Transverse-Momentum-Dependent Structures of the Proton from Lattice QCD

"Hadron Structure(s) from First Principles" Webinar,
Universidad Complutense de Madrid

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11/13/2020

BROOKHAVEN NATIONAL LABORATORY

TMD Collaboration

Outline

- Overview of TMD factorization
- Calculation of TMDPDFs from LaMET
    - Large-Momentum Effective Theory
    - Quasi-TMDPDF and TMDPDF
- Exploratory lattice results
    - The Collins-Soper evolution kernel
    - Soft factor

The Electron-Ion Collider

"A machine that will unlock the secrets of the strongest force in Nature"

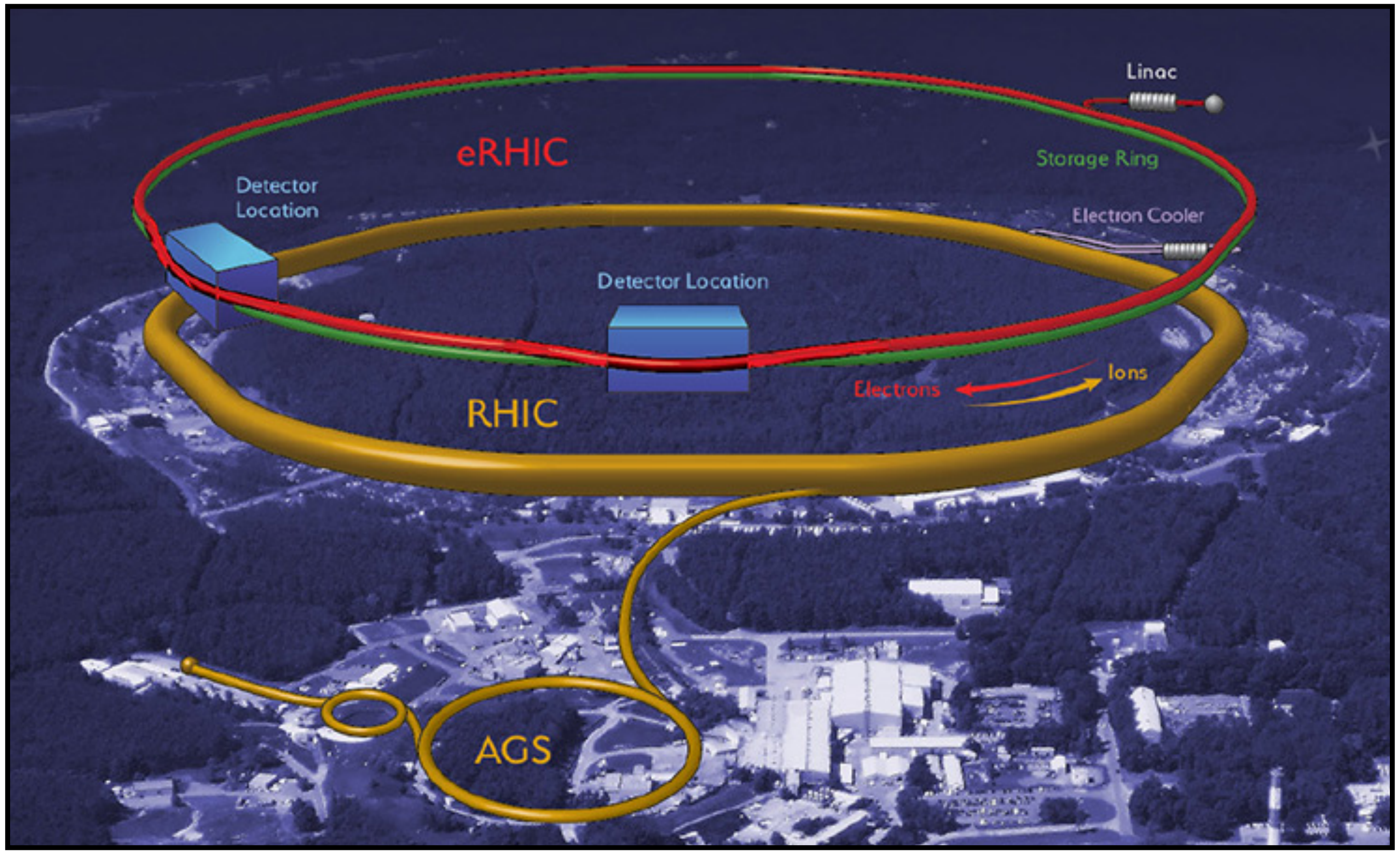
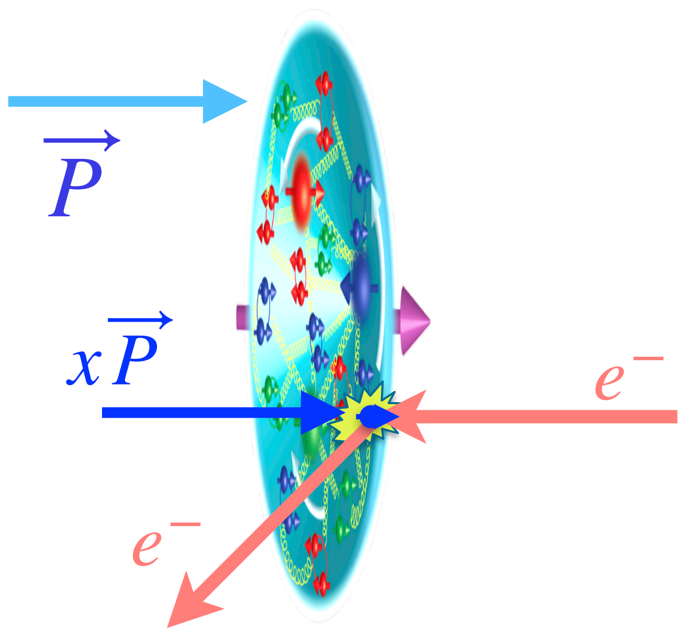


Image credit, BNL.

- Precision 3D imaging of protons and nuclei
- Solving the proton spin puzzle
- Search for saturation
- Quark and gluon confinement
- Quarks and gluons in nuclei

High-energy scattering and Feynman's parton model

- Deep inelastic (e.g., e⁻+p) scattering:



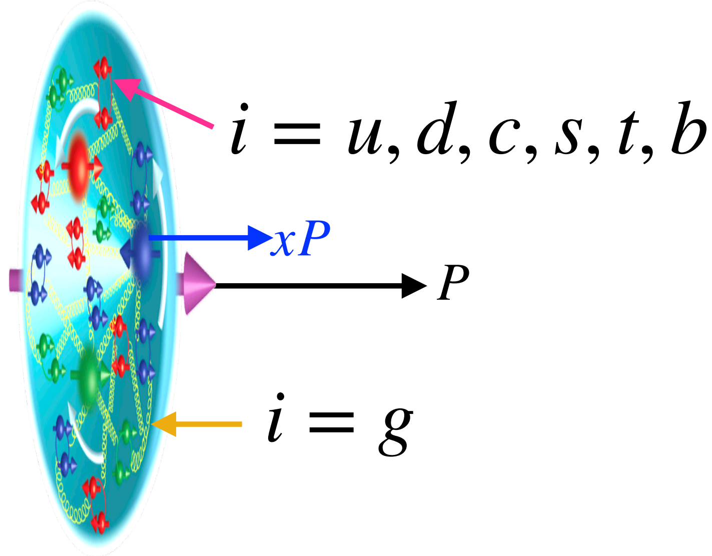
Richard P. Feynman

Feynman's parton model (1969):

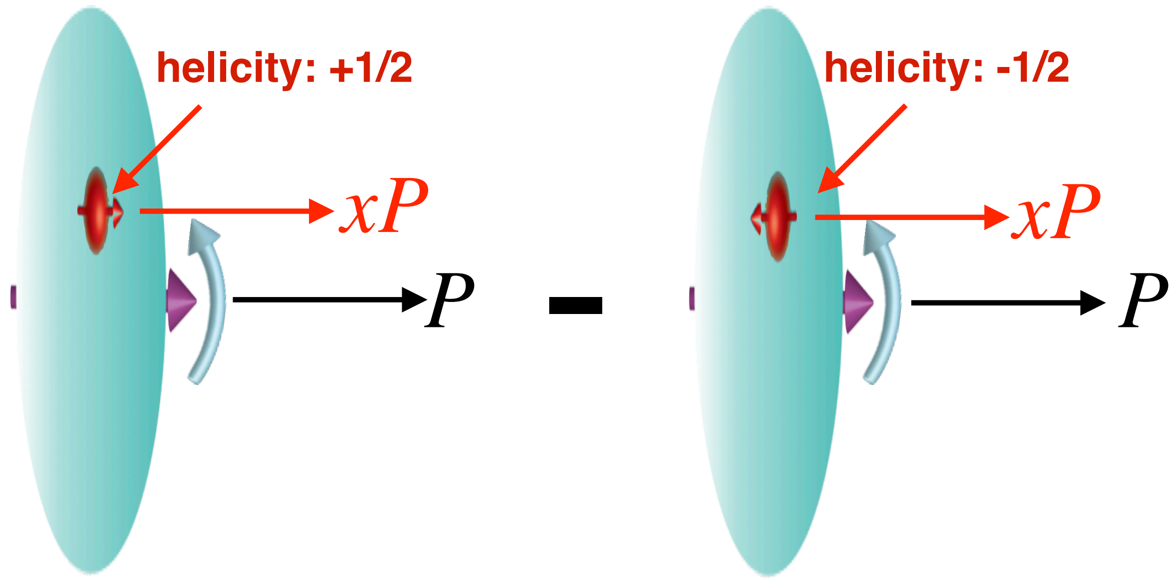
- Quarks and gluons are "frozen" in the transverse plane due to Lorentz contraction;
- During a hard collision, the struck quark/gluon (parton) appears to the probe that it does not interact with its surroundings.

Parton distribution functions (PDFs)

- Unpolarized PDF $f_i(x)$:

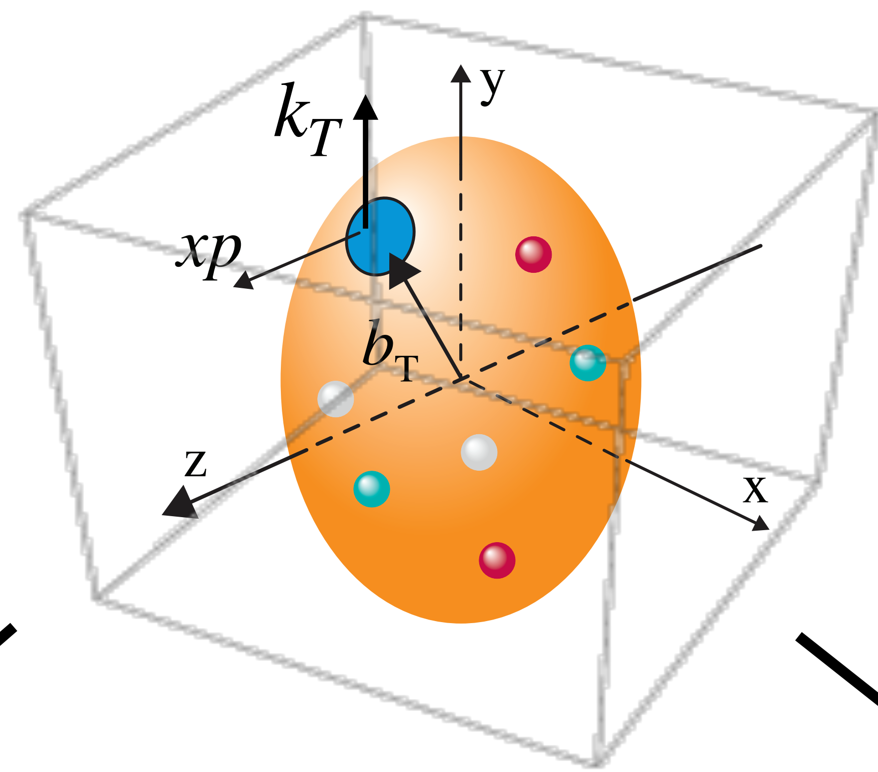


- Helicity PDF $\Delta f_i(x)$:



Phase-space distributions of partons

3D Tomography of the proton



Wigner distributions (or GTMDs):

$W(x, \overrightarrow{k}_T, \overrightarrow{b}_T)$

Momentum space: confined motion

$\int d^2 b_T$

Transverse momentum dependent (TMD) PDFs

$f(x, \overrightarrow{k}_T)$

Coordinate space: spatial distribution

$\int d^2 k_T$

Generalized Parton Distributions (GPDs)

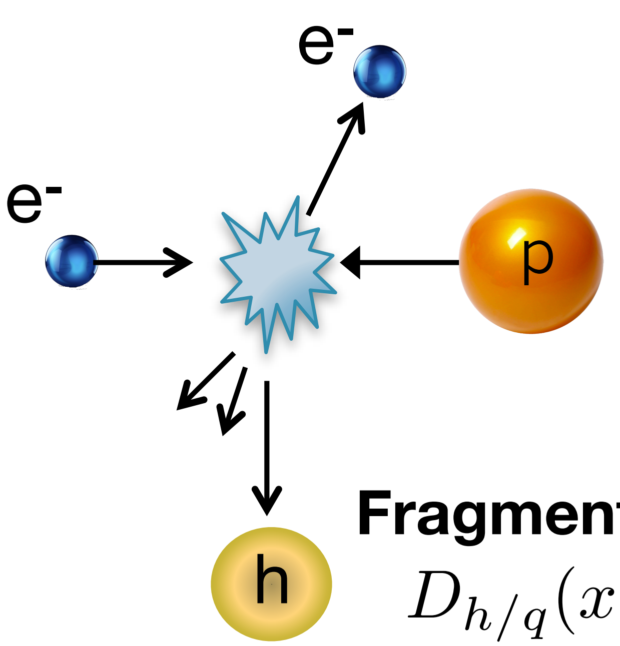
$f(x, \overrightarrow{b}_T)$

TMDPDFs from experiments

- TMD processes:

Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$

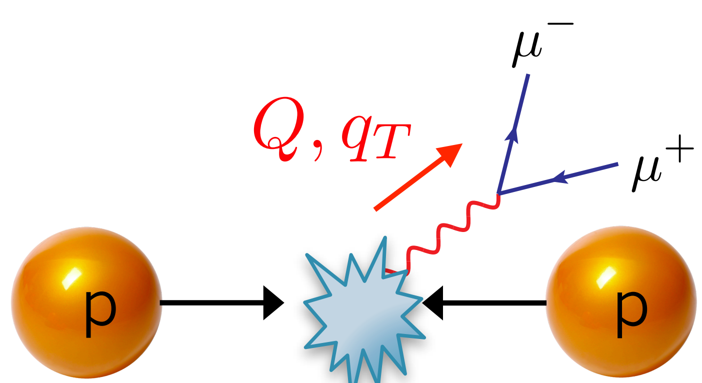


Fragmentation

$$D_{h/q}(x, k_T)$$

Drell-Yan

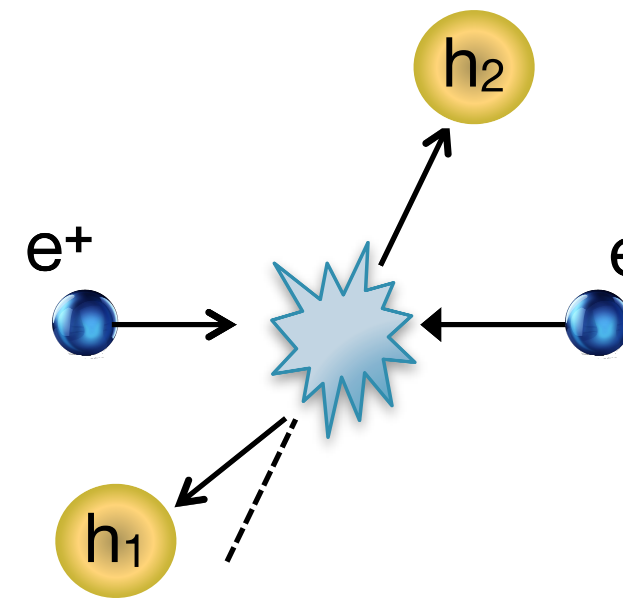
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



$$q_T \ll Q$$

Dihadron in e^+e^-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



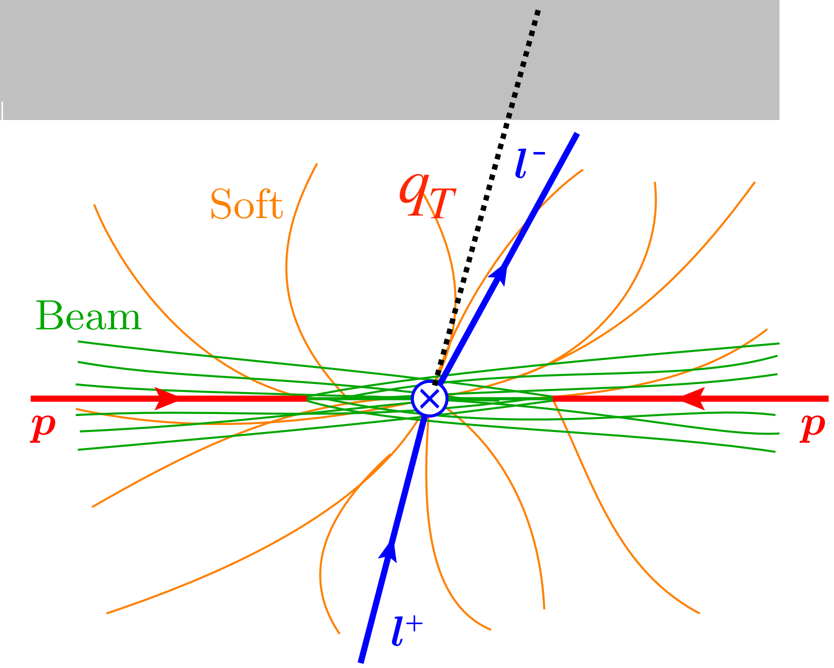
Many different schemes for TMD factorization in literature:

- Collins, Soper and Sterman, NPB250 (1985); Collins, 2011;
- Ji, Ma and Yuan, PRD71 (2005) 034005;
- Becher and Neubert, EPJC71 (2011);
- Echevarria, Idilbi and Scimemi, JHEP07 (2012), PLB726 (2013);
- Chiu, Jain, Neil and Rothstein, JHEP05 (2012), PRL108 (2012);
- Li, Neil and Zhu, arXiv: 1604.00392.

TMD Factorization

- Collinear factorization (e.g., for Drell-Yan):

$$\frac{d\sigma}{dQdY} = \sum_{i,j} \sigma_{ab}(Q, \mu, Y)\, f_i(x_a, \mu)\, f_j(x_b, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)\right]$$



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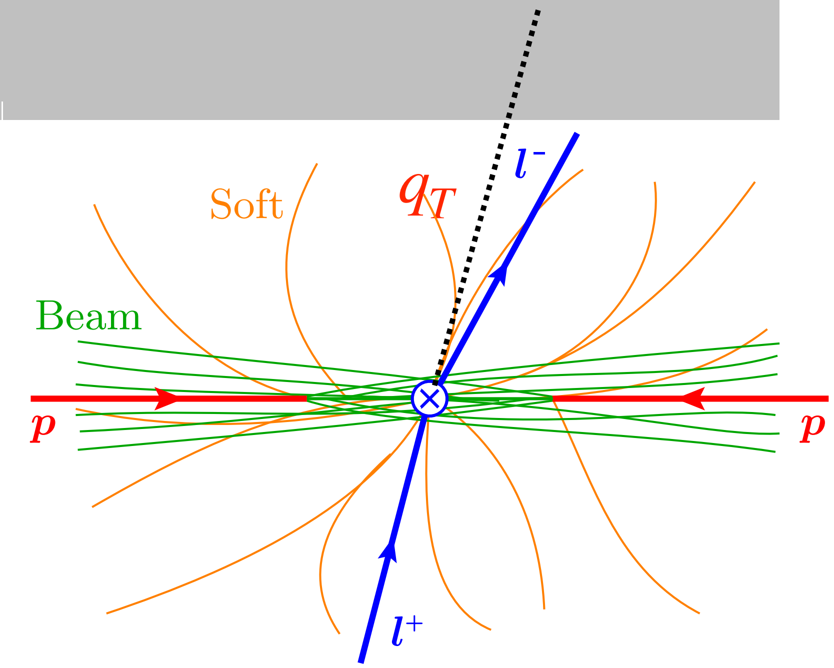


- TMDPDF factorization ($q_T \ll Q$):

$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{i,j} H_{ij}(Q,\mu)\int d^2b_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_i^{\rm TMD}(x_a,\vec{b}_T,\mu,\zeta_a) f_j^{\rm TMD}(x_b,\vec{b}_T,\mu,\zeta_b)\left[1+\mathcal{O}\left(\frac{q_T^2}{Q^2},\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)\right]$$

$ij = q\bar{q}$ for DY
$ij = gg$ for H

$$f(x,\vec{b}_T) = \int \frac{d^2k_T}{2\pi}\, e^{i\vec{k}_T\cdot\vec{b}_T} f(x,\vec{k}_T)$$

ζ: Collins-Soper Scale. $\zeta_a\zeta_b = Q^4 = x_a x_b (P_a^+ P_b^-)^2$

$$\zeta_a = (x_a P_a^+)^2 e^{-2y_n}, \qquad \zeta_b = (x_a P_b^-)^2 e^{2y_n}$$

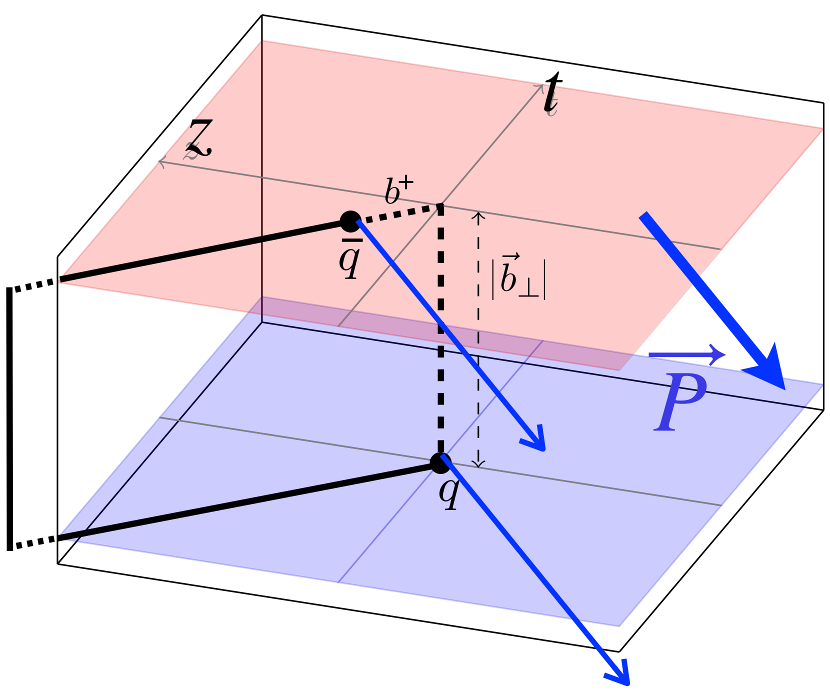
Definition of TMDPDF

Rapidity divergence regulator

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0,\tau\to 0} Z_{\mathrm{UV}}(\epsilon, \mu, xP^+) B_i(x, \vec{b}_T, \epsilon, \tau, xP^+)\sqrt{S^i(b_T, \epsilon, \tau)}$$

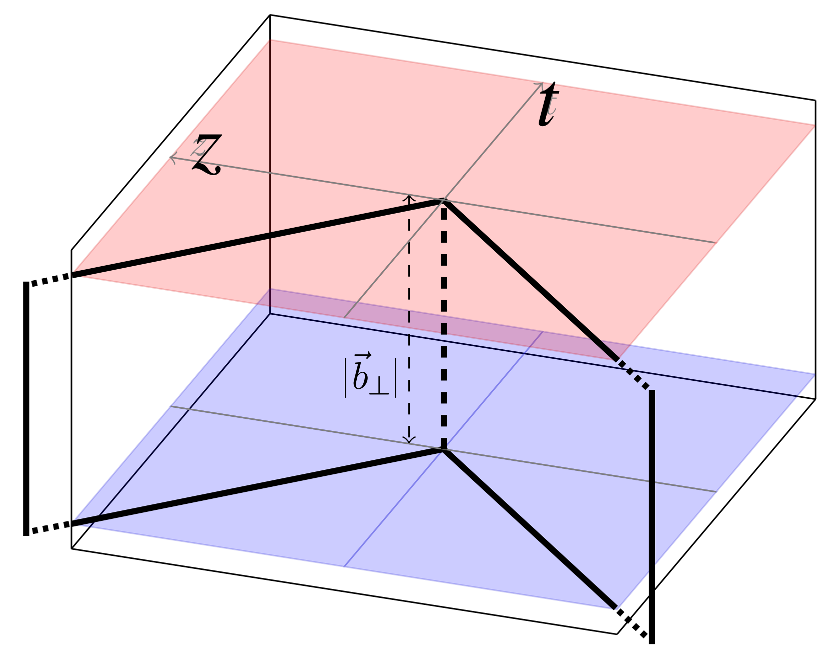
UV divergence regulator

- **Beam function :**



$$B^q(x, \vec{b}_T, \epsilon, \tau) = \int \frac{db^-}{2\pi} e^{-i(xP^+)b^-} \langle P | \bar{q}(b^\mu) W(b^\mu) \frac{\gamma^+}{2} \times W_T(-\infty\bar{n}; \vec{b}_T, \vec{0}_T) W^\dagger(0) q(0) \Big|_\tau | P\rangle$$

- **Soft function :**



$$S_q(b_T, \epsilon, \tau) = \frac{1}{N_c} \langle 0 | \mathbf{Tr}\big[S_n^\dagger(\vec{b}_T) S_{\bar{n}}(\vec{b}_T) S_T \times S_{\bar{n}}^\dagger(\vec{0}_T) S_n(\vec{0}_T) S_T^\dagger\big]\Big|_\tau |0\rangle$$

Definition of TMDPDF

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$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\mathrm{UV}}(\epsilon, \mu, xP^+) B_i(x, \vec{b}_T, \epsilon, \tau, xP^+) \sqrt{S^i(b_T, \epsilon, \tau)}$$

UV divergence regulator

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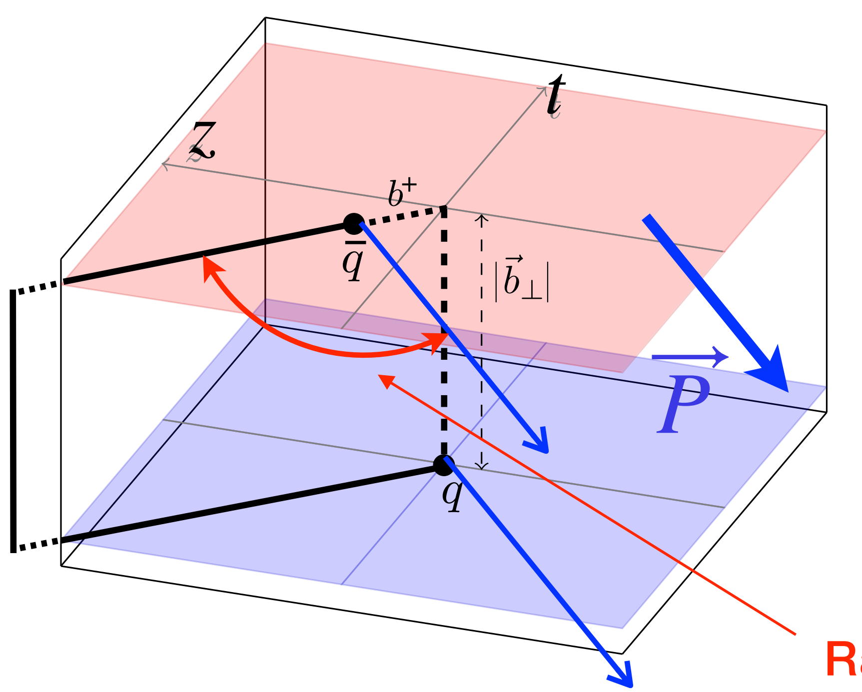
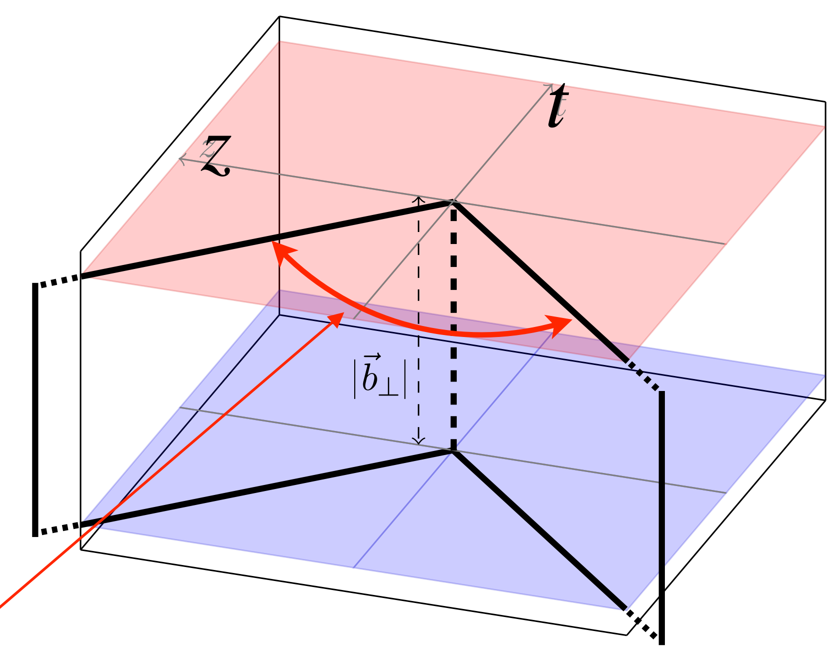


- **Soft function :**



Rapidity divergences

$$B^q(x, \vec{b}_T, \epsilon, \tau) = \int \frac{db^-}{2\pi} e^{-i(xP^+)b^-} \langle P | \bar{q}(b^\mu) W(b^\mu) \frac{\gamma^+}{2} \times W_T(-\infty\bar{n}; \vec{b}_T, \vec{0}_T) W^\dagger(0) q(0) \Big|_\tau | P \rangle$$

$$S_q(b_T, \epsilon, \tau) = \frac{1}{N_c} \langle 0 | \mathrm{Tr}\big[S_n^\dagger(\vec{b}_T) S_{\bar{n}}(\vec{b}_T) S_T \times S_{\bar{n}}^\dagger(\vec{0}_T) S_n(\vec{0}_T) S_T^\dagger \big] \Big|_\tau | 0 \rangle$$

TMDPDF Evolution

- Evolution of TMDPDF:
  - $\mu \sim Q,\ \zeta \sim Q^2 \gg \Lambda^2_{\rm QCD}$;
  - μ_0, ζ_0 : initial or reference scales, measured in experiments or determined from lattice (~2 GeV).

$$\mu \frac{d \ln f_i^{\rm TMD}}{d\mu} = \gamma_\mu^i(\mu, \zeta)$$ Anomalous dimension for μ evolution, perturbatively calculable;

$$\frac{1}{2}\zeta \frac{d \ln f_i^{\rm TMD}}{d\zeta} = \gamma_\zeta^i(\mu, b_T)$$ Collins-Soper kernel.

$$\frac{d\gamma_\zeta^i(\mu, b_T)}{d \ln \mu} = 2\frac{d\gamma_\mu^i(\mu, \zeta)}{d \ln \zeta} = -2\Gamma^i_{\rm cusp}[\alpha_s(\mu)]$$ **Analytical in the $\mu - \zeta$ plane.**

All order form: $$\gamma_\zeta^i(\mu, b_T) = -2\int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'}\Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(1/b_T)]$$

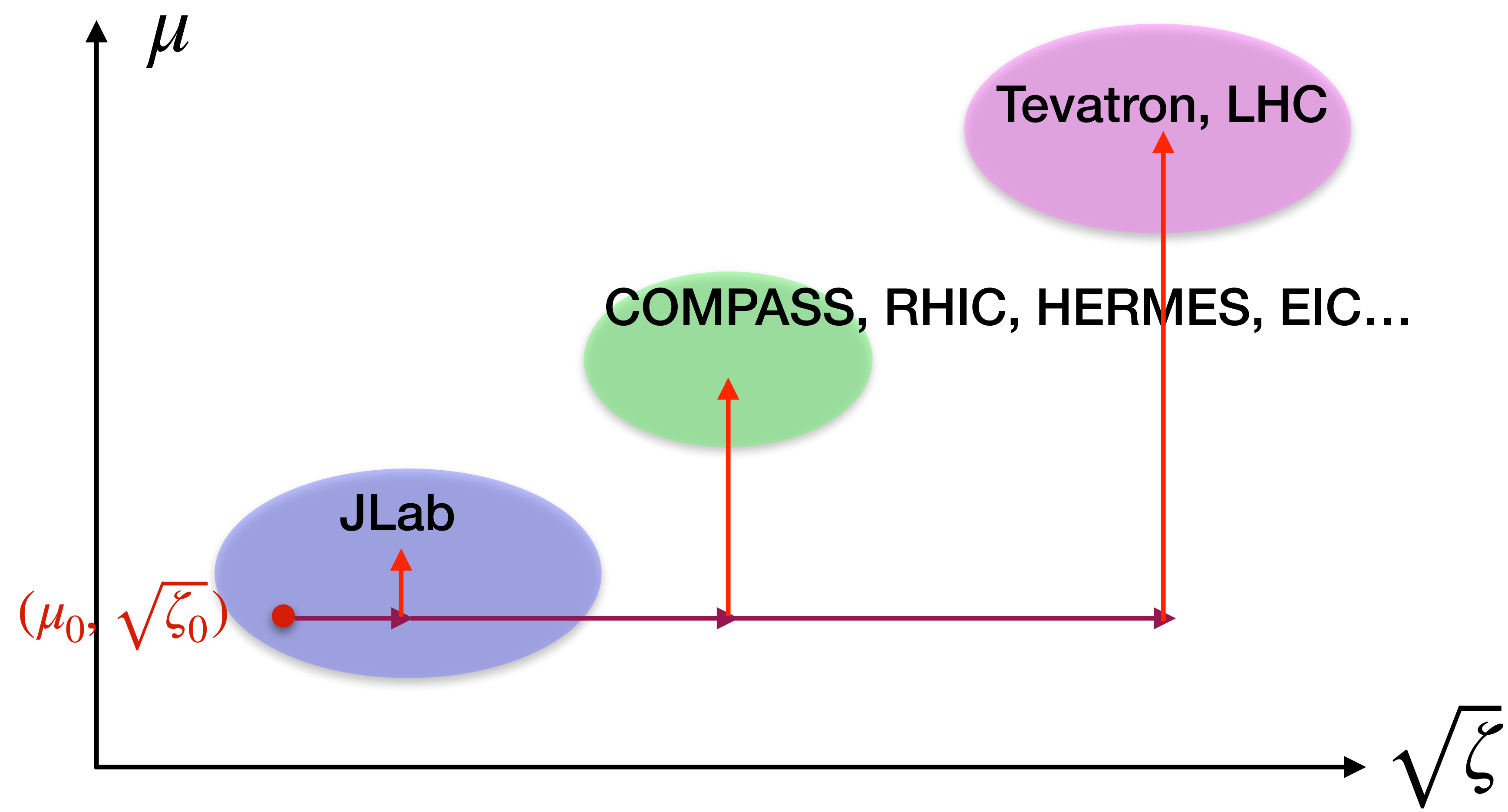
Nonperturbative when $b_T \sim 1/\Lambda_{\rm QCD}$.

Global analysis of TMDPDF

$$f_i^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) = f_i^{\rm TMD}(x, \vec{b}_T, \mu_0, \zeta_0)\ \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\gamma_\mu^i(\mu', \zeta_0)\right]\ \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T)\ln\frac{\zeta}{\zeta_0}\right]$$

Modelling　　　　Modelling

$\mu \sim Q, \zeta \sim Q^2$



For a systematic analysis of double scale evolution:
Vladimirov and Scimemi, JHEP 08(2018)

The Collins-Soper Kernel

- Collins Soper kernel:

Perturbatively Calculable

$$\gamma_\zeta^i(\mu, b_T) = -2\int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^i(\mu_b) + g^i(b_T, \mu_b)$$

$$\mu_b = \mu(b_T) \gg \Lambda_{\rm QCD} \qquad \mathrm{e.g.}, \mu_b = 2e^{-\gamma_E}\Big/\sqrt{\frac{b_T^2 B_{NP}^2}{(b_T^2 + B_{NP}^2)}} \qquad B_{NP} \ll \Lambda_{\rm QCD}^{-1}$$

- Modeling of the nonperturbative part, e.g., $g^i(b_T, \mu_b) = g_K^i b_T^2$

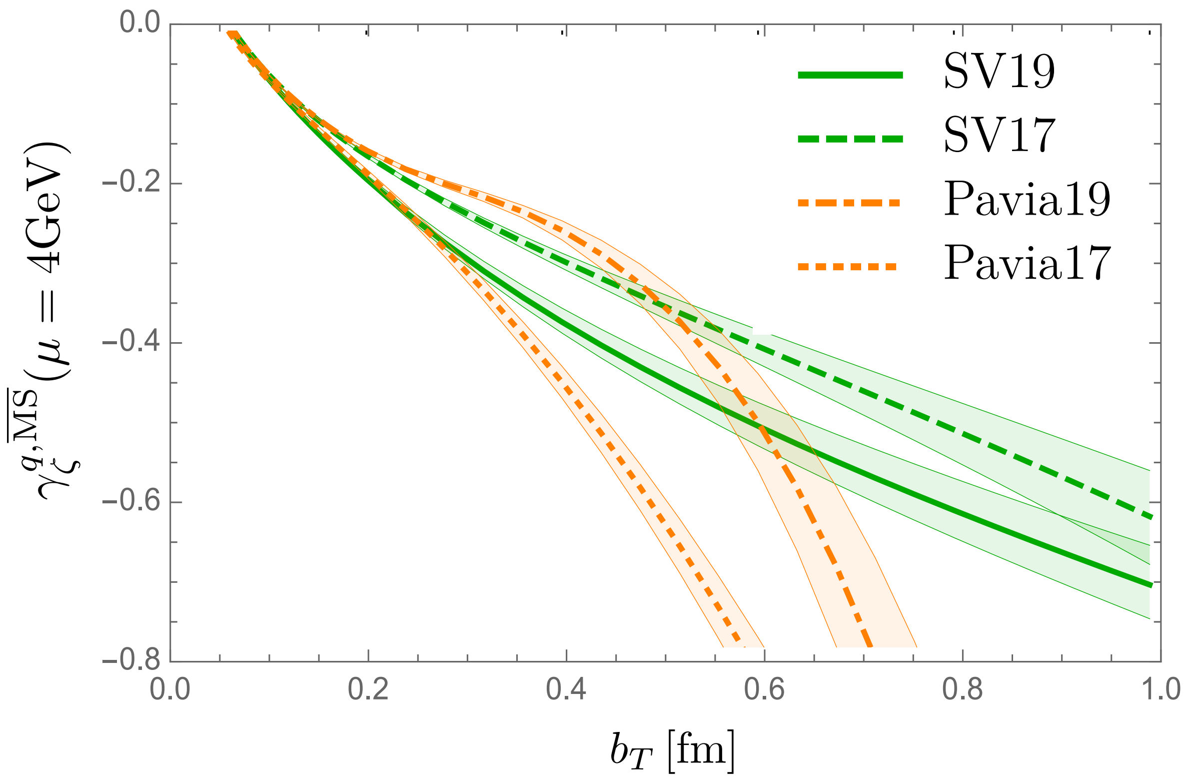


Figure:
- A. Vladimirov, 2003.02288.

SV 17&19: At large b_T, $g^i(b_T, \mu_b) \sim b_T$
- Scimemi and Vladimirov, EPJC 78 (2018);
- Scimemi and Vladimirov, JHEP 06 (2020);

Pavia 17&19: At large b_T, $g^i(b_T, \mu_b) \sim b_T^2$
- Bacchetta et al., JHEP1706 (2017)
- Bacchetta et al., JHEP 07 (2020)

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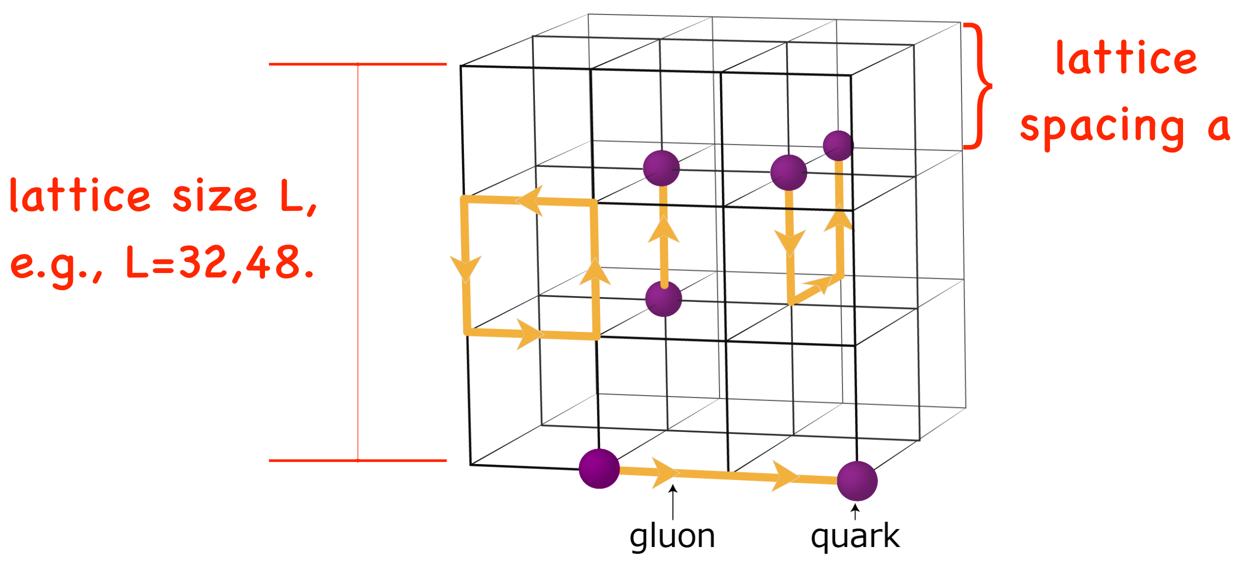
Lattice QCD

- Lattice gauge theory (1974): a systematically improvable approach to solve non-perturbative QCD.

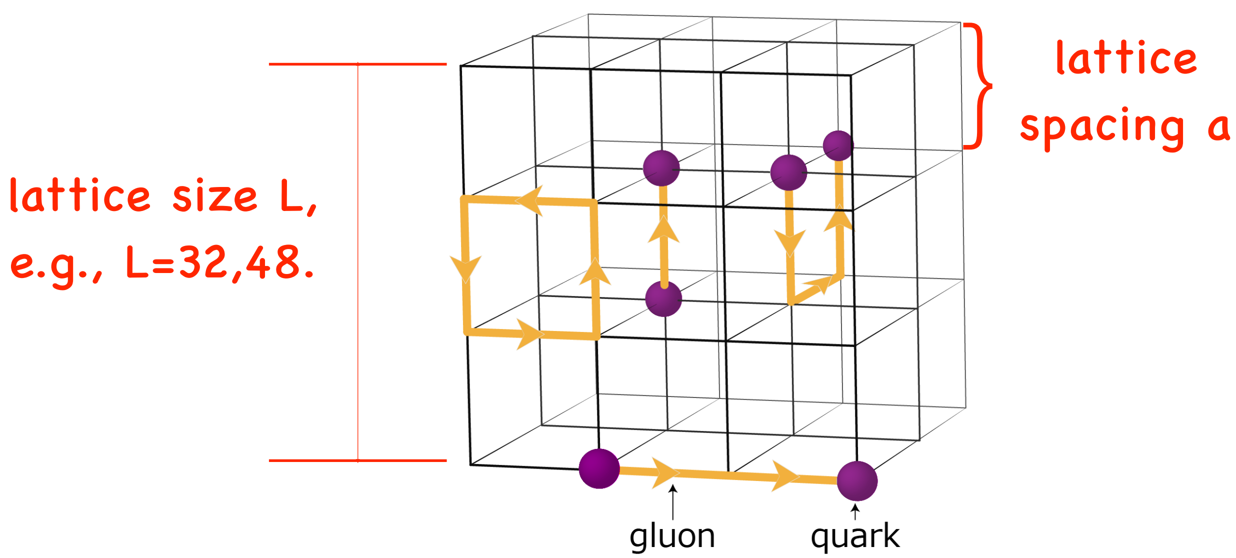


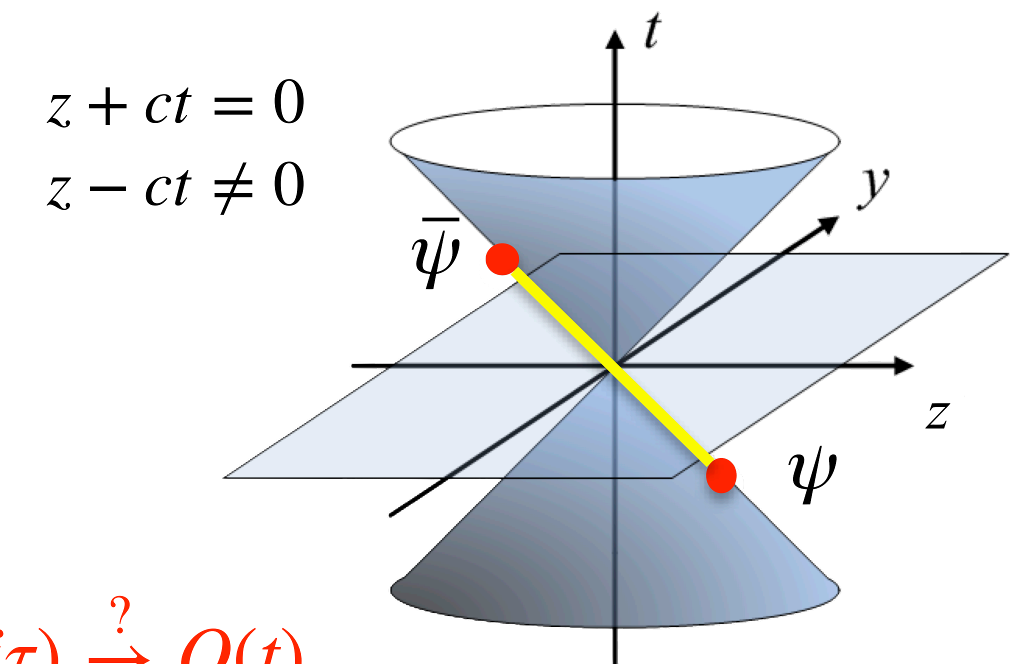
Imaginary time: $t \to i\tau \quad e^{iS} \to e^{-S}$

- Simulation of QCD in discrete Euclidean space and "time";
- Complex calculations enabled by high-performance supercomputers;
- Tremendous success in uncovering static properties like hadron spectroscopy, α_s, etc.

Lattice QCD

- Lattice gauge theory (1974): a systematically improvable approach to solve non-perturbative QCD.



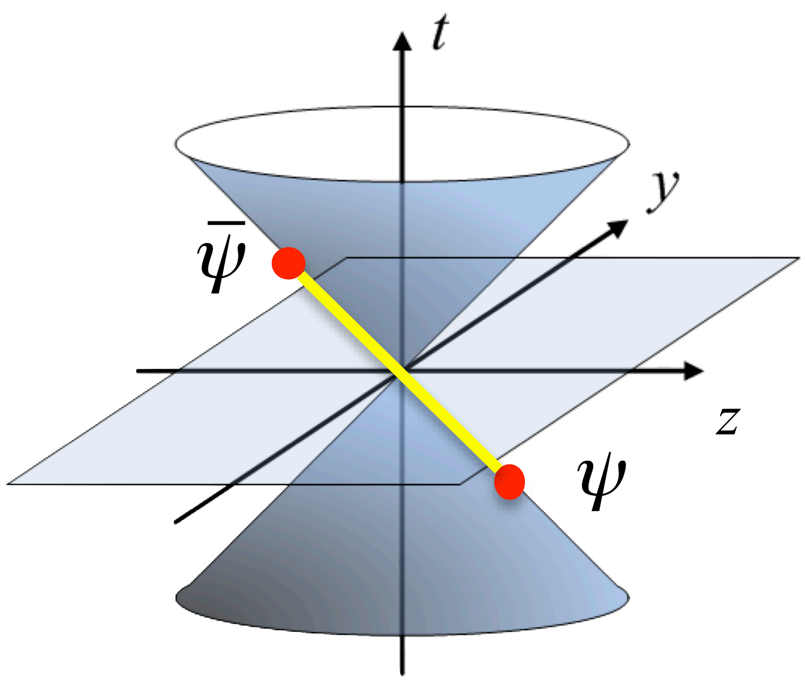


Imaginary time: $t \to i\tau$ $\quad O(i\tau) \overset{?}{\to} O(t)$

Simulating real-time dynamics has been extremely difficult due to the issue of analytical continuation. 🙁

Large-Momentum Effective Theory (LaMET)

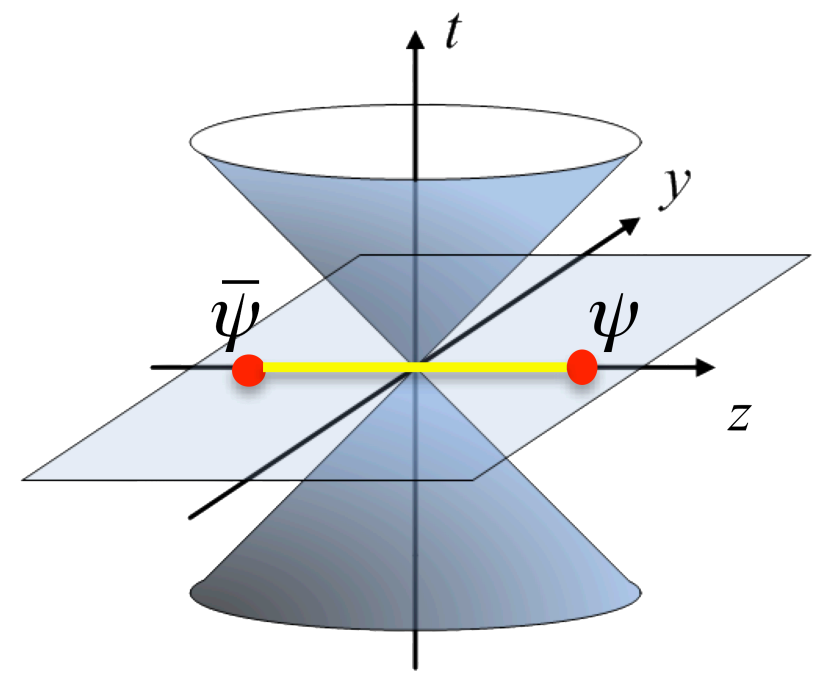
$z + ct = 0, \;\; z - ct \neq 0$



PDF $f(x)$:
Cannot be calculated
on the lattice

$$f(x) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(b^-) \times \frac{\gamma^+}{2} W[b^-, 0] \psi(0) | P \rangle$$

$t = 0, \;\; z \neq 0$



Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on
the lattice

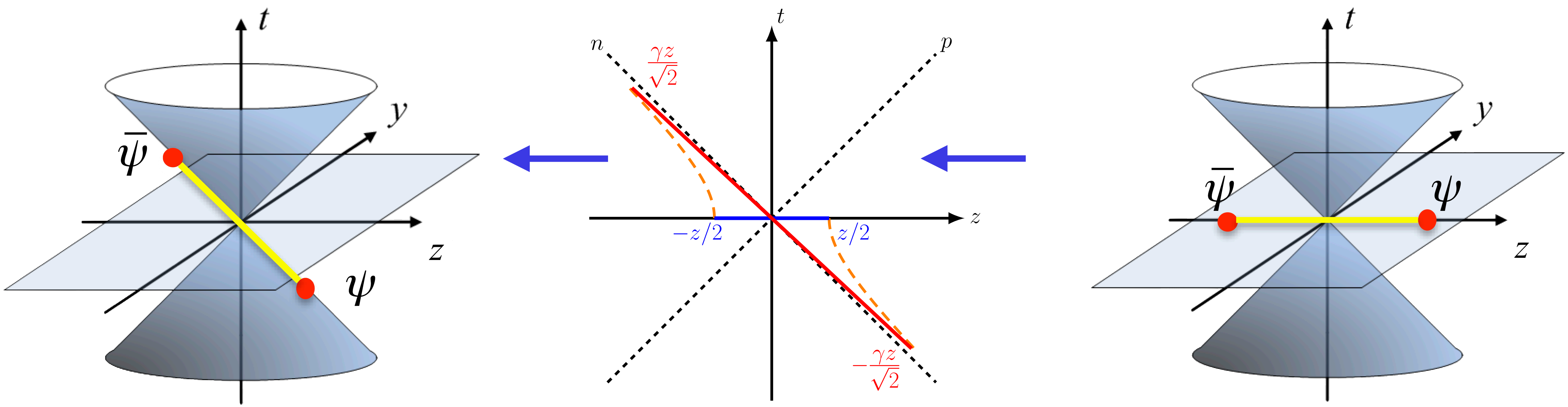
$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{ib^z(xP^z)} \langle P | \bar{\psi}(b^z) \times \frac{\gamma^z}{2} W[b^z, 0] \psi(0) | P \rangle$$

Large-Momentum Effective Theory (LaMET)

$z + ct = 0, \;\; z - ct \neq 0$

Related by Lorentz boost

$t = 0, \;\; z \neq 0$



PDF $f(x)$:
Cannot be calculated on the lattice

Equal in the infinite boost limit

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

$$f(x) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(b^-) \times \frac{\gamma^+}{2} W[b^-,0] \psi(0) | P \rangle$$

$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{ib^z(xP^z)} \langle P | \bar{\psi}(b^z) \times \frac{\gamma^z}{2} W[b^z,0] \psi(0) | P \rangle$$

Large-Momentum Effective Theory (LaMET)

$$\lim_{P^z\to\infty} \tilde{f}(x, P^z) \stackrel{?}{=} f(x)$$ ✘

- Quasi-PDF: $P^z \ll \Lambda$;

Λ: the ultraviolet lattice cutoff, $\sim 1/a$

- PDF: $P^z = \infty$, including $P^z \gg \Lambda$.
  - The limits $P^z \ll \Lambda$ and $P^z \gg \Lambda$ are not exchangeable;
  - As long as $P^z \gg \Lambda_{\text{QCD}}$, their infrared (nonperturbative) physics are the same, while their difference is perturbatively calculable.

Large-Momentum Effective Theory (LaMET)

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$$\tilde{f}(x,P^z) = \boxed{C\left(x, P^z/\mu\right)} \otimes f(x,\mu) + O\Big(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\Big)$$

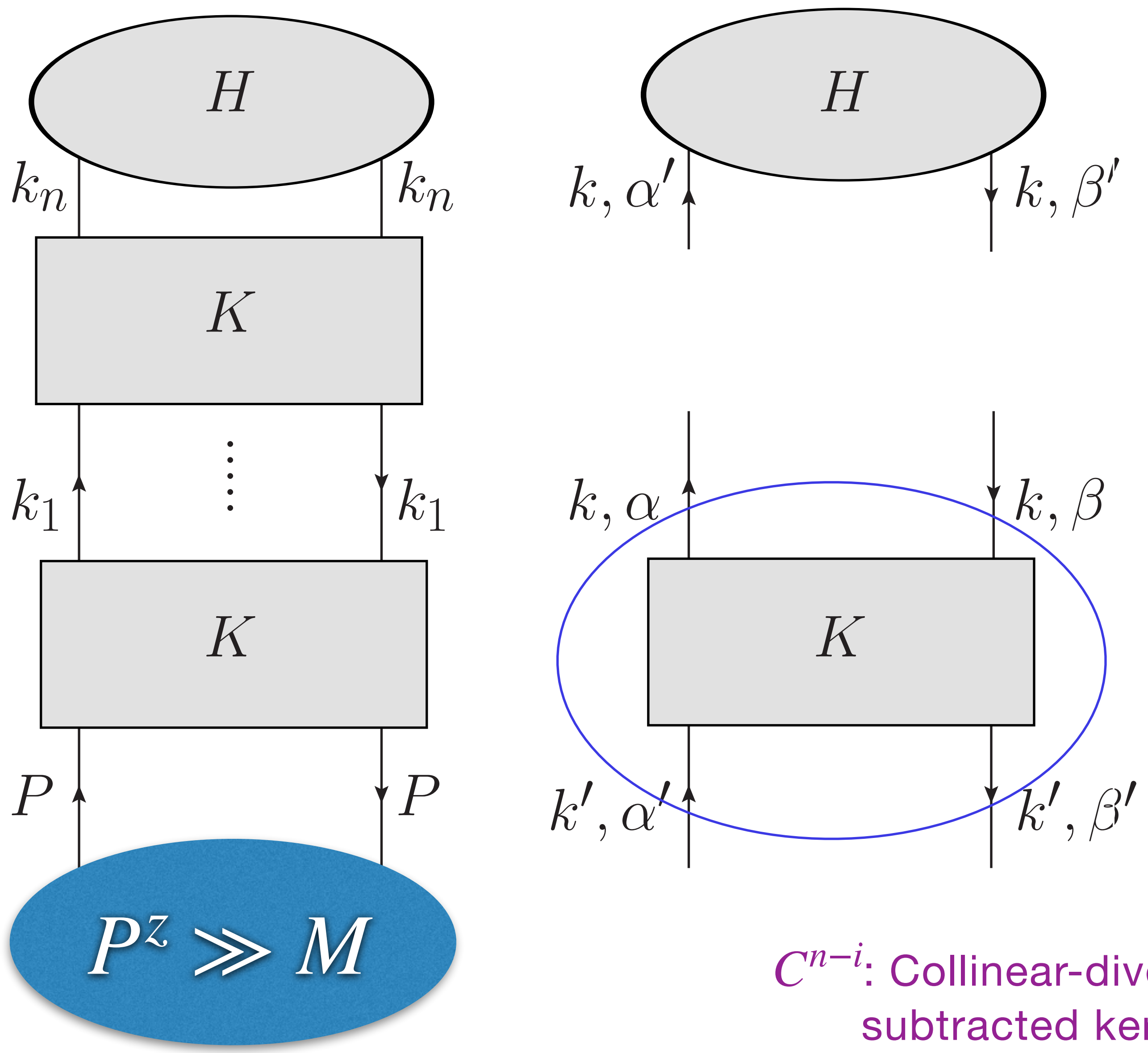
M: Proton mass

Perturbative matching

Power corrections

- X. Ji, PRL 110 (2013); SCPMA57 (2014).
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

Large-Momentum Effective Theory (LaMET)



$P^\mu \sim (P^z, 0, 0, P^z)$

- Large-momentum state, instead of the operator, filters out collinear modes $k \propto P$ in the field operators;
- Contribution from the collinear modes is identical to the PDF.

$$q^i(x, \epsilon_{IR}) = \int \frac{dk^- d^{d-2}k_\perp}{2(2\pi)^d} \mathrm{tr}\left[\gamma^+ K^i(xP^+, k^-, k_\perp; P)\not{P}\right],$$

$$\tilde{q}(y, P^z, \epsilon_{IR}) = \sum_{n=0}^{\infty} \sum_{i=0}^{n} \int \frac{dx}{x} C^{n-i}(y, x, P^z) q^i(x, \epsilon_{IR})$$

$$= \int \frac{dx}{x} C(y, x, P^z) q(x, \epsilon_{IR})$$

C^{n-i}: Collinear-divergence subtracted kernel

X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

Large-Momentum Effective Theory (LaMET)

- Factorization formula:

$$\tilde{f}(y, P^z) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-y)P^z)^2}\right)$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

- The formula can be inverted order by order in α_s:

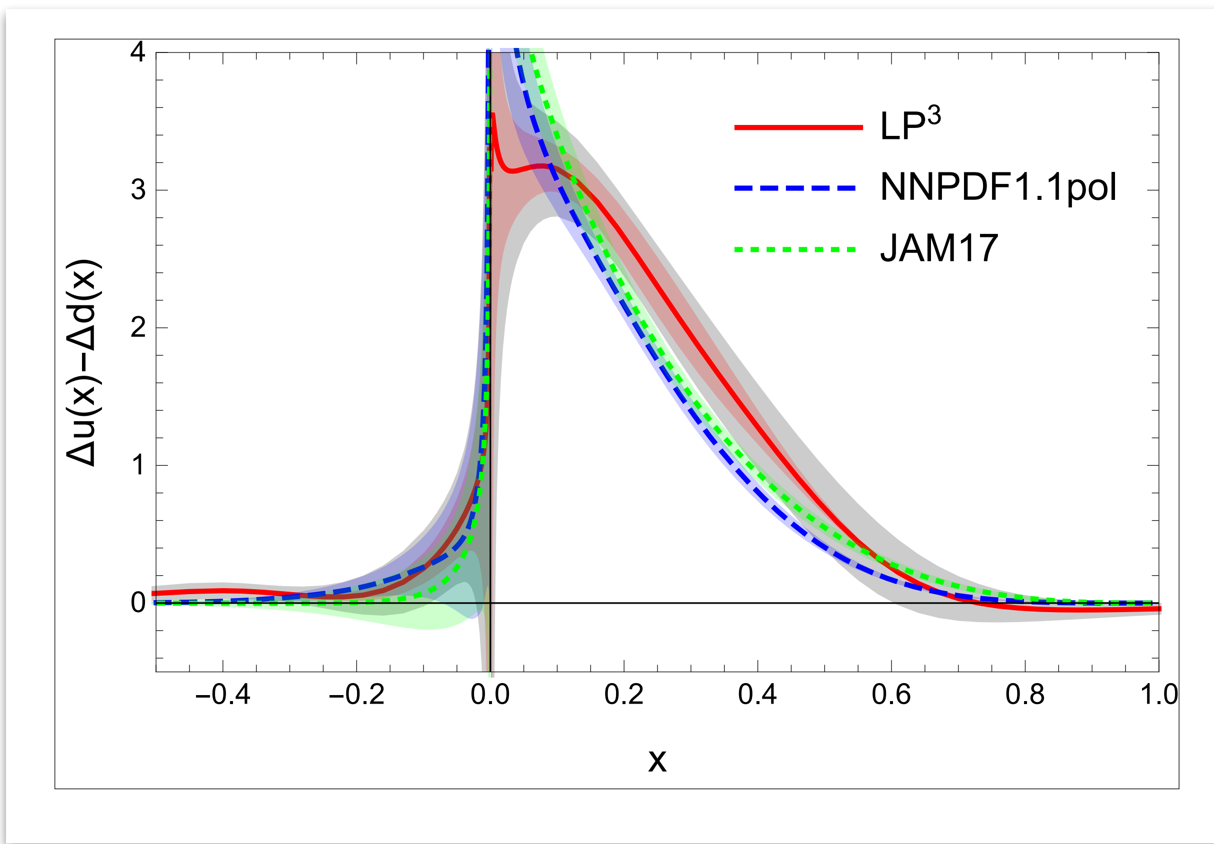
$$f(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Controlled power expansion for $x \in [x_{\rm min}, x_{\rm max}]$ at finite P^z

Systematic lattice calculations of the PDFs

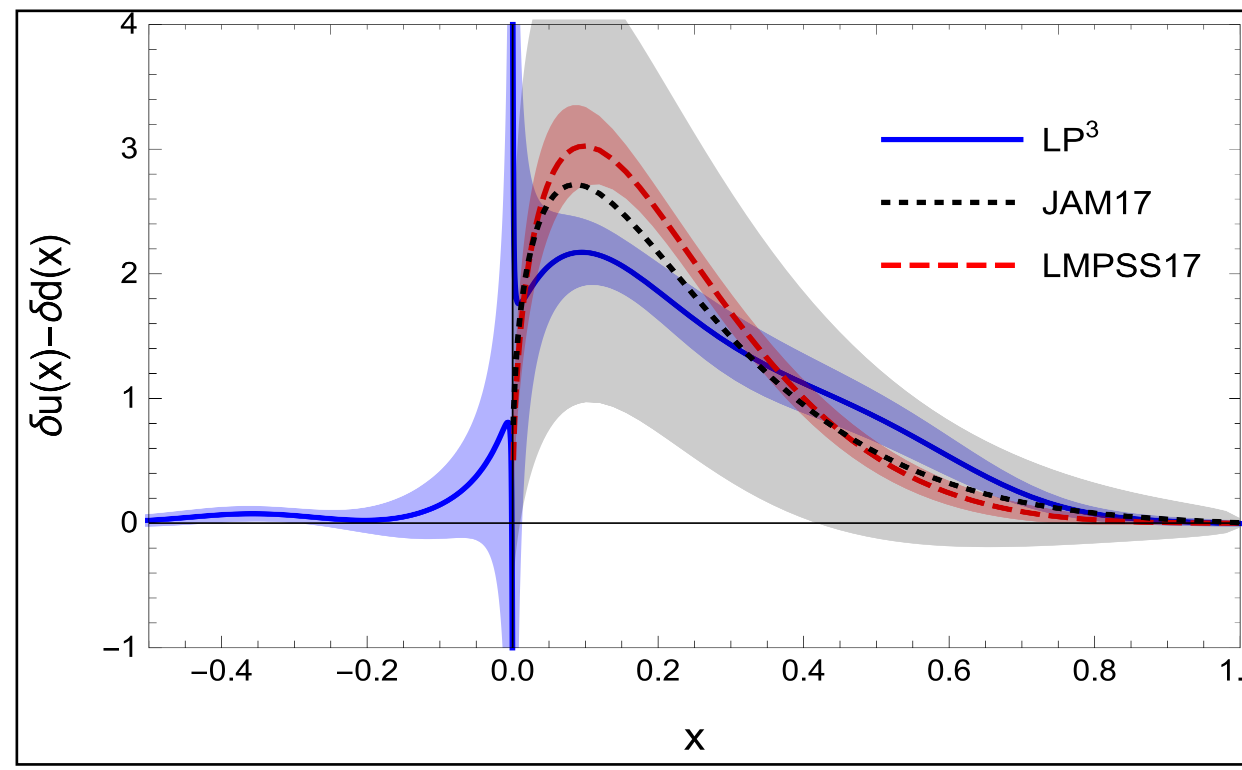
- For example, the isovector (u-d) PDFs of the proton:

Helicity PDF



H.W. Lin, YZ, et al. (LP3 Collaboration), Phys.Rev.Lett. 121 (2018)

Transversity PDF



Y.S. Liu, YZ, et al. (LP3), arXiv:1810.05043

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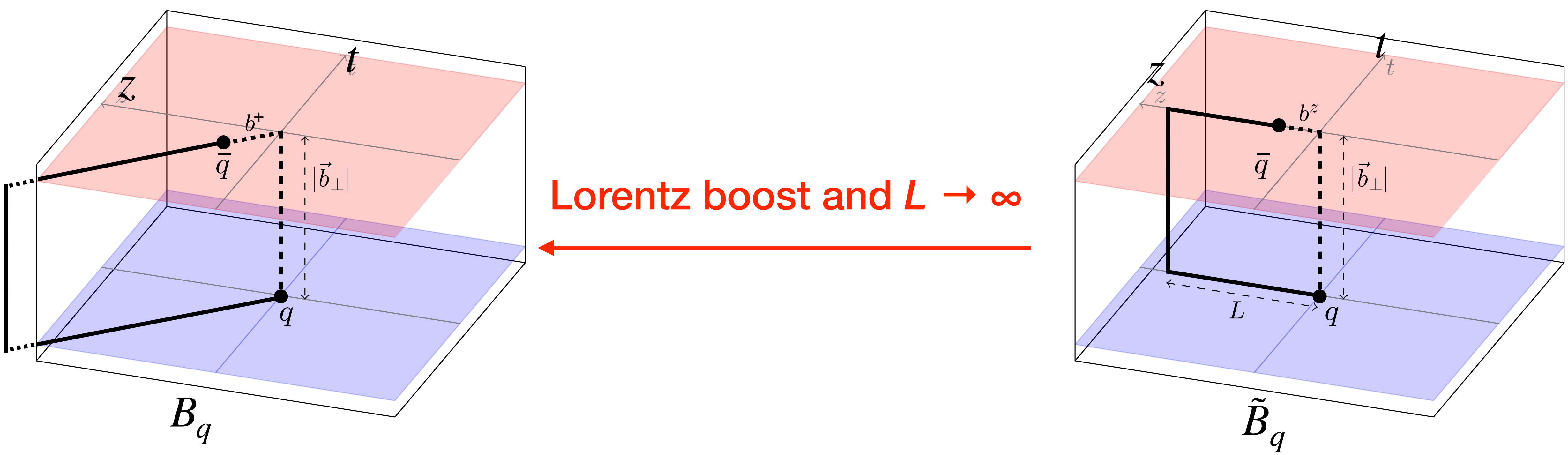
Construction of Quasi-TMDPDF

- Quasi-beam function on lattice:

$$\tilde{B}^q_\Gamma(x, \vec{b}_T, a, L, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$$

$$= \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \langle P \,|\, \bar{q}(b^\mu) W_{\hat{z}}(b^\mu; L - b^z) \frac{\Gamma}{2} W_T(L\hat{z}; \vec{b}_T, \vec{0}_T) W^\dagger_{\hat{z}}(0) q(0) \,|\, P \rangle$$

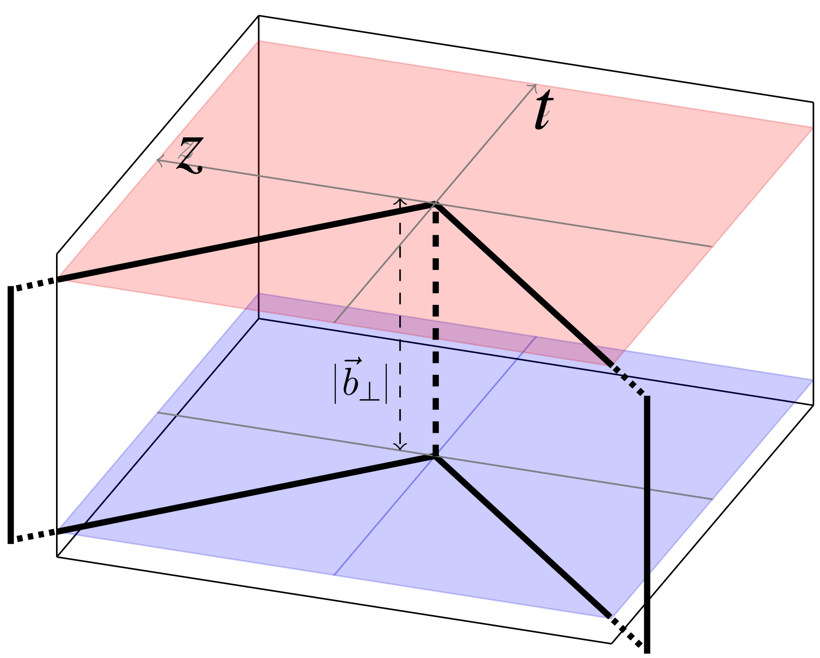


- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., PRD99 (2019), JHEP09 (2019) 037.
- Ji, Liu and Liu, arXiv: Nucl.Phys.B 955 (2020), 1911.03840.

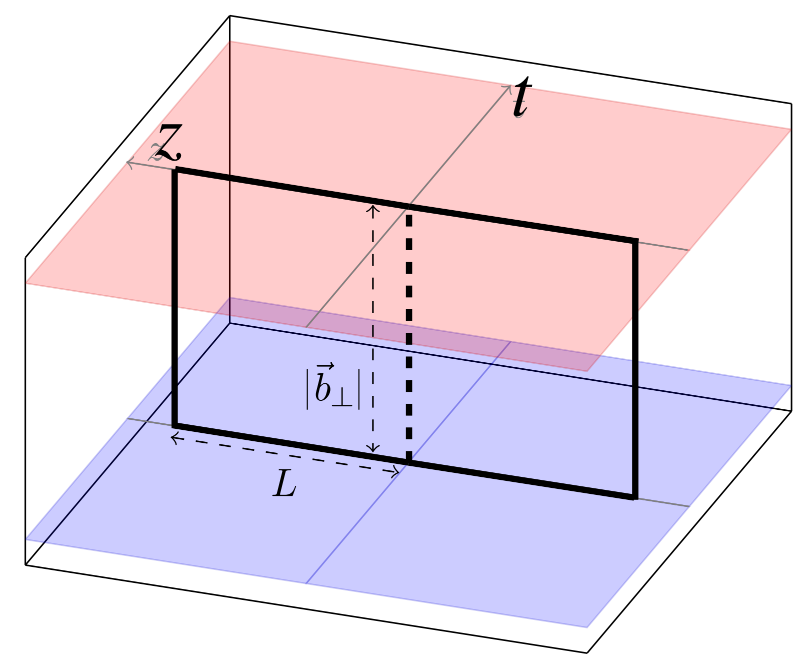
Construction of Quasi-TMDPDF

- Quasi-soft function on lattice (naive definition):

$$\tilde{S}_q(b_T, a, L) = \frac{1}{N_c}\langle 0 |\, \mathrm{Tr}\left[S^{\dagger}_{\hat{z}}(\vec{b}_T; L) S_{-\hat{z}}(\vec{b}_T; L) S_T(L\hat{z}; \vec{b}_T, \vec{0}_T) S^{\dagger}_{-\hat{z}}(\vec{0}_T; L) S_n(\vec{0}_T; L) S^{\dagger}_T(-L\hat{z}; \vec{b}_T, \vec{0}_T) \right] | 0 \rangle$$



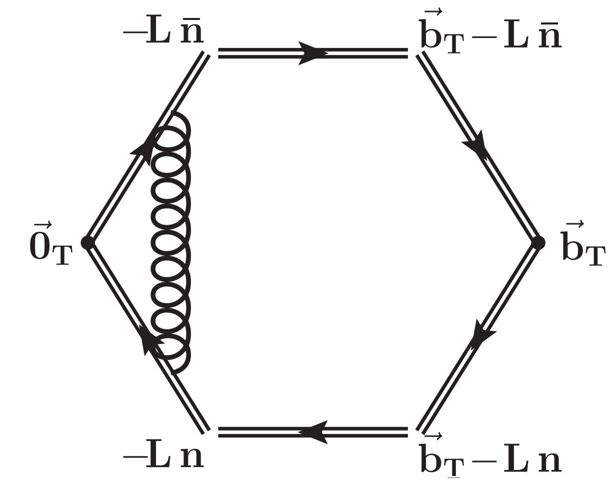
Cannot be related by Lorentz boost



Nevertheless, it can be used to cancel the linear divergences in the Wilson-line self-energy.

Impact of finite-length Wilson lines

- Linear power divergence under lattice regularization $\sim L/a$
- Finite L regulates rapidity divergences:



  - Light-like Wilson lines

$$g_s t^a n^\mu \frac{1-e^{ik^+L}}{k^+} \xrightarrow{L\to\infty} g_s t^a n^\mu \frac{1}{k^+}$$

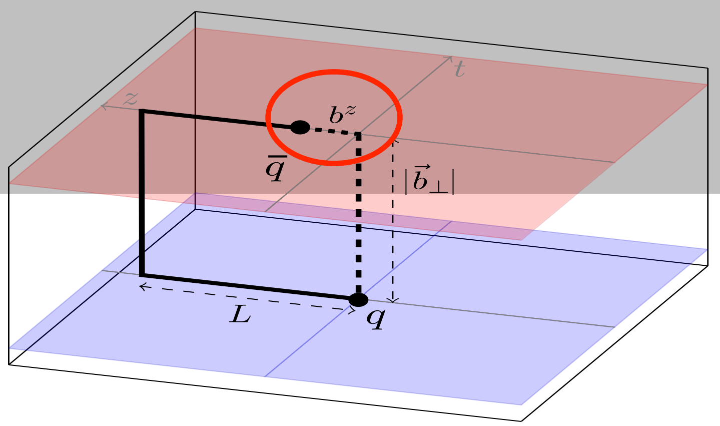
$$I_{\rm div} = \int dk^+ dk^- \frac{1}{(k^+k^-)^{1+\epsilon}} \longrightarrow \int dk^+ dk^- \frac{1}{(k^+k^-)^{\epsilon}} \frac{1-e^{ik^+L}}{k^+} \frac{1-e^{ik^-L}}{k^-}$$

  - Space-like Wilson lines

$$\tilde{I}_{\rm div} = \int dk_0 dk_z \frac{1}{(k_0^2-k_z^2)^\epsilon} \frac{1}{k_z^2} \longrightarrow \int dk_0 dk_z \frac{1}{(k_0^2-k_z^2)^\epsilon} \frac{1-e^{ik^zL}}{k^z} \frac{1-e^{-ik^zL}}{k^z}$$

- By construction the L-dependence has to be canceled out between the quasi-beam and soft functions.

Quasi-TMDPDF



- Quasi-TMDPDF in the MSbar scheme:

$$\tilde{f}_q^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \mu, a) \frac{\tilde{B}_q(b^z, \overrightarrow{b}_T, a, L, P^z)}{\sqrt{\tilde{S}_q(b_T, a, L)}}$$

- Relation to the TMDPDF:

$$\tilde{f}_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right) g_q^S(b_T, \mu) \ \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right]$$

$$\times f_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, \zeta) \ + \ \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

Hierarchy of scales: $b^z \sim \dfrac{1}{P^z} \ll b_T \ll L, \quad b_T \sim \Lambda_{\rm QCD}^{-1}$

$C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right)$ is independent of the spin structures!

- Vladimirov and Schaefer, arXiv: 2002.07527;
- Ebert, Schindler, Stewart, Y.Z., JHEP *JHEP* 09 (2020).

- M. Ebert, I. Stewart, Y.Z., JHEP 1909 (2019) 037;
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840;
- Vladimirov and Schaefer, arXiv: 2002.07527;
- Ji, Liu, YZ Liu, Zhang and Y.Z., arXiv: 2004.03543.

One-loop test

- Physical TMDPDF:

- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- Ebert, Stewart and Y.Z., JHEP 1909 (2019) 037.

$$f_q^{\mathrm{TMD}(1)}(x, \vec{b}_T, \epsilon, \zeta) = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\mathrm{IR}}} + \mathbf{L}_b\right)\frac{1+x^2}{1-x} + (1-x)\right]_+ \theta(x)\theta(1-x)$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[\frac{1}{\epsilon_{\mathrm{UV}}^2} + \frac{1}{\epsilon_{\mathrm{UV}}}\left(\frac{3}{2} + \mathbf{L}_\zeta\right) + \frac{1}{2} - \frac{\pi^2}{12}\right]$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[-\frac{1}{2}\mathbf{L}_b^2 + \frac{3}{2}\mathbf{L}_b + \mathbf{L}_b\mathbf{L}_\zeta\right]$$

$$b_0 = 2e^{-\gamma_E}$$

$$\mathbf{L}_b = \ln\frac{b_T^2\mu^2}{b_0^2}$$

$$\mathbf{L}_\zeta = \ln\frac{\mu^2}{\zeta}$$

- Naive quasi-TMDPDF:

$$\tilde{f}_q^{\mathrm{TMD}(1)}(x, \vec{b}_T, \epsilon, P^z) = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\mathrm{IR}}} + \mathbf{L}_b\right)\frac{1+x^2}{1-x} + (1-x)\right]_+ \theta(x)\theta(1-x)$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[\frac{3}{2}\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{2}\ln^2\frac{\mu^2}{(2xP^z)^2} - \ln\frac{\mu^2}{(2xP^z)^2} - \frac{3}{2}\right]$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[-\frac{1}{2}\mathbf{L}_b^2 + \frac{5}{2}\mathbf{L}_b + \mathbf{L}_b\ln\frac{\mu^2}{(2xP^z)^2}\right]$$

$$g_q^{S_{\mathrm{naive}}}(b_T, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\mathbf{L}_b$$

One-loop test

- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- Ebert, Stewart and Y.Z., JHEP 1909 (2019) 037.

- **Physical TMDPDF:**

$$f_q^{\mathrm{TMD}(1)}(x,\vec{b}_T,\epsilon,\zeta) = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\mathrm{IR}}}+\mathbf{L}_b\right)\frac{1+x^2}{1-x}+(1-x)\right]_+ \theta(x)\theta(1-x)$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[\frac{1}{\epsilon_{\mathrm{UV}}^2}+\frac{1}{\epsilon_{\mathrm{UV}}}\left(\frac{3}{2}+\mathbf{L}_\zeta\right)+\frac{1}{2}-\frac{\pi^2}{12}\right]$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[-\frac{1}{2}\mathbf{L}_b^2+\frac{3}{2}\mathbf{L}_b+\mathbf{L}_b\mathbf{L}_\zeta\right]$$

$$b_0 = 2e^{-\gamma_E}$$

$$\mathbf{L}_b = \ln\frac{b_T^2\mu^2}{b_0^2}$$

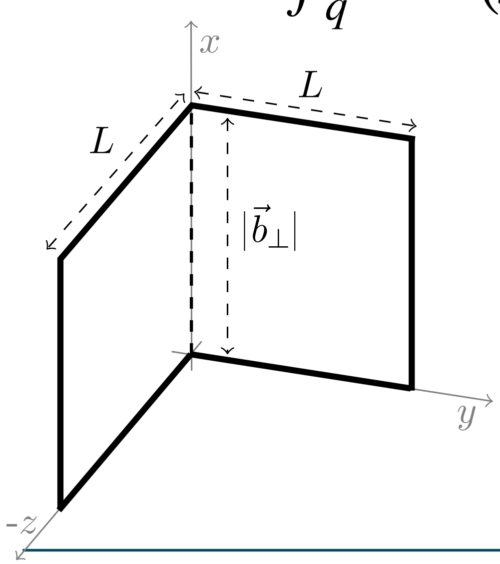
$$\mathbf{L}_\zeta = \ln\frac{\mu^2}{\zeta}$$

- **Bent soft function:**

$$\tilde{f}_q^{\mathrm{TMD}(1)}(x,\vec{b}_T,\epsilon,P^z) = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\mathrm{IR}}}+\mathbf{L}_b\right)\frac{1+x^2}{1-x}+(1-x)\right]_+ \theta(x)\theta(1-x)$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[\frac{3}{2}\frac{1}{\epsilon_{\mathrm{UV}}}-\frac{1}{2}\ln^2\frac{\mu^2}{(2xP^z)^2}-\ln\frac{\mu^2}{(2xP^z)^2}-\frac{3}{2}\right]$$

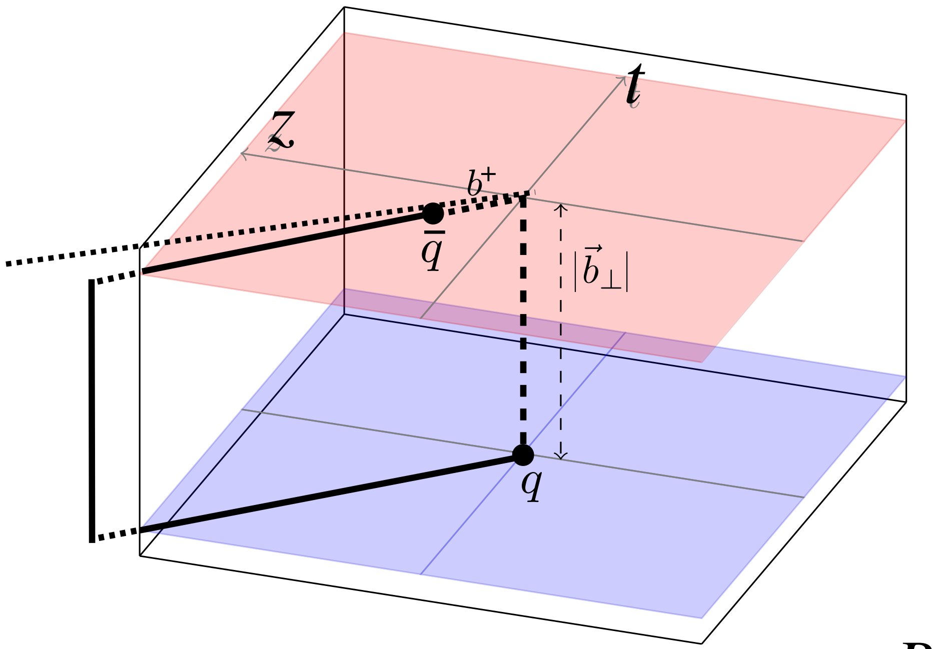
$$+\frac{\alpha_s C_F}{2\pi}\delta(1-x)\left[-\frac{1}{2}\mathbf{L}_b^2+\frac{3}{2}\mathbf{L}_b+\mathbf{L}_b\ln\frac{\mu^2}{(2xP^z)^2}\right]$$



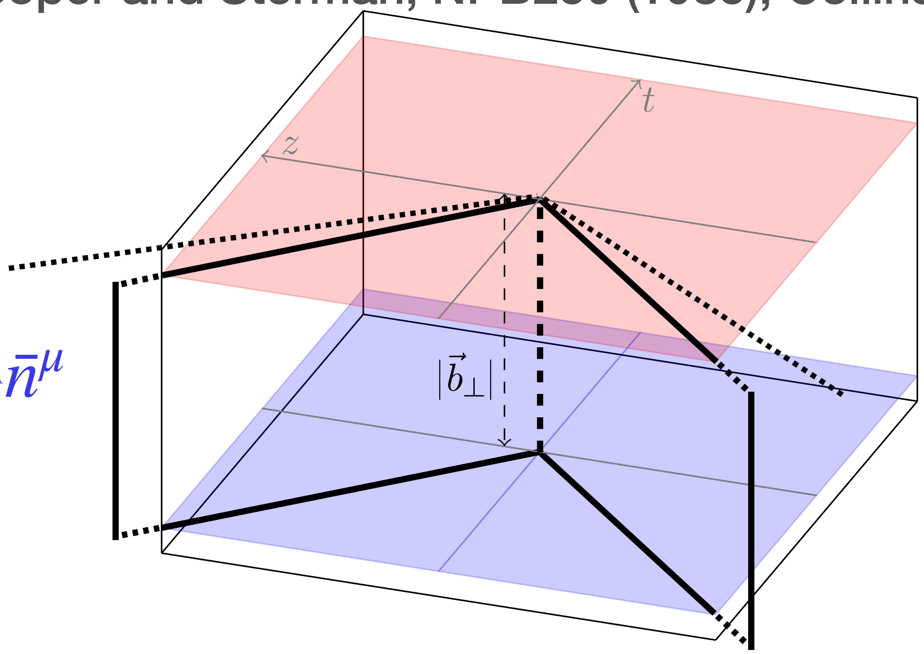
Comparison to Collins-Soper-Sterman Scheme

- Wilson lines off the light-cone:

Collins, Soper and Sterman, NPB250 (1985); Collins, 2011



$$n_A^\mu \equiv n^\mu - e^{-2y_A}\bar{n}^\mu$$



$$n_B^\mu \equiv \bar{n}^\mu - e^{2y_B}n^\mu$$

$$f_q^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{y_B \to -\infty} Z_{UV} \frac{B_q(x, \vec{b}_T, \epsilon, y_P - y_B)}{\sqrt{S^q(b_T, \epsilon, 2(y_n - y_B))}}$$

$$\lim_{y_P - y_B \to -\infty} B_q(x, \vec{b}_T, \epsilon, y_P - y_B) \propto e^{(y_P - y_B)\gamma_\zeta(b_T, \mu)}$$

$$\lim_{y_n - y_B \to -\infty} S^q(b_T, \mu, 2(y_n - y_B)) = e^{2(y_n - y_B)\gamma_\zeta(b_T,\mu) + \mathscr{D}(b_T,\mu)}$$

A. Vladimirov, JHEP 04 (2018).

- $e^{\mathscr{D}(b_T,\mu)}$ is what is missing in the quasi soft functions, which is intrinsically Minkowskian.
- The bent soft function does not work because of rapidity angle.

- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840;
- Ji, Liu, YZ Liu, Zhang and Y.Z., arXiv: 2004.03543.

Reduced soft function

$$\tilde{f}_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right) g_q^S(b_T, \mu) \ \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right]$$
$$\times f_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, \zeta) \ + \ \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

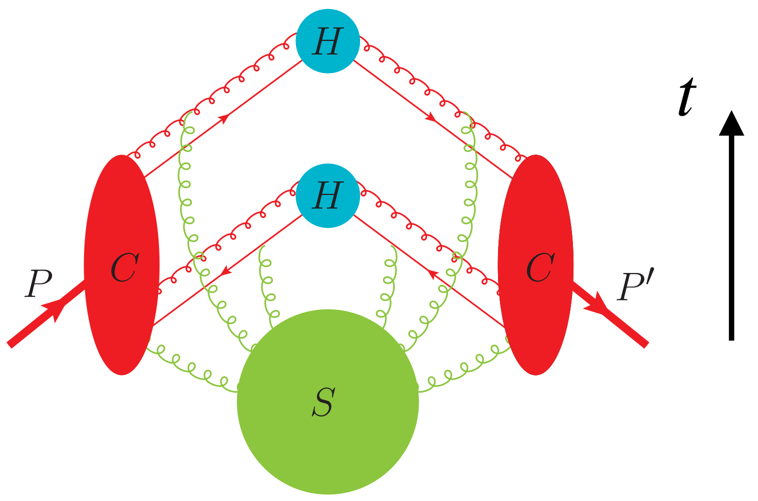
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840;

Related to the "instant jet function" in

- Vladimirov and Schaefer, arXiv: 2002.07527.

$$\boxed{g_q^S(b_T, \mu) = \sqrt{S_r^q(b_T, \mu)} = e^{\frac{1}{2}\mathscr{D}(b_T, \mu)}}$$

- $S_r^q(b_T, \mu)$ from a light-meson form factor:



$$F(b_T, P^z)$$
$$= \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle$$
$$= S_q^r(b_T, \mu) \ H(x, \mu) \otimes \Phi^\dagger(x, b_T, -P^z) \otimes \Phi(x, b_T, P^z)$$

Φ: Quasi-TMD distribution amplitudes

$$\langle 0 \,|\, \bar{q}(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger q(0) \,|\, \pi(P) \rangle$$

- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840.

Outline

- Overview of TMD factorization
- Calculation of TMDPDFs from LaMET
  - Large-Momentum Effective Theory
  - Quasi-TMDPDF and TMDPDF
- Exploratory lattice results
  - The Collins-Soper evolution kernel
  - Soft factor

Collins-Soper kernel from lattice QCD

- Collins-Soper kernel from momentum evolution of quasi-TMDs:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\,\tilde f_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\,\tilde f_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_2^z)}$$
$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\,\tilde B_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\,\tilde B_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_2^z)}$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ebert, Stewart and YZ, Phys.Rev.D 99 (2019).

- g^S as well as the quasi-soft function are canceled in the ratio;
- Does not depend on the external hadron state, could be calculated with pion for simplicity;
- One can also calculate ratios of TMDPDFs with different spin structures.

Collins-Soper kernel from lattice QCD

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$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

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- Does not depend on the external hadron state, could be calculated with pion for simplicity;
- One can also calculate ratios of TMDPDFs with different spin structures.

The idea of forming ratios has been used in the calculation of *x*-moments of TMDPDFs:

Hagler, Musch, Engelhardt, Yoon, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

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$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

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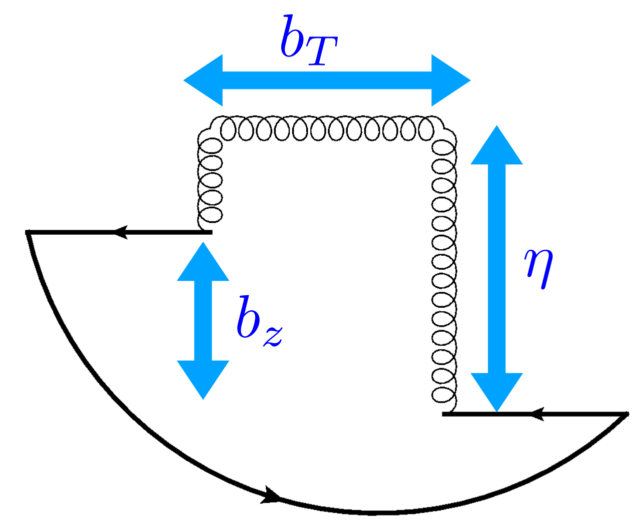
Collins-Soper kernel from lattice QCD

- Calculation of bare quasi beam function;
  - Quenched (Nf=0 for QCD evolution) Wilson gauge configurations;
  - Probe valence pion with $m_\pi \sim 1.2\ \mathrm{GeV}$;
  - Pion momentum $P^z = n^z 2\pi/L = \{2,3,4\}2\pi/L = \{1.3,1.9,2.6\}\mathrm{GeV}$.



Shanahan, Wagman, Y.Z., Phys.Rev.D 102 (2020).

- Lattice renormalization and matching to the MSbar scheme;

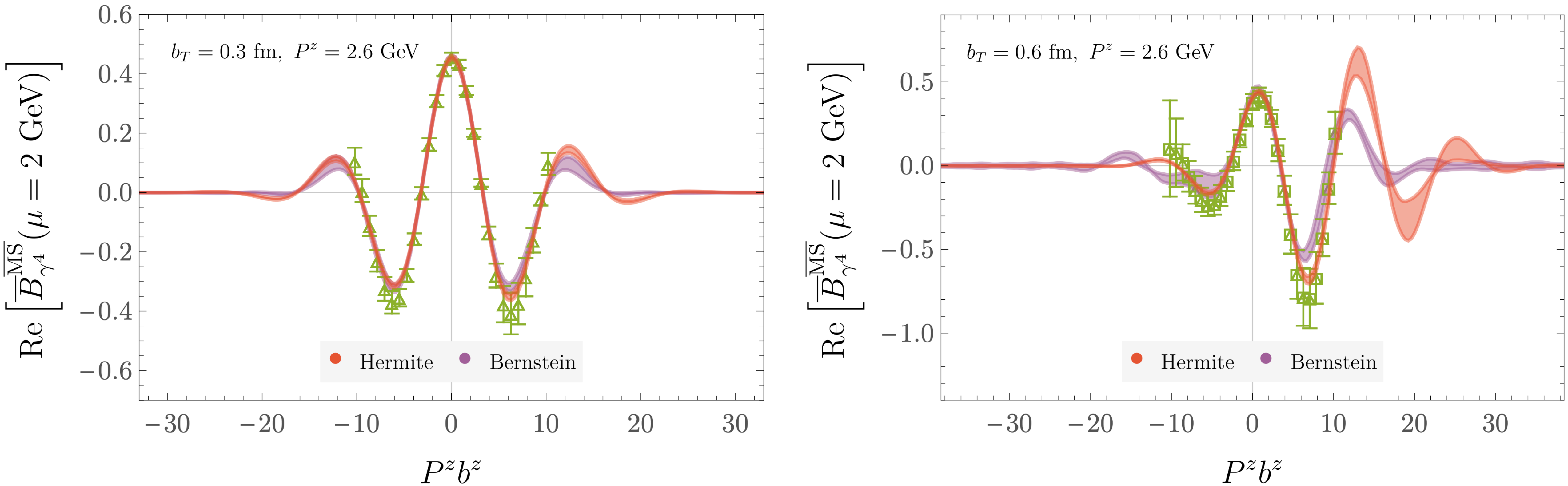
  - Constantinou, Panagopoulos and Spanoudes, PRD99 (2019) no.7, 074508;
  - Ebert, Stewart and Y.Z., JHEP 03 (2020) 099;
  - Shanahan, Wagman and Y.Z., Phys.Rev.D 101 (2020).

- Fourier transform to the momentum space and take ratios.

Extraction of the Collins-Soper kernel

- Fourier transform suffers from severe truncation effects

- Fitting the quasi beam function with models



Both models can fit to the quasi beam functions with good χ^2/d.o.f. with consistent values of the Collins-Soper kernel.

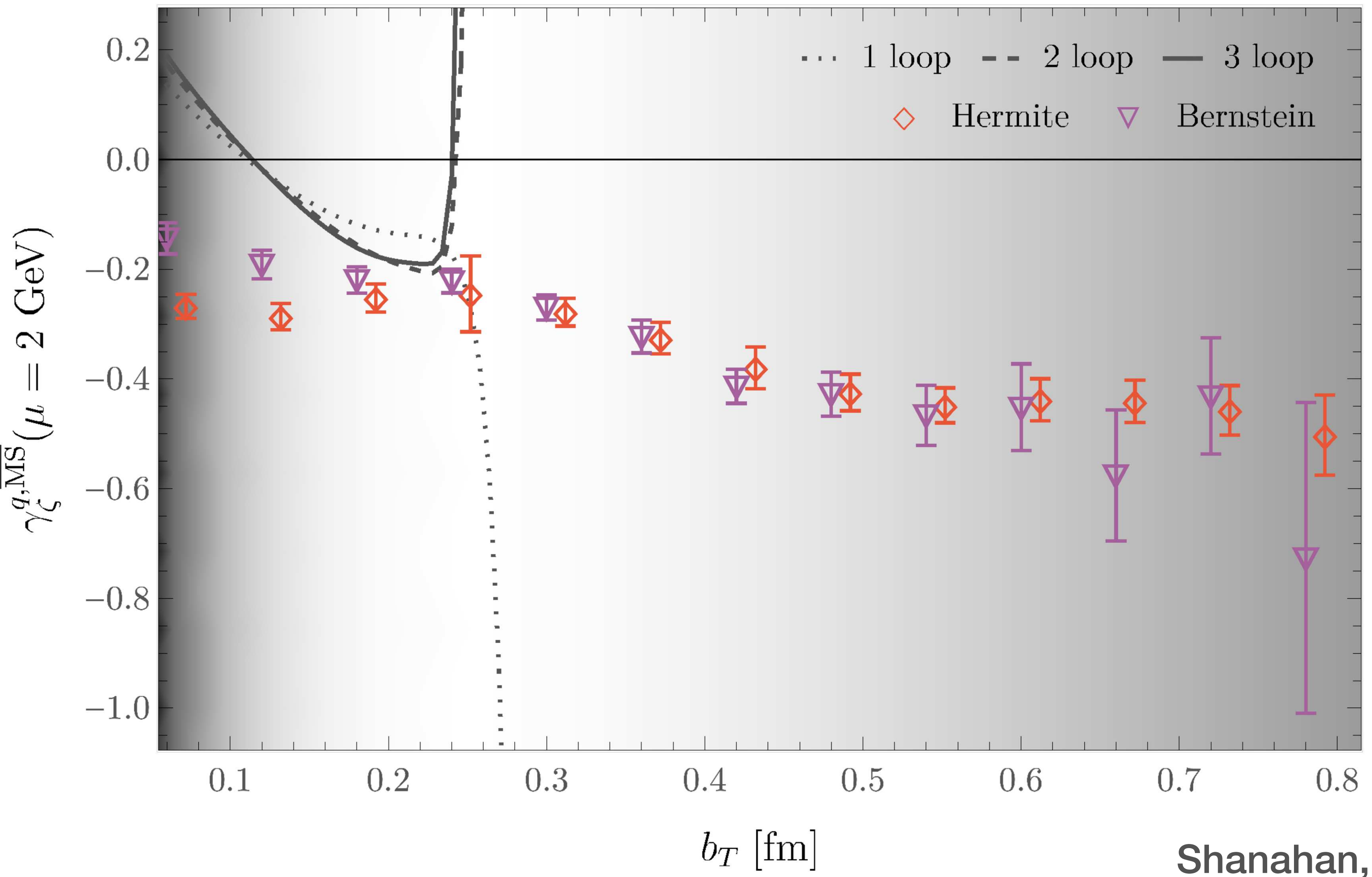
Shanahan, Wagman, Y.Z., Phys.Rev.D 102 (2020).

Extraction of the Collins-Soper kernel

- Collins-Soper kernel from the model fitting method:

Perturbative prediction with $N_f = 0$.

- Li and Zhu, PRL118 (2017) 2, 022004.
- A. A. Vladimirov, PRL118, 062001 (2017)



Note the power corrections

$$\mathcal{O}\left(\frac{b_T}{\eta}, \frac{1}{b_T P^z}, \frac{1}{P^z \eta}\right)$$

Background density

$$\sim \frac{b_T}{\eta} + \frac{1}{b_T P^z}$$

Shanahan, Wagman, Y.Z., Phys.Rev.D 102 (2020).

Extraction of the Collins-Soper kernel

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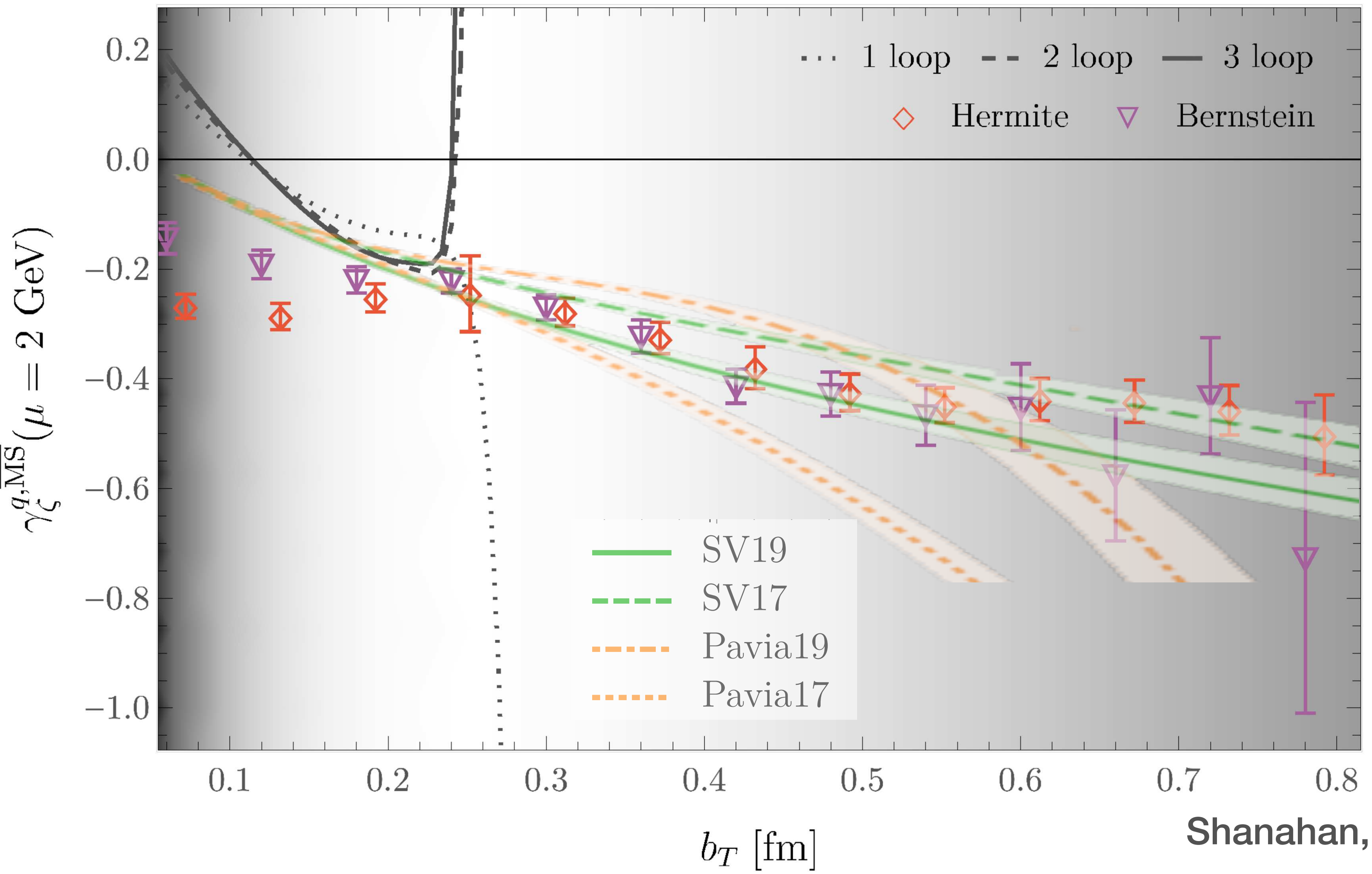
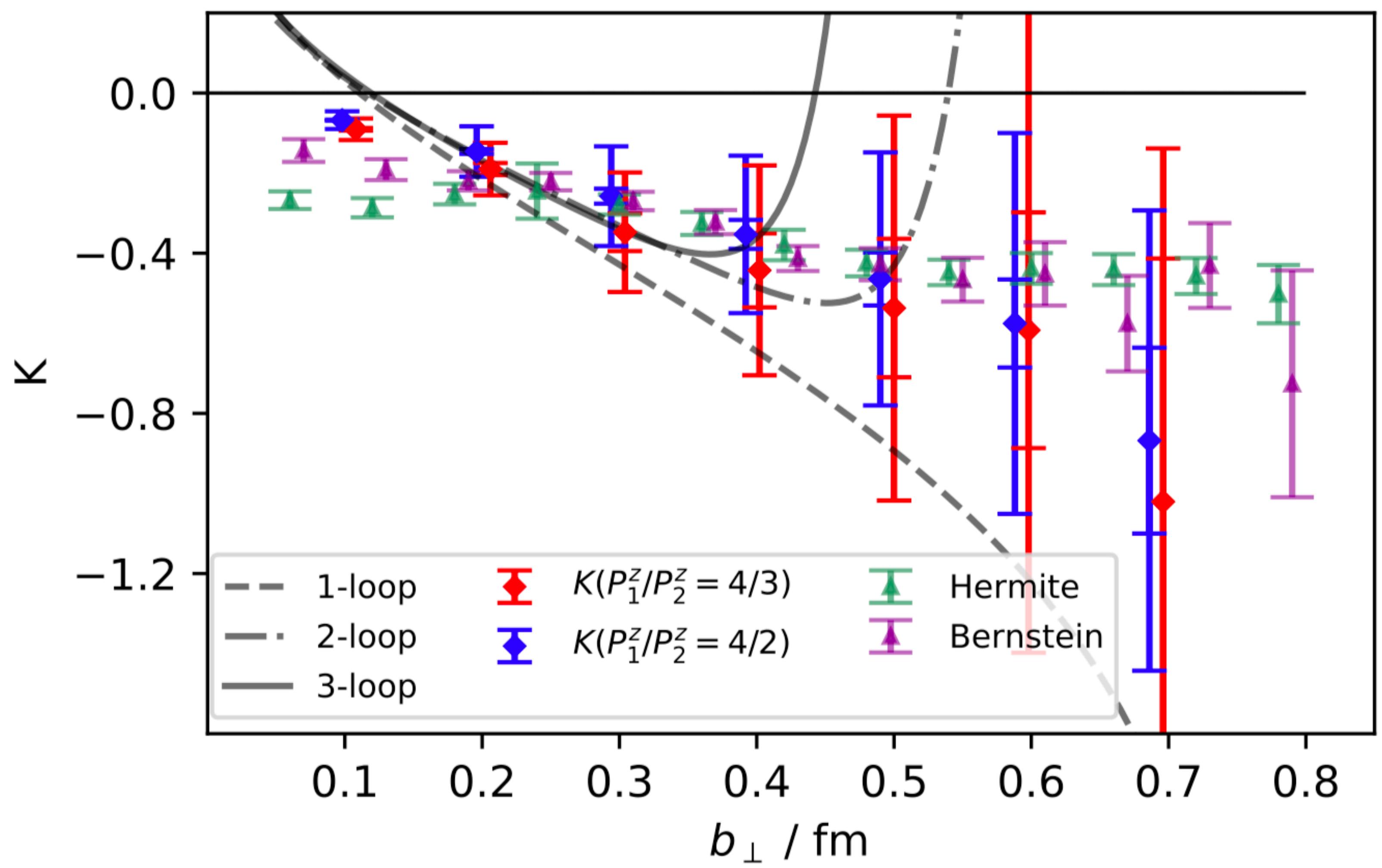


Shanahan, Wagman, Y.Z., Phys.Rev.D 102 (2020).

Extraction of the Collins-Soper kernel

- Comparison with CS kernel extracted from the quasi-TMD distribution amplitude:



Q.-A. Zhang, et al. (LP Collaboration), 2005.14572.

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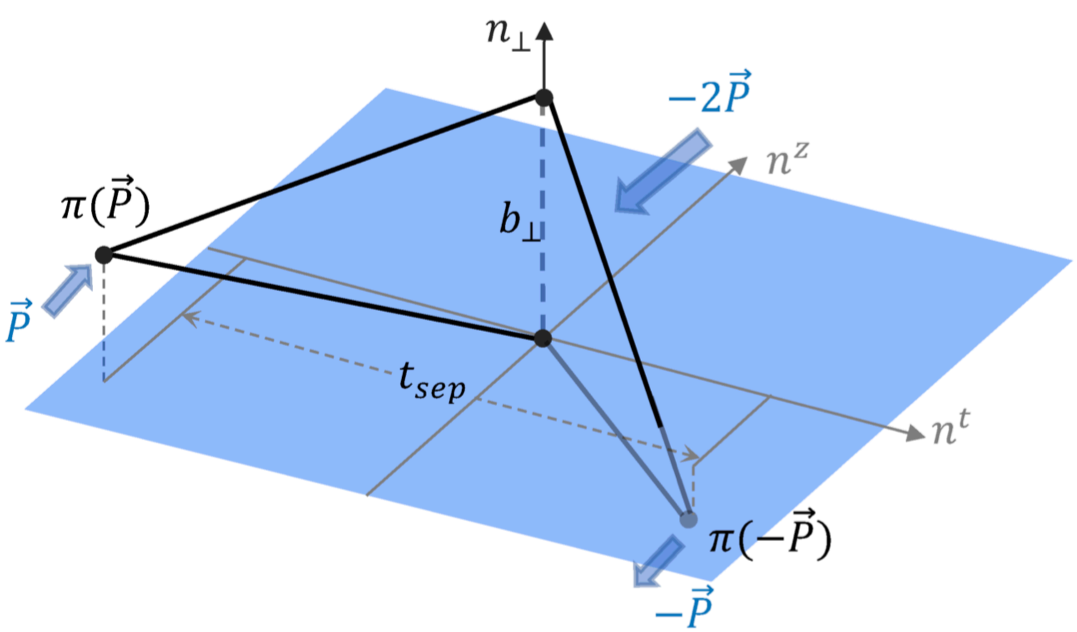
TMD soft function from lattice

$$F(b_T, P^z) = \langle \pi(-P)\,|\,j_1(b_T)j_2(0)\,|\,\pi(P)\rangle$$

$$= S_q^r(b_T,\mu)\; H(x,\mu) \otimes \Phi^\dagger(x, b_T, -P^z) \otimes \Phi^\dagger(x, b_T, P^z)$$

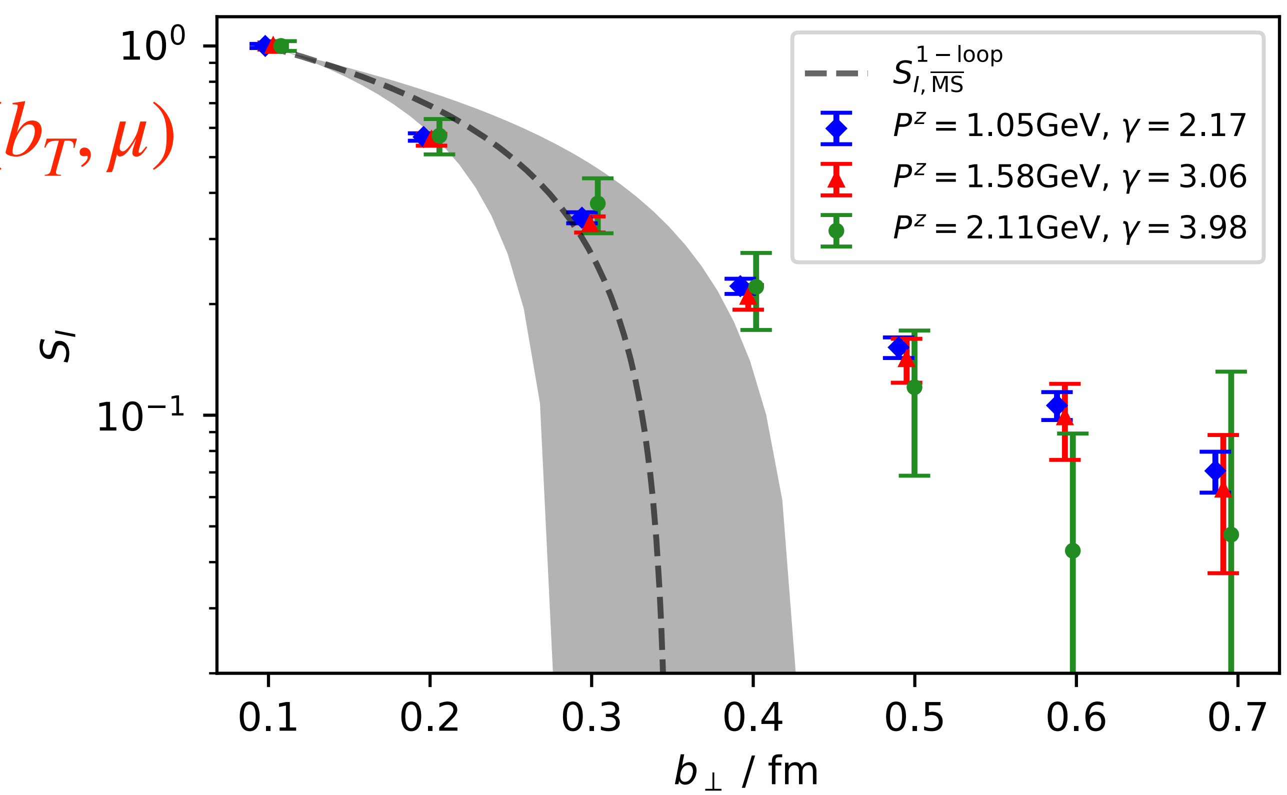
Quasi-TMD distribution amplitudes

Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840



$F(b_T, P^z)$

$S_q^r(b_T,\mu)$



First lattice calculation (with dynamical fermions) with tree-level matching:

Q.-A. Zhang, et al. (LP Collaboration), 2005.14572.

Summary and outlook

- Much progress has been made towards the lattice calculation of TMDPDFs and its evolution with LaMET recently;
- The Collins-Soper kernel and reduced soft factor for $0.2\ \mathrm{fm} < b_T < 0.8\ \mathrm{fm}$ can be robustly determined with contemporary lattice resources.
- Future work will include larger statistics, multiple lattice spacings, dynamical fermions and reduced Fourier transform truncation;
- Ultimate goal is to achieve precision calculation of the Collins-Soper kernel and TMDPDF, which can in principle be used to predict Drell-Yan cross sections.