When does causality constrain the monopole abundance?

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The causality bounds on monopole abundance are reexamined in the light of recently constructed models which display very rapid monopole annihilation. It is shown that the behavior displayed in these models is consistent with causality, and that the proper formulation of the causality bounds is very dependent on the representation content of the model. By invoking intermediate phases with appropriate types of symmetry breaking, it is possible to annihilate essentially all monopoles, or even to avoid their ever appearing.

It is well known that magnetic monopoles could have been produced during the course of certain cosmological phase transitions.[1,2] Specifically, consider a transition in which a symmetry group G is broken to a subgroup H by the development of a nonzero expectation value for some scalar field ϕ. The dynamics does not uniquely determine the value of ϕ, but only restricts it to a manifold $M=G/H$. Just after the transition, the actual value of ϕ will vary from place to place, with a characteristic correlation distance ξ. This twisting of ϕ in internal space can give rise to monopoles if $\Pi_2(G/H)$ is nontrivial.

A rough estimate of this initial monopole density is $n_{\text{init}} \sim p\xi^{-3}$, where p is a geometric factor expected to be of order 10^{-1}. While the calculation of ξ requires a detailed understanding of the dynamics, causality considerations imply that the orientation of the scalar field cannot be correlated over distances greater than the horizon distance d_H. This implies the horizon bound[3]

$$n(t) \gtrsim p d_H^{-3}(t) \ . \tag{1}$$

Although this bound was originally applied to the initial monopole density, the reasoning behind it should apply equally well at later times, as indicated in Eq. (1). It should therefore restrict the efficacy of any monopole-annihilation mechanism. However, two models[4,5] have been proposed in which a Langacker-Pi-type mechanism[6] appears to annihilate monopoles much more rapidly than allowed by Eq. (1). Furthermore, as we shall show, one can modify the models in such a way as to evade even the bound on the initial monopole production. Our aim in this paper is to resolve the apparent contradiction between causality and the behavior observed in these models.

We begin by recalling the essential features of the models. The first,[4] proposed by Everett, Vachaspati, and Vilenkin (EVV), has a U(1) symmetry and is set in a two-dimensional space, with vortices playing the same role as the three-dimensional monopoles. There are two scalar fields, which we shall denote ϕ and ψ, with the U(1) charge of the former being twice that of the latter. The dynamics is assumed to be such that the Universe, as it cools, passes through the following phases:

(I) unbroken U(1), with $\phi=\psi=0$;
(II) broken U(1), with $|\phi|\neq 0$, $\psi=0$;
(III) broken U(1), with $|\phi|\neq 0$, $|\psi|\neq 0$, and $\phi^*\psi^2$ real;
(IV) same as phase II.

(The fourth phase does not occur in the original version of the model.) Vortices are formed during the transition from phase I to II. When the Universe passes to phase III, the vortices become attached to strings. These can link either a vortex-antivortex pair or two vortices of the same sign. The contraction of these strings leads to vortex-antivortex annihilation in the former case and to the formation of doubly charged vortices in the latter. At the final transition, to phase IV, the strings disappear, thus liberating any remaining singly charged vortices. The doubly charged vortices may or may not dissociate at this time, depending on the dynamics; let us assume that they do not.

The model of Copeland *et al.*[5] (CHKMT) is a three-dimensional model with an SU(2) symmetry. The scalar fields comprise a triplet $\boldsymbol{\phi}$ and a doublet ψ. The sequence of phases is rather similar to that of the EVV model:

(I′) unbroken SU(2), with $\boldsymbol{\phi}=0$, $\psi=0$;
(II′) SU(2) broken to U(1), with $\boldsymbol{\phi}\neq 0$, $\psi=0$;
(III′) SU(2) completely broken, with $\boldsymbol{\phi}\neq 0$, $\psi\neq 0$, and $\psi^\dagger\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi}$ maximal;
(IV′) same as phase II′.

(Again, the fourth phase does not appear in the original version of the model.) Monopoles are produced during the transition to phase II′. At the next transition they become attached to strings; in contrast with the EVV model, these strings can only connect oppositely charged objects. As before, the final phase transition dissolves the strings, thus liberating the surviving monopoles.

In these models the rate of pair annihilation during the third phase is directly related to the distribution of string lengths at the onset of the phase. Both computer simulations[4,5,7] and analytic arguments[7] show that in both cases this initial distribution falls exponentially with length, implying that the density of monopoles or unit vortices falls exponentially in time. In the CHKMT model the Universe emerges into the final phase with a monopole density far below the bound of Eq. (1). In the EVV model the density of unit vortices in the final phase is similarly suppressed, although the density of doubly charged vortices is consistent with the horizon bound.

Although these models were constructed as counterexamples to the bound on the annihilation rate, they can easily be modified so as to violate even the bound on the initial density. This is accomplished by replacing the second phase in each model by a phase in which $\phi=0$ but $\psi\neq 0$, while leaving the other phases unchanged. In the EVV case this still results in a breaking of the U(1) symmetry, with ψ vortices being produced at the first phase transition. When the ϕ field becomes nonzero at the next transition, its phase is everywhere determined by the preexisting ψ phase. A doubly charged ϕ vortex is formed around every unit ψ vortex, but no singly charged ϕ vortices can be formed. Since the disappearance of the ψ field at the final phase transition should not disturb the ϕ vortices (recall that we are assuming that doubly charged vortices are stable against dissociation), the Universe can reach the final phase without any unit ϕ vortices ever being produced. The doubly charged vortices do, however, obey the horizon bound, just as in the original version of the model.

In the CHKMT case the development of a nonzero ψ field breaks the SU(2) symmetry completely. No monopoles can be produced at this stage, because $\Pi_2(G/H)=\Pi_2(\mathrm{SU}(2))$ is trivial. To be more explicit, the ψ field at this stage must be of the form

$$\psi(x)=G(x)\begin{bmatrix} v \\ 0 \end{bmatrix}, \tag{2}$$

where $G(x)$ is a 2×2 SU(2) matrix. Continuity of $\psi(x)$ implies continuity of $G(x)$. On any closed surface, the function $G(x)$ is topologically trivial; i.e., it can be continuously deformed to a constant. When $\boldsymbol{\phi}$ becomes nonzero at the next transition, its orientation in internal space is determined by that of ψ. Specifically, it must be such that

$$\boldsymbol{\phi}(x)\cdot\boldsymbol{\tau}=G(x)\tau_z G^{-1}(x)|\boldsymbol{\phi}(x)| \ . \tag{3}$$

Because $G(x)$ is topologically trivial on any closed surface, $\boldsymbol{\phi}(x)$ must also be trivial, and so no monopoles are formed.[8] Since there is no reason to expect monopoles to appear when ψ vanishes at the final transition, we have a mechanism for breaking SU(2) to U(1) without producing any monopoles, clearly contradicting Eq. (1).

These examples clearly demonstrate that the horizon bound as given above can be violated without introducing any nonlocal or noncausal elements into the dynamics. With this in mind, let us try to formulate more carefully the constraints imposed by causality.

Consider a theory with a symmetry group G and a set of scalar fields which we assemble into multiplets ϕ and ψ, each transforming according to a (possibly reducible) representation of G. The dynamics restricts ϕ to a manifold M, thereby breaking G to a subgroup H, with $M=G/H$. (This may be further broken by ψ, but we ignore this for the moment.) We are interested in objects characterized by nonzero values of the topological charge, which is given by an expression of the form

$$Q(\Sigma)=\int_{\Sigma} dS\, f(\phi,\nabla\phi) \ , \tag{4}$$

where the integration is over a surface Σ with the topology of an n-sphere S^n ($n=1$ or 2 in the models above). In particular, we want to consider the case where Σ is much larger than the horizon. The field ϕ at a given point x on Σ can take any value in M with equal probability. Furthermore, if two points x_1 and x_2 on Σ are causally disconnected, there will be no correlation between the values in M taken by $\phi(x_1)$ and $\phi(x_2)$. It might therefore seem that $Q(\Sigma)$ could take any value. However, simply imposing the requirement that $\phi(x)$ be continuous everywhere on Σ (which can certainly be done by local physics) restricts $Q(\Sigma)$ to a discrete set of values corresponding to the elements of $\Pi_n(M)$.

To say that local physics can distinguish between those functions $\phi(x)$ on Σ which correspond to elements of $\Pi_n(M)$ and those which do not (i.e., between functions which are everywhere continuous and those which are not) is hardly new. However, we will now show that local considerations can distinguish between certain elements of $\Pi_n(M)$. To do this, we must take into account ψ, which we have so far ignored. Let $\hat{M}$ be the manifold of dynamically allowed values of ϕ and ψ; it may be identified with $G/\hat{H}$, where $\hat{H}\subset H$ is the subgroup of G which leaves both ϕ and ψ invariant. By a simple extension of our previous remarks, the function from Σ to $\hat{M}$ defined by the combined choice of $\phi(x)$ and $\psi(x)$ can be forced by purely local mechanisms to correspond to an element of $\Pi_n(\hat{M})$. Now any continuous map from S^n to $\hat{M}$ clearly induces a continuous map from S^n to M. This correspondence implies that $\Pi_n(\hat{M})$ can be mapped into (although not necessarily onto) $\Pi_n(M)$, with the image of $\Pi_n(\hat{M})$ forming a subgroup of $\Pi_n(M)$. Since local physics can pick out continuous functions from $\hat{M}$ to S^n, it can distinguish those elements of $\Pi_n(M)$ which lie in the image of $\Pi_n(\hat{M})$ from those which do not. Therefore, causality alone can place no lower bound on the abundance of topological objects whose charges are not in the image of $\Pi_n(\hat{M})$; this remains true even if ψ vanishes at some later time.

Thus the correct formulation of the horizon bound depends on the representation content of the theory. It may be stated as follows. Let ϕ break the symmetry group G to a subgroup H. Now consider all possible breakings of H to an $\hat{H}\subset H$ which can be implemented by the remaining fields ψ. The intersection of the images of the various $\Pi_n(G/\hat{H})$ in $\Pi_n(G/H)$ forms a subgroup of $\Pi_n(G/H)$. The usual horizon bound applies—at all times—to those objects whose topological charge lies in this subgroup and only to those objects. Let us apply this formulation to the examples we considered above. In the EVV model the manifold M of allowed values of ϕ is a circle, and so $\Pi_1(M)=Z$, the additive group of the integers. $\hat{M}$, the set of allowed ϕ and ψ, is also a circle, but it is related to M in such a way that $\Pi_1(\hat{M})$ corresponds to the subgroup of Z formed by the even integers. The horizon bound therefore applies to doubly charged ϕ vortices, but not to singly charged ones; this is completely consistent with the behavior displayed by the model. In the CHKMT model, ϕ breaks SU(2) to U(1), which in turn can be broken completely by the doublet field ψ. We can therefore have $\hat{M}=G/\hat{H}=\mathrm{SU}(2)$; since $\Pi_2(\mathrm{SU}(2))$ is trivial, there is no horizon bound.

We have concentrated thus far on the horizon bound. However, causality considerations can also be used to bound the rate at which charge fluctuations can be dissipated, thus giving a more stringent bound on the annihilation rate.[9] This bound should be applicable to the same objects as the horizon bound and only to those.

It follows that an annihilation mechanism of the type proposed by Langacker and Pi is not limited by causality bounds. Such a mechanism may therefore be able to reduce the monopole abundance to an arbitrarily low level. Alternatively, the initial monopole production can be suppressed by a mechanism of the sort described above. As an example, consider the standard SU(5) model, in which monopoles are produced when the symmetry is broken to SU(3)×SU(2)×U(1) by an adjoint representation Higgs field ϕ. To avoid monopole production, we may introduce an additional field ψ in the ten-dimensional antisymmetric tensor representation. We then arrange that ψ develops a nonzero expectation value before ϕ does, with the potential being such that $\mathrm{tr}(\psi^\dagger\psi)^2/(\mathrm{tr}\psi^\dagger\psi)^2$ is maximized. This implies that ψ is of the form

$$\psi=U\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&v\\0&0&0&-v&0\end{bmatrix}U^t\,, \tag{5}$$

which breaks SU(5) to SU(3)×SU(2); this breaking does not support monopole solutions. We next allow ϕ to become nonzero, subject to the requirements that $(\mathrm{tr}\phi^4)/(\mathrm{tr}\phi^2)^2$ be minimized and $(\mathrm{tr}\psi^\dagger\phi^2\psi)/(\mathrm{tr}\phi^2)(\mathrm{tr}\psi^\dagger\psi)$ be maximized. With ψ given by Eq. (5), ϕ must then be

$$\phi=U\begin{bmatrix}w&0&0&0&0\\0&w&0&0&0\\0&0&w&0&0\\0&0&0&-\frac{3}{2}w&0\\0&0&0&0&-\frac{3}{2}w\end{bmatrix}U^{-1}\,. \tag{6}$$

Finally, we let ψ vanish again. Although this increases the unbroken-symmetry group to SU(3)×SU(2)×U(1), no monopoles are produced.

In summary, we have shown that local mechanisms can suppress the density of certain topological objects to much less than one per horizon volume. The horizon bound [Eq. (1)] is therefore very dependent on the representation content of the theory. By invoking intermediate phases with appropriate types of symmetry breaking, it is possible to annihilate essentially all monopoles or even to avoid their ever appearing. This provides the possibility of a noninflationary solution to the primordial monopole problem.

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