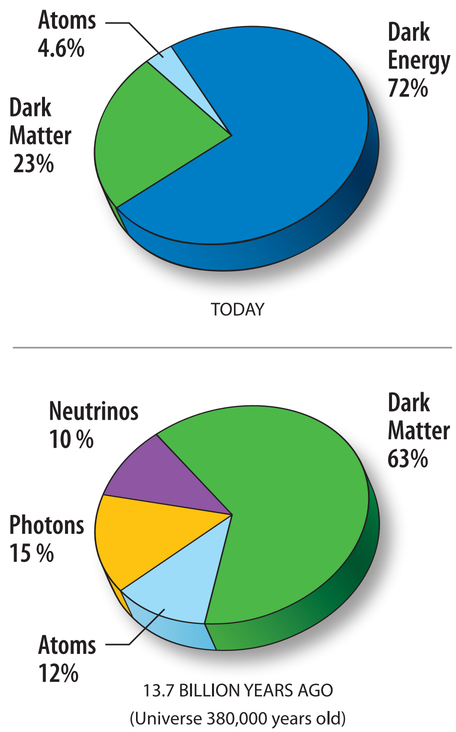
Contents of the universe



Mystery of Dark Energy

Acceleration of the universe

Most simple explanation is cosmological constant

However one needs the value

$$\Lambda \approx 10^{-120}$$

in units $8\pi G_N = \hbar = c = 1$

Quantum corrections are $O(1)$

Also, why-now problem.

Alternatives to Λ

- $\star$ cosmic fluid with exotic properties (quintessence)
- $\star$ Non-standard gravitation $f(R)$
- $\star$ $\ldots$

Mystery of Dark Energy has renewed interest on non-standard gravitation

To which extent quintessence and $f(R)$ are independent ?

The dark side of gravity

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Outline:

- Modified $f(R)$ gravity
- Graviton mass
- Shadow gravitons

Modified $f(R)$ gravity

R^2 example:

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\,(R + \alpha R^2)\;+\;S_m(g_{\mu\nu}, \Psi, A_\mu, \dots)$$

$(\Lambda = 0)$. Introduce ϕ= auxiliary field

$$\frac{1}{2}\int d^4x\,\sqrt{-g}\,(R + \alpha R^2 - \alpha(R - \frac{1}{2\alpha}\phi)^2)$$

Eqs. motion for ϕ are algebraic: $\frac{1}{2\alpha}\phi = R$

$$\frac{1}{2}\int d^4x\,\sqrt{-g}\,(R + R\phi - \frac{1}{4\alpha}\phi^2)$$

Brans-Dicke theory; it can be conformally transformed:

$$\bar{g}_{\mu\nu} = e^{\phi/\sqrt{3}}\, g_{\mu\nu} \qquad \text{(field redefinition)}$$

$$\begin{aligned} S = \int d^4x\, \sqrt{-\bar{g}} \quad [& \quad \frac{1}{2}\,\bar{R} \\ + & \quad \frac{1}{2}\,\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \\ + & \quad S_m(\bar{g}_{\mu\nu}e^{-\phi/\sqrt{3}}, \Psi, A_\mu, \ldots)] \end{aligned}$$

New scalar dof; dangerous $\bar{\Psi}\Psi\phi$ couplings
(Unless mechanism like chameleon)

This is a general feature

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\,[R + f(R)] + \ldots \qquad (JF)$$

$$S = \int d^4x\,\sqrt{-\bar{g}}\,[\frac{1}{2}\bar{R} + \frac{1}{2}\,\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)] + \ldots \qquad (EF)$$

Can be done when $f'' \neq 0$
Not possible for R and Λ

$\mathbf{f}(R) \neq \Lambda$ but leading to acceleration

$$\Downarrow$$

$\phi =$ quintessence field

Graviton mass

Naive way to modify gravity at large distances;
Not without problems

I shall restrict to the linear level

$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad h = h_\alpha^{\ \alpha}$

Linear Einstein eqs.

$$H_{\mu\nu} = T_{\mu\nu}$$

with

$$\begin{aligned} H_{\mu\nu} &= \frac{1}{2}(\partial_\alpha \partial_\nu h_\mu^{\ \alpha} + \partial_\alpha \partial_\mu h_\nu^{\ \alpha} - \partial_\mu \partial_\nu h - \Box\, h_{\mu\nu} \\ &- \eta_{\mu\nu} \partial_\alpha \partial_\beta h^{\alpha\beta} + \eta_{\mu\nu} \Box\, h) \end{aligned}$$

Two possible mass terms: $m^2 h_{\mu\nu},\ m^2 \eta_{\mu\nu} h$

Fierz-Pauli (No ghosts, 5 dof for a s=2 particle)

$$H_{\mu\nu} + m^2(h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}$$

Problems when non-linear interaction switch on;
Not treated here.

Limit $m \to 0$

Tensor very different from vector

massive γ:

$k^\mu = (E, k, 0, 0)$
extra polarization: $\epsilon^\mu = (k/m, E/m, 0, 0)$

$$\epsilon^\mu = \frac{k^\mu}{m} + \frac{m}{k+E}\,(-1, +1, 0, 0)$$

Use conservation $k^\mu J_\mu = 0$ in Amplitude $\epsilon^\mu J_\mu$

$m \to 0$ **matches** $m = 0$

massive g:

Extra polarizations: Vector and Scalar

- Vector polarization similar to massive γ case
- Scalar polarization is different

$$\epsilon^{\mu\nu} = \frac{2}{\sqrt{6}} \begin{pmatrix} k^2/m^2 & kE/m^2 & & \\ kE/m^2 & E^2/m^2 & & \\ & & -1/2 & \\ & & & -1/2 \end{pmatrix}$$

$$= \frac{2}{\sqrt{6}} \left[\frac{k^\mu k^\nu}{m^2} + \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & -1/2 & \\ & & & -1/2 \end{pmatrix} \right]$$

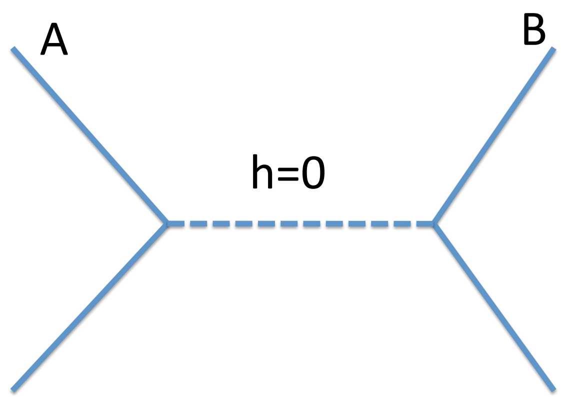
Use conservation $k^\mu T_{\mu\nu} = 0$ in Amplitude $\epsilon^{\mu\nu} T_{\mu\rho}$

finite part survives (scalar mode)

Discontinuity $m \to 0$

(Iwasaki-VanDam-Veltman-Zakharov)

Physical origin: $h = 0$ mode does not decouple.



Discontinuity can be seen calculating one-graviton exchange amplitude

Another way to see the discontinuity:

Eq of motion, Fierz-Pauli mass,

$$-\Box h_{\mu\nu} + m^2 h_{\mu\nu} = 2T_{\mu\nu} - \frac{2}{3}\eta_{\mu\nu}T^{\alpha}_{\alpha}$$

Metric around source $T^{00} = M\delta^3(\vec{r})$

$$ds^2 = -(1+2\phi)dt^2 + (1-\phi)(dx^2 + dy^2 + dz^2)$$

with $\phi = -\frac{GM}{r}\, e^{-mr} = \phi_N\, e^{-mr}$

Massless: $ds^2 = -(1+2\phi_N)dt^2 + (1-2\phi_N)(dx^2 + dy^2 + dz^2)$

Light bending/Time delay

Light path $ds^2 = 0$, consider effective refraction index n_{eff}

$m = 0$

$$n_{eff}^2 = \frac{dt^2}{dr^2} = \frac{1 - 2\phi_N}{1 + 2\phi_N} = 1 - 4\phi_N$$

$$n_{eff} = 1 - 2\phi_N$$

$m \neq 0$ (but small)

$$n_{eff}^2 = \frac{dt^2}{dr^2} = \frac{1 - \phi}{1 + 2\phi} = 1 - 3\phi$$

$$n_{eff} = 1 - \frac{3}{2}\phi$$

Bending angle α is proportional to global coefficient of $n_{eff} - 1$.

$$\alpha_{(m=0)} = \frac{3}{4}\alpha_{(m\to 0)}$$

Same factor in radar-time-delay.

observation excludes massive graviton
(in the linear theory)

Possible way out: non-linear effects (Vainstein)
Non-perturbative effects on extra scalar mode

Shadow gravitons

Work in progress, with Z. Berezhiani, O. Pujolas

$$
\begin{aligned}
L \;=\; & -\frac{1}{2}\begin{pmatrix} h & h' \end{pmatrix}\begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}\begin{pmatrix} H \\ H' \end{pmatrix} \\
& -\frac{1}{4}\begin{pmatrix} \hbar & \hbar \end{pmatrix}\begin{pmatrix} c^2 & c\,s \\ c\,s & s^2 \end{pmatrix}\begin{pmatrix} \hbar \\ \hbar' \end{pmatrix}^* \\
& + hT + \lambda h' T'
\end{aligned}
$$

($c = \cos\phi$, $s = \sin\phi$). The mass matrix is arranged so that there is zero eigenvalue.

Have defined $\hbar^{\mu\nu} = h^{\mu\nu} - a\eta^{\mu\nu}$
(FP corresponds to $a = (1 \pm \sqrt{3}i)/2$)

Put $\lambda = 1$

Relation between propagation and interaction basis

$$U = \begin{pmatrix} c - \epsilon s^3 & -s + \epsilon c^3 \\ s + \epsilon c s^2 & c + \epsilon c^2 s \end{pmatrix}$$

To simplify, put $\epsilon = 0$, have only one parameter: mixing angle

Potential energy among two (standard) masses M_{1A} and M_{1B}

$$V = -\frac{GM_{1A}M_{1B}}{r}(c^2 + \frac{4}{3}s^2 e^{-mr})$$

Light bending by an angle α in presence of a mass M_1:

- for $b \gg m$

$$\alpha = \frac{4GM_1}{b}\, c^2$$

- for $b \ll m$,

$$\alpha = \frac{4GM_1}{b}\,(c^2 + s^2) = \frac{4GM_1}{b}$$

Conclusion

Mixing with shadow graviton leads to
modification of Newton potential due to graviton mass
but in agreement with light bending & time delay
for small enough s.

Work in progress:

- $\star$ limits to m and mixing angles
- $\star$ Consider $\lambda \neq 0$