Relaxing a large cosmological constant

Florian Bauer

Departament d'Estructura i Constituents de la Matèria (ECM),
Institut de Ciències del Cosmos (ICC),
Universitat de Barcelona (UB)

Theoretical miniWorkshop on Dark Matters
IFT-UAM/CSIC, Madrid, 16.-18.09.2009

References

- FB, Joan Solà, Hrvoje Štefančić, PLB 678 (2009) 427, arXiv:0902.2215
- FB, arXiv:0909.2237

Introduction

- Accelerating cosmos
- Dark energy
- Cosmological constant (CC), scalar fields, modified gravity $f(R)$, ...
- Always: CC
  - Quantum field zero-point energy
  - Phase transitions (electro-weak, GUT, ...)
  - Non-zero minimum of scalar potential (inflaton)

CC problems

- Large CC expected: $\rho_\Lambda^i \sim \pm \left(10^2 \dots 10^{19}\,\mathrm{GeV}\right)^4$
- Problem: No Big Bang universe.
  - de Sitter with large Hubble rate $H \gg H_0$
  - Big Crunch at early times
- Standard way to get rid of ρ_Λ^i: Add a counter-term

$$\rho_\Lambda^i + \rho_\Lambda^{ct} \leq \rho_\Lambda^0 \sim \left(10^{-12}\,\mathrm{GeV}\right)^4$$

- Extreme fine-tuning needed: $\rho_\Lambda^i = -\rho_\Lambda^{ct}$ almost perfectly $\rightsquigarrow$ CC Relaxation
- Why $\rho_m \sim \rho_\Lambda$ now? Coincidence problem $\rightsquigarrow$ Tracking

Outline

1. CC Relaxation mechanism
2. Tracking
3. Perturbations
4. Evolution of perturbations
5. Conclusions

CC Relaxation mechanism

- CC Relaxation: Add ρ_{DE} component without fine-tuning

$$\left|\rho_\Lambda^i + \rho_{DE}\right| \ll \left|\rho_\Lambda^i\right|$$

- Model in the ΛXCDM framework
  - Variable CC, $\dot{\rho_\Lambda} \neq 0$
  - Equation of state EOS$= -1$
  - Interaction with another component: X
  - Here: X is identified with CDM

Relaxation model

- Variable CC: Large constant initial CC + dynamical part

$$\rho_\Lambda = \rho_\Lambda^i + \frac{\beta}{f}, \quad \beta = \text{const.}$$

- Example: $f = H^2$ for today (close to de Sitter cosmos with Hubble rate $H_* \approx H_0 \sim 10^{-42}$ GeV)
- Einstein equation:

$$\rho_c = \frac{3H^2}{8\pi G} = \left(\rho_X + \rho_\Lambda^i + \frac{\beta}{f}\right)$$

- Fix parameter $\beta = -\rho_\Lambda^i H_*^2$

Relaxation model

- Solution

$$H^2 = -\frac{\beta}{\rho_\Lambda^i}\left(1 + \frac{\rho_X - \rho_c}{\rho_\Lambda^i}\right) \approx -\frac{\beta}{\rho_\Lambda^i} = H_*^2$$

- What happened?
  - $$\rho_c = \frac{3H^2}{8\pi G} = \left(\rho_X + \rho_\Lambda^i + \frac{\beta}{H^2}\right)$$
  - $f = H^2$ becomes small at late times
  - β/H^2 gets large and compensates ρ_Λ^i
- Only the magnitude of β is important $\rightsquigarrow$ No fine-tuning

Counter-term method

- Without fine-tuning in ρ_Λ^{ct}

$$\rho_c = \frac{3H^2}{8\pi G} = \rho_X + \rho_\Lambda^i \left(1 + \frac{\rho_\Lambda^{ct}}{\rho_\Lambda^i}\right) = \mathcal{O}\left(\rho_\Lambda^i\right)$$

- No Big Bang universe!

Stability

- Einstein equation in the case $\rho_\Lambda^i < 0, \ (\beta > 0)$

$$\rho_c = \frac{3H^2}{8\pi G} = \left(\rho_{\text{other}} + \rho_\Lambda^i + \frac{\beta}{H^2}\right)$$

- $\rho_\Lambda^i < 0$ and $\beta/H^2 > 0$ act in different directions until a stable point is achieved:

$\left|\rho_\Lambda^i\right| \gg \frac{\beta}{H^2}$

$\rho_\Lambda^i < 0 \rightsquigarrow$ Hubble rate $H \searrow 0$ until $\frac{\beta}{H^2} \approx \left|\rho_\Lambda^i\right|$

$\left|\rho_\Lambda^i\right| \ll \frac{\beta}{H^2}$

$\frac{\beta}{H^2} > 0 \rightsquigarrow$ Hubble rate $H \nearrow$ until $\frac{\beta}{H^2} \approx \left|\rho_\Lambda^i\right|$

Dynamical relaxation without fine-tuning.

Matter and radiation epochs

Late-time de Sitter cosmos

$f = H^2$ works well

Matter era

Scale factor $a \propto t^{\frac{2}{3}}$, deceleration $q = -\frac{\ddot{a}}{a}/H^2 = \frac{1}{2} \rightsquigarrow f \sim (q - \frac{1}{2})$

Radiation era

Scale factor $a \propto t^{\frac{1}{2}}$, $q = 1 \rightsquigarrow f \sim (q - 1)$

Stabilisation similar to $f = H^2$

$(q - \frac{1}{2})$ (or $(q - 1)$ for radiation) will change slightly to compensate ρ_Λ^i with $q \approx \frac{1}{2}$

Full model

- Complete ΛCDM-like cosmos

$$f = 4\frac{(2-q)}{(1-q)} \cdot H^2\left(\frac{1}{2}-q\right) + y \cdot 72(1+q^2) \cdot H^6(1-q).$$

- Term $H^2(\frac{1}{2}-q)$: matter era for small $(\frac{1}{2}-q)$, and final de Sitter era for small H^2
- Term $y \cdot H^6(1-q)$ for the radiation era for large H. Parameter $y \sim H_{eq}^{-4}$ with the Hubble scale $H_{eq} \sim 10^5\, H_0$ at the radiation-matter transition.

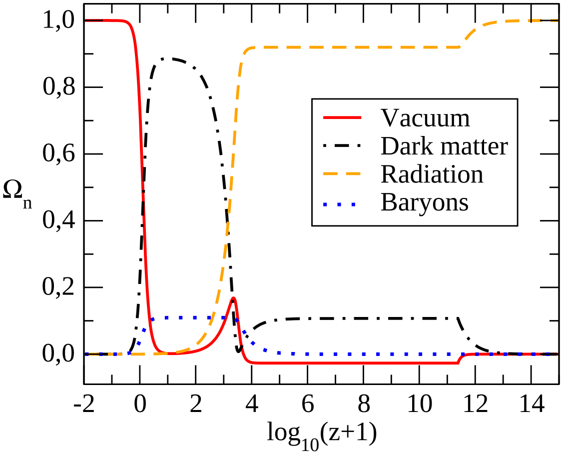
Covariant form

- Function f in covariant form

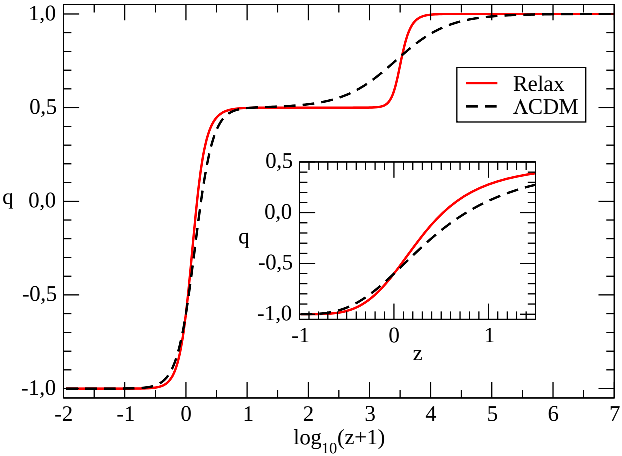
$$\rho_\Lambda = \rho_\Lambda^i + \beta \frac{R}{B} \quad \text{with} \quad B := R^2 - S + y \cdot R^2 T .$$

  - Ricci scalar $R = g^{ab} R_{ab} = 6H^2(1-q)$
  - Ricci tensor squared $S = R_{ab} R^{ab} = 12H^4(q^2 - q + 1)$
  - Riemann tensor squared $T = R_{abcd} R^{abcd} = 12H^4(q^2 + 1)$
  - Here: S and T can be replaced by R^2 and the Gauß-Bonnet term G: $S_* = \frac{1}{3}R^2 - \frac{1}{2}G$, $T_* = \frac{1}{3}R^2 - G$.

$\rho_\Lambda^i = -10^{40}\,\rho_c^0$



$\rho_\Lambda^i = -10^{40}\,\rho_c^0$



Parameter β

- Mass scale M

$$|\beta| = M^6$$

- β is fixed in the final de Sitter era with Hubble rate H_*, $q = -1$

$$\beta = -\rho_\Lambda^i 9 H_*^2$$

- Example: $\rho_\Lambda^i = \mathcal{O}(M_{Pl}^4)$, $H_* \approx H_0 \sim 10^{-42}\,\text{GeV}$

$$M = |\beta| = \mathcal{O}(100\,\text{MeV})$$

Friedmann equations

- Friedmann equations for H and q:

$$\begin{aligned} H^2 &= \frac{H_0^2}{\rho_c^0}(\rho_X + \rho_\Lambda + \rho_r) \\ qH^2 &= \frac{H_0^2}{\rho_c^0}\left(\frac{1}{2}\rho_X - \rho_\Lambda + \rho_r\right) \end{aligned}$$

- Bianchi identity for X/dark matter (dust) and Λ:

$$\dot{\rho}_\Lambda + \dot{\rho}_X + 3H\rho_X = 0$$

- Radiation

$$\rho_r = \rho_{r0}\, a^{-4}$$

Relaxation regime

- CC Relaxation means $|\rho_\Lambda| = |\rho_\Lambda^i + \frac{\beta}{f}| \ll |\rho_\Lambda^i|$
- Minimum of $f = B/R$ in the radiation era ($q \approx 1$):

$$B = -12H^4 + 2y \cdot 24H^4R^2 = 12H^4(2yR^2 - 1)$$

  B minimal for $R \longrightarrow 1/\sqrt{2y} \sim H_{eq}^2$
- R from Friedmann equations

$$R = \frac{6H_0^2}{\rho_c^0}\left(\frac{1}{2}\rho_X + 2\rho_\Lambda\right).$$

- R, B in ρ_Λ :

$$\rho_\Lambda = \rho_\Lambda^i + \beta\frac{R}{B} = \rho_\Lambda^i + \frac{\gamma}{\epsilon \cdot (\frac{1}{2}\rho_X + 2\rho_\Lambda) - 1}$$

  Solve for ρ_Λ!

Tracking solutions

Two solutions for ρ_Λ

$$\rho_\pm \simeq \frac{1}{8}\left[4\rho^i_\Lambda - \rho_X \pm \left|4\rho^i_\Lambda + \rho_X\right|\right]$$

$\rho^i_\Lambda < 0$

- Early times: $\rho_X \gg |\rho^i_\Lambda|$

$$\rho_\Lambda = \rho_+ \simeq \rho^i_\Lambda = \text{const.}$$

- Late times: $\rho_X \ll |\rho^i_\Lambda|$

$$\rho_\Lambda = \rho_+ \simeq -\frac{1}{4}\rho_X$$

Tracking!

$\rho^i_\Lambda > 0$

- Always tracking:

$$\rho_\Lambda = \rho_- \simeq -\frac{1}{4}\rho_X$$

Conservation equation

ΛXCDM framework

- Dark/X matter and the vacuum are interacting
- Bianchi identity $\rightsquigarrow$ Conservation equation

$$\dot{\rho}_\Lambda + \dot{\rho}_X + 3H\rho_X = 0$$

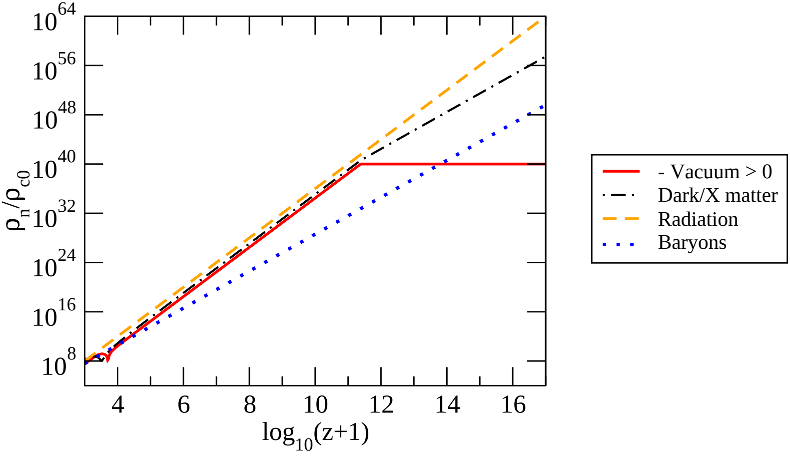
- Tracking regime in the radiation era:

$$\rho_\Lambda \simeq -\frac{1}{4}\rho_X$$

X and the vacuum scale like radiation

$$\rho_\Lambda, \rho_X \propto \rho_r = \rho_{r0}\, a^{-4}$$

Example: $\rho_\Lambda^i = -10^{40}\,\rho_c^0$



Relaxation regime

Matter era: $q \approx \frac{1}{2}$

- Friedmann equations

$$H^2\left(\frac{1}{2}-q\right)=\frac{H_0^2}{\rho_c^0}\left(\frac{3}{2}\rho_\Lambda-\frac{1}{2}\rho_r\right),$$

- Vacuum energy

$$\begin{aligned}\rho_\Lambda &= \rho_\Lambda^i+\frac{\beta}{f}\approx\rho_\Lambda^i+\frac{\beta}{12H^2(\frac{1}{2}-q)+45H^6y}\\ &= \rho_\Lambda^i+\frac{1}{d(\frac{3}{2}\rho_\Lambda-\frac{1}{2}\rho_r)+c}\end{aligned}$$

$d := 12H_0^2/(\rho_c^0\beta),\ \ (d\rho_\Lambda^i)<0$

Tracking solution

Two solutions for ρ_Λ

$$\rho_\pm \simeq \frac{1}{6d}\left[3d\rho_\Lambda^i \pm |3d\rho_\Lambda^i| \pm \frac{6d(-d\rho_\Lambda^i)}{|3d\rho_\Lambda^i|}\cdot\rho_r\right]$$

Only ρ_+ relaxes the CC

$$\rho_\Lambda \simeq \frac{1}{3}\rho_r$$

Tracking in the matter era!

Conservation equation

$$\dot\rho_\Lambda + \dot\rho_X + 3H\rho_X = 0 \implies \rho_X = \rho_{X0}\, a^{-3} - \frac{4}{3}\rho_r$$

Deviation from ΛCDM are of order $\rho_r \propto a^{-4}$.

Newtonian gauge

- Perturbed metric $g_{ab} = \overline{g_{ab}} + \delta g_{ab}$ in linear order

$$g_{00} = 1 + 2\Psi(t, \vec{x}), \quad g_{ij} = -\delta_{ij} a^2(t)(1 + 2\Phi(t, \vec{x})), \quad g_{0i} = 0$$

Gravitational scalar potentials: Φ and Ψ

- Perturbed versions of

$$\Gamma^a_{bc}, R, R_{ab}, G_{ab}, R_{abcd}, S, T, \ldots$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} - 2a^{-2}\nabla^2\Psi - 4a^{-2}\nabla^2\Phi - 12H^2(1-q)\Psi$$

Varying CC term

Energy-momentum tensor of the CC

$$T^\mu_\nu = \rho_\Lambda \, \delta^\mu_\nu \quad \text{with} \quad \rho_\Lambda = \rho^i_\Lambda + \beta/f$$

$$\delta T^\mu_\nu = \delta^\mu_\nu \, \delta\rho_\Lambda$$

Vacuum energy perturbation

$$\delta\rho_\Lambda = \beta \, \delta \left(\frac{R}{B}\right) = \frac{\beta R}{B} \cdot \frac{N}{BR}$$

$$N := -(R^2 + S)\delta R + R\delta S - y(R^2 T\delta R + R^3 \delta T)$$

Varying CC term

Relaxation regime

$$|\rho_\Lambda| = |\rho_\Lambda^i + \beta R/B| \ll |\rho_\Lambda^i|$$

$$\delta\rho_\Lambda = \frac{\beta R}{B} \cdot \frac{N}{BR} \overset{\text{Relax}}{=} -\mathcal{O}(\rho_\Lambda^i)\frac{N}{BR}$$

Final de Sitter era with Hubble rate H_*

Parameter $\beta = -\rho_\Lambda^i 9 H_*^2$

$$BR \overset{\text{Relax}}{=} 9H_*^2 R^2$$

Interacting X-Λ components

- Covariantly conserved energy-momentum tensor

$$T^a_{\ b} = \rho_X u^a u_b + \rho_\Lambda g^a_{\ b}.$$

- Perturbed version

$$\rho_i \to \rho_i(t) + \delta\rho_i(t,\vec{x}), \quad i = X, \Lambda; \quad u^a \to u^a(t) + \delta u^a(t,\vec{x}),$$

- 4-velocity u^a, 3-velocity perturbation v^j

$$u^a = \delta^a_0, \quad \delta u^0 = -\Psi = -\delta u_0, \quad \delta u^j = \frac{v^j}{a}, \quad \delta u_j = -a v^j$$

Conservation equations

- Wave number $k = |\vec{k}|$ in Fourier space

$$\Psi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \exp(-i\vec{k}\vec{x})\Psi(t,\vec{k})$$

- Bianchi identity $T^a_{\ b;a} = 0$

$$\begin{aligned} \delta\dot{\rho}_\Lambda + \delta\dot{\rho}_X + 3\dot{\Phi}\rho_X + 3H\delta\rho_X - i\rho_X v\frac{k}{a} &= 0 \\ \delta\rho_\Lambda\left(\frac{k}{a}\right)^2 - \rho_X\Psi\left(\frac{k}{a}\right)^2 - i\frac{d}{dt}\left(\rho_X v\frac{k}{a}\right) - i5H\left(\rho_X v\frac{k}{a}\right) &= 0 \end{aligned}$$

Radiation

- No changes in the baryon, photon and neutrino sector.
- Radiation perturbations: Monopole Θ_0, dipole Θ_1, ...

$$\delta T^0_0 = 4\rho_r \Theta_0, \quad \delta T^0_j = 4i\rho_r \Theta_1 k^j \frac{a}{k}.$$

- The evolution equations for $\Theta_{0,1}$ read

$$\begin{aligned} \dot{\Theta}_0 &= -\dot{\Phi} + \frac{k}{a}\Theta_1, \\ \dot{\Theta}_1 &= -\frac{1}{3}\frac{k}{a}(\Psi + \Theta_0) + \frac{2}{3}\frac{k}{a}\Theta_2. \end{aligned} \tag{1}$$

Einstein equations

- Perturbed $\delta G^a_{\ b} = 8\pi G\, \delta T^a_{\ b}$

$$\begin{aligned}
6H\dot{\Phi} - 6H^2\Psi + 2\left(\frac{k}{a}\right)^2 \Phi &= 8\pi G(\delta\rho_\Lambda + \delta\rho_X + \delta\rho_r), \\
\rho_X v \frac{k}{a} &= 4i\rho_r\Theta_1 \frac{k}{a} - \frac{2i}{8\pi G}\left(\frac{k}{a}\right)^2 (\dot{\Phi} - H\Psi) \\
\frac{1}{8\pi G}\frac{2}{3}\left(\frac{k}{a}\right)^2 (\Psi + \Phi) &= -\frac{8}{3}\rho_r\Theta_2 = \text{anisotropies}
\end{aligned}$$

- Neglect anisotropies, $\Theta_2 \rightsquigarrow \Psi = -\Phi$

How large is $\delta\rho_\Lambda$?

- Einstein equations + conservation equations

$$\begin{aligned}\delta\rho_\Lambda &= \rho_X\Psi + \frac{4}{3}\rho_r(\Theta_0 + \Psi - 2\Theta_2) \\ &+ \frac{2}{8\pi G}[\ddot{\Phi} + 3H\dot{\Phi} - H\dot{\Psi} - \dot{H}\Psi - 3H^2\Psi]\end{aligned}$$

1. $\Psi, \Theta_0 \leq \mathcal{O}(\Phi)$ and $\rho_r, \rho_X, \rho_\Lambda \leq \mathcal{O}(\rho_c)$

$$\delta\rho_\Lambda = \mathcal{O}(\rho_c\Phi)$$

2. Relaxation regime

$$\delta\rho_\Lambda = \frac{\beta R}{B}\cdot\frac{N}{BR} \overset{\text{Relax}}{=} -\mathcal{O}(\rho_\Lambda^i)\frac{N}{BR}$$

How large is $\delta\rho_\Lambda$?

$$\delta\rho_\Lambda = \frac{\beta R}{B} \cdot \frac{N}{BR} \overset{\text{Relax}}{=} -\mathcal{O}(\rho_\Lambda^i)\frac{N}{BR} = \mathcal{O}(\rho_c \Phi)$$

$$\frac{N}{BR} = \mathcal{O}\left(\frac{\rho_c \Phi}{\rho_\Lambda^i}\right) \lll 1$$

in addition $BR \overset{\text{Relax}}{=} 9H_*^2 R^2$

Evolution equation for Φ

$$0 = N = -(R^2 + S)\delta R + R\delta S - y(R^2 T\delta R + R^3 \delta T)$$

Gravitational Potential

Matter era

$q = \frac{1}{2}$, $H = 2/(3t) \ll H_{eq}$, $R = 3H^2$, $S = 9H^4$, $T = 15H^4$,
$y = H_{eq}^{-4}$ part neglected.

$$\begin{aligned} N &= -2R^2\delta R + R\delta S \\ &= -18H^4[6\ddot{\Phi} + 18H\dot{\Phi} - 6H\dot{\Psi} - 2a^{-2}\nabla^2(\Psi + \Phi)] \\ &= -6 \cdot 18H^4[\ddot{\Phi} + 4H\dot{\Phi}] \quad (\text{with } \Psi = -\Phi) \end{aligned}$$

$0 = N \propto \ddot{\Phi} + 4H\dot{\Phi}$ has two solutions

$$\Phi = \text{const.}, \quad \Phi \sim t^{-\frac{5}{3}} \text{ will die out.}$$

Vacuum perturbations $\delta\rho_\Lambda$

- Einstein, conservation and tracking ($\rho_\Lambda = \frac{1}{3}\rho_r$) relations

$$\begin{aligned}\delta\rho_\Lambda &= \rho_X\Psi + \frac{4}{3}\rho_r(\Theta_0 + \Psi - 2\Theta_2) + \\ &+ \frac{2}{8\pi G}[\ddot{\Phi} + 3H\dot{\Phi} - H\dot{\Psi} - \dot{H}\Psi - 3H^2\Psi] \\ &= 4\rho_\Lambda\Theta_0\end{aligned}$$

Radiation equations $\langle\Theta_0\rangle = \Phi \rightsquigarrow$ Vacuum energy contrast

$$\frac{\delta\rho_\Lambda}{\rho_\Lambda} = 4\Phi = \text{const.}$$

X/dark matter perturbations $\delta\rho_X$

$$\delta\rho_X = -\rho_X\Psi - \frac{4}{3}\rho_r(4\Theta_0 + \Psi) - \frac{2}{8\pi G}\left[\ddot{\Phi} - H\dot{\Psi} - \dot{H}\Psi - (k/a)^2\,\Phi\right]$$

$$\approx \rho_X\Phi\left[2 + \frac{2k^2}{3H_0^2\Omega_{X0}}\cdot a\right]$$

Sub-horizon modes, $k/(aH) \gg 1$: ΛCDM result

$$\frac{\delta\rho_X}{\rho_X} = \frac{2k^2\Phi}{3H_0^2\Omega_{X0}}\cdot a$$

Dark/X matter density contrast grows with scale factor a

Gravitational Potential

Final de Sitter era

$q = -1$, $H = H_* =$ const., $R = 12H^2$, $S = 36H^4$, $T = 24H^4$,
$y = H_{eq}^{-4}$ terms neglected

$$N = -18 \cdot 36H^4 \left[\ddot{\Phi} + 5H\dot{\Phi} + \frac{1}{3}\left(\frac{k}{a}\right)^2 \Phi + 4H^2\Phi\right]$$

$N = 0$ has two decaying solutions

$$\Phi_1 = x^2(\sin x + x^{-1}\cos x)$$

$$\Phi_2 = x^2(x^{-1}\sin x - \cos x), \quad x := \frac{k}{\sqrt{3}aH} \propto \frac{1}{a}$$

Vacuum and X/DM perturbations $\delta\rho_{\Lambda,X}$

For $\Phi = \Phi_1$ (Φ_2 similar) we find

$$\begin{aligned}\delta\rho_\Lambda &= -\rho_X\Phi_1 + \frac{3H^2}{8\pi G}\left(\frac{2}{3}x^3\cos x - \frac{2}{3}x^2\Phi_1\right),\\ \delta\rho_X &= \rho_X\Phi_1 + \frac{3H^2}{8\pi G}\left(-\frac{8}{3}x^3\cos x + \frac{8}{3}x^2\Phi_1\right).\end{aligned}$$

Λ density contrast is decreasing

$$\frac{\delta\rho_\Lambda}{\rho_\Lambda} = -\frac{2}{3}(x^4\sin x).$$

No perturbative instabilities expected in the future

Gravitational Potential

Radiation era

$q = 1$, $S = 12H^4$ and $T = 24H^4$

Relaxation regime

$$R \approx \frac{1}{\sqrt{2y}} \sim H_{eq}^2 \ll H^2, \quad BR = 9H_*^2 R^2 \approx \frac{9H_*^2}{2y}.$$

Numerator N

$$\begin{aligned} N &= -\left(R^2 + S + \frac{1}{2}T\right)\delta R + R\delta S - \frac{1}{2}R\delta T \\ &\approx -24H^4\,\delta R \\ \delta R &= 6\left(\ddot{\Phi} + 5H\dot{\Phi} + \frac{1}{3}\left(\frac{k}{a}\right)^2\Phi\right) \end{aligned}$$

Gravitational Potential

$N = 0$ has two solutions

$$\begin{aligned}\Phi_1 &= 3\Phi_0 x^{-2}(\sin x + x^{-1}\cos x) = \Phi_0\left(3x^{-3} + \mathcal{O}(x^{-1})\right)\\ \Phi_2 &= 3\Phi_0 x^{-2}(x^{-1}\sin x - \cos x) = \Phi_0\left(1 + \mathcal{O}(x^{2})\right)\end{aligned}$$

$$x := \sqrt{\frac{k^2}{3a^2H^2}} \propto a \propto \sqrt{t}$$

The same solutions exist in ΛCDM, where radiation determines Φ.

Only Φ_2 has a regular early-time behaviour $\Phi(t \to 0) = \Phi_0$

$$\Phi_1 \sim t^{-\frac{3}{2}}, \quad \Phi_2 \to \Phi_0 = \text{const.}$$

Radiation, Vacuum and X/DM perturbations $\delta\rho_{r,\Lambda,X}$

Radiation multipoles

$$\begin{aligned}\Theta_0(x) &= c_1\cos x + c_2 \sin x - \Phi(x) + 3\Phi_0 x^{-1}\sin x\\ \Theta_1(x) &= \left(-c_1 \sin x + c_2\cos x - x\Phi(x)\right)/\sqrt{3}\end{aligned}$$

Tracking relation for $\delta\rho_{X,\Lambda}$

$$\delta\rho_X = 4\Theta_0\rho_X - \frac{2}{3}\rho_c\left((8c_1 + 12\Phi_0) + 8c_2\sin x\right) = -4\delta\rho_\Lambda$$

Relaxation regime + ΛCDM initial conditions $(c_1 = -\frac{3}{2}\Phi_0,\ c_2 = 0)$

$$\frac{\delta\rho_X}{\rho_X} = 4\Theta_0 = \frac{\delta\rho_\Lambda}{\rho_\Lambda} = \frac{\delta\rho_r}{\rho_r}$$

Perturbations of Λ, X and radiation evolve in the same manner.

Conclusions

- Dynamical CC Relaxation model without fine-tuning

$$|\rho_\Lambda| = \left|\rho_\Lambda^i + \frac{\beta}{f}\right| \ll |\rho_\Lambda^i|$$

- Tracking relations
  - Radiation era

$$\rho_\Lambda = -\frac{1}{4}\rho_X \propto \rho_r$$

  - Matter era

$$\rho_\Lambda = \frac{1}{3}\rho_r$$

- Linear perturbations
  - Gravitational potentials Φ controlled by the vacuum
  - No unexpected instabilities
  - Matter era: Similar to ΛCDM with growing dark/X matter density contrast

Effective equation of state

EOS ω_{eff} of a self-conserved DE component ρ_{DE}

- Hubble rate $H(z)$ of the relaxation model
- DM with standard dust scaling law $\tilde{\rho}_X \propto (z+1)^3$
- $\rho_{\text{DE}} = \rho_{\text{tot}} - \tilde{\rho}_X - \rho_r - \rho_b$

$$\omega_{\text{eff}} = -1 + \frac{1+z}{3} \frac{1}{\rho_{\text{DE}}(z)} \frac{d\rho_{\text{DE}}(z)}{dz}$$

Effective equation of state