The Geometry of Flavor in F-theory GUTs

Jonathan J. Heckman

Based on work with C. Vafa, as well as:

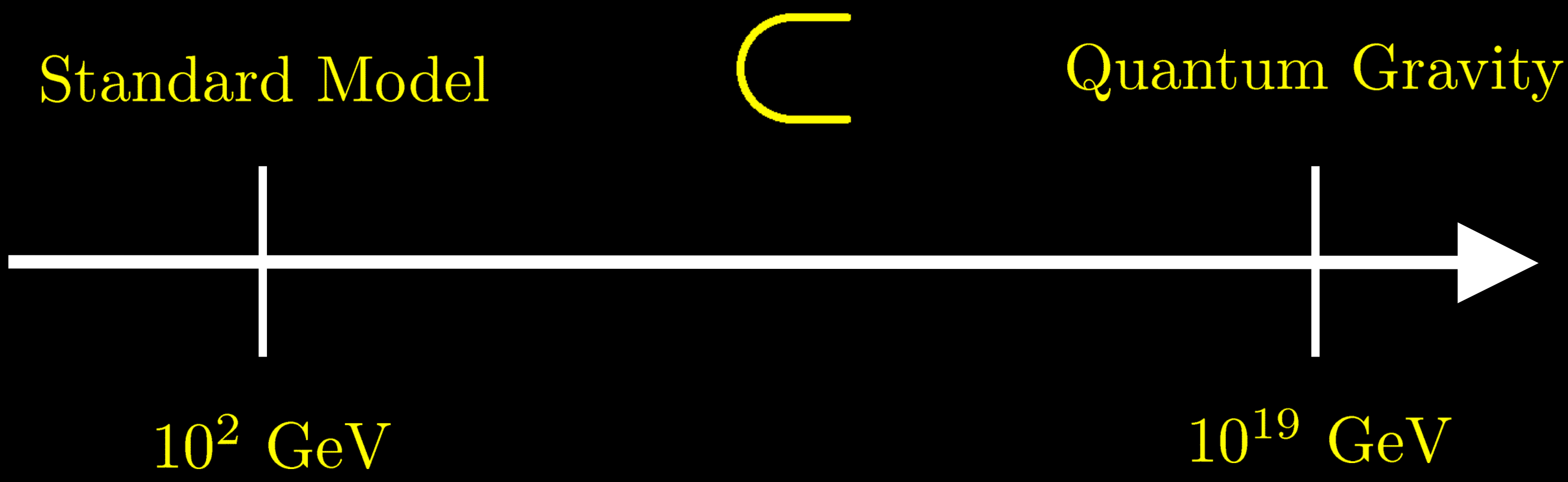
C. Beasley, V. Bouchard, S. Cecotti, M. Cheng, J. Seo, A. Tavanfar

Upcoming Review: arxiv:10??.???? [hep-th]

Outline

- Motivation
- F-theory GUTs
- Quark Models
- Lepton Models
- Conclusions

Motivation



High Energy Constraints $\Rightarrow$ Low Energy Predictions?

Low Energy Observations $\Rightarrow$ High Energy Constraints?

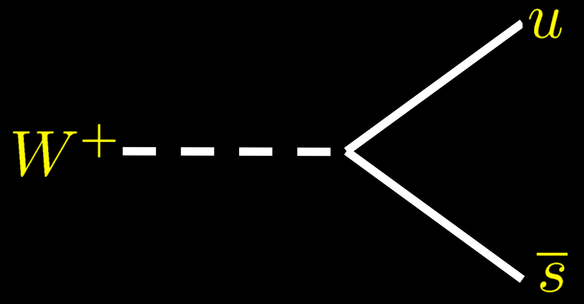
Focus for this talk: Flavor in the Standard Model/MSSM

SM/MSSM Flavor

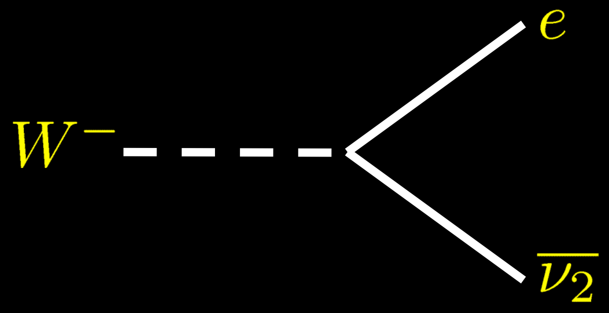
$$L_{eff} \supset m_u^{ij} \cdot U_L^i U_R^j + m_d^{ij} \cdot D_L^i D_R^j + m_l^{ij} \cdot E_L^i E_R^j + m_\nu^{ij} \cdot N_L^i N^j$$

Diagonalize: $V_L \cdot m \cdot V_R^\dagger = \mathrm{diag}(\widetilde{m}_1, \widetilde{m}_2, \widetilde{m}_3)$

$$V_{CKM}^{(quark)} = V_u^L \cdot V_d^{L\dagger}$$

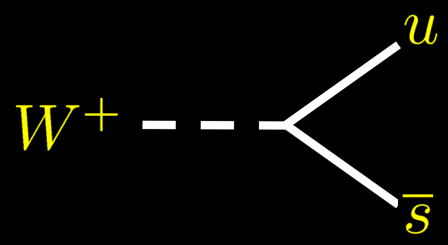


$$V_{PMNS}^{(lepton)} = V_l^L \cdot V_\nu^{L\dagger}$$



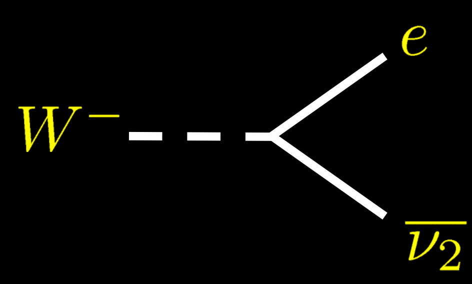
Some Flavor Questions:

Why do quarks mix so little?



$$
\begin{array}{ccc}
 & \begin{array}{ccc} \quad d \quad & \quad s \quad & \quad b \quad \end{array} & \\
|V_{CKM}| \sim & \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{bmatrix} & \begin{array}{c} u \\ c \\ t \end{array}
\end{array}
$$

Why do leptons mix so much?



$$
\left|V_{PMNS}^{obs(3\sigma)}\right| \sim \begin{bmatrix} 0.77 - 0.86 & 0.50 - 0.63 & 0.00 - 0.22 \\ 0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\ 0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \end{bmatrix}
$$

Some More Flavor Questions:

What about quark mass hierarchies?

$$(m_u, m_c, m_t) \sim (0.003 \text{ GeV}, 1.3 \text{ GeV}, 170 \text{ GeV})$$

$$(m_d, m_s, m_b) \sim (0.004 \text{ GeV}, 0.1 \text{ GeV}, 5 \text{ GeV})$$

Why are charged leptons similar but neutrinos so different?

$$(m_e, m_\mu, m_\tau) \sim (0.0005 \text{ GeV}, 0.1 \text{ GeV}, 1.8 \text{ GeV})$$

$$m_\nu \sim 0.05 \text{ eV}$$

+ Strings?

There is an entire landscape of string vacua

Presumably some reproduce the Standard Model

But which ones?

A Strategy:

1) Focus on UV motivated gauge theories

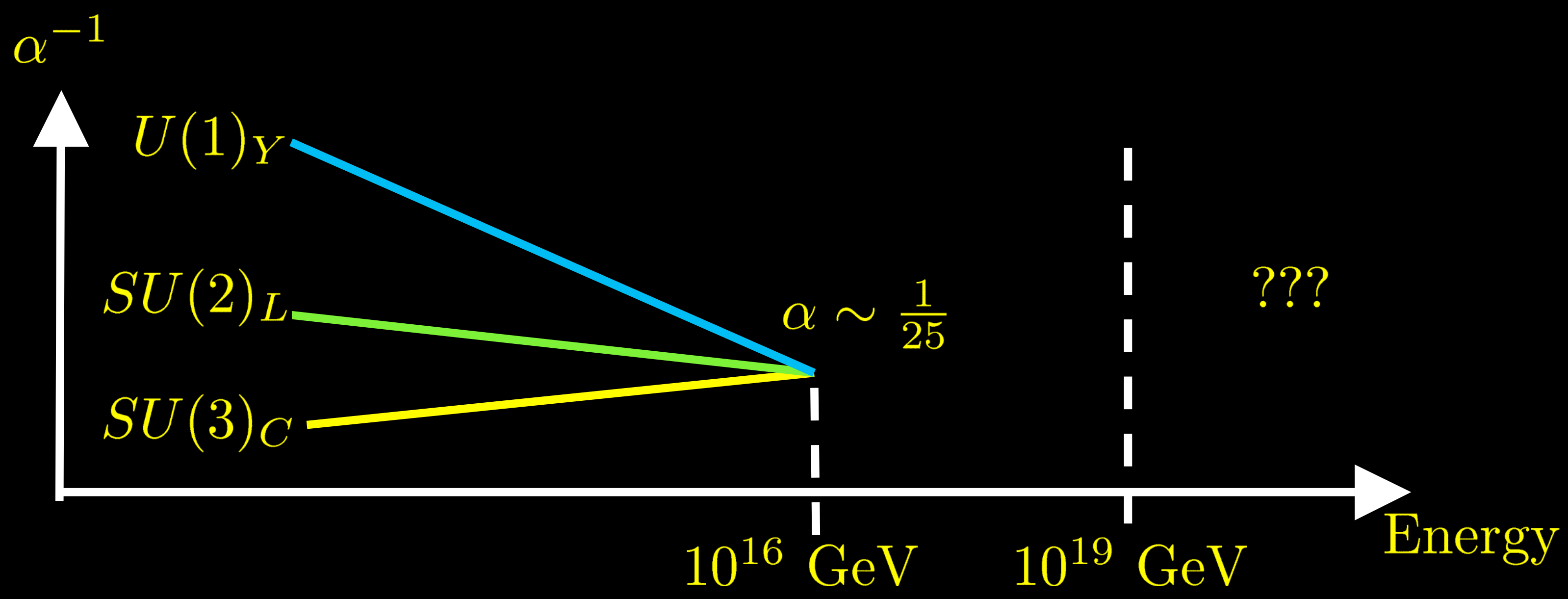
2) Worry about gravity later

UV Motivated Models

String theory predicts supersymmetry

Assume it persists to TeV scale

Supersymmetric Grand Unification:



SUSY GUT Structures

$SU(5)_{GUT} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$

$$10_M = \begin{bmatrix} 0 & U & U & Q & Q \\ -U & 0 & U & Q & Q \\ -U & -U & 0 & Q & Q \\ -Q & -Q & -Q & 0 & E \\ -Q & -Q & -Q & -E & 0 \end{bmatrix} \qquad 5_H = \begin{bmatrix} T_u \\ T_u \\ T_u \\ H_u \\ H_u \end{bmatrix}$$

$$\overline{5}_M = \begin{bmatrix} D & D & D & L & L \end{bmatrix}$$

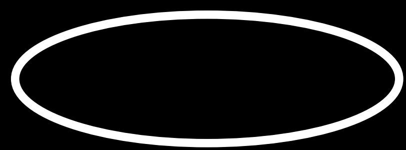
$$\overline{5}_H = \begin{bmatrix} T_d & T_d & T_d & H_d & H_d \end{bmatrix}$$

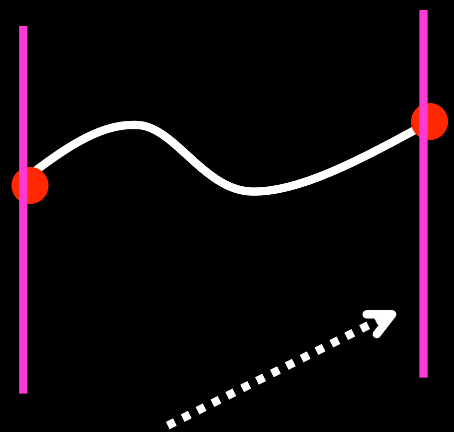
$L_{GUT} \supset 5_H \times 10_M \times 10_M \Rightarrow t$ quark mass

$L_{GUT} \supset \overline{5}_H \times \overline{5}_M \times 10_M \Rightarrow b$ quark & τ lepton mass

Focussing on Particle Physics

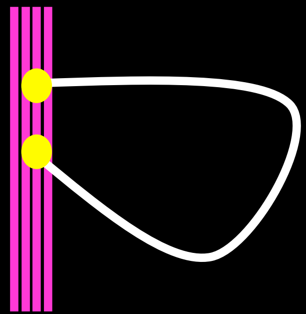
Gravity and Gauge Fields from Different Strings:

Closed Strings:  Spin 2 (gravity)

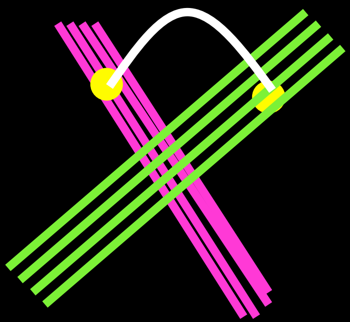
Open Strings:  Spin $0, \frac{1}{2}, 1$ (matter)

Dirichlet Branes

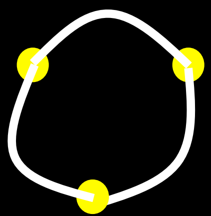
Open String Building Blocks



$\Rightarrow$ Gauge Groups: $U(N), SO(2N), USp(2N)$



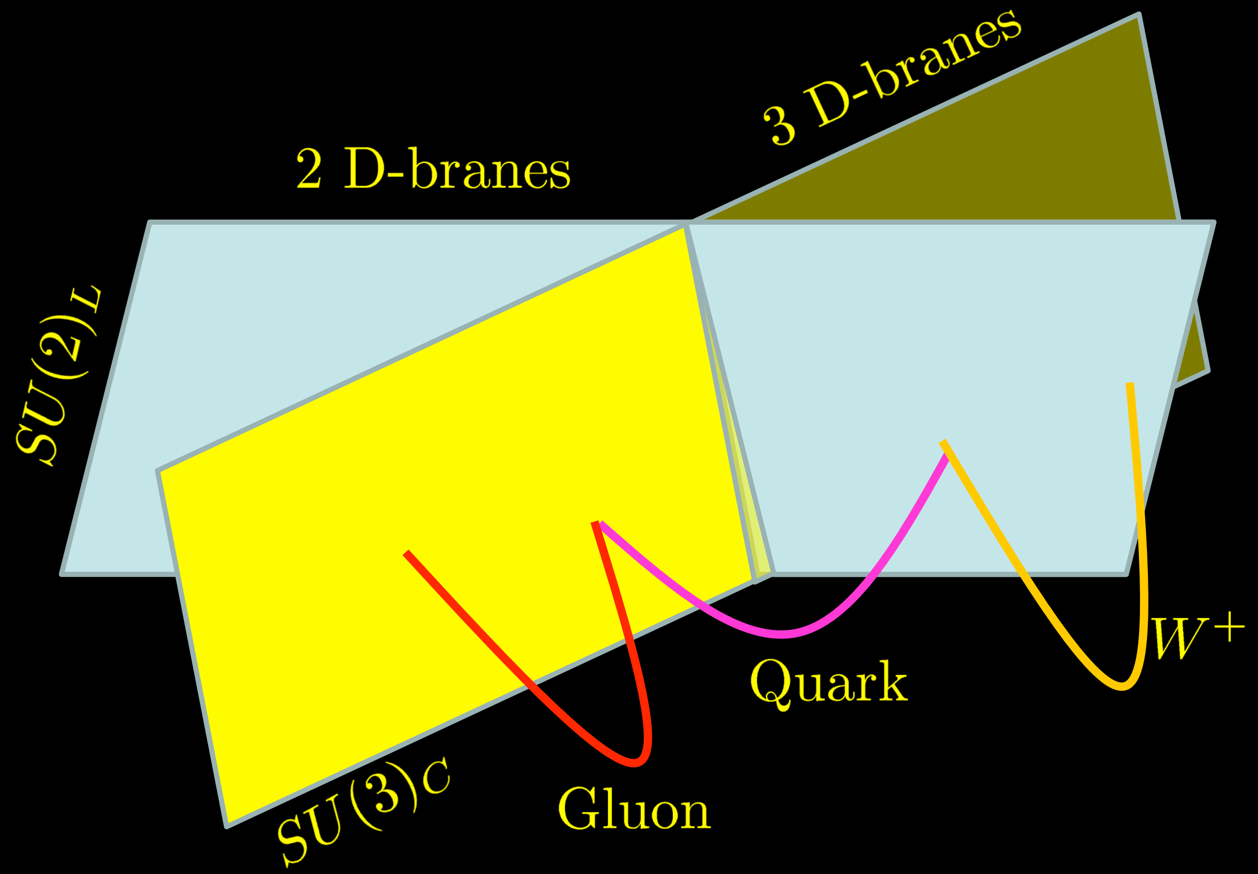
$\Rightarrow$ Matter in $\Box \times \overline{\Box}$ $\quad \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$ $\quad \begin{array}{|c|c|}\hline & \\ \hline \end{array}$



$\Rightarrow$ Interactions Link $\Box$ and $\overline{\Box}$

Qualitative Features

Aldazabal Ibanez
Quevedo Uranga '00
+ many others

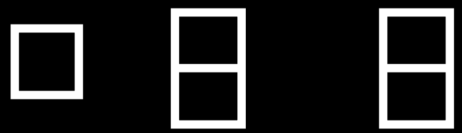


Can this be combined with Grand Unification?

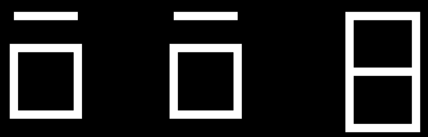
GUTs and Open Strings

Open strings for gauge theory $\Rightarrow$ Problems with GUTs:

No $5_H \times 10_M \times 10_M \Rightarrow$ pert. massless t quark



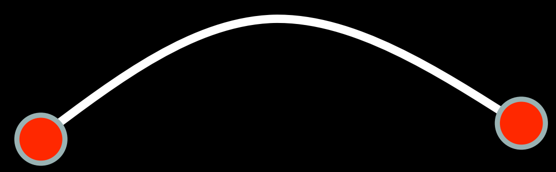
But $\bar{5}_H \times \bar{5}_M \times 10_M \Rightarrow$ massive b quark



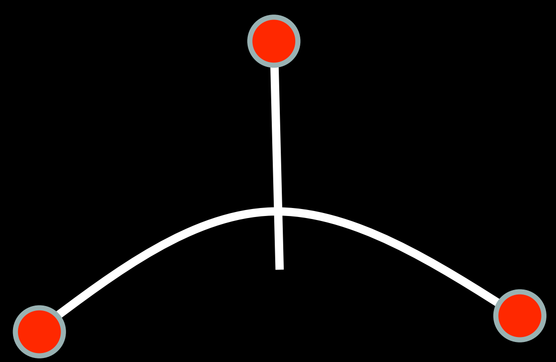
Wrong Prediction: $m_b > m_t$

The Main Idea:

Perturbative open strings somewhat limited



Increasing $g_s \rightarrow O(1)$ allows new bound states



Roadmap

- Motivation

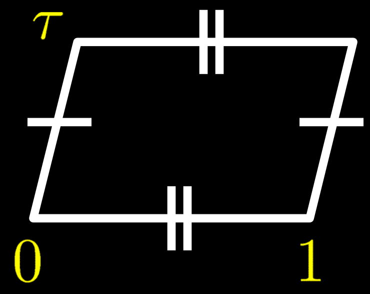
- F-theory GUTs

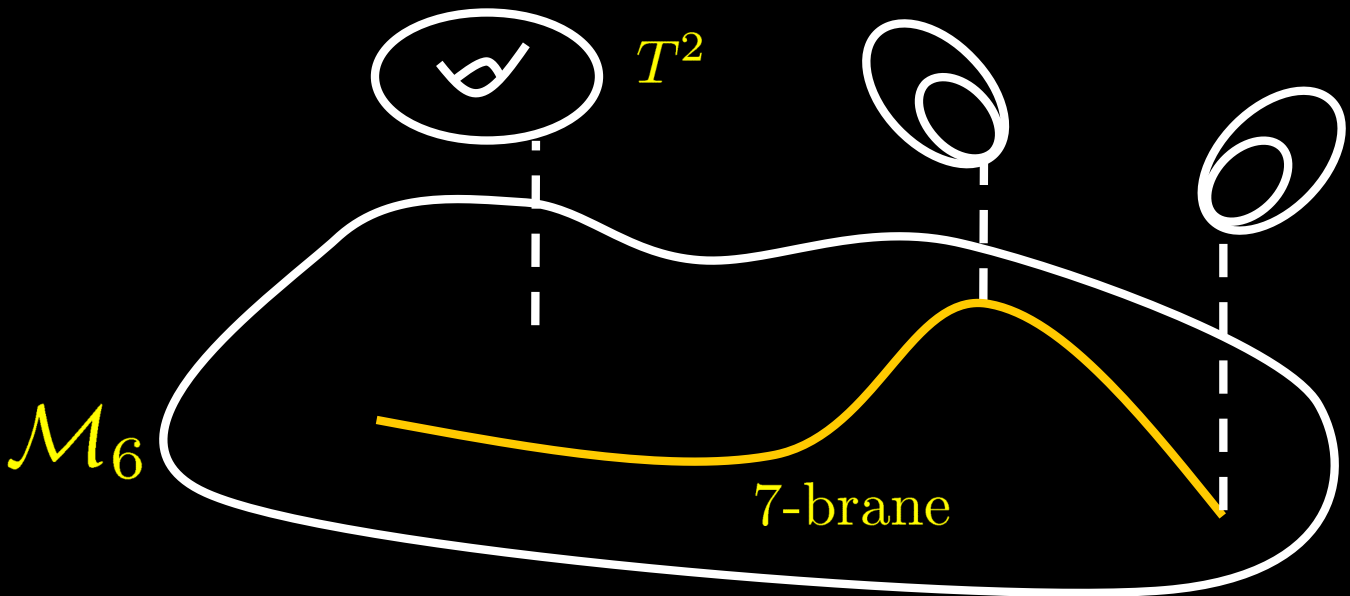
F-theory Review

Vafa '96

F-theory = Strongly Coupled Formulation of IIB in 12d

$\tau(y_6) = C_0 + \frac{i}{g_s}$ is shape of a T^2:





Terminology: p-brane = extended object in p spatial directions

$g_s \sim O(1) \Rightarrow$ Extra Ingredients

Gauge Groups: $\underbrace{SU(N), SO(N), USp(2N)}_{g_s \ll 1}, \underbrace{E_6, E_7, E_8, G_2, F_4}_{g_s \sim O(1)}$

Matter: $\underbrace{5, 10 \text{ of } SU(5)}_{g_s \ll 1}, \underbrace{16 \text{ of } SO(10), 27 \text{ of } E_6}_{g_s \sim O(1)}$

Interactions: $\underbrace{\overline{5} \times \overline{5} \times 10 \text{ of } SU(5)}_{g_s \ll 1}, \underbrace{5 \times 10 \times 10 \text{ of } SU(5)}_{g_s \sim O(1)}$

$\cap$ 7-branes

	$R^{3,1}$				$\mathcal{M}_6$					
	0	1	2	3	4	5	6	7	8	9
7_{GUT}	×	×	×	×	×	×	×	×		
$7'$	×	×	×	×	×	×			×	×
$7''$	×	×	×	×			×	×	×	×

$10D$: Gravity

$8D$: Gauge Group (7)

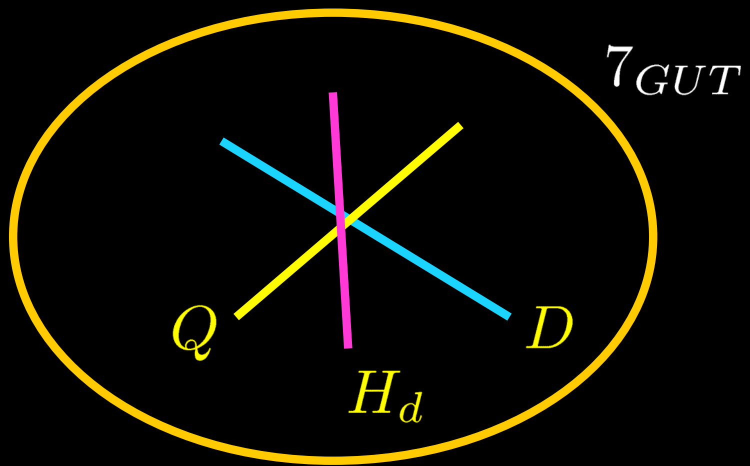
$6D$: Matter $(7 \cap 7')$

$4D$: Yukawas $(7 \cap 7' \cap 7'')$

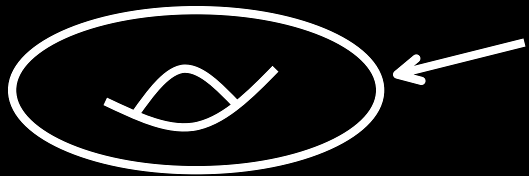
Example: Quarks

					Q		D			
	0	1	2	3	4	5	6	7	8	9
7_{GUT}	×	×	×	×	×	×	×	×		
$7'_{\perp}$	×	×	×	×	×	×			×	×
$7''_{\perp}$	×	×	×	×			×	×	×	×

View from 7_{GUT}:



Getting Chiral Matter

6d Matter: $\mathbb{R}^{3,1}\times$  + gauge field flux on Σ

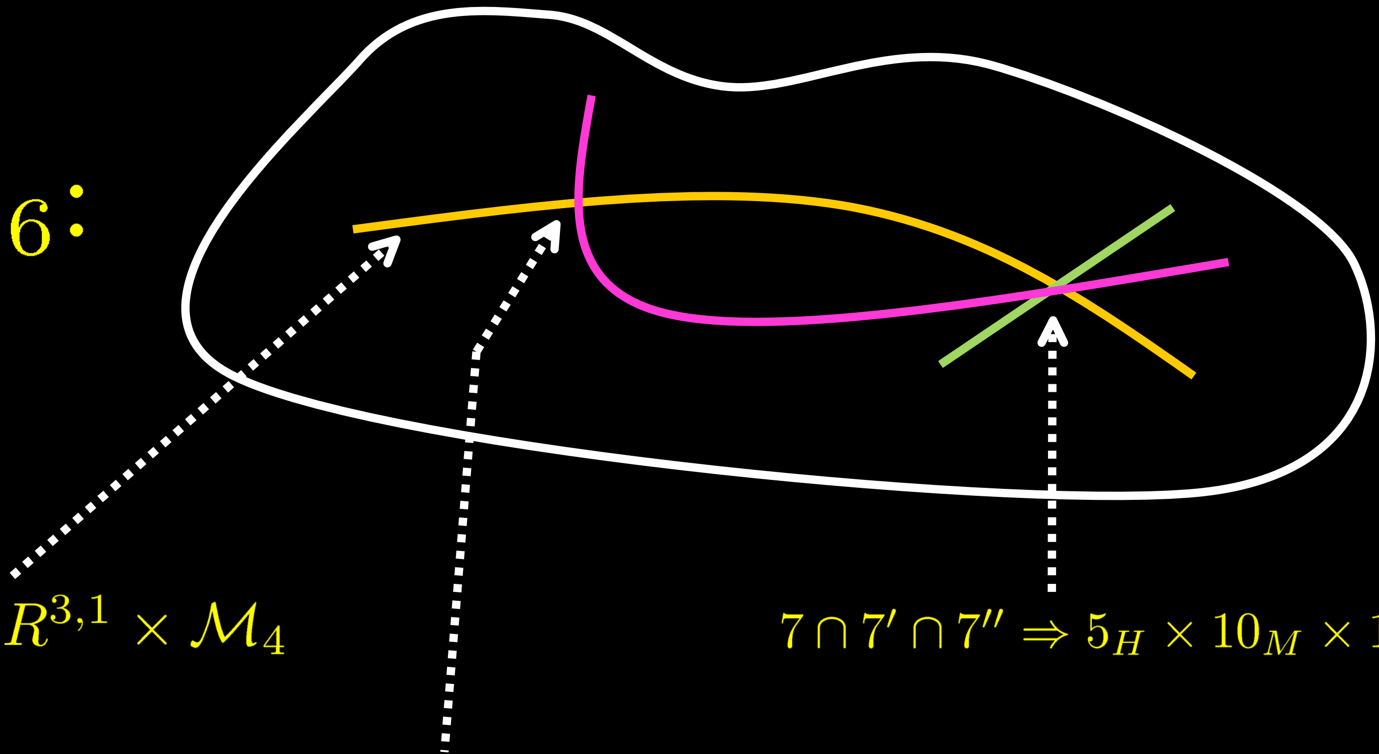
$$(\not{D}_{\mathbb{R}^{3,1}} + \not{D}_{\Sigma})\Psi_{6d} = 0 : \text{Massless modes} \iff \not{D}_{\Sigma}\Psi_{(0)} = 0$$

$$\#\ \text{Generations} = \frac{1}{2\pi}\int_{\Sigma} F$$

F-theory GUTs

Beasley JJH Vafa I II '08, Donagi Wijnholt I II '08
(see also Hayashi et al. '08 '09)

$\mathcal{M}_6$:



7_{GUT} on $R^{3,1} \times \mathcal{M}_4$

$7 \cap 7' \cap 7'' \Rightarrow 5_H \times 10_M \times 10_M \ldots$

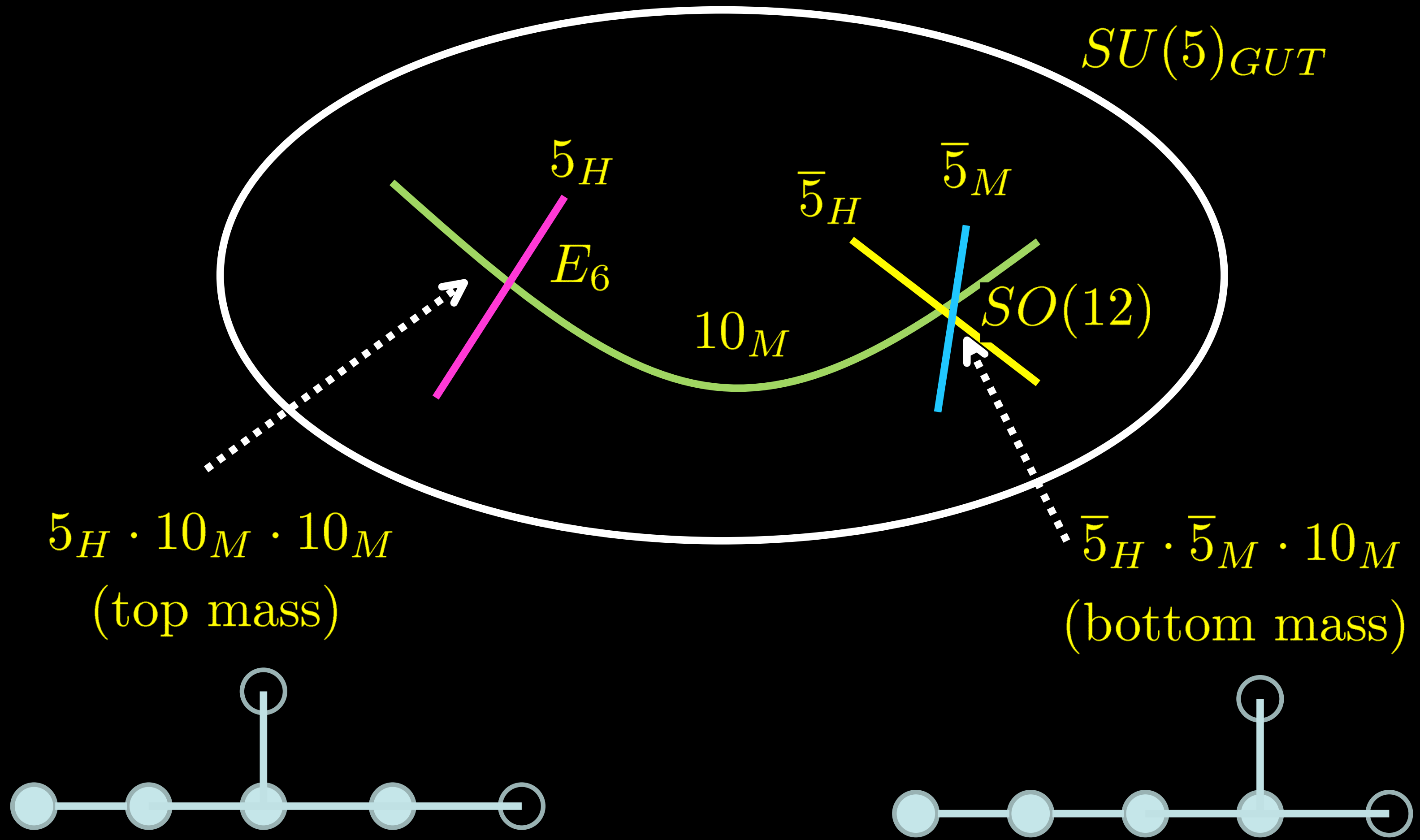
$7 \cap 7' \Rightarrow \overline{5}, 10 \in SU(5),\ 16 \in SO(10) \ldots$

Roadmap

- F-theory GUTs
- Quark Models

Minimal Ingredients

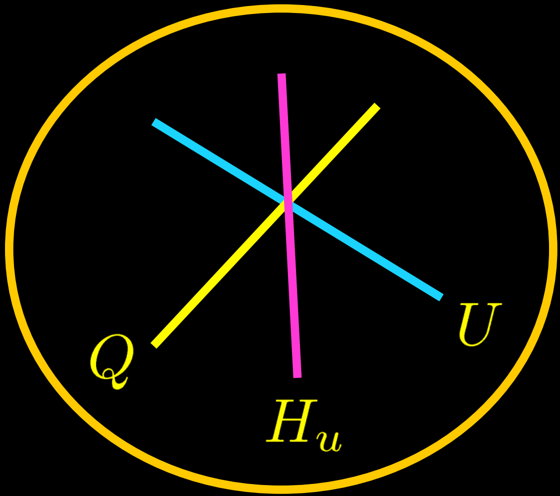
On 7_{GUT} worldvolume need (at least):



Quark Yukawas:

$R^{3,1}$: $W \supset \lambda_u^{ij} \cdot Q^i U^j H_u + \cdots$

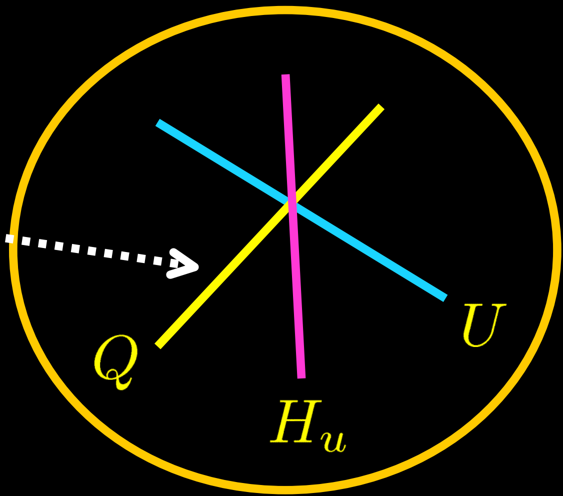
$\mathcal{M}_6$: $\overline{\mathcal{D}}\Psi = 0$: $\Psi_Q^i,\ \Psi_U^i,\ \Psi_{H_u},\ \cdots$



Overlap Integral: $\lambda_u^{ij} = \int \Psi_{H_u} \Psi_Q^i \Psi_U^j$

Geometry $\Rightarrow$ One Heavy Gen

Ψ_Q has sharp falloff off of curve



$$\Downarrow$$

$$\lambda_u^{ij} = \Psi_Q^i(p)\Psi_U^j(p)\Psi_{H_u}(p) + \cdots$$

See Beasley JJH Vafa II ‘08
And Hayashi et al. ‘09

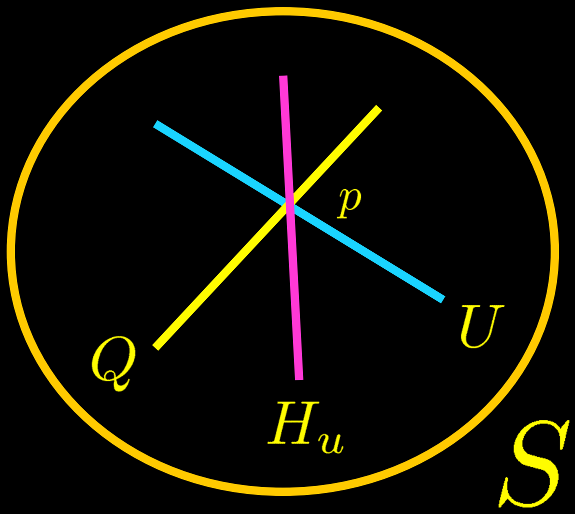
(outer product)

$$\begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} = \begin{bmatrix} 0 & & \\ & 0 & \\ & & m \end{bmatrix}$$

Minimality

Adding more points leads to higher rank:

$$\lambda_u^{ij} = \sum_p \Psi_Q^i(p)\Psi_U^j(p)\Psi_{H_u}(p) + \cdots$$



$\boxed{\text{One heavy generation} \Rightarrow \#\ \text{p} = 1}$

In principle can tune $p_i \to p$ to maintain nearly rank one

Getting Hierarchies

Geometry $\Rightarrow$ Rank 1; Hierarchy = Higher order corrections:

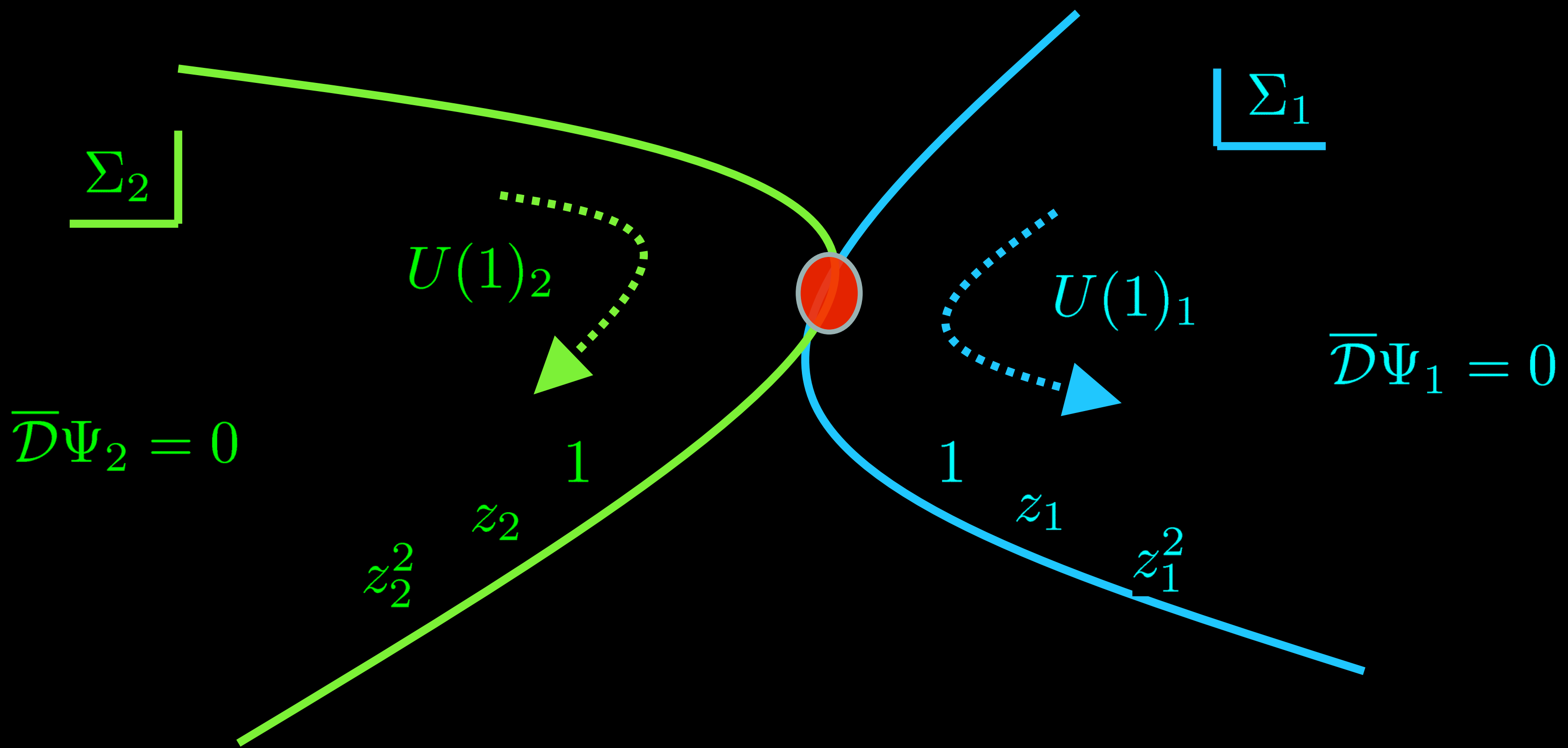
Hierarchy = Higher order corrections:

$$\lambda \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \varepsilon^i \\ 0 & \varepsilon^i & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \varepsilon^j \\ 0 & \varepsilon^j & \varepsilon^i \\ \varepsilon^j & \varepsilon^i & 1 \end{bmatrix} + \cdots$$

Main Idea

$\Psi^i \sim z^i$ exhibits rephasing symmetry

JJH Vafa '08
see also Froggatt Nielsen '79



Fluxes violate internal Lorentz symmetry $\Rightarrow$ hierarchical corrections

Which Fluxes?

Geometry $\Rightarrow$ Rank 1

Cecotti, Cheng, JJH, Vafa ’09

Flux $\Rightarrow$ Rank 3:

Available gauge potentials: $A_I \; B_{IJ}, \ldots$

$\Rightarrow$ Fluxes: F_{IJ}, H_{IJK}

F_{IJ} alone does nothing to Yukawas

CCHV (see also Font Ibanez ‘09
And Conlon Palti ‘09)

But $F'_{IJ} = F_{IJ} + B_{IJ}$ does distort Yukawas

CCHV

H-flux & Yukawas

Crude estimates suggest two structures: JJH, Vafa '08

$$\lambda(\partial^N \text{Flux}) \sim \begin{bmatrix} \varepsilon^5 & \varepsilon^4 & \varepsilon^3 \\ \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \;\&\; \lambda((\partial\ \text{Flux})^N) \sim \begin{bmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{bmatrix}$$

$$\varepsilon^2 \sim \text{Flux}^2/M_*^4 \sim \alpha_{GUT} \sim 1/25$$

Explicit computations in toy models corroborate much of this

Compute H-flux in terms of Non-Commutative Geometry:

Cecotti, Cheng, JJH, Vafa '09

$$x * y - y * x = \theta(x, y)$$

See also Marchesano Martucci '09

Quark Masses

Crude estimates suggest $\sqrt{\alpha_{GUT}} \sim 0.2$ which is close:

One parameter fit of up masses to $\lambda((\partial \text{ Flux})^N)$:

$$(m_u, m_c, m_t) \sim (\varepsilon_U^8, \varepsilon_U^4, 1) \cdot m_t \Rightarrow \varepsilon_U \sim 0.26$$

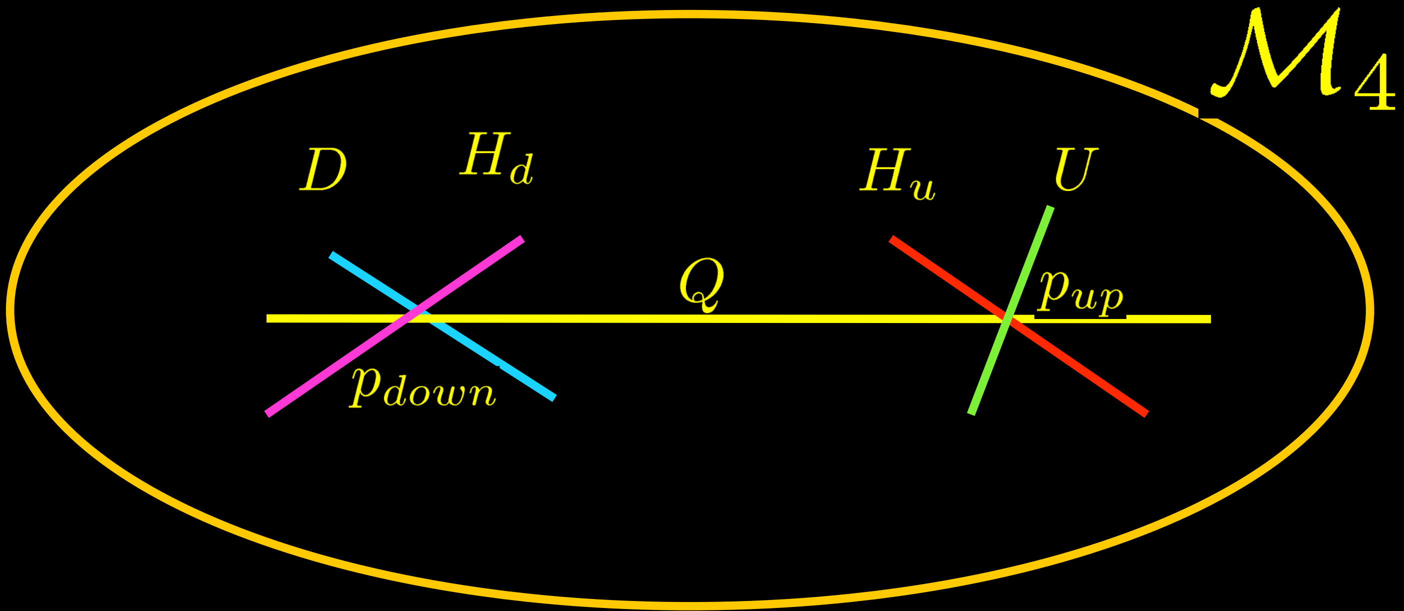
$$\left(m_u^{F-th}, m_c^{F-th}, m_t^{F-th}\right) \sim (0.004, 0.8, 170) \text{ GeV}$$

$$\left(m_u^{obsrv}, m_c^{obsrv}, m_t^{obsrv}\right) \sim (0.003, 1.3, 170) \text{ GeV}$$

Down quarks similar (fitting to $\lambda(\partial^N \text{Flux})$)

Quark Mixing

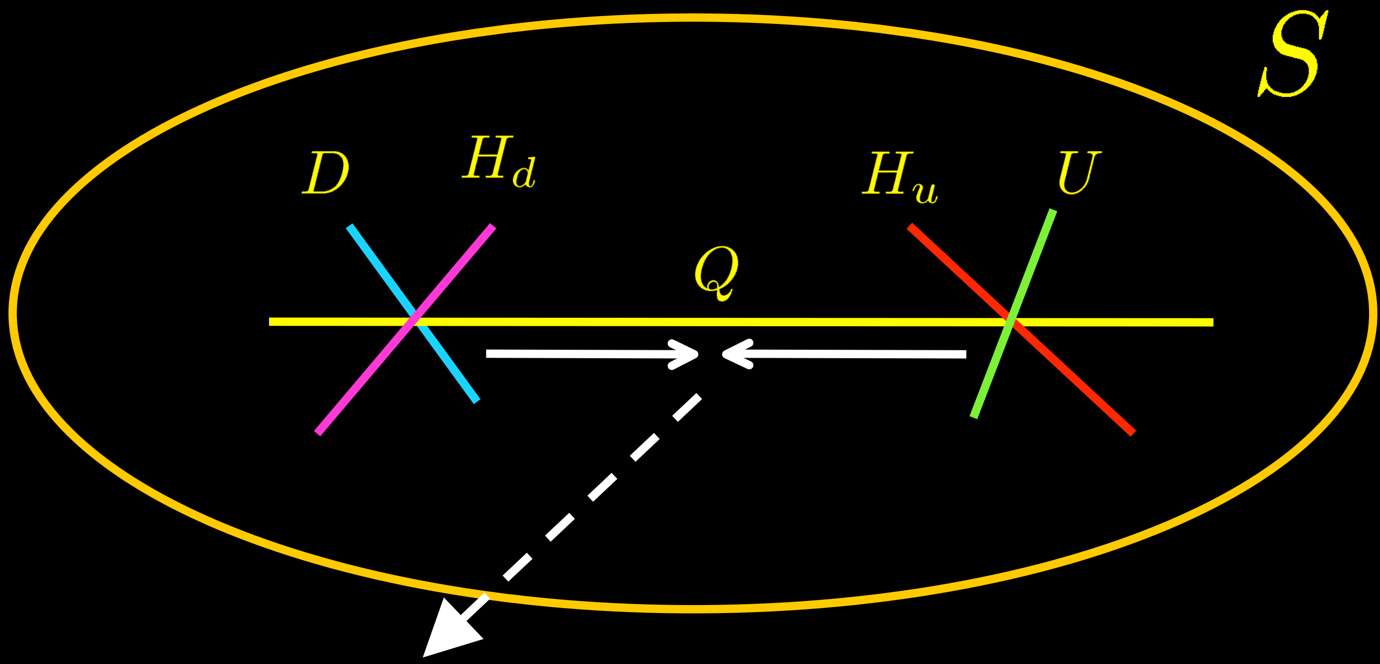
Mixing is more subtle; determined by local Ψ overlaps



Problem: $\Psi^{Q}_{near\ p_{down}} \neq \Psi^{Q}_{near\ p_{up}} \Rightarrow O(1)$ Mixing

$$p_{down} \longrightarrow p_{up}$$

Solution: $\Psi^{Q}_{near\ p_{down}} \longrightarrow \Psi^{Q}_{near\ p_{up}}$



$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix}$$

$E_6 : 5_H 10_M 10_M \rightarrow E_7$

$SO(12) : \bar{5}_H \bar{5}_M 10_M \rightarrow E_7$

Numerology

$$|V_{CKM}^{F-th}| \sim \begin{bmatrix} 1 & \alpha_{GUT}^{1/2} & \alpha_{GUT}^{3/2} \\ \alpha_{GUT}^{1/2} & 1 & \alpha_{GUT} \\ \alpha_{GUT}^{3/2} & \alpha_{GUT} & 1 \end{bmatrix}$$

$$\Downarrow$$

$$|V_{CKM}^{F-th}| \sim \begin{bmatrix} 1 & 0.2 & 0.008 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{bmatrix}$$

$$|V_{CKM}^{obs}| \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.99 \end{bmatrix}$$

Roadmap

- Quark Models
- Lepton Models

Charged Leptons vs Neutrinos

$L \subset \overline{5}_M$ and $E \subset 10_M \Rightarrow$ Similar to ups and downs

What about neutrinos?

Majorana and Dirac can both have $m_\nu \sim M_{weak}^2/\Lambda_{UV}$:

Majorana: $\int d^2\theta \frac{(H_u L)^2}{\Lambda_{UV}} \longrightarrow m_\nu N_L N_L$

$$\langle H \rangle \sim M_w + M_w^2 \theta^2$$

Dirac: $\int d^4\theta \frac{H_d^\dagger L N_R}{\Lambda_{UV}} \longrightarrow m_\nu N_L N_R$

Majorana Scenarios

$L_{eff} \supset \int d^2\theta \frac{(H_u L)^2}{\Lambda_{UV}}$ from $L \supset \int d^2\theta H_u L N_R + \Lambda_{UV} N_R N_R$

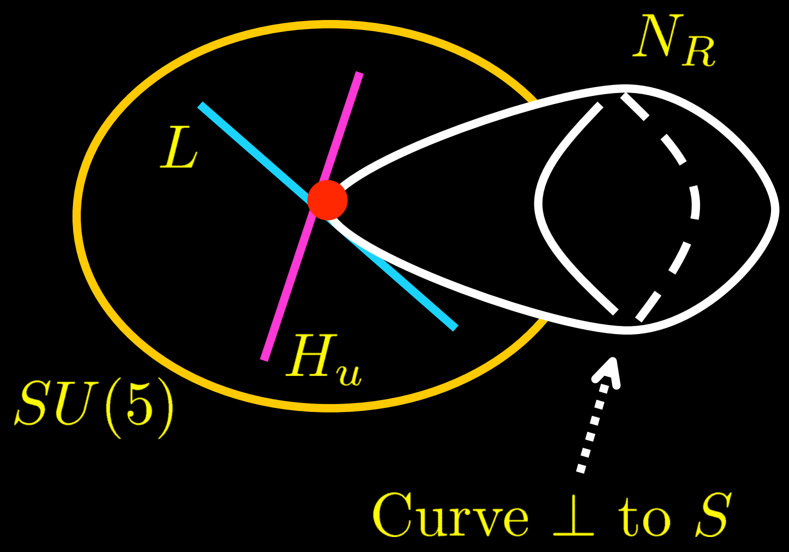
N_R are heavy $SU(5)_{GUT}$ singlets given by:

1) Moduli: Λ_{UV} set by flux

Tatar, Tsuchiya, Watari '09

2) KK modes: $\Lambda_{UV} \sim 10^{15}$ GeV

Bouchard, JJH, Seo Vafa '09

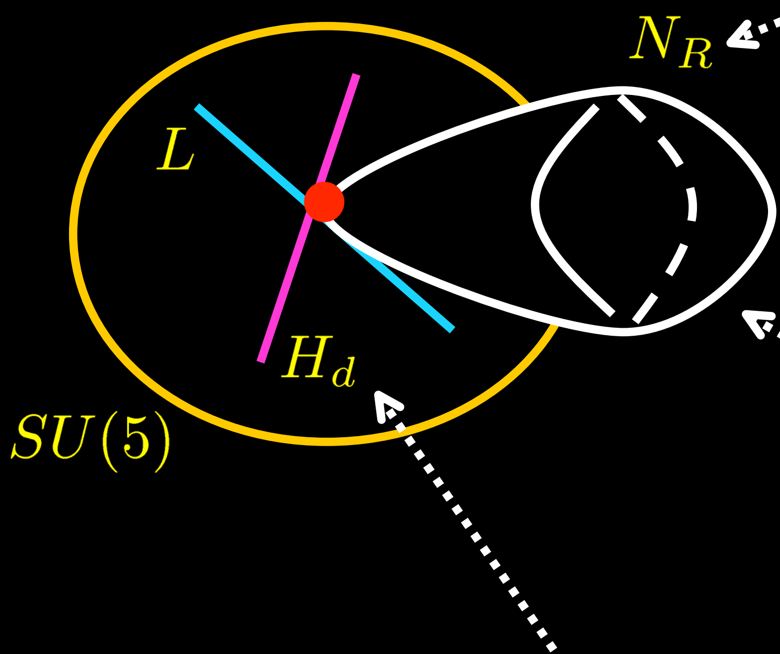


Dirac Scenario

$L_{eff} \supset \int d^4\theta \frac{H_d^\dagger L N_R}{\Lambda_{UV}}$ is equally natural:

N_R now a zero mode

Bouchard, JJH, Seo Vafa '09



Curve $\perp$ to S

Majorana has H_u instead of H_d

Heavy States & $U(1)$

Integrating out Heavy States $\Rightarrow$ Neutrinos Light

Bouchard, JJH, Seo Vafa '09

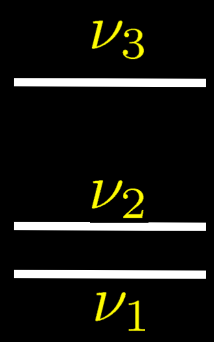
$\overline{\mathcal{D}}\Psi_{HEAVY} \neq 0 \Rightarrow$ Bigger $U(1)$ Violation

$\boxed{\Rightarrow \text{Less Hierarchy:}}$

$$\lambda_\nu \sim \begin{bmatrix} \varepsilon_N^2 & \varepsilon_N^{3/2} & \varepsilon_N^1 \\ \varepsilon_N^{3/2} & \varepsilon_N^1 & \varepsilon_N^{1/2} \\ \varepsilon_N^1 & \varepsilon_N^{1/2} & 1 \end{bmatrix} \qquad T_L \cdot \lambda_l \cdot T_R^\dagger \sim \begin{bmatrix} \varepsilon_L^8 & \varepsilon_L^6 & \varepsilon_L^4 \\ \varepsilon_L^6 & \varepsilon_L^4 & \varepsilon_L^2 \\ \varepsilon_L^4 & \varepsilon_L^2 & 1 \end{bmatrix}$$

ν Masses

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \varepsilon_N^2 : \varepsilon_N : 1 \Rightarrow$$



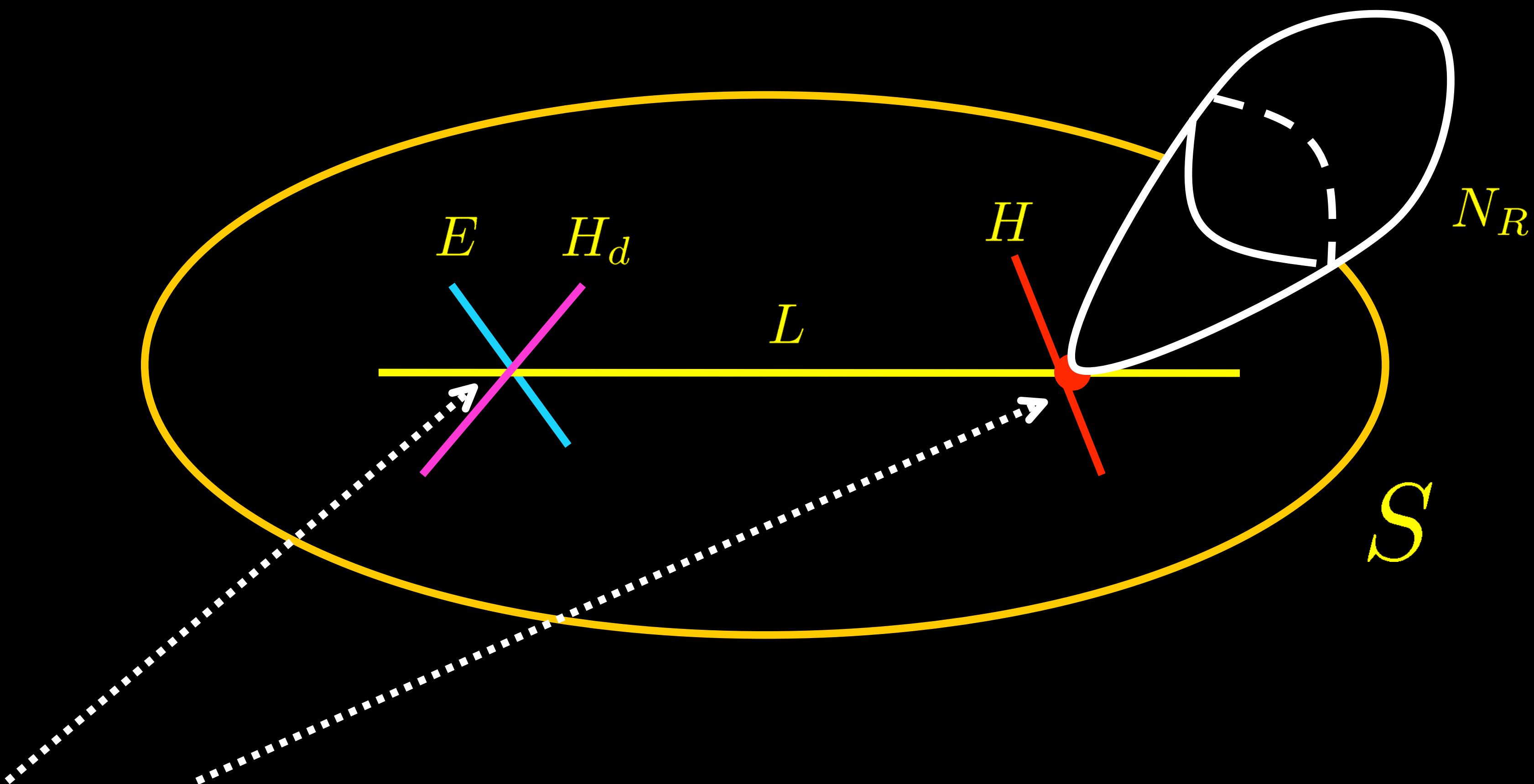
"Normal Hierarchy"

Predict: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} \sim \alpha_{GUT} \sim 0.04$

Close!

Observe: $\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} = \frac{m_{sol}^2}{m_{atm}^2} \sim 0.03$

Neutrino Mixing

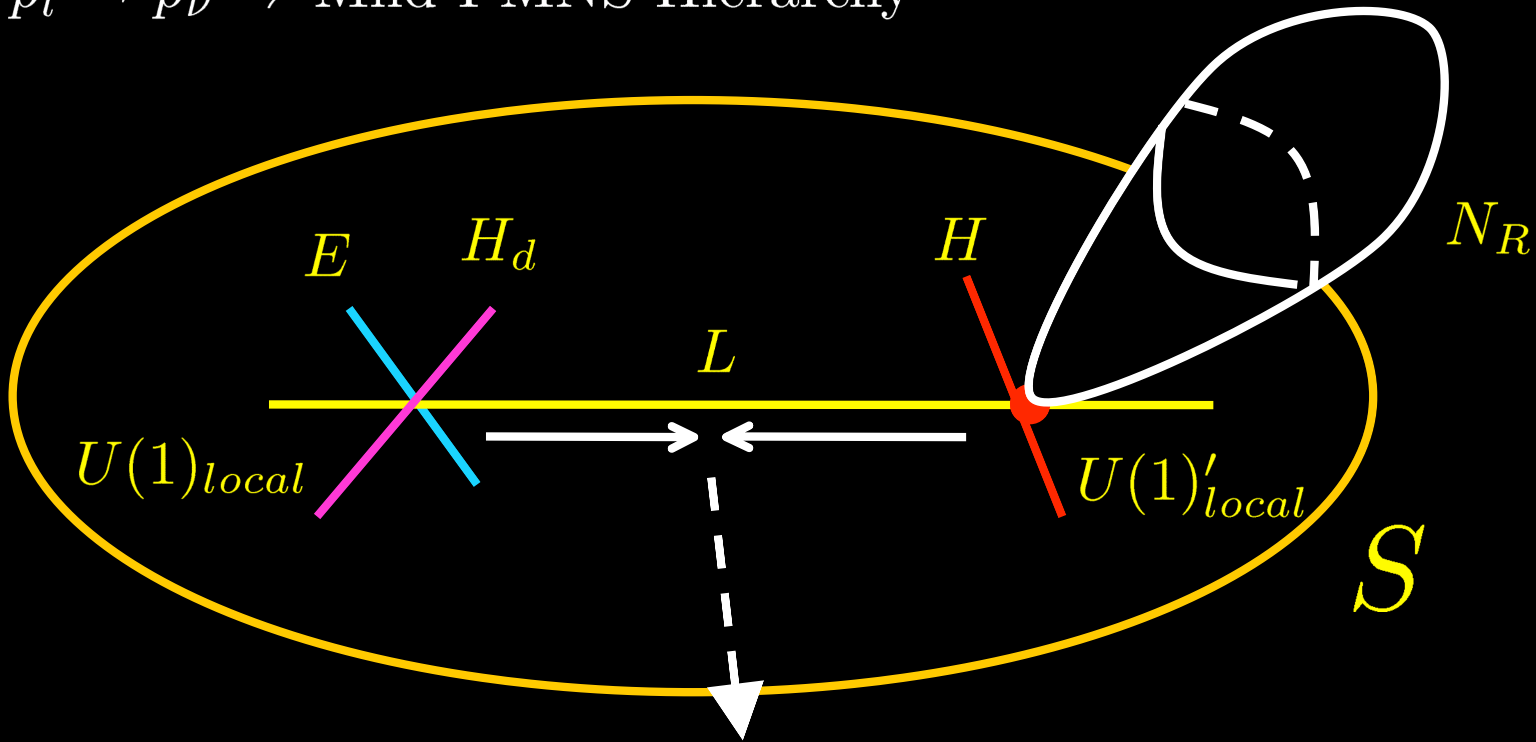


p_l and p_ν Far Apart $\Rightarrow$ Misaligned Eigenbases $\Rightarrow$ Large Mixing

$\boxed{\text{But tension with } V_{PMNS}^{1,3} < 0.2}$

ν Mixing Hierarchy

$p_l \longrightarrow p_\nu \Rightarrow$ Mild PMNS Hierarchy



$$|V_{PMNS}| \sim \begin{bmatrix} U_{e1} & \varepsilon^{1/2} & \varepsilon \\ \varepsilon^{1/2} & U_{\mu 2} & \varepsilon^{1/2} \\ \varepsilon & \varepsilon^{1/2} & U_{\mu 3} \end{bmatrix}$$

PMNS Matrix

Bouchard, JJH, Seo Vafa '09

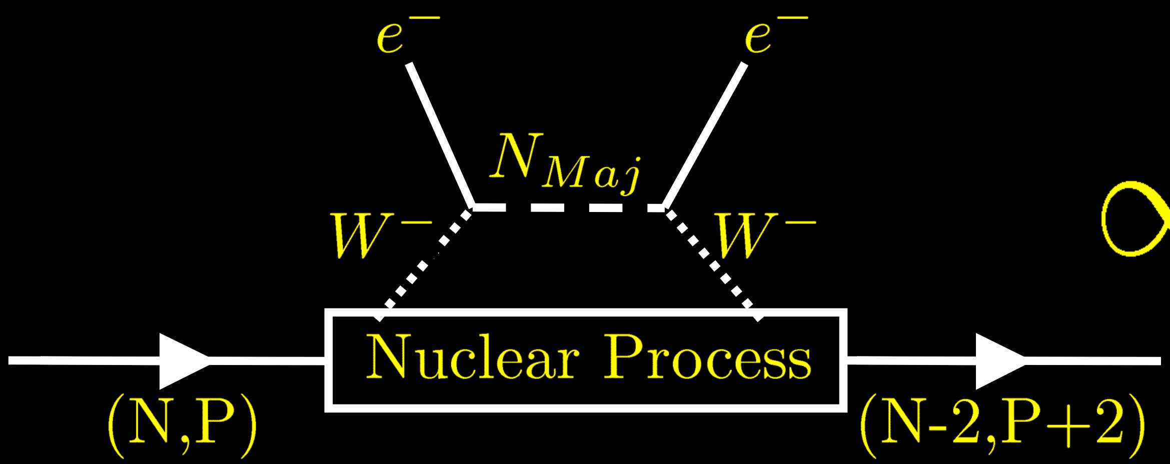
$$\begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ |V_{PMNS}^{F-th}| \sim & \left[\begin{array}{ccc} 0.87 & 0.45 & \boxed{0.2} \\ 0.45 & 0.77 & 0.45 \\ 0.2 & 0.45 & 0.87 \end{array}\right] \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$

$$|V_{PMNS}^{obs(3\sigma)}| \sim \left[\begin{array}{ccc} 0.77-0.86 & 0.50-0.63 & \boxed{0.00-0.22} \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.80 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{array}\right]$$

$\boxed{\Rightarrow \text{Predict } V_{PMNS}^{1,3} \text{ close to current bound}}$

Dirac or Majorana?

ν-less $\beta\beta$ decay for Majorana Neutrinos:



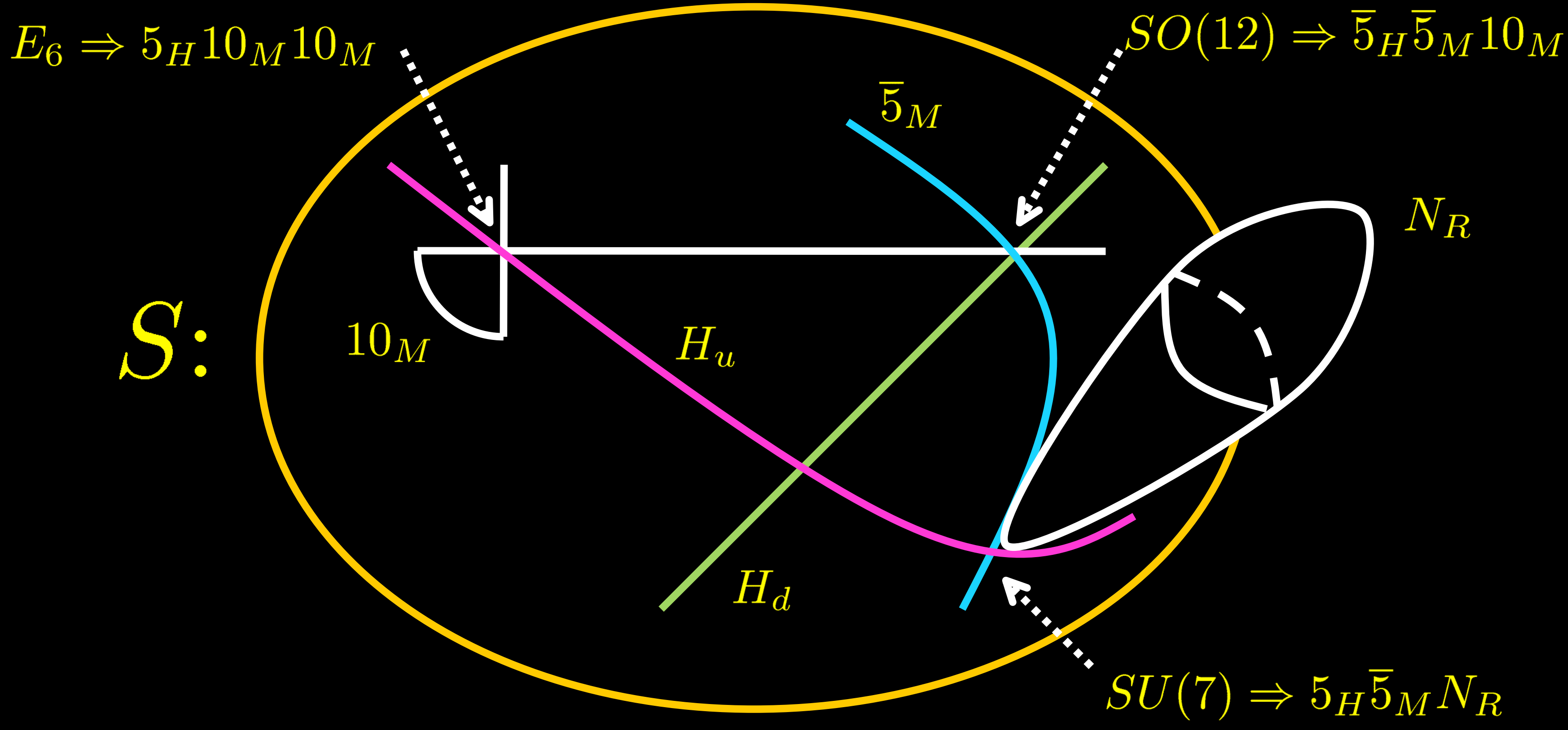
$$\propto m_{\beta\beta}$$

$m_{\beta\beta} \lesssim 6$ meV $\Rightarrow$ Predict No Detection

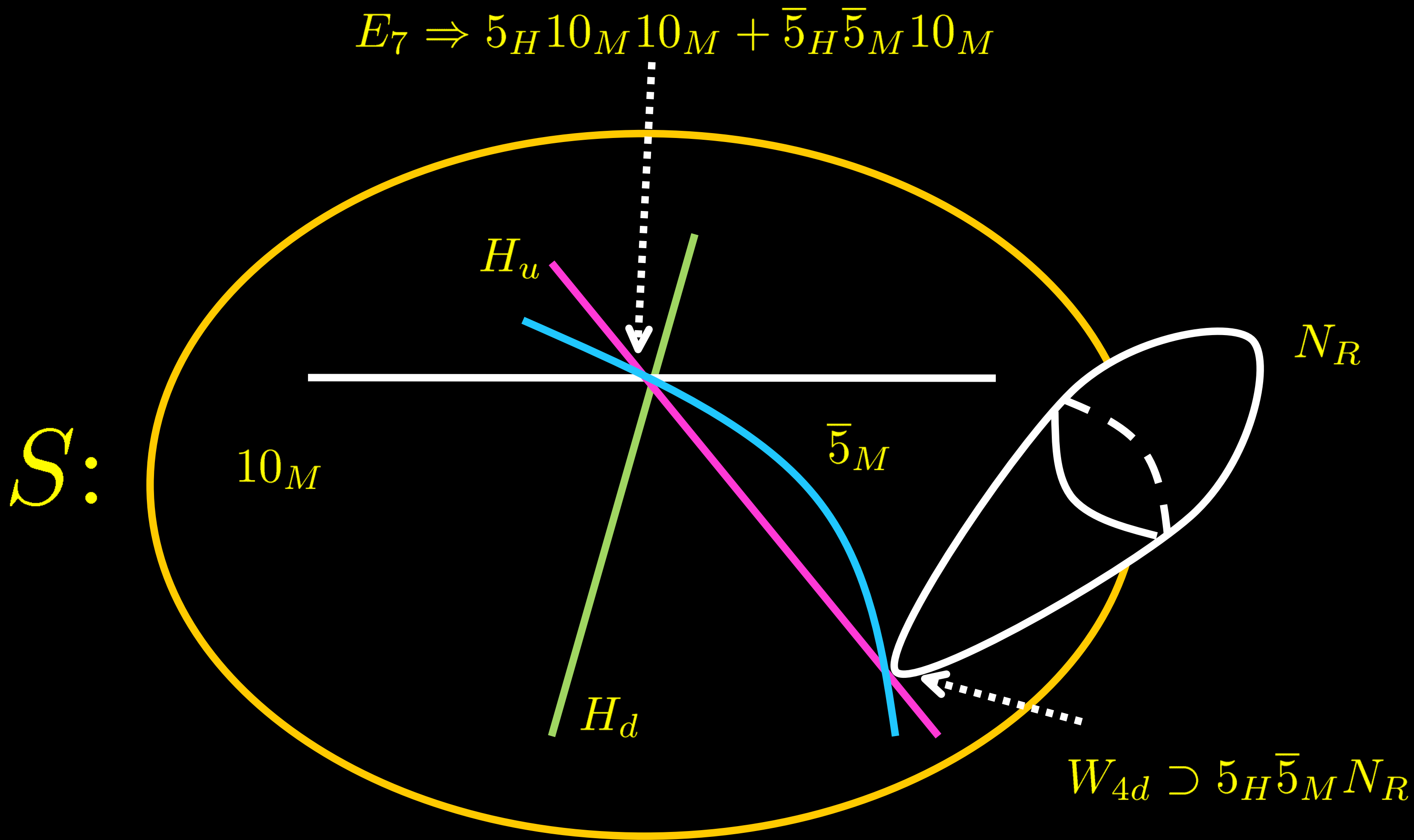
Near Future Limits: ~ 50 meV

Point Unification

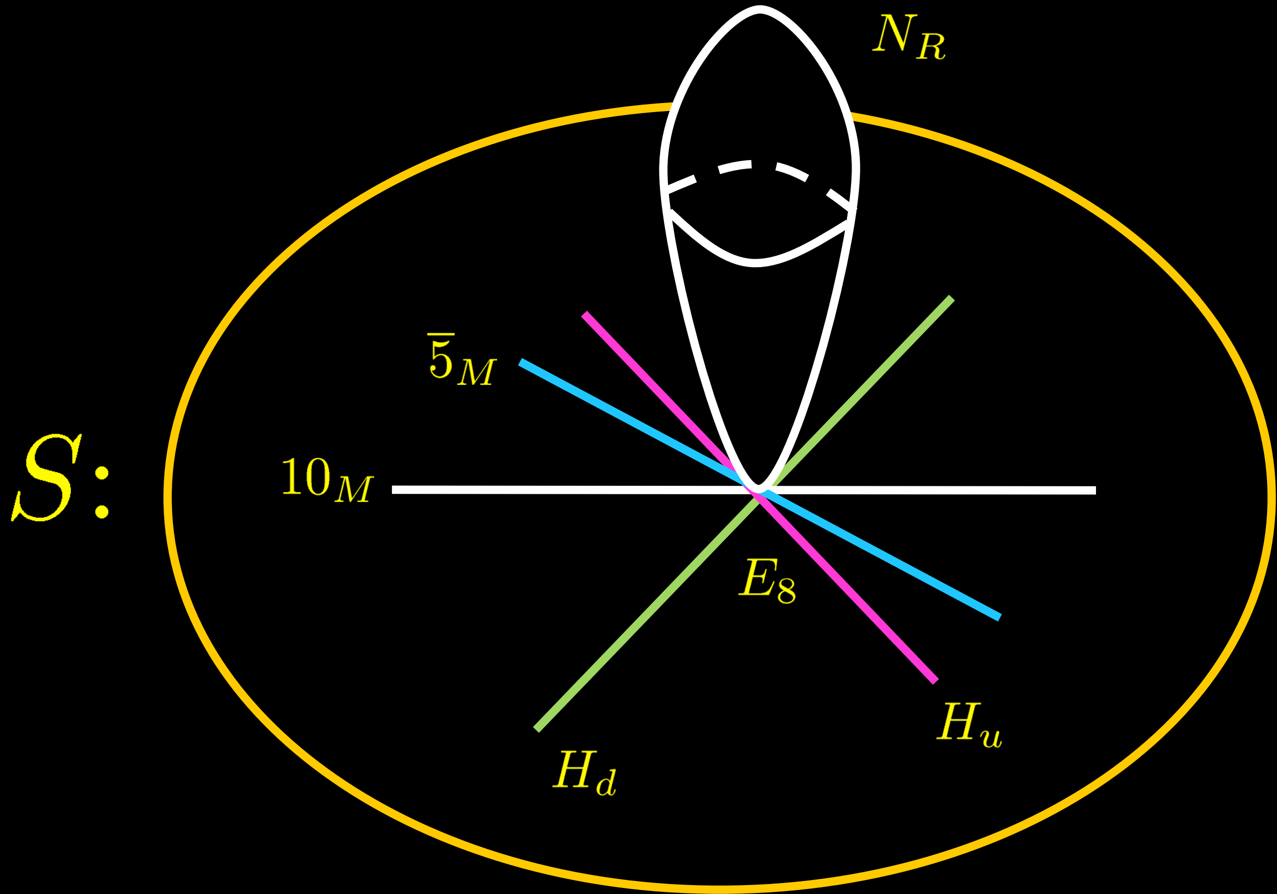
Beasley JJH Vafa II '08



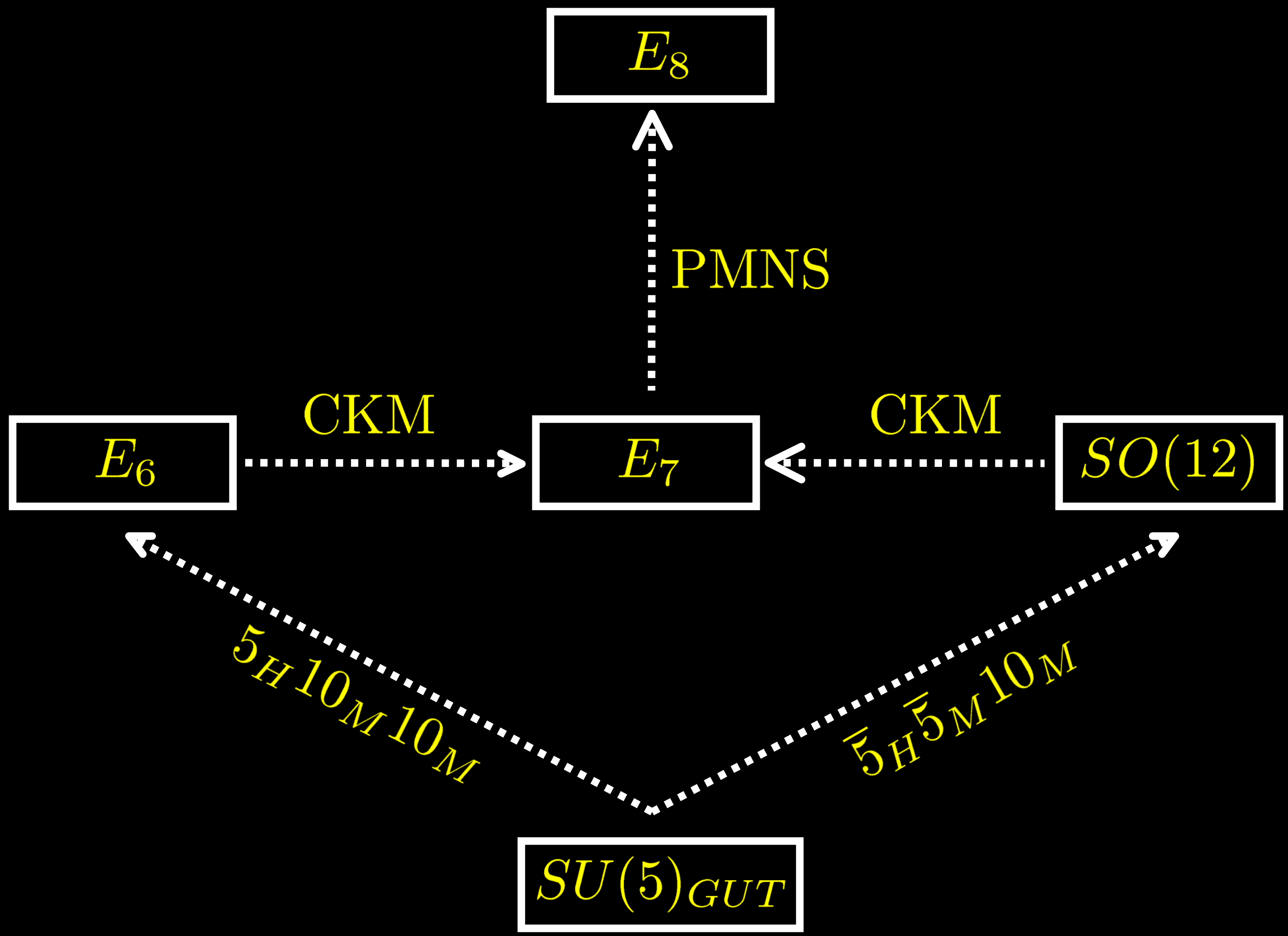
Point Unification



Point Unification



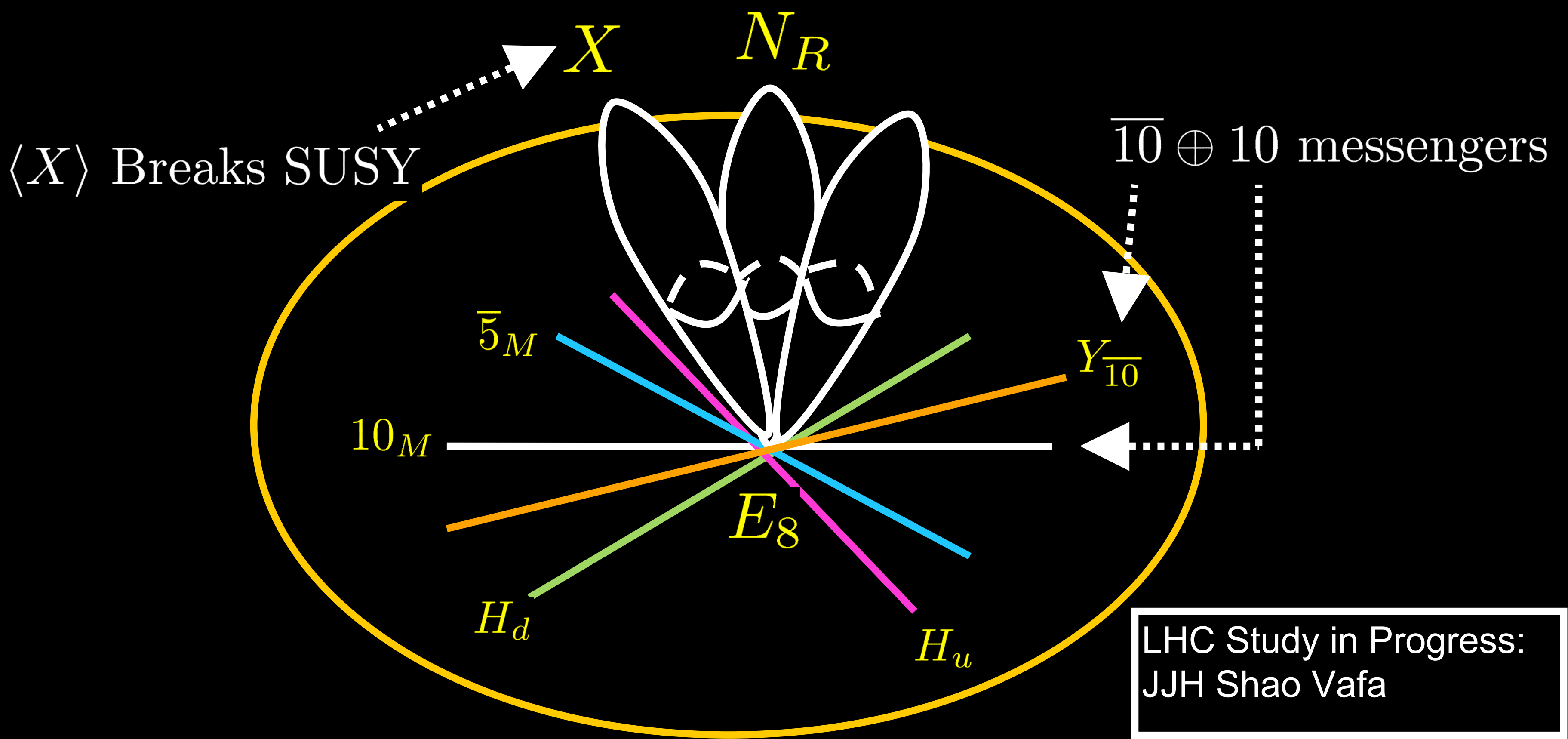
CKM + PMNS $\Rightarrow E_8$



Minimal Scenario

Minimal E_8 is very constraining: MSSM

$+$ deform^n of min. gauge med^n



Conclusions

- Bottom Up GUTs and F-theory
- Geometry + H-flux $\Rightarrow$ Flavor
- Quark and Lepton Masses and Mixing
- Flavor and E_8