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Cosmology with a time dependent vacuum: expansion and structure formation

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Guidelines of the Talk

- Cosmological constant and Dark Energy
- Variable CC and effective EOS
- "ΛXCDM" cosmologies
- DE perturbations and cosmic coincidence
- Variable CC models versus experiment
- Conclusions

Λ in QFT: the Vacuum Energy

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x \sqrt{-g}\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi\, \partial^\mu \phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2 \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu} \mathcal{L}$$

$$= \left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\ \partial_\alpha \phi\, \partial^\alpha \phi \right] + g_{\mu\nu} V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$< T_{\mu\nu} >= g_{\mu\nu}\ < V_{eff} >$$

The many Cosmological Constant Problems

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$

- **Problem I**:

The "Classic" CC Problem: $\left(\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55}\right)$

Why the induced and vacuum counterparts of the CC cancel each other with such a huge precision?

F. Bauer, JS, H. Stefancic
PLB 678:427-433,2009.
see Florian's talk

- **Problem II**:

The (first) "Coincidence" CC Problem:

J.Grande, A. Pelinson, J. Solà,
Phys.Rev.D79:043006,2009.
JCAP 0712:007,2007.
JCAP 0608:011,2006.

Why the observed CC in the present-day Universe is so close to the matter density ρ?

coincidence ratio now: $r \equiv \frac{\rho_\Lambda^0}{\rho_M^0} = \frac{\Omega_\Lambda^0}{\Omega_M^0} \simeq \frac{7}{3} = \mathcal{O}(1)$

- **Problem III**:

The "nature" of the the CC Problem:

In more recent times the notion of Λ has been superseded by that of the DE. The latter is more general and involves a variety of models leading to an accelerated expansion of the universe in which the DE itself is a time-evolving entity. These models include dynamical scalar fields (quintessence... and the like), phantom fields, braneworld models, Chaplygin gas, holographic dark energy, cosmic strings, domain walls...

What is, then, the true dynamical cause responsible for the DE?

- **Problem IV**:

The (second) "Coincidence" Problem:

Present observations seem to indicate an evolving DE with a potential phantom phase near our time.

If the dark energy behaves phantom-like, why just now?

➢ ''Canonical'' definition of Dynamical Dark Energy

One popular possibility is the idea of quintessence, where there is no "true" Λ

The total energy-momentum tensor on the *r.h.s.* of Einstein eqs. is the sum

$$\tilde{T}_{\mu\nu} \equiv T^M_{\mu\nu} + T^D_{\mu\nu}\,.$$

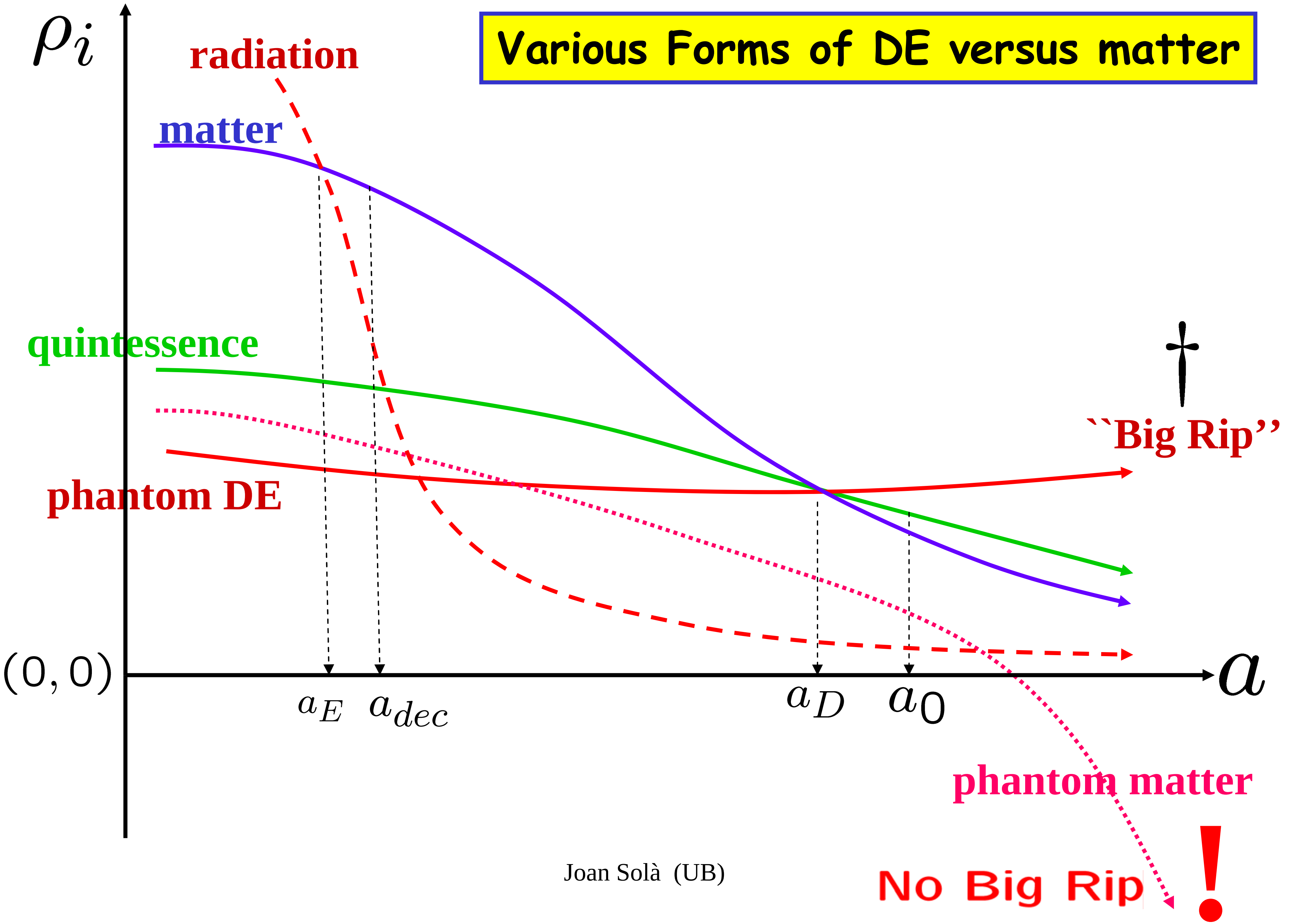
One assumes that both tensors are separately conserved, and so $\bigtriangledown^\mu \tilde{T}_{\mu\nu} = 0$ is equivalent to

$$\bigtriangledown^\mu T^M_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

and (unmixed conservation laws)

$$\bigtriangledown^\mu T^D_{\mu\nu} = 0 \Longleftrightarrow \frac{d\rho_D}{dt} + 3\,H\,(\rho_D + p_D) = 0$$

Various Forms of DE versus matter



Question:

Can a dynamical DE still be Λ?...

Need running Λ !!

Variable Λ

- For variable Λ, the conserved quantity is not the matter energy-momentum tensor $T_{\mu\nu}$, but the sum

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\,\rho_\Lambda(t)\,, \qquad \nabla^\mu \tilde{T}_{\mu\nu} = 0\,.$$

By the Bianchi identities, Λ is constant $\Longleftrightarrow$ the matter $T_{\mu\nu}$ is individually conserved ($\nabla^\mu T_{\mu\nu} = 0$)– in particular, $\rho_\Lambda = $ const. if $T_{\mu\nu} = 0$ (e.g. during inflation).

- From FLRW metric

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - k\,r^2} + r^2\,d\theta^2 + r^2\,\sin^2\theta\,d\phi^2\right)\,,$$

we may compute explicitly the local energy-conservation law $\nabla^\mu \tilde{T}_{\mu\nu} = 0$. The result is an equation allowing transfer of energy between ordinary matter and the dark energy associated to the Λ term :

$$\frac{d\rho_\Lambda}{dt} + \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

(<u>mixed</u> conservation law!)

Running Λ from Planck Scale Physics

- One may expect that the **RGE** of Λ is totally dominated by sub-Planckian masses:

$$\frac{d\Lambda}{d\ln\mu} = \frac{1}{(4\pi)^2}\sum_i c_i\,\mu^2 M_i^2 + \ldots. = \frac{1}{(4\pi)^2}\sum_i c_i\,H^2 M_i^2 + \ldots$$

$$(\mu = H)$$

$$= \frac{1}{(4\pi)^2}\,\sigma\,H^2 M^2 + \ldots..$$

I.L.Shapiro, J. Solà.
JHEP 0202 (2002) 6
Phys.Lett.B475 (2000) 236.

I.L.Shapiro, et al
Phys. Lett. B574 (2003) 149

J. Solà
J.Phys.A4 (2008) 164066

with

$$M \equiv \sqrt{\sum_i c_i\,M_i^2}\,.$$

- Provides a natural explanation for the geometric mean puzzle:

$$\Lambda \simeq \sqrt{\rho_P\,\rho_H} = \sqrt{M_P{}^4\,H^4} = M_P{}^2\,H^2$$

A semiclassical FLRW with running Λ

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) + H_0^2\,\Omega_K^0(1+z)^2$$

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0\,.$$

$$\frac{d\Lambda}{d\ln H} = \frac{1}{(4\pi)^2}\sum_i c_i\, M_i^2\, H^2 + \ldots = \frac{3\nu}{4\pi} M_P{}^2 H^2.$$

$$\nu = \frac{\sigma\,M^2}{12\pi M_P{}^2}$$

$$\rho_\Lambda \equiv \Lambda = C_1 + C_2\,H^2\,.$$

Running cosmological ``constant''

The (bare) vacuum-to-vacuum part or ZPE in dimensional regularization (one-loop):

$$\bar{V}^{(1)}_{vac} = \sum_k \frac{1}{2}\hbar\,\omega_k \equiv \frac{1}{2}\,\mu^{4-n}\int \frac{d^{n-1}k}{(2\pi)^{n-1}}\,\hbar\,\sqrt{\vec{k}^2+m^2}\,.$$

$$(n\to 4) \qquad = \frac{m^4\,\hbar}{64\,\pi^2}\left(-\frac{2}{4-n} - \ln\frac{4\pi\mu^2}{m^2} + \gamma_E - \frac{3}{2}\right)\,.$$

In the $\overline{MS}$ subtraction scheme: $\delta\rho_\Lambda^{\rm vac} = \frac{m^4}{64\,\pi^2}\left(\frac{2}{4-n} + \ln 4\pi - \gamma_E\right)\,.$

$$V_{vac} = \rho_\Lambda^{\rm vac}(\mu) + \delta\rho_\Lambda^{\rm vac} + \bar{V}^{(1)}_{vac}(\mu)$$

$$= \rho_\Lambda^{\rm vac}(\mu) + \frac{m^4\,\hbar}{64\,\pi^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$\overline{MS}$-renormalized vacuum value where $$\rho_\Lambda^0 = \mu^{n-4}\left(\rho_\Lambda^{\rm vac}(\mu) + \delta\rho_\Lambda^{\rm vac}\right) \qquad (n \to 4) \quad \Rightarrow$$

$$= \rho_\Lambda^{\rm vac}(\mu) + \frac{m^4\,\hbar}{64\pi^2}\left(\frac{2}{4-n} + \ln 4\pi - \gamma_E\right) - \frac{m^4\,\hbar}{32\pi^2}\ln\mu$$

Since $d\rho_\Lambda^0/d\ln\mu = 0 \quad \Rightarrow$

$$\boxed{\frac{d\rho_\Lambda^{\rm vac}}{d\ln\mu} = \frac{m^4}{32\pi^2}\,\hbar \equiv \beta_\Lambda^{(1)}}$$

RGE for ρ_Λ !!

Induced contribution:

$$U(\phi) = \frac{m^2\phi^2}{2} + \frac{\lambda\phi^4}{4!} \qquad (m^2 < 0)$$

$$\langle\phi\rangle = \sqrt{\frac{-6\,m^2}{\lambda}} \qquad\qquad \langle U(\phi)\rangle = -\frac{3\,m^4}{2\lambda} \qquad\qquad \textbf{(SSB)}$$

$$m_H^2 = -2\,m^2$$

$$\frac{1}{8}\,m_H^2\,v^2 \gtrsim 10^8\ GeV^2$$

$$\lambda = 3\,m_H^2/v^2$$

Quantum effects$\longrightarrow$ μ-dependence:

$$V_{\text{eff}}(\phi) = U(\phi) + \hbar\,V^{(1)}(\phi)$$

$$V^{(1)}(\phi) = \frac{1}{64\pi^2}\left(m^2 + \frac{\lambda\phi^2}{2}\right)^2\left[\ln\frac{\left(m^2 + \lambda\phi^2/2\right)}{\mu^2} - \frac{3}{2}\right]$$

RG method looks for relations of the form

$$\frac{d\rho_\Lambda^i}{d\ln\mu} = \beta_\Lambda(P,\mu)\,,$$

for the various running parts of ρ_Λ.

I.L. Shapiro, J. Solà, arXiv:0808.0315 [hep-th]

I.L. Shapiro, J. Solà, Phys.Lett.B475 (2000) 236
JHEP 0202 (2002) 006

This may eventually lead to fundamental explanation for phenomenological time-evolution

$$\frac{d\rho_\Lambda}{dt} = F_\Lambda(H,\rho_M,\rho_\Lambda,...)\,,$$

Usually this kind of expressions are postulated in the literature, for some suitable F.

However, **"running" is NOT just μ-dependence...**

In particular, for the ZPE

$$V_{vac}(\mu) = \rho_\Lambda^{\rm vac}(\mu) + \frac{m^4\,\hbar}{64\,\pi^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

In the absence of a non-trivial metric, i.e. **in flat spacetime**, μ-dependence reflects no evolution, just **RG**-invariance under renormalization

$$\frac{d\rho_\Lambda^{\rm vac}}{d\ln\mu} = \frac{m^4}{32\pi^2}\hbar \quad \Rightarrow \quad \rho_\Lambda^{\rm vac}(\mu) = \rho_\Lambda^{\rm vac}(\mu_0) - \frac{m^4\,\hbar}{64\pi^2}\ln\frac{\mu_0^2}{\mu^2}$$

$$\Downarrow$$

$$V_{vac} = \rho_\Lambda^{\rm vac}(\mu_0) + \frac{m^4\,\hbar}{64\,\pi^2}\left(\ln\frac{m^2}{\mu_0^2} - \frac{3}{2}\right)$$

$\lambda\phi^4$ for $m = 0$

$$V_{\text{eff}}(\phi) = \frac{\lambda\phi^4}{4!} + \frac{\lambda^2\phi^4}{256\pi^2}\left[\ln\frac{\lambda\phi^2/2}{\mu^2} - \frac{3}{2}\right]$$

$$\boxed{\mu\frac{d\lambda}{d\mu} = \beta_\lambda^{(1)} = \frac{3\lambda^2}{(4\pi)^2}} \Rightarrow \quad \lambda(\mu') = \frac{\lambda(\mu)}{1 - \frac{3\,\lambda(\mu)}{32\,\pi^2}\ln\frac{\mu'^2}{\mu^2}} \simeq \lambda(\mu) + \beta_\lambda^{(1)}\,\tau\,,$$

$$(\tau = \tfrac{1}{2}\ln\tfrac{\mu'^2}{\mu^2})$$

Explicit and implicit μ-dependences cancel out!

...but the implicit one carries dynamics: $\boxed{\mu \longrightarrow q}$

Reasonable...but

not so easy in cosmology !!!

$$\mu \longrightarrow f(q, \rho, R, H, 1/t, z, ..).$$

↖ QED, $\lambda\phi^4$...

Let us move to the cosmological case, within **QFT in curved space-time**. Let us show possible sources of dynamical quantum effects,

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = -12\,H^2 - 6\,\dot{H} = -3\,\Lambda - 3\,H^2\,.$$

Evolving background, $H = H(t)$

$$\Downarrow$$

Dynamical curvature $\Rightarrow$ **enables physical running from quantum effects**

$$\boxed{\mu \longrightarrow q} \quad \Rightarrow \quad \boxed{\mu \longrightarrow H}$$

in cosmology?,,,

Renormalizable action in QFT in a curved background:

$$S_{vac} = S_{EH} + S_{HD}$$

$$S_{EH} = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi\,G_N}\,R + \rho_\Lambda\right)$$

$$S_{HD} = \int d^4x\sqrt{-g}\left\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\right\}$$

$$S = S_{vac} + S_\phi$$

$$S_\phi = \frac{1}{2}\int d^4x\,\sqrt{-g}\,\left\{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2 + \xi R\phi^2\right\}$$

Local quantum effects :

Quantum effects generalizing the ZPE from flat QFT:

$$\bar{\Gamma}^{(1)}_{vac} = \frac{1}{2(4\pi)^2}\int d^4x\,\sqrt{-g}\,\left\{\frac{m^4}{2}\cdot\left(\frac{1}{\epsilon}+\frac{3}{2}\right)+\left(\xi-\frac{1}{6}\right)m^2R\left(\frac{1}{\epsilon}+1\right)\right\}$$

↑

flat QFT part found earlier ($\longrightarrow$ ZPE)

$$\frac{1}{\epsilon} = \frac{2}{4-n}+\ln\left(\frac{4\pi\mu^2}{m^2}\right)-\gamma_E$$

We expect from covariance that the general form is

$$\rho_\Lambda^{ph}(H) = \rho_\Lambda^0 + \beta\, M_P^2\,(H^2 - H_0^2) + \mathcal{O}(H^4)$$

I.L. Shapiro, J. Solà,
arXiv:0808.0315 [hep-th]

Analogy with conformaly anomaly case
(Starobinsky's inflation), also suggest it $\Rightarrow$

J. Solà,
J. Phys. A 41 (2008)164066,

"DE picture" versus "CC picture"

- Observations leading to the **EOS** of the **DE** are sensitive to the function $H = H(z)$.

- We can describe a variable CC model with mixed energy-conservation law as if it would be a dynamical DE model with unmixed EC-law.

- Let us assume there is an underlying fundamental dynamics

$$\rho_\Lambda(z) = \rho_\Lambda(\rho(z), H(z), ...)\,, \quad G(z) = G(\rho(z), H(z), ...)$$

⇓

$$H_\Lambda^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda)$$

H. Stefancic, J.S.
Mod. Phys. Lett. A 21 (2006) 479

Solving the conservation equations in the DE picture:

$$\rho_m(z) = \rho_m(0)\,(1+z)^{\alpha}$$

$$\alpha = 3(1+\omega_m) \ (\omega_m = 1/3 \text{ or } \omega_m = 0)$$

and

$$\rho_D(z) = \rho_D(0)\,\zeta(z)$$

$$\zeta(z) \equiv \exp\left\{3\int_0^z dz' \frac{1+\omega_D(z')}{1+z'}\right\}$$

Hence

$$\omega_D(z) = -1 + \frac{1}{3}\,\frac{1+z}{\zeta}\,\frac{d\zeta}{dz}$$

$$H_D^2(z) = H_0^2\left[\tilde{\Omega}_M^0\,(1+z)^{\alpha} + (1-\tilde{\Omega}_M^0)\,\zeta(z)\right]$$

(flat space)

$$(\Delta\Omega_M \equiv \Omega_M^0 - \tilde{\Omega}_M^0)$$

⇒ "Matching condition" of the two pictures:

$$H_D^2(z) = H_\Lambda^2(z)$$

$$H_0^2 \left[\tilde{\Omega}_M^0 \, (1+z)^\alpha + (1-\tilde{\Omega}_M^0) \, \zeta(z)\right] = H_\Lambda^2(z)$$

Matching generates an "effective **EOS**" for Λ:

$$\omega_{\rm eff}(z) = -1 + \frac{1}{3} \frac{1+z}{\zeta} \frac{d\zeta}{dz}$$

$$= -1 + \frac{\alpha}{3} \left(1 - \frac{\xi_\Lambda(z)}{\rho_D(z)}\right)$$

Effective equation of state for the variable Λ as a function of the redshift: $\omega_{\rm eff} = \omega_{\rm eff}(z;\nu)$

H. Stefancic, J.S.
Phys. Lett. B624 (2005) 147

$\Delta\Omega_M \neq 0$

$$\rho_\Lambda = \rho_\Lambda^0 + \frac{3\nu}{4\pi} M_P^2 (H^2 - H_0^2)$$

$$\omega_{\rm eff}(z) = -1+(1-\nu)\frac{\Omega_M^0 (1+z)^{3(1-\nu)} - \tilde{\Omega}_M^0 (1+z)^3}{\Omega_M^0 [(1+z)^{3(1-\nu)} - 1] - (1-\nu) [\tilde{\Omega}_M^0 (1+z)^3 - 1]} .$$

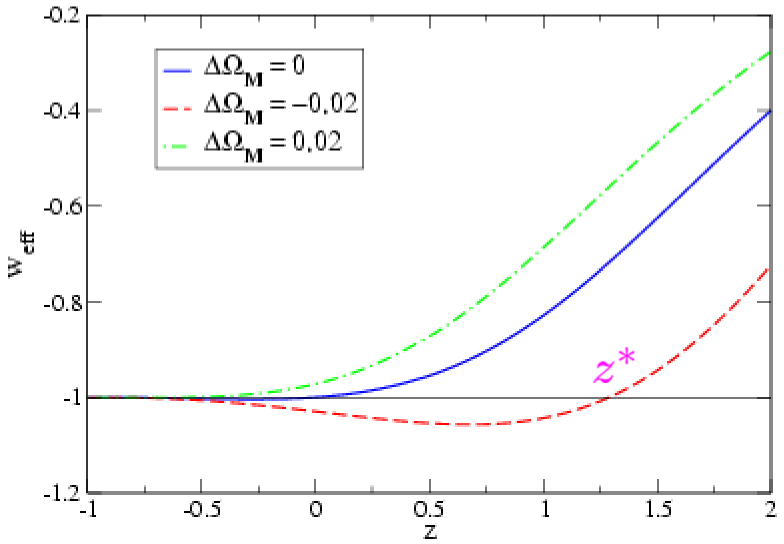
$\Delta\Omega_M = 0$

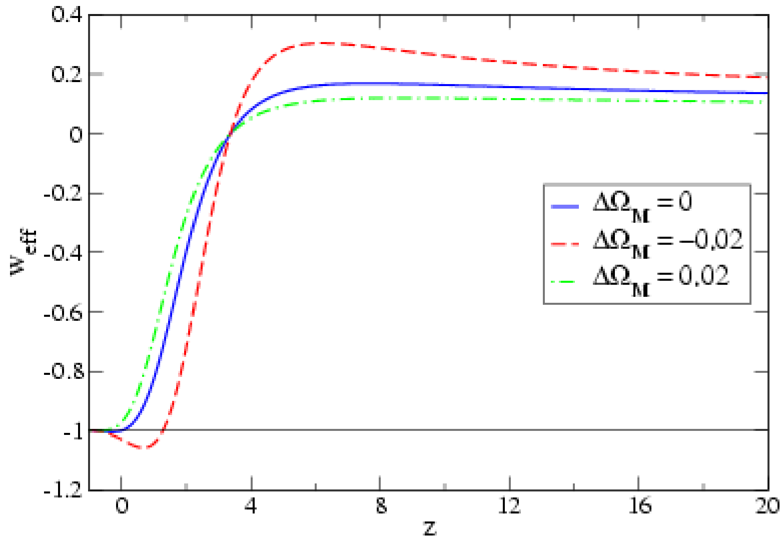
$$\omega_{\rm eff}(z) = -1+(1-\nu)\frac{\Omega_M^0 (1+z)^3 \left[(1+z)^{-3\nu} - 1\right]}{1-\nu-\Omega_M^0 + \Omega_M^0 (1+z)^3 \left[(1+z)^{-3\nu} - 1 + \nu\right]} .$$

$$\simeq -1 - 3\,\nu \frac{\Omega_M^0}{\Omega_\Lambda^0} (1+z)^3 \ln(1+z) .$$

$$(\text{case } \nu < 0 \,; \quad \nu = -\nu_0)$$

$\Delta\Omega_M = 0 \quad \Delta\Omega_M \neq 0$!!





$$\left(\nu_0 = \frac{1}{12\pi} \simeq 0.026\right)$$

H. Stefancic, J.S.
Phys. Lett. B 624 (2005) 147

NEXT STEP?...........

Running cosmological parameters and dynamical dark energy component?

↓

The "ΛXCDM" model

J. Grande, H. Stefancic, J.S.
JCAP 0608:011,2006
Phys.Lett.B645:236-245,2007.

In the ΛXCDM case, with $G = const.$ and separate conservation of matter and DE:

$$\dot{\rho}_m + \alpha_m\,\rho_m\,H = 0\,, \qquad \alpha_m = 3(1+\omega_m)$$

$$\dot{\rho}_D + \alpha_D\,\rho_D\,H = 0\,, \qquad \alpha_D = 3(1+\omega_e)$$

with

$$\rho_D = \rho_\Lambda + \rho_X$$

cosmological "constant" contribution

cosmon contribution

$$\omega_e = \frac{p_\Lambda + p_X}{\rho_\Lambda + \rho_X} = -1 + \frac{1}{3}\,\frac{\alpha_X\,\rho_X}{\rho_D}$$

➢ General stopping condition

$$r(a) = \frac{\rho_D}{\rho_M}$$

$$r(a) = \frac{(\Omega_\Lambda^0 - \nu)\, a^3}{(1-\nu)\,\Omega_M^0} + \frac{\epsilon}{w_X - \epsilon} + \left[\frac{1-\Omega_\Lambda^0}{\Omega_M^0\,(1-\nu)} - \frac{w_X}{w_X - \epsilon}\right] a^{-3(w_X - \epsilon)}$$

$\epsilon \equiv \nu(1 + w_X)$ (nucleosynthesis parameter)

(Maximizing)

$$\frac{\Omega_\Lambda^0 - \nu}{w_X\,(\Omega_X^0 + \nu\,\Omega_M^0) - \epsilon\,(1-\Omega_\Lambda^0)} > 0\,, \quad (1 + w_X)\left(\Omega_\Lambda^0 - \nu\right) < 0$$

J. Grande, H. Stefancic, J.S.
JCAP 0608:011,2006

Nucleosynthesis Constraints

At temperatures $T \lesssim 0.1\,MeV$ the weak interactions (responsible for neutrons and protons to be in equilibrium) freeze-out. The expansion rate is sensitive to the amount of DE, hence primordial nucleosynthesis can place stringent bounds on the parameters of the ΛXCDM model

Define $$r = \frac{\rho_D}{\rho_m} = \frac{\rho_\Lambda + \rho_X}{\rho_m}$$

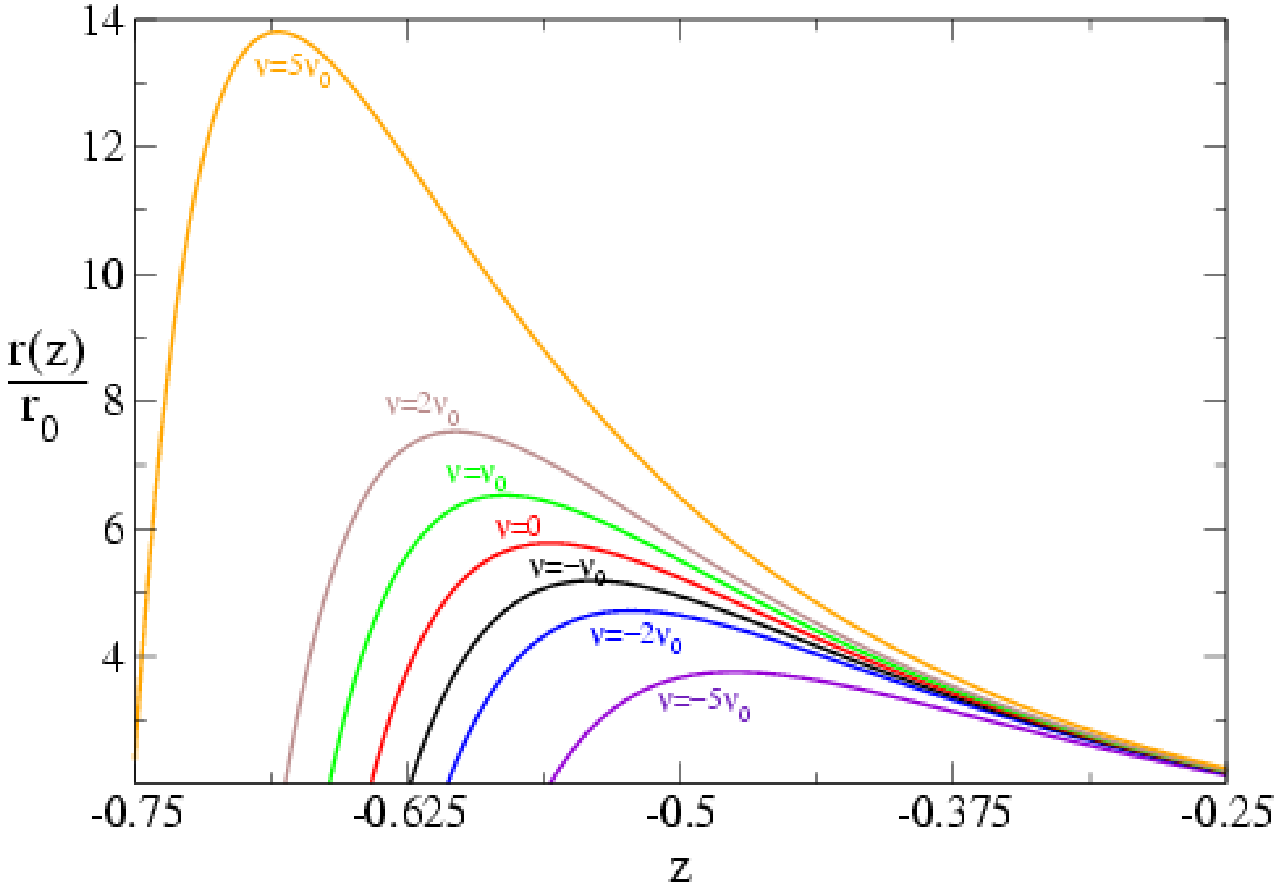
$$(\Omega_K^0 = 0)$$

$$\tilde{\Omega}_D = \frac{r}{1+r}$$

Then $$\tilde{\Omega}_D \lesssim 10\% \iff r \lesssim 10\%$$

$$\epsilon \equiv \nu\,(1 + \omega_X) \lesssim 10\%$$

Evolution of the Ratio $r = \rho_D / \rho_m$

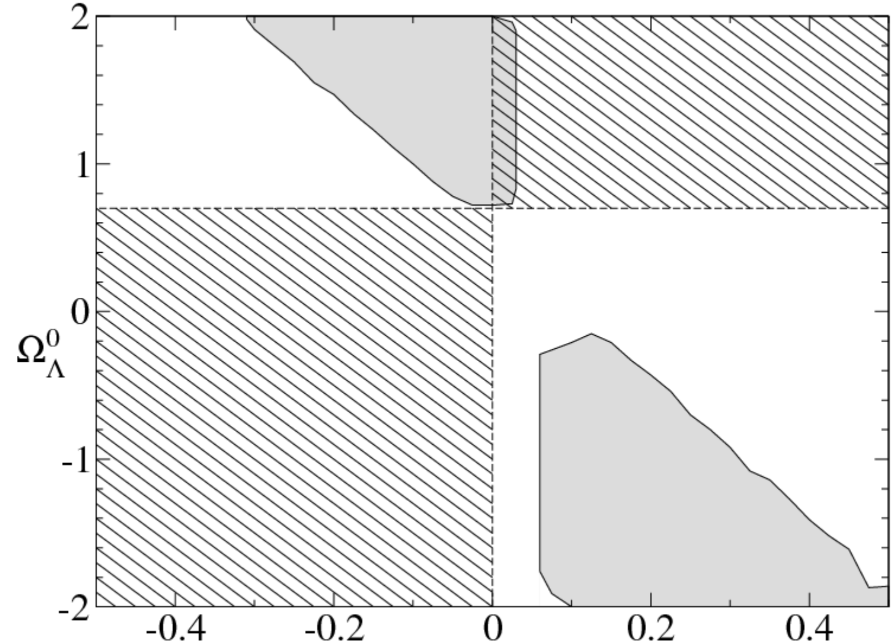


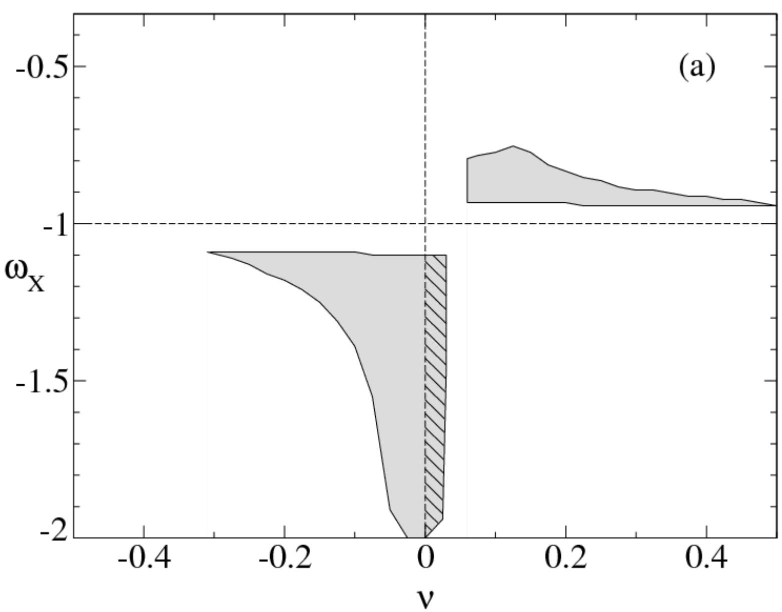
Evolution of r for $\omega_X = -1.85$, $\Omega_\Lambda = 0.75$ and different ν

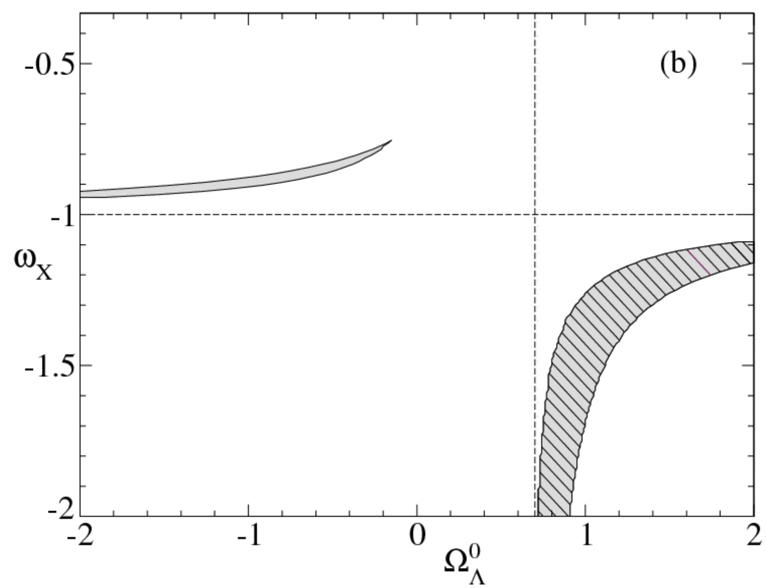
Cosmological perturbations (II)

(including δ_D)

J.Grande, A. Pelinson, J. Solà,
Phys.Rev.D79:043006,2009.







Generic time-varying CC models versus observation

S. Basilakos, M. Plionis, JS,
arXiv:0907.4555 [astro-ph], to appear in PRD

Model types (units $8\pi G = 1$):

1) **Quantum field vacuum (Λ_{RG})**

$$\Lambda(H) = n_0 + n_2 H^2 = \Lambda_0 + 3\gamma\,(H^2 - H_0^2)$$

I.L.Shapiro, JS.
JHEP 0202 (2002) 6
Phys.Lett.B475 (2000) 236.

2) **Power series model (Λ_{PS1})**

$$\Lambda(H) = n_1 H + n_2 H^2$$

S. Basilakos
MNRAS 395 (2009) 2347

3) **Linear model (Λ_{PS2})**

$$\Lambda(H) \propto H$$

R. Schutzhold, PRL 89 (2002) 081302
S. Carneiro et al. (2008), F. Klinkhammer and G.E. Volovik (2009) etc

4) **Quadratic model**

$$\Lambda(H) \propto H^2 \propto \rho_T$$

J.C. Carvalho et al (1992),
R.C. Arcuri and I. Waga (1994) etc.

5) **Power law model (Λ_n)** $\Lambda(H) \propto a^{-n}$

M. Ozer and O. Taha (1987), O. Bertolami (1986), W. Chen and Y.S. Wu (1990)

Geometric probes: standard rulers and standard candles

Standard Rulers: CMB+BAO

$$R = \sqrt{\Omega_m} \int_{a_{ls}}^{1} \frac{da}{a^2 E(a)} \ , \qquad E(a) = \frac{H(a)}{H_0}$$

Angular scale of the sound horizon at the last scattering surface (location first peak)

Direct measurement at $z = z_{CMB} \simeq 1090$

$$A(\mathbf{p}) = \frac{\sqrt{\Omega_m}}{[z_s^2 E(a_s)]^{1/3}} \left[\int_{a_s}^{1} \frac{da}{a^2 E(a)} \right]^{2/3} ,$$

Geometric mean of the visual distortion of an speherical object (due to FLRW geometry) in two orthogonal directions

Indirect measurement of BAO at low $z_s \simeq 0.35$

Standard Candles: SNIa

'Constitution set' of 397 type Ia supernovae of Hicken et al.(2009)

$$\mu = m - M = 5\log d_L + 25 \ , \qquad d_L(a, \mathbf{p}) = \frac{c}{a}\int_a^1 \frac{\mathrm{d}y}{y^2 H(y)}$$

(distance modulus)

(luminosity distance for flat space)

Joint likelihood analysis:

$$\mathcal{L}_{tot}(\mathbf{p}) = \mathcal{L}_{\mathrm{BAO}} \times \mathcal{L}_{\mathrm{cmb}} \times \mathcal{L}_{\mathrm{SNIa}} \ \Leftrightarrow \ \boxed{\chi^2_{tot}(\mathbf{p}) = \chi^2_{\mathrm{BAO}} + \chi^2_{\mathrm{cmb}} + \chi^2_{\mathrm{SNIa}}}$$

$$\chi^2_{\mathrm{cmb}}(\mathbf{p}) = \frac{[R(\mathbf{p}) - 1.71]^2}{0.019^2} \qquad \chi^2_{\mathrm{BAO}}(\mathbf{p}) = \frac{[A(\mathbf{p}) - 0.469]^2}{0.017^2}$$

$$\chi^2_{\mathrm{SNIa}}(\mathbf{p}) = \sum_{i=1}^{351}\left[\frac{\mu^{\mathrm{th}}(a_i, \mathbf{p}) - \mu^{\mathrm{obs}}(a_i)}{\sigma_i}\right]^2$$

Indicator of structure formation: galaxy clustering rate

We know that at distances below the sound the δ_D perturbations are neglile.

J.Grande, A. Pelinson, JS, Phys.Rev.D79:043006,2009.

$$D \equiv \delta\rho_m/\rho_m$$

For $\delta_D \to 0$,

$$\ddot{D} + (2H+Q)\dot{D} - \left[\frac{\rho_m}{2} - 2HQ - \dot{Q}\right] D = Q\,\theta_m \qquad Q(t) \equiv -\dot{\Lambda(t)}/\rho_m(t)$$

where θ_m is the gradient of velocities for the DM particles

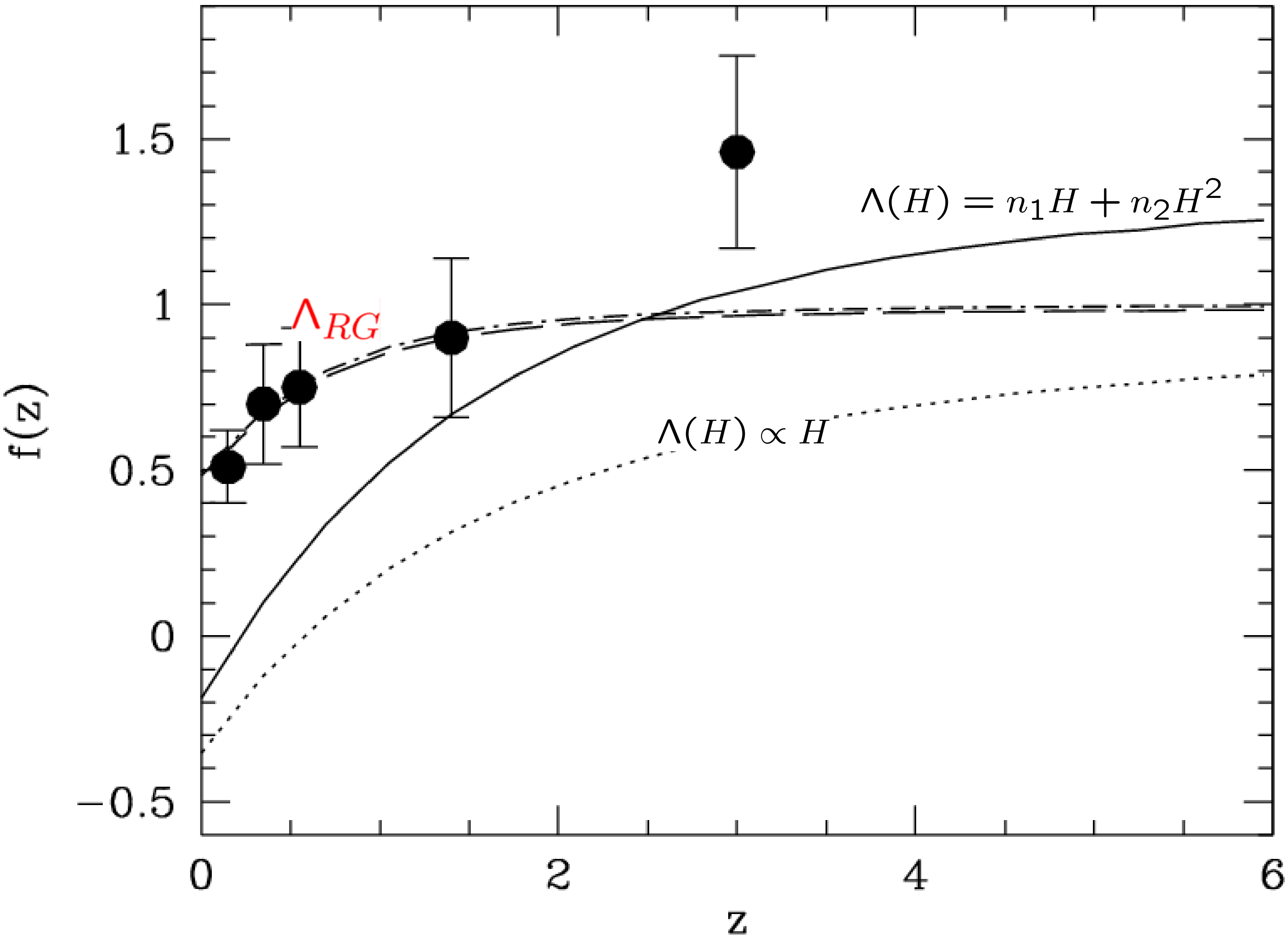
Thus, for negligible DE perturbations and and negligible velocities of the DM particles,

$$\ddot{D} + (2H+Q)\dot{D} - \left[\frac{\rho_m}{2} - 2HQ - \dot{Q}\right] D = 0$$

Clustering rate:

$$f(a) = \frac{d\ln D(a)}{d\ln a} \quad \Leftrightarrow \quad f(z) = -(1+z)\frac{d\ln D(z)}{dz}$$

S. Basilakos, M. Plionis, JS,
arXiv:0907.4555 [astro-ph], to appear in PRD



S. Basilakos, M. Plionis, JS,
arXiv:0907.4555 [astro-ph], to appear in PRD

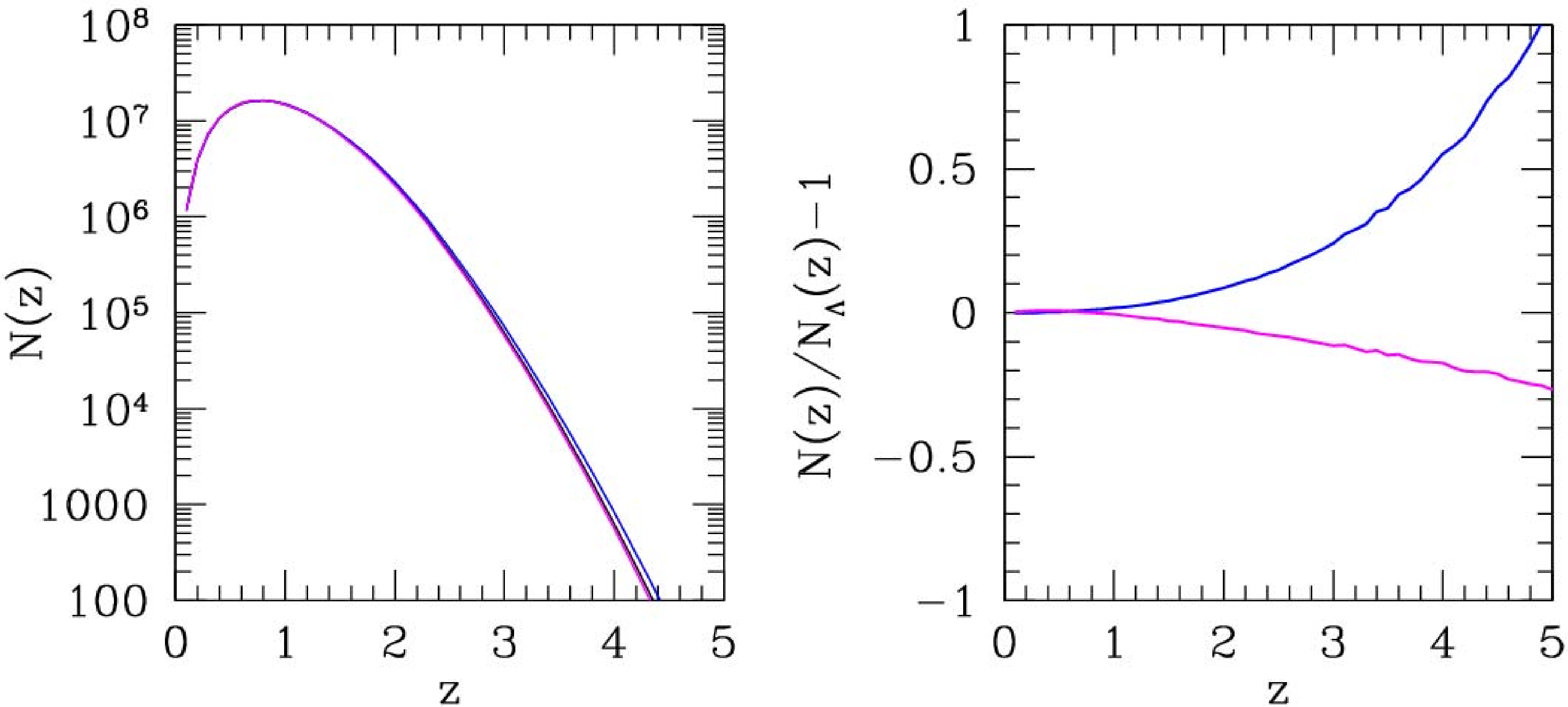


FIG. 4: The expected cluster redshift distribution (left panel) and the corresponding fractional difference (right panel) of the Λ_{RG} (upper blue curve) and Λ_n (lower magenta curve) models with respect to the standard Λ model.

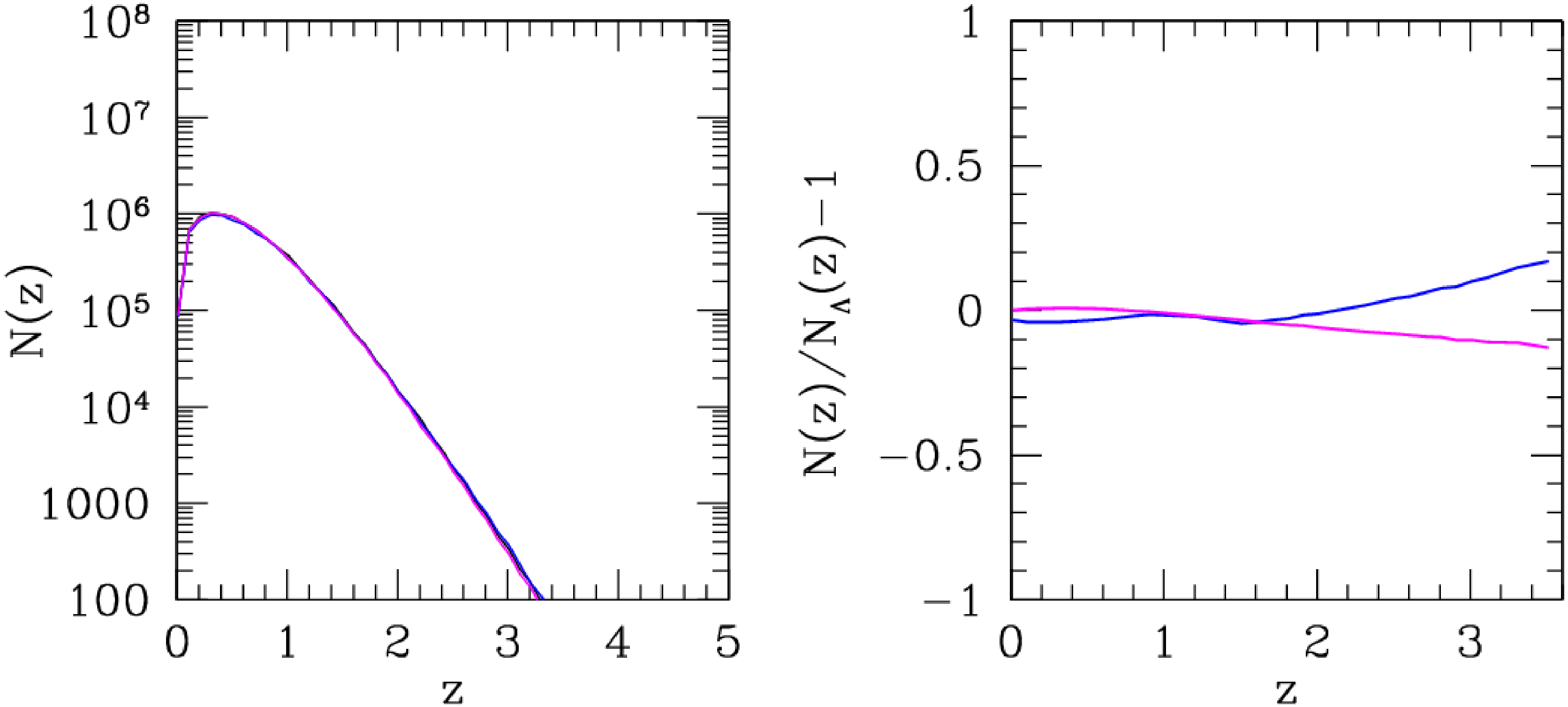


FIG. 6: The expected cluster redshift distribution (left panel) and the corresponding fractional difference (right panel) of the Λ_{RG} (upper blue curve) and Λ_n (lower magenta curve) models with respect to the standard Λ model for the case of a realistic (future) SZ survey with a flux limit of 5 mJy.