Holographic non-Fermi liquids

many-body physics through a gravitational lens

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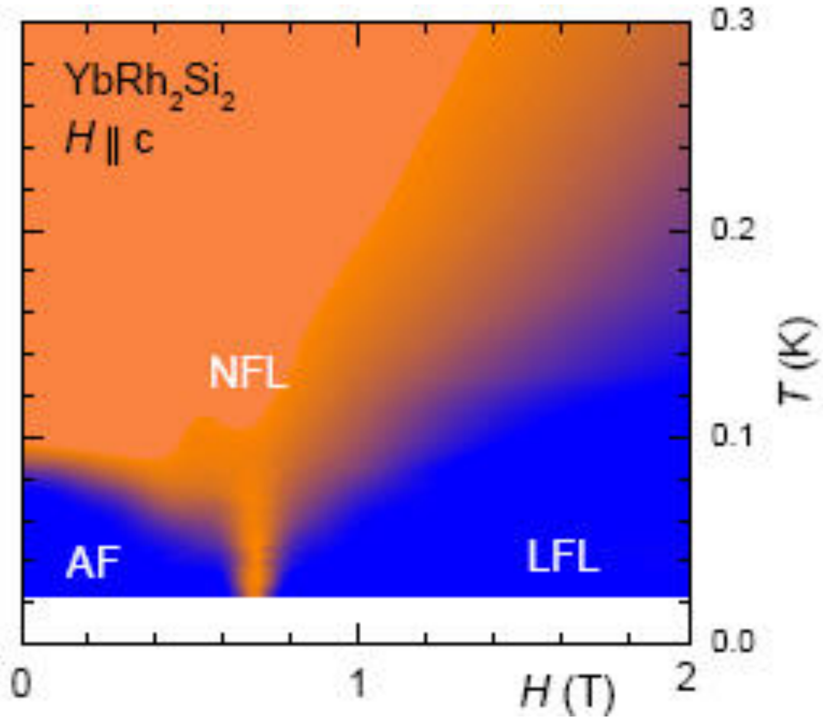
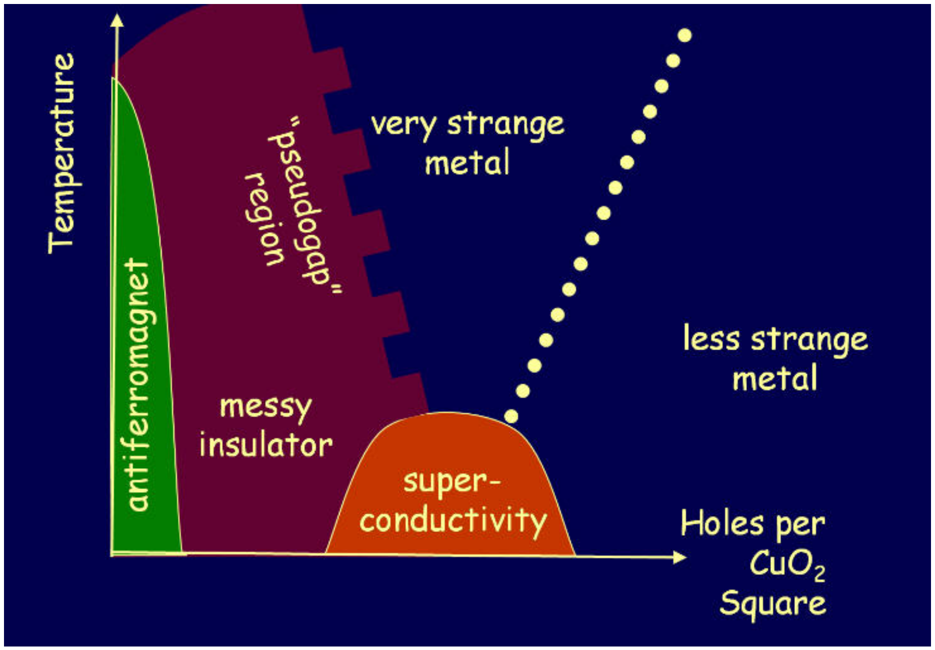




Two Pillars of condensed matter physics

1. Landau's Fermi liquid theory

Almost all metals, semiconductors, Helium 3, superconductors …..

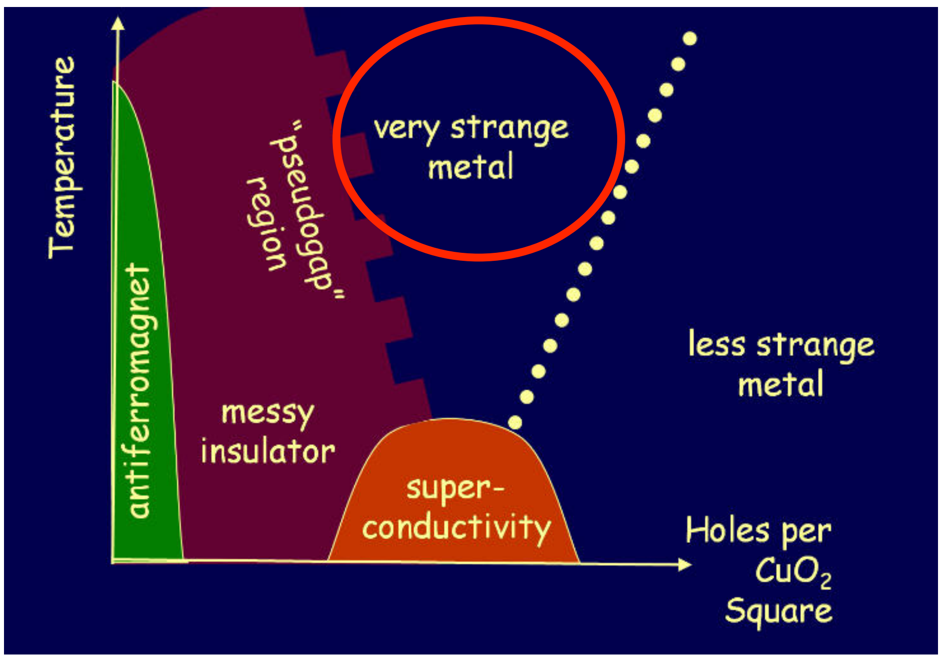
2. Landau's theory of order and Landau-Ginsburg-Wilson paradigm for phase transitions

Different orders characterized different symmetries

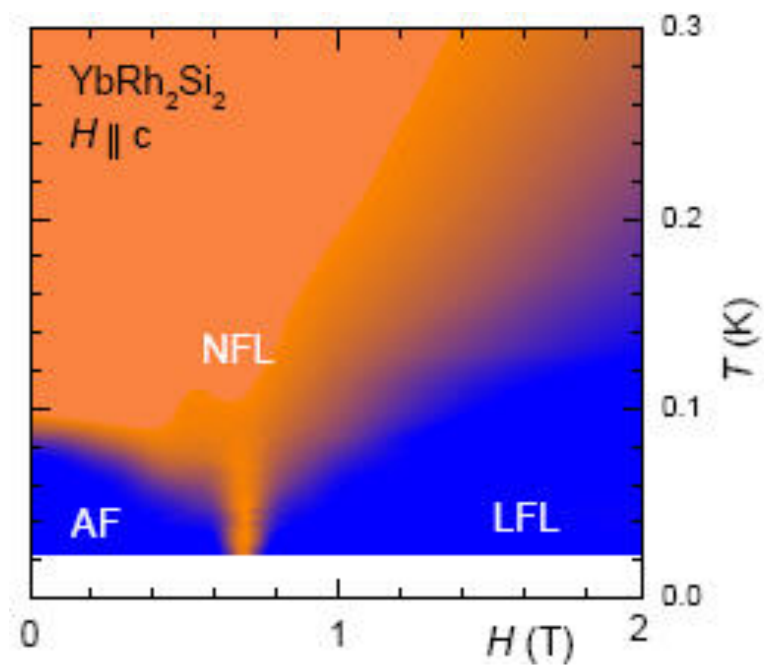
Phase transitions: symmetry breaking

Strongly correlated fermionic systems at finite density

During the last two decades, these pillars are challenged at both experimental and theoretical level.



High Tc cuprates



Quantum phase transitions of heavy fermion metals

Plan

1. Holographic non-Fermi liquids:

   • Experimental motivation

   • Gauge/gravity duality for a finite density system

   • Holographic non-Fermi liquids

2. Holographic phase transitions (a separate talk)

   • Experimental motivation

   • Holographic quantum phase transitions going beyond Landau-Ginsburg-Wilson paradigm

Holographic non-Fermi liquids:

HL, McGreevy, David Vegh, 0903.2477
Tom Faulkner, HL, JM, DV, 0907.2694
TF, Nabil Iqbal, HL, JM, DV, 1003.1728,
Science 329, 1043 (2010)
Sung-Sik Lee, 0809.3402
Cubrovic, Zaanen, Schalm, 0904.1933
Faulkner, Polchinski, arXiv:1001.5049

Holographic quantum phase transitions:

Nabil Iqbal, HL, Mark Mezei, to appear

Nabil Iqbal, HL, Mark Mezei, Qimiao Si arxiv:1003.0010

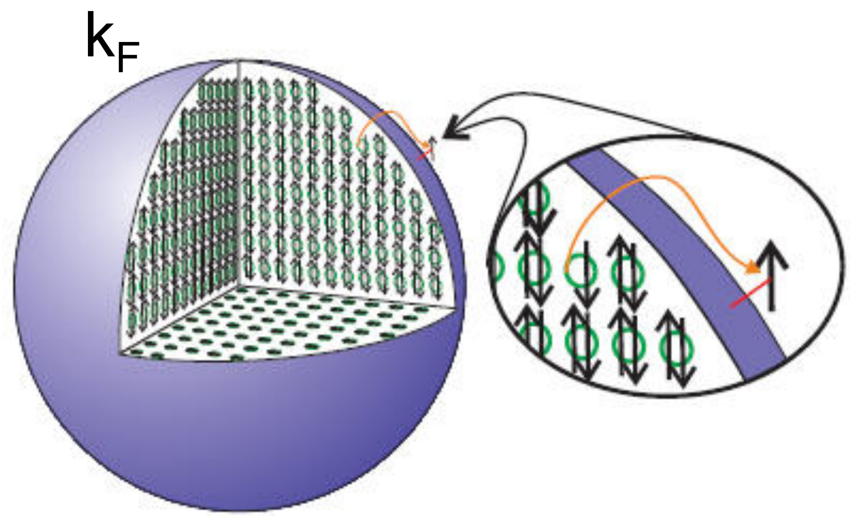
Faulkner, Horowitz, Roberts, arxiv:1003.0010

Jensen, Karch, Son, Thompson, arxiv: 1002.3159

Holographic non-Fermi liquids

Fermi Liquids theory

Landau: a finite density of interacting fermions



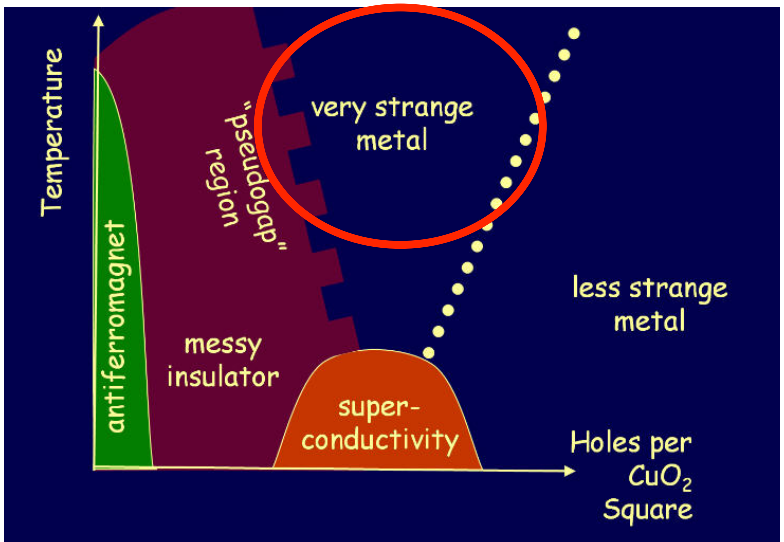
1. ground state: characterized by a sharp Fermi surface in momentum space

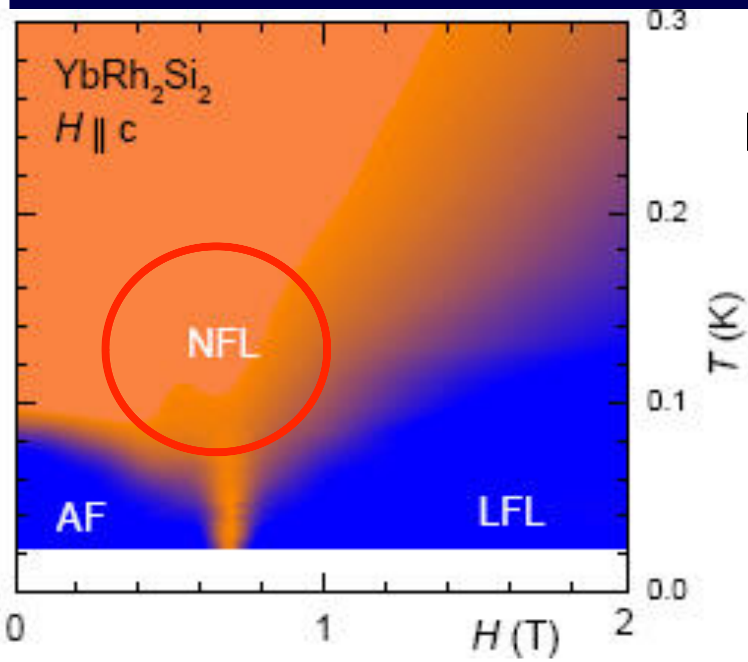
2. Low energy excitations: weakly interacting quasi-particles around the Fermi surface.

→ Thermodynamic, collective behavior, transports

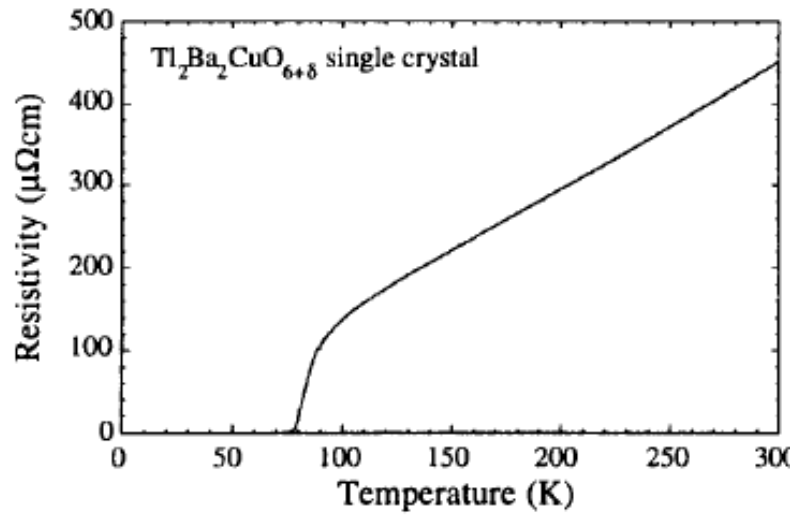
Does not depend on specific microscopic dynamics of an individual system.

Non-Fermi Liquids: Strange metals





Resistivity linear in temperature:



Mackenzie 97

In sharp contrast with that of a Fermi Liquid:

$$\rho = \rho_0 + cT^2$$

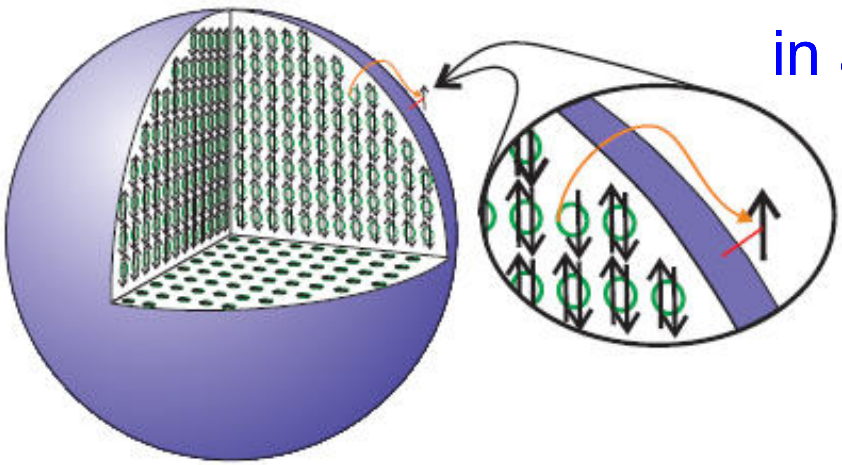
Simple, robust, universal, long standing puzzle

Other anomalous behavior: specific heat, scattering rate, …….

Does one or both Laudau’s postulates for Fermi liquids break down?

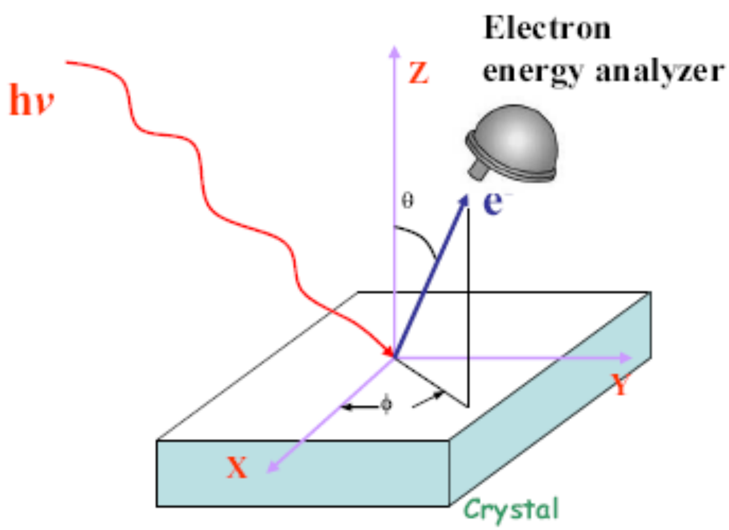
1. Fermi surface
2. Quasi-particles

Signature of Fermi surfaces (I)



How to characterized a Fermi surface in an interacting system?

Fermi surface: nonanalyticity in the small frequency behavior of A(ω,k) near some finite momentum shell in momentum space.



ARPES

spectral function

$$A(\omega,k) \equiv \frac{1}{\pi}\,\mathrm{Im}\,G_R(\omega,k)$$

$$G_R(t,\vec{x}) = i\theta(t)\langle\{\psi(t,\vec{x}),\psi(0,0)\}\rangle$$

Ψ : electron operator

Signature of Fermi surfaces (II)

Fermi liquids:

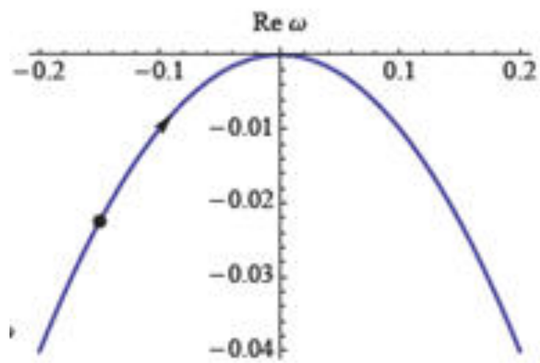
$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F k_\perp + i\Gamma}$$

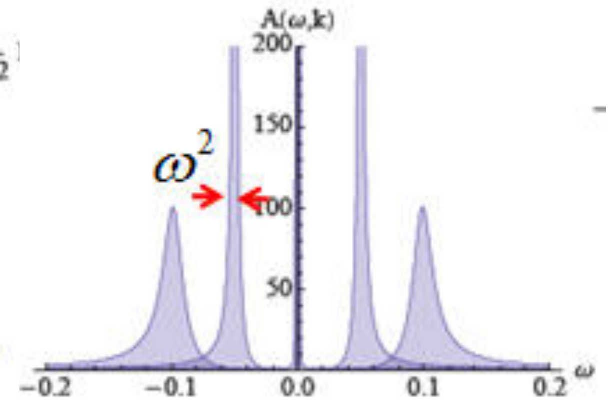
with Fermi surface at:

$$\omega = 0, \quad k_\perp \equiv k - k_F = 0$$

Quasi-particle decay rate $\Gamma \propto \omega^2$

Z: quasi-particle weight





``Marginal Fermi liquid'' for cuprates (Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein 1989)

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

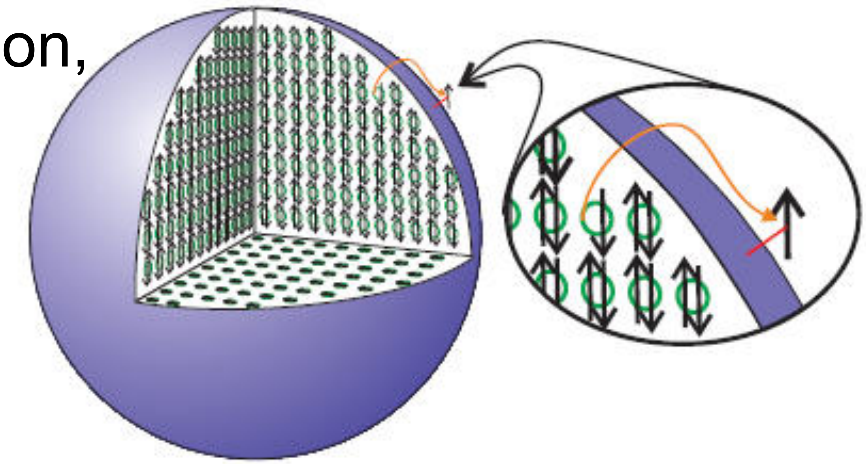
$\tilde{c}_1$: real

c_1 : complex

Quasi-particle decay rate $\Gamma \propto \omega$ weight vanishes as $\frac{1}{|\log \omega|}$

Fermi surface without quasi-particles

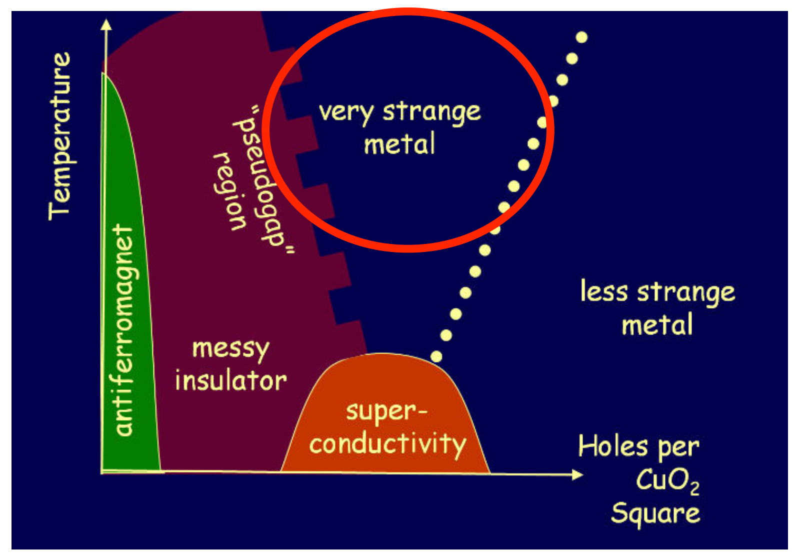
From transport and spectral function,



Strange metals:

Has a sharp Fermi surface

Quasi-particle picture breaks down



Theoretical challenges:

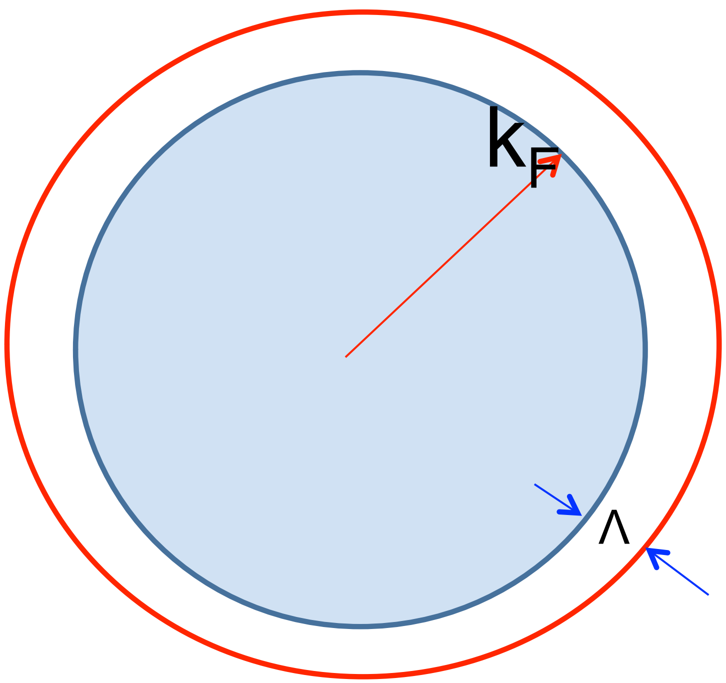
How to describe a Fermi surface without quasi-particles?

Can we find a general theory for strange metals?

RG perspective

Landau Fermi Liquid: free fermion fixed point of the RG toward the Fermi surface.

Shankar, Polchinski
Benfatto, Gallavotti



Λ : RG scale

non-Fermi liquids:
likely controlled by some interacting fixed points.

Unusual: gapless excitations at a finite momentum shell.

Need to develop a proper language to think about such fixed points

Summary

Strange metals and other non-Fermi liquids:

no systematic theoretical understanding of their properties

not clear what are organizing principles

Strongly correlated systems, famous theoretical challenge

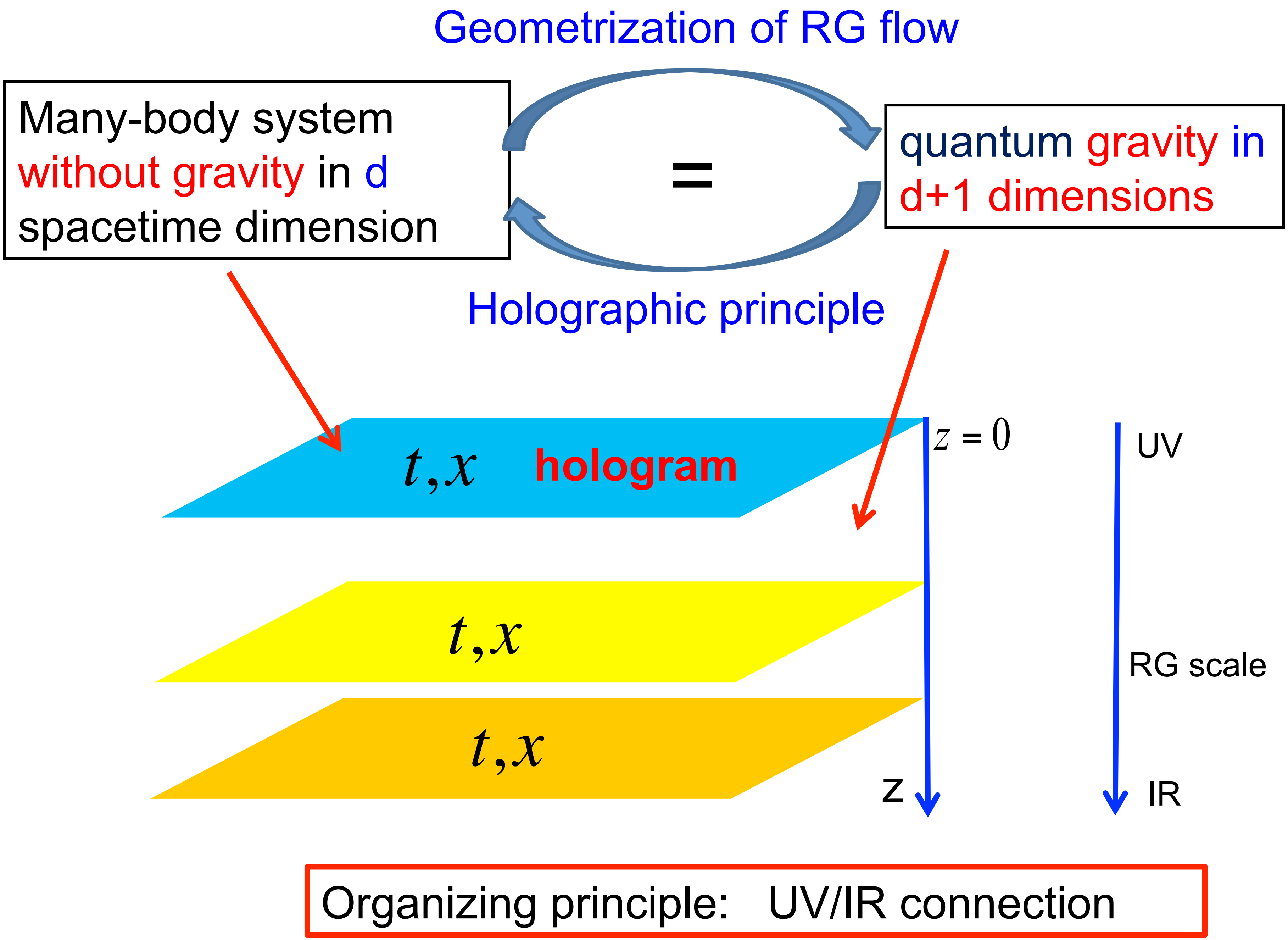
Numerical: fermionic sign problem (NP-hard) Troyer. Wiese (04)

Important:

high Tc cuprates

Novel quantum phase transitions

....

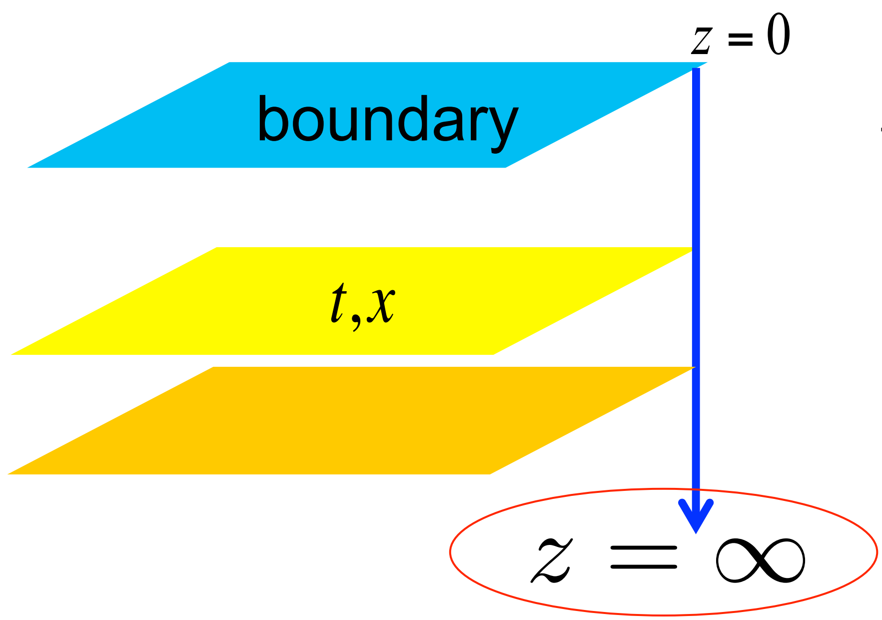

Geometrization of RG flow
Many-body system
without gravity in d
spacetime dimension
=
quantum gravity in
d+1 dimensions
Holographic principle
t, x hologram
z = 0
UV
t, x
RG scale
t, x
z
IR
Organizing principle: UV/IR connection

Power of holographic approach:

Large N and strong coupling limit ⟷ Classical gravity

highly quantum mechanical, strong coupling phenomena ⟷ simple geometric picture or gravitational dynamics

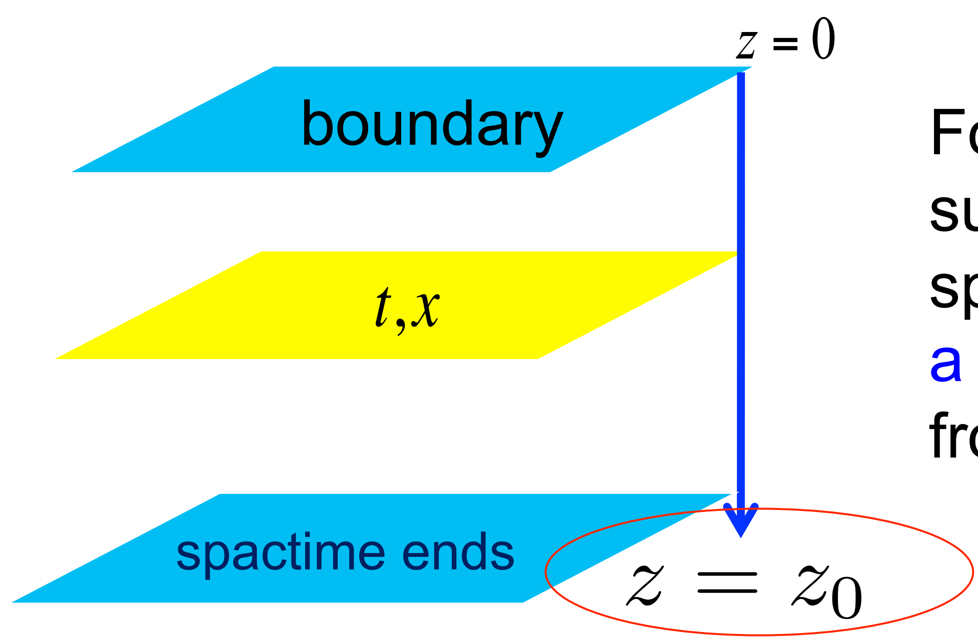
Many dynamical/geometric features do not depend on specific theories under consideration.



CFT: Scale invariance of the boundary theory requires that the bulk metric is invariant under scaling:

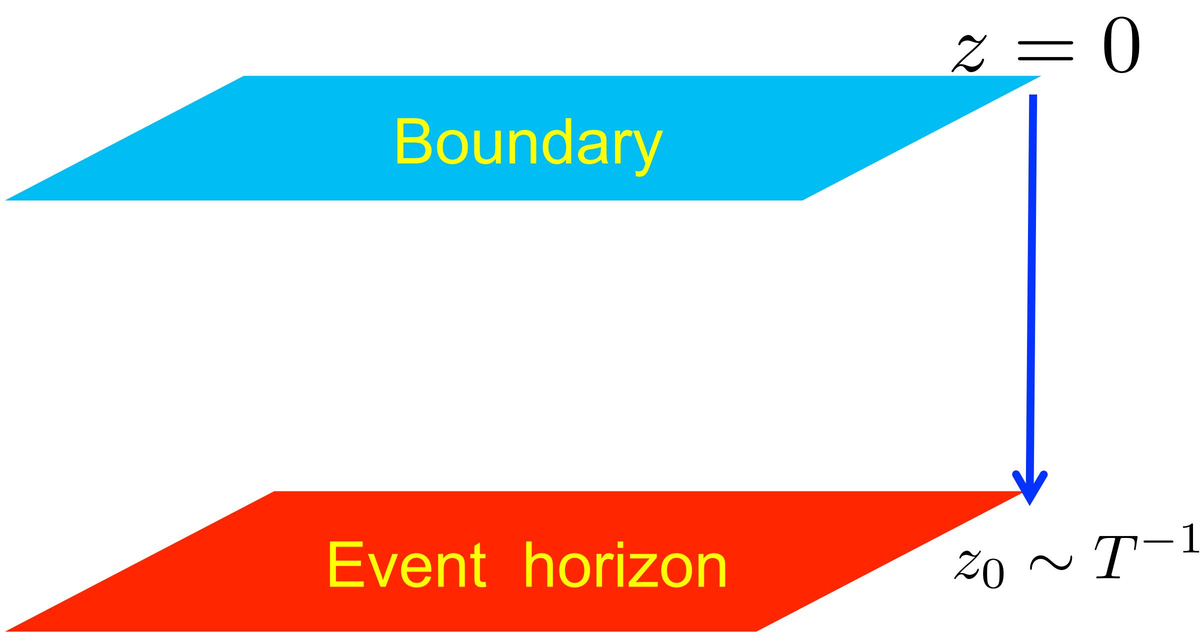
$$(t,x) \to \lambda\,(t,x), \;\; z \to \lambda\, z$$

⇒ AdS spacetime



For a theory with a mass gap, such as a confining theory, spacetime ends smoothly at a finite proper distance from any interior point.

Finite temperature



boundary system at
a finite temperature

⇕

Black hole

Entropy, energy,
Euclidean action of BH

⇔

Entropy, energy,
free energy of YM
theory

Finite chemical potential (finite density)

Start with your favorite field theories with a gravity dual:

D=3+1: N=4 super-Yang-Mills theory

D=2+1: ABJM

Non-Abelian gauge fields coupled to scalars and fermions. Gauge group: SU(N)

Take a U(1) global symmetry. Put the system at a finite chemical potential for this U(1).

J_μ ⟺ A_M

Field theory at finite chemical potential ⟺ A charged black hole

Chamblin, Emparan, Johnson, Myers

Gravity description : large N limit

charge density: $O(N^2)$

Fermi surfaces from AdS/CFT?

Start with your favorite field theories with a gravity dual:

Take a U(1) global symmetry. Put the system at a finite chemical potential for this U(1), which is described by a charge BH.

This generates a metallic (finite density) state:

does it have a Fermi surface?

Not obvious: since both scalars and fermions carry the same U(1) charge.

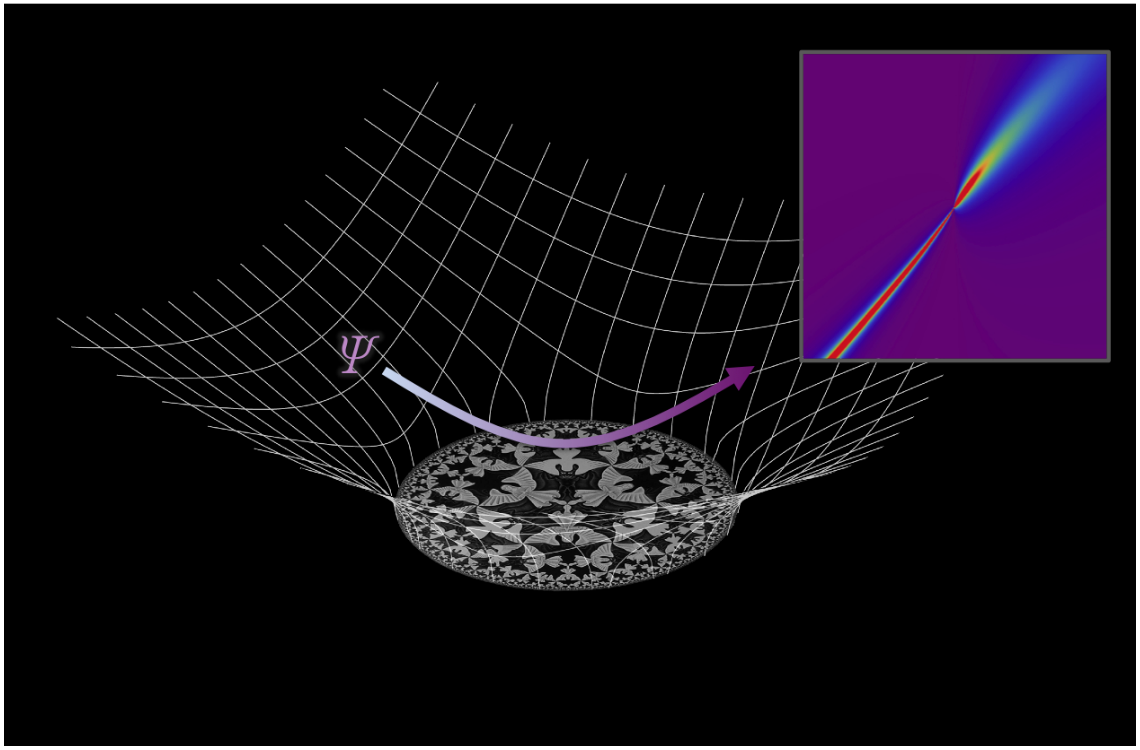
At strong coupling: dual gravity should tell us.

We want to compute: O: some fermionic operator Ψ (bulk spinor field)

$$G_R(t,\vec{x}) = i\theta(t)\langle\{\mathcal{O}(t,\vec{x}),\mathcal{O}(0,0)\}\rangle \quad A(\omega,\vec{k}) = \mathrm{Im}\, G_R(\omega,\vec{k})$$

“Photoemission experiments” on black holes

S-S Lee
HL, McGreevy, Vegh
Cubrovic, Zaanen, Schalm



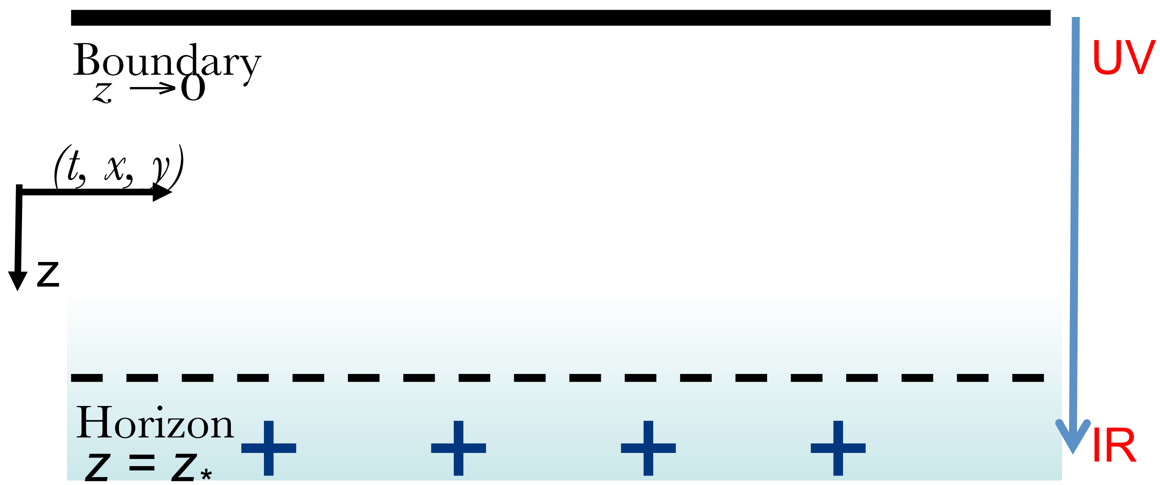
Solving Dirac equation for ψ, extracting boundary values

Universality of 2-point functions: (controlled by Dirac equation)

do not depend on which specific theory and operator we use.
Results will only depend on charge q and dimension m .

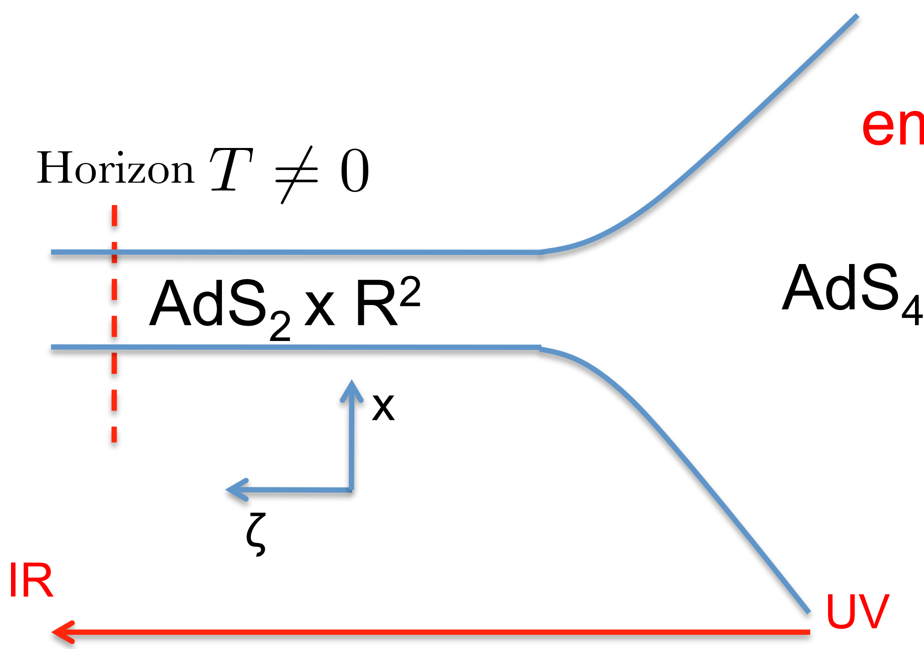
Will now use q and m as input parameters

Extremal charged black hole



T=0: extremal charged BH with a degenerate horizon at $z = z_*$

$$\zeta \equiv \frac{z_*^2}{6(z_* - z)}$$



emergent scaling symmetry in IR

$$t \to \lambda t, \quad \zeta \to \lambda \zeta$$

When T << μ, only AdS_2 region is heated up.

An emergent IR CFT

Metric for $AdS_2 \times R^2$

$$ds^2 = \frac{R_2^2}{\zeta^2}(-dt^2 + d\zeta^2) + \frac{R^2\mu^2}{3}d\vec{x}^2$$

Gravity in the AdS_2 region ⟺ a (0+1)-d CFT (QM)

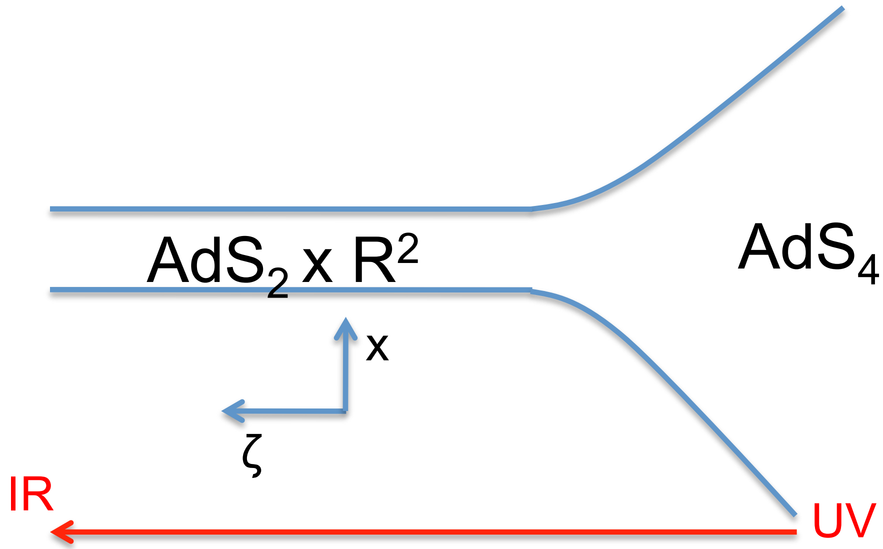
At low frequencies, the parent theory at finite density should be controlled by an emergent IR CFT !

Scaling symmetry is only in the time direction, spatial directions become labels.

Each operator will develop new scaling dimensions in the IR.

AdS_2 gravity ⇒ Operator dimensions, correlation functions

Conformal dimension in IR CFT



Conformal dimension of O in the vacuum

$$\Delta = \frac{3}{2} + \nu\,, \quad \nu = \sqrt{m^2 + \frac{9}{4}}$$

In the IR $O_{\vec{k}}$ match to an operator $\mathcal{O}_{\vec{k}}$ in the IR CFT.

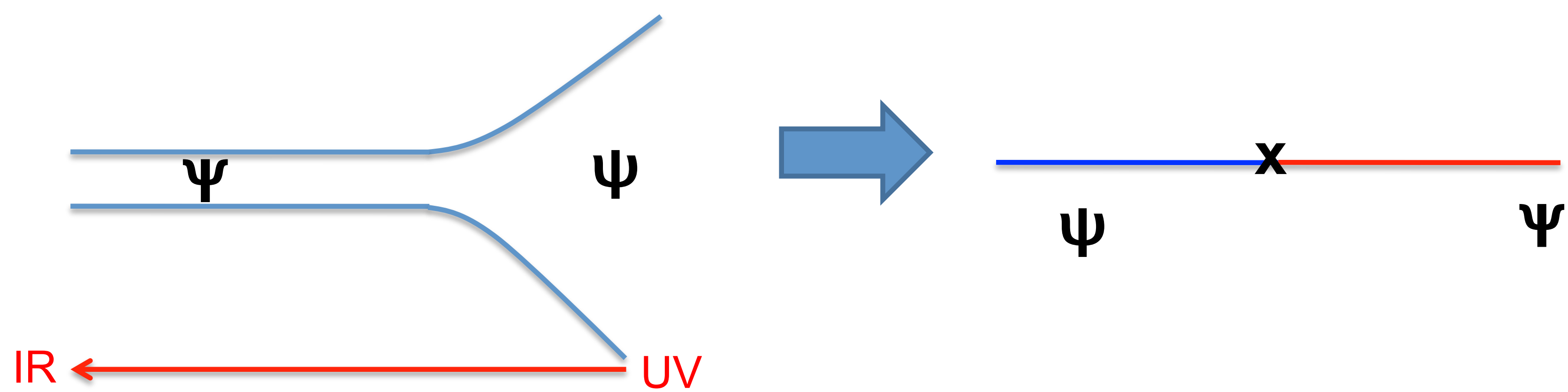
IR scaling dimensions for $\mathcal{O}_{\vec{k}}$ $\quad \delta_k = \frac{1}{2} + \nu_k \quad \nu_k = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2} + \frac{3}{2}}$

IR correlation functions for $\mathcal{O}_{\vec{k}}$ $\quad \mathcal{G}_k(\omega) = c(\nu_k)\omega^{2\nu_k}$

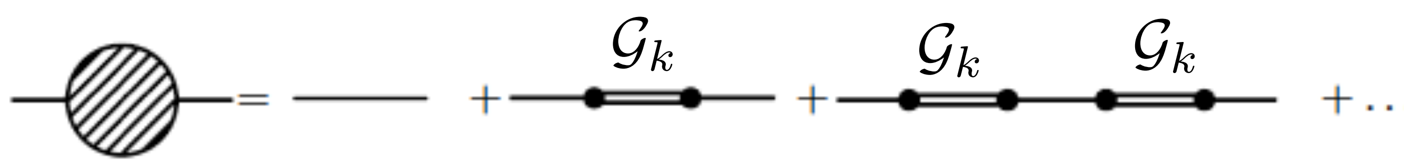
This insight now allows us to obtain analytically the low frequency behavior of the retarded function for the full theory

$$G_R(\omega, \vec{k})$$

in terms of that of the AdS_2 region $\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$

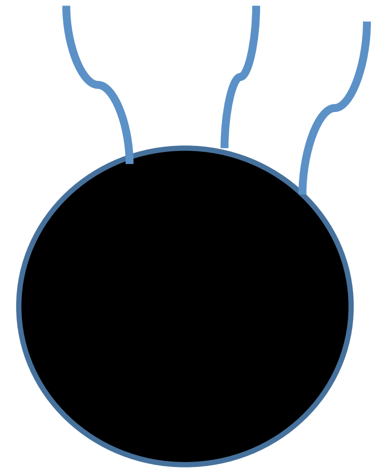


$G_R(\omega, \vec{k})$ =



Faulkner, HL, McGreevy, Vegh; Faulkner, Polchinski

Fermionic black hole hair and Fermi surface

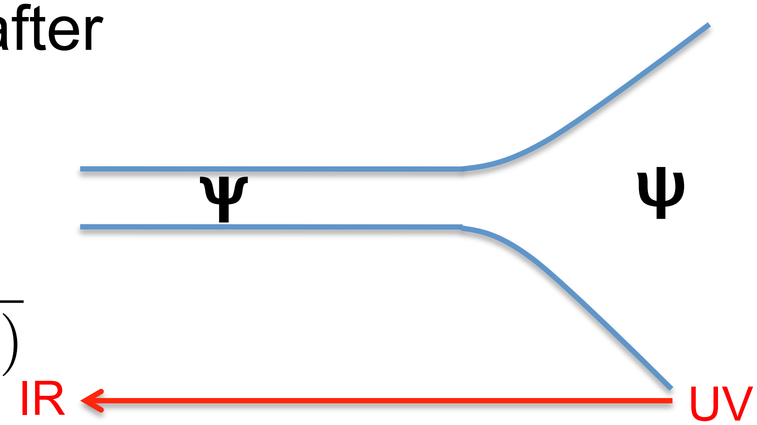


An extremal charged black hole can admit fermionic hair of nonzero momentum at some finite $k=k_F$.

When this happens **ψ** develops a free fermion Fermi surface, but after coupling to AdS_2 region

$$G_R(\omega, k) = \frac{h}{\omega - v_F(k-k_F) + \Sigma(\omega)}$$

$$\Sigma(\omega) = h_2 \mathcal{G}_{k_F}(\omega)$$



Small excitations at the Fermi surface

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \cdots$$

competition

Will treat ν_{k_F} as a tunable parameter

(k_F: controlled by UV physics)

Quasi-particle decay rate:

$$\Gamma \propto \omega^{2\nu}$$

(decay by falling into the black hole)

$\nu > \frac{1}{2}$ long-lived quasi-particles,

$\nu \leq \frac{1}{2}$ No long-lived quasi-particles

$\nu = \frac{1}{2}$ $\Gamma \propto \omega$ as in high Tc cuprates !

Finite temperature: Replace $\omega^{2\nu}$ by a universal scaling function $T^{2\nu} g(\omega / T)$ (known analytically)

Marginal Fermi liquid

For $\nu_{k_F} = \frac{1}{2}$

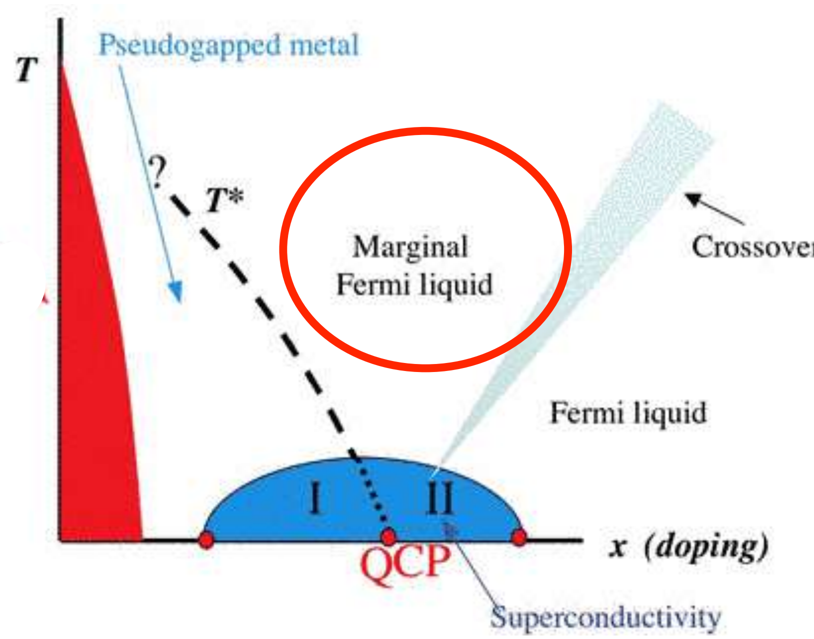
$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

$\tilde{c}_1$: real

c_1 : complex

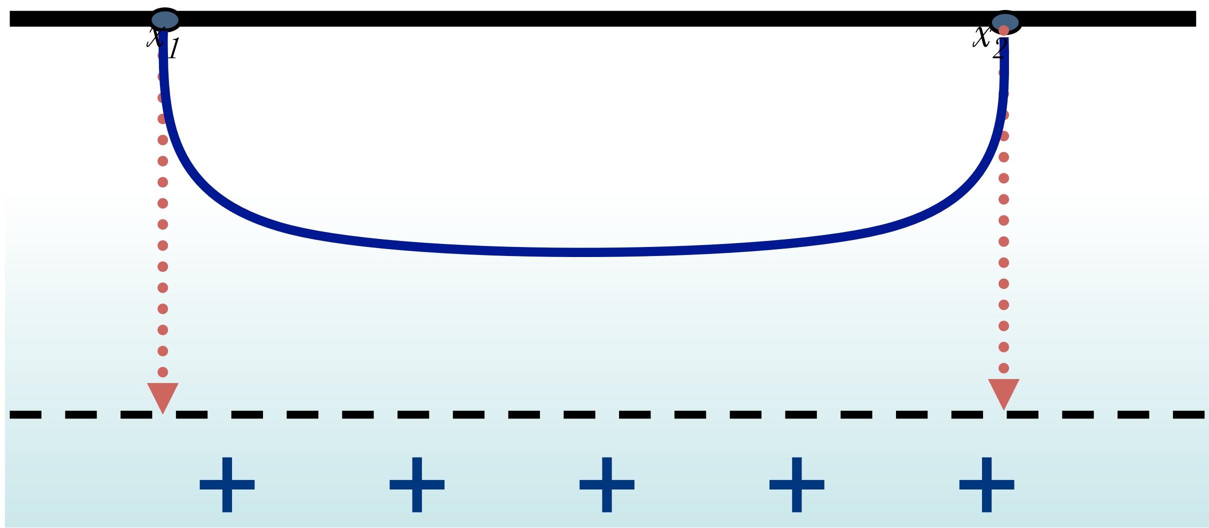
Precisely that for
``Marginal Fermi liquid''
proposed on phenomenological
 ground for high Tc cuprates
near optimal doping.

Varma, Littlewood, Schmitt-Rink,
Abrahams, Ruckenstein (89)



Bulk physical picture

Single-particle decay rate



How fast it can decay depends on the scaling dimension in the AdS_2 region.

Summary

Operator dimensions in the IR CFT ⟺ Scaling exponents near the Fermi surface

Self-energy is analytic in k/μ: local quantum criticality

Depending on values of m and q, we can have

- Fermi surface with stable quasi-particles ($\nu_{k_F} > 1/2$)
- Fermi surface without quasi-particles ($\nu_{k_F} < 1/2$)

Marginal Fermi liquid for high Tc cuprates arises

for $\nu_{k_F} = \frac{1}{2}$

How about resistivity?

Important:

None of the leading order (in 1/N) thermodynamical or transport properties of the system will be sensitive to the presence of the Fermi surface.

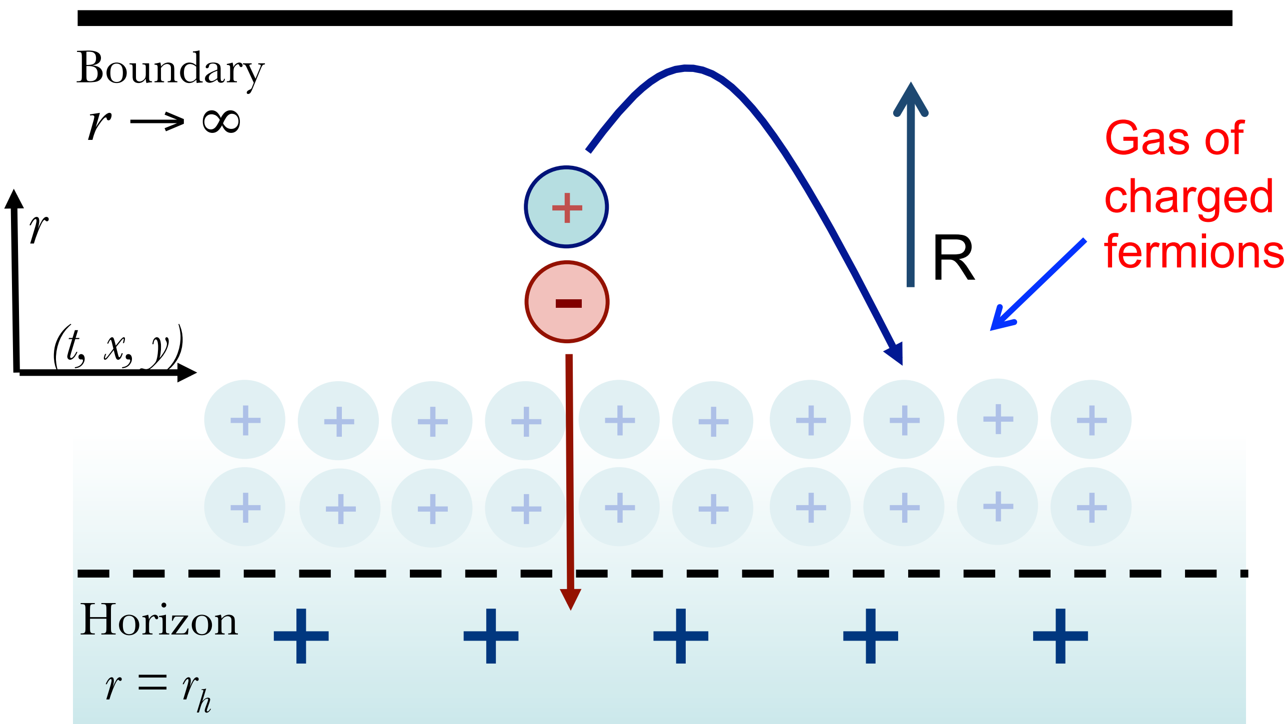
k_F is of order O(1), which implies a charge density of order O(1).

The total charge density is $O(N^2)$

Thus the charge density associated with the Fermi surface is only a tiny bit of the whole system.

Finite fermion density

Consider a charged fermionic field outside the black hole:



bulk charged fermion gas

a finite O(1) density of fermions in a strongly interacting boundary theory

Fermion: reflection probability R < 1 , leading to an equilibrium

Scalar: R > 1 (superradiance) , will grow and condense.

In boundary theory:

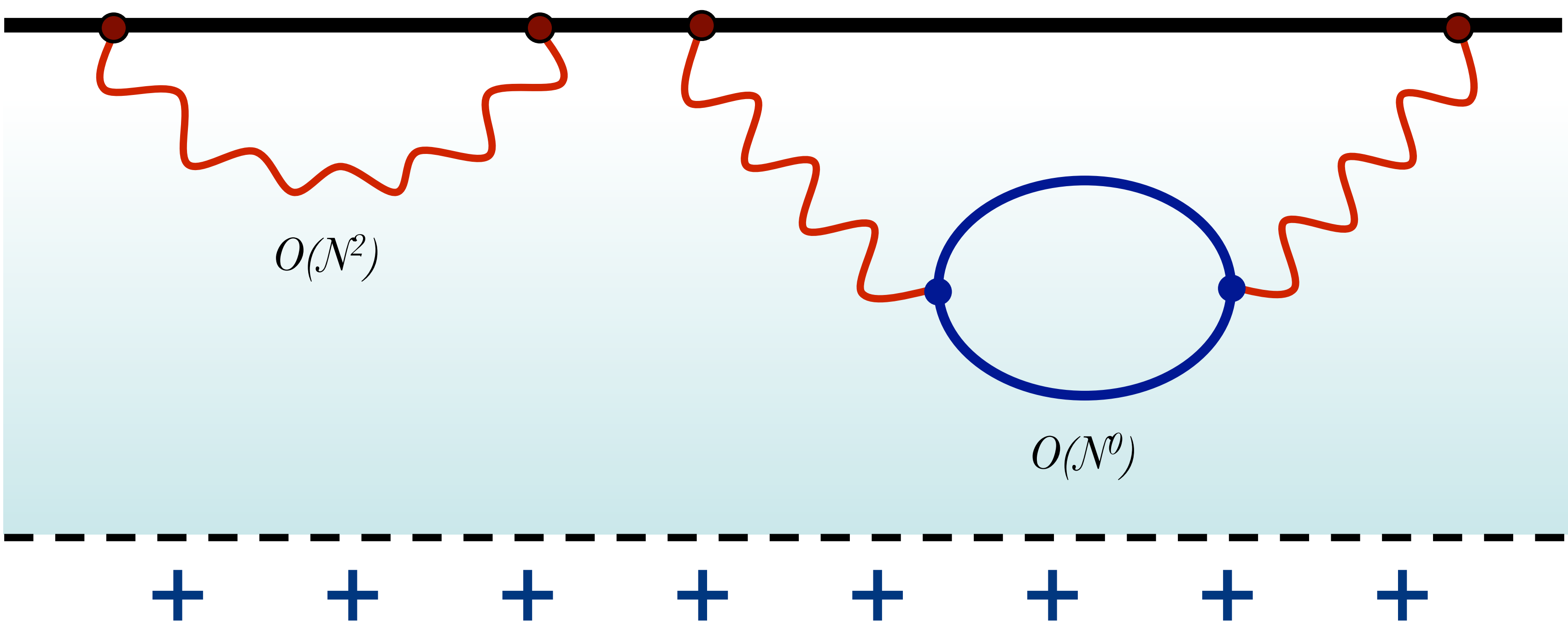
$$\mathcal{O}(\epsilon)^\dagger \mathcal{O}(0) \sim \frac{1}{\epsilon^{2\Delta}} + \frac{c(\Delta)}{N^2}\frac{qJ^0}{\epsilon^{2\Delta-2}} + \cdots$$

$$\Rightarrow \quad n_{\mathcal{O}} = \left\langle \mathcal{O}(\epsilon)^\dagger \mathcal{O}(0) \right\rangle \sim \frac{1}{\epsilon^{2\Delta}} + \frac{c(\Delta)}{N^2}\frac{q\langle J^0 \rangle}{\epsilon^{2\Delta-2}} + \cdots$$

Conductivity

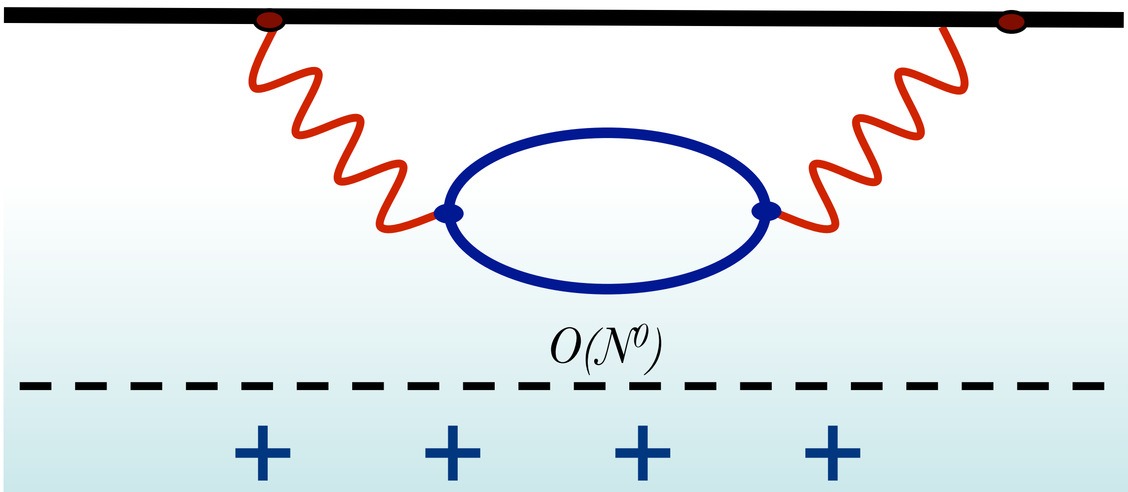
$$\sigma(\omega) = \frac{1}{i\omega} \langle J_x(\omega) J_x(-\omega) \rangle_{\text{retarded}}$$

$$J_x \Leftrightarrow A_x$$



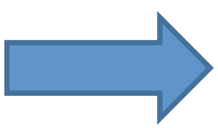
Conductivity from fermions

Faulkner, Iqbal, HL, McGreevy, Vegh
Science 329, 1043 (2010)



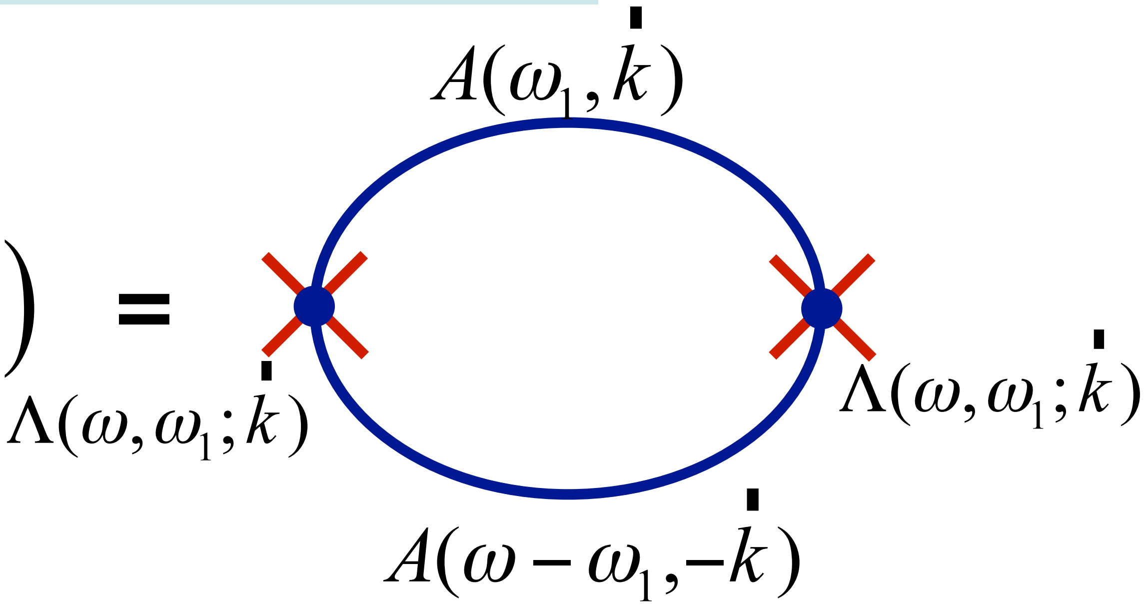
One-loop calculation in gravity:

many subtleties and potential pitfalls ……



after an epic calculation

$$\sigma(\omega) =$$



In the low temperature limit, the contribution near the Fermi surface dominates, for which

$$\Lambda(\omega, \omega_1; \vec{k}) \sim O(1) \quad \Rightarrow$$

$$\sigma_{FS} \propto T^{-\alpha} \qquad \text{with} \qquad \alpha = 2\nu_{k_F}$$

For marginal fermi liquid (relevant for cuprates) $\nu_{k_F} = \frac{1}{2}$

$$\sigma_{FS} \propto T^{-1}$$ leading to linear resistivity !

The precise prefactor can also be calculated (in progress)

Optical conductivity

$$\nu_{k_F} < \frac{1}{2}: \ \sigma(\omega) = T^{-2\nu_{k_F}} F_1\left(\frac{\omega}{T}\right) \Rightarrow a(i\omega)^{-2\nu_{k_F}}, \quad T \ll \omega \ll \mu$$

Scaling form, no quasi-particle

$\nu_{k_F} > \frac{1}{2}$ quasi-particle lifetime $\Gamma^{-1} \sim T^{-2\nu_{k_F}} \gg T^{-1}$

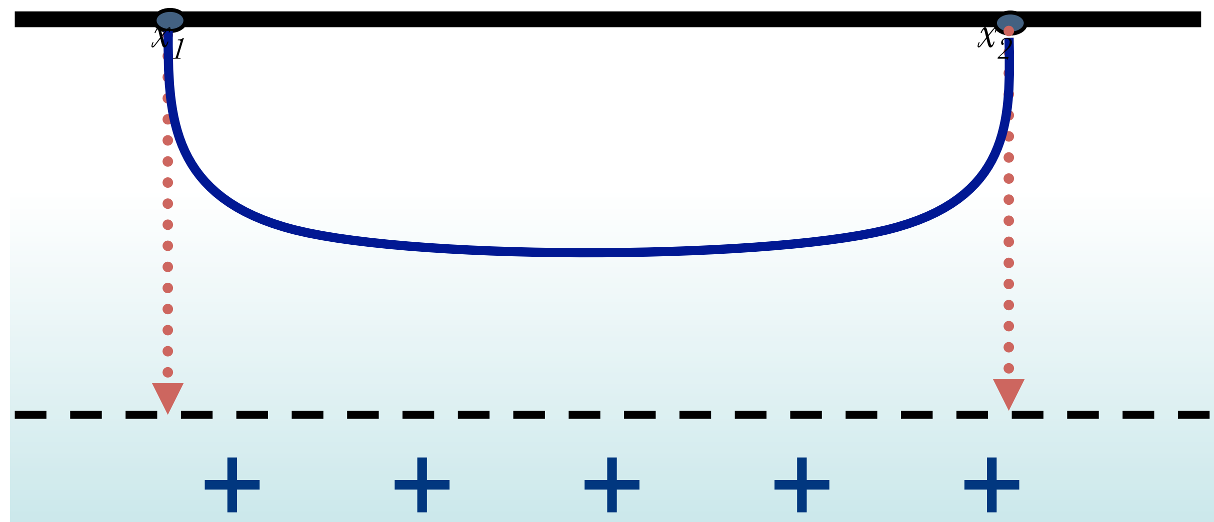
$$\sigma(\omega) \sim \begin{cases} \frac{\omega_p^2}{\frac{1}{\tau} - i\omega} & \omega \sim \tau^{-1} \sim \Gamma \\ \frac{i\omega_p^2}{\omega} + b(i\omega)^{2\nu_{k_F}-2} & T \ll \omega \ll \mu \end{cases}$$

$$\nu_{k_F} = \frac{1}{2}: \ \sigma(\omega) = T^{-1} F_2\left(\frac{\omega}{T}, \log\frac{T}{\mu}\right)$$

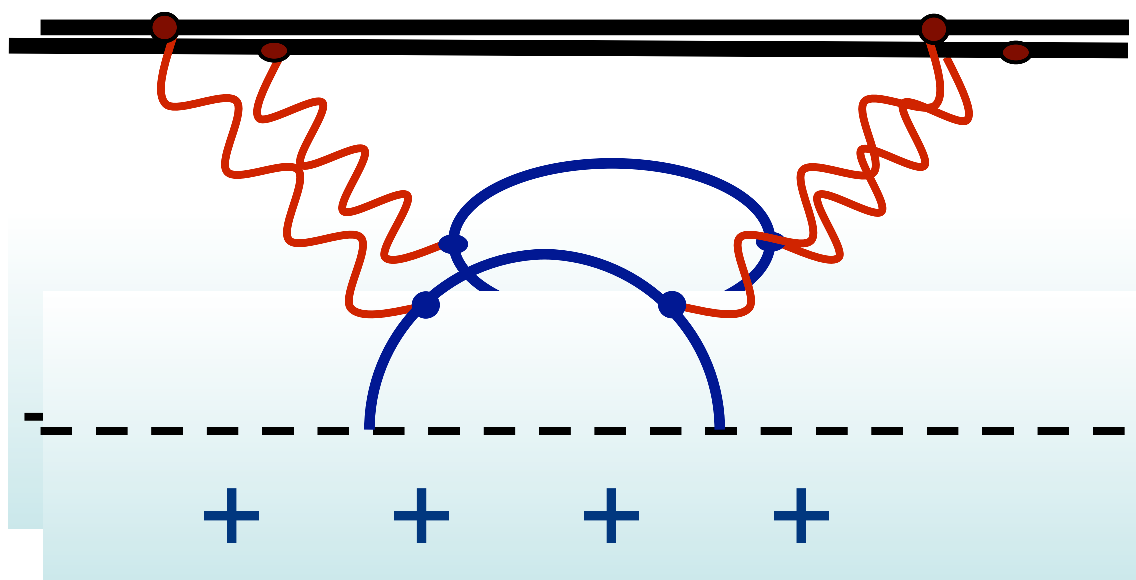
$$\sigma(\omega) \propto \frac{i}{\omega}\left(\frac{1}{\log\frac{\omega}{\mu}} + \frac{1}{(\log\frac{\omega}{\mu})^2}\frac{1+i\pi}{2}\right) + \cdots, \quad \omega \gg T$$

Bulk physical picture

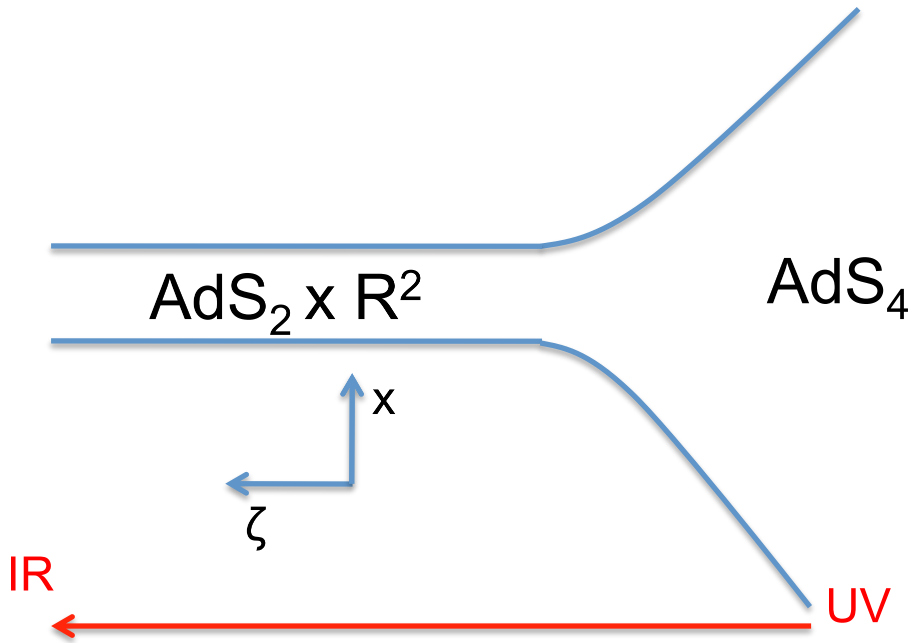
Single-particle decay rate



Decay of a current



Summary



Fermi surface with or without quasi-particles

and

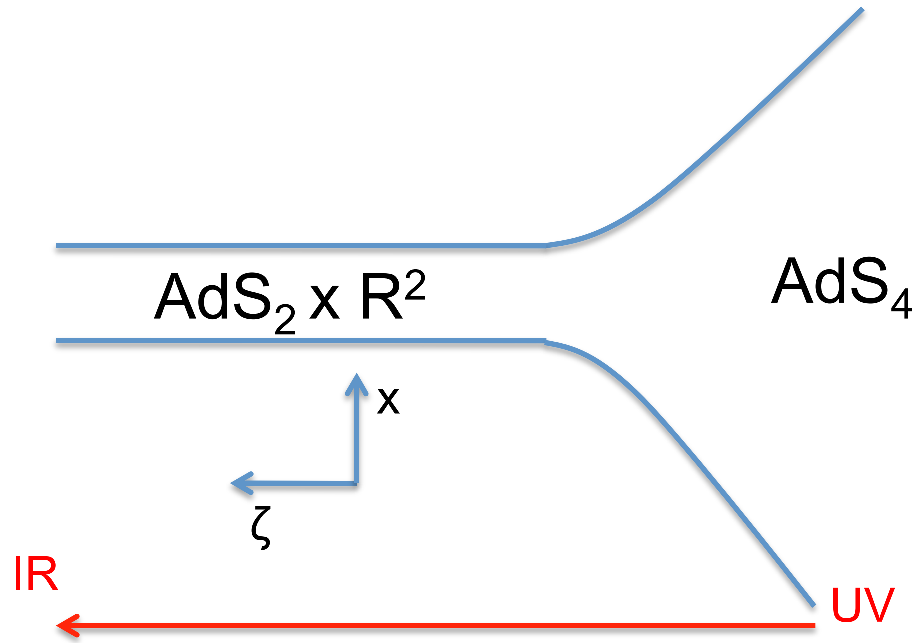
Transport behavior

All boil down to the interplay between

Strong coupled physics in AdS$_2$ (local, non-analytic, dissipative)

hybridized with

IR physics outside AdS$_2$ (analytic, mean-field like, non-dissipative)

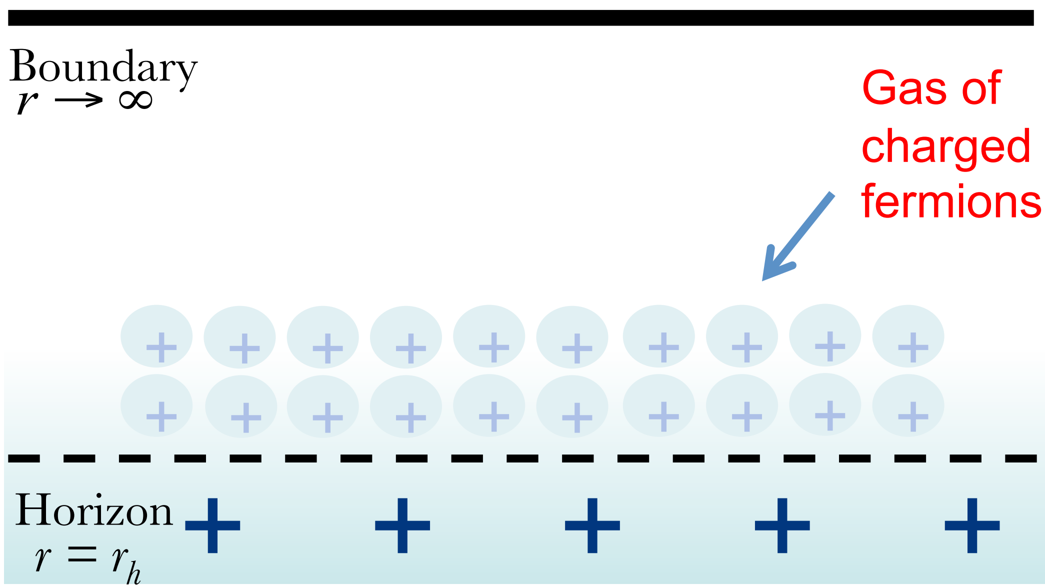


When consider a bosonic field in this geometry,
it could condense when dialing external parameters,
leading to a new phase.

Exactly the same kind of interplay between the AdS_2 and outside region leads to novel quantum phase transitions of non-Landau type!

Finite N and back reaction of fermionic gas

Hartnoll, Polchinski, Silverstein, Tong



Backreaction of the fermionic gas: the spacetime becomes Lifshitz at a sufficiently small scale)

$$\Lambda \sim e^{-cN^2}$$

Genuine vacuum physics below Λ appears to be a Fermi liquid.

Generalizations:

1. Turn on a magnetic field, quantum oscillations

   Albash and Johnson; Basu, He, Mukherjee, Shieh;
   Denef, Hartnoll and Sachdev; Hartnoll, Hofman, ….

2. Couple fermions to a superconducting condensate

   Chen, Kao, Wen; Faulkner et al; Gubser, Rocha, Talavera,
   Gubser, Rocha, Yarom, ….

3. Pairing instability of (non)-Fermi liquids

   Hartman, Hartnoll, …

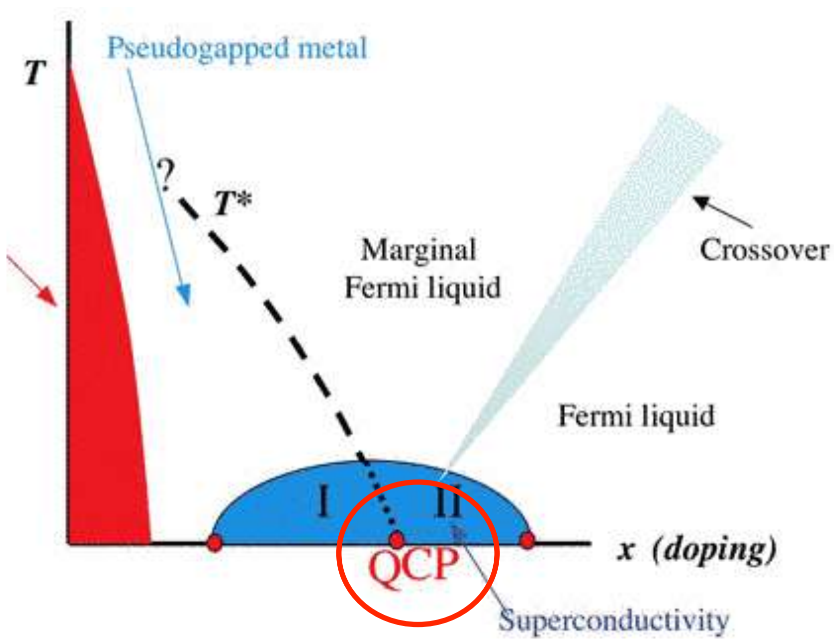
There are many other questions to explore:

specific heat, hydrodynamics of non-Fermi liquids, thermal conductivity, Hall conductivitiy, …. …..

Some perspective

I have talked about two aspects of the gravity example at $\nu_{k_F} = \frac{1}{2}$ which matches perfectly with high Tc cuprates.

→ a good laboratory for studying many other questions related to high Tc or other materials



Could it be that our IR CFT lie in the same universality class of the (conjectured) quantum critical point for high Tc cuprates?

Thank You

Additional materials

Imaginary exponent

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k} \qquad\qquad \delta_k = \frac{1}{2} + \nu_k$$

$\nu_k = -i\lambda_k$ is pure imaginary for small enough k when

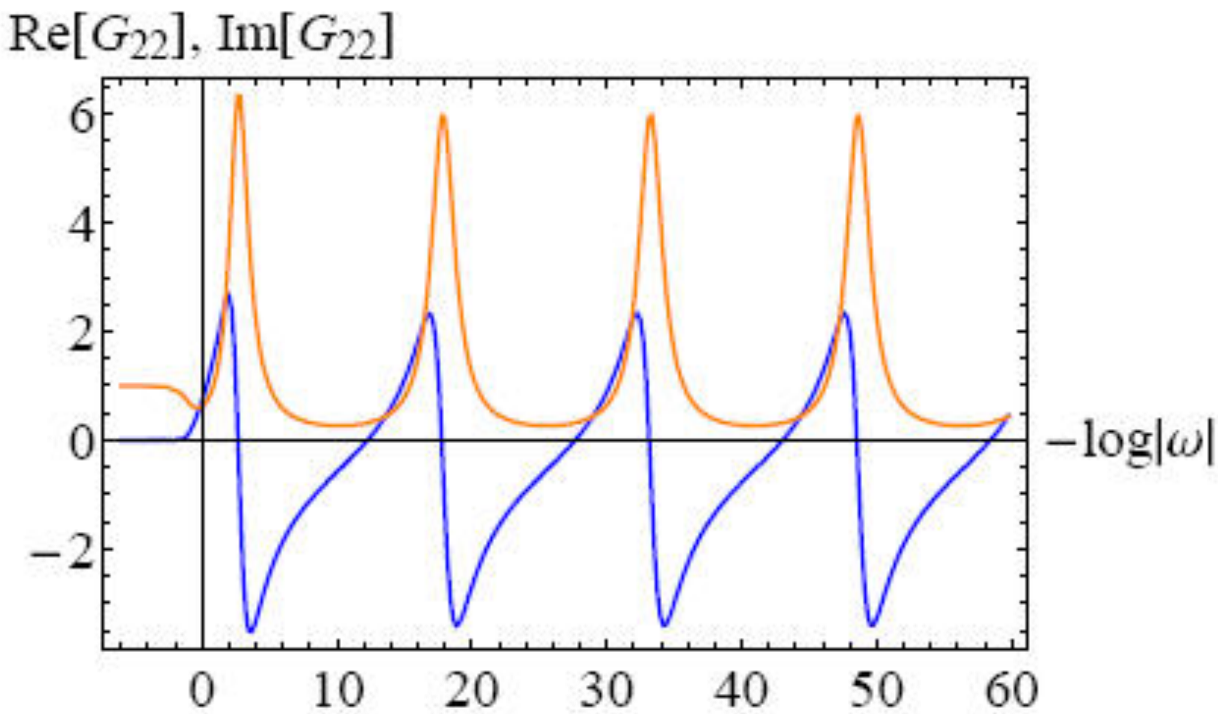
$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

$$G_R(\omega, k) \approx \frac{b_+^{(0)} + b_-^{(0)} c(k)\omega^{-2i\lambda_k}}{a_+^{(0)} + a_-^{(0)} c(k)\omega^{-2i\lambda_k}} + O(\omega)$$

Note: no instability

Log-periodic behavior

This leads to a discrete scaling symmetry and



Conditions for Fermi surface

For what values of q and Δ, are Fermi surfaces allowed? i.e. when fermionic hair exists

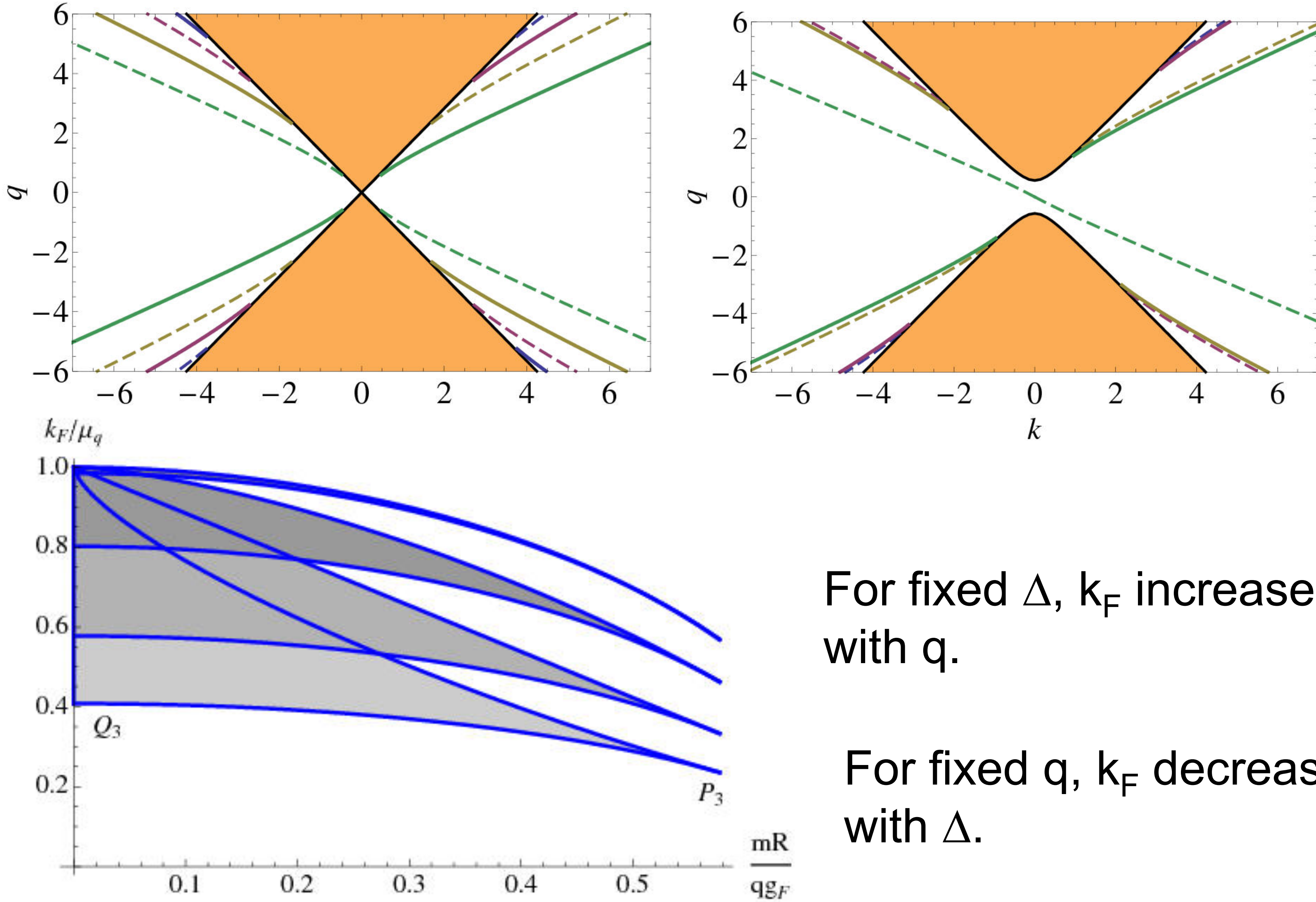
$$\Delta < \frac{|q|}{\sqrt{3}} + \frac{d}{2}$$

It always lies inside the region which allows log-periodic behavior

$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

Except for $$\frac{d-1}{2} < \Delta < \frac{d}{2} - \frac{|q|}{\sqrt{2}}$$ (alternative quantization)

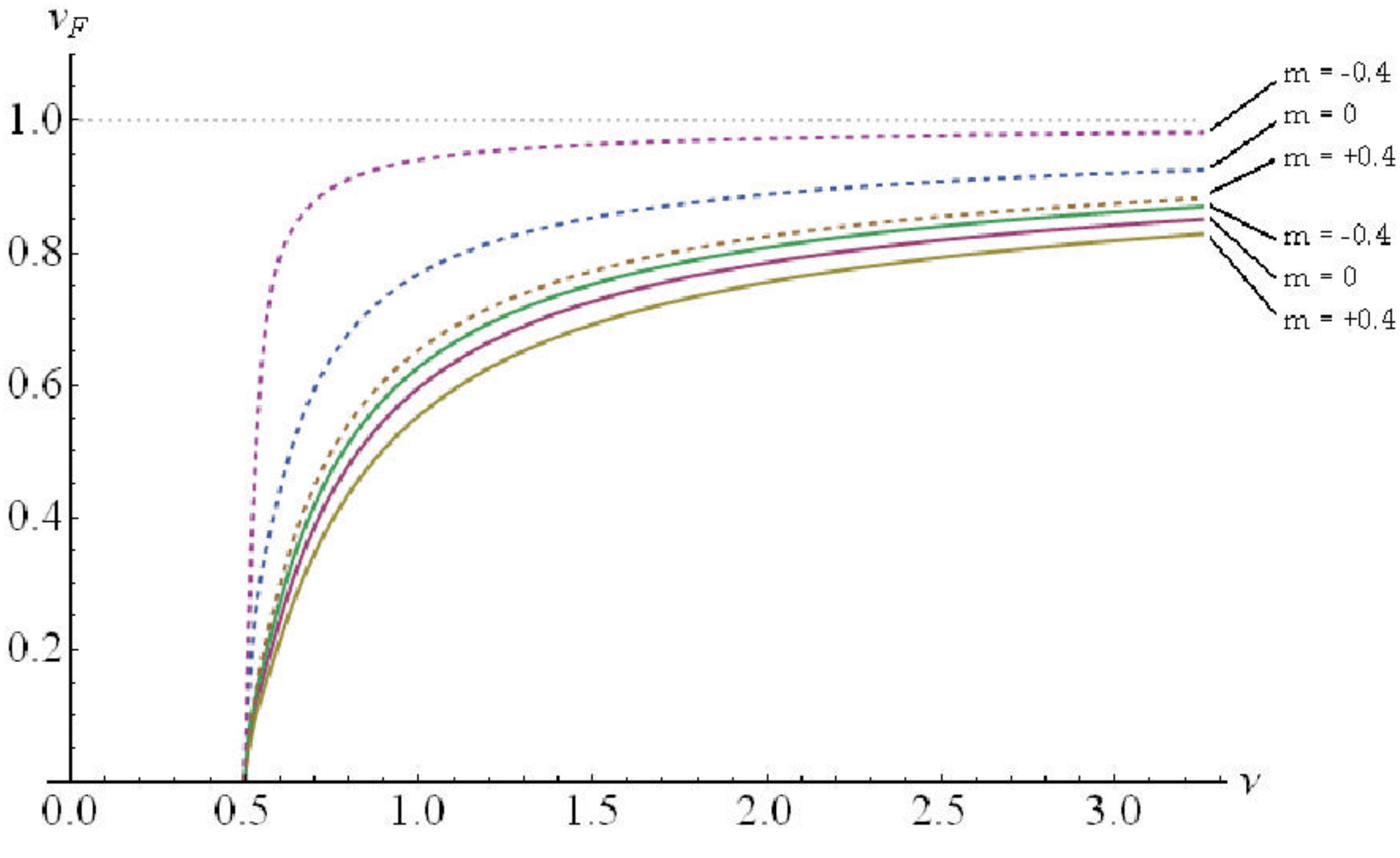
How does k_F depend on q and Δ ?



For fixed Δ, k_F increases with q.

For fixed q, k_F decreases with Δ.

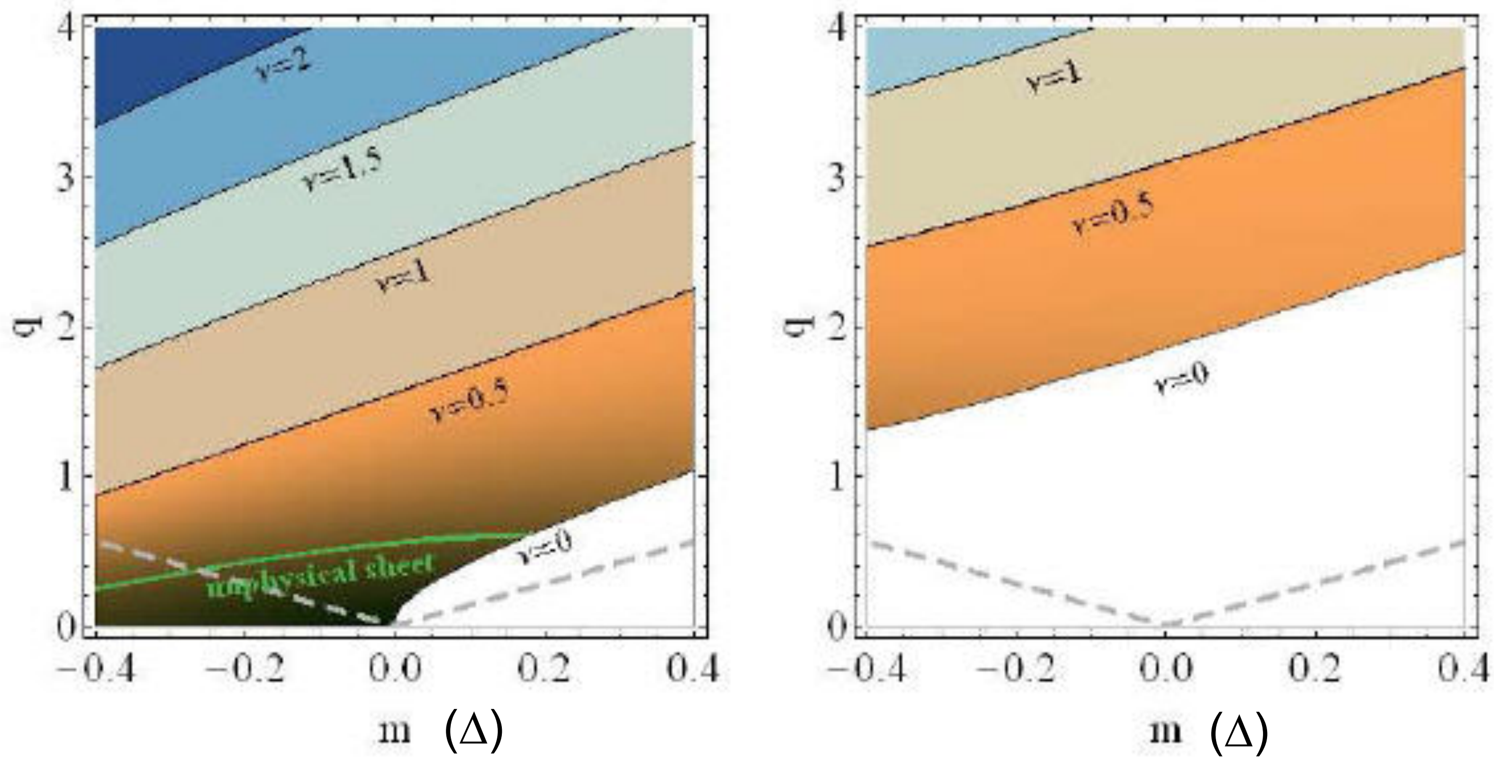
UV data: Fermi Velocity



Fermi velocity goes to zero as the marginal limit is approached, so does the residue.

Landscape of exponents

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}}, \quad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}}$$



Small frequency expansion

$$G_R(\omega, k) = \frac{b_+(\omega, k) + \mathcal{G}_k(\omega) b_-(\omega, k)}{a_+(\omega, k) + \mathcal{G}_k(\omega) a_-(\omega, k)}$$

$G_k(\omega)$: retarded function for $O_{\vec{k}}$ in the IR CFT, depending only on the AdS_2 region. (IR data)

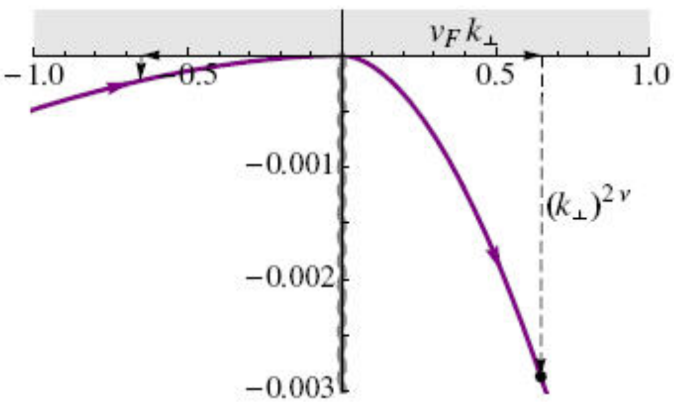
(generically) non-analytic in ω and complex (dissipative)

$a_\pm, b_\pm$: from solving the Dirac equation in the UV region

Real, analytic in ω and k, expressed in power series of ω.

(UV data)

For $\nu_{k_F} > \frac{1}{2}$ pole $\omega = \omega_* - i\Gamma$



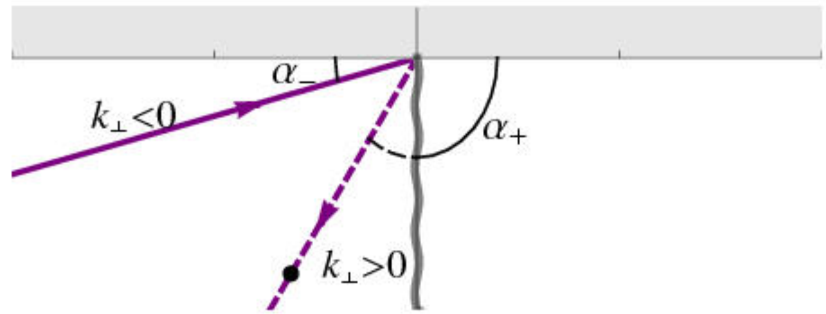
$$\omega_*(k) = v_F k_\perp + \cdots , \qquad \frac{\Gamma(k)}{\omega_*(k)} \propto k_\perp^{2\nu_{k_F}-1} \to 0, \quad Z = h_1 v_F$$

Linear dispersion relation, the quasi-particle becomes stable approaching the Fermi surface,
non-vanishing residue at the Fermi surface.

Quasi-particle picture applies, like in Fermi liquids.

But $\Gamma \propto \omega^{2\nu_k}$

For $\nu_{k_F} < 1/2$ pole $\omega = \omega_* - i\Gamma$



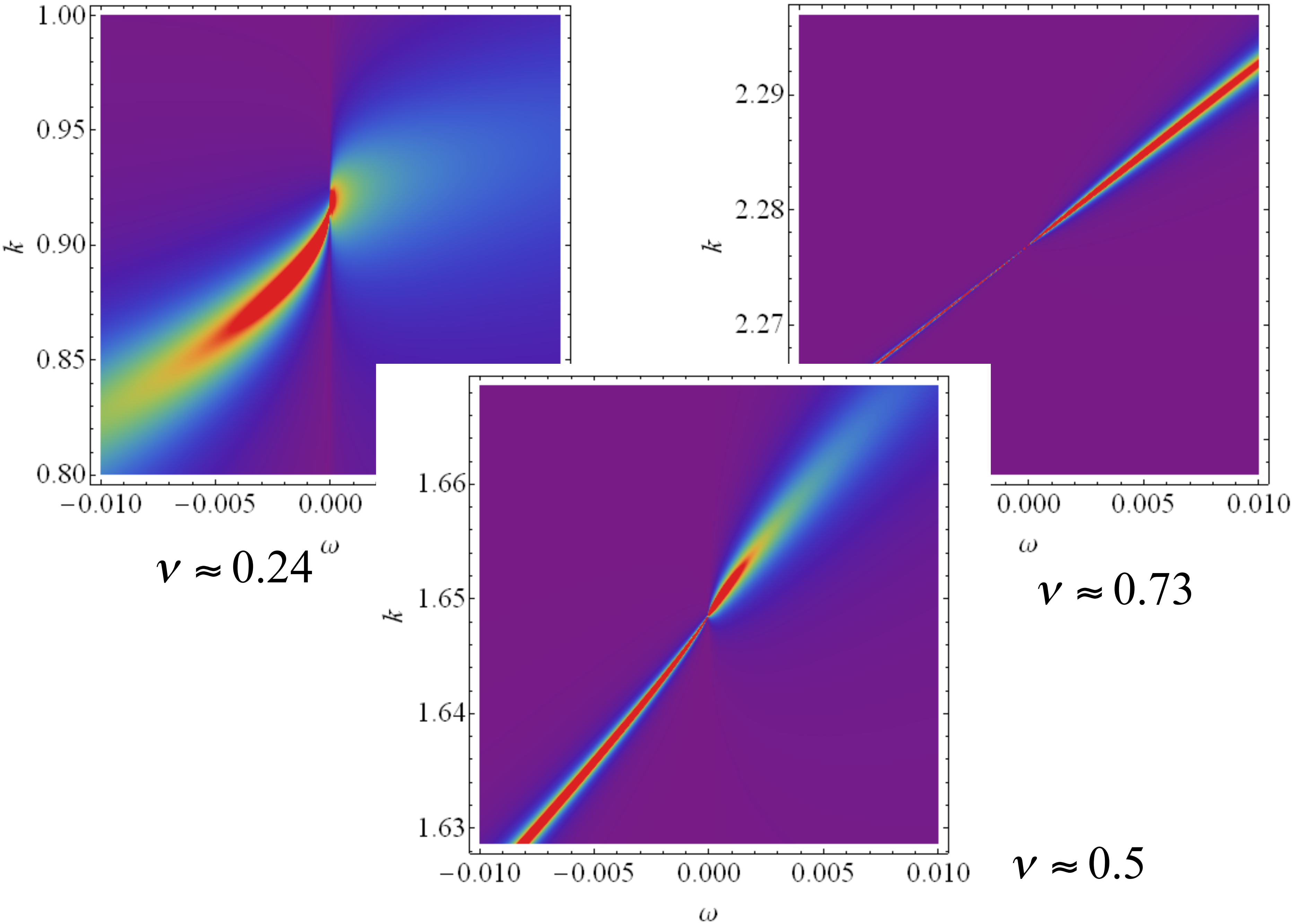
$$\omega_*(k) \sim k_\perp^z, \quad z = \frac{1}{2\nu_{k_F}} > 1, \quad \frac{\Gamma(k)}{\omega_*(k)} = \text{const}$$

Imaginary part is always comparable to the real part
(quasi-particle never stable)

$$Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \to 0, \quad k_\perp \to 0$$

Residue of the pole vanishes
at the Fermi surface

Fermi surface without sharp quasi-particles !



k
1.00
0.95
0.90
0.85
0.80
−0.010
−0.005
0.000
ω
ν ≈ 0.24
k
2.29
2.28
2.27
0.000
0.005
0.010
ω
ν ≈ 0.73
k
1.66
1.65
1.64
1.63
−0.010
−0.005
0.000
0.005
0.010
ω
ν ≈ 0.5