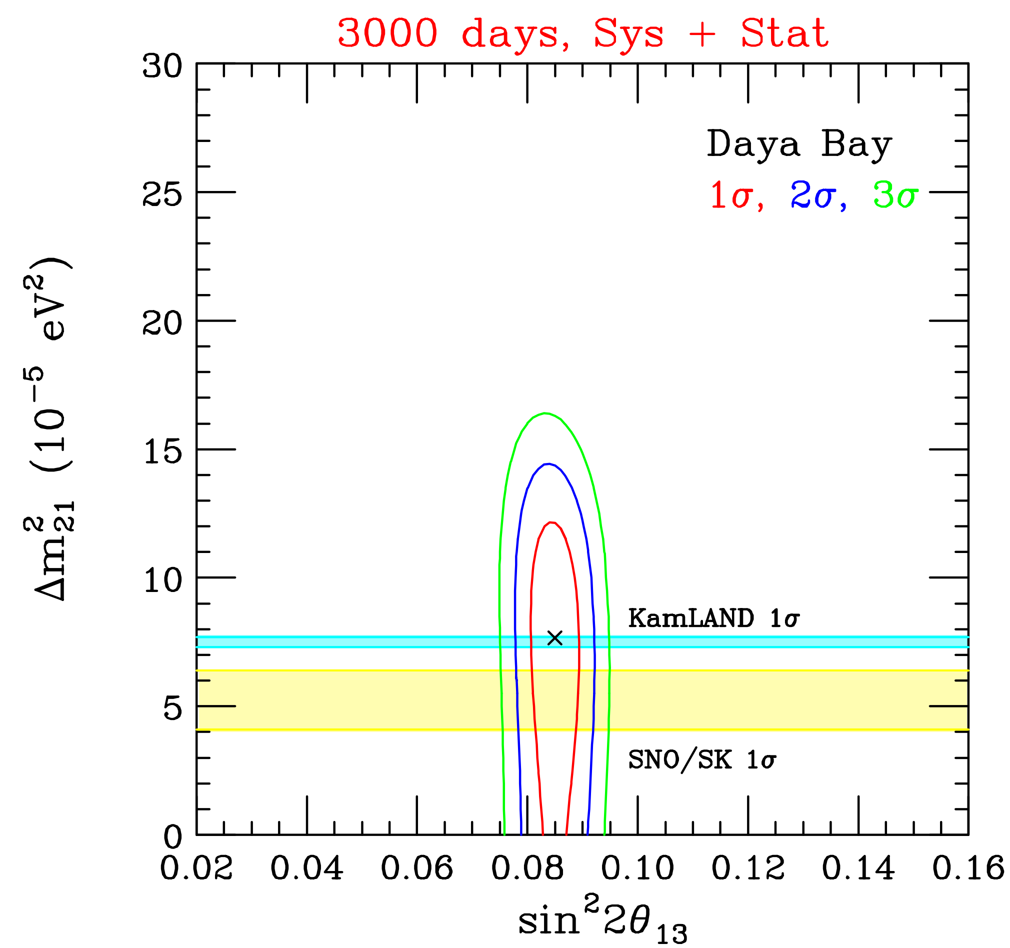
1 - 10 KamLAND

$$\Delta m^2_{21}$$

Simulation

S.H. Seo and SP arXiv:1808.09150





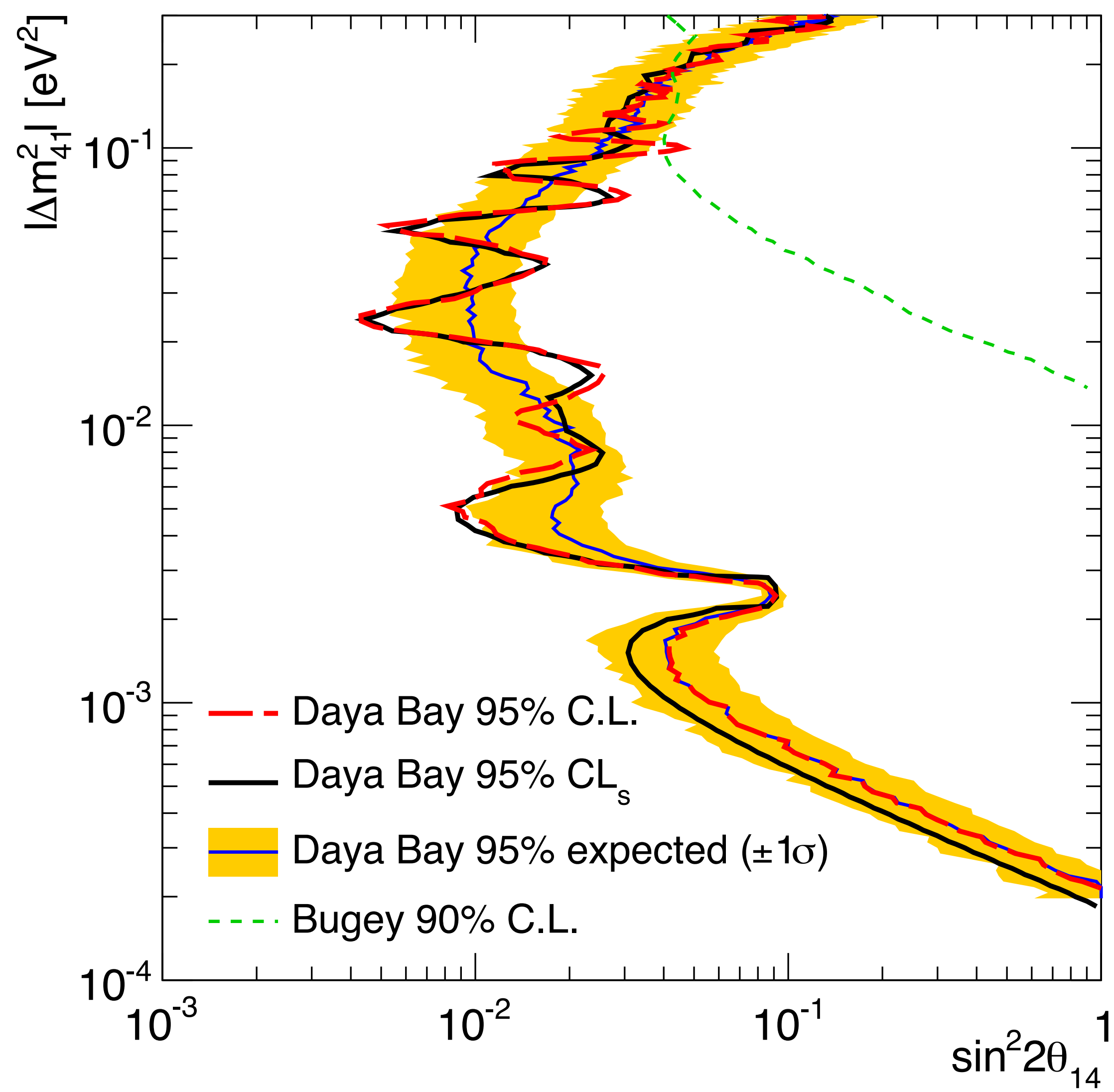
$L/E \sim 0.5$ km/MeV compared to KamLAND $L/E \sim 50$ km/MeV

T2K is at 0.5 km/MeV

Daya Bay
Sterile Neutrino Search

$$\approx 1 - \sin^2 2\theta_{14} \sin^2 \Delta_{41} - \sin^2 2\theta_{13} \sin^2 \Delta_{31}.$$

1607.01174
404 days



$\mathbf{3}\times$

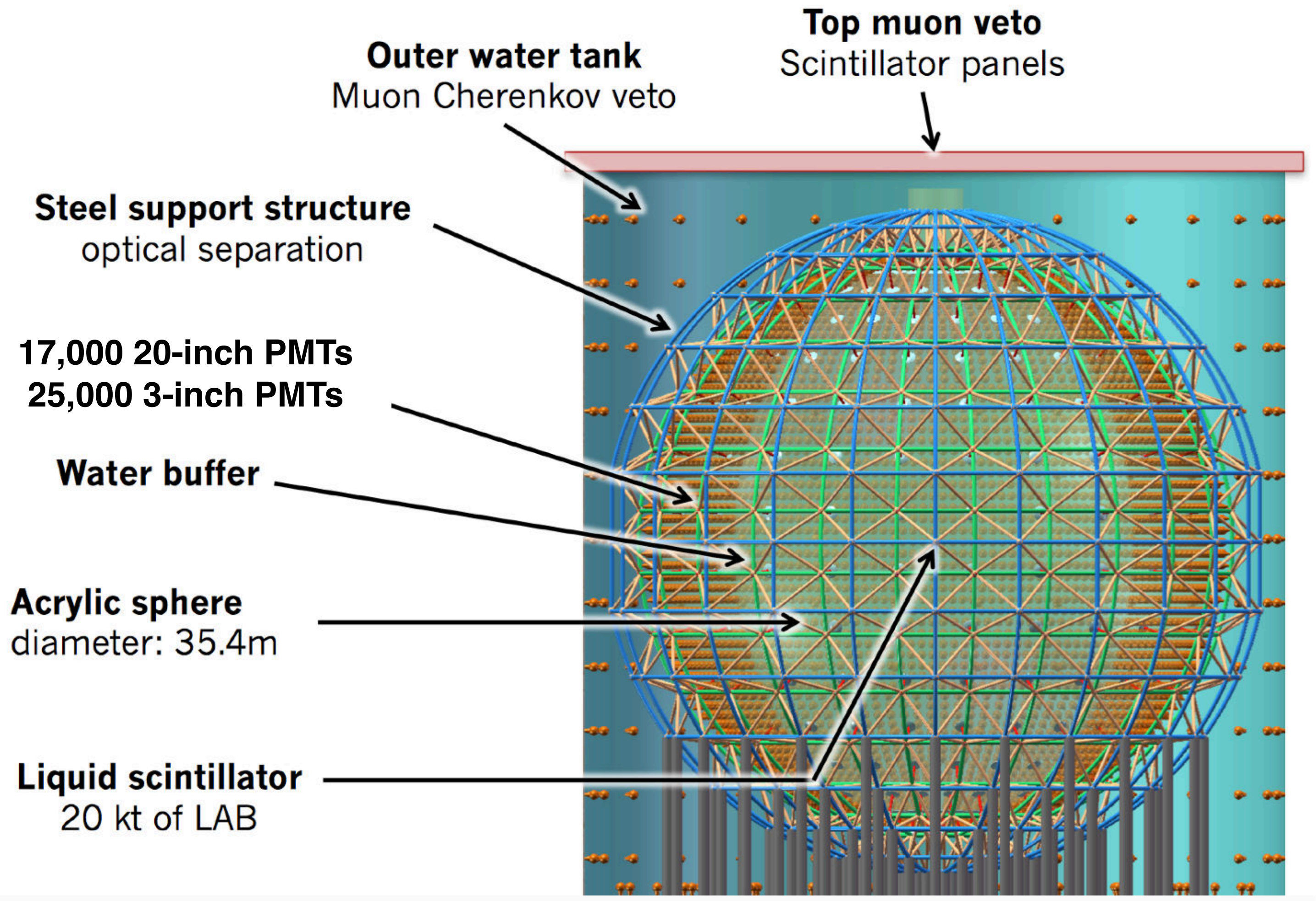
KamLAND value

Reinterpretation !

JUNO

JUNO

LS Detectors	Daya Bay	Borexino	KamLAND	JUNO
Target Mass	**20 t x 8**	**300 t**	**1 kt**	**20 kt**



Similar in concept to previous LS experiments, but much LARGER

In fact, JUNO will be the largest liquid scintillator (LS) detector so far in history!

JUNO precision ~2025

$$\sin^2\theta_{12},\ \Delta m^2_{21}\ \text{and}\ |\Delta m^2_{ee}|$$

0.5%

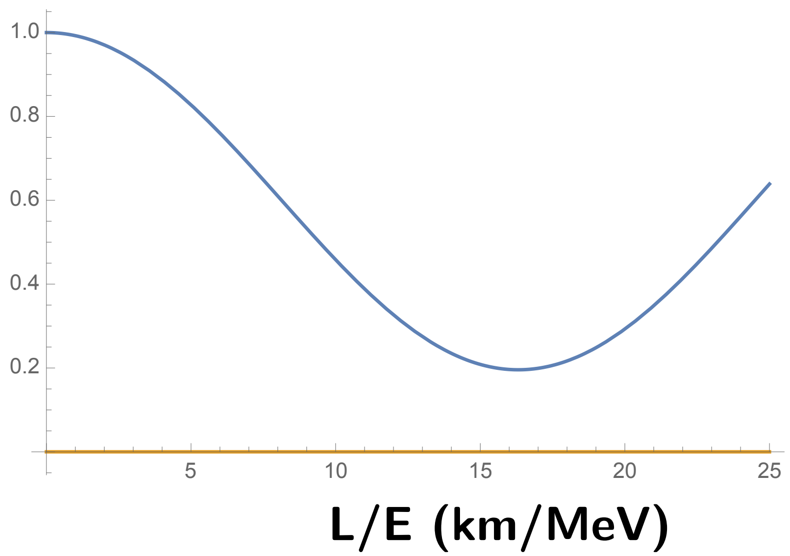
	Nominal	+ B2B (1%)	+ BG	+ EL (1%)	+ NL (1%)
$\sin^2\theta_{12}$	0.54%	0.60%	0.62%	0.64%	0.67%
Δm^2_{21}	0.24%	0.27%	0.29%	0.44%	0.59%
$\lvert\Delta m^2_{ee}\rvert$	0.27%	0.31%	0.31%	0.35%	0.44%

Table 3-2: Precision of $\sin^2\theta_{12}$, Δm^2_{21} and $|\Delta m^2_{ee}|$ from the nominal setup to those including additional systematic uncertainties. The systematics are added one by one from left to right.

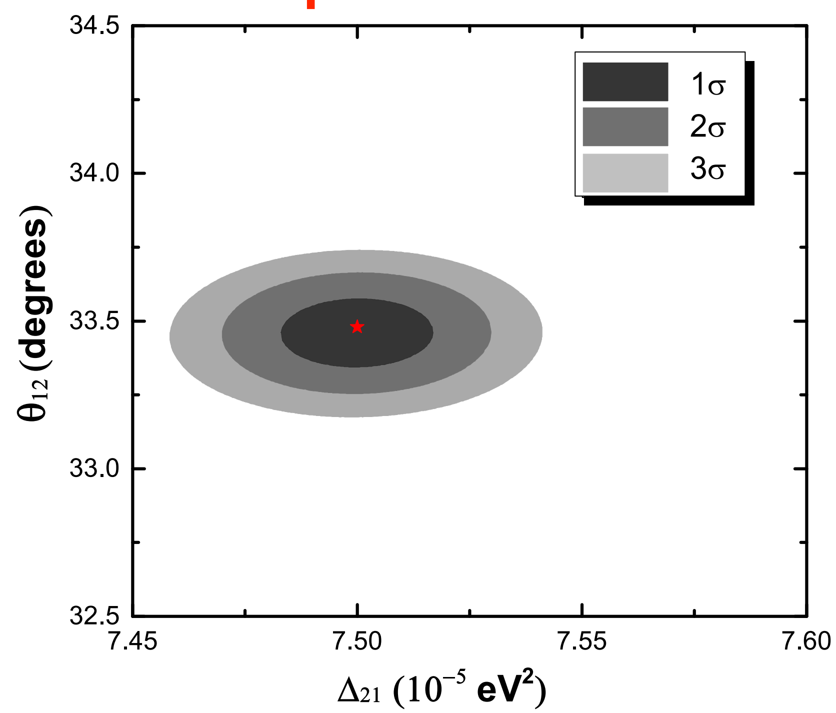
Matter Effects in JUNO

Li, Wang, Xing 1605.00900

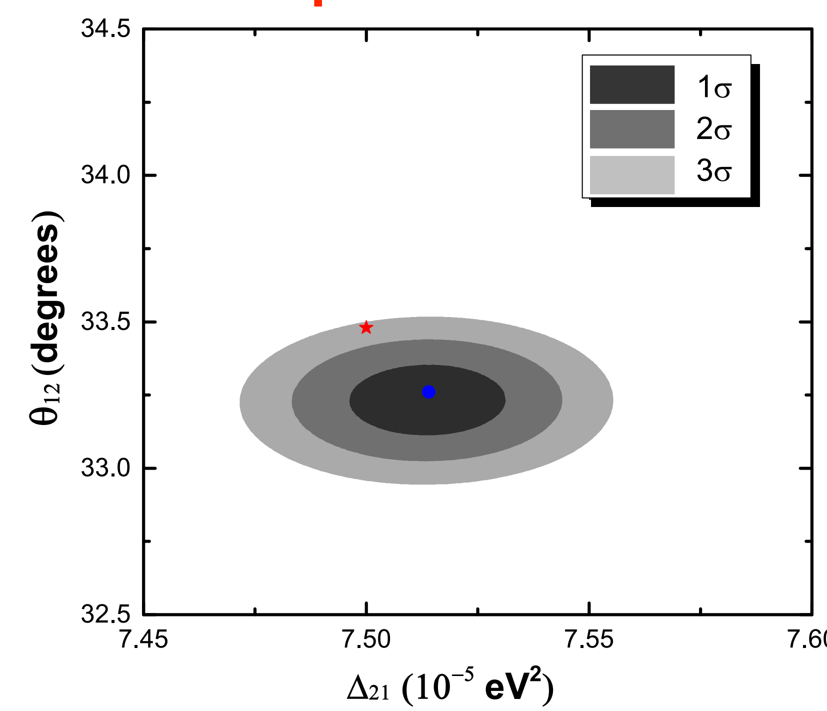
$1 - P_{12}$



Matter Input/Matter Fit



Matter Input/Vacuum Fit

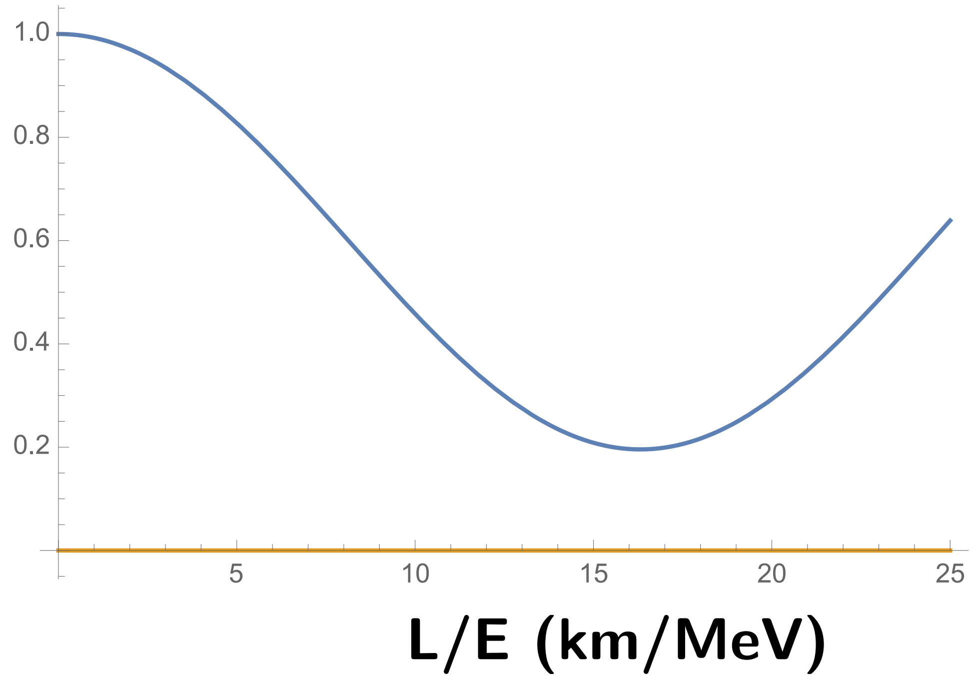


Shift 1σ in Δm^2_{21} and 3σ in θ_{12}

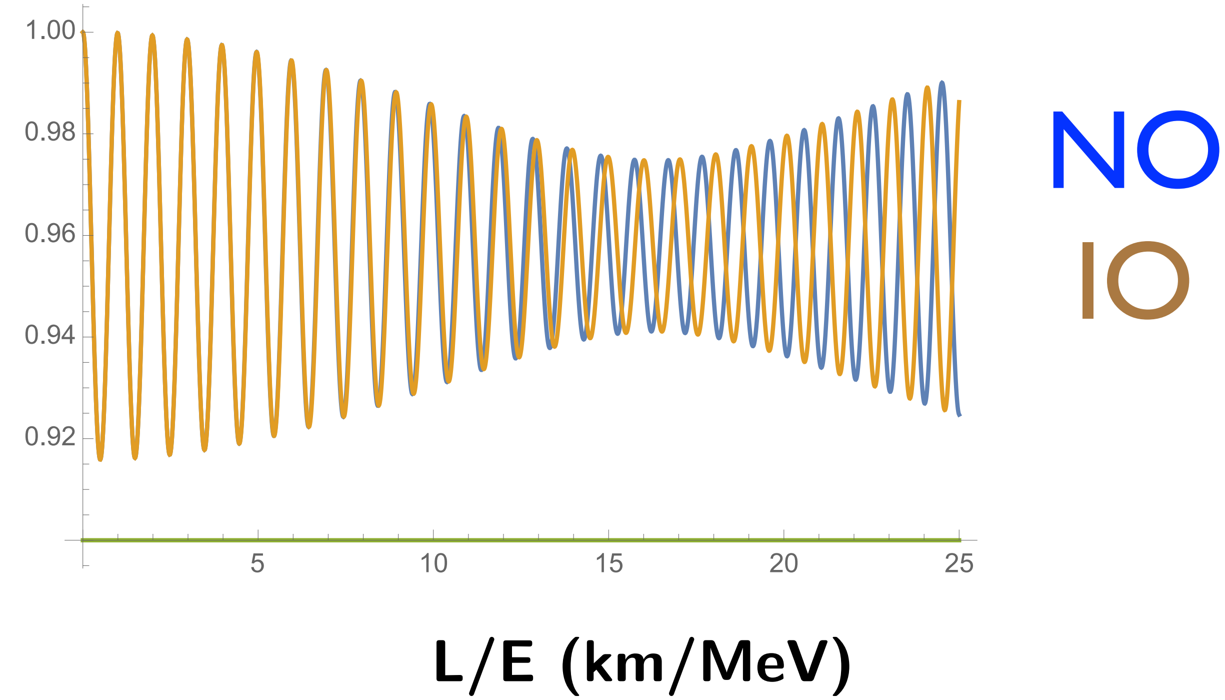
Size of shift unexplained in 1605.00900:

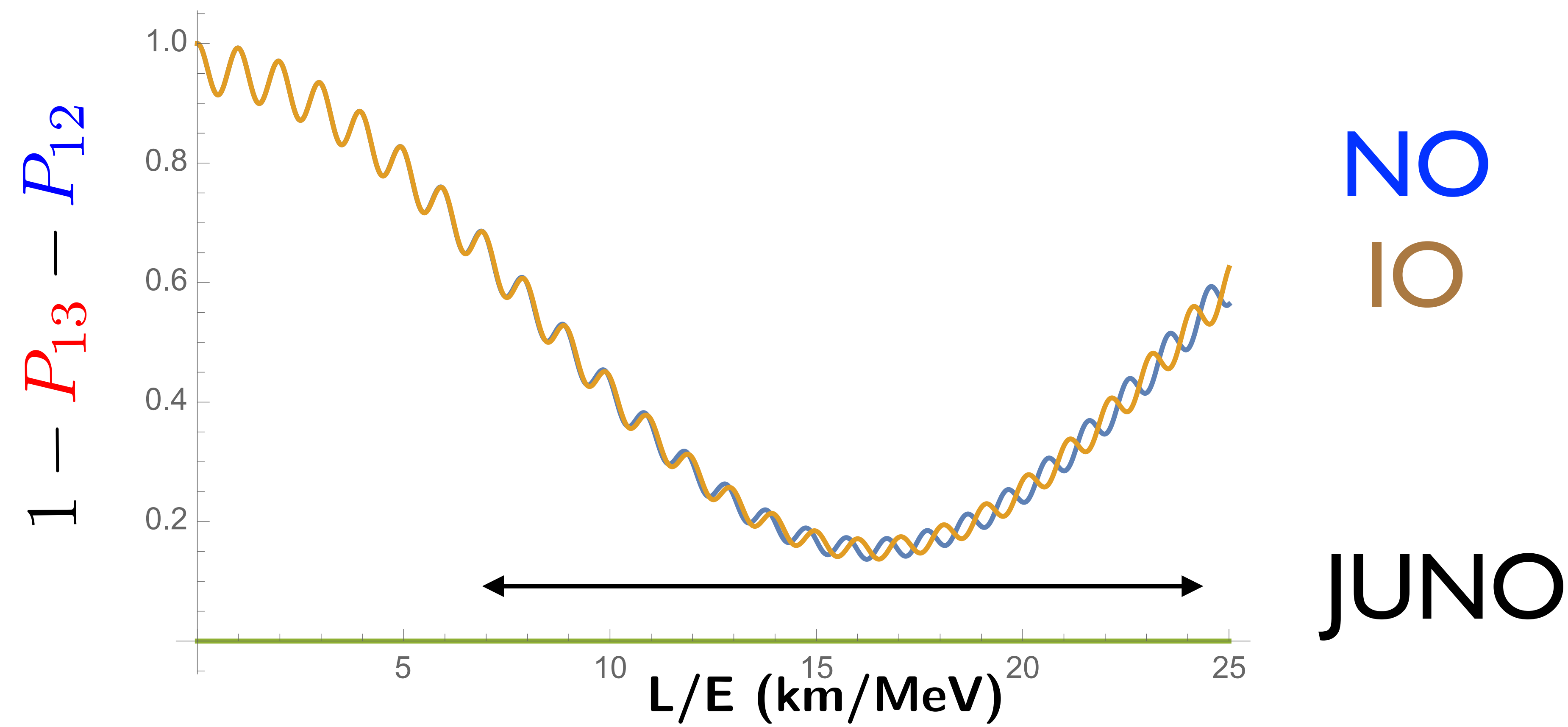
Khan, Nunokawa, SP upcoming 1810.?? or 1811.??

$1 - P_{12}$



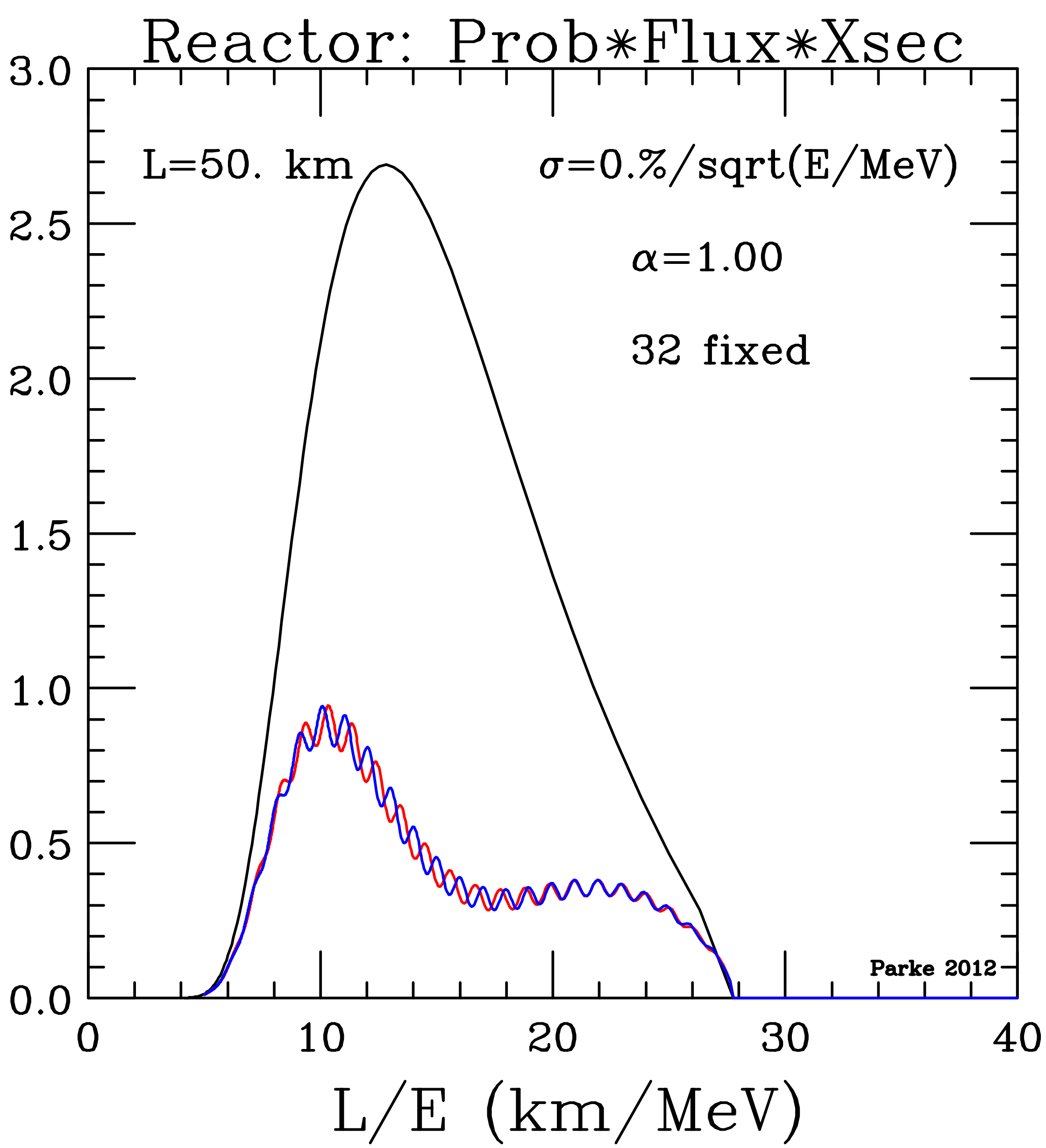
$1 - P_{13}$





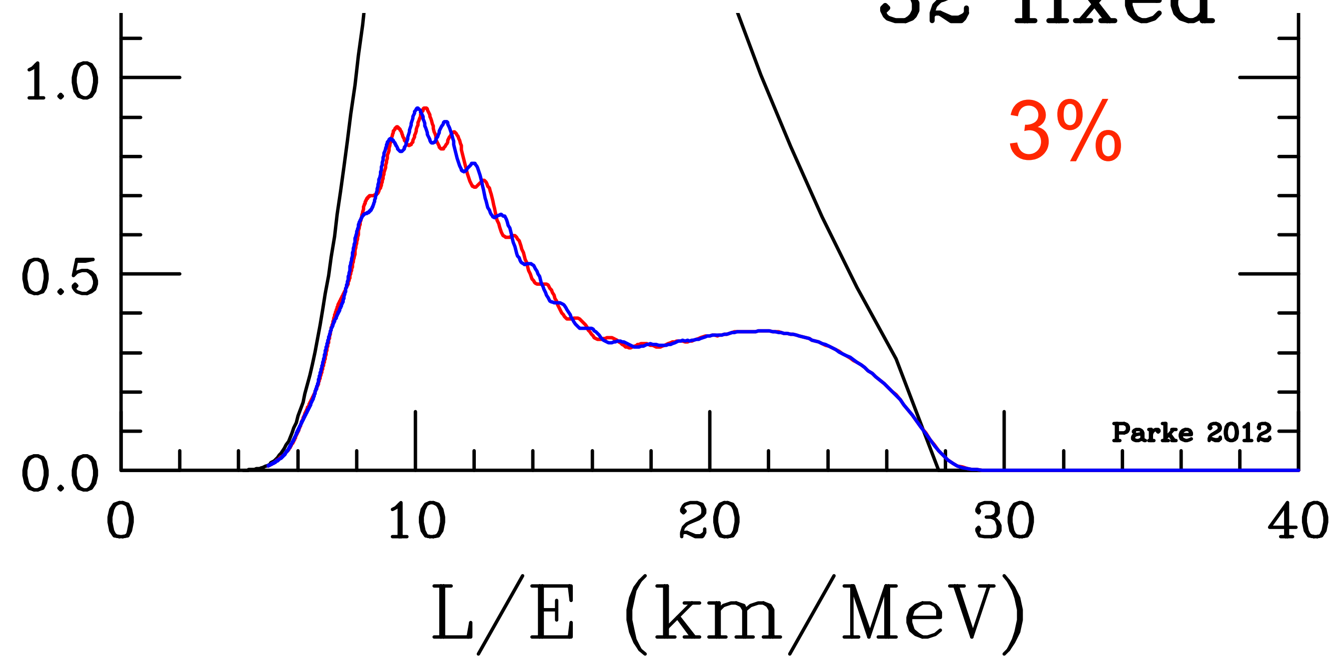
perfect resolution, not $3\%/\sqrt{E/1\ \text{MeV}}$

JUNO and the Mass Ordering:



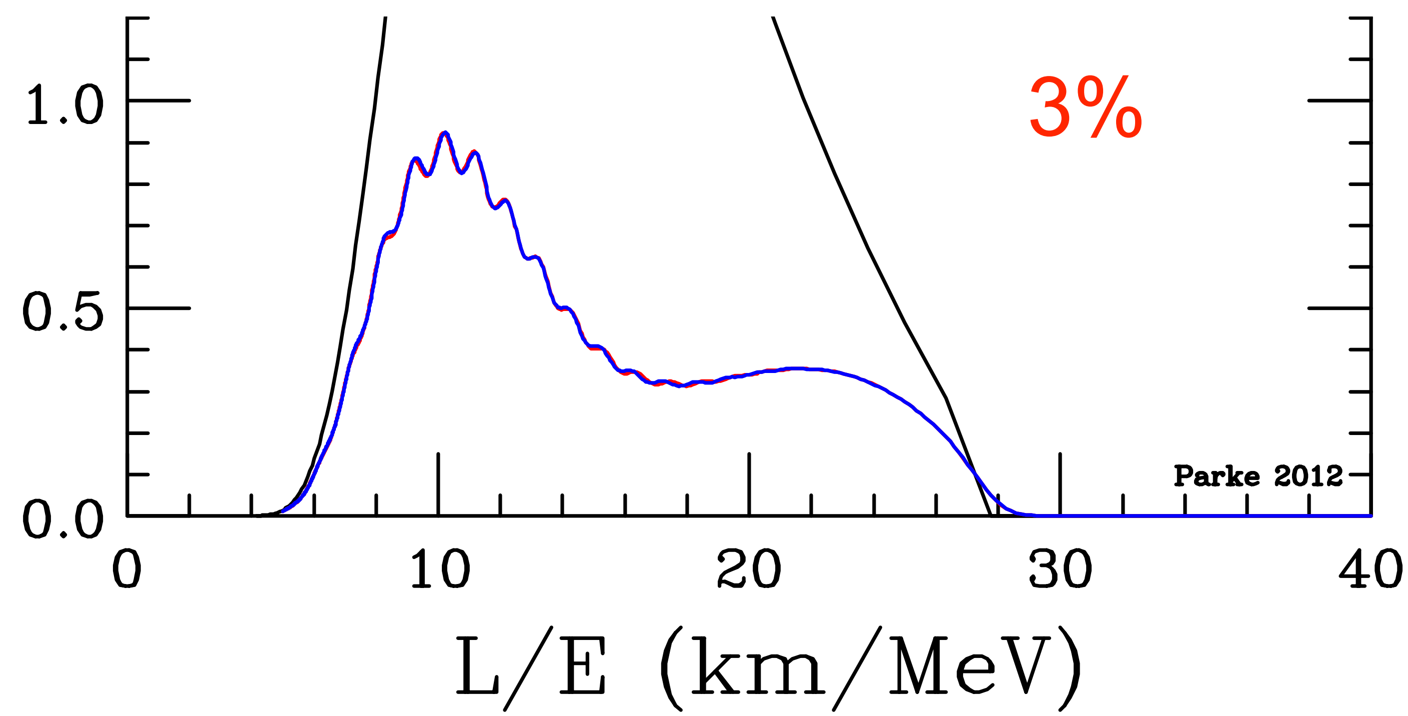
32 fixed

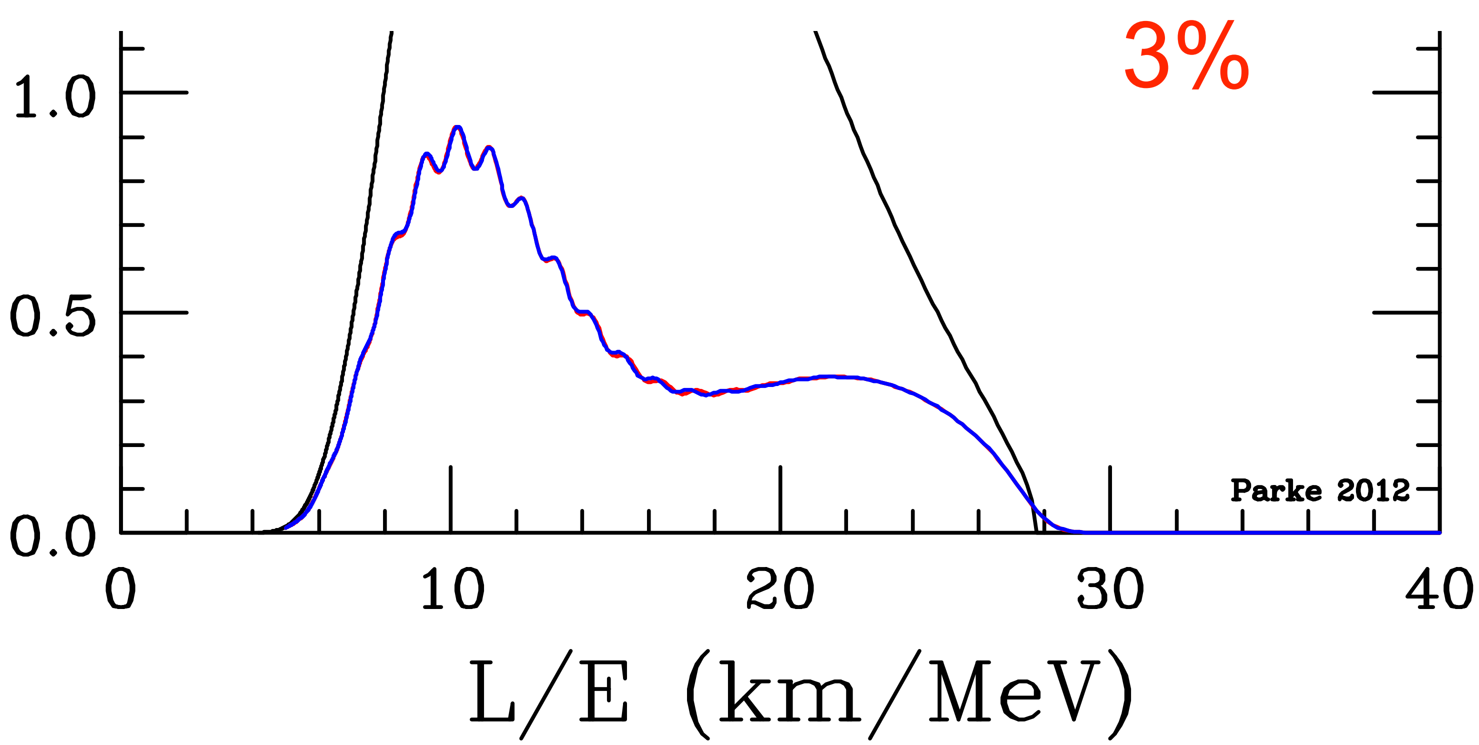
3%

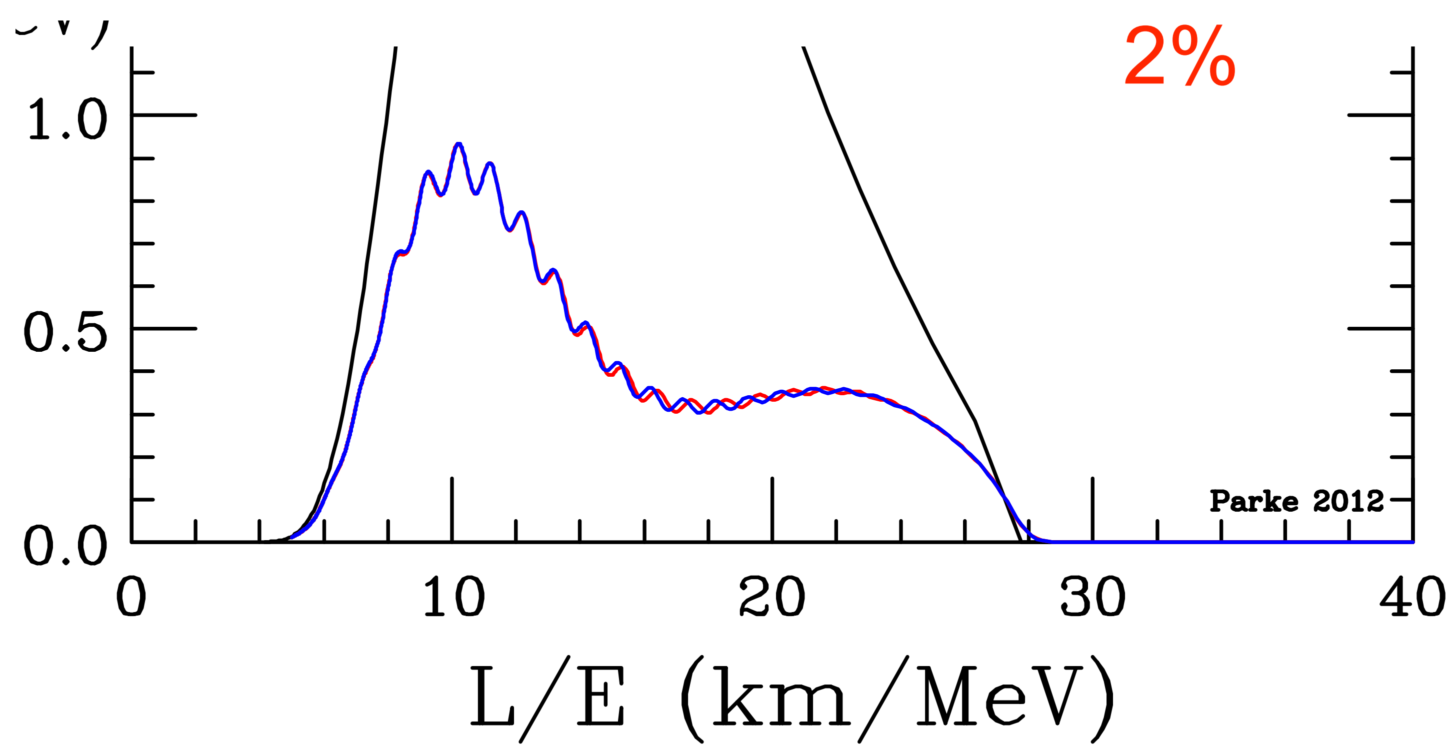


31 fixed

3%




3%
1.0
0.5
0.0
0
10
20
30
40
Parke 2012
L/E (km/MeV)


2%
1.0
0.5
0.0
0
10
20
30
40
Parke 2012
L/E (km/MeV)

$$P_x(\bar\nu_e \to \bar\nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{1-\sin^2 2\theta_{12}\sin^2\Delta_{21}}\;\cos\Omega\right)$$

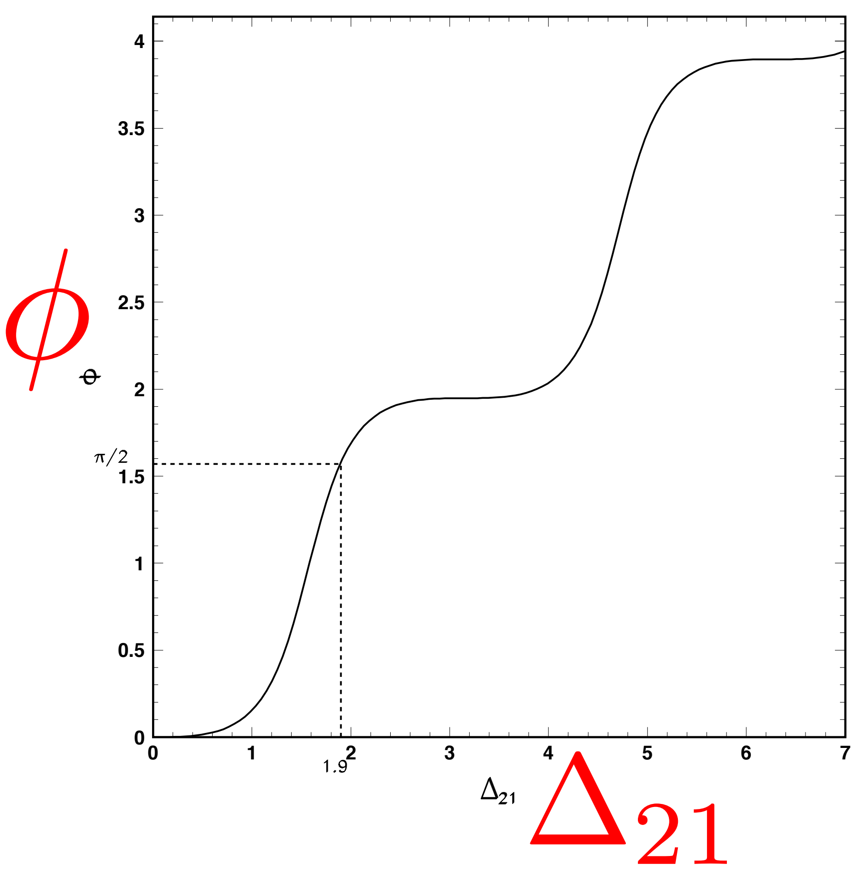
$$\text{with}\quad \Omega = (\Delta_{31}+\Delta_{32}) + \arctan(\cos 2\theta_{12}\tan\Delta_{21}).$$

$$\Omega = 2|\Delta_{ee}| \pm \phi$$

1875

$$\Delta m^2_{ee} \equiv \frac{\partial\,\Omega}{\partial(L/2E)}\bigg|_{\frac{L}{E}\to 0} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

$$\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

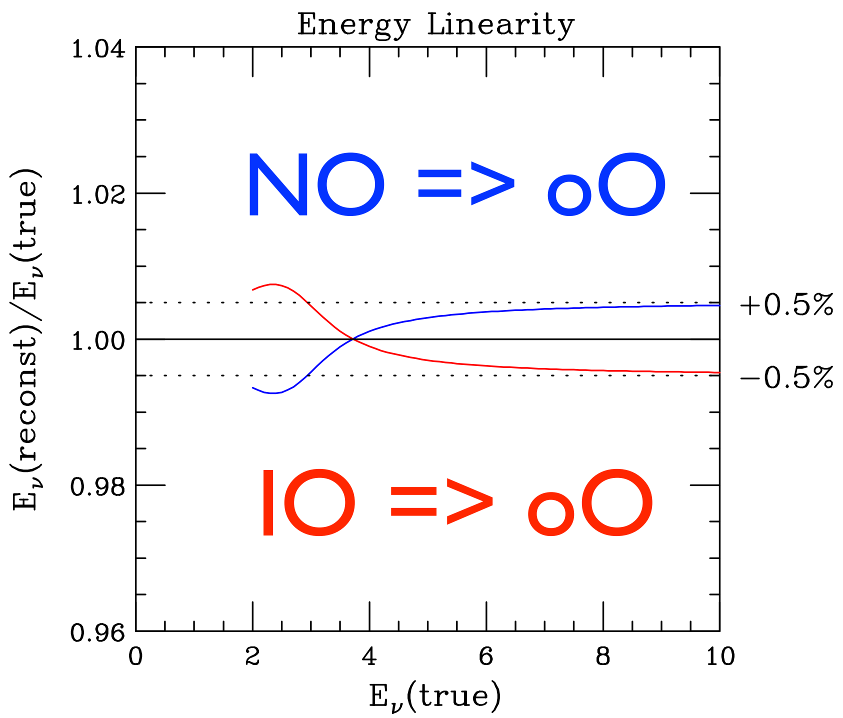


$$\phi(\Delta_{21} \pm \pi) = \phi(\Delta_{21}) \pm 2\pi\sin^2\theta_{12},$$

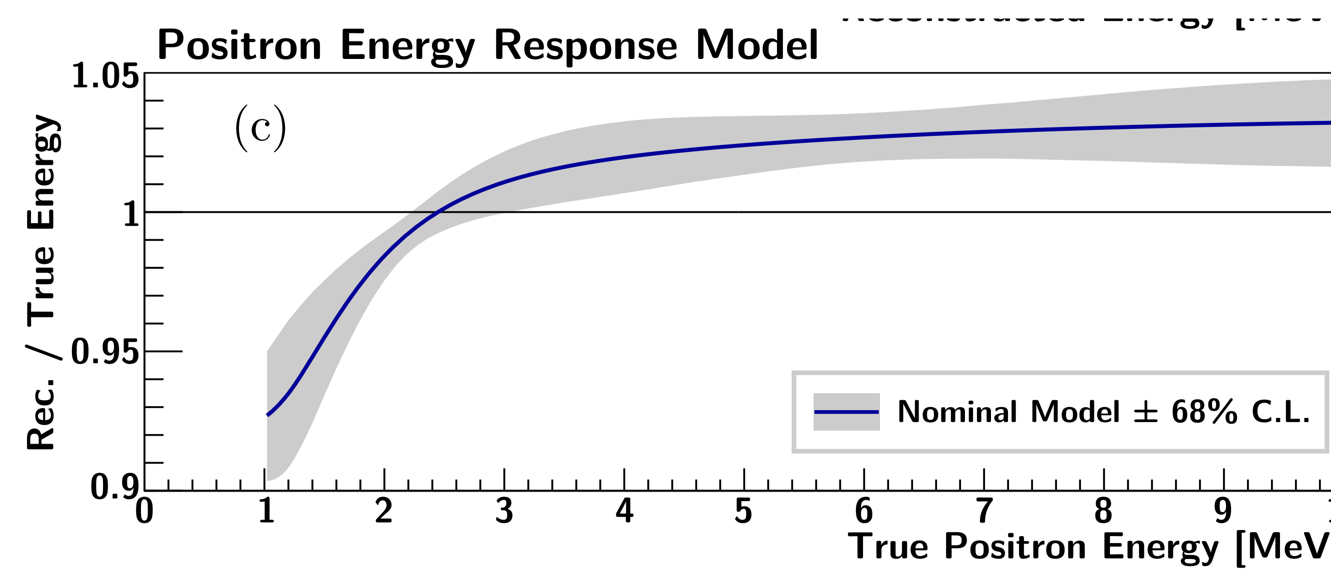
NO phase advance

IO phase retardation

Neutrino Energy Reconstruction



RENO

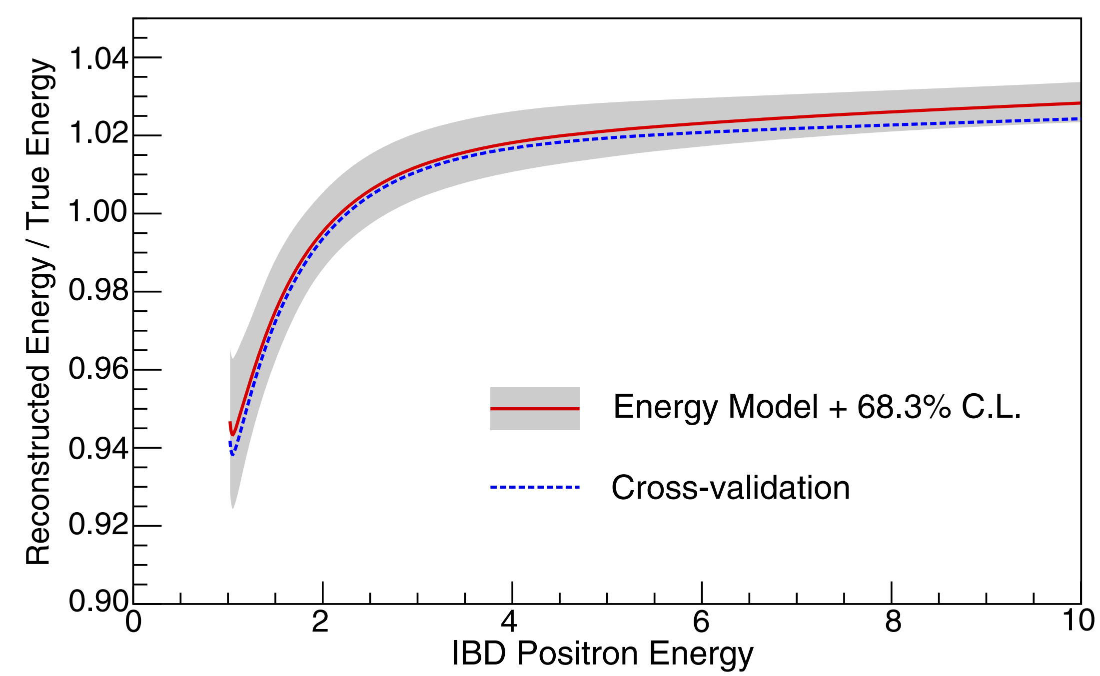


$$\Omega = 2|\Delta_{ee}| + \eta\,\phi$$

(NO, oO, IO) given by η=(1, 0, -1)

$$\delta\,(\Delta m^2_{ee}) \sim 0.5\%$$

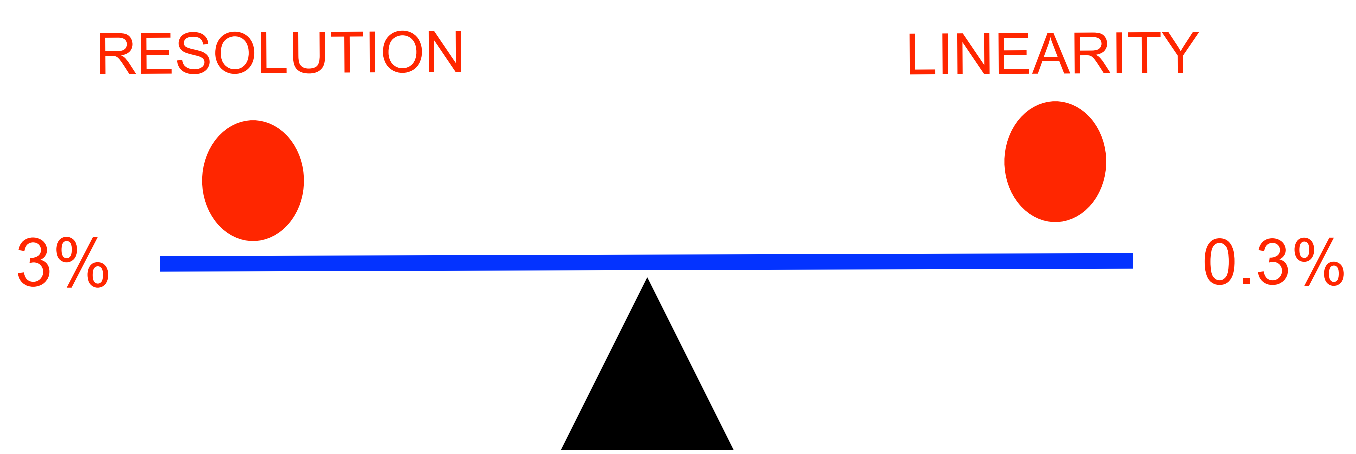
Daya Bay



Parke @ NOW 2008

- Energy Resolution to 3% or lower at 1 MeV
- Linearity to sub 1% precision for the reconstructed neutrino energy

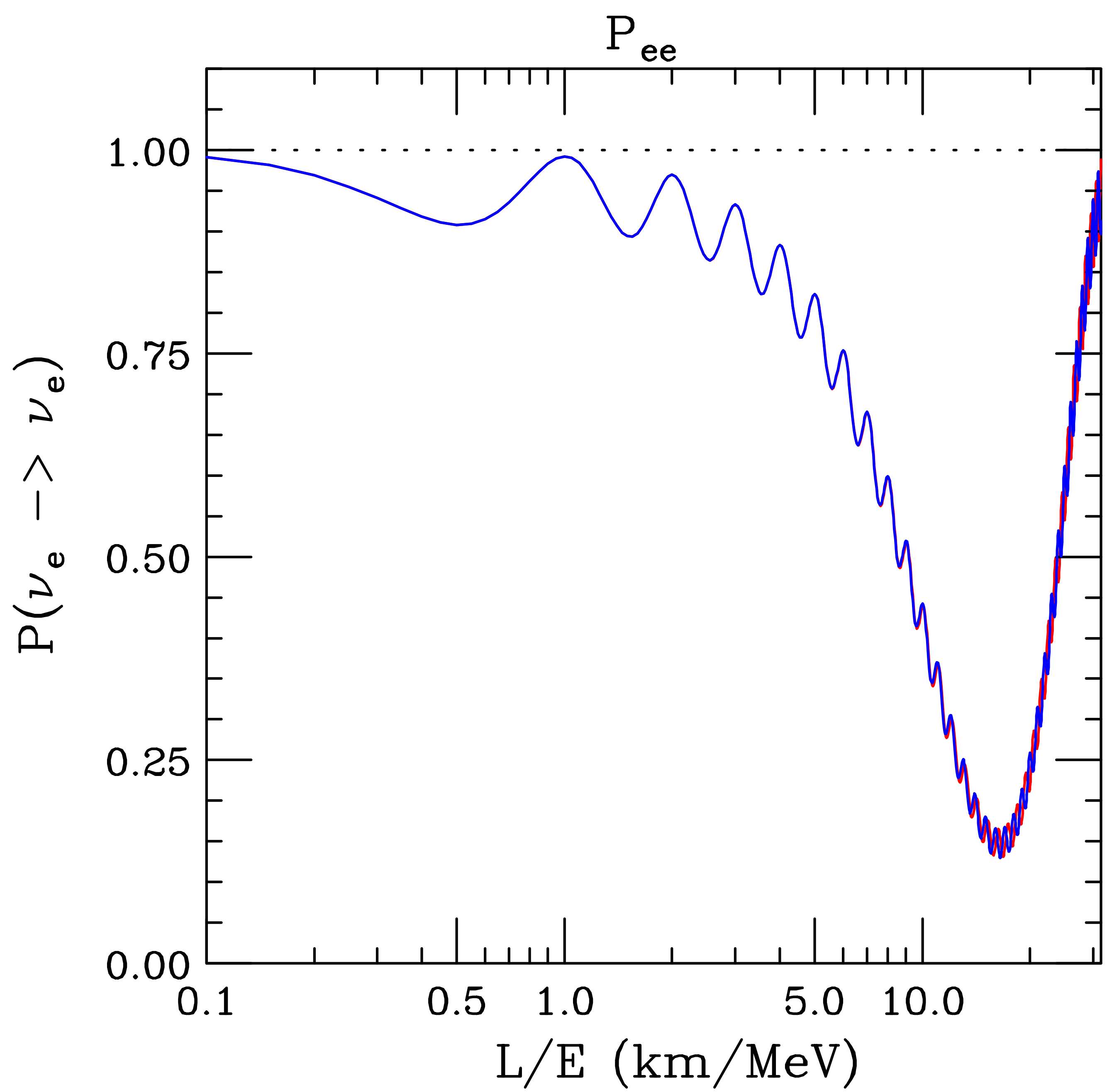
	KamLAND	JUNO	RENO-50	
LS mass	~1 kt	20 kt	18 kt	20 x
Energy Resolution	6%/	~3%/	~3%/	
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV	4 x
Linearity	1.9%	< 0.5%	< 0.5%	**> 4 x**



Other Prospects:

Ultra-Short Baseline: 10 m baseline

NEOS
PROSPECT
DANSS
STEREO
Nu-4
……
0.001 - 0.01



Matter Effects:

Neutrino Propagation in Matter:

$$i\frac{d}{dx}\nu \;=\; H\nu \qquad\qquad \boldsymbol{\nu} \equiv \begin{pmatrix} \boldsymbol{\nu_e} \\ \boldsymbol{\nu_\mu} \\ \boldsymbol{\nu_\tau} \end{pmatrix}$$

$$H = \frac{1}{2E}\left\{ U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger + \begin{bmatrix} a(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$$a \;=\; 2\sqrt{2}G_F N_e E \approx 1.52 \times 10^{-4} \left(\frac{Y_e \rho}{\mathrm{g.cm^{-3}}}\right)\left(\frac{E}{\mathrm{GeV}}\right) \mathrm{eV}^2.$$

if $\rho Y_e = 1.5$ g/cm^3 and $E = 10$ GeV then $a \approx \Delta m^2_{31}$

$E = 300\ MeV$ then $a \approx \Delta m^2_{21}$

2 flavor mixing in matter

$$ax^2 + bx + c = 0$$

simple, intuitive, useful

3 flavor mixing in matter

$$ax^3 + bx^2 + cx + d = 0$$

complicated, counter intuitive, ...

Hamiltonian:

H. Minakata + SP arXiv:1505.01826
P. Denton + H. Minakata + SP arXiv:1604.08167

$$H = \frac{1}{2E}\left\{ U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

Rewrite as $\boldsymbol{H} = \boldsymbol{H_0} + \boldsymbol{H_1}$

solvable perturbation

where H_0 is diagonal

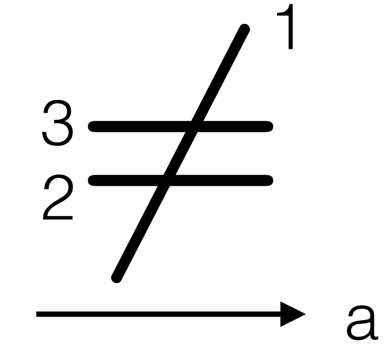
and H_1 is off-diagonal.

Neutrino Evolution in Matter (conti):

$$U_{23}^\dagger(\theta_{23},\delta)\, H\, U_{23}(\theta_{23},\delta) = H_D + H_{OD}$$

D=diagonal OD= off-diagonal

$$(2E)\,H_D = \begin{bmatrix} a + s_{13}^2 \Delta m_{ee}^2 & & \\ & (c_{12}^2 - s_{12}^2)\Delta m_{21}^2 & \\ & & c_{13}^2 \Delta m_{ee}^2 \end{bmatrix}$$



$$\Delta m_{ee}^2 \equiv \cos^2\theta_{12}\Delta m_{31}^2 + \sin^2\theta_{12}\Delta m_{32}^2$$

!!! level crossing !!!

$$(2E)\,H_{OD}/\Delta m_{ee}^2 = s_{13}c_{13}\begin{bmatrix} & & 1 \\ & 0 & \\ 1 & & \end{bmatrix}$$

0.15

$$+\, c_{13}\; s_{12}c_{12}\left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right)\begin{bmatrix} & 1 & \\ 1 & & 0 \\ & 0 & \end{bmatrix}$$

0.015

$$-\, s_{13}\; s_{12}c_{12}\left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right)\begin{bmatrix} & 0 & \\ 0 & & 1 \\ & 1 & \end{bmatrix}$$

0.002

DMP 0th order in a nutshell:

$$\text{Vacuum} \quad \Rightarrow \quad \text{Matter}$$

$$P_{\nu_\alpha \to \nu_\beta}(\Delta m^2_{31}, \Delta m^2_{21}, \theta_{13}, \theta_{12}, \theta_{23}, \delta) \quad \Rightarrow \quad P_{\nu_\alpha \to \nu_\beta}(\Delta \widetilde{m^2}_{31}, \Delta \widetilde{m^2}_{21}, \widetilde{\theta}_{13}, \widetilde{\theta}_{12}, \theta_{23}, \delta)$$

(same function just use matter variables)

$$\cos 2\widetilde{\theta}_{13} = \frac{\Delta m^2_{ee} \cos 2\theta_{13} - a}{\Delta \widetilde{m^2}_{ee}}$$ where $a = 2\sqrt{2} G_F N_e E_\nu$ and

two 2 flavor rotations

$$\Delta \widetilde{m^2}_{ee} = \Delta m^2_{ee} \sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}$$

with $\Delta m^2_{ee} \equiv \cos^2 \theta_{12} \Delta m^2_{31} + \sin^2 \theta_{12} \Delta m^2_{32}$

$$\cos 2\widetilde{\theta}_{12} = \frac{\Delta m^2_{21} \cos 2\theta_{12} - a'}{\Delta \widetilde{m^2}_{21}}$$ where $a' \equiv (a + \Delta m^2_{ee} - \Delta \widetilde{m^2}_{ee})/2$

(the effective matter potential for 1-2 sector.)

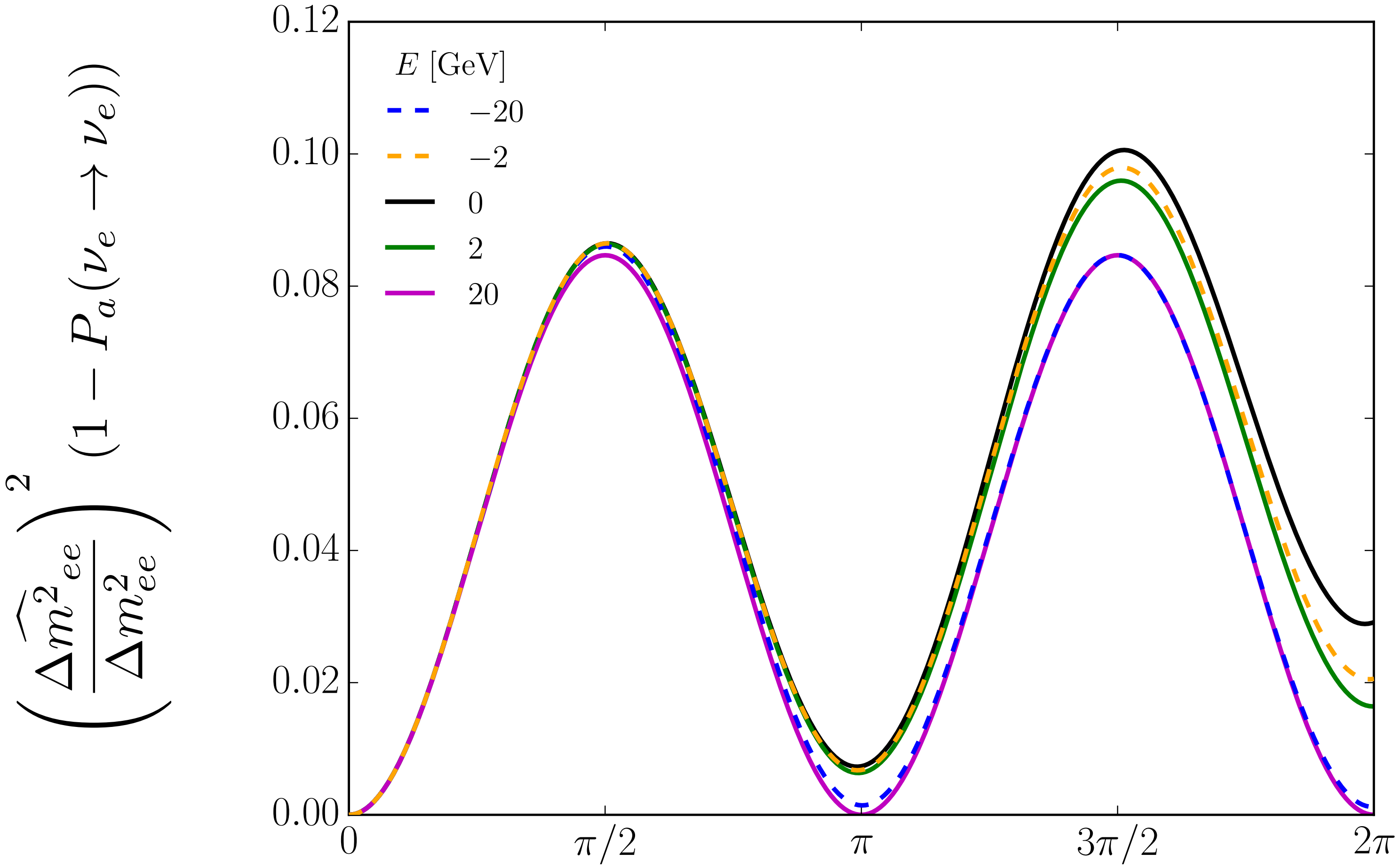
$$\Delta \widetilde{m^2}_{21} = \Delta m^2_{21} \sqrt{(\cos 2\theta_{12} - a'/\Delta m^2_{21})^2 + \cos^2(\widetilde{\theta}_{13} - \theta_{13}) \sin^2 2\theta_{12}}$$

$$\Delta \widetilde{m^2}_{31} = \Delta m^2_{31} + \tfrac{1}{4} a + \tfrac{3}{4}(\Delta \widetilde{m^2}_{ee} - \Delta m^2_{ee}) + \tfrac{1}{2}(\Delta \widetilde{m^2}_{21} - \Delta m^2_{21})$$

$\nu_e \rightarrow \nu_e$

$$P_a(\nu_e \rightarrow \nu_e) \approx 1 - \sin^2 2\theta_{13} \left(\frac{\Delta m^2_{ee}}{\Delta \widehat{m^2}_{ee}} \right)^2 \sin^2 \widehat{\Delta}_{ee} \,, \qquad \widehat{\Delta}_{ee} \equiv \Delta \widehat{m^2}_{ee} L/(4E) \,,$$

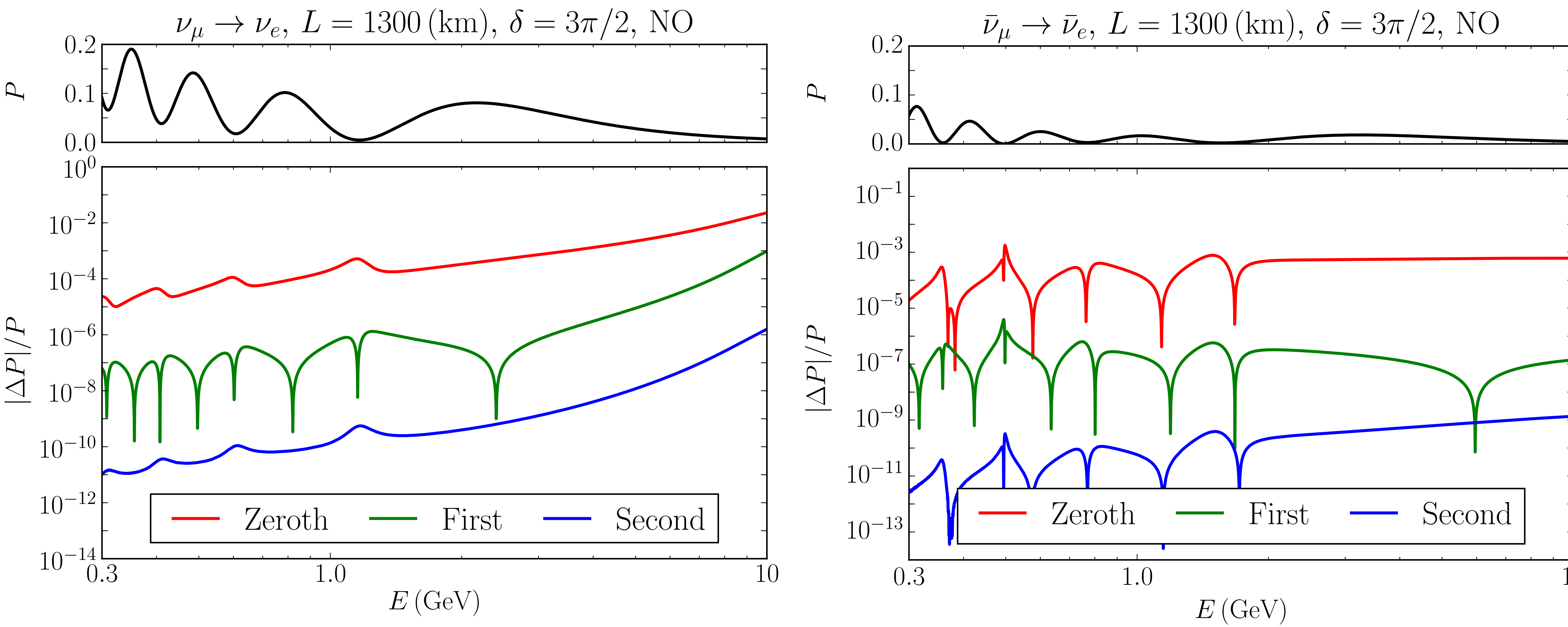
$$\Delta \widehat{m^2}_{ee} \approx \Delta m^2_{ee} \sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}} \,,$$



$|\widehat{\Delta}_{ee}|$

Denton, SP 1808.09453

New Perturbation Theory for Osc. Probabilities



systematic expansion

Summary:

- Observation of Solar Neutrinos and Reactor Neutrinos have taught us a great deal the electron row of the PMNS matrix, about U_ei and Delta m^2 's.

- the concept of an effective Delta m^2, Delta m^2_ee, is useful for the shape analysis of reactor neutrinos.

  Δm^2_{ee} is ν_e average of Δm^2_{31} and Δm^2_{32}

- Short baseline reactor experiments can constrain (maybe measure) Delta m^2_21 at twice the KamLAND value.

- the generalization of Delta m^2_ee into matter, is useful for understanding neutrino oscillations in matter for DUNE and T2HK(K)

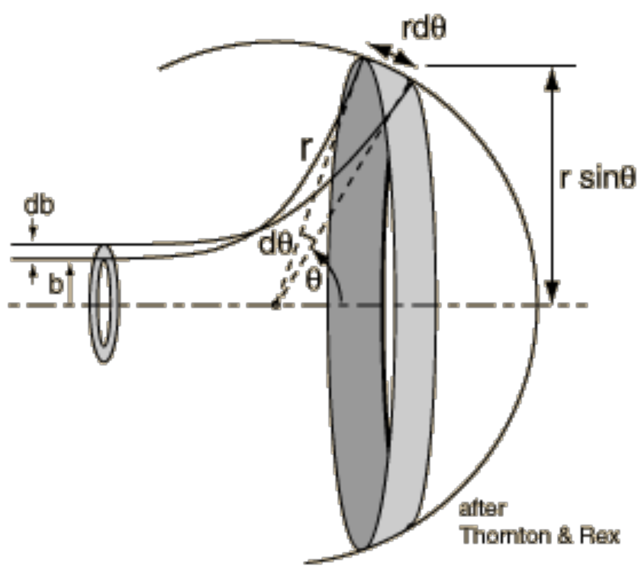
$$\Delta m^2_{ee}\sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}$$

Ernest Rutherford, master of simplicity

By Ashutosh Jogalekar | August 30, 2013 | 6

Ernest Rutherford, emperor of the atomic domain (Image: Wikipedia Commons)

"theorists play games with their symbols while we discover truths about the universe". And yet



$$\sigma = \pi Z^2 \left(\frac{ke^2}{KE}\right)^2 \left(\frac{1+\cos\theta}{1-\cos\theta}\right)$$

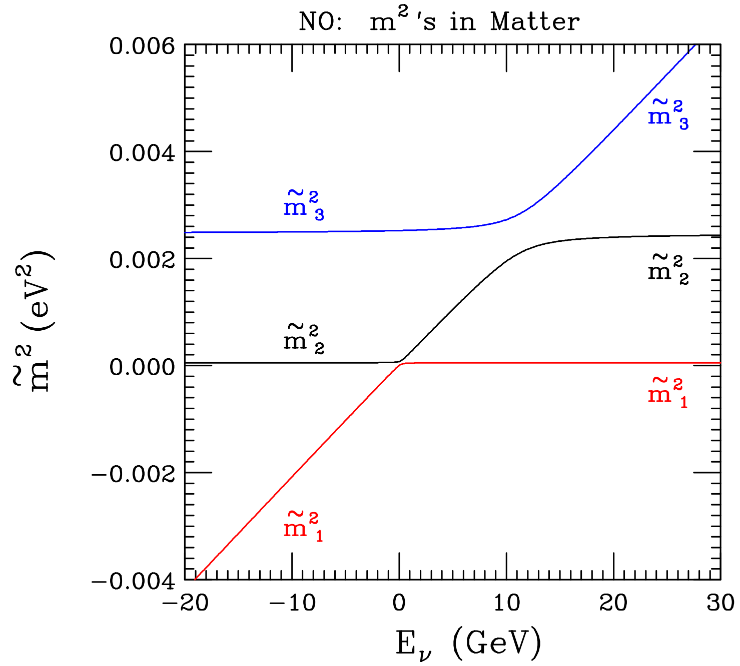
he had an eye for theoretical talent that allowed him to nurture Niels Bohr, as dyed-in-the-wool a theoretician and philosopher as you could find.

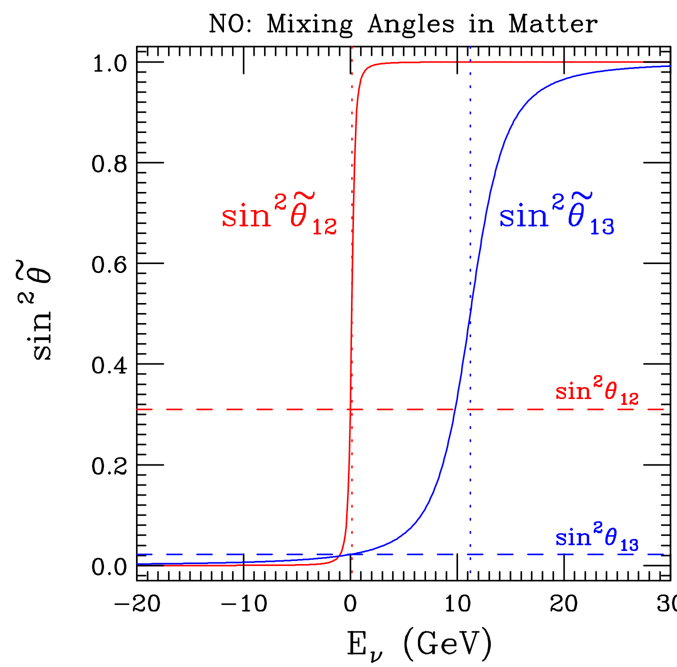
extras

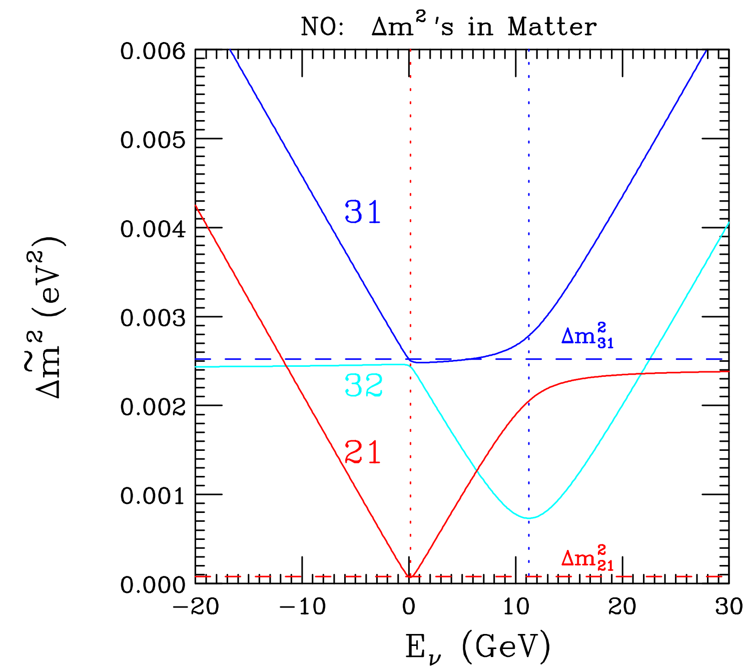
Mixings and Masses in Matter:

0th Order:

θ_{23}, δ unchanged







NO:
for IO figures see 1604.08167

$$\widetilde{\theta}_{13}, \widetilde{\theta}_{12} \qquad \Delta \widetilde{m^2}_{31}, \Delta \widetilde{m^2}_{21}$$

After 2 rotations:

$\tilde{s}_{12} \equiv \sin\tilde{\theta}_{12}$, etc

$$(2E)\, H_{OD}/\Delta m^2_{ee} \quad = \quad \sin(\tilde{\theta}_{13}-\theta_{13})\; s_{12}c_{12}\left(\frac{\Delta m^2_{21}}{\Delta m^2_{ee}}\right)\begin{bmatrix} & & -\tilde{s}_{12} \\ & & \tilde{c}_{12} \\ -\tilde{s}_{12} & \tilde{c}_{12} & \end{bmatrix}$$

$$\sin(\tilde{\theta}_{13}-\theta_{13}) \approx s_{13}c_{13}\left(\frac{a}{\Delta m^2_{ee}}\right)$$

4×10^{-4}

for $E = 2$ GeV and $\rho = 3$ g.cm^{-3}

zero in vacuum

How Does Daya Bay Define Δm^2_{EE} ?

arXiv:1310.6732

ν_j and ν_i. Since $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ [1], the short-distance ($\sim$km) reactor $\overline{\nu}_e$ oscillation is due primarily to the Δ_{3i} terms and naturally leads to the definition of the effective mass-squared difference $\sin^2 \Delta_{ee} \equiv \cos^2\theta_{12} \sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}$ [11].

1505.03456v1

[8] $\sin^2\Delta_{ee} \equiv \cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}$, where $\Delta_{ji} \equiv 1.267\Delta m^2_{ji}(\mathrm{eV}^2)[L(\mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i.