Horizon formation and far-from-equilibrium isotropization in strongly coupled plasma

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based on work with P. Chesler

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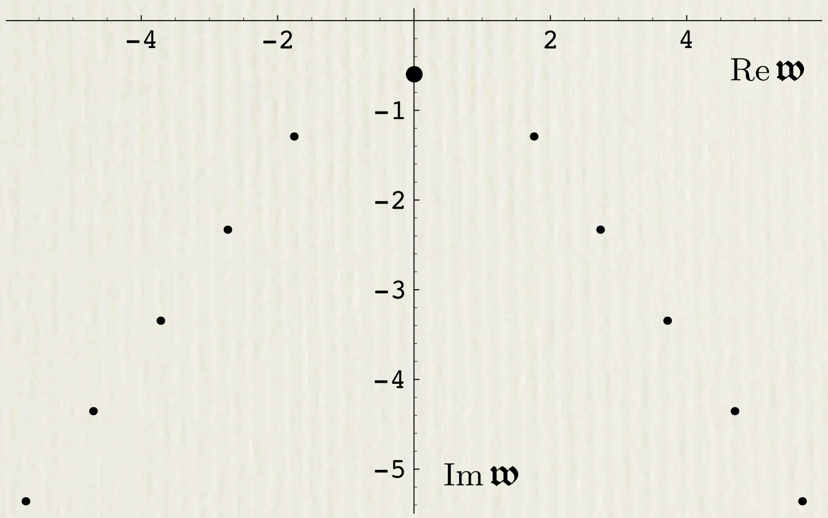
Thermal plasma physics from AdS/CFT

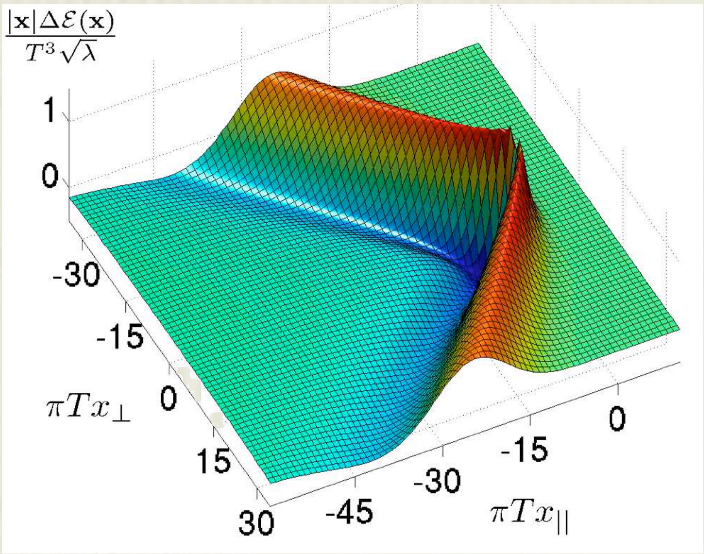
- Equilibrium ($\mathcal{N}$ = 4 SYM) (static, Euclidean signature)
  - equation of state
  - correlation lengths, screening
  - flavor physics
  - finite volume
    - confinement/deconfinement
    - chemical potentials
    - rotation

SUGRA mode	$\mathcal{J}^{CR_t}_{R_y}$	SYM operator	mass/πT
G_{00}	0^{++}_{+}	T_{00}	2.3361
a	0^{+-}_{-}	$\mathrm{tr}\, E \cdot B$	3.4041
G_{ij}	2^{++}	T_{ij}	3.4041
ϕ	0^{++}_{+}	$\mathcal{L}$	3.4041
G_{i0}	1^{+-}	T_{i0}	4.3217
B_{ij}	0^{-+}_{-}	$\mathcal{O}_{ij}$	5.1085
C_{ij}	0^{--}_{+}	$\mathcal{O}_{30}$	5.1085
B_{i0}	1^{--}	$\mathcal{O}_{i0}$	6.6537
C_{i0}	1^{-+}	$\mathcal{O}_{3j}$	6.6537
G^{a}_{a}	0^{++}_{+}	$\mathrm{tr}\, F^4$	7.4116

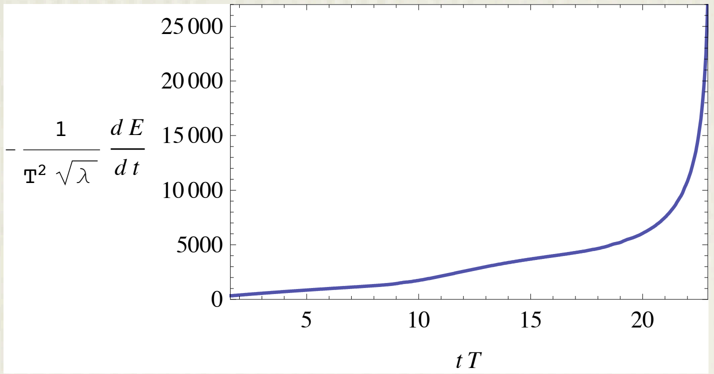
- Near-equilibrium (real-time response, Minkowski signature)
  - viscosity, diffusion
  - quasi-normal modes, late time expansions
  - photo-emission
  - second-order transport coefficients
  - non-linear conductivity



Thermal plasma physics from AdS/CFT



- Probe dynamics (classical string dynamics)
  - heavy quark drag
  - wakes, Brownian motion
  - heavy meson stability, dispersion
  - light quark jets

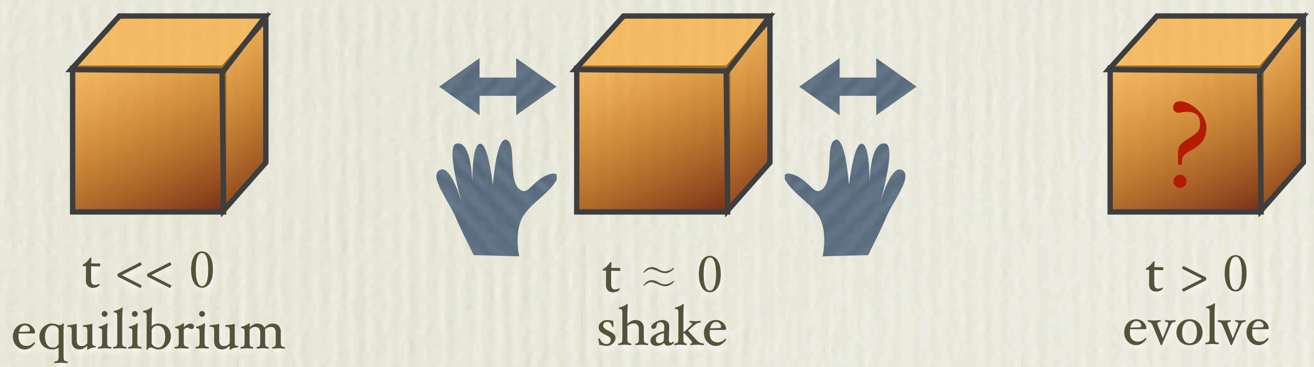


- Far-from-equilibrium dynamics ???
  - plasma formation
  - early thermlization
  - turbulence

} initial value problems with non-trivial time-dependent bulk geometry

Non-equilibrium initial states

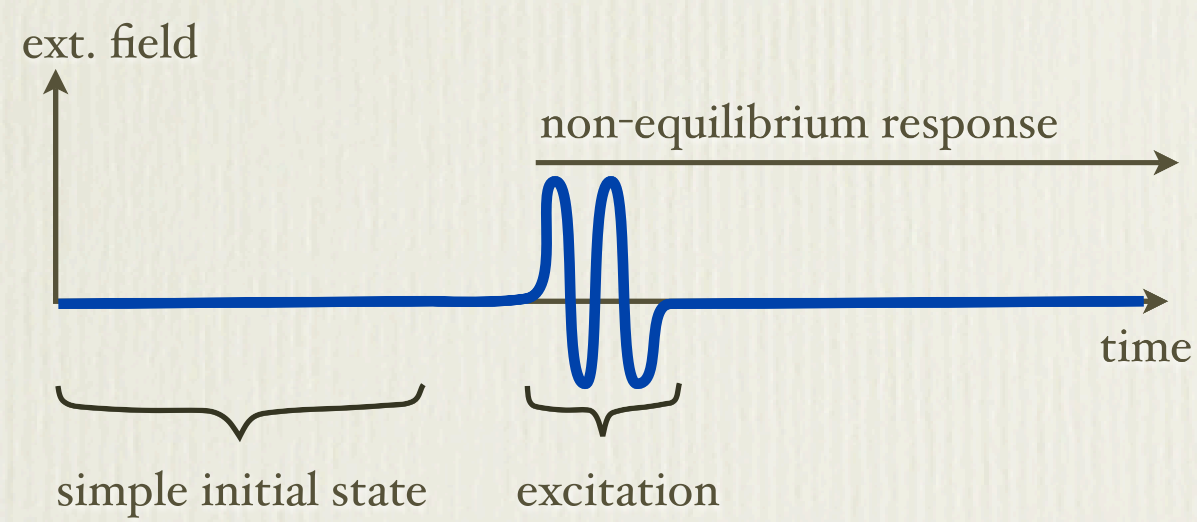
- Specify complete density matrix $\boldsymbol{\rho}$? Ugh!
- Pick geometry on initial Cauchy surface ? Ugh!
- Want "operational" description:



∴ Specify time-dependent external fields

➡ time-dependent dynamics

➡ external work done on system



Anisotropy dynamics

- Metric $g^{\mu\nu}$ = external field coupling to stress-energy $T^{\mu\nu}$

  ∴ time-dependent geometry ➡ non-equilibrium $\langle T^{\mu\nu} \rangle$

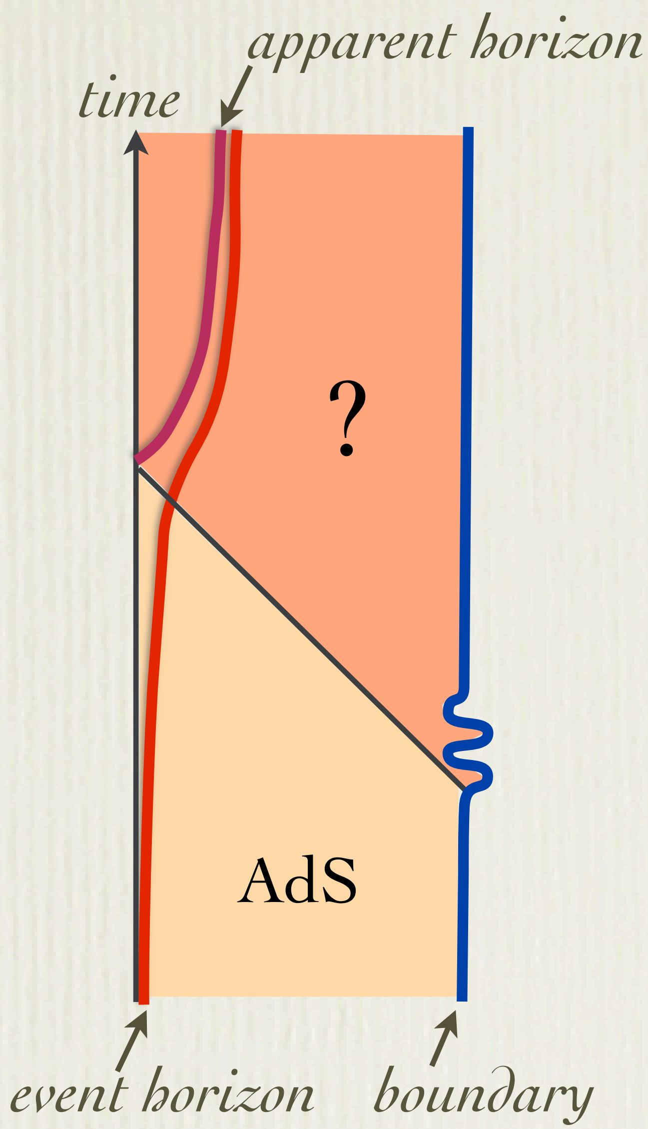
- "Simple" case: perfect spatial homogeneity, arbitrary anisotropy

  Ex: $ds^2 = -dt^2 + e^{f(t)}(dx^2 + dy^2) + e^{-2f(t)}\, dz^2$

$$\Rightarrow \quad \langle T^{\mu\nu}(t, \mathbf{x}) \rangle = \begin{bmatrix} \varepsilon(t) & & & \\ & p_\perp(t) & & \\ & & p_\perp(t) & \\ & & & p_\parallel(t) \end{bmatrix}$$

Gravitational description

- Solve 5-*d* Einstein equations with time-dependent boundary condition $G^{AB} \to g^{\mu\nu}$ and simple initial condition (AdS or AdS-BH)
- Extract $\langle T^{\mu\nu} \rangle$ from sub-leading near-boundary asymptotics
- Note:
  - time-dependent boundary conditions produce dynamic event horizon
  - "Teleological" event horizon growth occurs outside causal future of boundary time dependence
  - ➡ event horizon area (pulled back to boundary) *cannot* represent entropy in non-equilibrium setting



Practical issues (I)

- Coordinate choice:

  ✘ Bad: Fefferman-Graham or similar ($r, t, \boldsymbol{x}$)

  ✓ Good: Incoming Eddington-Finklestein

$$ds^2 = -A(v,r)\,dv^2 + 2\,dv\,dr + \Sigma(v,r)^2\left[e^{B(v,r)}(dx^2+dy^2) + e^{-2B(v,r)}dz^2\right]$$

  - v = const. on incoming (radial) null geodesics
  - $dv/dr = \frac{1}{2}A$ on outgoing (radial) null geodesics
    - $g' \equiv \partial_r g$ = directional derivative along incoming null geodesics,
    - $\dot{g} \equiv \partial_v g + \frac{1}{2}A\,\partial_r g$ = directional derivative along outgoing null geodesics

- Boundary conditions as $r \to \infty$: $A \to r^2$, $\Sigma \to r$, $B \to f(v)$

Einstein equations

- $R_{MN} - \frac{1}{2} G_{MN}(R + 2\Lambda) = 0$
- Non-trivial components: *vv, rr, vr, zz, xx+yy*
  - ➡ *5 equations, 3 unknown functions* (A,B,Σ)
- Need to separate dynamics from constraints
  - ➡

$$
\begin{aligned}
0 &= \Sigma\,(\dot{\Sigma})' + 2\,\Sigma'\,\dot{\Sigma} - 2\,\Sigma^2 \\
0 &= \Sigma\,(\dot{B})' + \tfrac{3}{2}\big(\Sigma'\,\dot{B} - B'\,\dot{\Sigma}\big) \\
0 &= A'' + 3\,B'\,\dot{B} - 12\,\Sigma'\,\dot{\Sigma}/\Sigma^2 + 4 \\
0 &= \ddot{\Sigma} + \tfrac{1}{2}\,(\dot{B})^2\,\Sigma - \tfrac{1}{2}\,A'\,\dot{\Sigma} \quad \longleftarrow \text{boundary value constraint} \\
0 &= \Sigma'' + \tfrac{1}{2}\,(B')^2\,\Sigma \quad \longleftarrow \text{initial value constraint}
\end{aligned}
$$

  - N.B.: A = non-dynamical auxillary field

Practical issues (II)

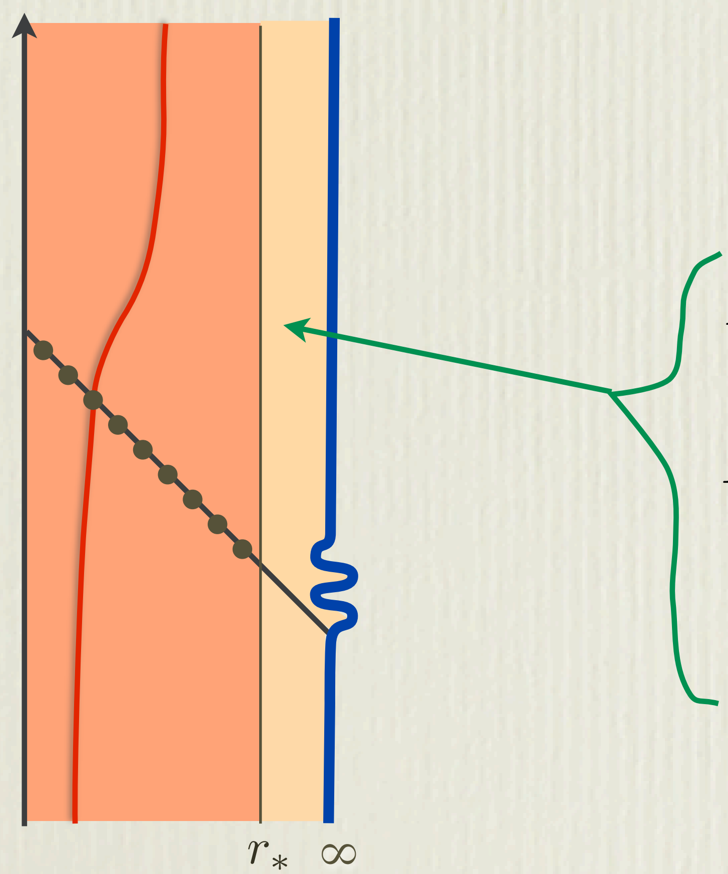
- Need to solve for "velocities," $\partial_v B$, $\partial_v \Sigma$, and auxillary field A

$$\dot{\Sigma}(r,v) = -\frac{2}{\Sigma(r,v)^2}\int_r dw\,\Sigma(w,v)^3$$

$$\dot{B}(r,v) = -\frac{3}{\Sigma(r,v)^{3/2}}\int_r dw\,\frac{B'(w,v)}{\Sigma(w,v)^{3/2}}\int_w d\bar{w}\,\Sigma(\bar{w},v)^3$$

- Discretize r → system of coupled ODEs

- Must treat near-boundary behavior accurately
  - ➡ match discretized numerics to large r asymptotics



$$A(r,v) = \sum_{n=0}[a_n(v) + \alpha_n(v)\log r]\,r^{2-n}\,,$$

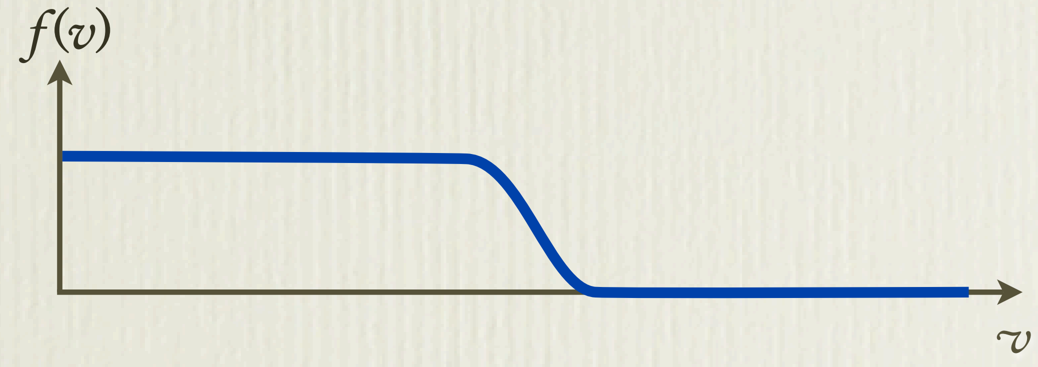
$$B(r,v) = \sum_{n=0}[b_n(v) + \beta_n(v)\log r]\,r^{-n}\,,$$

$$\Sigma(r,v) = \sum_{n=0}[s_n(v) + \sigma_n(v)\log r]\,r^{1-n}\,.$$

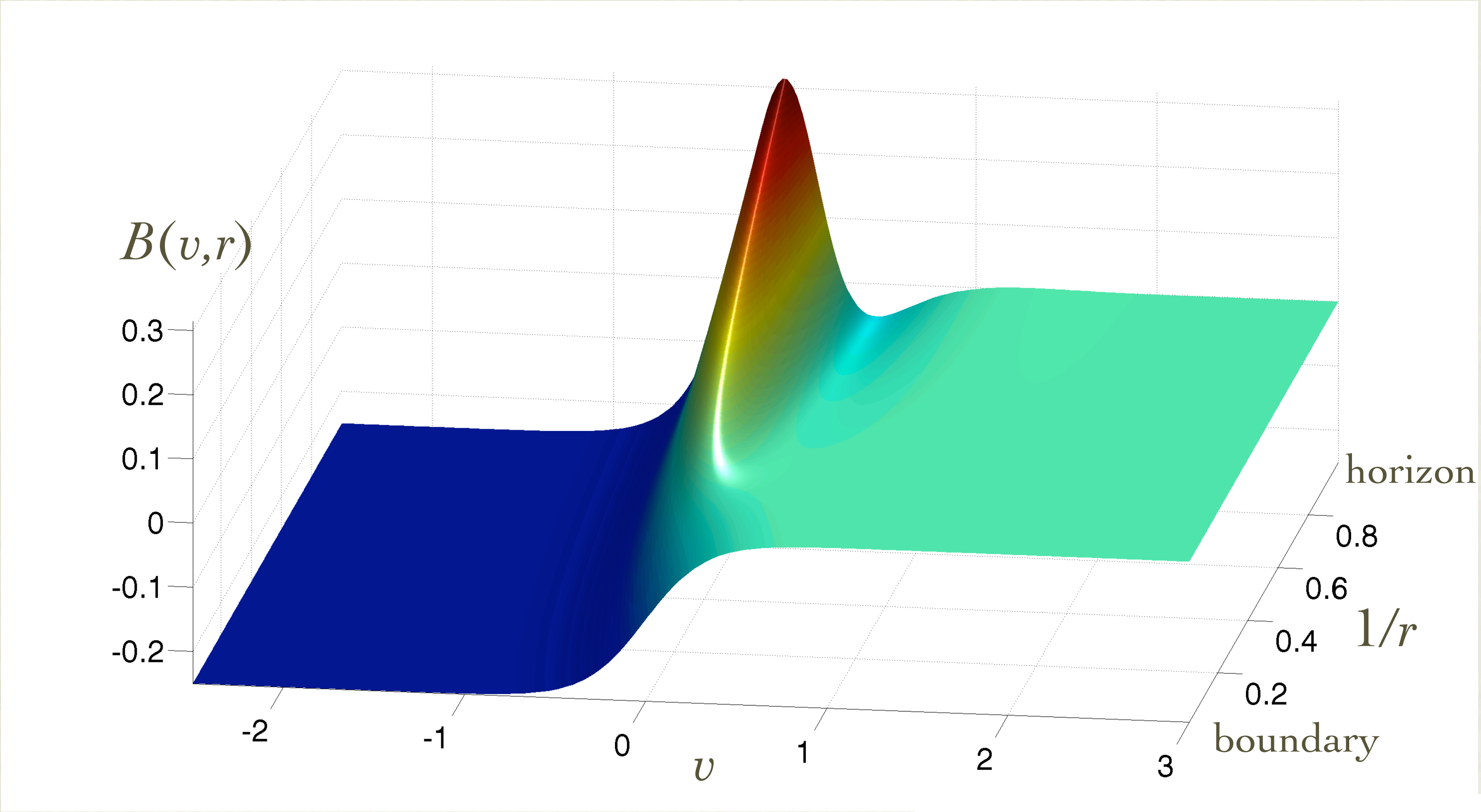
Practical issues (III)

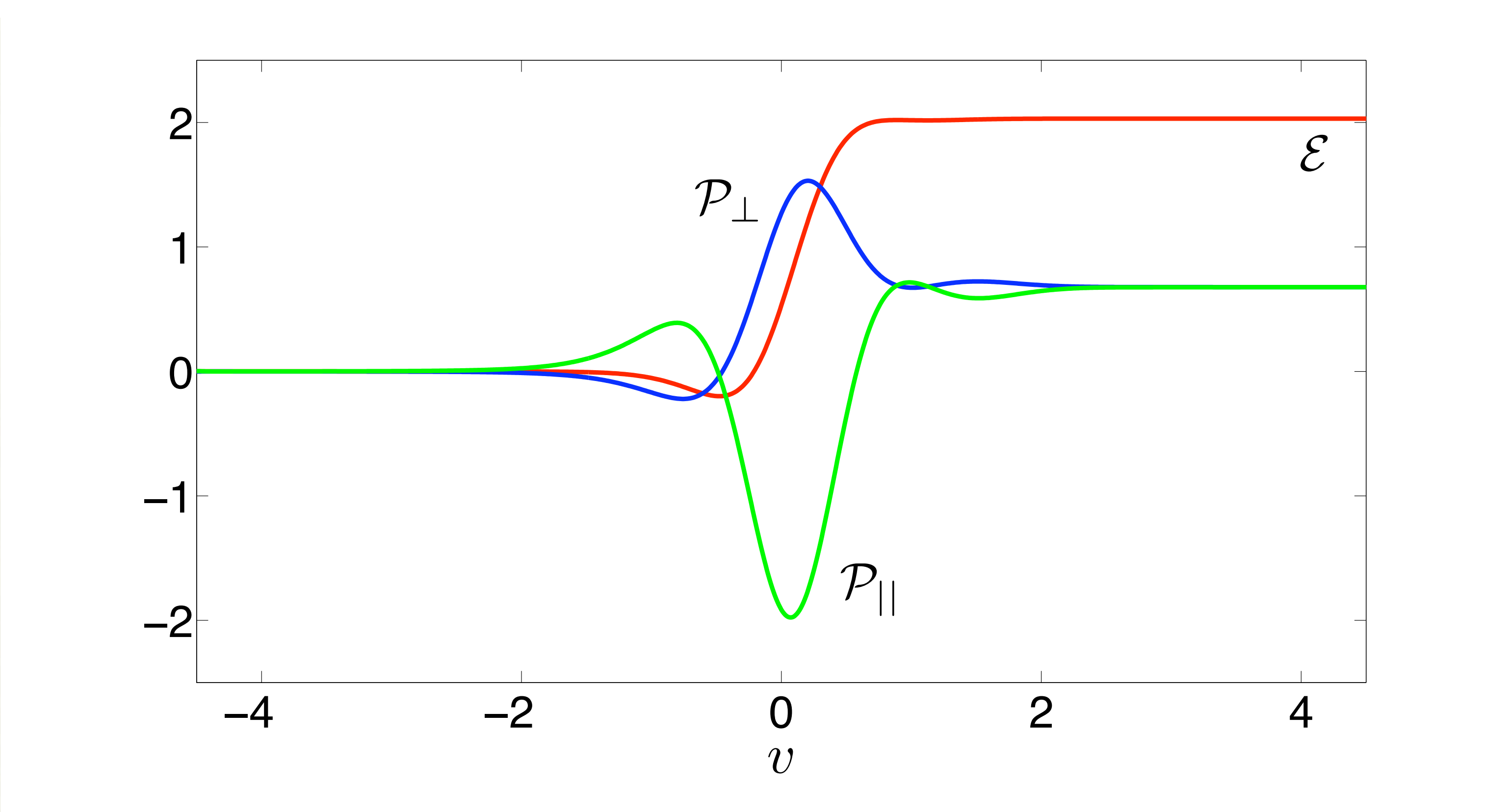
- Must remove residual reparameterize freedom: $r \to r + \alpha(v)$
  - ✘ Bad: fix coordinate location of event horizon
  - ✓ Good: fix $a_1 = 0$
- Must excise region surrounding singularity: $r < r_{\min}(v) < r_{\text{horizon}}(v)$
- Must choose specific boundary time dependence

  Ex: $f(v) = \frac{1}{2}\, c\, [\, 1 - \tanh\,(t/\tau)\,]$

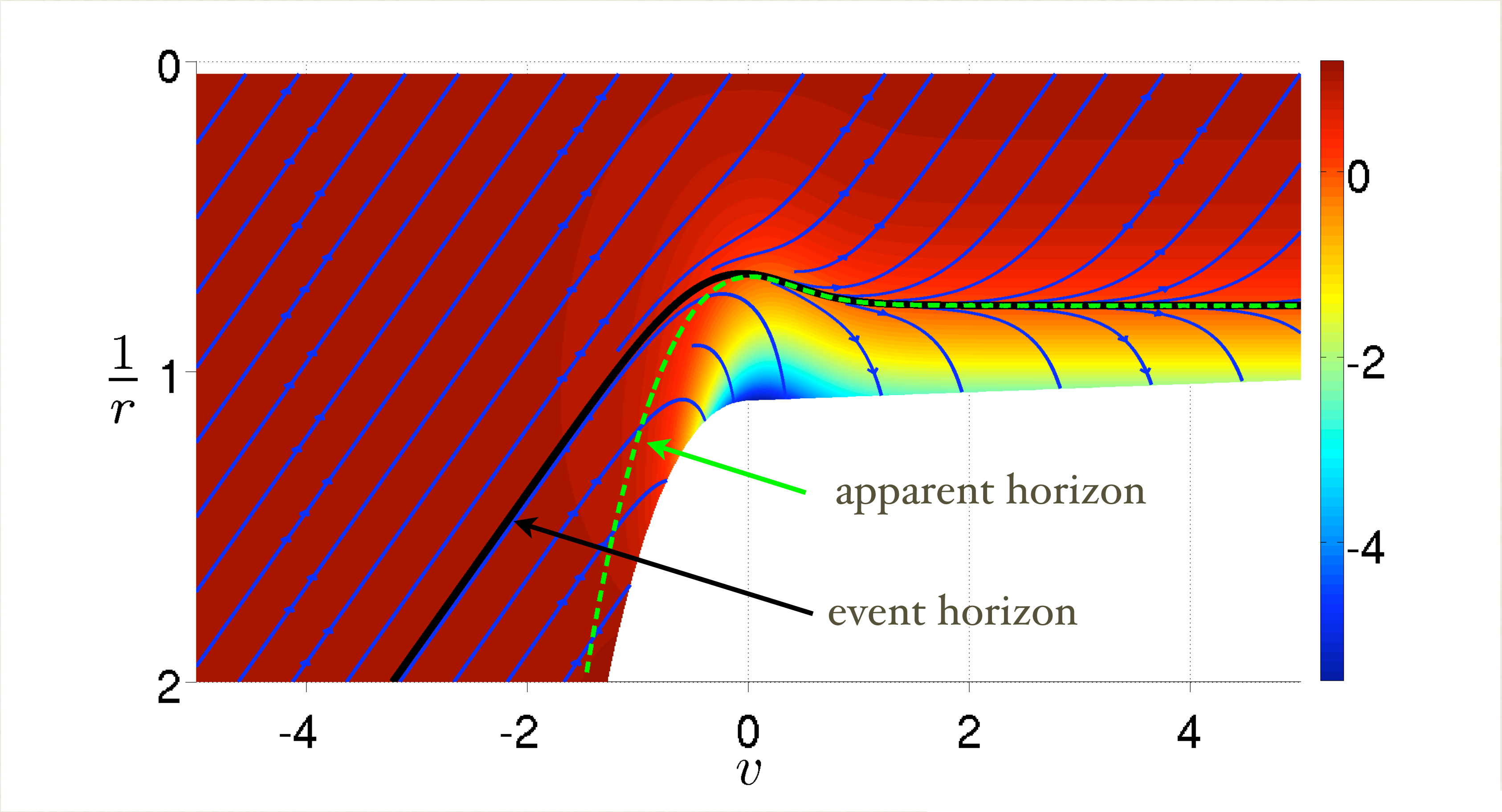


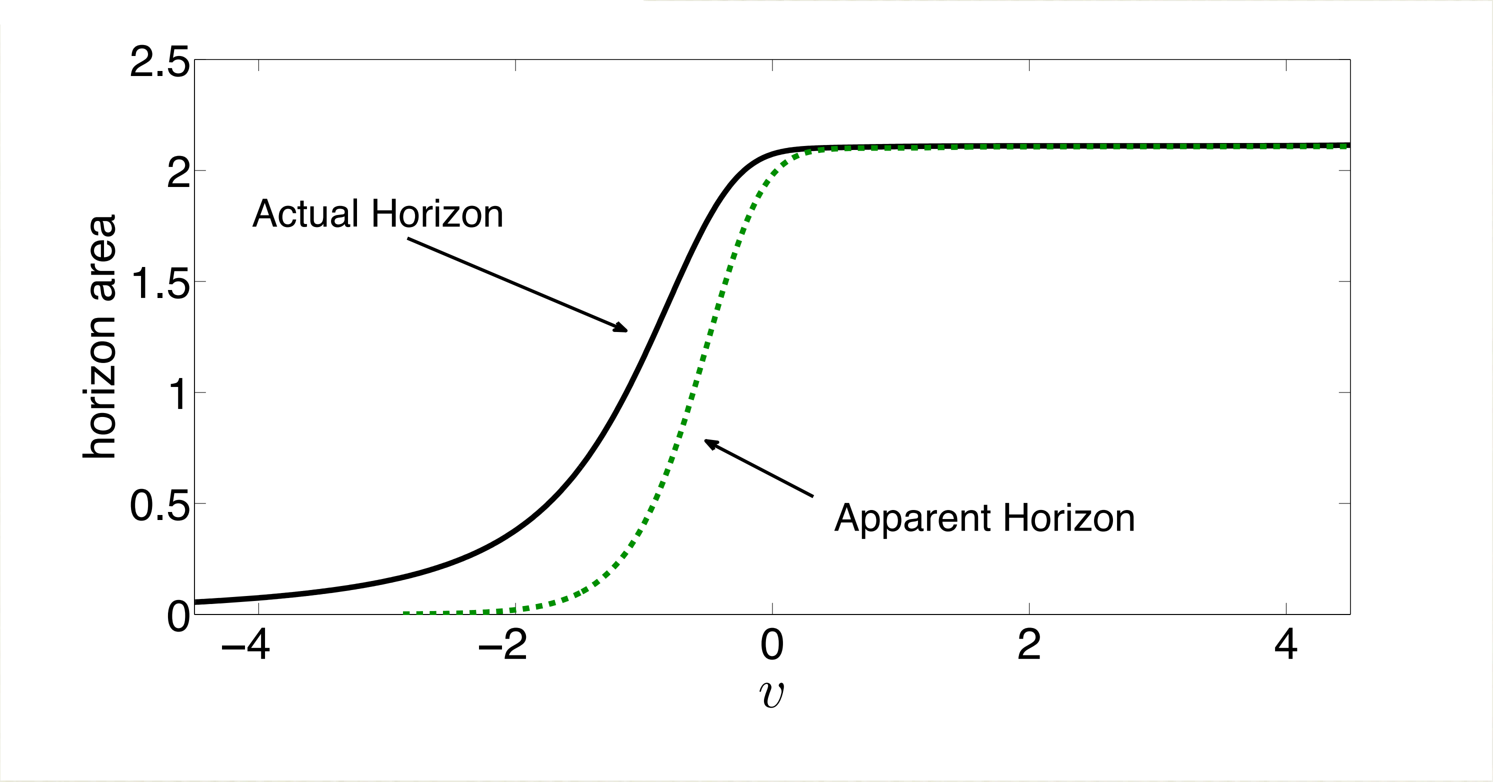
Results (I)


B(v,r)
0.3
0.2
0.1
0
-0.1
-0.2
-2
-1
0
1
2
3
v
horizon
0.8
0.6
0.4
0.2
1/r
boundary


2
1
0
−1
−2
−4
−2
0
2
4
v
$\mathcal{E}$
$\mathcal{P}_\perp$
$\mathcal{P}_{||}$

Results (II)


apparent horizon
event horizon
0
1
2
$\frac{1}{r}$
-4
-2
0
2
4
v
0
-2
-4


Actual Horizon
Apparent Horizon
horizon area
2.5
2
1.5
1
0.5
0
−4
−2
0
2
4
v

Results (III)

$\lvert c\rvert$	1	1.5	2	2.5	3	3.5	4
$\tau\, T$	0.23	0.31	0.41	0.52	0.65	0.79	0.94
$\tau_{\rm iso}\, T$	0.67	0.68	0.71	0.92	1.2	1.5	1.8
$\tau_{\rm iso}/\tau$	3.0	2.2	1.7	1.8	1.8	1.9	1.9

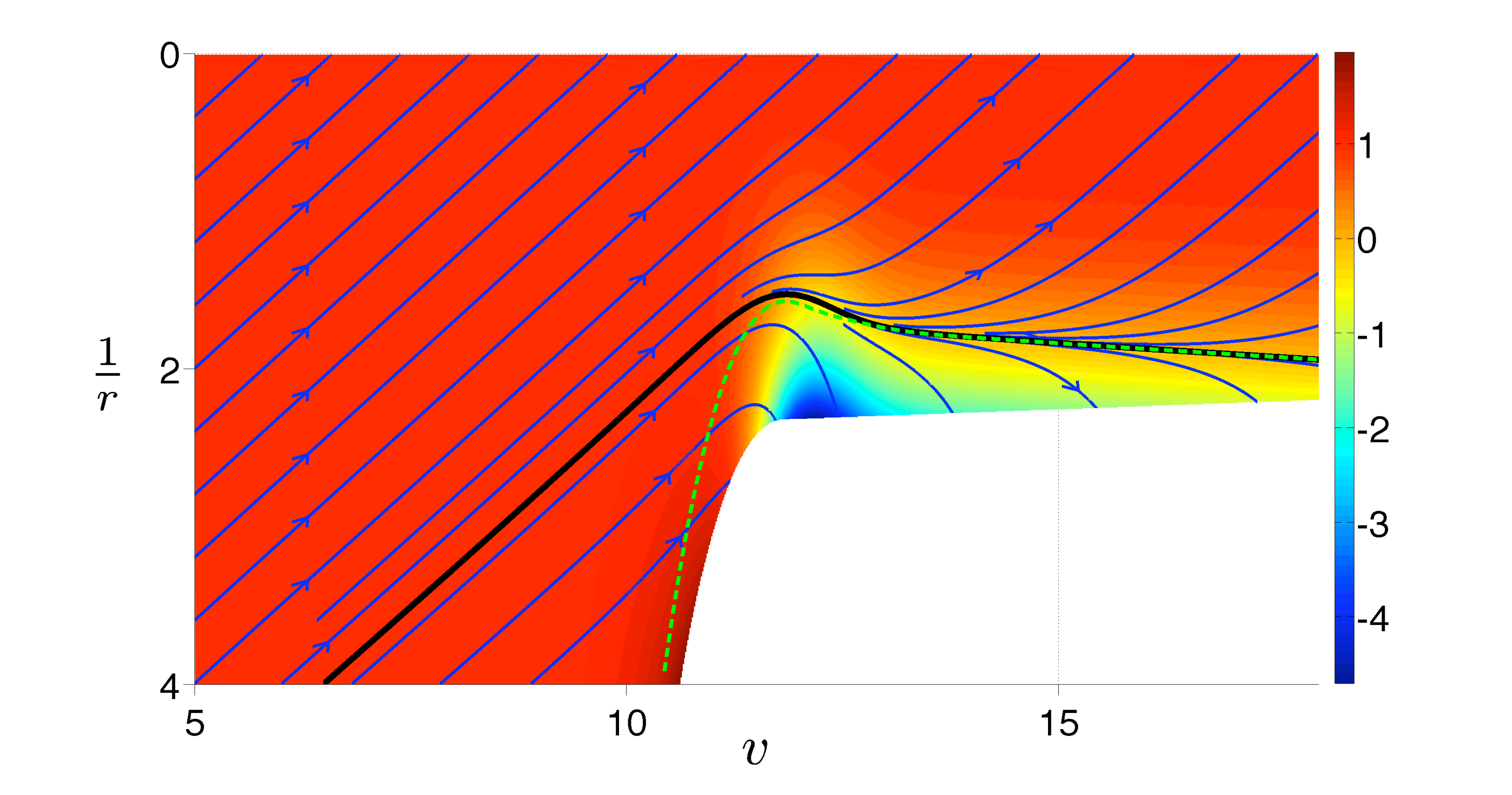
T = final equilibrium temperature

$\tau_{\rm iso}$ = isotropization time

τ = plasma creation time scale

$\tau_{\rm iso} \approx 0.7/T$ ➡ $\tau_{\rm iso} \approx 0.5$ fm/c at $T \approx 350$ MeV --- relevant at RHIC???

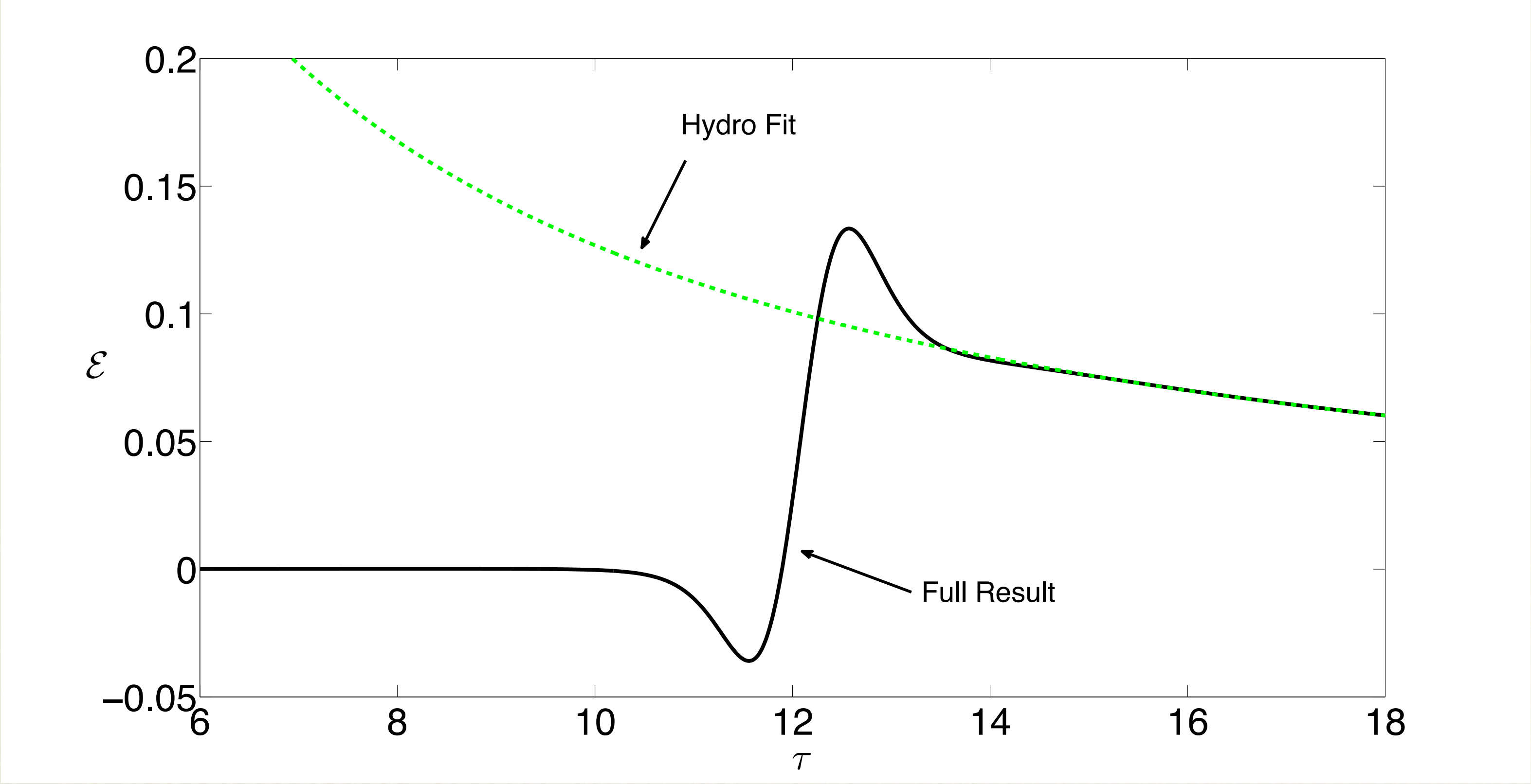
Boost invariant expansion



boundary geometry:

$$ds^2 = -d\tau^2 + \tau^2 e^{-2f(\tau)}\, d\eta^2 + e^{f(\tau)}\, d\mathbf{x}_\perp^2$$

$$f(\tau) = \tfrac{1}{2} c\,[1 - \tanh(\tau - \tau_0)]$$



Open questions

- Sensitivity to choice of boundary time dependence?
  - wider range of amplitudes
  - periodic forcing
- Precise connection between entropy & apparent horizon area?
  - ambiguities in definition of non-equilibrium entropy
  - foliation dependence of apparent horizon area
- Feasibility of evolving anisotropic & inhomogeneous geometries?
  - finite expanding fluids
  - turbulent driven systems
- Relevance for heavy ion collisions?