# Hard probes for a strongly coupled plasma

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Collaboration with Yoshitaka Hatta and Al Mueller

(lecture notes arXiv:0812.0500)

# Introduction

- Experimental results at RHIC suggest that the deconfined hadronic matter ('Quark–Gluon Plasma') produced in a $AA$ collision at high energy might be strongly interacting
- A challenge for the theory: lattice QCD cannot be used for such dynamical phenomena
- New method: string theory via AdS/CFT correspondence
  - not yet QCD: conformal symmetry, no confinement
  - at high energy and/or finite temperature, such issues are (presumably) less important, even in QCD
- A vigourous activity with many interesting results
  - conceptually interesting relations between particle physics, string theory, gravity, black holes
  - physical interpretation of the results is very challenging

# Outline



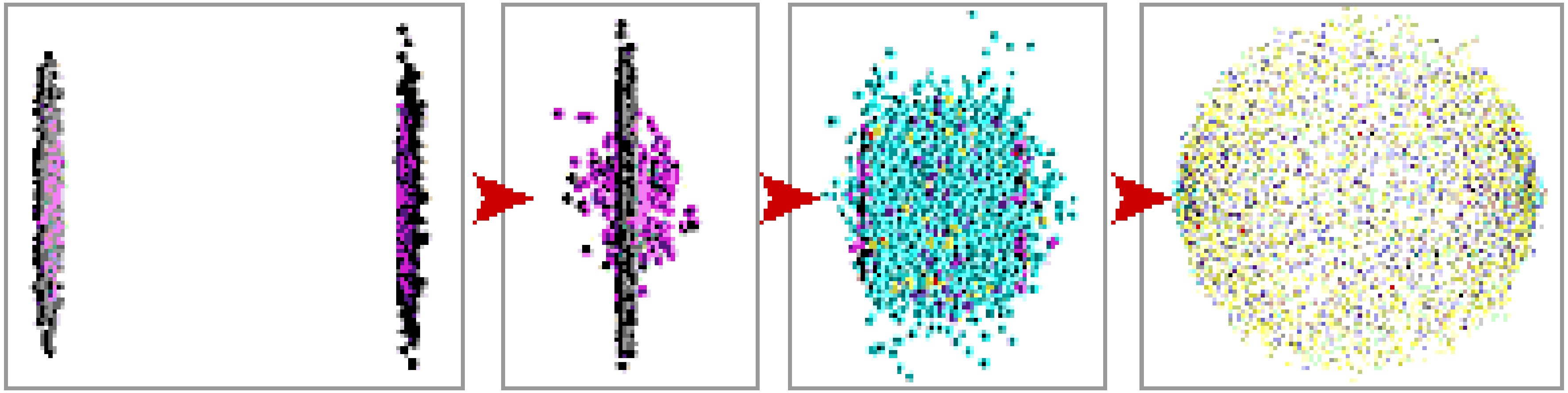
- Motivation : Heavy Ion Collisions at RHIC and LHC
- Weak coupling: Partons and jets in perturbative QCD
- Strong coupling: Partons and jets from AdS/CFT
- Vacuum case: electron–positron annihilation at strong coupling
- Finite–$T$ plasma: Deep inelastic scattering & Parton saturation
- Finite–$T$ plasma: Jet quenching & Momentum broadening

  *(new developments in collaboration with Grégory Giecold and Al Mueller)*

# The Little Bang



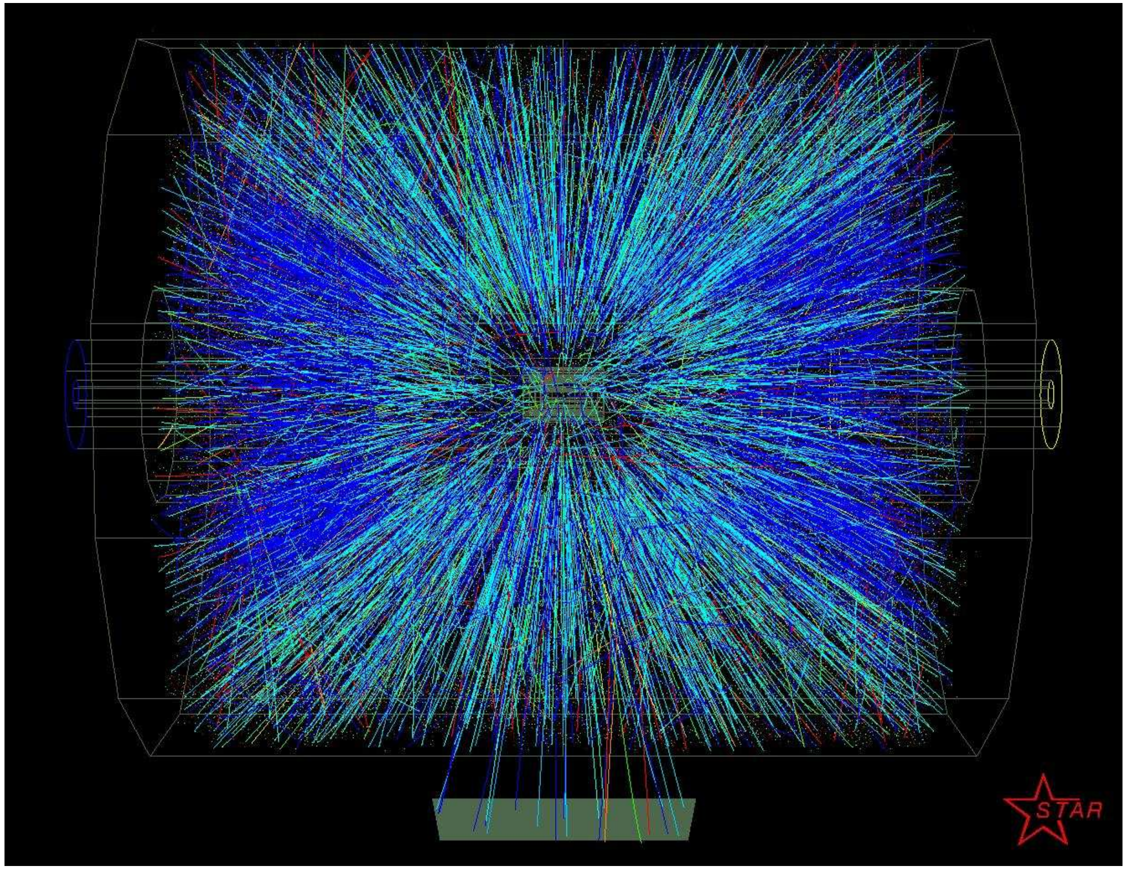
- Ultrarelativistic heavy ion collisions @ RHIC and LHC



- Extremely complex phenomena
  - high density partonic systems in the initial wavefunctions
  - multiple interactions during the collisions
  - complicated, non-equilibrium, dynamics after the collision
  - expansion, thermalization, hadronisation
- Is there any place for strong–coupling dynamics ?

# Hadron production at RHIC

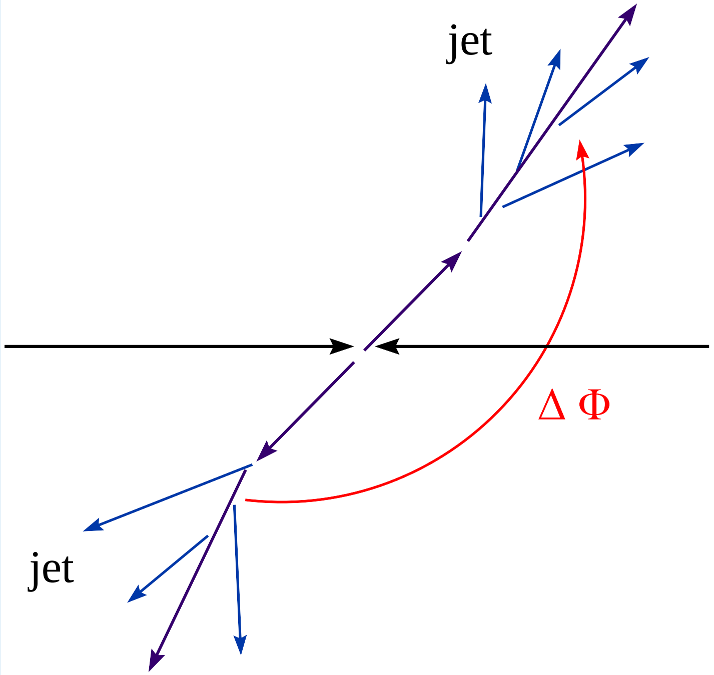


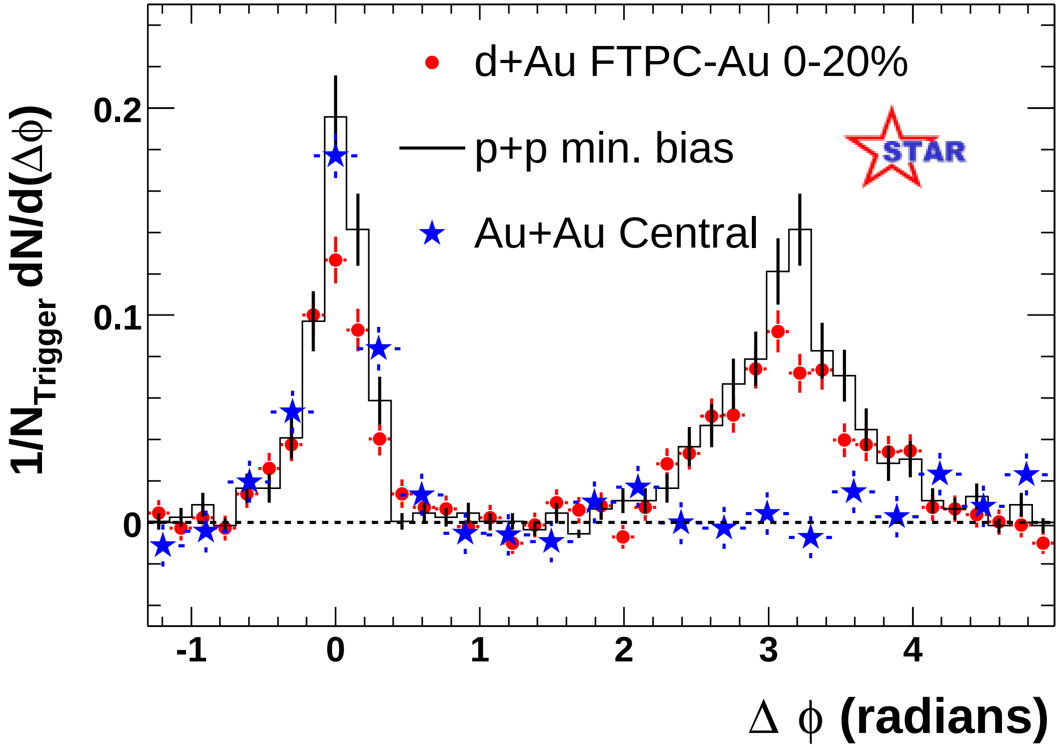



- $\sim 3000$ hadrons in the final state vs. 400 nucleons in $AA$
- Most of them arise as hadronized partons
- Particle correlations are essential to disentangle phenomena

# Jets in proton–proton collisions





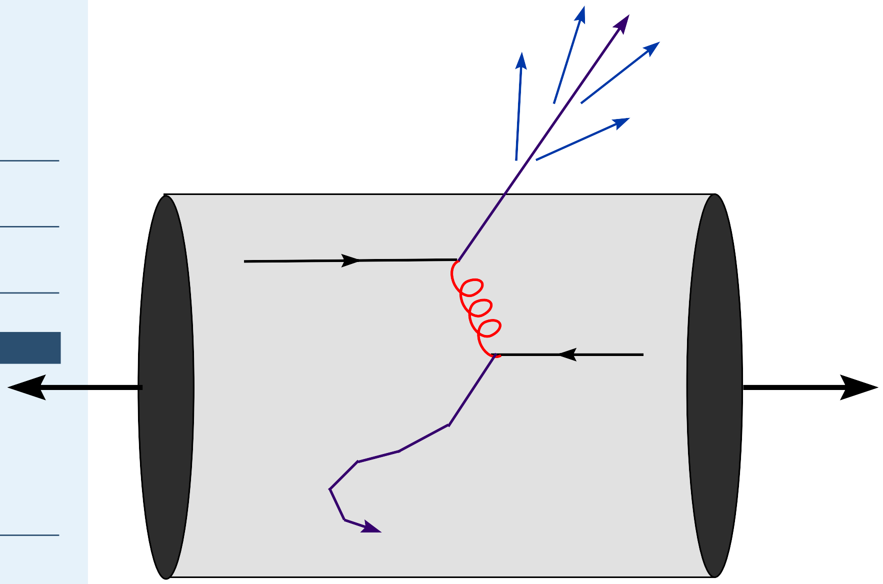



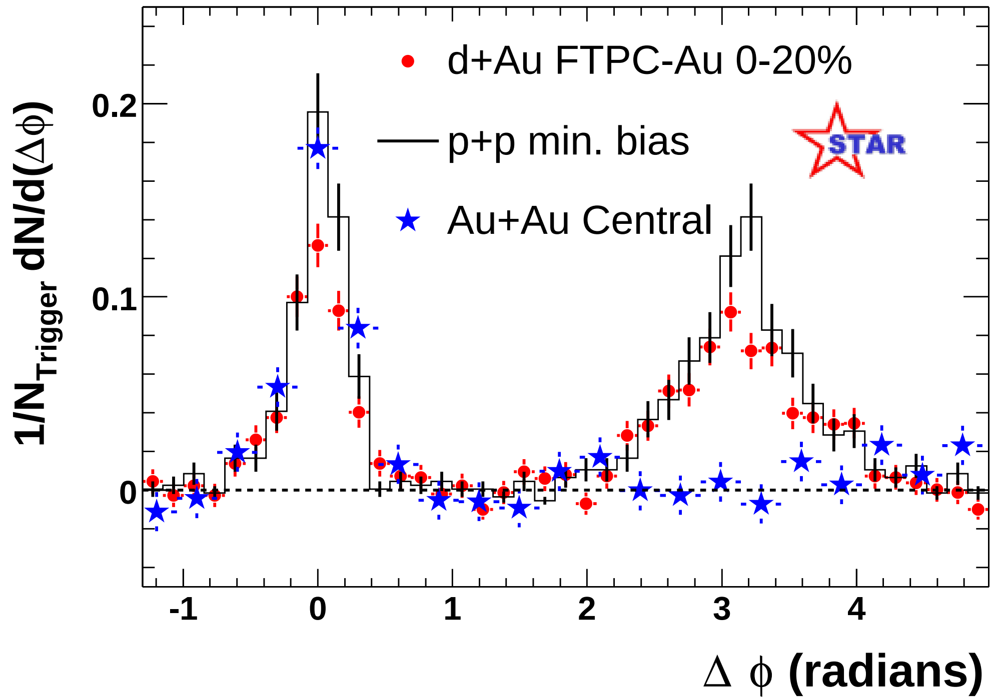

- Azimuthal correlations between the produced jets:

  a peak at $\Delta\Phi = 180°$

# Nucleus–nucleus collision








- The “away–side” jet has disappeared !

  absorbtion (or energy loss, or “jet quenching”) in the medium

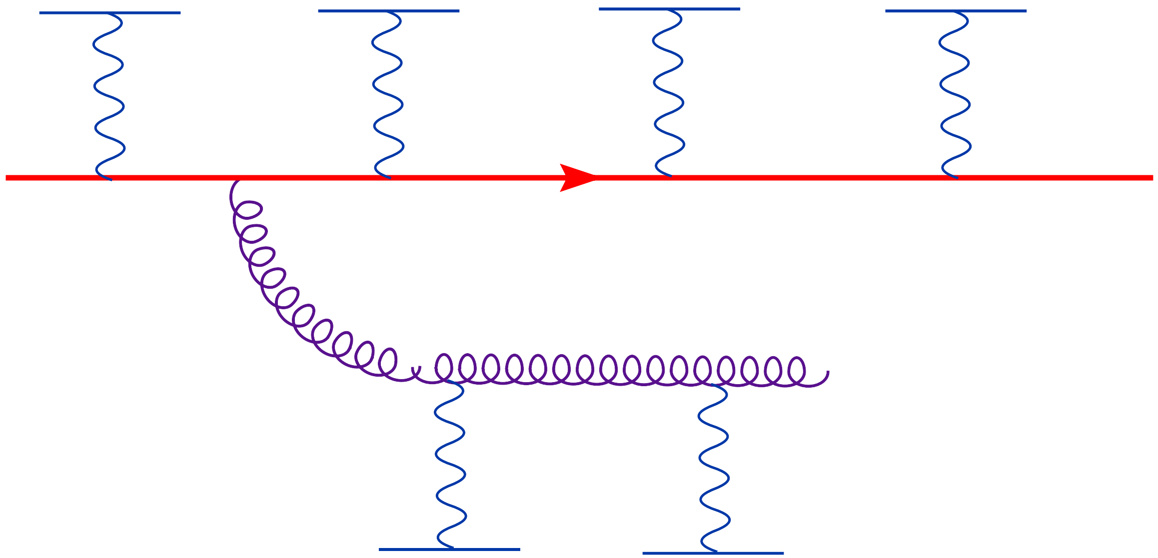
- The matter produced in a heavy ion collision is opaque

  high density, strong interactions, ... or both

# Jet quenching



- Energy loss for a heavy quark: Medium induced radiation

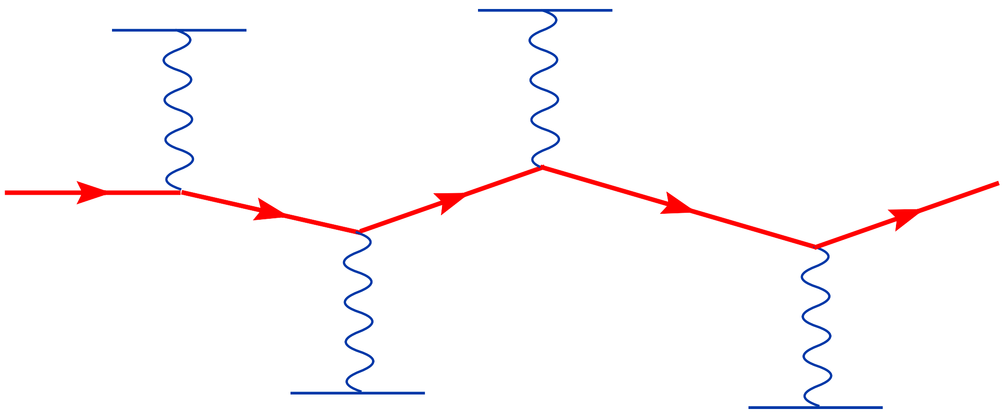


$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \alpha_s N_c \langle p_\perp^2 \rangle$$ : relation to 'momentum broadening'

# Transverse momentum broadening



- A parton ('heavy quark') scatters off the plasma constituents on its own, hard, resolution scale



$$\frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \frac{xg(x,Q^2)}{N_c^2-1}$$

- $xg(x,Q^2)$ : gluon distribution per unit volume in the medium
- Weakly–coupled QGP : incoherent sum of the gluon distributions produced by thermal quarks and gluons

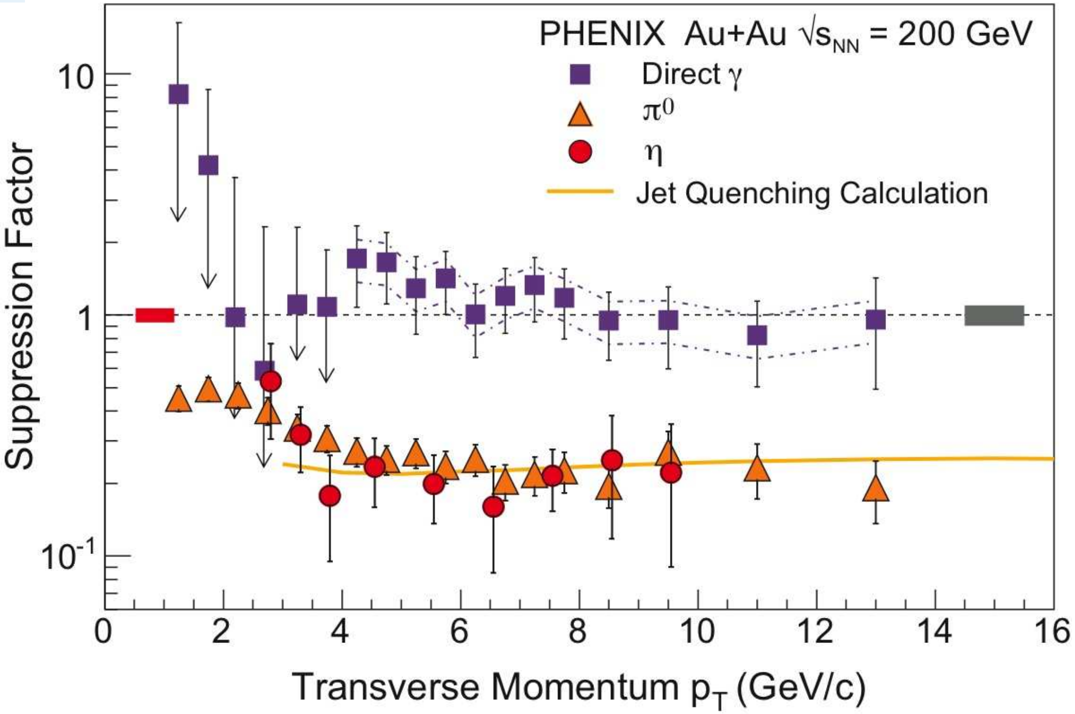
$$xg(x,Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g\,, \quad \text{with} \quad n_{q,g}(T) \propto T^3$$

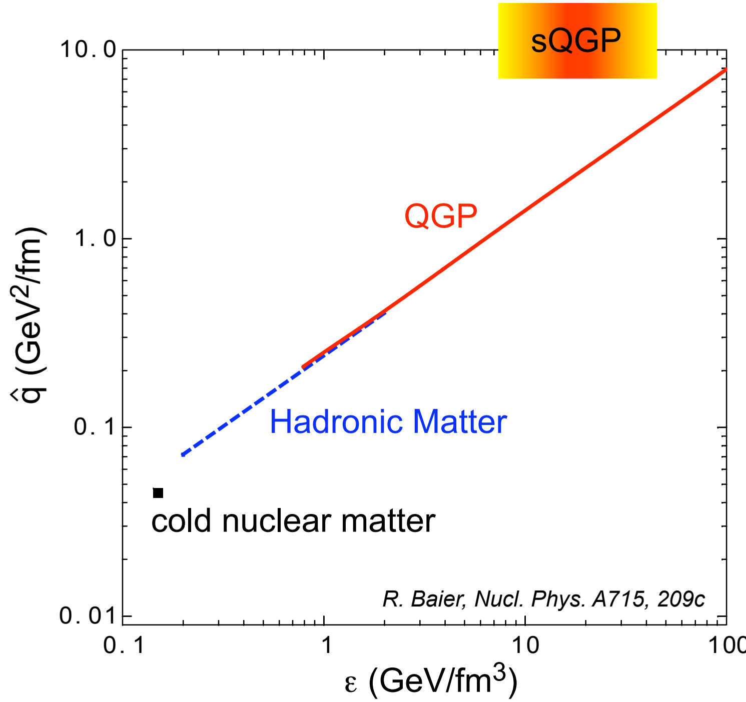
# Nuclear modification factor



- How to measure $\hat{q}$ ? Compare $AA$ collisions at RHIC to $pp$

$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p)\ \times\ A^2}$$



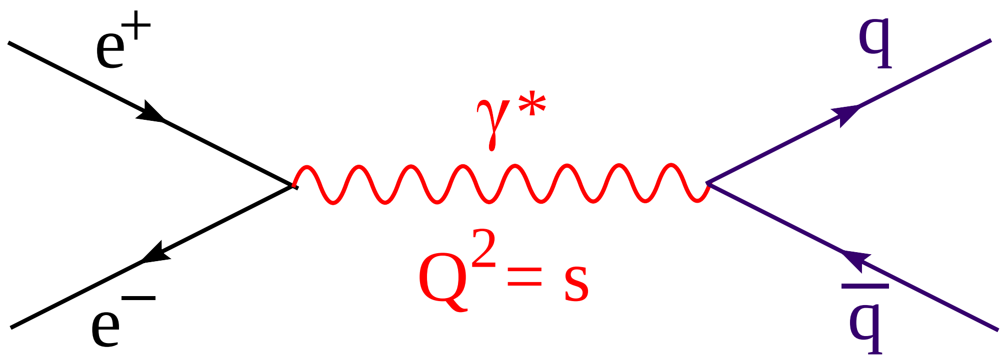




- RHIC data seem to prefer $\hat{q} \simeq 10$ GeV$^2$/fm, which is too large to be accounted for by weakly–coupled QGP (??)

# $e^+e^-$ annihilation: Jets in pQCD



- How would a high–energy jet interact in a strongly coupled plasma ?
- How to produce jets in the first place ?
- Guidance from perturbative QCD: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$




- Decay of a time–like photon: $Q^2 \equiv q^\mu q_\mu = s > 0$

# $e^+e^-$ annihilation: Jets in pQCD



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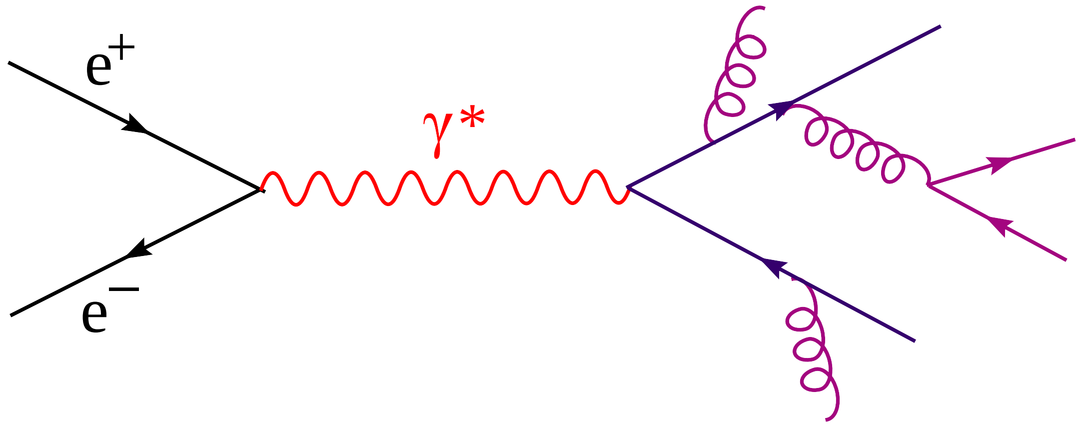
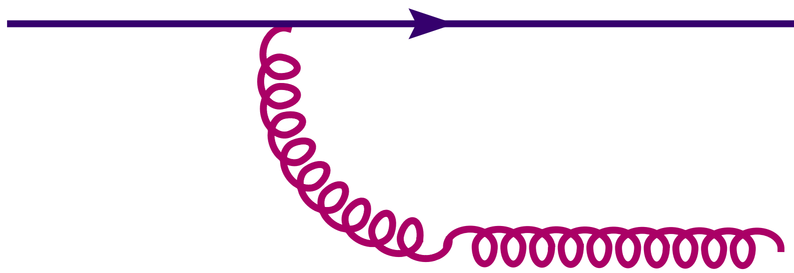



- The structure of the final state is determined by
  - parton branching & hadronisation

# Bremsstrahlung



- Gluon emission to lowest order in perturbative QCD:



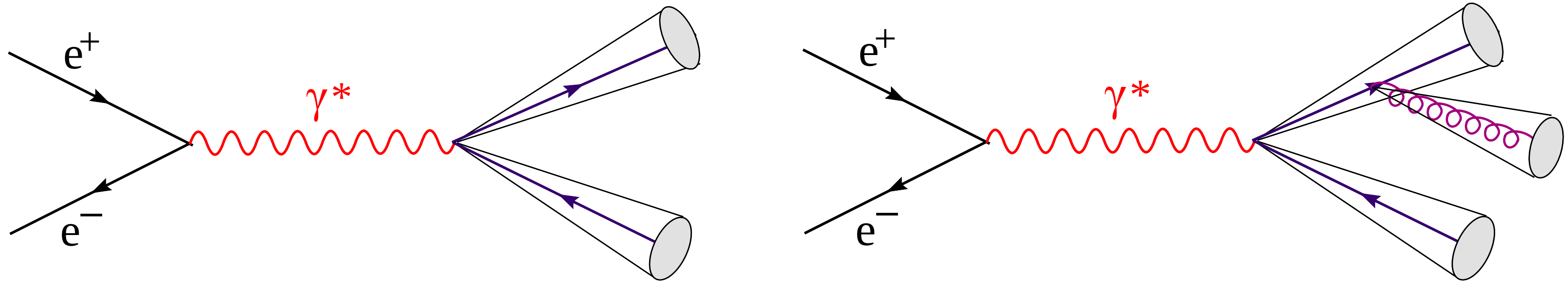

$$\mathrm{d}\mathcal{P}_{\mathrm{Brem}} \sim \alpha_s(k_\perp^2)\, N_c \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

- Phase–space enhancement for the emission of
  - collinear $(k_\perp \to 0)$
  - and/or low–energy $(x \to 0)$ gluons
- Parton lifetime (or 'gluon formation time') : $\Delta t \sim \frac{k_z}{k_\perp^2}$

  Soft partons ($k_\perp \sim \Lambda_{\mathrm{QCD}}$) are produced later

# Jets in perturbative QCD






- Few, well collimated, jets
- $e^+e^-$ cross–section computable in perturbation theory

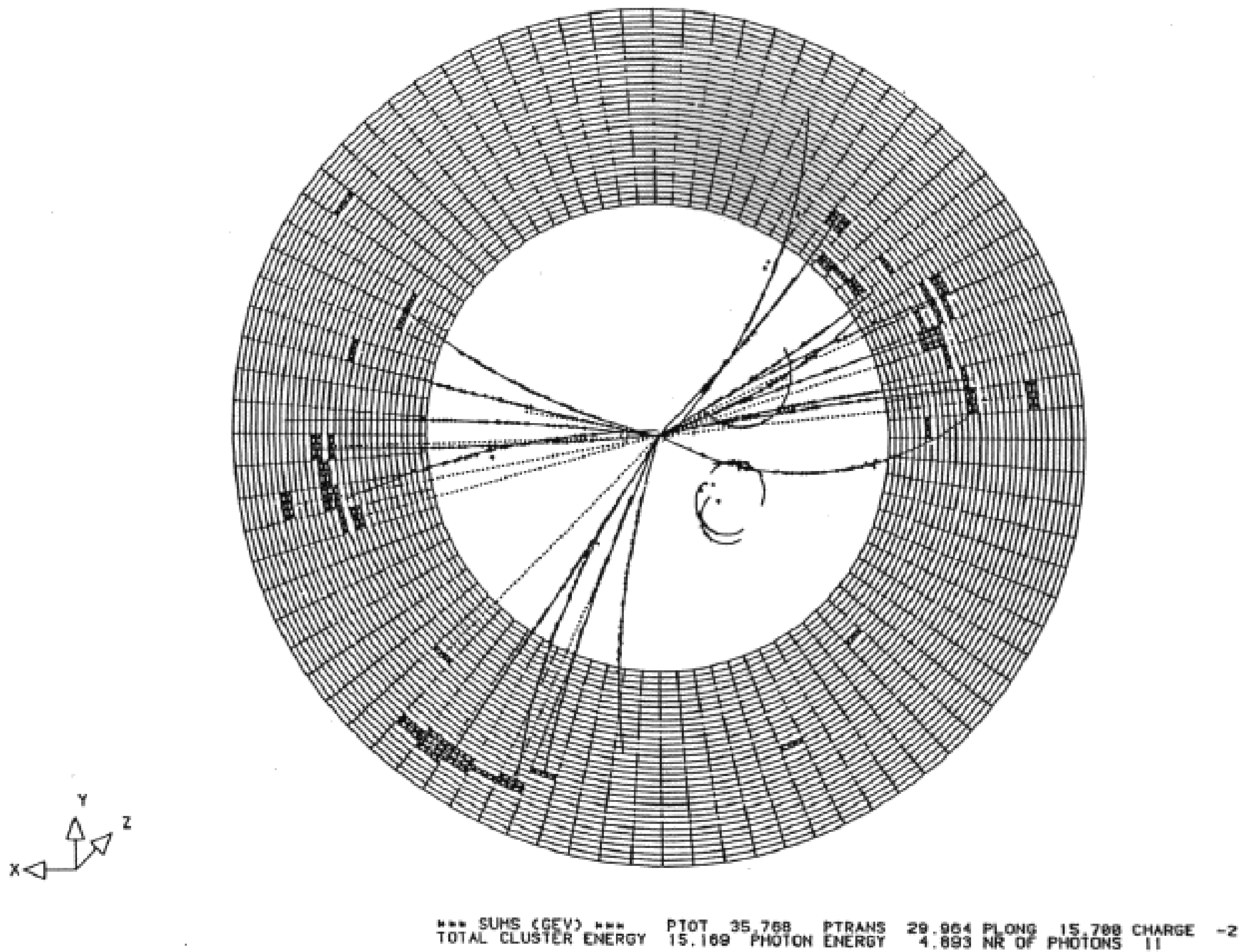
$$\sigma(s) = \sigma_{\rm QED} \times \Big(3\sum_f e_f^2\Big)\left(1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right)$$

$\sigma_{\rm QED}$ : cross–section for $e^+e^- \to \mu^+\mu^-$

- Multi–jet ($n \geq 3$) events appear, but are comparatively rare

# 3–jet event at OPAL (CERN)



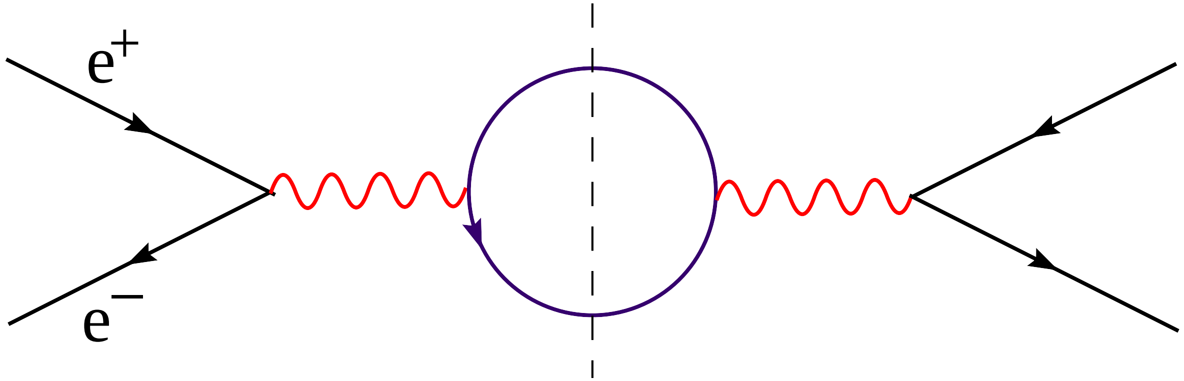


# Optical theorem



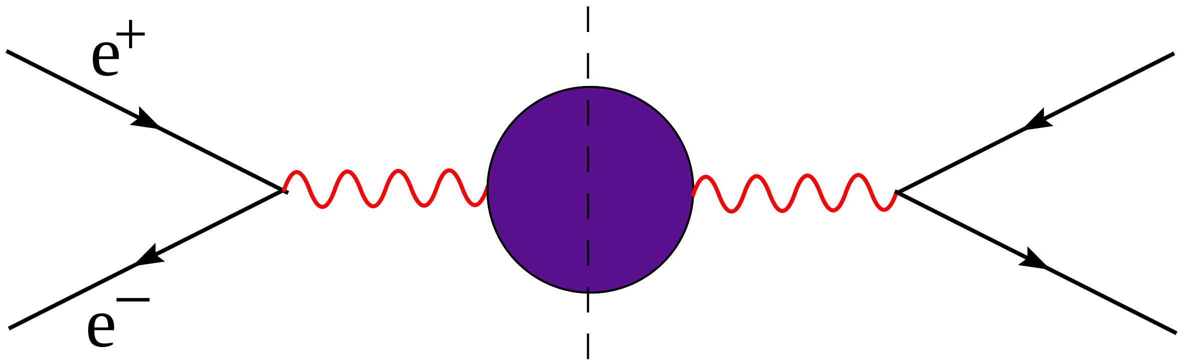
- Total cross–section given by the optical theorem

$$\sigma(e^+e^-) = \frac{1}{2s}\,\ell^{\mu\nu}\,\mathrm{Im}\,\Pi_{\mu\nu}(q)$$




- The quark loop: The vacuum polarization tensor $\Pi_{\mu\nu}$ for a time–like photon (here, evaluated at one–loop order)
- This can be generalized to all–orders

$$\sigma(e^+e^-) = \frac{1}{2s}\,\ell^{\mu\nu}\,\mathrm{Im}\,\Pi_{\mu\nu}(q)$$



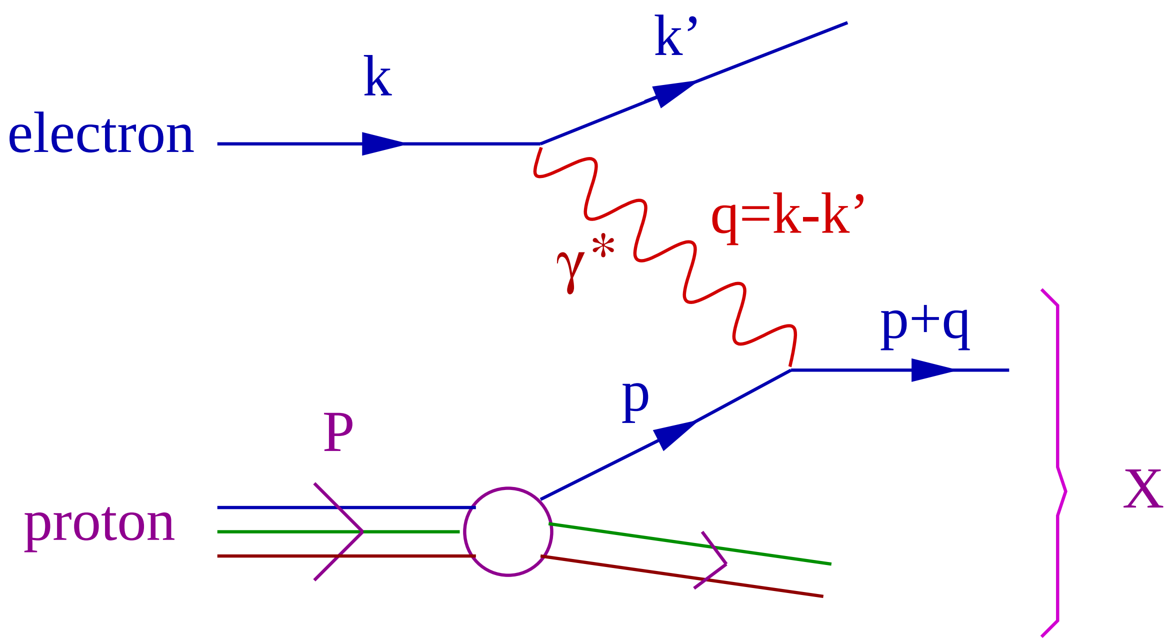

- $\Pi_{\mu\nu}$ = current–current correlator to all orders in QCD

$$\Pi_{\mu\nu}(q) \equiv i\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,\langle 0\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,0\rangle$$

$$J^\mu = \sum_f e_f\,\bar{q}_f\,\gamma^\mu\,q_f$$ : quark electromagnetic current

- Valid to leading order in $\alpha_{\rm em}$ but all orders in $\alpha_s$

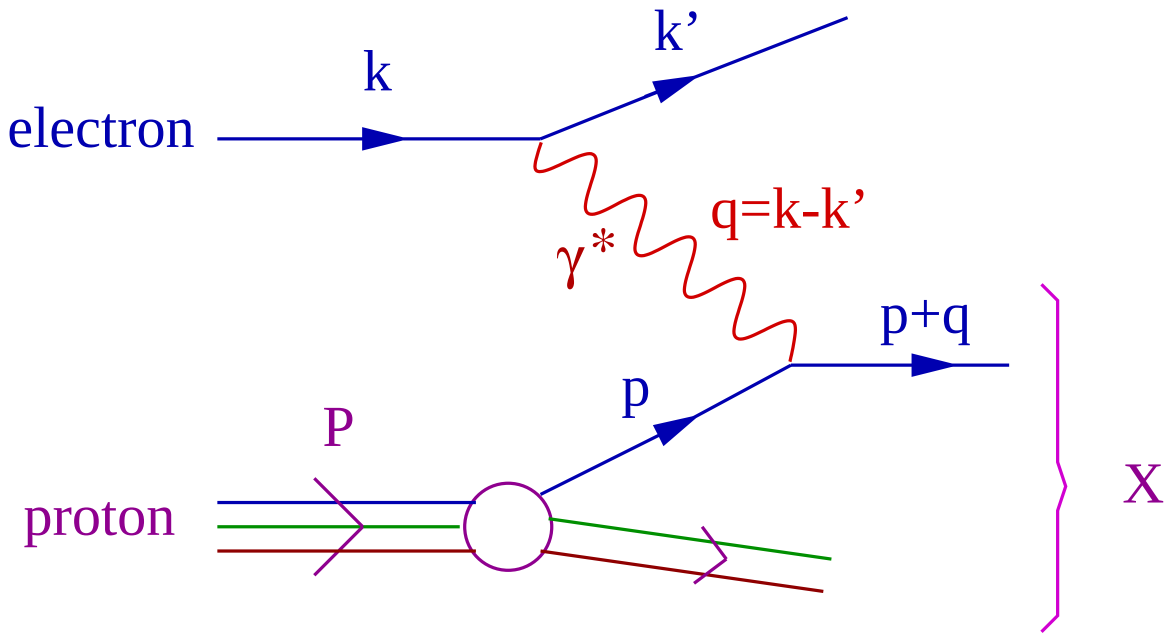
# Deep inelastic scattering




- Space–like current: $Q^2 \equiv -q^\mu q_\mu \geq 0$ and $x \equiv \frac{Q^2}{2P\cdot q}$
- Physical picture: $\gamma^*$ absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

# The proton structure function

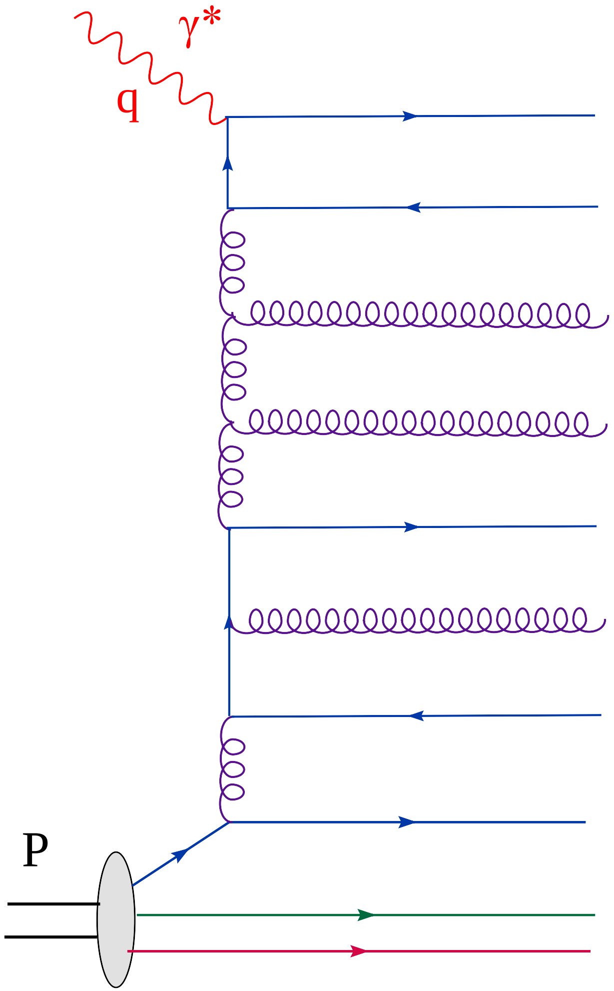


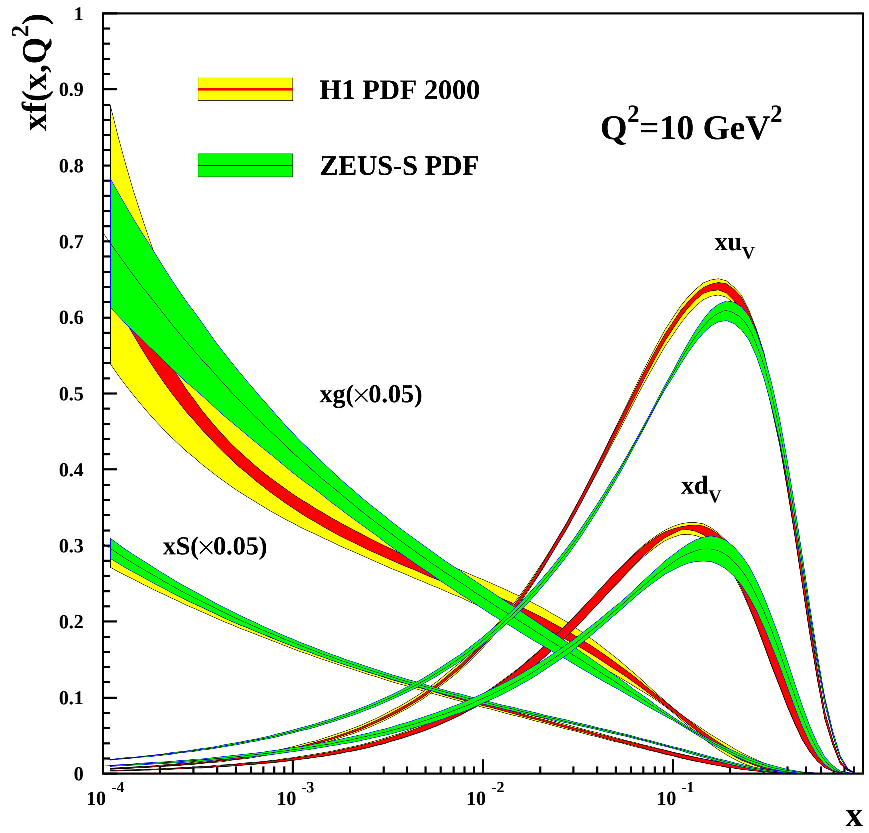



$$\sigma_{\gamma^* p}(x, Q^2) \;=\; \frac{4\pi^2\alpha_{\rm em}}{Q^2}\, F_2(x, Q^2)$$

- $F_2(x, Q^2)$ : 'quark distribution' = number of quarks with longitudinal momentum fraction $x$ and transverse area $1/Q^2$

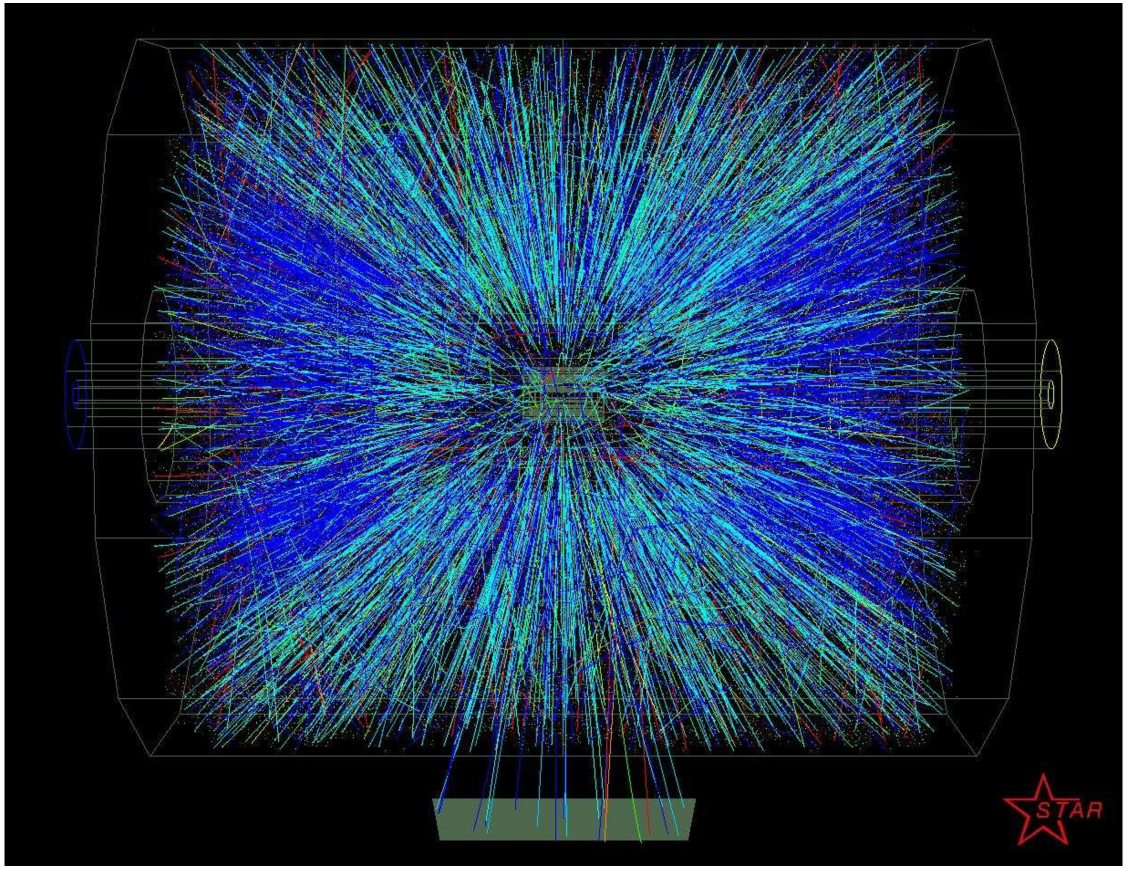
- Gluons are implicitly seen in DIS, via parton evolution




- Bremsstrahlung favors the emission of gluons with $x \ll 1$

# Partons at RHIC




- Partons are actually ‘seen’ (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–$x$ partons
  - forward/backward rapidities: large–$x$ partons

# Electromagnetic current in a plasma



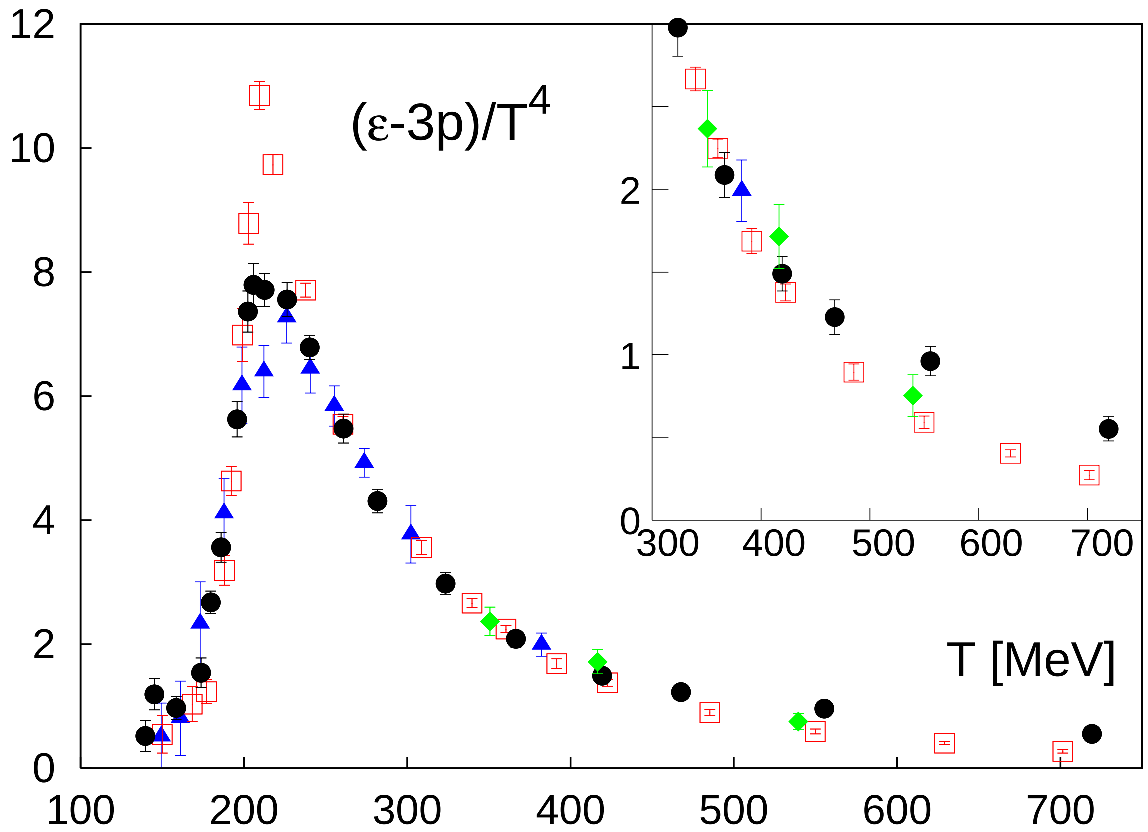
- Thermal expectation value (retarded polarization tensor) :

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\,\langle\, [J_\mu(x), J_\nu(0)]\,\rangle_T$$

- 'Hard probe' : large virtuality $Q^2 \equiv |q^2| \gg T^2$
  - time–like current ($q^2 > 0$) : jets
  - space–like current ($q^2 < 0$) : DIS, partons
- Strong coupling $\Longrightarrow$ AdS/CFT correspondence
- A 'cousin' of QCD: $\mathcal{N} = 4$ Super Yang–Mills theory
  - conformal invariance: coupling is fixed !
  - no confinement, no fundamental quarks ...
- Perhaps better suited for QCD at finite temperature

# Trace anomaly from lattice QCD






$$\beta(g)\,\frac{\mathrm{d}p}{\mathrm{d}g} = \langle T^{\mu}_{\mu}\rangle = \mathcal{E} - 3p$$

- $(\mathcal{E}-3p)/\mathcal{E}_0 \lesssim 10\%$ for any $T \gtrsim 2T_c \simeq 400$ MeV

# A Gauge/Gavity duality *(Maldacena, 1997)*



- A conformal gauge field theory in $D = 4$ : $\mathcal{N} = 4$ SYM
- Type IIB string theory living in $D = 10$ : $AdS_5 \times S^5$

$$\mathrm{d}s^2 = \frac{R^2}{\chi^2}(-\mathrm{d}t^2 + \mathrm{d}\vec{x}^2) + \frac{R^2}{\chi^2}\mathrm{d}\chi^2 + R^2\mathrm{d}\Omega_5^2$$

  - $0 \leq \chi < \infty$ : 'radial', or '5th', coordinate
  - gauge theory lives at the Minkowski boundary $\chi = 0$
- Strong 't Hooft coupling (more properly, $N_c \to \infty$) :

$\lambda \equiv g^2 N_c \gg 1$ with $g^2 \ll 1 \Longrightarrow$ classical supergravity

- Generating functional for the correlations of an operator $\hat{\mathcal{O}}$

$$\left\langle \mathrm{e}^{i\int \mathrm{d}^4x \hat{\mathcal{O}}\phi} \right\rangle_{4D} \approx \mathrm{e}^{iS_{\mathrm{SUGRA}}[\phi_{cl}]} \quad \text{with} \quad \phi_{cl}(x, \chi = 0) = \phi(x)$$

# Heating $AdS_5$



- $\mathcal{N}=4$ SYM at finite temperature $\Longleftrightarrow$ Black Hole in $AdS_5$

$$\mathrm{d}s^2 = \frac{R^2}{\chi^2}\big(-f(\chi)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2\big) + \frac{R^2}{\chi^2 f(\chi)}\mathrm{d}\chi^2 + R^2\mathrm{d}\Omega_5^2$$

where $f(\chi) = 1-(\chi/\chi_0)^4$ and $\chi_0 = 1/\pi T$ = BH horizon

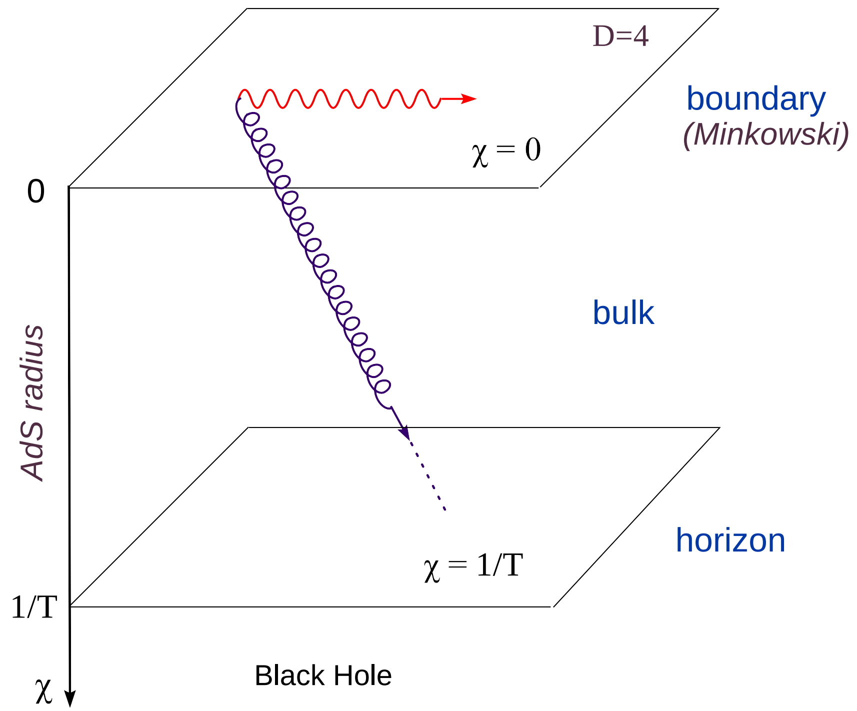
- Example: compute the plasma entropy density for $\lambda \to \infty$
- The black hole entropy: Bekenstein–Hawking formula

$$S_{\rm BH} = \frac{A}{4G} \quad \text{with } A = \text{horizon area}$$

- ... is identified with the entropy of the $\mathcal{N}=4$ plasma:

$$\Longrightarrow s \equiv \frac{S_{\rm BH}}{V_{3D}} = \frac{\pi^2}{2}N_c^2T^3 = \frac{3}{4}s_0$$

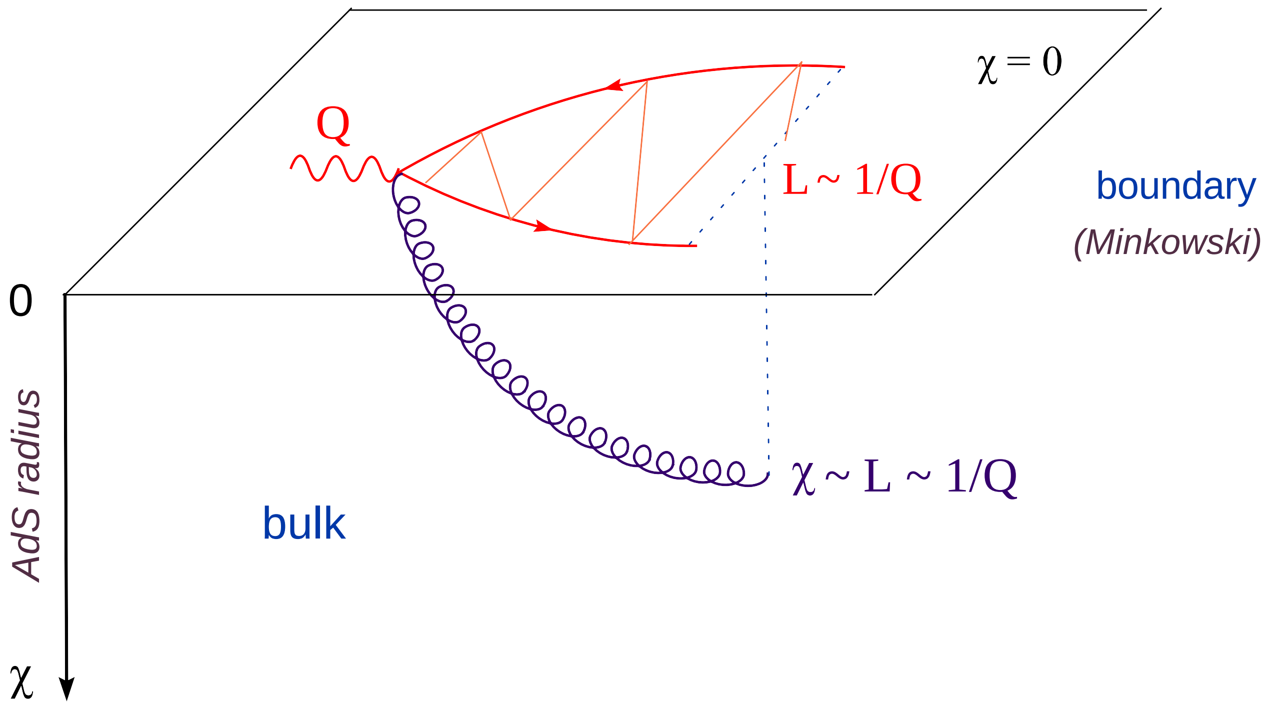
- Abelian current $J_\mu$ in 4D $\longleftrightarrow$ Maxwell wave $A_\mu$ in $AdS_5$ BH
- Im $\Pi_{\mu\nu}$ $\longleftrightarrow$ absorption of the wave by the BH




- Maxwell equations in a curved space–time

$$\partial_m\left(\sqrt{-g}g^{mn}g^{pq}F_{nq}\right) = 0 \qquad \text{where} \qquad F_{mn} = \partial_m A_n - \partial_n A_m$$

- 'Holography' : A quantum field theory in $D = 3 + 1$ $\Longleftrightarrow$ A theory with gravitation in higher dimensions

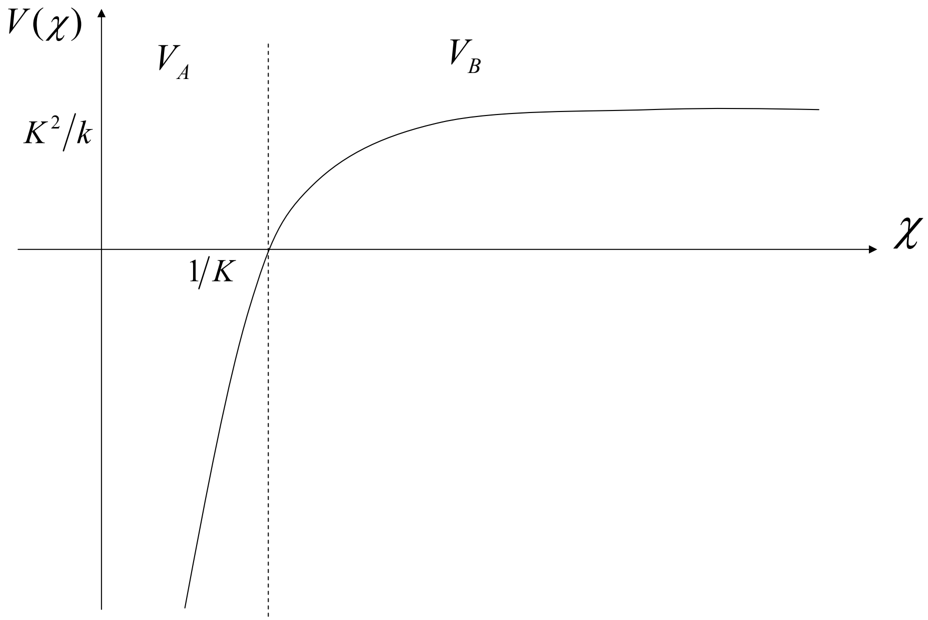



- Rôle of the 5th dimension: a reservoir of quantum flucts.
- Radial penetration $\chi$ of the wave packet in $AdS_5$ $\longleftrightarrow$ transverse size $L$ of the partonic fluctuation on the boundary

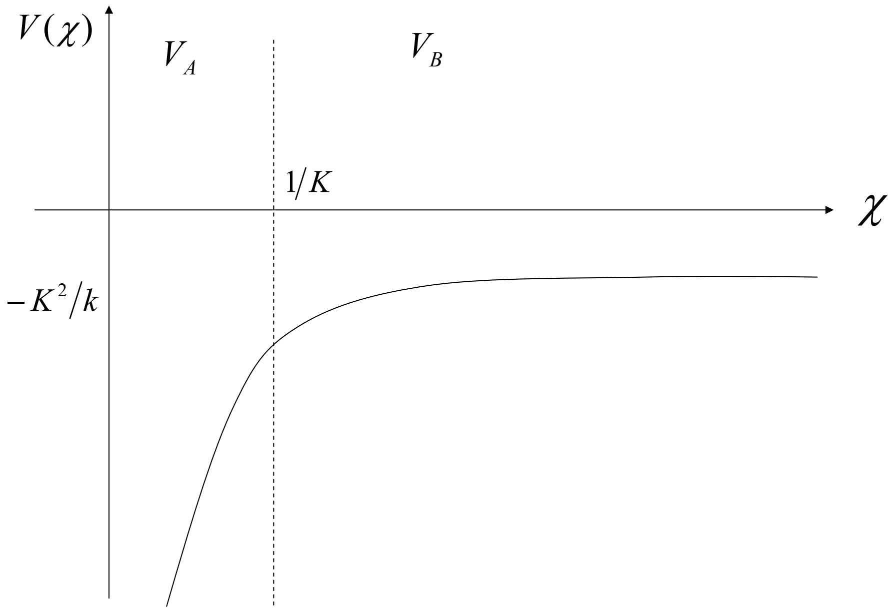
# The vacuum case as a warm up



- $A_\mu(t, \boldsymbol{x}, \chi) \,=\, \mathrm{e}^{-i\omega t + ikz}\, A_\mu(\chi) \;\Longrightarrow\; \Pi_{\mu\nu}(\omega, k)$
- Maxwell eq. for $A_0 \longleftrightarrow$ Schrödinger eq. for $\psi(\chi) \equiv \sqrt{\chi}\, A_0$
- “Time–independent Schrödinger eq.” : $-\psi'' + V(\chi)\psi \,=\, 0$




space–like

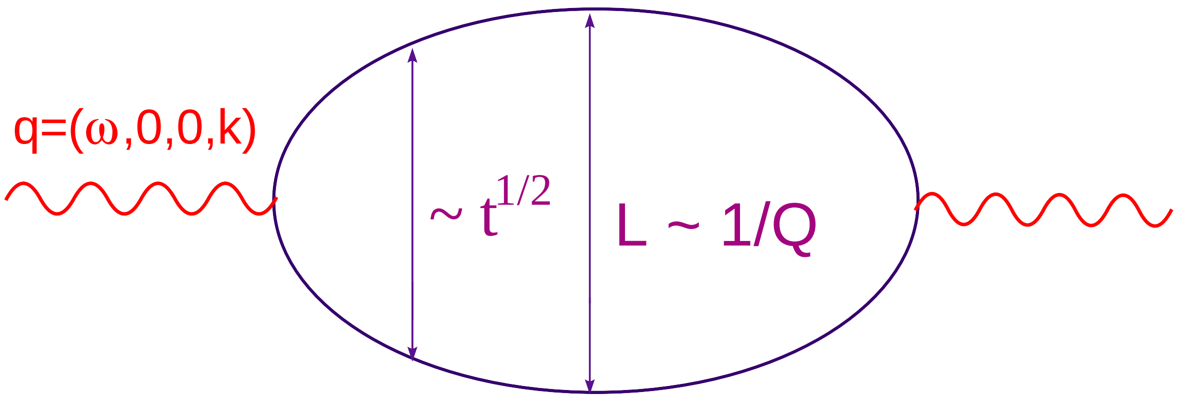



time–like

($K \,\equiv\, Q$ with $Q^2 = |\omega^2 - k^2|$ in all the figures)

# Space–like current in the vacuum

- Potential barrier $\Longrightarrow$ the wave is trapped at $\chi \lesssim 1/Q$
- By energy–momentum conservation, a space–like current cannot decay (in the vacuum)
- It can develop a virtual partonic fluctuation



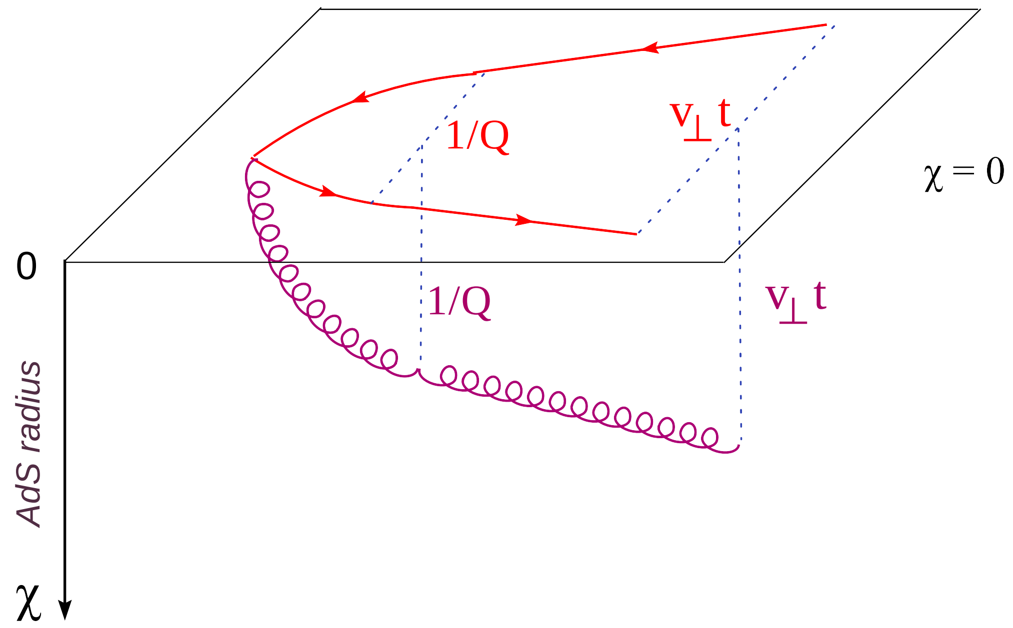

- By uncertainty principle, this has a transverse size $L \sim 1/Q$

and a lifetime $\Delta t \sim \frac{1}{Q} \times \frac{\omega}{Q} \sim \frac{\omega}{Q^2}$

# Time–like current in the vacuum



- No potential barrier (flat potential)
  - the wave can escape towards large values of $\chi$
  - free streaming with radial velocity $v_\chi = Q/\omega$

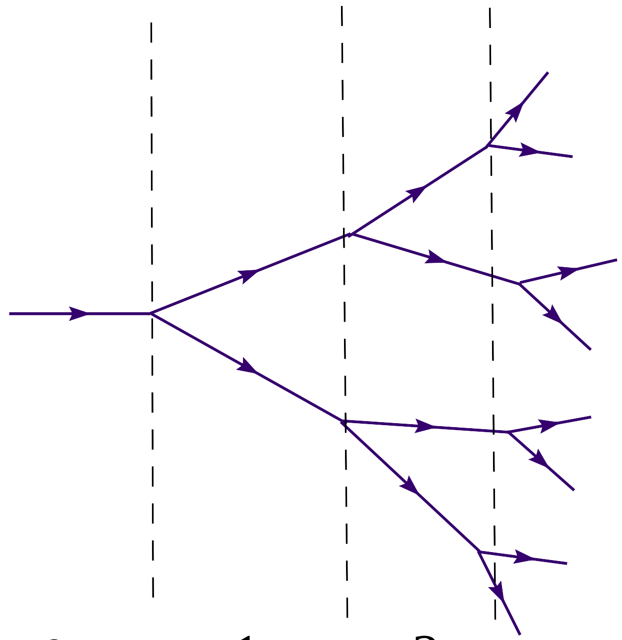



- Physics: The current decays into massless partons
  - common longitudinal velocity $v_z = k/\omega$
  - transverse velocity $v_\perp = \sqrt{1 - v_z^2} = Q/\omega$

# Quasi–democratic parton branching



- No reason why branching should stop at 2 parton level !
- No reason to favour special corners of phase–space !




$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$Q_n \sim \frac{Q_{n-1}}{2} \sim \frac{Q}{2^n}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

$$\frac{Q_n}{\omega_n} \sim \frac{Q}{\omega} = \frac{1}{\gamma}$$

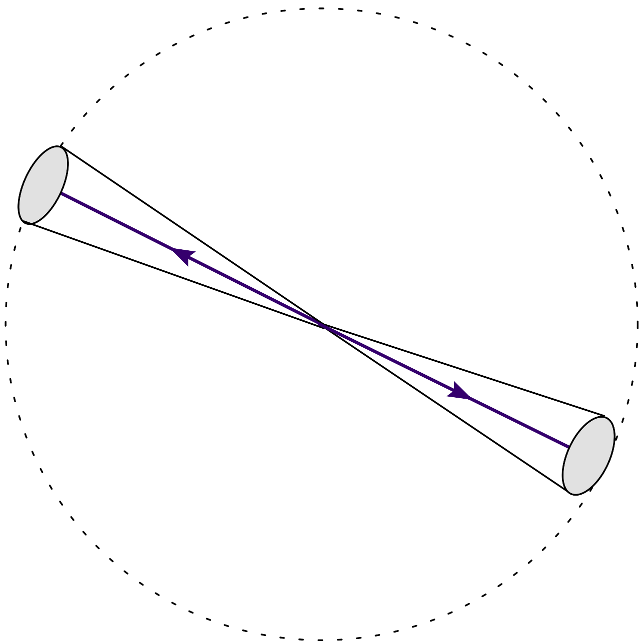
$$\frac{Q_n - Q_{n-1}}{\Delta t_n} \sim -\frac{Q}{\omega} Q_n^2 \qquad \Longrightarrow \qquad \frac{\mathrm{d}Q(t)}{\mathrm{d}t} \simeq -\frac{Q^2(t)}{\gamma}$$

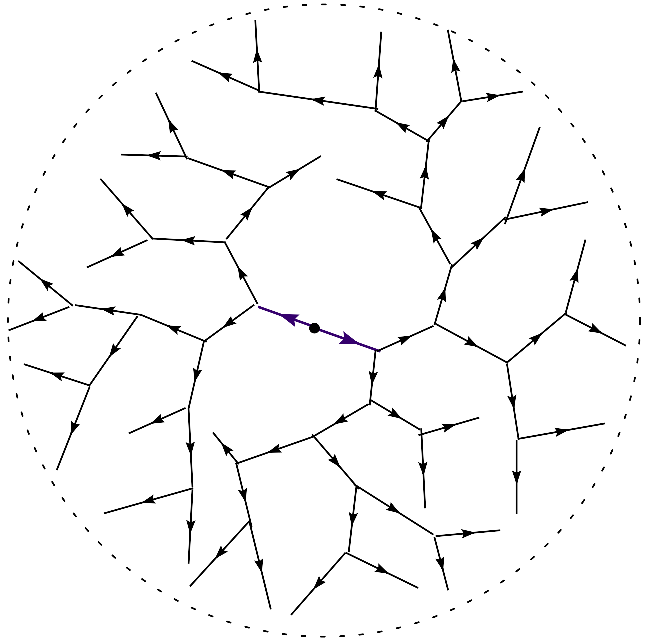
- $L(t) \sim 1/Q(t) \Longrightarrow L(t) \sim t/\gamma = \sqrt{1-v_z^2}\, t$ ✓

# $e^+e^-$ at strong coupling



- Time–like current in the vacuum

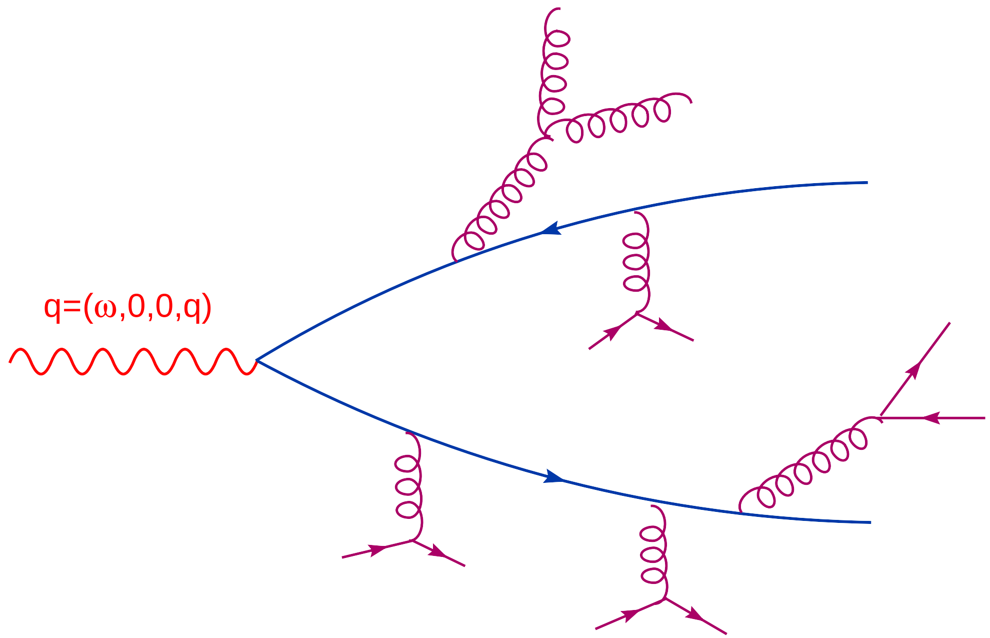




- Infrared cutoff $\Lambda \longrightarrow$ splitting continues down to $Q \sim \Lambda$
- In the COM frame $\longrightarrow$ spherical distribution $\Longrightarrow$ no jets !
  *(similar conclusion by Hofman and Maldacena, 2008)*
- Final state looks very different as compared to pQCD !

# Finite–$T$ plasma : Space–like current

- The current can now decay due to the parton interactions in the plasma $\Longrightarrow$ Im $\Pi_{\mu\nu}$ : a contribution to $F_2(x, Q^2)$
  - thermal rescattering
  - medium–induced radiation

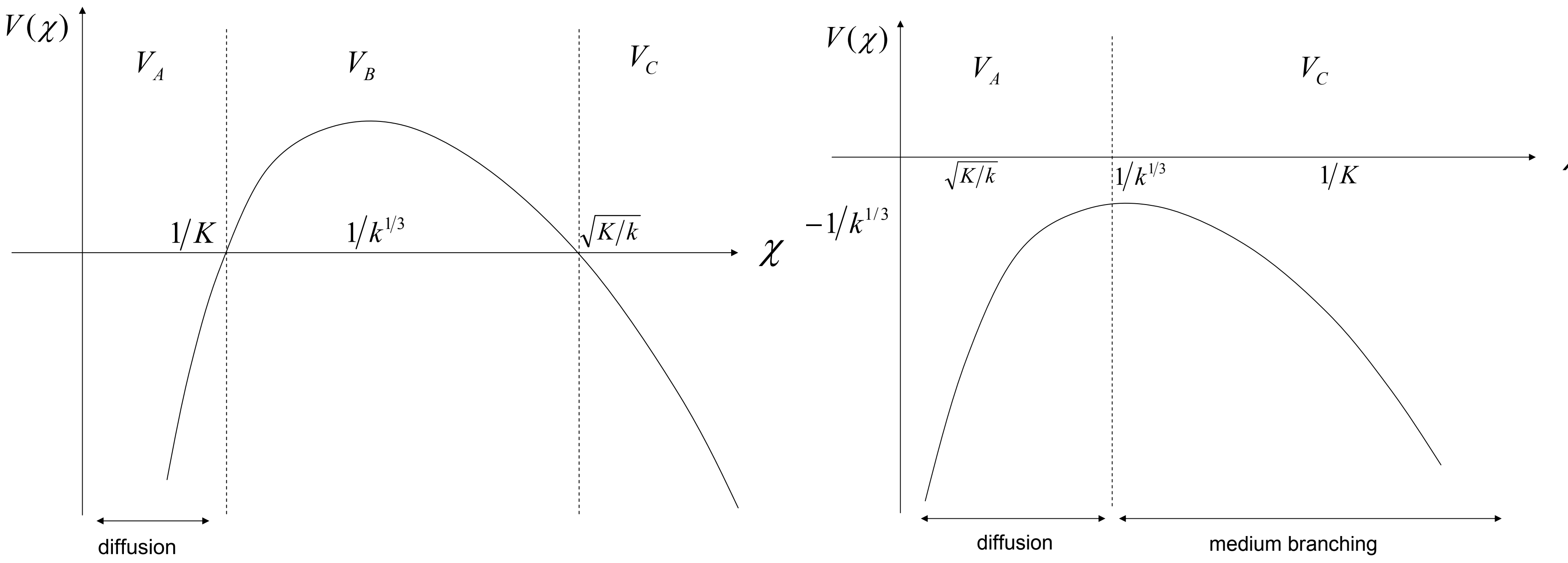



- What is the situation at strong coupling ?

# Space–like current with $Q \gg T$



- Competition between repulsion (energy conservation) and attraction (by the black hole)




moderate energy

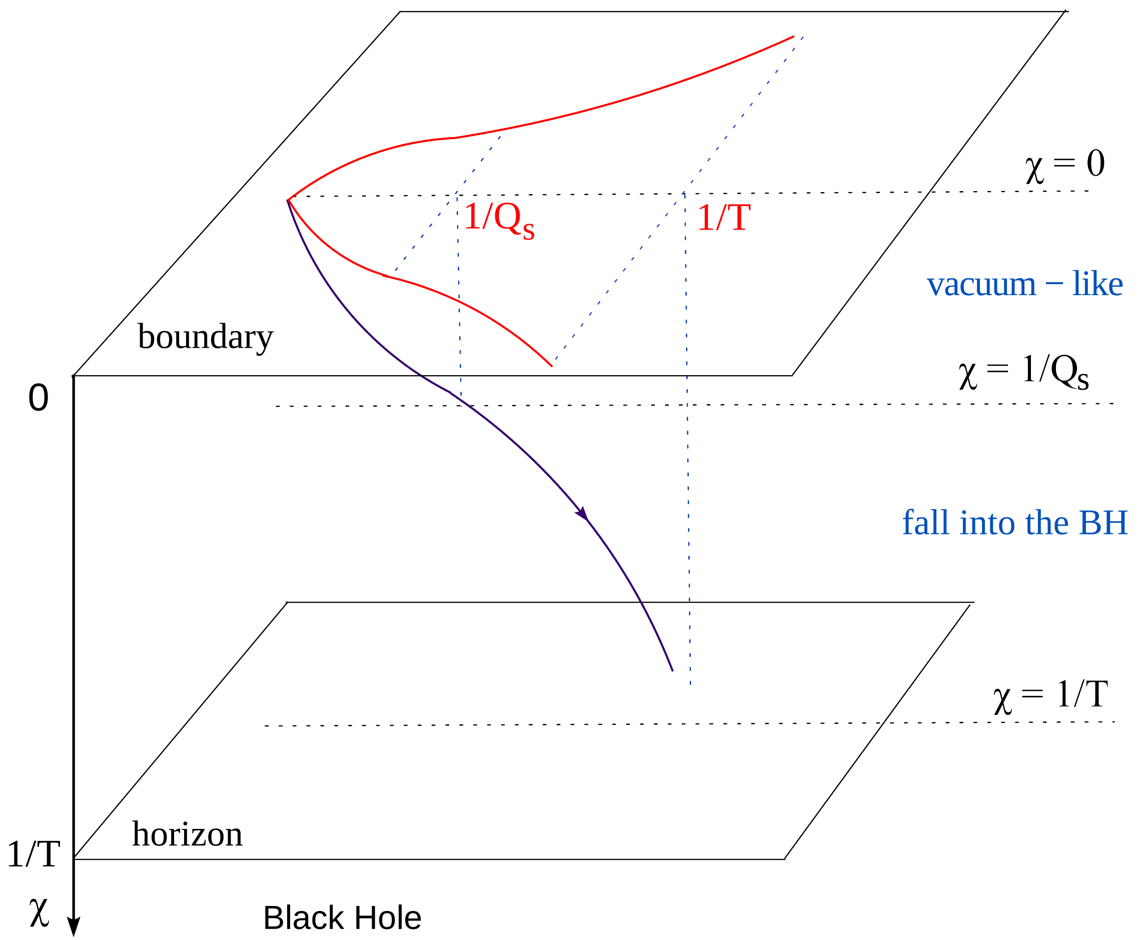
high energy

*($K \equiv Q$ in these figures)*

- Gravitational interaction grows with the energy $\omega \sim k$

# High energy: The fall

- Gravitational attraction becomes stronger with increasing energy and eventually washes out the repulsive barrier




- The wave falls into the BH along a massless geodesics

# Saturation momentum



- Gravitational interactions are proportional to the energy density in the wave ($\omega$) and in the plasma ($T$)
- The criterion for strong interaction within the plasma

$$\underbrace{Q}_{\text{potential barrier}} \lesssim \underbrace{\frac{\omega T^2}{Q^2}}_{\text{gravitational potential}}$$

- Gravitational attraction must overcome the barrier due to energy conservation
- $Q_s(x)$ : plasma saturation momentum

# Saturation momentum



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- The criterion for strong interaction within the plasma

$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

- The partonic fluctuation must live long enough to feel the effects of the plasma
- $Q_s(x)$ : plasma saturation momentum

# Saturation momentum



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- The criterion for strong interaction within the plasma

$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

- High energy, or high $T$, or low $Q$ : $Q \lesssim Q_s$ with

$$Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x} \quad \text{where} \quad x \equiv \frac{Q^2}{2\omega T}$$

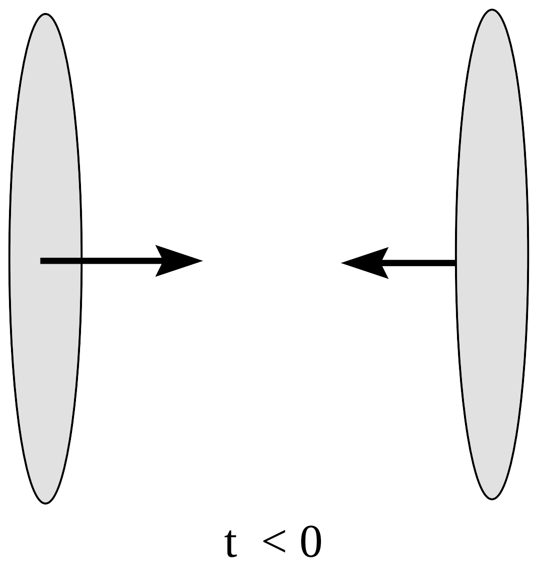
Recall: the parton picture involves 2 variables : $x$ and $Q^2$

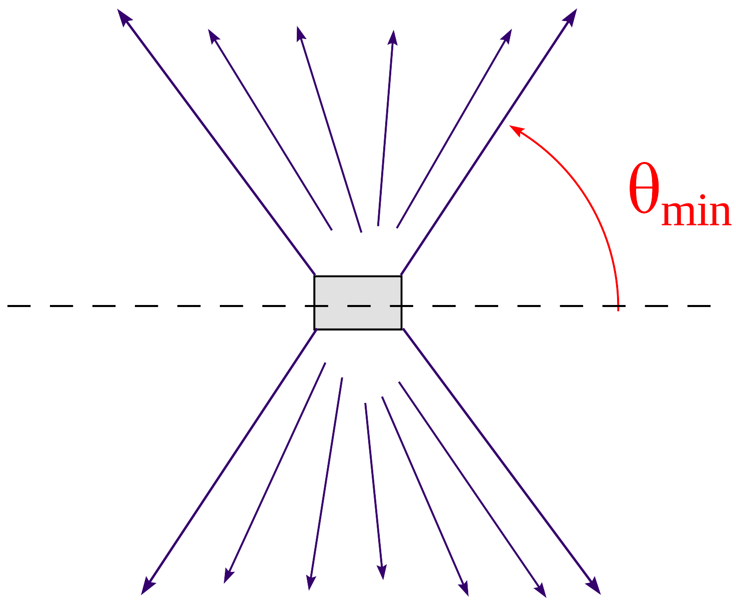
- $Q_s(x)$ : plasma saturation momentum

# DIS at large $x$ : No partons !

- Low energy, or large $x$ : $x > x_s(Q) \simeq T/Q$
- No scattering (except through tunneling) $\Longrightarrow F_2(x, Q^2) \approx 0$

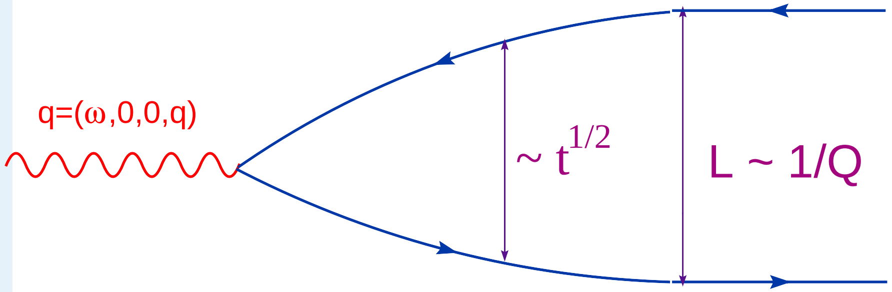
  $\Longrightarrow$ no partons with large momentum fractions $x > x_s$
- No forward/backward jets in hadron–hadron collisions !

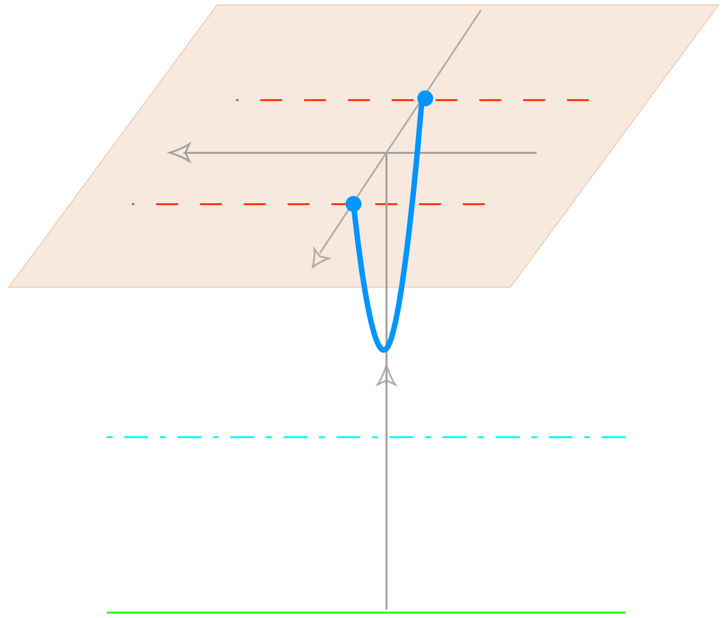


# “No drag force on a *small meson* in the plasma”





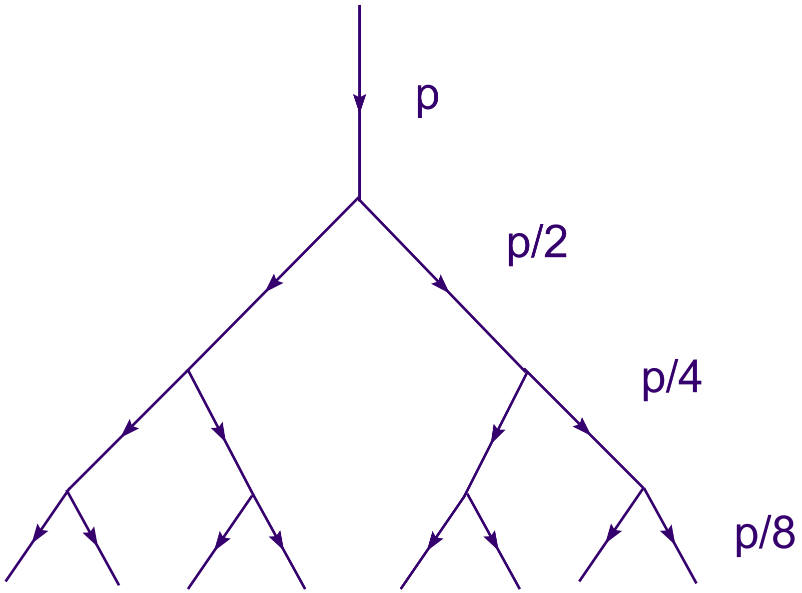
- Larger mesons melt in the plasma: critical size $L_s \sim 1/Q_s$

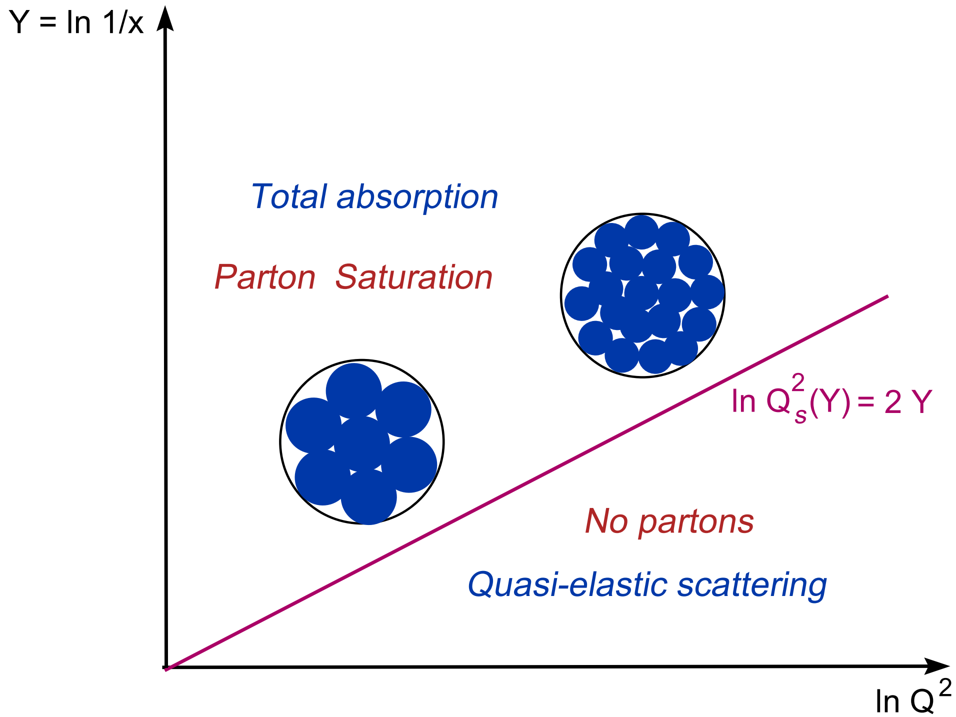
$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Longrightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma}T} = \frac{(1-v_z^2)^{1/4}}{T}$$

*[cf. Liu, Rajagopal, Wiedemann; Chernicoff et al; Caceres et al (2006)]*

# Low $x$ : Parton saturation

- $x \lesssim x_s = T/Q$ : strong scattering $\Longrightarrow$ $F_2(x,Q^2) \sim xN_c^2Q^2$
- Parton occupation numbers of $\mathcal{O}(1) \Longrightarrow$ 'saturation' (CGC)
- Physical interpretation: 'Quasi–democratic' parton branching

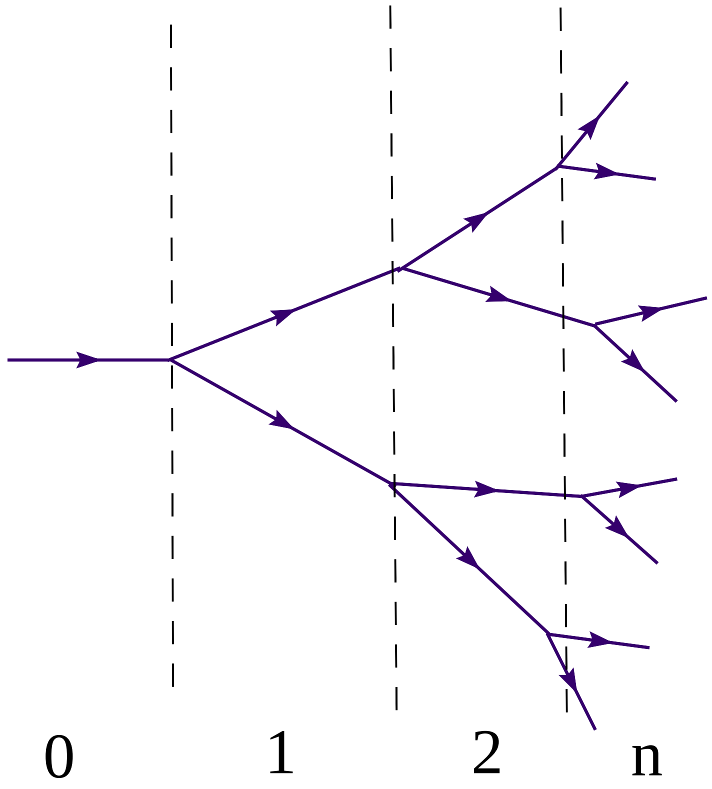



- All partons have branched down to small values of $x$ !

# Medium induced parton branching

- Quasi–democratic branching in the presence of the uniform transverse force $\sim T^2$ exerted by the plasma




$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

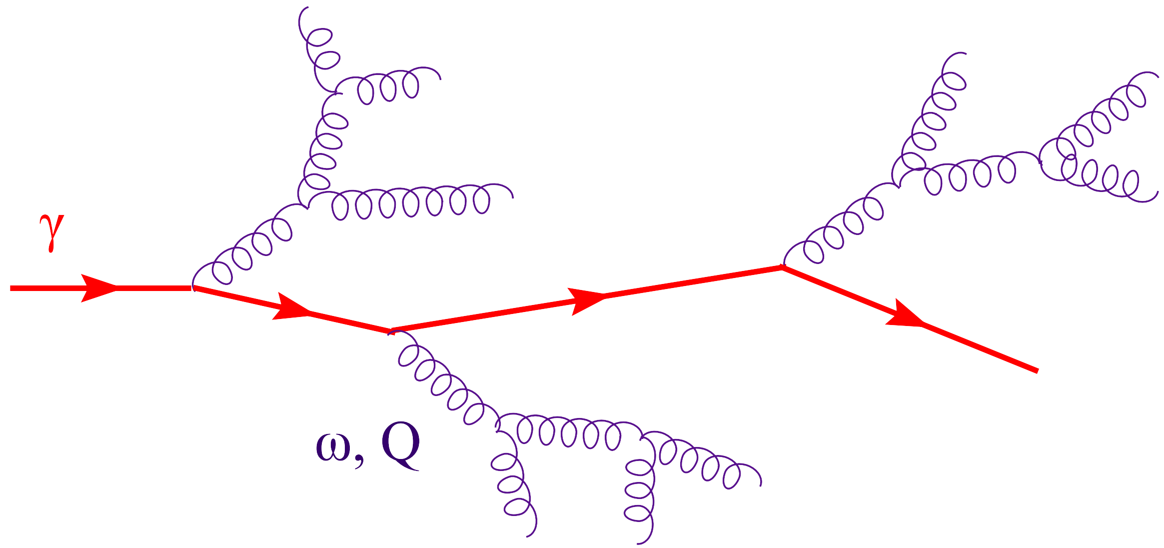
$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2$$

$$Q_n \sim \frac{\omega_n}{Q_n^2} T^2 \sim (\omega_n T^2)^{1/3}$$

Lifetime of the current: $\Delta t \sim \frac{\omega}{Q_s^2} \ll \frac{\omega}{Q^2}$

# Heavy Quark: Energy loss






- Virtual quanta with $Q \lesssim Q_s$ are absorbed by the plasma
- Maximal energy loss: $\omega \sim \gamma Q_s$

$$Q_s \simeq \frac{\omega}{Q_s^2} T^2 \simeq \frac{\gamma}{Q_s} T^2 \implies Q_s^2 \sim \gamma T^2$$

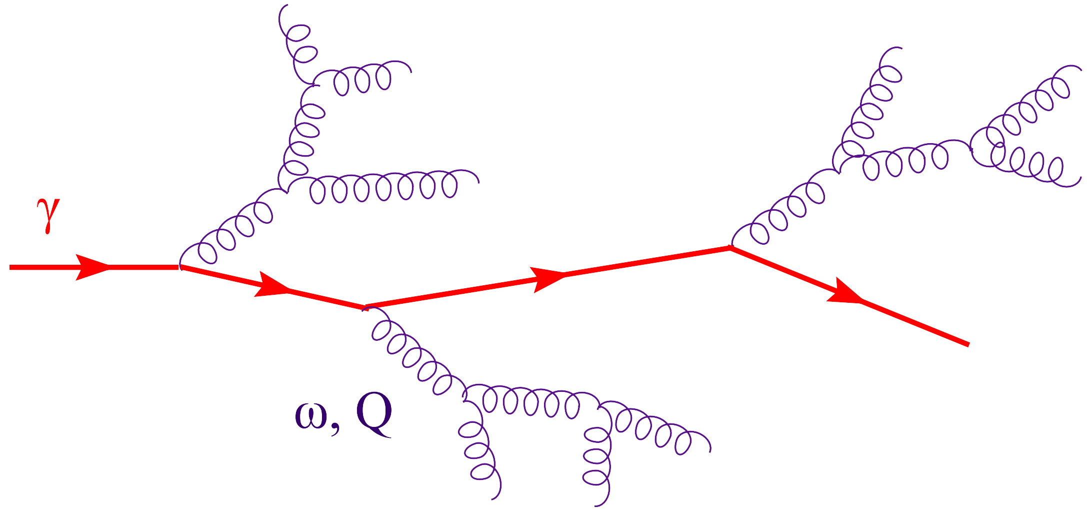
$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\,Q_s^2 \simeq \sqrt{\lambda}\,\gamma\,T^2$$

*Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser, 2006 (trailing string)*

# Momentum broadening



- Fluctuations in the medium–induced emission process




$$\frac{\mathrm{d}\langle p_T^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{Q_s^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\frac{Q_s^4}{\gamma Q_s} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,T^3$$

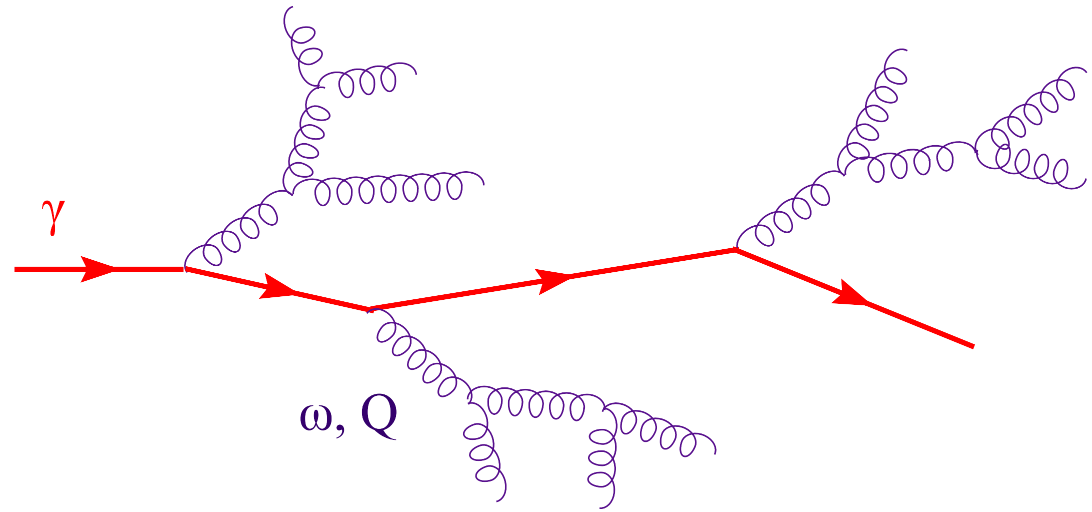
$$\frac{\mathrm{d}\langle p_L^2\rangle}{\mathrm{d}t} \sim \sqrt{\lambda}\,\frac{\omega^2}{(\omega/Q_s^2)} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,\gamma^2\,T^3$$

*Casalderrey-Solana, Teaney; Gubser, 2006 (from trailing string)*

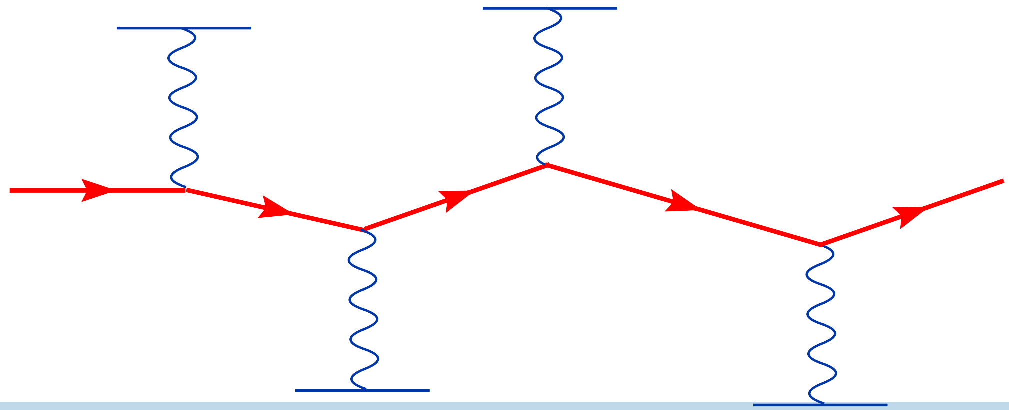
# Momentum broadening



- Strong coupling : fluctuations in the emission process



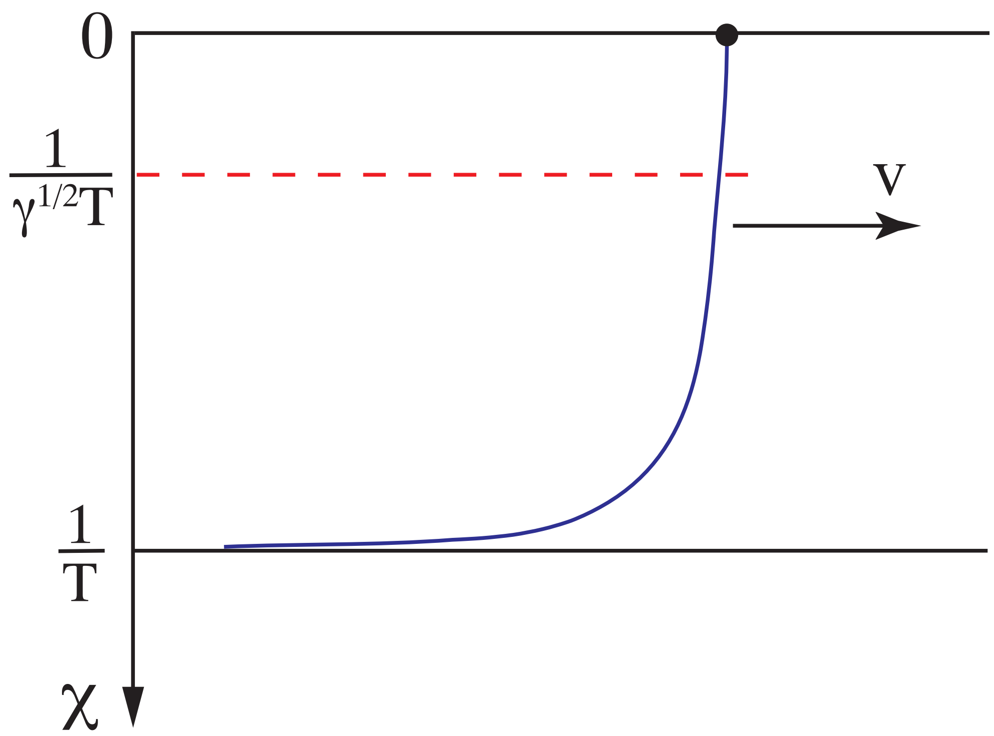

- pQCD : thermal rescattering (different physics !)



# Stochastic trailing string



- How are quantum–mechanical (as opposed to thermal) fluctuations encoded in AdS/CFT ?



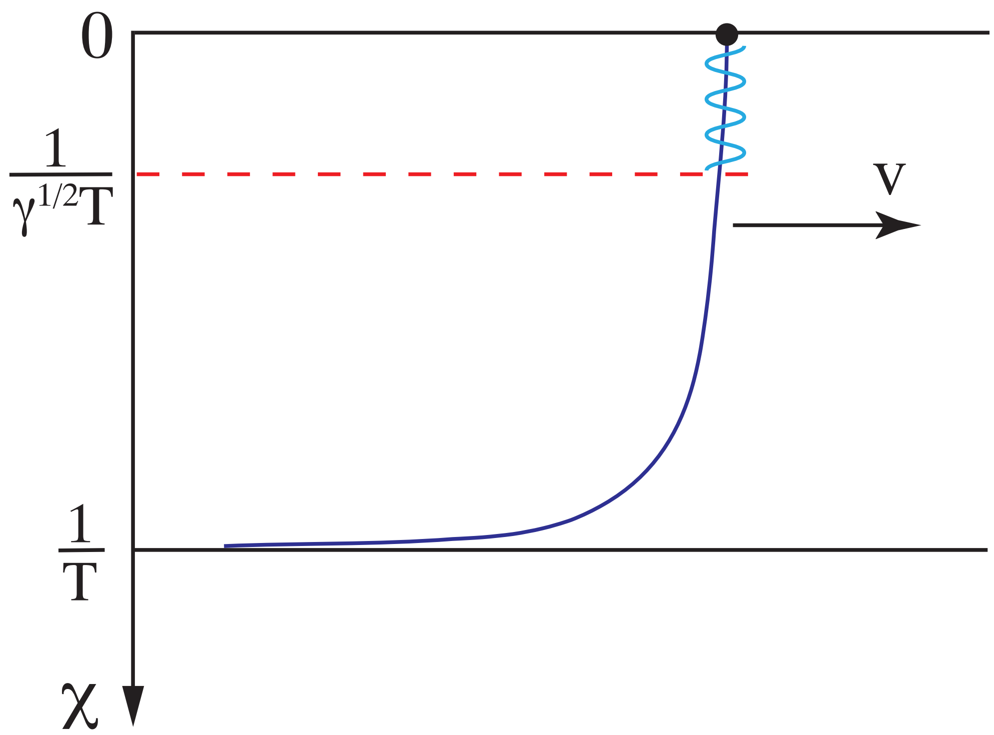

- World–sheet horizon at $\chi_s = 1/Q_s \sim 1/(\sqrt{\gamma}T) \ll 1/T$
- Hawking radiation ( = thermal flucts.) plays no role
  (in contrast to a static string; cf. talk by Rangamani)

# Stochastic trailing string



- Fluctuations on top of the world–sheet horizon $\chi_s$

  $\Longrightarrow$ noise term on the 'stretched horizon' at $\chi = \chi_s + \epsilon$




- Langevin equation for the upper part of the string & the heavy quark *(G. Giecold, E.I., A. Mueller: to appear)*
- Physics: Fluctuations in the parton cascades

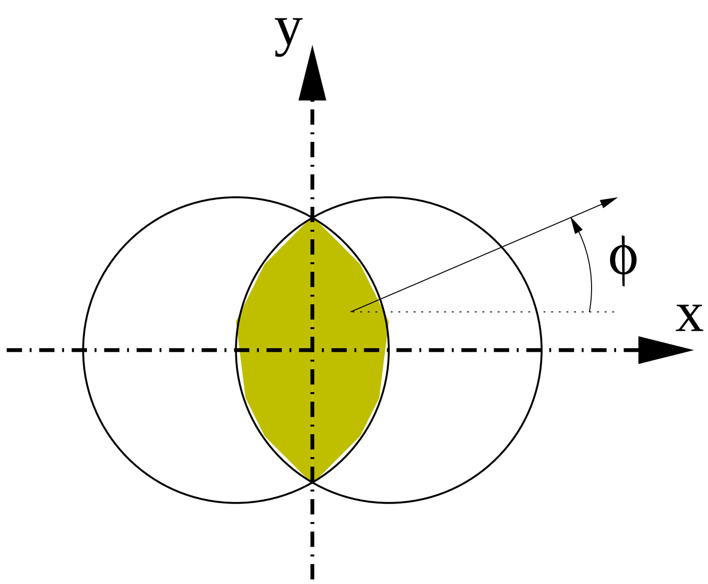
# Conclusions

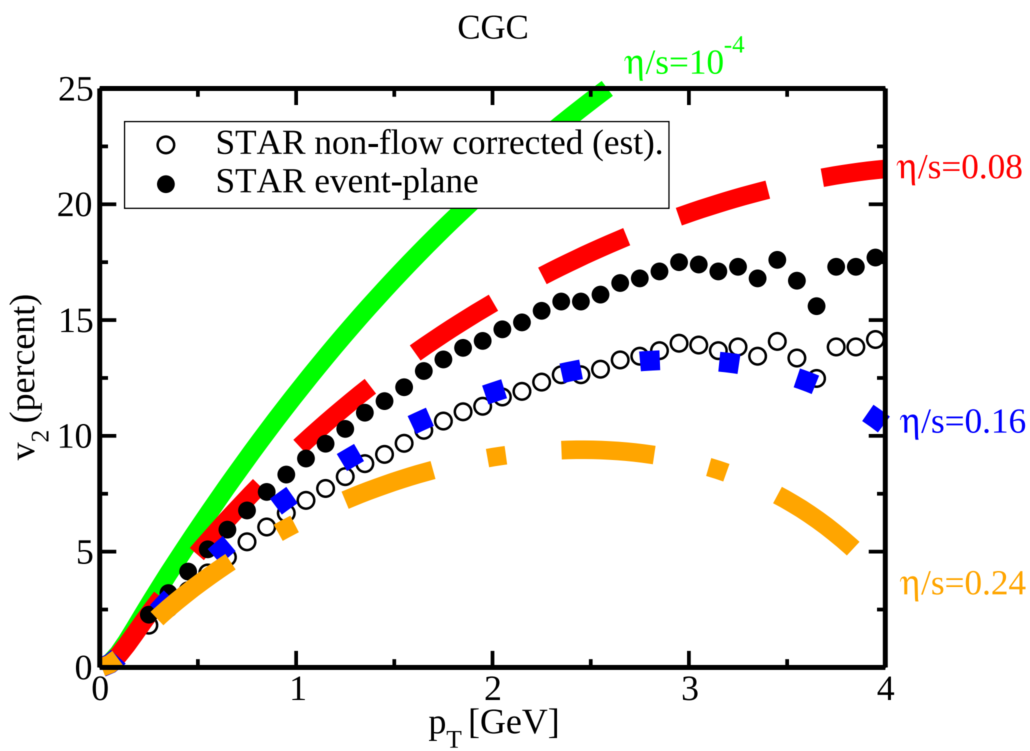


- Hard probes & high-energy physics appears to be quite different at strong coupling as compared to QCD
  - no forward/backward particle production in HIC
  - no jets in $e^+e^-$ annihilation
  - different mechanism for jet quenching
- Not so surprising: by asymptotic freedom, hard & high–energy physics in QCD is weakly coupled
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - some observables receive contributions from several scales, from soft to hard: use AdS/CFT in the soft sector
  - long–range properties (hydro, thermalization, etc) might be controlled by strong coupling
- Many (simple) physical ideas appear to smoothly interpolate from weak to strong coupling !

# Elliptic flow at RHIC: The perfect fluid






- Non–central $AA$ collision: Pressure gradient is larger along $x$

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2\phi\,, \qquad v_2 = \text{“elliptic flow”}$$

- Well described by hydrodynamical calculations with very small viscosity/entropy ratio: “perfect fluid”

# Viscosity over entropy density ratio



- Viscosity/entropy density ratio at RHIC (in units of $\hbar$)

$$\frac{\eta}{s} = 0.1 \pm 0.1(\mathrm{theor}) \pm 0.08(\mathrm{exp}) \quad [\hbar]$$

- This ratio is small when the coupling is strong !
- Kinetic theory: viscosity is due to collisions among molecules

$$\eta \sim \rho\, v\, \ell = \text{mass density} \times \text{velocity} \times \text{mean free path}$$

- Conjecture (from AdS/CFT) : *[Kovtun, Son, Starinets, 2003]*

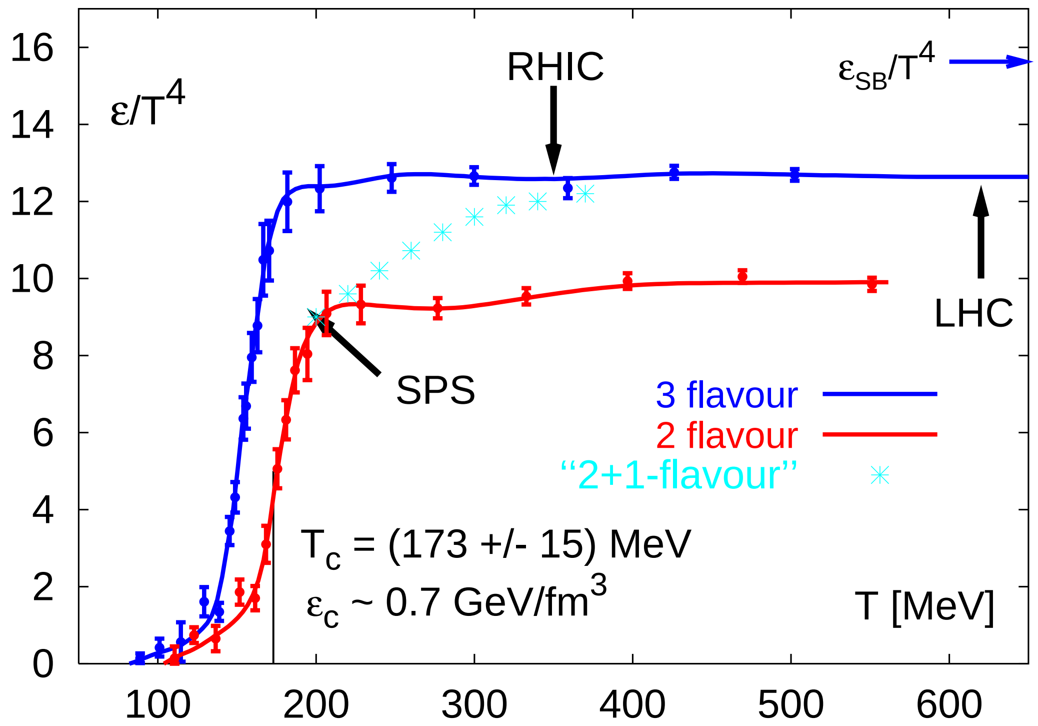
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \qquad \text{[lower limit = infinite coupling]}$$

- The RHIC value is at most a few times $\hbar/4\pi$ !

# Heating QCD : Lattice results



- Energy density as a function of $T$ (Bielefeld Coll.)




$\mathcal{E}/\mathcal{E}_0 \approx 0.85$ for $T = 3T_c$

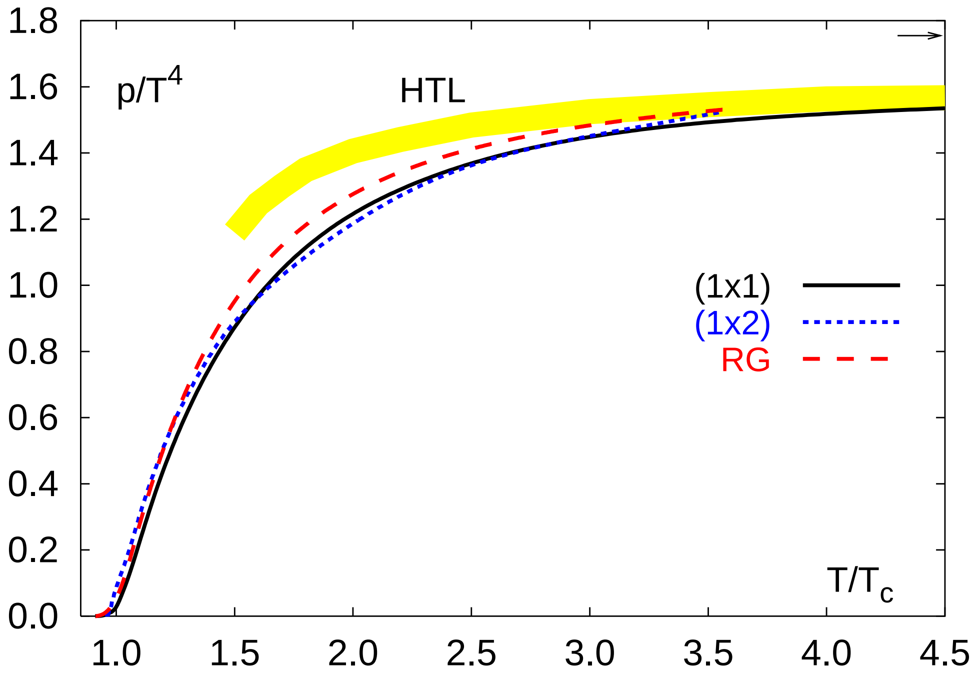
- Is this deviation from ideal gas small ? Or is it large ?
- AdS/CFT : $\mathcal{E}/\mathcal{E}_0 \to 3/4$ when $\lambda \to \infty$ ($\mathcal{N} = 4$ SYM)

# Finite–$T$ : Resummed perturbation theory

- This ratio $p/p_0 \approx 0.85$ can be also explained by resummed perturbation theory

  (collective phenomena: screening, thermal masses)

  *(J.-P. Blaizot, A. Rebhan, E. Iancu, 2000)*




- First principle calculation without free parameter

# The 'perfect fluid'



- Uncertainty principle applied to viscosity:

$$\eta \sim \rho v \lambda_f\,, \qquad S \sim n \sim \frac{\rho}{m}$$

$$\frac{\eta}{S} \sim m v \lambda_f \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

- Weakly interacting systems have $\eta/S \gg \hbar$
- Strongly coupled $\mathcal{N}=4$ SYM plasma

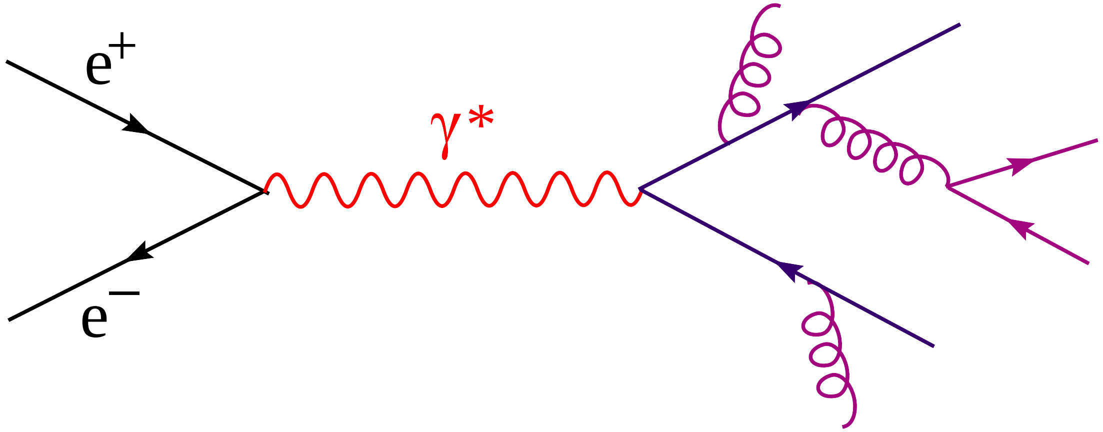
$$\frac{\eta}{S} \to \frac{\hbar}{4\pi} \qquad \text{when} \qquad \lambda \to \infty$$

*(Policastro, Son, and Starinets, 2001)*

- This bound is believed to be universal : $\eta/S \geq \hbar/4\pi$
- The data at RHIC are consistent with the lower limit being actually reached : 'sQGP'

# Jets






- ‘Multi–jet event’ : large emission angle & $x \sim \mathcal{O}(1)$

$$k_\perp \sim k \sim \sqrt{s} \implies \mathcal{P}_{\rm Brem} \sim \alpha_s(s) \ll 1$$

small probability for emitting an extra gluon jet !

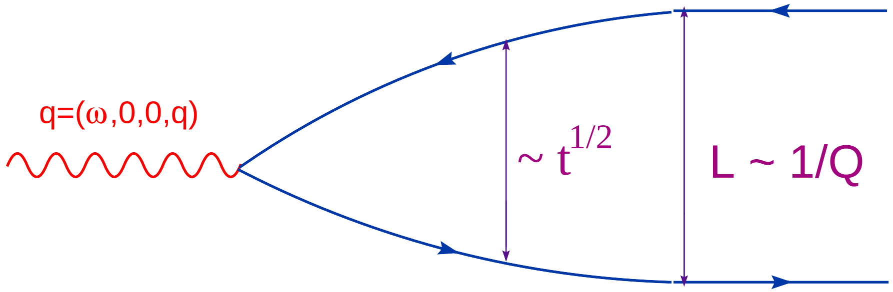
- ‘Intra–jet activity’ : collinear and/or soft gluons

$$\Lambda_{\rm QCD} \ll k_\perp \ll k \ll \sqrt{s} \implies \mathcal{P}_{\rm Brem} \sim \alpha_s \ln^2 \frac{\sqrt{s}}{\Lambda_{\rm QCD}} \sim \mathcal{O}(1)$$

modifies particle multiplicity but not the number of jets

# Screening length

- A small color dipole ('meson') with transverse size $L \ll 1/Q_s$ propagates through the strongly–coupled plasma with almost no interactions !




- Larger dipoles with $L \gtrsim 1/Q_s$ cannot survive in the plasma

$$L_s \sim \frac{1}{Q_s} \quad \& \quad \gamma \sim \frac{\omega}{Q} \quad \Longrightarrow \quad L_s \sim \frac{1}{\sqrt{\gamma}\, T} \ll \frac{1}{T}$$

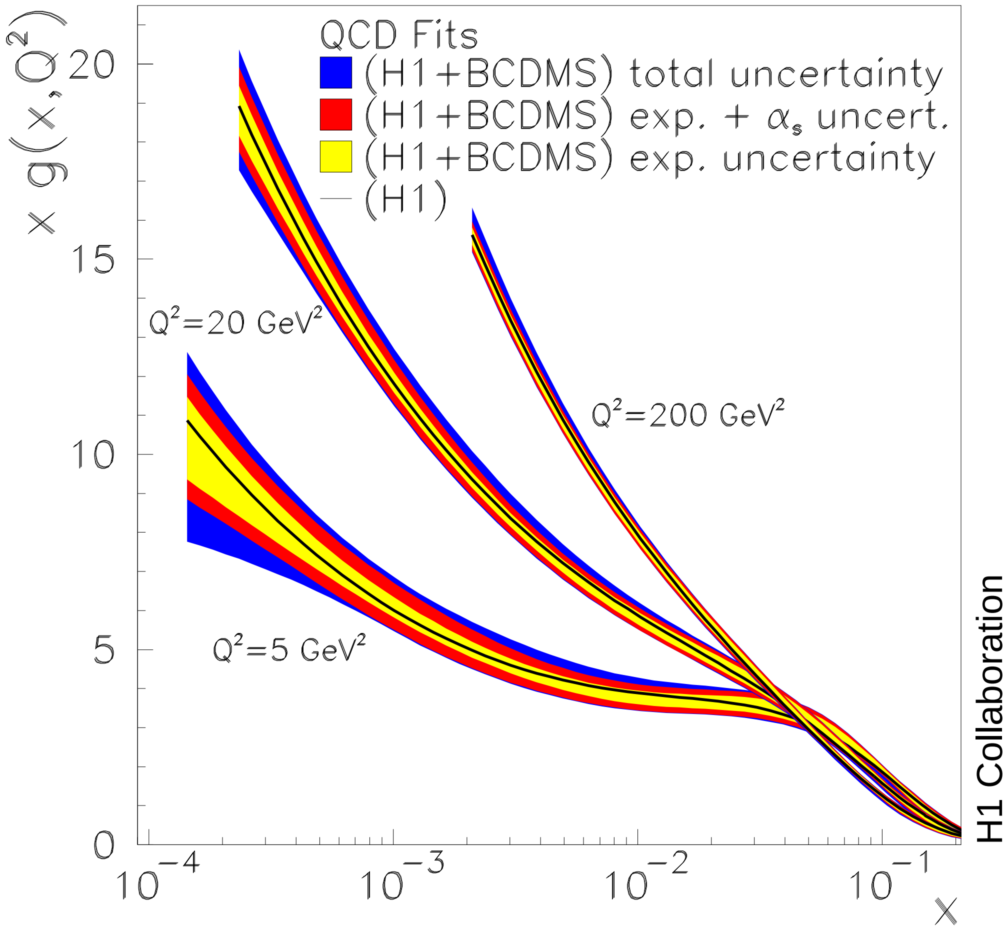
- The dipole lifetime is short on natural time scales:

$$\Delta t \sim \frac{\omega}{Q_s^2} \sim \frac{\sqrt{\gamma}}{T} \ll \frac{\gamma}{T}$$

# Gluons at HERA



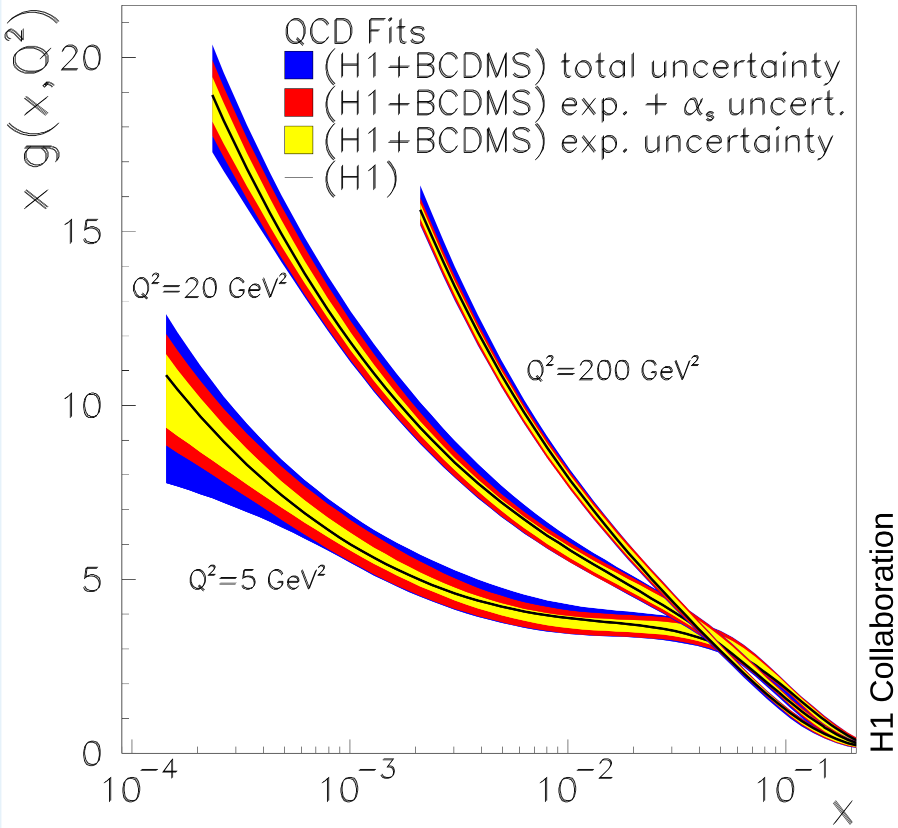
$xg(x, Q^2)$ $=$ # of gluons with transverse area $\sim 1/Q^2$ and $k_z = xP$

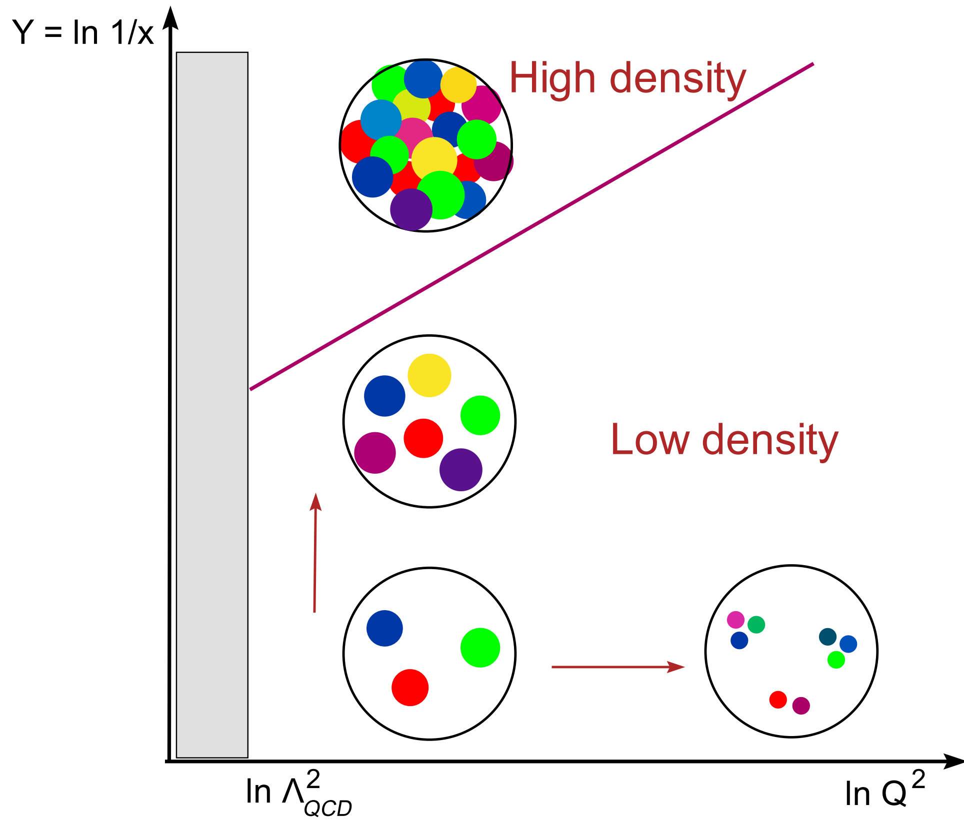



▷ Rapid rise with $1/x$: $xg(x, Q^2) \sim 1/x^{\lambda}$ with $\lambda = 0.2 \div 0.3$

# Gluons at HERA





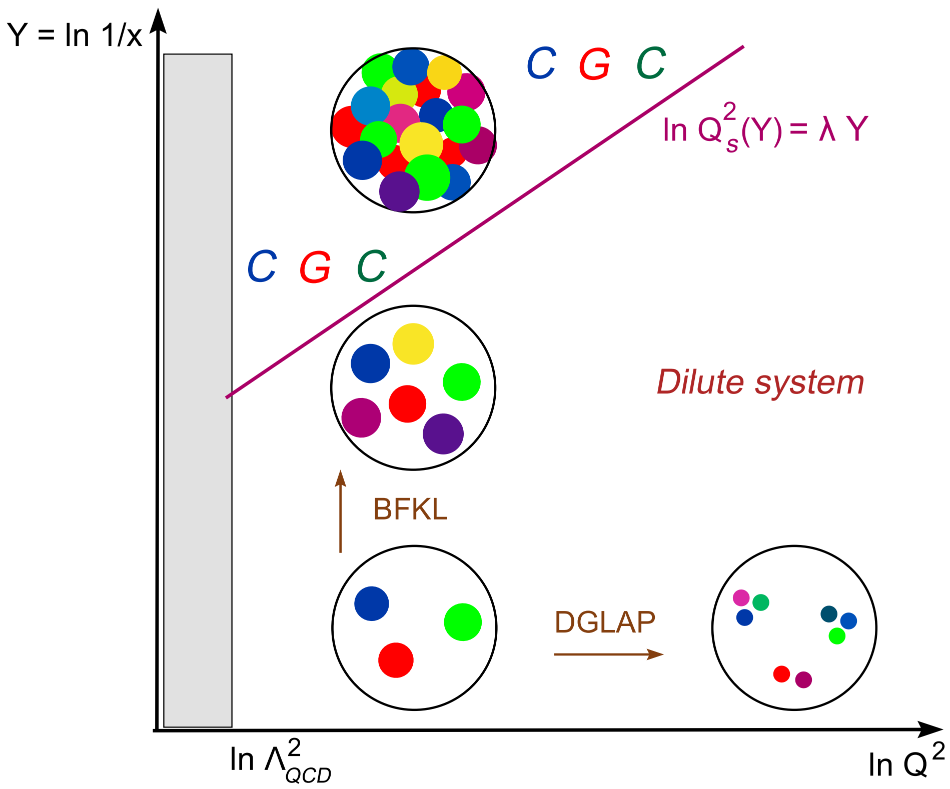

- High–$Q^2$ evolution : The parton density is decreasing
- Small–$x$ evolution: An evolution towards increasing density
- The gluon density cannot become arbitrarily high !

# Color Glass Condensate



- The gluon occupation number cannot be larger than $1/\alpha_s$ :

$$n(x, Q^2) \sim \frac{1}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2}$$



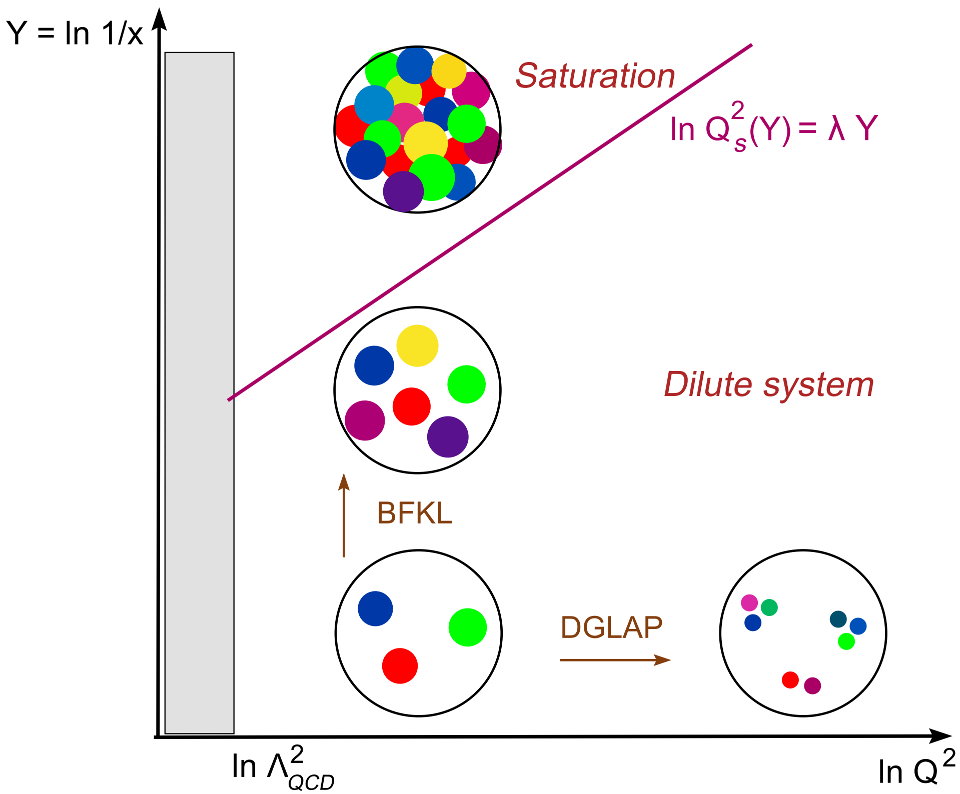

- When $n \sim 1/\alpha_s$, gluons form a Bose condensate: CGC

# The Saturation Momentum



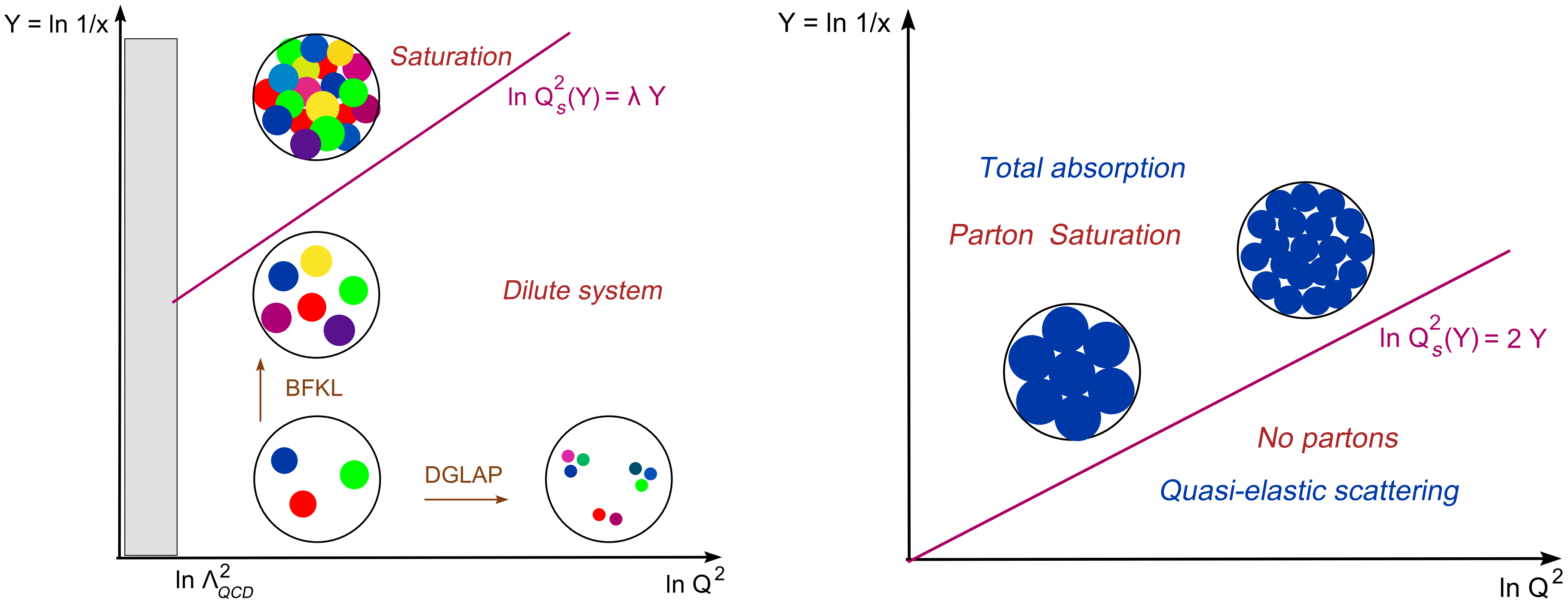
- $n(x, Q^2) \sim 1/\alpha_s \Longrightarrow$ the saturation line $Q^2 = Q_s^2(x)$

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^{\lambda_s}}$$



# Saturation line: weak vs. strong coupling



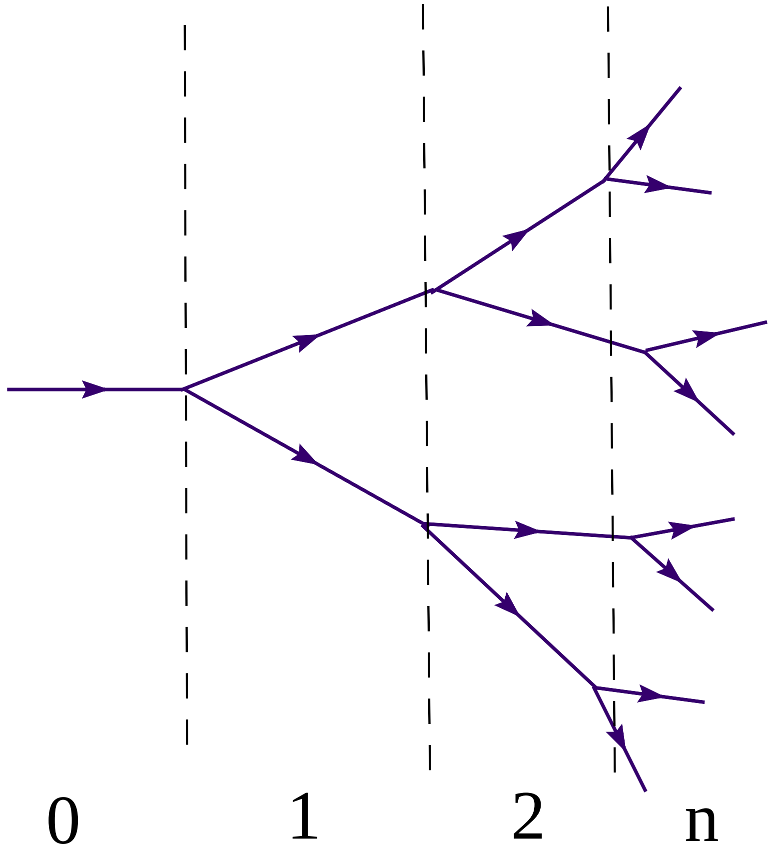



- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv e^{\lambda_s Y}$
  - weak coupling (LO pQCD): $\lambda_s \approx 0.12\, g^2 N_c$
  - phenomenology & NLO pQCD: $\lambda_s \approx 0.2 \div 0.3$
  - strong coupling (plasma): $\lambda_s = 2$ (graviton)

# Quasi–democratic parton branching



- No reason why branching should stop at 2 parton level !
- No reason to favour special corners of phase–space !




$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$Q_n \sim \frac{Q_{n-1}}{2} \quad \text{(vacuum)}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

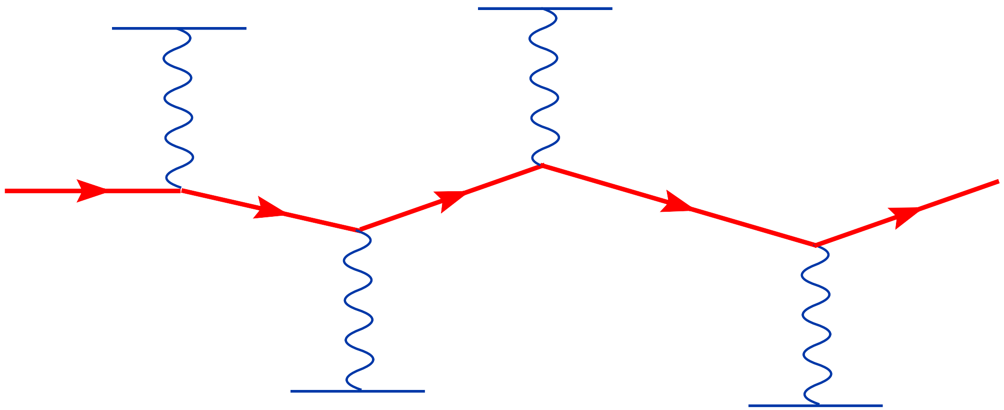
$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2 \quad \text{(plasma)}$$

- Qualitative agreement with all the results from AdS/CFT

# Transverse momentum broadening



- A parton (say, heavy quark) undergoes multiple scattering (random kicks) off the plasma constituents



$$\frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \frac{xg(x,Q^2)}{N_c^2-1}$$

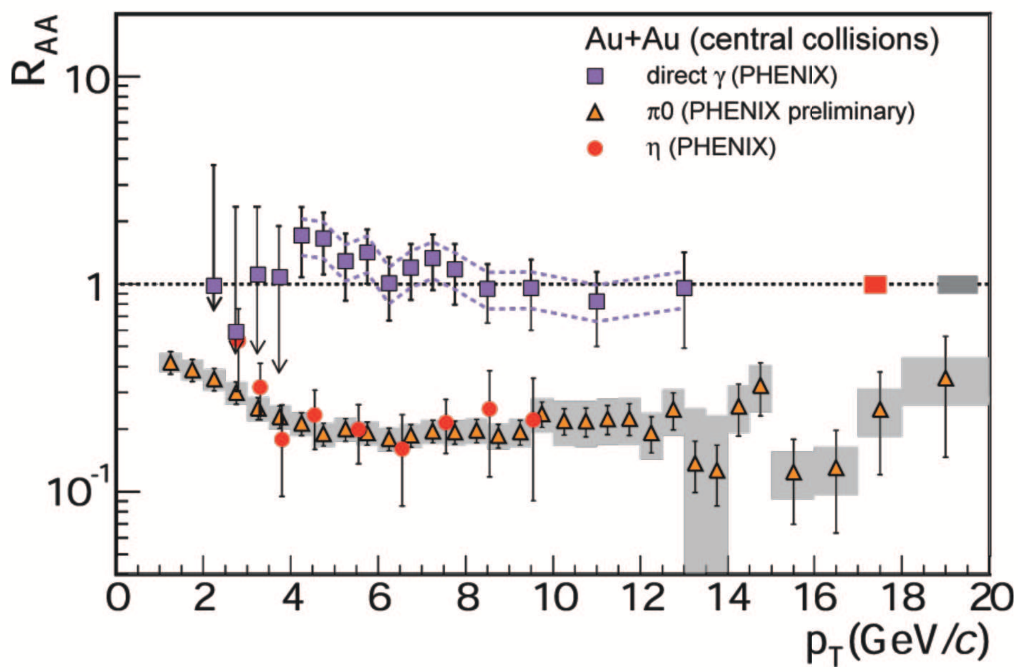
- $xg(x,Q^2)$ : gluon distribution per unit volume in the medium
- Weakly–coupled QGP : incoherent sum of the gluon distributions produced by thermal quarks and gluons

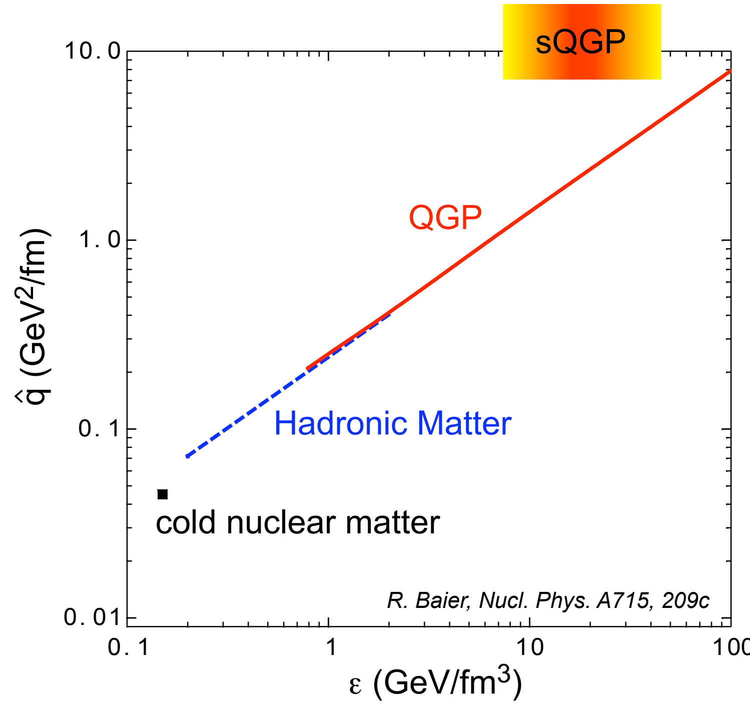
$$xg(x,Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g\,, \quad \text{with} \quad n_{q,g}(T) \propto T^3$$

# Nuclear modification factor

- How to measure $\hat{q}$ ? Compare $AA$ collisions at RHIC to $pp$

$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p) \times A^2}$$

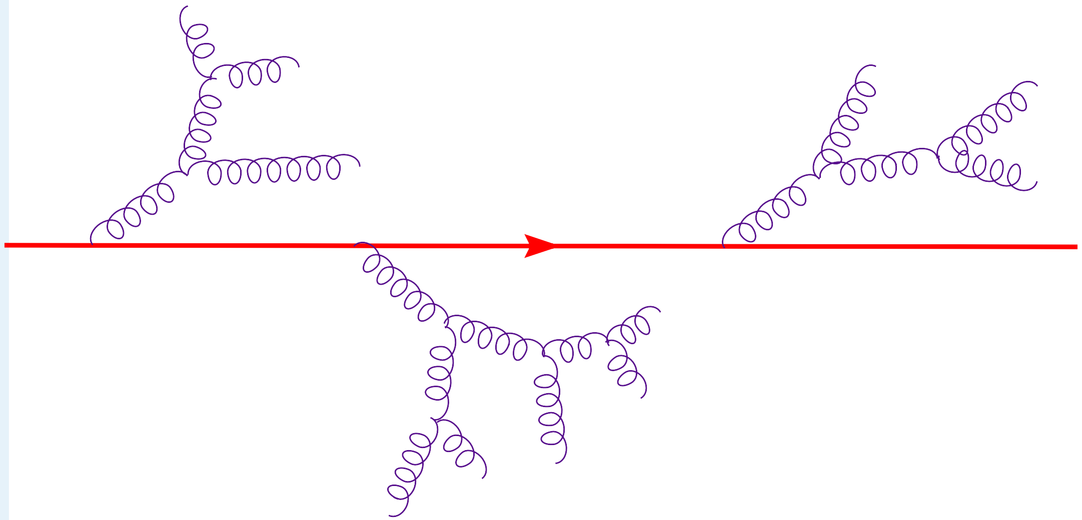



- RHIC data seem to prefer $\hat{q} \simeq 10$ GeV$^2$/fm, which is too large to be accounted for by weakly–coupled QGP (??)

# Heavy Quark: Energy loss



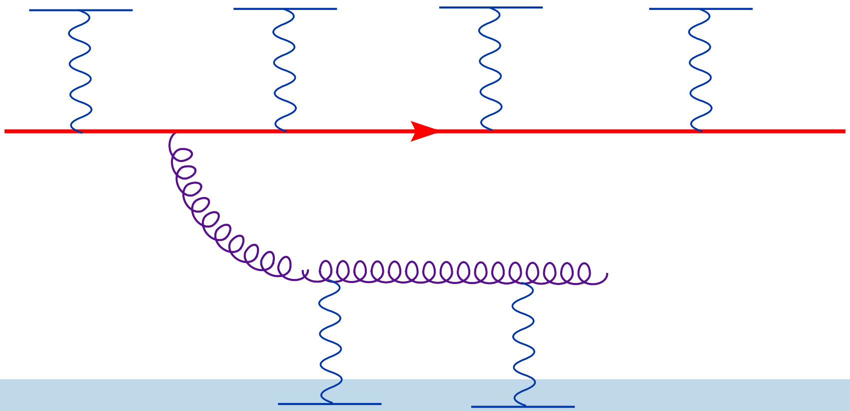


$$Q_s^2 \sim \left(t_{\rm br} T^2\right)^2 \sim \gamma\, T^2$$

$$-\frac{{\rm d}E}{{\rm d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\,Q_s^2$$

*(Herzog et al; Gubser, 2006)*

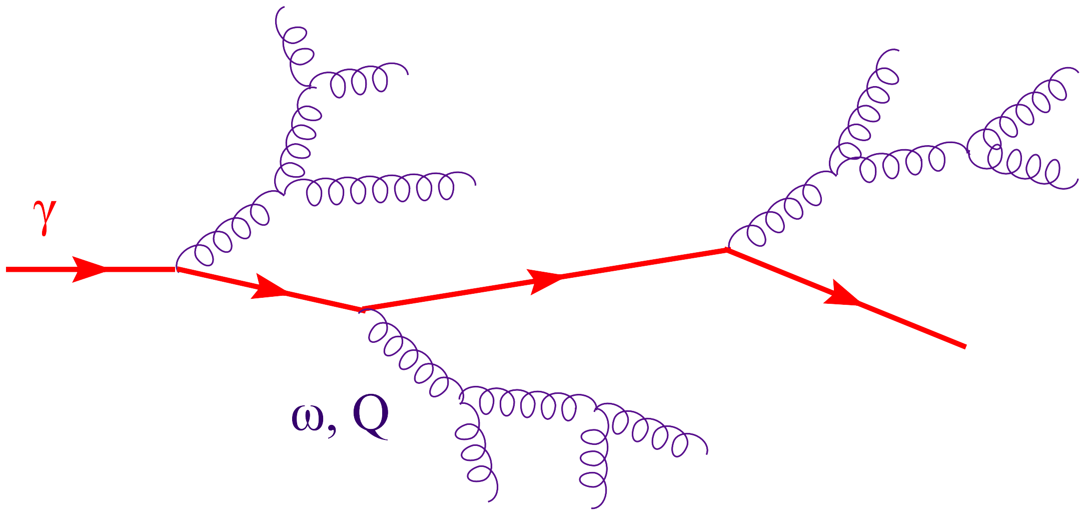
- pQCD : same formula, but with $\sqrt{\lambda} \to g^2 N_c$ and different $Q_s$



$$Q_s^2 \sim t_{\rm br}\,\hat{q}$$ (quenching param.)

# Transverse momentum broadening



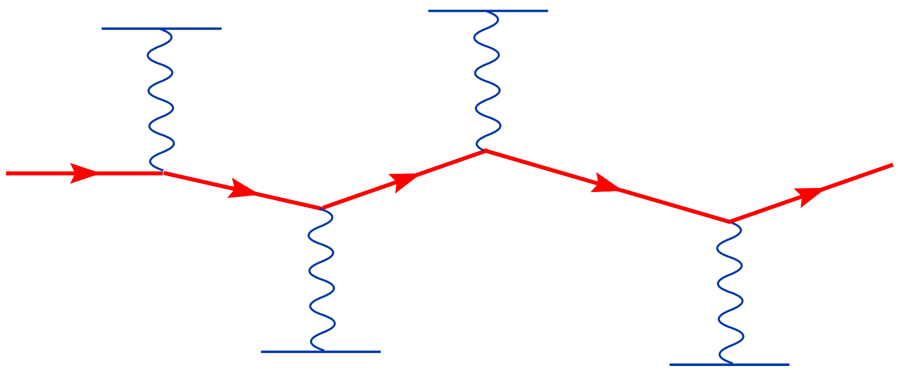



$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \sqrt{\lambda}\,T^2 Q_s$$

*Casalderrey-Solana, Teaney, 2006; Gubser, 2006; Dominguez et al, 2008*

see talks by Al Mueller and Cyrille Marquet

- pQCD : different physics ! thermal rescattering



$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \hat{q}$$