Can we learn something from heavy ion collisions for cosmology?

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I. Overall equation of state

Long ago it looked like there could be interesting cosmology associated with basic QCD thermodynamics:

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Cosmic separation of phases

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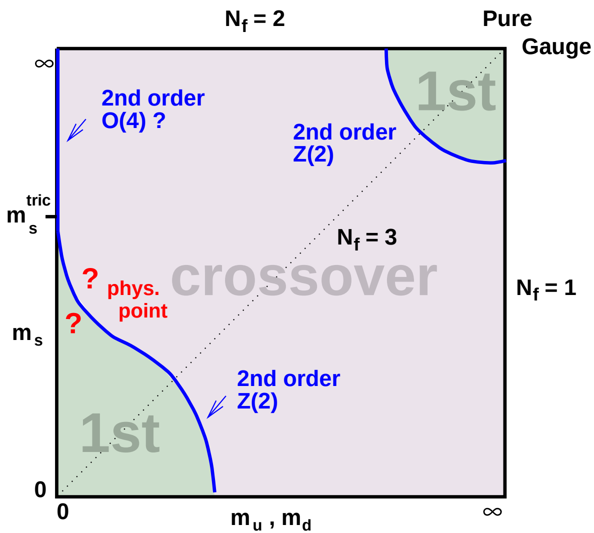
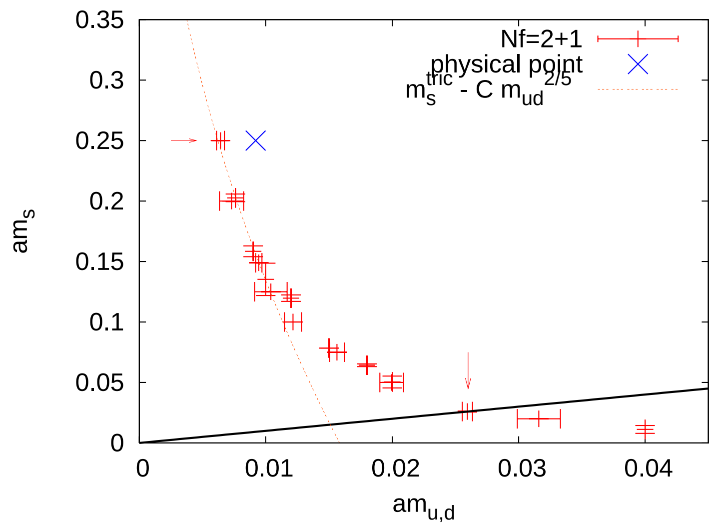
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

But these hopes have been dashed by lattice simulations:

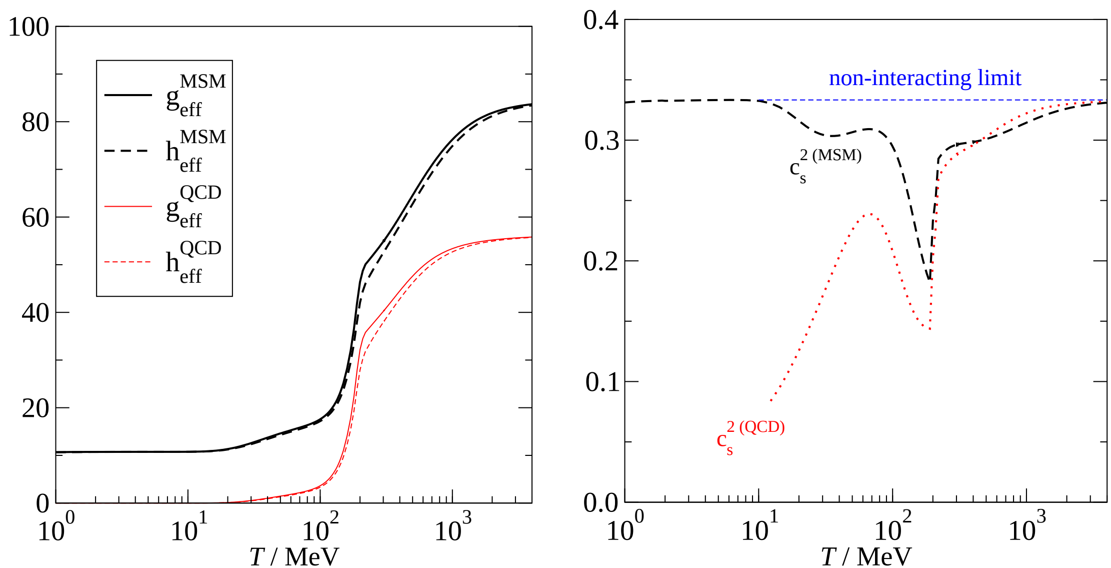




de Forcrand, Philipsen hep-lat/0607017
Aoki et al hep-lat/0611014

Yet the crossover is steep, as can be guessed even without lattice, by interpolating resonance gas at $T < T_c$ to pQCD at $T > T_c$:

$$g_{\text{eff}} \equiv \frac{e(T)}{\left[\frac{\pi^2 T^4}{30}\right]}\,, \quad h_{\text{eff}} \equiv \frac{s(T)}{\left[\frac{2\pi^2 T^3}{45}\right]}\,, \quad c_s^2 \equiv \frac{\mathrm{d}p}{\mathrm{d}e}\,.$$



ML Schröder hep-ph/0603048

In principle this could affect Cold Dark Matter decoupling at $T \sim m_{\text{CDM}}/25 \sim$ few GeV, or Warm Dark Matter.

Srednicki et al NPB 310 (1988) 693;

Hindmarsh Philipsen hep-ph/0501232;

ML Shaposhnikov 0804.4543.

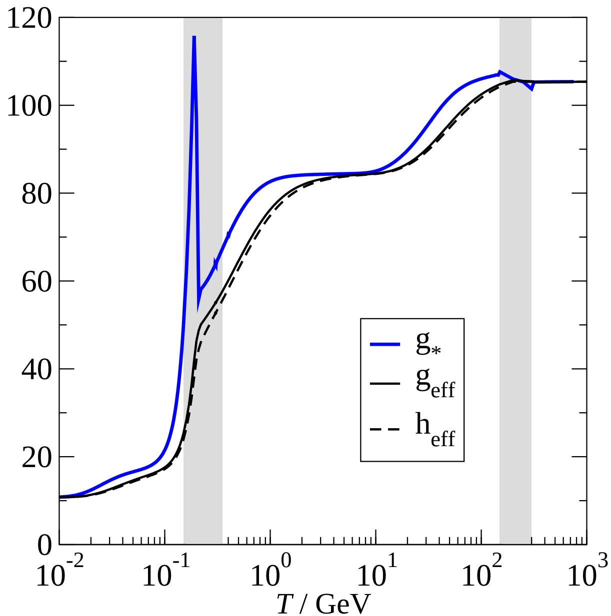
Evolution equation for $Y \equiv n_{\rm dm}/s$: Gondolo Gelmini NPB 360 (1991) 145

$$\frac{\mathrm{d}Y}{\mathrm{d}T} \simeq \sqrt{\frac{\pi g_*(T)}{45}}\, m_{\rm Pl}\, \langle \sigma v_{\rm rel} \rangle (Y^2 - Y_{\rm eq}^2) \;,$$

where Hindmarsh Philipsen hep-ph/0501232

$$g_*(T) \simeq \frac{h_{\rm eff}^2(T)}{g_{\rm eff}(T)} \left[\frac{1}{3 c_s^2(T)} \right]^2 \;.$$

The function $g_*(T)$ has a relatively rich structure:



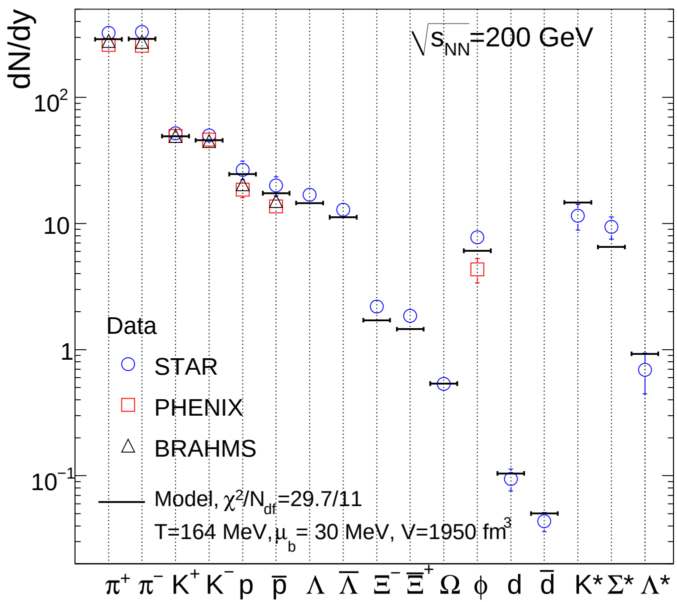
Physically:
heat capacity has a peak $\Rightarrow$ it takes extra time to dilute all the heat released.

Effects on the final Y on the percent level?

$\Rightarrow$ A detailed understanding of QCD and MSM thermodynamics is indeed a background ingredient in Dark Matter computations.

II. Shear and bulk viscosities

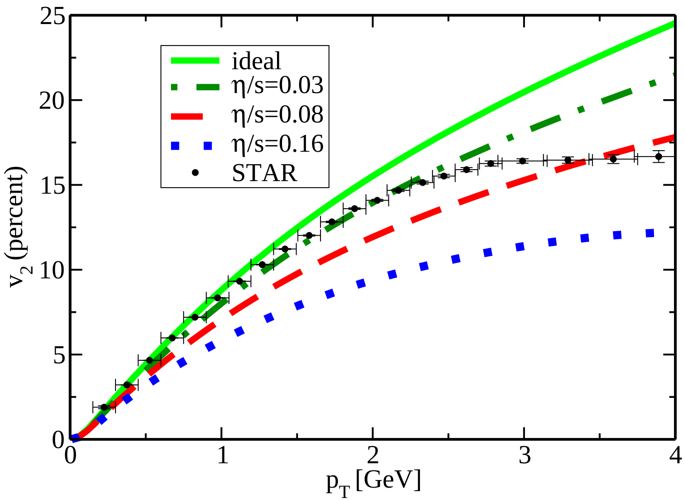
Just looking at the total multiplicity in heavy ion collisions suggests that the system does thermalize:



Andronic et al 0901.2909

Here all information about the evolution before hadronization is lost, because thermal equilibrium has no memory.

However differential observables – like momentum distribution in various directions – may allow to extract refined characteristics:



2×Romatschke 0706.1522

Here η is the "shear viscosity", and the data supposedly suggests a saturation of the famous AdS/CFT lower limit $\eta/s \geq 1/4\pi$.

The reason that a viscosity can (perhaps) be extracted is that it yields a gradient correction to the energy-momentum tensor:

$$T^{\mu\nu} = [p(T) + e(T)]u^\mu u^\nu - p(T)g^{\mu\nu} + \mathcal{O}((\eta, \zeta)\partial^\mu u^\nu) \ ,$$

where u^μ is the flow velocity, and $\partial_\mu T^{\mu\nu} = 0$.

In contrast, the Universe is very homogeneous, and gradient corrections have no direct effect on the overall expansion.[1]

It turns out, however, that the bulk viscosity ζ makes a formal appearance in a completely different context!

Bödeker hep-ph/0605030

[1] Evolution of density perturbations may be a different story.

Consider some very weakly coupled scalar field, φ:

$$\mathcal{L} \sim \frac{1}{2}\varphi(-\Box - m^2)\varphi + \frac{\varphi}{M}F^{\mu\nu}F_{\mu\nu} \ ,$$

where $m \sim m_{\rm SUSY}$, $M \sim m_{\rm Pl}$.

This leads to a "moduli problem": after inflation $\langle \varphi(0) \rangle \gg m$, and φ decays slowly, $\Gamma \sim m^3/M^2$, whereby its energy density eventually dominates over radiation.

Q: could the vacuum rate $\Gamma \sim m^3/M^2$ be modified by thermal corrections associated with the "normal" degrees of freedom, here represented by $F_{\mu\nu}$?

Equation of motion in a non-trivial environment:

$$\Box\varphi + V'_{\rm eff}(\varphi) = -\Gamma\dot{\varphi} + \mathcal{O}(\dot{\varphi}^2, (\nabla\varphi)^2) \ .$$

A standard tool in thermal field theory is a "Kubo formula", allowing to determine such "response" coefficients:

$$\Gamma = \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,\Pi(\omega + i0^+, \mathbf{0}) \ ,$$

where for $\mathcal{L}_{\rm int} = \varphi H_{\rm int}$:

$$\Pi(\omega,\mathbf{p}) = \int_{t,\mathbf{x}} e^{i\omega t - i\mathbf{p}\cdot\mathbf{x}} \langle H_{\rm int}(x) H_{\rm int}(0)\rangle_{\rm ret} \ .$$

Now we may recall the trace anomaly of pure Yang-Mills,

$$\Theta = T^{\mu}{}_{\mu} \sim \frac{\beta_{g^2}}{g^2} F^{\mu\nu} F_{\mu\nu} \ ,$$

as well as the definition of the bulk viscosity:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \int_{t,\mathbf{x}} e^{i\omega t} \langle \Theta(x) \Theta(0) \rangle_{\mathrm{ret}} \ .$$

Moreover, motivated by heavy ion collisions, the weak-coupling expression for ζ has been worked out: $\zeta \sim \alpha_s^2 T^3 / \ln(1/\alpha_s)$.

Arnold Dogan Moore hep-ph/0608012

Now $H_{\text{int}} = \frac{1}{M} F^{\mu\nu} F_{\mu\nu} \sim \frac{g^2}{M \beta_{g^2}} \Theta \sim \frac{\pi}{M \alpha_s} \Theta$.

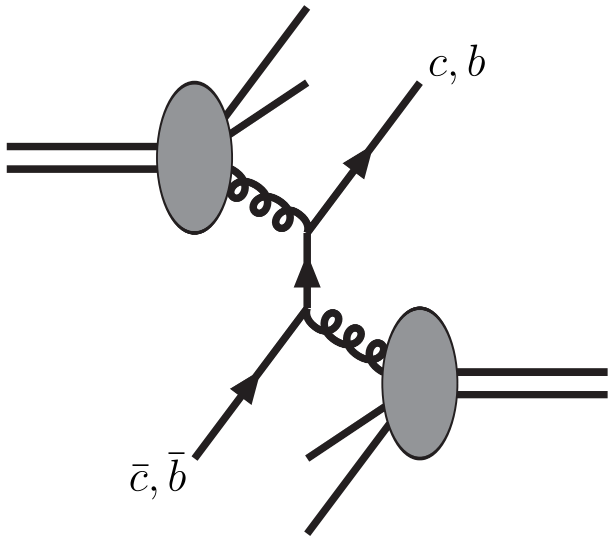
In conclusion, the vacuum decay rate, $\Gamma \sim \frac{m^3}{M^2}$, is overtaken at $T \gg m$ by a thermal correction:

$$\Gamma \sim \frac{9\pi^2 \zeta}{(M\alpha_s)^2} \sim \frac{9\pi^2}{\ln(1/\alpha_s)} \frac{T^3}{M^2} \,.$$

In practice, the effect is probably not large enough to solve the moduli problem, but at least heavy ion collision inspired computations have found an “exciting” application!

III. Heavy quark jet quenching

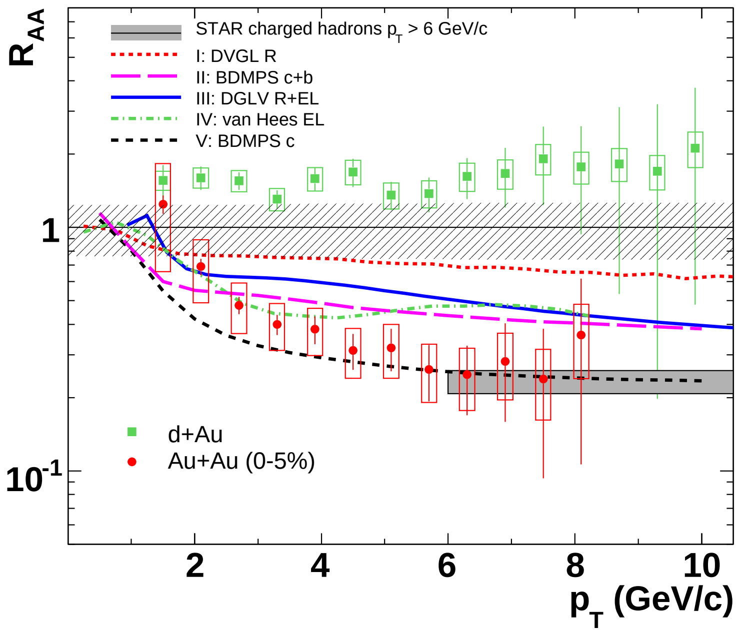
Initial production of heavy quarks is supposedly relatively well understood for p + p, d + Au, Au + Au. Cacciari et al hep-ph/0502203



Subsequently the heavy quarks decay, often semi-leptonically as $c \longrightarrow \ell \nu X$, and ℓ can be observed.

But in Au + Au less ℓ observed than expected:

STAR nucl-ex/0607012, PHENIX nucl-ex/0611018



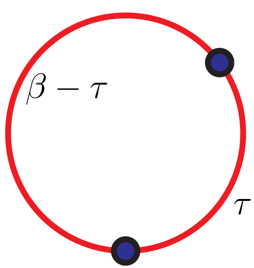
Heavy quarks get stopped by scatterings — they **thermalize**, behaving much like particles in non-relativistic Brownian motion.

The (kinetic) thermalization rate, η_D, can be fluctuation-dissipation-related to a force-force transport coefficient, κ:

$$\eta_D = \frac{\kappa}{2M_{\mathsf{kin}}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{\mathsf{kin}}}\right)\right) \quad , \quad \kappa = \lim_{\omega\to 0}\frac{2T\rho_E(\omega)}{\omega} \; .$$

Here ρ_E is the spectral function corresponding to the Euclidean correlator

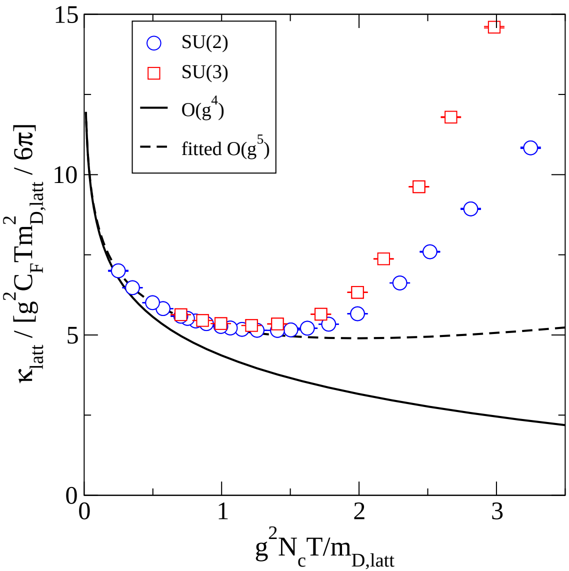
$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;\tau}\, gE_i(\tau,\mathbf{0})\, U_{\tau;0}\, gE_i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U_{\beta;0}]\rangle} \; .$$



Casalderrey-Solana Teaney hep-ph/0605199;

Caron-Huot ML Moore 0901.1195

So far κ has only been measured within "classical lattice gauge theory", but the result is interesting if plotted in terms of a quantity having an analogue in QCD, $m^2_{\text{D,latt}} \sim g^2 T/a$:



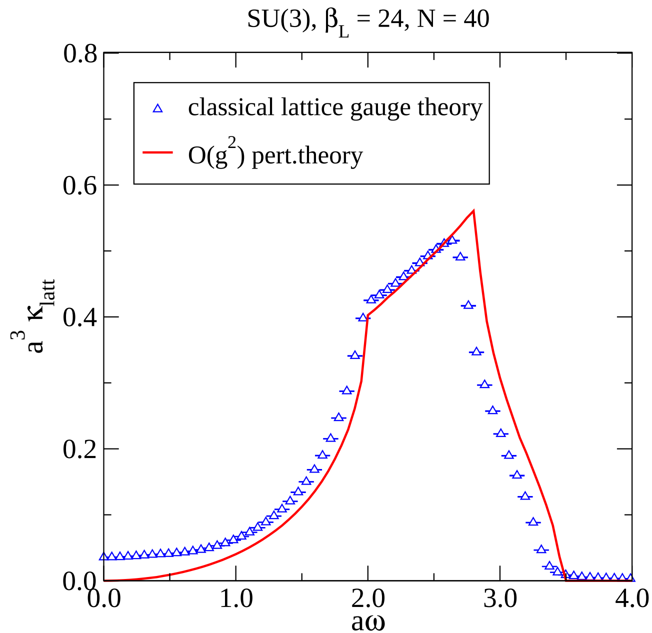
$O(g^5)$ in QCD: Caron-Huot Moore, 0708.4232

data: ML Moore Philipsen Tassler, 0902.2856

$\Rightarrow$ the kinetic thermalization rate could be unexpectedly large — as appears to be required by phenomenology.

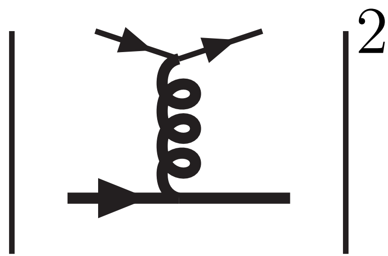
e.g. Akamatsu et al 0809.1499

The combination $\kappa(\omega) \equiv 2T\rho_E(\omega)/\omega$ is rather flat at small frequencies:



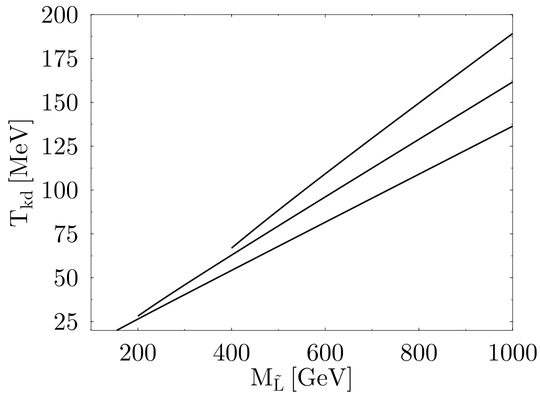
In particular **no transport peak** around $\omega = 0$, so analytic continuation from $G_E(\tau)$ might be feasible $\Rightarrow$ lattice?

Again, qualitative similar physics may play a role in cosmology: Cold Dark Matter is by definition non-relativistic, and kinetically decouples when elastic scatterings, albeit with weak interactions, cease to be active.



Then the phase space density no longer behaves as $e^{-p^2/2M_{\mathsf{kin}}T}$. This may be visible in the large scale structures that can form.

In practice, though, kinetic decoupling happens at a very low temperature, so probably a Fermi model treatment is sufficient, and then the analogy with a gauge theory like QCD is feeble.



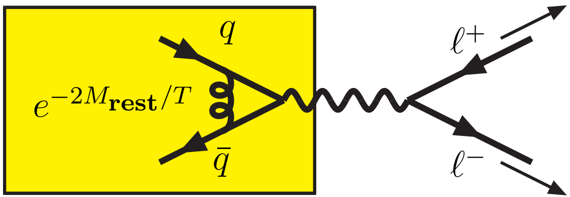
Hofmann Schwarz Stöcker astro-ph/0104173

But at least a conceptual link exists, and could perhaps play a more significant role e.g. in connection with leptogenesis.

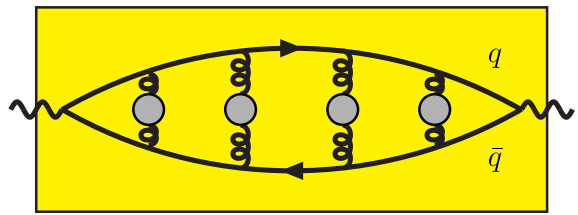
IV. Quarkonium dissociation

Dileptons produced by quarkonium decays are important probes for heavy ion collision experiments:

amplitude:



dilepton rate:



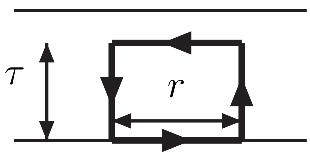
Matsui Satz 1986; McLerran Toimela 1985; Weldon 1990; Gale Kapusta 1991

Can the ladders be resummed into a "static potential" that the heavy particles feel while propagating through the medium?

within pert.theory: ML et al hep-ph/0611300; Beraudo et al 0712.4394; Escobedo Soto 0804.0691; Brambilla et al 0804.0993

Euclidean $\beta = 1/T$ is "small", Minkowskian t is "large"

$\Rightarrow$ define potential from analytic continuation:



$$\begin{aligned} C_E(\tau, r) &\equiv \langle \mathrm{Tr}[W_E(\tau, r)] \rangle \ , \\ i\partial_t C_E(it, r) &\equiv V_>(t, r) C_E(it, r) \ . \end{aligned}$$

The static limit $V_>(\infty, r)$ through **spectral analysis**.

Hatsuda Sasaki Rothkopf 0910.2321

Position of the spectral peak: average energy, $\mathrm{Re}\, V_>(\infty, r)$.
Its width: Coulomb scattering/Landau damping, $\mathrm{Im}\, V_>(\infty, r)$.

Explicitly:

$$\mathrm{Re}\,V_>(\infty,r) \;=\; -\frac{g^2C_F}{4\pi}\left[m_{\mathsf{D}}+\frac{\exp(-m_{\mathsf{D}}r)}{r}\right]\;,$$

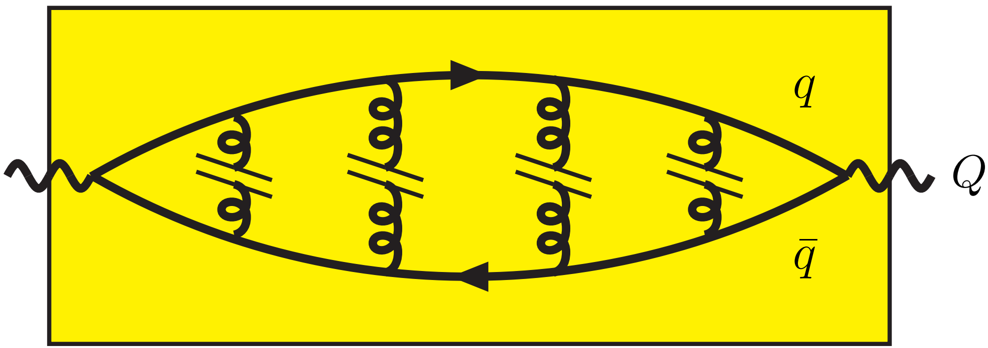
$$\mathrm{Im}\,V_>(\infty,r) \;=\; -\frac{g^2TC_F}{4\pi}\,\phi(m_{\mathsf{D}}r)\;,$$

where $m_{\mathsf{D}} \sim gT$ is the Debye mass, $C_F \equiv 4/3$, and

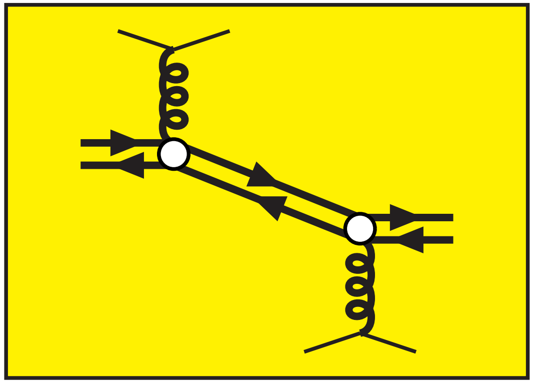
$$\phi(x) = 2\int_0^\infty \frac{\mathrm{d}z\,z}{(z^2+1)^2}\left[1-\frac{\sin(zx)}{zx}\right]\;.$$

The real part is characterized by the Debye screening of the colour-electric field binding together the quark and antiquark:

Matsui Satz 1986

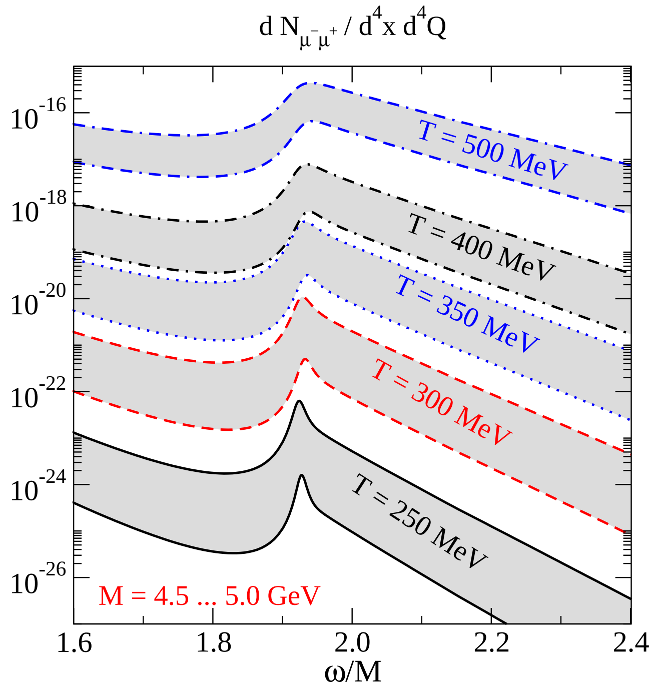


The imaginary part can be related to "collisional broadening" (momentum transfer) due to hard particles in the plasma:



Bound state is charge-neutral but has a finite size and hence electric dipole moment; it gets kicked / experiences drag.

Result for the thermal dilepton rate from $\bar{b}b$ with such ingredients:



Burnier et al 0812.2105

In cosmology, a somewhat similar rate is relevant for the annihilation (decoupling) of dark matter heavy neutralinos:

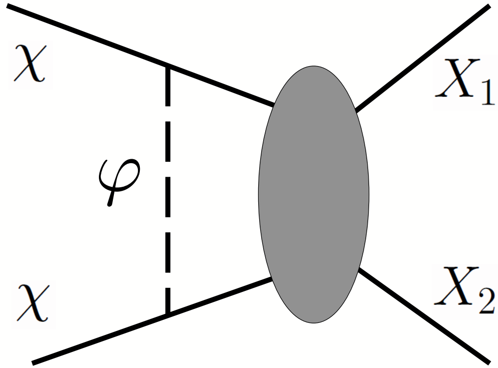


figure from Drees Kim Nagao 0911.3795

Another formal counterpart might be the initial chemical thermalization ($\longleftarrow$) and/or final decoupling ($\longrightarrow$) of heavy Majorana neutrinos relevant for leptogenesis.

Conclusions

Hot QCD is a simple but non-trivial theory for which systematic theoretical tools are being developed, and to some extent tested against Heavy Ion Collisions as well as lattice QCD.

Some of these tools may find use also in cosmology, perhaps with the modification gluons $\longrightarrow W^{\pm}, Z^{0}$ or gluons $\longrightarrow$ Higgs.