Georges de la Tour (1593-1652)

Dark energy: an electromagnetic relic from inflation

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What is the nature of dark energy?

The standard cosmological constant explanation suffers from an important <u>naturalness</u> problem

$\rho_\Lambda^{1/4} \sim (10^{-3}\ \mathrm{eV}) \ll M_P$

Alternative models based on new physics plagued by:

- classical or quantum instabilities,
- fine tuning problems,
- inconsistencies with local gravity constraints.

Large-distance modifications of gravity suggested

What about electromagnetism on large scales?

EM quantization in Minkowski space-time

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu \right)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$
$$\partial_\mu J^\mu = 0$$

Gauge invariance

$$\partial_\nu F^{\mu\nu} = J^\mu$$

Maxwell's equations

BUT ....

- **Photon propagator?**
- **"Unphysical" temporal and longitudinal photons**

Coulomb's gauge

Covariant quantization

EM quantization in Minkowski space-time

Coulomb's gauge

$$\partial_\mu A^\mu = 0$$

Lorenz condition

$$A_\mu \rightarrow A_\mu + \partial_\mu \theta$$
$$\Box\theta = 0$$

Residual gauge symmetry

$$\Box A_\mu = J_\mu$$

Maxwell's equations

$$A_0 = 0$$
$$\vec{\nabla}\cdot\vec{A} = 0$$

Residual gauge (free fields)

2 physical states Positive energies

Quantize

Covariant quantization

$$S = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{2}(\partial_\mu A^\mu)^2 + A_\mu J^\mu\right)$$

Modified action: only residual symmetry

$$\partial_\nu F^{\mu\nu} + \lambda\partial^\mu(\partial_\nu A^\nu) = J^\mu$$

$$\Box(\partial_\nu A^\nu) = 0$$

Free field + boundary conditions= Lorenz condition

$$\partial_\nu A^{\nu\,(+)}|\phi\rangle = 0$$
$$\Updownarrow$$
$$(\mathrm{a}_0(\vec{k}) + \mathrm{a}_{||}(\vec{k}))|\phi\rangle = 0$$
$$\Updownarrow$$
$$n_0(k) = n_{||}(k)$$

2 physical states. Positive energies

EM quantization in an expanding universe

Covariant quantization

$$S = \int d^4x\sqrt{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{2}(\nabla_\mu A^\mu)^2 + A_\mu J^\mu\right)$$

$$\nabla_\nu F^{\mu\nu} + \lambda\nabla^\mu(\nabla_\nu A^\nu) = J^\mu \longrightarrow \Box(\nabla_\nu A^\nu) = 0$$

Non-conformally coupled to gravity

$$\mathbf{a}_\lambda^{(in)}(\vec{k}) \rightarrow \sum_{\lambda'=0,\|}\left[\alpha_{\lambda\lambda'}(\vec{k})\mathbf{a}_{\lambda'}^{(in)}(\vec{k}) + \overline{\beta_{\lambda\lambda'}(\vec{k})}\mathbf{a}_{\lambda'}^{(in)\dagger}(-\vec{k})\right]$$

$\Updownarrow$

$\nabla_\nu A^\nu$ can be amplified from quantum vacuum fluctuations by the expanding background (e.g. during inflation) $\rightarrow$ **Lorenz condition?**

A toy model

Flat Robertson-Walker metric

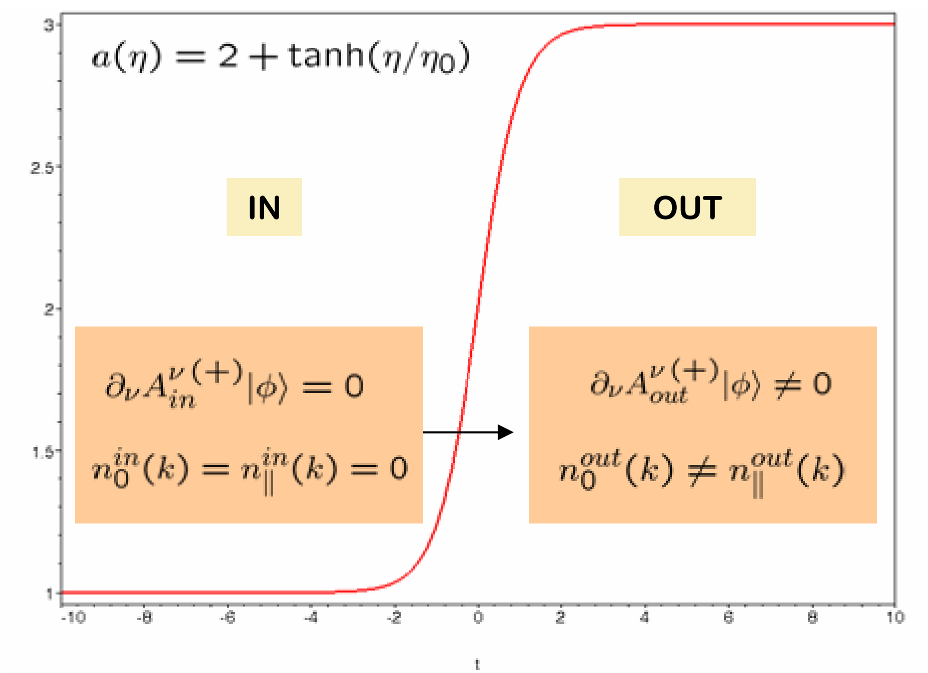
$$ds^2 = a(\eta)^2(d\eta^2 - d\vec{x}^2)$$

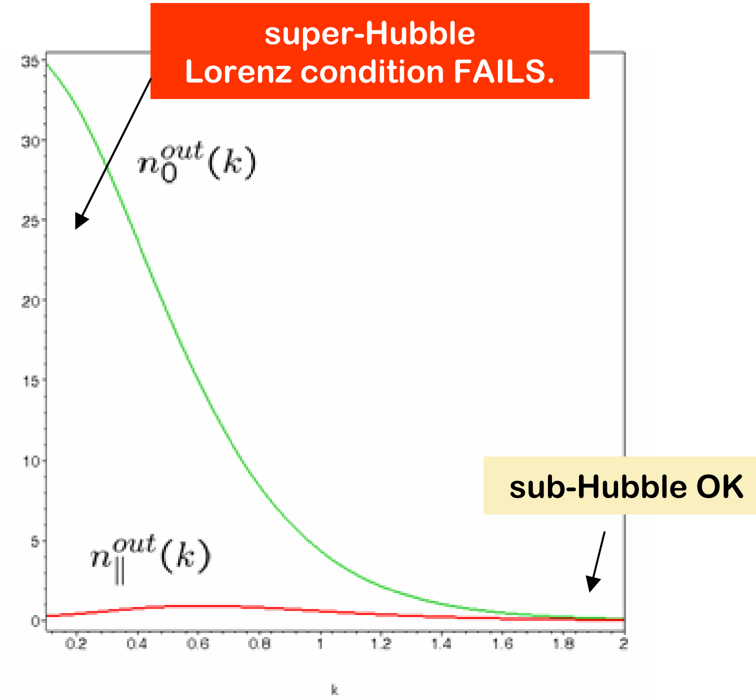
Mode equations

$$\mathcal{A}''_{0k} - \left[\frac{k^2}{\lambda} - 2\mathcal{H}' + 4\mathcal{H}^2\right]\mathcal{A}_{0k} - 2ik\left[\frac{1+\lambda}{2\lambda}\mathcal{A}'_{\parallel k} - \mathcal{H}\mathcal{A}_{\parallel k}\right] = 0$$

$$\mathcal{A}''_{\parallel k} - k^2\lambda\mathcal{A}_{\parallel k} - 2ik\lambda\left[\frac{1+\lambda}{2\lambda}\mathcal{A}'_{0k} + \mathcal{H}\mathcal{A}_{0k}\right] = 0$$

$$\vec{\mathcal{A}}''_{\perp k} + k^2\vec{\mathcal{A}}_{\perp k} = 0$$





Problems with covariant quantization

An initial physical state is not necessarily physical at a later time

$$\partial_\nu A^{\nu\,(+)}_{in} |\phi\rangle = 0 \quad \longrightarrow \quad \partial_\nu A^{\nu\,(+)}_{out} |\phi\rangle \neq 0$$

A possible solution: define the physical states as:

$$\nabla_\nu A^{\nu\,(+)} |\phi\rangle = 0, \quad \forall \eta$$

Pfenning, 2002

But this requires a previous knowledge of the universe geometry at all times

Another solution: introduce ghosts

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda}{2} (\nabla_\mu A^\mu)^2 + g^{\mu\nu}\, \partial_\mu \bar{c}\, \partial_\nu c + A_\mu J^\mu \right)$$

Appropriate boundary conditions:
(Adler, Lieberman and Ng, 1977)

$$\langle \phi | T^\lambda_{\mu\nu} + T^g_{\mu\nu} | \phi \rangle = 0$$

What if EM is not a gauge invariant theory?

Assume the fundamental theory of electromagnetism is not gauge invariant (no need for Lorenz condition):

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right)$$

General solution

$$A_\mu = \underbrace{A_\mu^{\perp(1)} + A_\mu^{\perp(2)}}_{\text{Photon}} + \underbrace{A_\mu^S}_{\text{New scalar state}} + \underbrace{\partial_\mu \theta}_{\text{Pure residual gauge}}$$

The pure gauge mode can be eliminated so that all the physical states have positive norm and positive energy

Flat Robertson-Walker metric

$$\Box(\nabla_\nu A^\nu) = 0 \longrightarrow \nabla_\nu A_k^\nu = \begin{cases} \dfrac{C}{a(\eta)} e^{-ik\eta} & k\eta \gg 1 \\ const. & k\eta \ll 1 \end{cases}$$

$k\eta \gg 1 \rightarrow$ **Negligible: Maxwell's eq. OK**

$k\eta \ll 1 \rightarrow$ **DARK ENERGY**

EM quantization without the Lorenz condition

Ordinary QED recovered in Minkowski space-time

QED effective action: gauge fixing procedure

$$e^{iW} = \int [dA][dc][d\bar{c}][d\psi][d\bar{\psi}]e^{i\int d^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\lambda}{2}(\partial_\mu A^\mu)^2+\eta^{\mu\nu}\partial_\mu\bar{c}\,\partial_\nu c+\mathcal{L}_F\right)}$$

Ghosts decoupled

$$e^{iW} = \int [dA][d\psi][d\bar{\psi}]e^{i\int d^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\lambda}{2}(\partial_\mu A^\mu)^2+\mathcal{L}_F\right)}$$

Gauge non-invariant EM effective action:
no gauge fixing required

A consistent gauge non-invariant EM theory

$$S=\int d^4x\sqrt{g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\lambda}{2}(\nabla_\mu A^\mu)^2+A_\mu J^\mu\right)$$

Consistent EM quantization with THREE physical states

- New scalar state only coupled to gravity
- Electric charge is conserved (only EM gauge sector modified)
- No negative norm (energy) states
- Classical Maxwell's equations recovered on sub-Hubble scales (electromagnetism only tested below 1.3 AU)
- QED recovered in Minkowski space-time (ghosts play no role)

Cosmological electromagnetic fields

$$\ddot{A}_0 + 3H\dot{A}_0 + 3\dot{H}A_0 = 0$$
$$\ddot{\vec{A}} + H\dot{\vec{A}} = 0$$

$H = p/t$ $\Rightarrow$

$$A_0(t) = A_0^+ t + A_0^- t^{-3p}$$
$$\vec{A}(t) = \vec{A}^+ t^{1-p} + \vec{A}^-$$

$$\frac{d}{dt}(\nabla_\mu A^\mu) = \frac{d}{dt}(\dot{A}_0 + 3HA_0) = 0$$

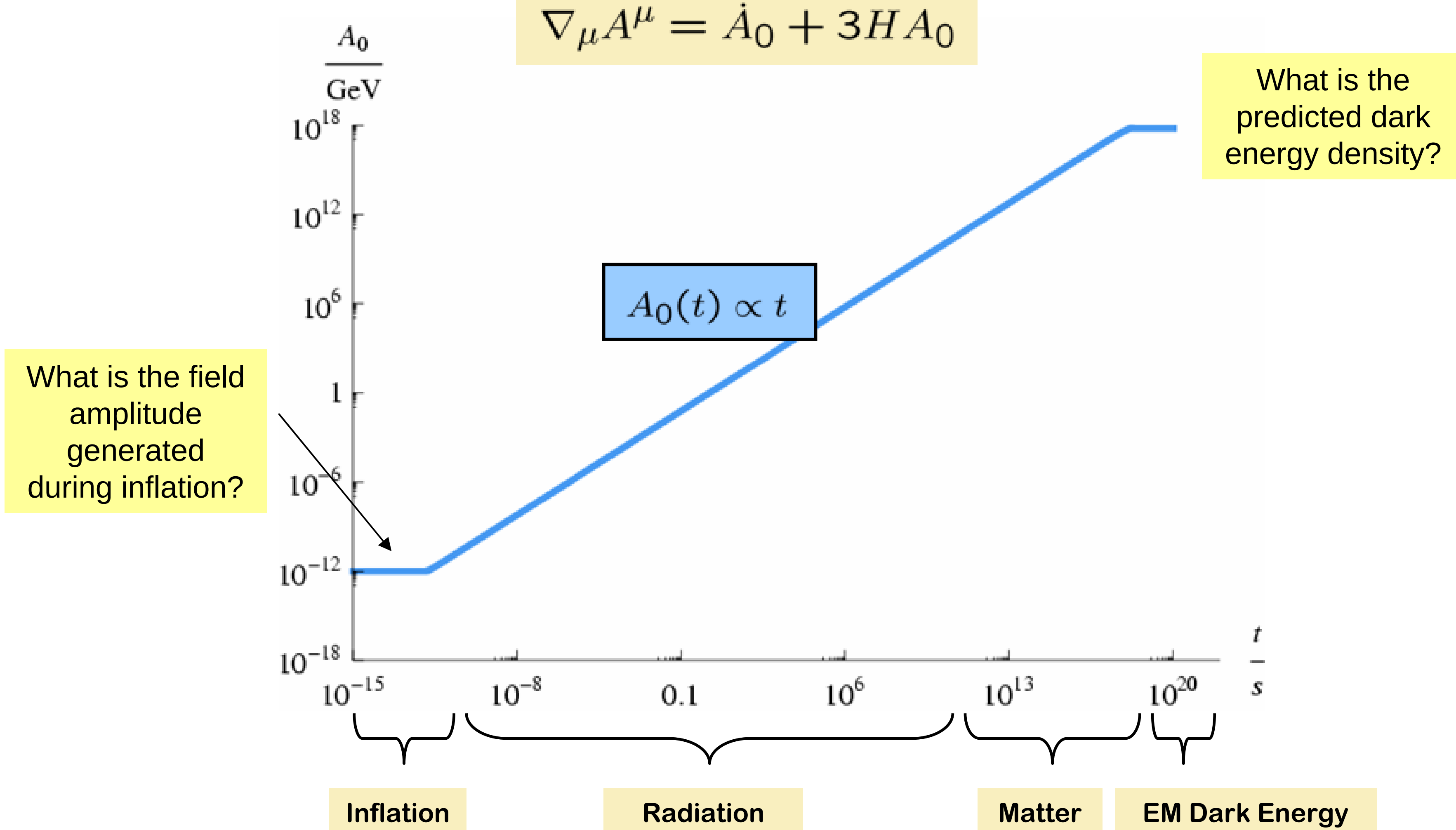
$$A_\mu = (A_0(t), \vec{A}(t))$$

$$\rho_{A_0} = \frac{\lambda}{2}\left(\dot{A}_0 + 3HA_0\right)^2 = const.$$ → Cosmological constant

$$\rho_{\vec{A}} = \frac{1}{2a^2}\left(\dot{\vec{A}}\right)^2 \propto a^{-4}$$ → Isotropy OK

The absolute cosmic electric potential

$$\nabla_\mu A^\mu = \dot{A}_0 + 3HA_0$$



Quantum fluctuations during inflation

De Sitter inflation

$$a(\eta) = -\frac{1}{H_I \eta}$$

$$\mathcal{A}_\mu = \int d^3\vec{k} \sum_{\lambda=1,2,s} \left[\mathbf{a}_\lambda(k) \mathcal{A}_{\mu k}^{(\lambda)} + \mathbf{a}_\lambda^\dagger(k) \overline{\mathcal{A}_{\mu k}^{(\lambda)}} \right]$$

$$\left[\mathbf{a}_\lambda(\vec{k}), \mathbf{a}_{\lambda'}^\dagger(\vec{k}')\right] = \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}'), \quad \lambda, \lambda' = 1, 2, s$$

Normalized scalar mode

$$\mathcal{A}_{0k}^{(s)} = -\frac{1}{(2\pi)^{3/2}} \frac{i}{\sqrt{2k}} \left\{ k\eta e^{-ik\eta} + \frac{1}{k\eta} \left[\frac{1}{2}(1 + ik\eta) e^{-ik\eta} - k^2 \eta^2 e^{ik\eta} E_1(2ik\eta) \right] \right\} e^{i\vec{k}\vec{x}}$$

$$\mathcal{A}_{\| k}^{(s)} = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}} \left\{ (1 + ik\eta) e^{-ik\eta} - \left[\frac{3}{2} e^{-ik\eta} + (1 - ik\eta) e^{ik\eta} E_1(2ik\eta) \right] \right\} e^{i\vec{k}\vec{x}}$$

Power spectrum on super-Hubble scales

$$\mathcal{P}_{A_0}(k) \equiv 4\pi k^3 |A_{0k}|^2 = \frac{H_I^2}{16\pi^2}$$

Initial conditions from inflation

Initial conditions from vacuum fluctuations during inflation

$$\langle A_0^2 \rangle \sim H_I^2$$

$$\rho_{A_0} \sim H^2 A_0^2 \sim \; const. \longrightarrow \rho_{A_0} \sim H_I^4 \longrightarrow \rho_{A_0} \sim \left(\frac{M_I^2}{M_P}\right)^4$$

Predicted dark energy density

Friedmann equation (M_I scale of inflation)

$$H_I^2 \sim \frac{M_I^4}{M_P^2}$$

$$\rho_{A_0} \sim \left(\frac{M_I^2}{M_P}\right)^4 \sim (10^{-3}\mathrm{e}V)^4 \Rightarrow M_I \sim 1\,\mathrm{T}eV$$

The cosmological constant scale can be explained by physics at the electroweak scale

Natural solution to the coincidence problem

Stability and local gravity tests

PPN parameters

$$I = (16\pi G)^{-1} \int [R + \omega K_\mu K^\mu R + \eta K^\mu K^\nu R_{\mu\nu} - \varepsilon F_{\mu\nu} F^{\mu\nu} + \tau K_{\mu;\nu} K^{\mu;\nu}](-g)^{1/2}\, d^4x + I_{NG}(q_A, g_{\mu\nu})$$

Will, '81

$$\gamma = \frac{1 + K^2[\omega - 2\omega(2\omega + \eta - \tau)/(2\varepsilon - \tau)]}{1 - K^2[\omega + 8\omega^2/(2\varepsilon - \tau)]},$$

$$\beta = \tfrac{1}{4}(3 + \gamma) + \tfrac{1}{2}\sigma[1 + \gamma(\gamma - 2)/G],$$

$$\xi = 0,$$

$$\alpha_1 = 4(1 - \gamma)[1 - (2\varepsilon - \tau)\Delta] + 4\omega K^2 \Delta a,$$

$$\alpha_2 = 3(1 - \gamma)[1 - \tfrac{2}{3}(2\varepsilon - \tau)\Delta] + 2\omega K^2 \Delta a - \tfrac{1}{2} b K^2/G,$$

$$\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$$

All parameters agree with GR for arbitrary A_0

$$\gamma = \beta = 1$$

$$\alpha_1 = \alpha_2 = \alpha_3 = 0$$

Classical and quantum stability

v* = *c for scalar, vector and tensor perturbations. No ghosts.

Cosmological perturbations

Same background as ΛCDM, but different perturbations

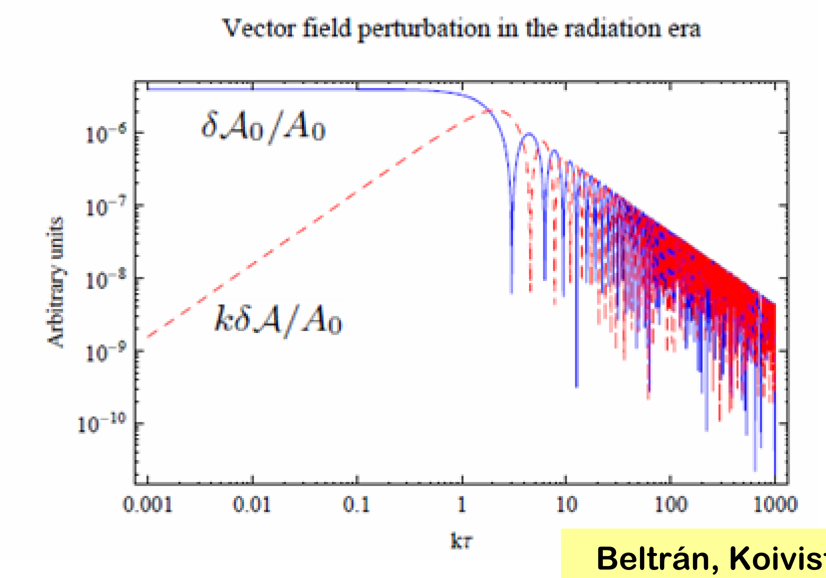
Scalar perturbations:

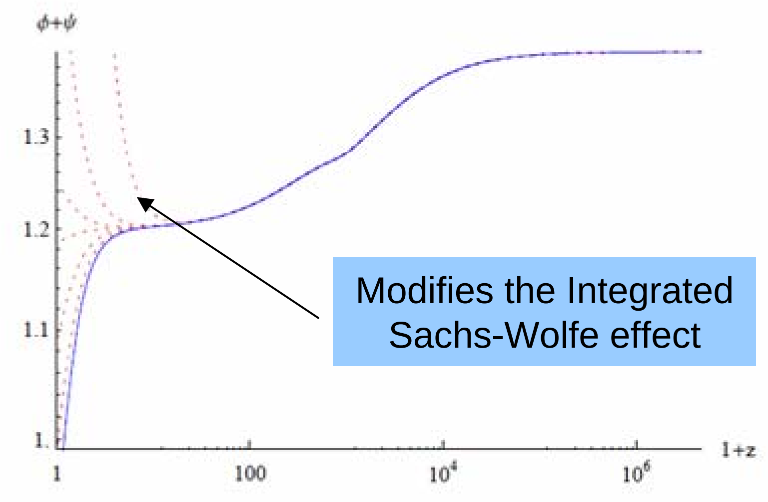
$$ds^2 = a^2(\eta)\left[(1+2\phi(\eta,\mathbf{x}))d\eta - (1-2\psi(\eta,\mathbf{x}))d\mathbf{x}^2\right]$$

Perturbed fields

$$\delta\mathcal{A}_\mu = (\delta\mathcal{A}_0(\eta,\mathbf{x}), \nabla\delta\mathcal{A}(\eta,\mathbf{x}))$$

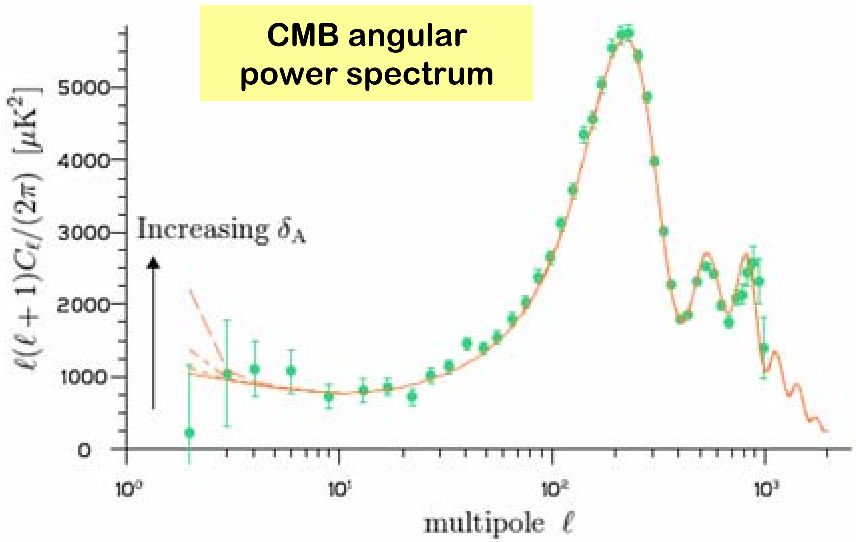
$$\phi = \psi$$

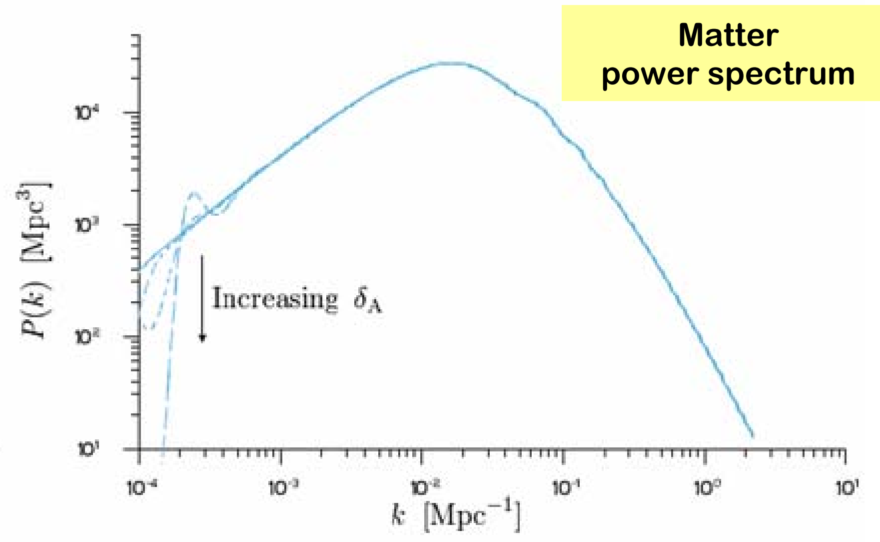




Beltrán, Koivisto, Maroto, Mota, arXiv 0907.3648

Cosmological perturbations





Modifications only in the quadrupole

Compatible with CMB and LSS for:

$$\frac{\delta \mathcal{A}_{0k}}{\mathcal{A}_0} \lesssim 5 \times 10^{-3}$$

EM dark energy perturbation

$$\delta_A \equiv \frac{\mathcal{P}_k^{1/2}}{\rho_{\mathcal{A}0}}$$

$$\mathcal{P}_k = \frac{k^3}{2\pi^2} |\delta\rho_k|^2$$

Conclusions

Consistent EM quantization with <u>three</u> physical states

The new scalar state generates an effective cosmological constant

Compatible with local gravity tests and free from classical or quantum instabilities

Cosmological constant value <u>naturally</u> explained in the context of inflationary cosmology

Consistent with CMB and LSS data with the same number of free parameter as ΛCDM

Nature of dark energy is established without resorting to new physics

Vault of the old Salamanca University Library
Fernando Gallego (1440-1507)

EM quantization without the Lorenz condition

Quantizing the three physical modes:

$$\mathcal{A}_\mu = \int d^3\vec{k} \sum_{\lambda=1,2,s} \left[\mathbf{a}_\lambda(k)\mathcal{A}^{(\lambda)}_{\mu k} + \mathbf{a}^\dagger_\lambda(k)\overline{\mathcal{A}^{(\lambda)}_{\mu k}} \right]$$

Positive normalization:

$$\begin{aligned} \left(\mathcal{A}^{(\lambda)}_k, \mathcal{A}^{(\lambda')}_{k'}\right) &= i\int_\Sigma d\Sigma_\mu \left[\overline{\mathcal{A}^{(\lambda)}_{\nu k}}\, \Pi^{(\lambda')\mu\nu}_{k'} - \overline{\Pi^{(\lambda)\mu\nu}_k}\, \mathcal{A}^{(\lambda')}_{\nu k'} \right] \\ &= \delta_{\lambda\lambda'}\delta^{(3)}(\vec{k}-\vec{k}'), \qquad \lambda, \lambda' = 1, 2, s \end{aligned}$$

Canonical commutators: no negative norm states

$$\left[\mathbf{a}_\lambda(\vec{k}), \mathbf{a}^\dagger_{\lambda'}(\vec{k}')\right] = \delta_{\lambda\lambda'}\delta^{(3)}(\vec{k}-\vec{k}'), \quad \lambda, \lambda' = 1, 2, s$$