Neutrino oscillations: theory and phenomenology

Evgeny Akhmedov

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A bit of history...

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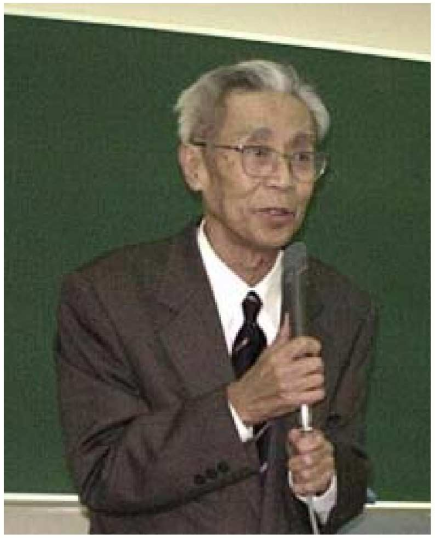
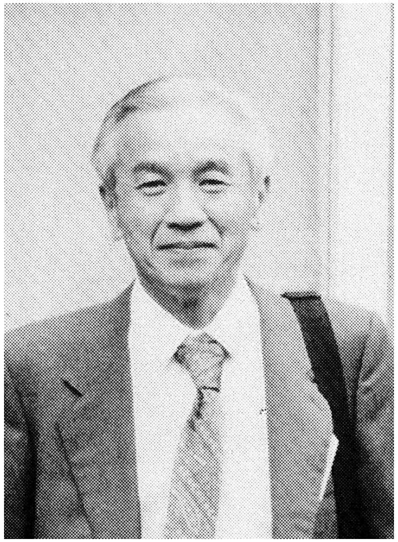
B. Pontecorvo
1913 - 1993

S. Sakata
1911 – 1970

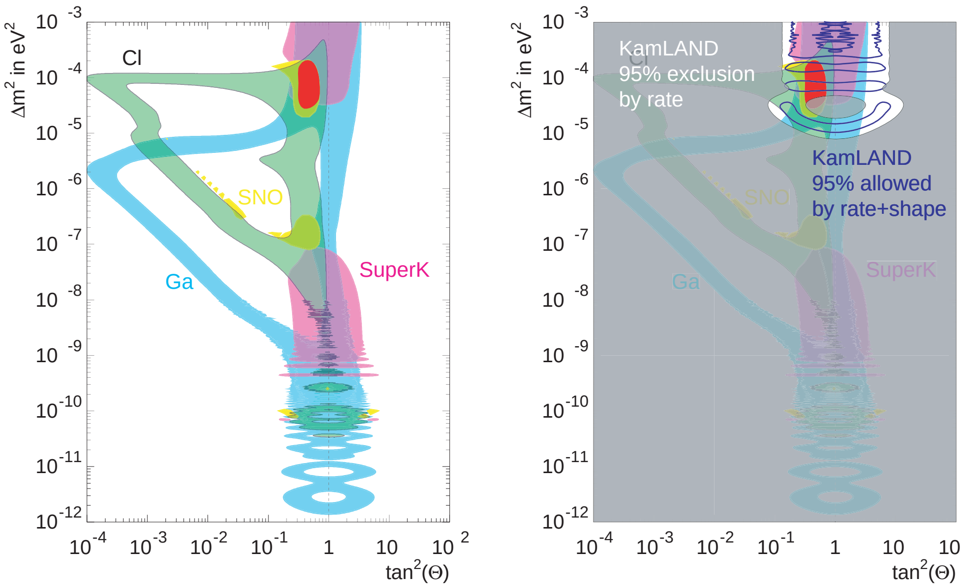


Z. Maki
1929 – 2005

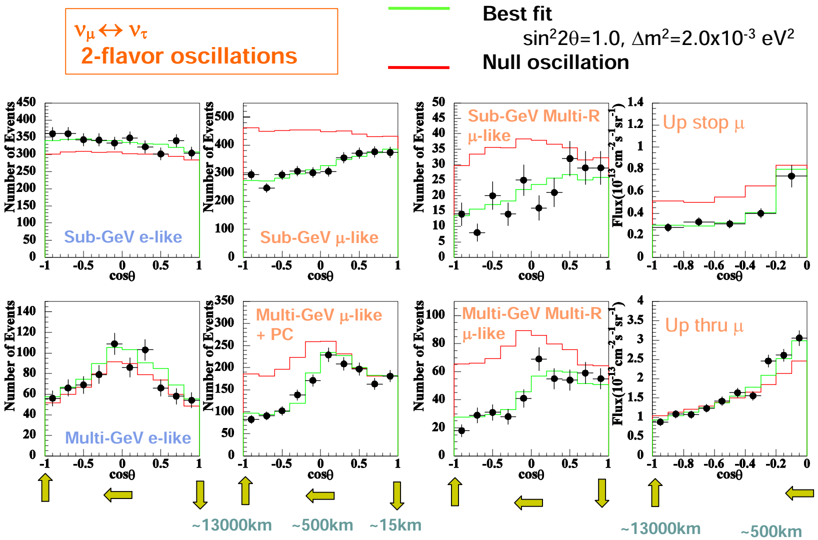


M. Nakagawa
1932 – 2001

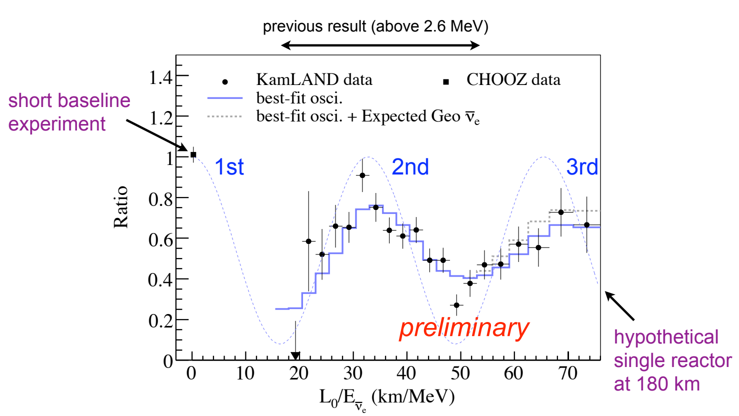
Oscillations discovered experimentally !



Zenith angle distributions



Neutrino Oscillation

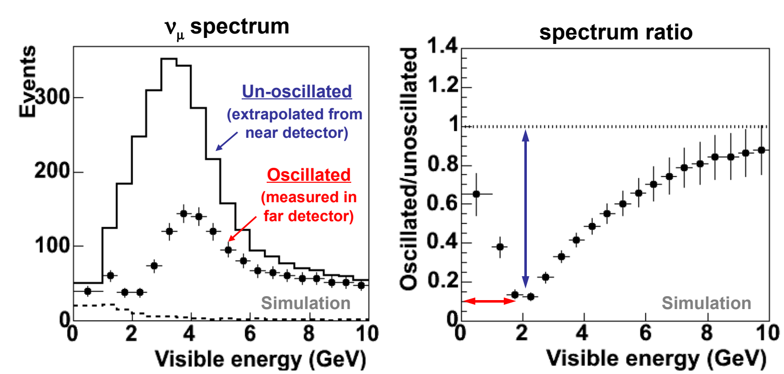


KamLAND covers the 2nd and 3rd maximum

→ **characteristic of neutrino oscillation**

ν_μ Disappearance Measurement

Look for ν_μ deficit : $P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{E}\right)$



Andy Blake, Cambridge University

The MINOS Experiment, slide 7

I. Phenomenology

Leptonic mixing

For $m_\nu \neq 0$ weak eigenstate neutrinos ν_e, ν_μ, ν_τ do not coincide with mass eigenstate neutrinos ν_1, ν_2, ν_3

Diagonalization of leptonic mass matrices:

$$e_L \;\to V_L\, e_L\,, \qquad \nu_L \;\to U_L\, \nu_L\; \ldots \qquad \Rightarrow$$

$$-\mathcal{L}_{w+m} \;=\; \frac{g}{\sqrt{2}}(\bar{e}_L \gamma_\mu\, V_l^\dagger U_L\, \nu_L)\, W^\mu \;+\; \text{diag.}\;\; \text{mass}\;\; \text{terms}$$

Leptonic mixing matrix: $U \;=\; V_l^\dagger U_L$

$$\diamondsuit \qquad |\nu_a^{\text{fl}}\rangle = \sum_i U_{ai}^*\, |\nu_i^{\text{mass}}\rangle$$

Oscillation probability in vacuum

For relativistic neutrinos: $E \simeq p + \frac{m^2}{2p}$, $\quad L \simeq t$,

$\diamondsuit$

$$P(\nu_a \to \nu_b; L) = \left| \sum_i U_{bi}\, e^{-i\frac{\Delta m_{ij}^2}{2p}L}\, U_{ai}^* \right|^2$$

– standard oscillation formula. For 2-flavor oscillations (good first approximation in many cases):

$$|\nu_e\rangle \;=\; \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$\diamondsuit$ $\quad P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}\,L\right)$

3ν vs $N_\nu \geq 4$ oscillation schemes

Most of the current ν data can be explained in terms of oscillations between the 3 known neutrino species $(\nu_e, \nu_\mu, \nu_\tau)$.

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta 0\nu$ decay ...

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However, the evidences are not strong!

Still, even if e.g. LSND result is disproved, this would not exclude the possibility of light ν_s – an intriguing possibility with important implications to particle physics, astrophysics and cosmology

3f neutrino mixing and oscillations

For 3 neutrino species: mixing matrix $\tilde{U}$ depends on θ_{12}, θ_{23}, θ_{13}, $\delta_{\rm CP}$, $\sigma_{1,2}$. Majorana-type $\not{C}\not{P}$ phases can be factored out in the mixing matrix:

$$\tilde{U} = U K \,, \qquad\qquad K = {\rm diag}(1\,, e^{i\sigma_1}\,, e^{i\sigma_2})$$

$\Rightarrow$ Majorana-type phases do not affect neutrino oscillations.

The relevant part of the mixing matrix:

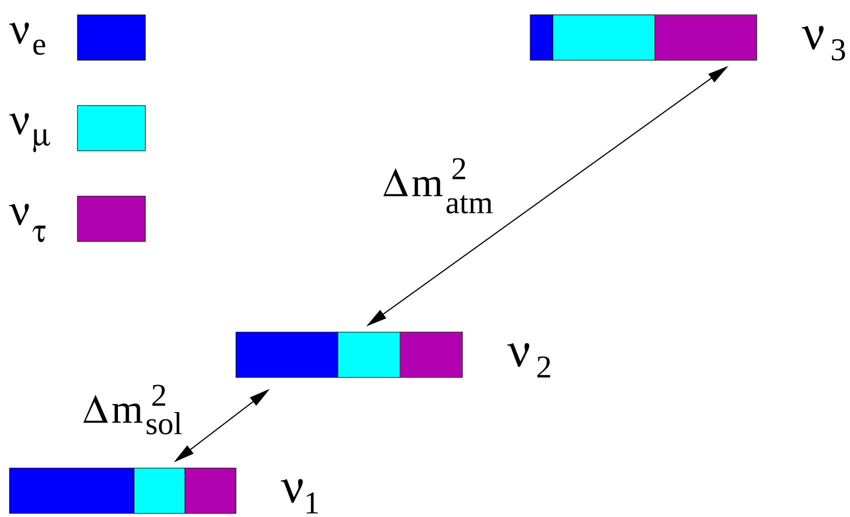
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= O_{23}\,(\Gamma_\delta\, O_{13}\, \Gamma_\delta^\dagger)\; O_{12}\,, \qquad\qquad \Gamma_\delta \equiv {\rm diag}(1\,,\; 1\,,\; e^{i\delta_{\rm CP}})$$

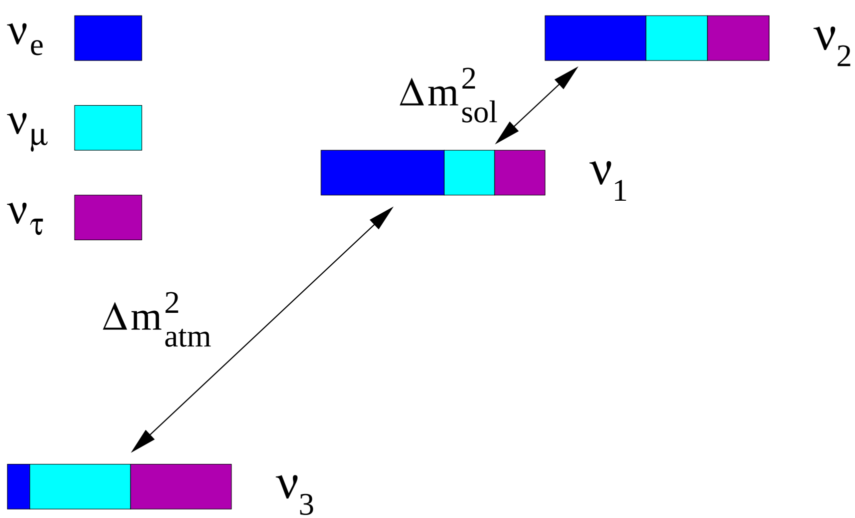
Leptonic mixing

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Normal hierarchy:



Inverted hierarchy:



Neutrino oscillations in matter

The MSW effect (Wolfenstein, 1978; Mikheyev & Smirnov, 1985)

Matter can change the pattern of neutrino oscillations drastically

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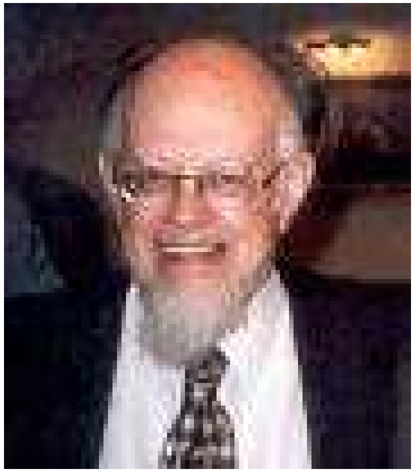
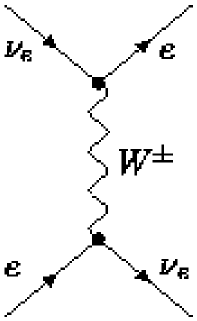
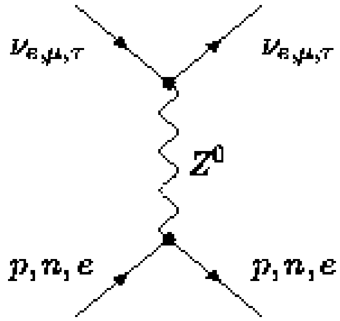


Neutrino oscillations in matter

Coherent forward scattering on the particles in matter





$$V_e^{\rm CC} \equiv V = \sqrt{2}\, G_F\, N_e$$

2f neutrino evolution equation:

◇ $$i\frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

Mixing in matter

♢ $$\sin^2 2\theta_m = \frac{\sin^2 2\theta \cdot (\frac{\Delta m^2}{2E})^2}{[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e]^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}$$

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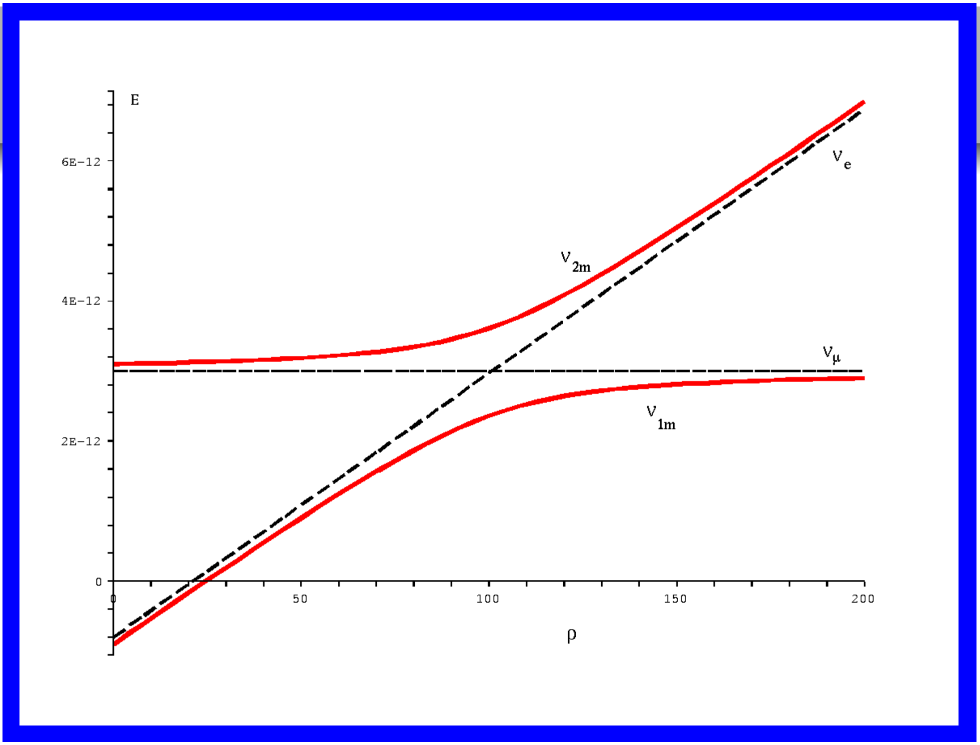
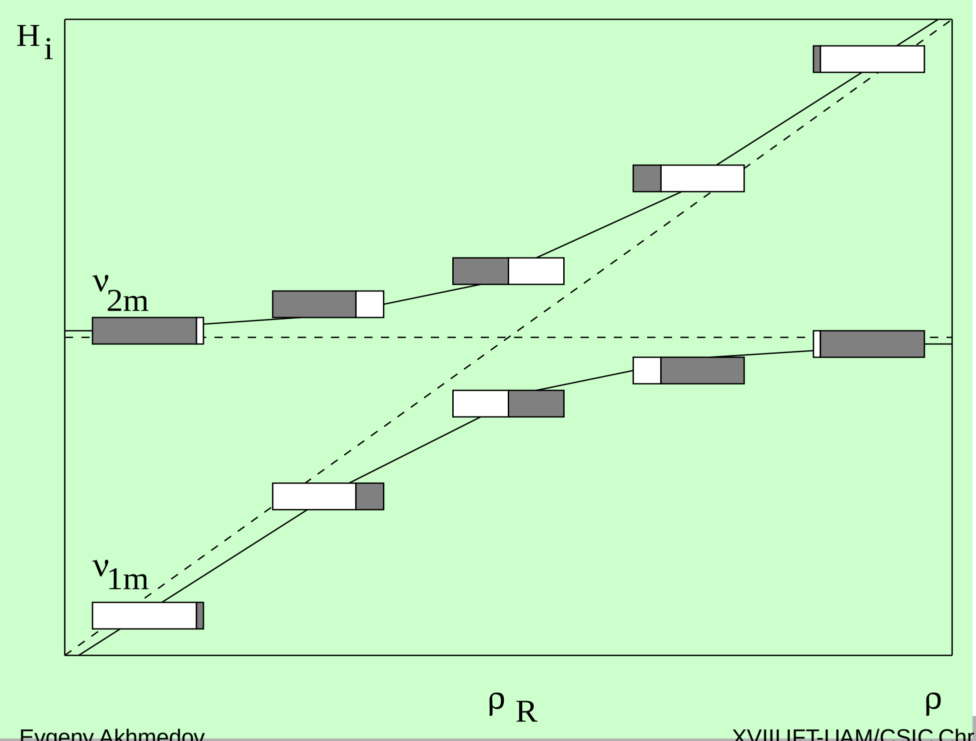
$$N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90^\circ$$

$$N_e = (N_e)_{\rm res}: \quad \theta_m = 45^\circ$$

$$N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$$

$|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – eigenstates of H in matter (matter eigenstates)





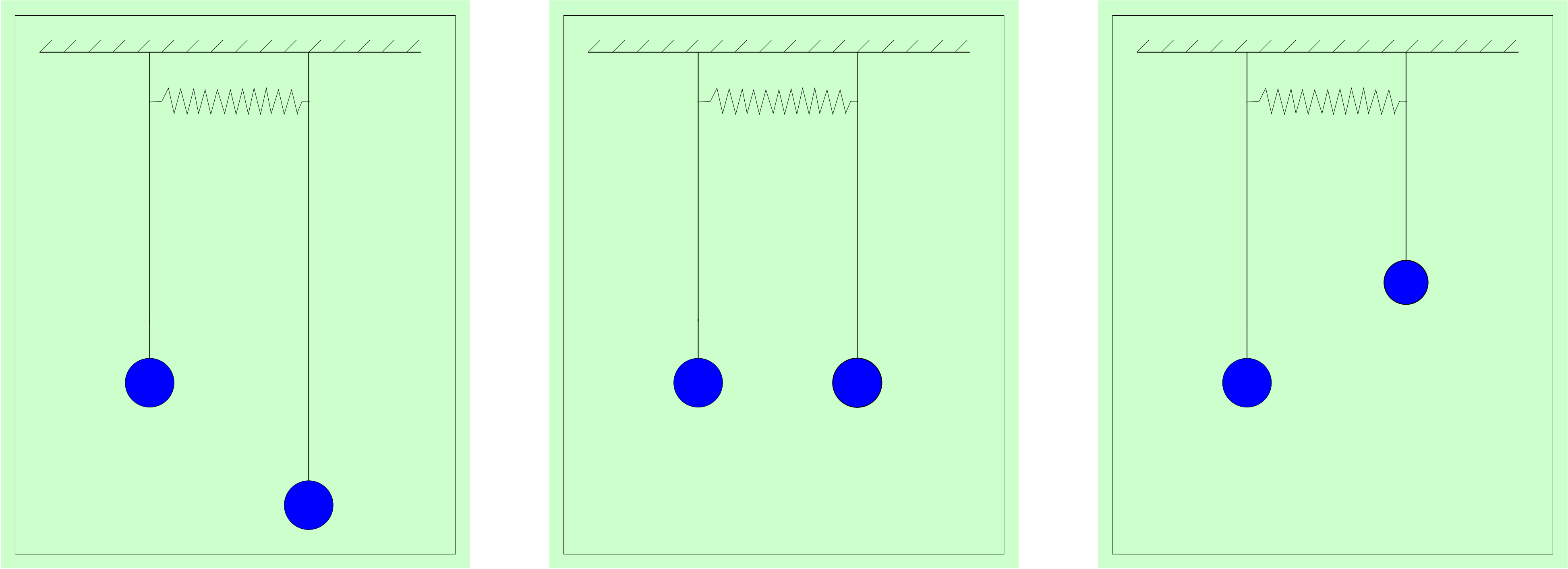
Adiabaticity: slow density change along the neutrino path

$$\frac{\sin^2 2\theta}{\cos 2\theta}\,\frac{\Delta m^2}{2E}\,L_\rho \gg 1$$

L_ρ – electron density scale hight:

$$L_\rho = \left|\frac{1}{N_e}\frac{dN_e}{dx}\right|^{-1}$$

Analogy: Two coupled pendula



Mechanical model of the MSW effect

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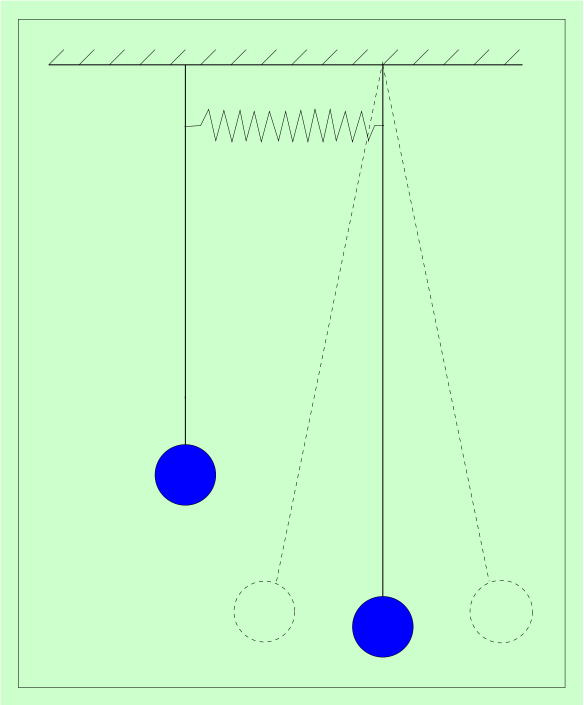

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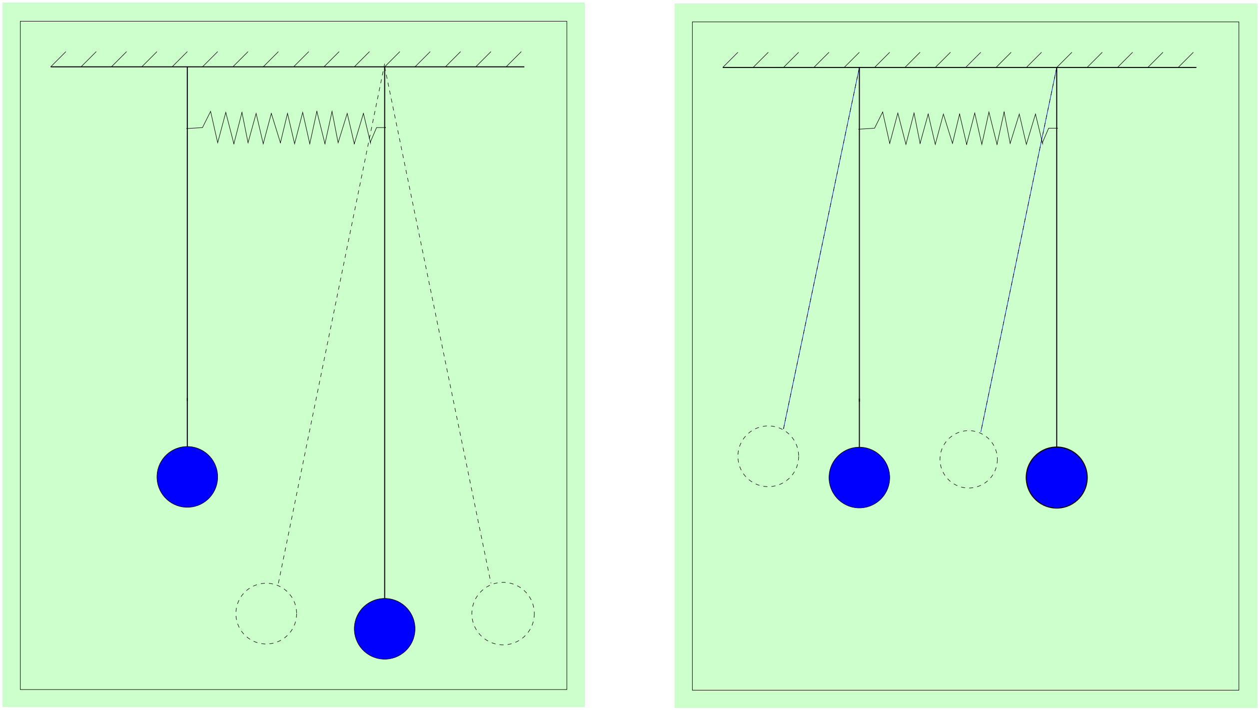

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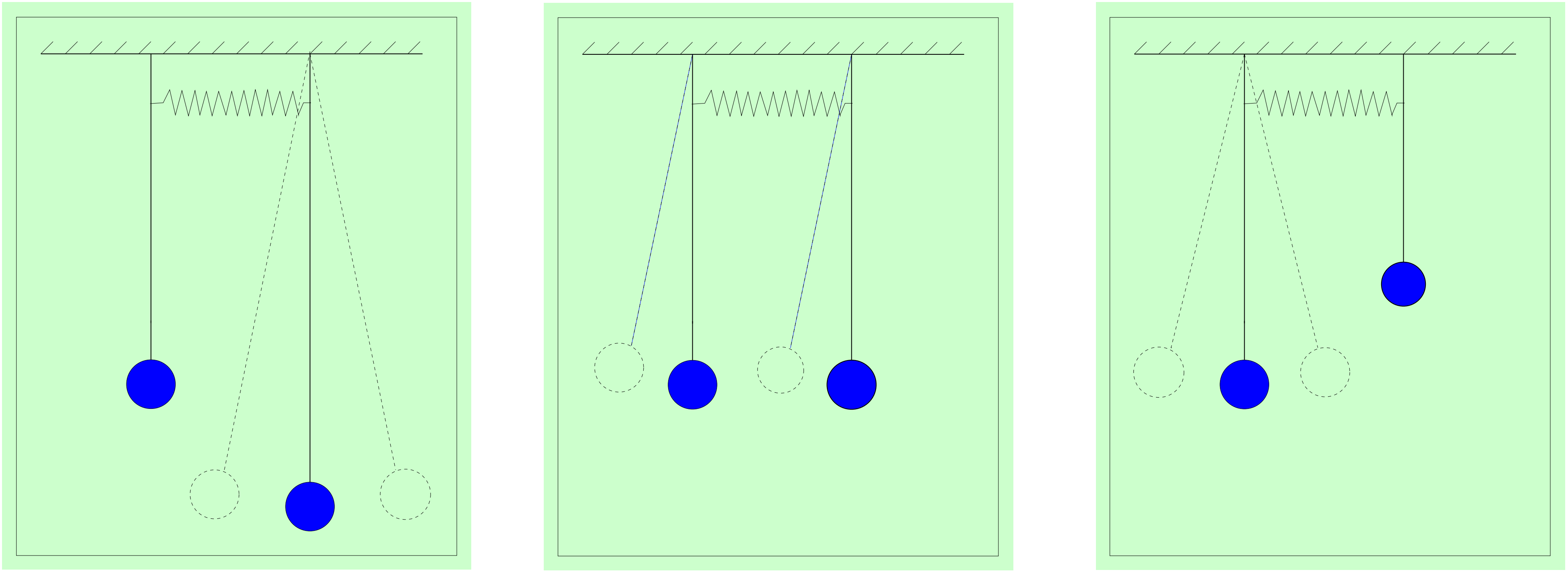
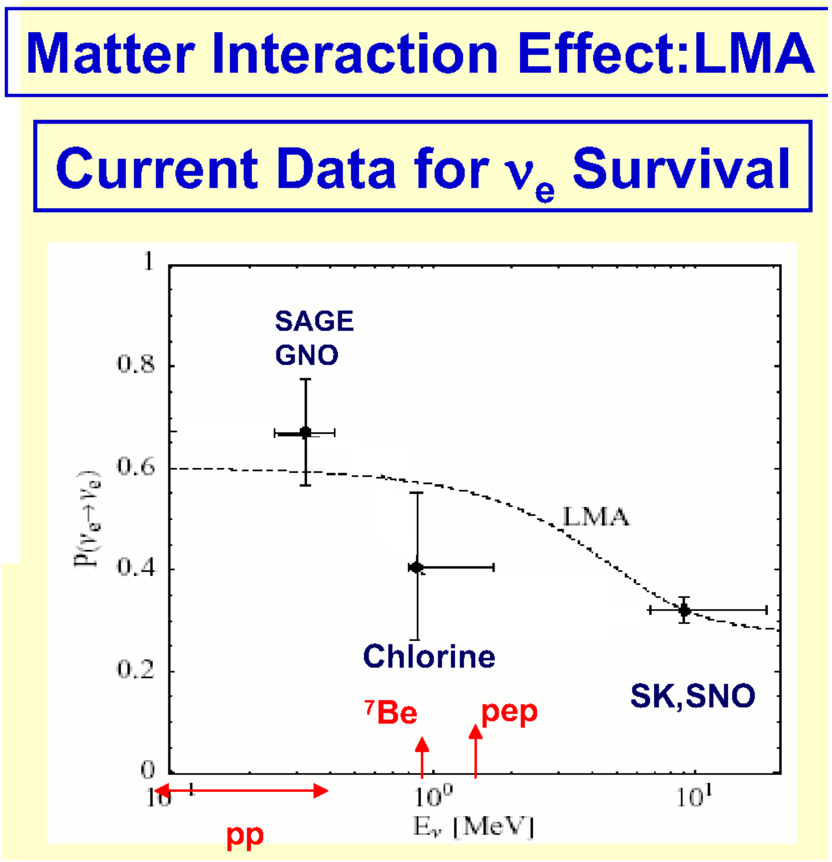
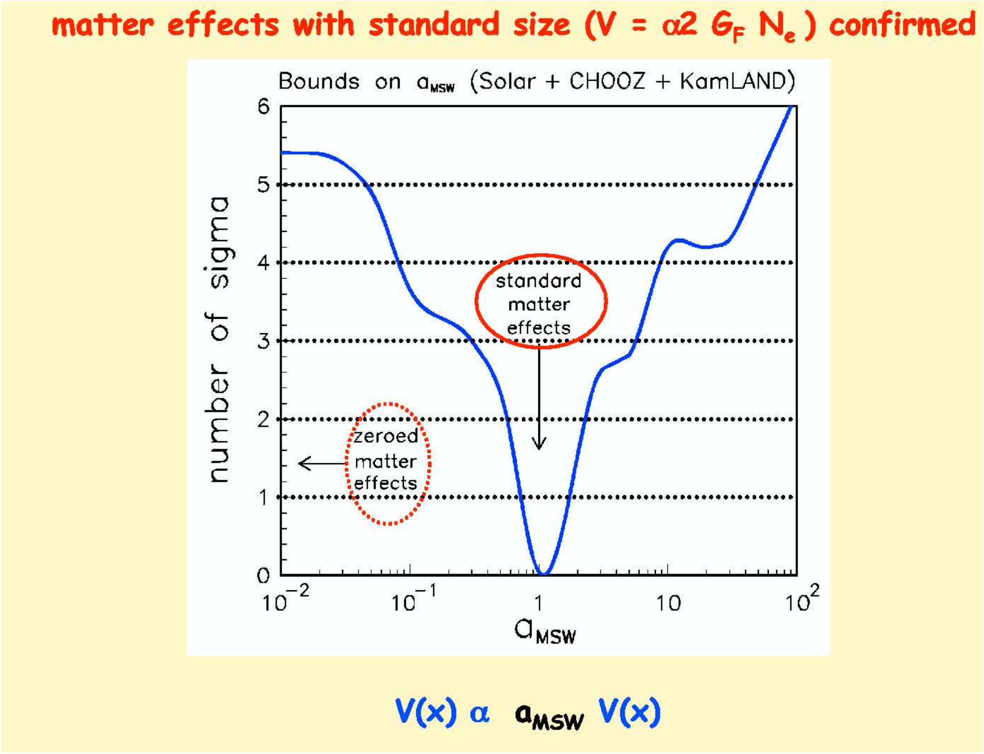


Mechanical model of the MSW effect

Evidence for the MSW effect



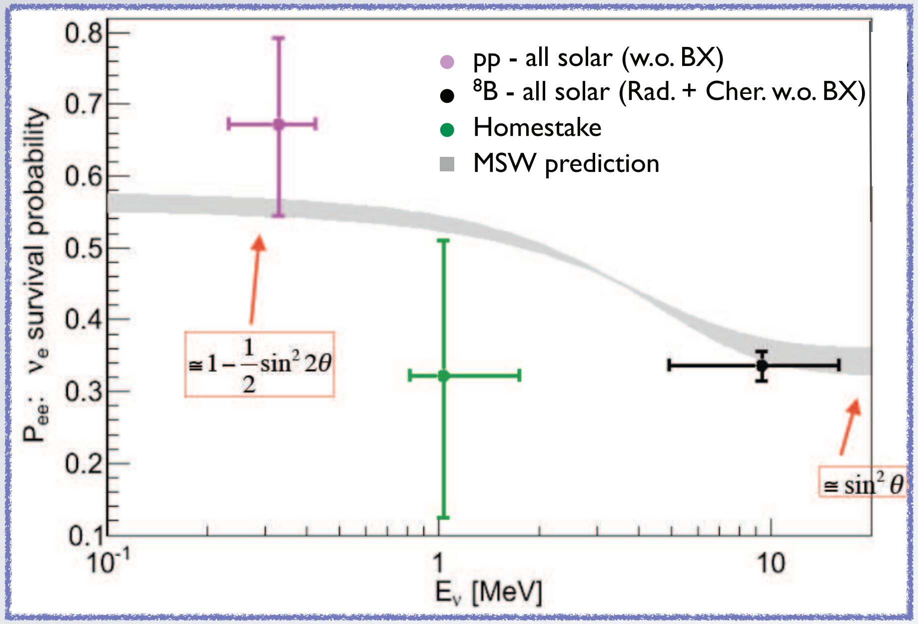


$V(x) \Rightarrow a_{\rm MSW} V(x);\quad a_{\rm MSW} = 1$ strongly favoured

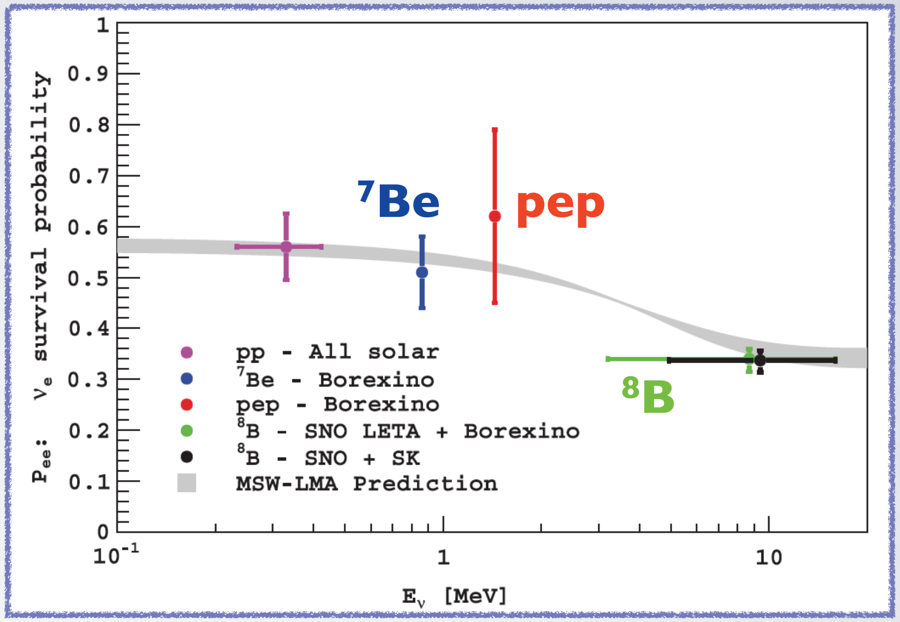
(Fogli et al. 2003, 2004; Fogli & Lisi 2004)

BOREXINO IMPACT ON SOLAR NEUTRINO PHYSICS

Before Borexino

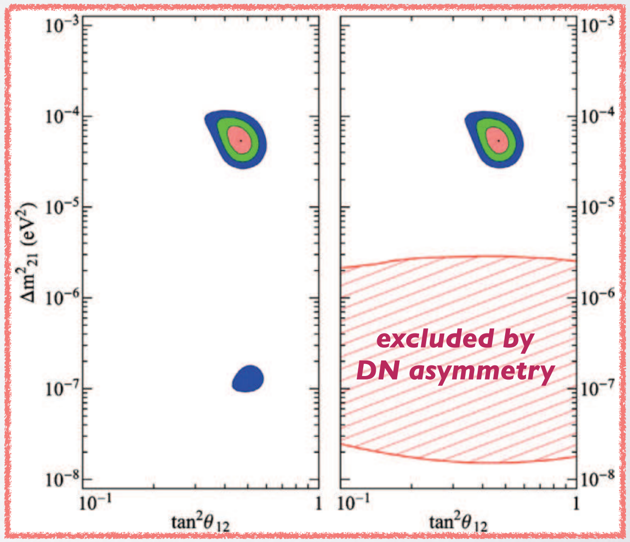


Borexino 2012



In the near future (Phase 2: 2012-2013)

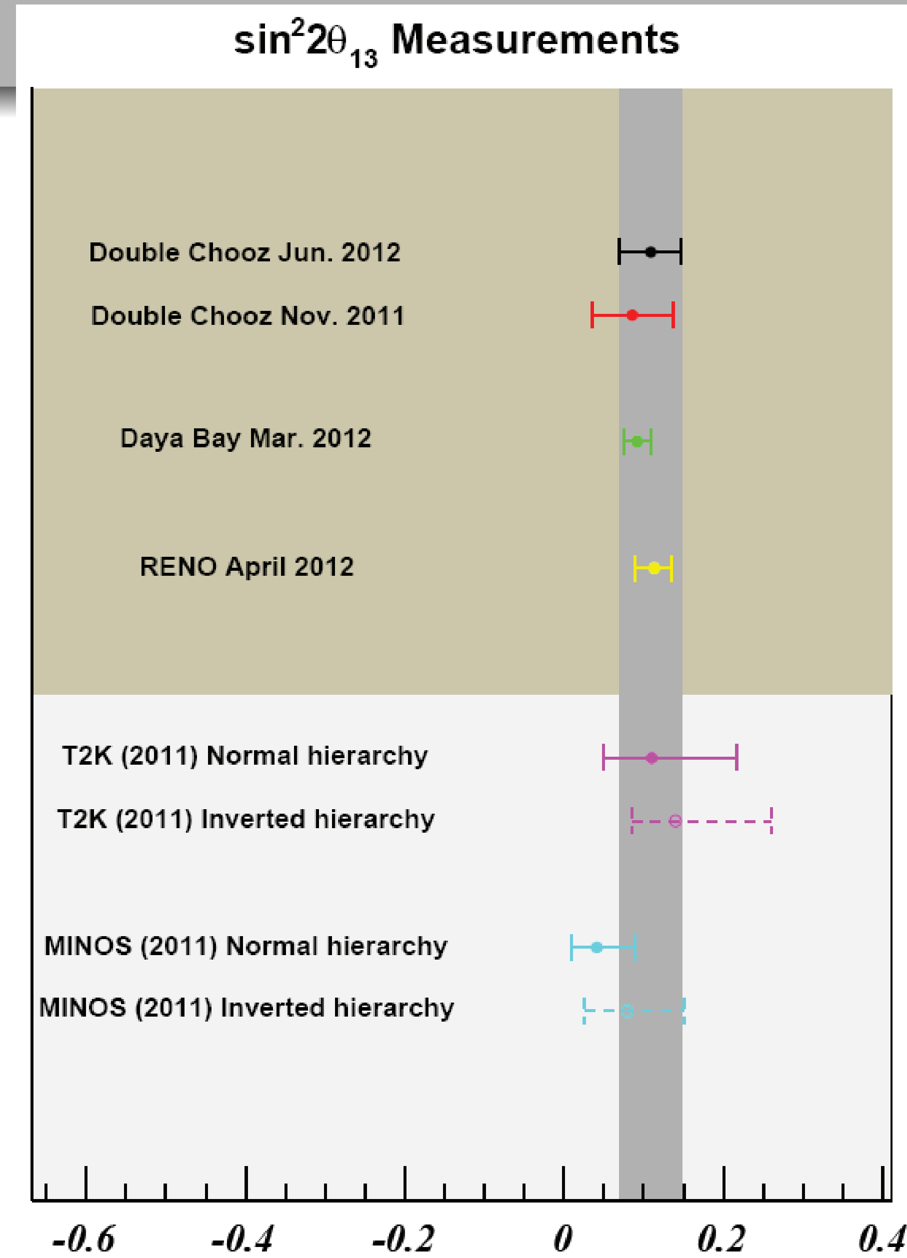
- Improve ^{7}Be, ^{8}B → test of MSW
- Confirm pep at more than 3σ and reduce error
- Improve upper limit on CNO → probe metallicity
- Attempt direct pp measurement



θ_{13} revolution

Conclusions

- Up to 2010 only upper bounds on θ_{13}
- In 2011 we had 3σ evidence for $\theta_{13} \neq 0$ from fits, but not from any one experiment
- The situation in 2012 is completely different:
  - Two accelerator-based experiments see $\nu_\mu \to \nu_e$ appearance (T2K: 3.2σ)
    - Should also be confirmed in near future by NOνA (not discussed here)
  - Three reactor-based experiments see $\overline{\nu}_e$ disappearance (Daya Bay >> 5σ)
- Measurement of $\sin^2 2\theta_{13}$ to a precision of 5% very likely in the next 2 years



$\sin^2 2\theta_{13}$ is LARGE

→ Good prospect for δ_{CP} searches in next 10-20 years

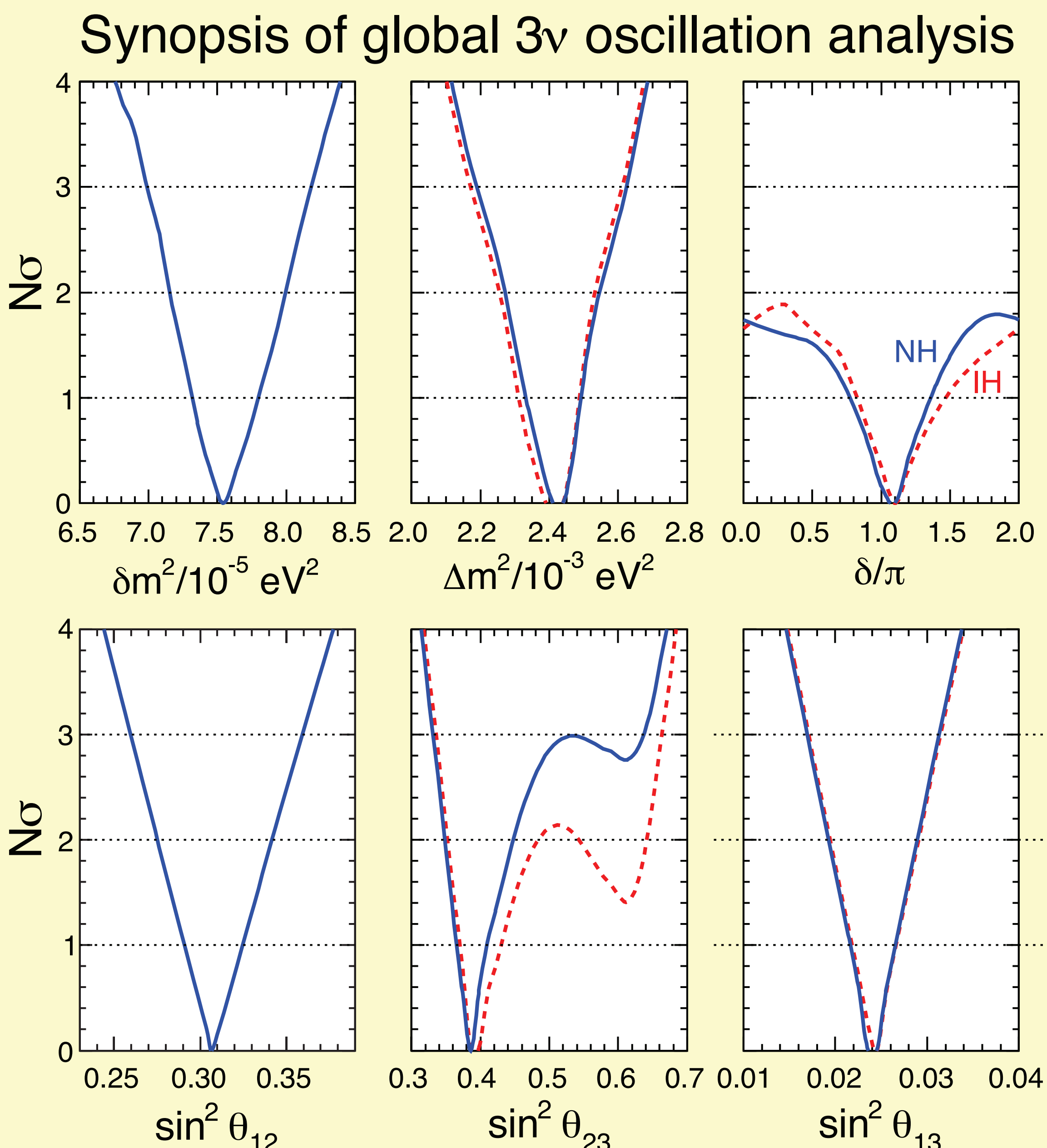
Matt Toups, MIT -- BENE 2012 33

Neutrino parameter determination – global fits

- Madrid-Barcelona-Heidelberg group
  (Gonzalez-Garcia, Maltoni, Salvado & Schwetz)
- Valencia group (Tortola, Valle et al.)
- Bari group (Fogli, Lisi et al.)

(With some inter-relations between the 1st and the 2nd groups)

Global fits – Bari group:



Previous hints of $\theta_{13} > 0$
are now measurements!
(and basically independent
of old/new reactor fluxes)

Some hints of $\theta_{23} < \pi/4$
are emerging at $\sim 2\sigma$,
worth exploring by means
of atm. and LBL+reac. data

A possible hint of $\delta_{CP} \sim \pi$
emerging from atm. data
[Is the PMNS matrix real?]

So far, no hints for
NH ⟷ IH

Global fits – Bari group:

Numerical 1σ, 2σ, 3σ ranges:

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

Fractional 1σ accuracy [defined as 1/6 of $\pm 3\sigma$ range]

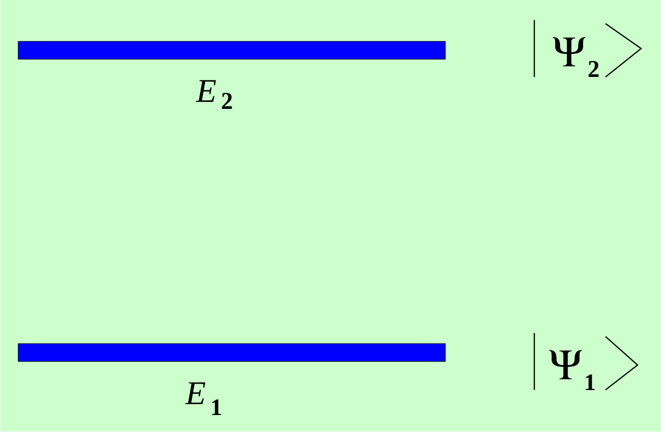
δm^2	Δm^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
2.6%	3.0%	5.4%	10%	14%

Note: above ranges obtained for "old" reactor fluxes. For "new" fluxes, ranges are shifted (by ~ 1/3 σ) for two parameters only: $\Delta \sin^2\theta_{12}/10^{-1} \simeq +0.05$ and $\Delta \sin^2\theta_{13}/10^{-2} \simeq +0.08$

Hierarchy differences well below 1σ for various data combinations

II. Theory

Oscillations: a well known QM phenomenon



$$\Psi_1(t) = e^{-i\,E_1\,t}\,\Psi_1(0)$$

$$\Psi_2(t) = e^{-i\,E_2\,t}\,\Psi_2(0)$$

$$\Psi(0) = a\,\Psi_1(0) + b\,\Psi_2(0) \quad (|a|^2 + |b|^2 = 1)\ ; \qquad \Rightarrow$$

$$\Psi(t) = a\,e^{-i\,E_1\,t}\,\Psi_1(0) + b\,e^{-i\,E_2\,t}\,\Psi_2(0)$$

Probability to remain in the same state $|\Psi(0)\rangle$ after time t:

♢ $$P_{\rm surv} = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \left||a|^2\,e^{-i\,E_1\,t} + |b|^2\,e^{-i\,E_2\,t}\right|^2$$

$$= 1 - 4|a|^2|b|^2\sin^2[(E_2 - E_1)\,t/2]$$

Neutrino oscillations

Appear to be a simple QM phenomenon

But: A closer look reveals a number of subtle and even paradoxical issues

There are some “damned questions”

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There are some "damned questions"

- Why should one assume same energies or same momenta of ν mass eigenstates?

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But: A closer look reveals a number of subtle and even paradoxical issues

There are some “damned questions”

- Why should one assume same energies or same momenta of ν mass eigenstates?
- If neither is the same, how should one calculate the oscillation phase?

Neutrino oscillations

Appear to be a simple QM phenomenon

But: A closer look reveals a number of subtle and even paradoxical issues

There are some “damned questions”

- Why should one assume same energies or same momenta of ν mass eigenstates?
- If neither is the same, how should one calculate the oscillation phase?
- How can one reconcile neutrino oscillations with energy-momentum conservation? – entanglement?

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- How can one make sure that the oscillation probability is Lorentz invariant?

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- When is the emission of neutrinos from different space-tme points in extended sources coherent?
- Is the standard oscillation formula correct? If yes, what is the domain of its applicability?

Debating the basics of neutrino oscillations ...

Lipkin arXiv:0801.1465, arXiv:0905.1216, arXiv:0910.5049, Ivanov & Kienle arXiv:0909.1287, Merle arXiv:0907.3554, Peshkin arXiv:0804.4891, Faber arXiv:0801.3262, Gal arXiv:0809.1213, Giunti arXiv:0805.0431, Flambaum arXiv:0908.2039, Kienert, Kopp, Lindner & Merle arXiv:0808.2389, Walker Nature 453 (2008) 864, Giunti arXiv:0807.3818, Kleinert & Kienle ("Neutrino-pulsating vacuum") arXiv:0803.2938, Lambiase, Papini & Scarpeta arXiv:0811.2302, Burkhardt, Lowe, Stephenson, Goldman & McKellar, arXiv:0804.1099
Bilenky, v. Feilitzsch & Potzel arXiv:0804.3409, arXiv:0803.0527, J. Phys. G36 (2009) 078002, EA, Kopp & Lindner arXiv:0802.2513, arXiv:0803.1424,

Cohen, Glashow & Ligety arXiv:0810.4602, Visinelli & Gondolo arXiv:0810.4132, Keister & Polizou arXiv:0908.1404, Nishi & Guzzo arXiv:0803.1422, Lychkovskiy arXiv:0901.1198, Adhikari & Pal arXiv:0912.5266, Giunti arXiv:1001.0760, Ahluwalia & Schritt arXiv:0911.2965, Schmidt-Parzefall arXiv:0912.3620, Robertson arXiv:1004.1847 and many others.

Clarification of some of these issues and some apparent paradoxes of neutrino oscillations in:

EA, arXiv:0706.1216; EA & Smirnov, arXiv:0905.1903; arXiv:1008.2077; EA, Hernandez & Smirnov. arXiv:1201.4128; EA & Kopp arXiv:1001.4815

Leptonic mixing

For $m_\nu \neq 0$ weak eigenstate neutrinos ν_e, ν_μ, ν_τ do not coincide with mass eigenstate neutrinos ν_1, ν_2, ν_3

Diagonalization of leptonic mass matrices:

$$e_L \to V_L e_L\,, \qquad \nu_L \to U_L \nu_L \ \ldots \qquad \Rightarrow$$

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma^\mu V_L^\dagger U_L \nu_L) W_\mu^- + \text{diag. mass terms} + h.c.$$

Here e_L and ν_L are mass eigenstates!

Leptonic mixing matrix: $U = V_L^\dagger U_L$

$$\diamondsuit \qquad \nu_\alpha = \sum_i U_{\alpha i}\,\nu_i \qquad \Rightarrow \qquad |\nu_\alpha\rangle = \sum_i U_{\alpha i}^*\,|\nu_i\rangle$$

$$(\alpha = e\,,\ \mu\,,\ \tau, \qquad i = 1\,,\ 2\,,\ 3)$$

The standard formula for the oscillation probability of relativistic neutrinos in vacuum:

♢ $$P(\nu_\alpha \to \nu_\beta; L) = \left|\sum_i U_{\beta i}\, e^{-i\frac{\Delta m_{i1}^2}{2p}L}\, U_{\alpha i}^*\right|^2$$

How is it usually derived?

Assume at time $t = 0$ and coordinate $x = 0$ a flavour eigenstate $|\nu_a\rangle$ is produced:

$$|\nu(0,0)\rangle = |\nu_\alpha^{\rm fl}\rangle = \sum_i U_{\alpha i}^* \, |\nu_i^{\rm mass}\rangle$$

For plane-wave particles:

$$|\nu(t,\vec{x})\rangle = \sum_i U_{\alpha i}^* \, e^{-ip_i x} |\nu_i^{\rm mass}\rangle$$

After time T at the position $\vec{L}$, mass eigenstates pick up the phase factors $e^{-i\phi_i}$ with

$$\phi_i \equiv p_i \, x = ET - \vec{p}\vec{L}$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \langle \nu_\beta^{\rm fl} | \nu(T,\vec{L})\rangle \right|^2$$

How is it usually derived?

Consider $\vec{p}\,||\,\vec{L} \quad \Rightarrow \quad \vec{p}\vec{L} = \mathrm{p}L \quad (\mathrm{p} = |\vec{p}|, \quad L = |\vec{L}|)$

Phase differences between different mass eigenstates:

$$\Delta\phi = \Delta E \cdot T - \Delta \mathrm{p} \cdot L$$

Shortcuts to the standard formula

1. Assume the emitted neutrino state has a well defined momentum (same momentum prescription) $\Rightarrow \quad \Delta \mathrm{p} = 0.$

   For ultra-relativistic neutrinos $E_i = \sqrt{\mathrm{p}^2 + m_i^2} \simeq \mathrm{p} + \frac{m_i^2}{2\mathrm{p}} \quad \Rightarrow$

$$\Delta E \simeq \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}\,; \qquad T \approx L \qquad (\hbar = c = 1)$$

$\Rightarrow$ The standard formula is obtained

How is it usually derived?

2. Assume the emitted neutrino state has a well defined energy (same energy prescription) $\Rightarrow$ $\Delta E = 0$.

$$\Delta\phi = \Delta E \cdot T - \Delta \mathrm{p} \cdot L \quad \Rightarrow \quad - \Delta \mathrm{p} \cdot L$$

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$$-\Delta \mathrm{p} \equiv \mathrm{p}_1 - \mathrm{p}_2 \approx \frac{\Delta m^2}{2p};$$

$\Rightarrow$ The standard formula is obtained

Stand. phase $\Rightarrow$ $$(l_{\rm osc})_{ik} = \frac{4\pi E}{\Delta m_{ik}^2} \simeq 2.5\ m\, \frac{E\,(\mathrm{MeV})}{\Delta m_{ik}^2\, \mathrm{eV}^2}$$

Same E and same p approaches

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Allow one to quickly arrive at the desired result

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Trouble: they are both wrong

Kinematic constraints

Same momentum and same energy assumptions: contradict kinematics!

Pion decay at rest $(\pi^+ \to \mu^+ + \nu_\mu, \;\; \pi^- \to \mu^- + \bar{\nu}_\mu)$:
For decay with emission of a massive neutrino of mass m_i:

$$E_i^2 \; = \; \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 \; = \; \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \;\; \mathrm{MeV}$

To first order in m_i^2:

$$E_i \; \simeq \; E + \xi\frac{m_i^2}{2E}\,, \qquad p_i \; \simeq \; E-(1-\xi)\frac{m_i^2}{2E}\,, \qquad \xi \; = \; \frac{1}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \; \approx \; 0.2$$

Kinematic constraints

Same momentum or same energy would require
$\xi = 1$ or $\xi = 0$ – not the case!

Also: would violate Lorentz invariance of the oscillation probability

How can wrong assumptions lead to the correct oscillation formula ?

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Plane waves mean that energies and momenta of particles have sharp values
$\Rightarrow$ When applied to neutrino production and detection processes: neutrino E and $\vec{p}$ can be determined from those of the accompanying particles .

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Neutrinos propagate macroscopic distances between their source and detector, i.e. are on the mass shell $\Rightarrow$ their energy and momentum satisfy

$$E^2 = p^2 + m^2$$

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By knowing the neutrino energy and momentum one can determine its mass
But: Mass eigenstates do not oscillate !

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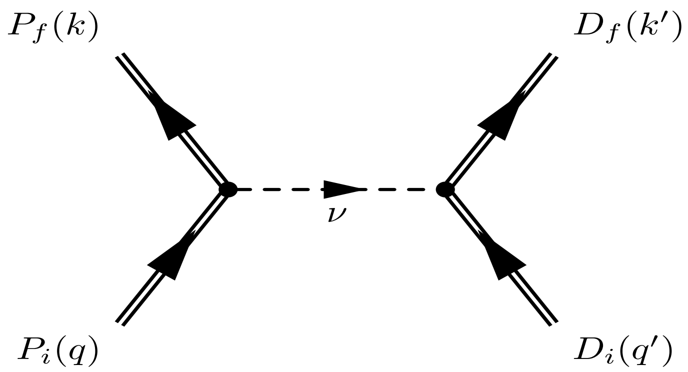
- QM wave packet approach – neutrinos described by wave packets rather than by plane waves
- QFT approach: neutrino production and detection explicitly taken into account. Neutrinos are intermediate particles described by propagators



Oscillation phase

Oscillations are due to phase differences of different mass eigenstates:

$$\Delta\phi = \Delta E \cdot T - \Delta p \cdot L \qquad \left(E_i = \sqrt{p_i^2 + m_i^2}\right)$$

Consider the case $\Delta E \ll E$ (relativistic or quasi-degenerate neutrinos) $\Rightarrow$

$$\Delta E = \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 = v\Delta p + \frac{1}{2E}\Delta m^2$$

$$\Delta\phi = (v\Delta p + \frac{1}{2E}\Delta m^2)\, T - \Delta p \cdot L$$

$$= -(L - vT)\Delta p + \frac{\Delta m^2}{2E} T$$

In the center of wave packet $(L - vT) = 0$. In general, $|L - vT| \lesssim \sigma_x$;
if $\sigma_x \Delta p \ll 1$ (i,e, $\Delta p \ll \sigma_p$), $|L - vT|\Delta p \ll 1$ $\Rightarrow$

$$\Delta\phi \;=\; \frac{\Delta m^2}{2E}\,T\,, \qquad\qquad L\;\simeq vT\;\simeq T$$

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$$\diamondsuit \qquad \Delta\phi = -\frac{1}{v}(L - vT)\Delta E + \frac{\Delta m^2}{2p}\,L \qquad \Rightarrow \qquad \frac{\Delta m^2}{2p}\,L$$

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The reasons why wrong assumptions give the correct result:

- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$
- Neutrino energy uncertainty $\sigma_E \gg \Delta E$ (typically this means $\sigma_x \ll l_{\rm osc}$)

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^* \, e^{-i\frac{\Delta m_{ik}^2}{2\bar{P}}L}\, \tilde{I}_{ik}$$

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$$\tilde{I}_{ik} = N \int \frac{dq}{2\pi} f_i^S(r_k q - \Delta E_{ik}/2v + P_i) f_i^{D*}(r_k q - \Delta E_{ik}/2v + P_i)$$
$$\times f_k^{S*}(r_i q + \Delta E_{ik}/2v + P_k) f_k^D(r_i q + \Delta E_{ik}/2v + P_k)\, e^{i\frac{\Delta v}{v}qL}$$

Here: $v \equiv \frac{v_i + v_k}{2}\,, \quad \Delta v \equiv v_k - v_i\,, \quad r_{i,k} \equiv \frac{v_{i,k}}{v}\,, \quad N \equiv 1/[2E_i(P)2E_k(P)v]$

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- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

The standard osc. probability?

The standard formula for the oscillation probability corresponds to $\tilde{I}_{ik} = 1$.

If the two above conditions are satisfied, $\tilde{I}_{ik}$ is not suppressed and is L-, T-and i, k-independent (i.e. a constant).

The standard probability is obtained when this constant is 1 (normalization necessary!)

The normalization prescription

Oscillation probability calculated in QM w. packet approach is not automatically normalized! Can be normalized "by hand" by imposing the unitarity condition:

$$\sum_\beta P_{\alpha\beta}(L) \;=\; 1\,.$$

This gives

$$\int dT|\mathcal{A}_i(L,T)|^2 = 1 \quad \Rightarrow \quad \tilde{I}_{ii} = N_1 \int \frac{dp}{2\pi v} |f_i^S(p)|^2\, |f_i^D(p)|^2 \;=\; 1$$

– important for proving Lorentz invariance of the oscillation probability.

Depends on the overlap of $f_i^S(p)$ and $f_i^S(p)$ $\Rightarrow$ no independent normalization of the produced and detected neutrino wave function would do!

In QFT approach the correctly normalized $P_{\alpha\beta}(L)$ is automatically obtained and the meaning of the normalization procedure adopted in the w. packet approach clarified

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The dichotomy led to a significant confusion in the literature.
How can it be resolved?

Possible solution: entanglement

Consider e.g. $\pi \to \mu + \nu$ decay.
Suppose that the 4-momentum of the pion p_π is well defined but the muon 4-momentum is correlated with that of the emitted ν_i:

$$p_{\nu i} + p_{\mu i} = p_\pi, \qquad i = 1, 2, 3$$

State produced in the pion decay: a coherent superposition of different neutrino mass eigenstates accompanied by the muon states with correlated 4-momenta (entangled state):

$$|\mu\,\nu\rangle = \sum_i U_{\mu i}^* |\mu(p_{\mu i})\rangle |\nu_i(p_{\nu i})\rangle.$$

If muon 4-momentum is measured very accurately (e.g. $p_\mu = p_{\mu 1}$) $\Rightarrow$ neutrino detector should observe only ν_1 with 4-momentum $p_{\nu 1}$.
A realization of the Einstein-Podolsky-Rosen correlation.

<u>But:</u> in this case no oscillations would occur!

Entanglement – contd.

⇒ Disentanglement is necessary.

Entanglement – contd.

$\Rightarrow$ Disentanglement is necessary.

Assumed to be achieved through a measurement of the muon momentum with a sufficiently large intrinsic uncertainty ($\Leftrightarrow$ sufficiently good localization of the measurement process). Leads to a violation of the strict correlation between the muon and neutrino 4-momenta $\Rightarrow$ to a separation of the muon and neutrino parts of $|\mu\,\nu\rangle$. Oscillations become possible.

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$\diamondsuit$ Kinematic entanglement is irrelevant to neutrino oscillations!

Wave packets

- ♢ Wave packets are necessary for describing localization of neutrino production and detection processes $\Rightarrow$ of neutrinos themselves!

WPs necessary for a proper definition of S-matrix

Neutrino energy and momentum have some uncertainties, σ_E and σ_p.

This does not mean that energy-momentum conservation is violated!
E-p conserv. is exact for closed systems. Satisfied exactly when applied to all particles in the system (including those that localize particles participating in ν production and detection in given space-time regions).

Energy and momentum uncertainties do not contradict E-p conservation!

At the technical level:

$$\mathcal{A}_i = \prod_j \int \frac{d\vec{p}_j}{(2\pi)^3} \tilde{f}_j(\vec{p}_j, \bar{\vec{p}}_j; T_S, \vec{X}_S) \prod_l \int \frac{d\vec{p}_l}{(2\pi)^3} \tilde{f}_l(\vec{p}_l, \bar{\vec{p}}_l; T_D, \vec{X}_D)\, \mathcal{A}_i^{pw}(\{p_j\}, \{p_l\})$$

$$\mathcal{A}_i^{pw}(\{p_j\}, \{p_l\}) \propto \delta^{(4)}\Big(\sum_f p_f - \sum_i p_i\Big)\,.$$

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates (Kayser, 1981)
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation (Nussinov, 1976)

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E \;=\; p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim$ (a few)$\times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \quad \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E \;=\; p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\rm prod},\, (E/p)\sigma_E^{\rm prod}\} = \min\{\sigma_p^{\rm prod},\, (1/v_g)\sigma_E^{\rm prod}\}$$

Which uncertainty is smaller at production, $\sigma_p^{\rm prod}$ or $\sigma_E^{\rm prod}$?

Consider neutrino production in decays of an unstable particle localized in a box of size L_S. Time between two collisions with the walls of the box: T_S.

- If $T_S < \tau$ (τ – lifetime of the parent unstable particle) $\Rightarrow$ $\sigma_E \simeq T_S^{-1}$ (collisional broadening). Mom. uncertainty: $\sigma_p \simeq L_S^{-1}$.

  But: $L_S = v_S T_S \;\Rightarrow\; \sigma_E < \sigma_p$ (a consequence of $v_S < 1$)

- If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow$ $\sigma_E \simeq \tau^{-1} = \Gamma$.

  $\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\boxed{\sigma_E^{\rm prod} < \sigma_p^{\rm prod}}$ $\Leftarrow$ also when $\nu's$ are produced in collisions.

$$\Longrightarrow \qquad \sigma_{p\ \rm eff} \simeq \frac{\sigma_E}{v_g}\,, \qquad\qquad \boxed{\sigma_x \simeq \frac{v_g}{\sigma_E}}$$

In the stationary limit $(\sigma_E \to 0)$ one has $\sigma_{p\ \rm eff} \to 0$ even though σ_p is finite!
Therefore $\sigma_x \to \infty$ and so the coherence length $l_{\rm coh} \to \infty$
– a well known result.

Lorentz invariance of oscillation probability

1. "Paradox" of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi\,, \qquad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} \;(= v_g \tau)$$

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For decay in flight: $\Gamma'_\pi = (m_\pi/E_\pi)\Gamma_\pi$. One might expect

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<u>The solution:</u> pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ ("chases" the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1+v_g u)}\,,$$

[the relativ. law of addition of velocities: $v'_g = (v_g+u)/(1+v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma_x' = \frac{\sigma_x}{\gamma_u(1 + v_g u)}$$

For relativistic neutrinos $v_g \approx v_g' \approx 1 \quad \Rightarrow$

$$\sigma_x' = \sigma_x \sqrt{\frac{1-u}{1+u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission $(u > 0)$ the neutrino wave packet gets contracted; when it is boosted in the opposite direction $(u < 0)$ – the wave packet gets dilated.

Lorentz invariance issues – contd.

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The stand. osc. formula results when (i) production and detection and (ii) propagation are coherent; for neutrinos from conventional sources (i) implies $\sigma_x \ll l_{\rm osc}$. $\Rightarrow$ one can consider neutrinos pointlike and set $L = v_g t$. $\Rightarrow$ $L' = \gamma_u L(1 + u/v_g)$.

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$\Rightarrow$

$$\boxed{L'/p' = L/p}$$

Lorentz invariance issues – contd.

A more general argument (applies also to Mössbauer neutrinos which are not pointlike): Consider the phase difference

$$\diamondsuit \qquad \Delta\phi = -\frac{1}{v_g}(L - v_g\, t)\Delta E + \frac{\Delta m^2}{2p}\, L$$

– a Lorentz invariant quantity, though the two terms are in not in general separately Lorentz invariant.

<u>But:</u> If the 1st term is negligible in all Lorentz frames, the second term is Lorentz invariant by itself $\Rightarrow$ L/p is Lorentz invariant.

The 1st term can be neglected when the production/detection coherence conditions are satisfied. In particular, it vanishes in the limit of pointlike neutrinos $L = v_g t$. N.B.:

$$L' - v_g' t' = \gamma_u \left[(L + ut) - \frac{v_g + u}{1 + v_g u}(t + uL)\right] = \frac{L - v_g t}{\gamma_u (1 + v_g u)}\,,$$

i.e. the condition $L = v_g t$ is Lorentz invariant. MB neutrinos: $\Delta E \simeq 0$.

Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant even when the coherence conditions are not satisfied !
Lorentz invariance is enforced by the normalization condition.

$$P_{ab}(L) \;=\; \sum_{i,k} U_{ai}U_{bi}^{*}U_{ak}^{*}U_{bk}\, I_{ik}(L)\,, \qquad \text{where}$$

$$I_{ik}(L) \;\equiv\; \int dt\, \mathcal{A}_i(L,t)\mathcal{A}_k^{*}(L,t)e^{-i\Delta\phi_{ik}}$$

From the norm. cond. $\int dt\, |\mathcal{A}_i(L,t)|^2 \;=\; 1 \qquad \Rightarrow$

$$|\mathcal{A}_i|^2 dt \;=\; inv. \quad \Rightarrow \quad |\mathcal{A}_i||\mathcal{A}_k|dt \;=\; inv. \quad \Rightarrow \quad \mathcal{A}_i\mathcal{A}_k^{*}dt \;=\; inv.$$

The phase difference $\Delta\phi_{ik} = \Delta E_{ik}t - \Delta p_{ik}L$ is also Lorentz invariant $\quad\Rightarrow$ so is $I_{ik}(L)$, and consequently $P_{ab}(L)$.

Do charged leptons oscillate?

What do we mean by charged leptons?

The usual $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$ are mass eigenstates $\Rightarrow$ do not oscillate.

Is that the full answer?

Can we imagine a situation when one creates a coherent superposition of e, μ and τ and then also detects their coherent superposition (the same or different) rather than individual mass-eigenstate charged leptons?

Charged - current weak interactions look completely symmetric w.r.t. neutrinos and charged leptons!

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}(\bar{e}_{aL}\gamma^\mu U_{ai}\nu_{iL})W_\mu^- + h.c.\,, \qquad U = V_L^\dagger V_\nu$$

Why do we say that charged leptons are emitted and detected in mass eigenstates and neutrinos in flavour states (superpositions of mass eigenstates) and not vice versa? Or not both as some superpositions of mass eigenstates? E.g.

$|e_1\rangle = U_{1e}|e\rangle + U_{1\mu}|\mu\rangle + U_{1\tau}|\tau\rangle$ is emitted or detected together with ν_1,
$|e_2\rangle = U_{2e}|e\rangle + U_{2\mu}|\mu\rangle + U_{2\tau}|\tau\rangle$ is emitted or detected together with ν_2,
$|e_3\rangle = U_{3e}|e\rangle + U_{3\mu}|\mu\rangle + U_{3\tau}|\tau\rangle$ is emitted or detected together with ν_3.

Why do neutrinos oscillate?

Because they are emitted (and absorbed) alongside charged leptons of definite mass $e^{\pm}$, $\mu^{\pm}$ or $\tau^{\pm}$. (This "measures" the flavour of neutrinos). How do we know that charged leptons are in mass eigenstates?

(1) Beta decay: only electrons are emitted together with neutrinos. Emission of $\mu^{\pm}$ and $\tau^{\pm}$ is forbidden by energy conservation.

(2) Decays $\pi^{\pm} \to \mu^{\pm}\nu$, $\pi^{\pm} \to e^{\pm}\nu$ (or $K^{\pm} \to \mu^{\pm}\nu$, $K^{\pm} \to e^{\pm}\nu$). Here emission of both muons and electrons is allowed.

Assume a coherent superposition of e and μ is produced in pion decay (nearly) at rest. The energy uncertainty of the charged lepton:

$$\sigma_E \simeq \Gamma_\pi = 2.5 \cdot 10^{-8}\ \mathrm{eV}$$

Uncertainty in the mass determination $\left(\sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2]} \simeq 2\sqrt{2}E\sigma_E\right)$:

$$\sigma_{m^2} \sim 2\sqrt{2}E\sigma_E \simeq 2\sqrt{2} \cdot (90\ \mathrm{MeV}) \cdot (2.5 \cdot 10^{-8}\ \mathrm{eV}) \simeq 6.4\ \mathrm{eV}^2$$

Do charged leptons oscillate?

This has to be compared with $m_\mu^2 - m_e^2 \simeq (106 \text{ MeV})^2 \quad \Rightarrow$
Different mass-eigenstate charged leptons are emitted incoherently!

This provides a "measurement" of the flavour of the emitted neutrino

For pion decay in flight: assume pion's energy is E_0. The energies of the produced charged leptons are rescaled as $E \to E\,(E_0/m_\pi)$, but the pion decay width (and so σ_E) is rescaled as $\Gamma_\pi \to \Gamma_\pi(m_\pi/E_0) \quad \Rightarrow$ $[(2E\sigma_E)^2 + (2p\sigma_p)^2]^{1/2}$ remains the same (σ_{m^2} a Lorentz invariant quantity).

$$\Downarrow$$

- ♢ Charged leptons produced in $\pi^\pm \to l^\pm \nu$ and $K^\pm \to l^\pm \nu$ decays are always emitted as mass eigenstates and not as coherent superpositions of different mass eigenstates because of their very large Δm^2.

- ♢ Therefore even oscillations between e_1, e_1 and e_3 (or any other superpositions of e, μ and τ) are not possible.

Do charged leptons oscillate?

The masses and decay widths of $\pi^\pm$, $K^\pm$ are rather small $\Rightarrow$ σ_{m^2} small.

How about decays of $W^\pm$? For $W^\pm \to l^\pm \nu$ decays at rest:

$$\Gamma^0_{W\to l_a \nu} \simeq \frac{G_F m_W^3}{6\sqrt{2}\pi} \simeq 230\ \text{MeV}$$

$$\Rightarrow \qquad \sigma_{m^2} \sim 2\sqrt{2}\, E\sigma_E \simeq 2\sqrt{2}\cdot 40\ \text{GeV}\cdot 230\ \text{MeV} \simeq (5\ \text{GeV})^2\,.$$

Thus

$$\sigma_{m^2} \gg m_\mu^2 - m_e^2\,, \qquad \sigma_{m^2} > m_\tau^2 - m_\mu^2 \simeq (1.77\ \text{GeV})^2\,,$$

$\Rightarrow$ all three charged leptons are produced *coherently* in $W^\pm$ decays.

Can one then observe oscillations between their different coh. superpositions?

Coherence length $l_{\rm coh} \simeq \sigma_x/\Delta v_g$:

$$(l_{\rm coh})_{\rm max} \simeq [\Gamma^0_{W\to l_a\nu}(\Delta v_g)_{\rm min}]^{-1} \simeq \frac{3\sqrt{2}\pi}{G_F m_W (m_\mu^2 - m_e^2)} \simeq 2.5\times 10^{-8}\ \text{cm}\,.$$

$\Rightarrow$ $l^\pm$ loose their coherence almost immediately after their production

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Production and detection coherence $\Leftrightarrow$ localization cond.:

$$l_{\rm prod} \ll l_{\rm osc}\,, \qquad\qquad l_{\rm det} \ll l_{\rm osc}$$

Usually satisfied with large margins.

Propagation coherence: $L < l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\,v\sigma_x$

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim U_{e1} = \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim U_{e2} = \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) = \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \ < \ 1$$

A universal oscillation probability?

Q.: When are the oscillations described by a universal (production and detection independent) oscillation probability?

A.: When neutrinos are relativistic or quasi-degenerate in mass and the conditions of coherent neutrino emission and detection

$$\Delta E \ll \sigma_E, \qquad \Delta p \ll \sigma_p$$

are satisfied.

Under these conditions the rate of the overall neutrino production-propagation-detection process can be factorized into the production rate $d\Gamma_\alpha^{\rm prod}(E)/dE$, propagation (oscillation) probability $P_{\alpha\beta}(E,L)$ and detection cross section $\sigma_\beta(E) \quad \Rightarrow \quad P_{\alpha\beta}(E,L)$ can be extracted.

Coherence of ν production in different points

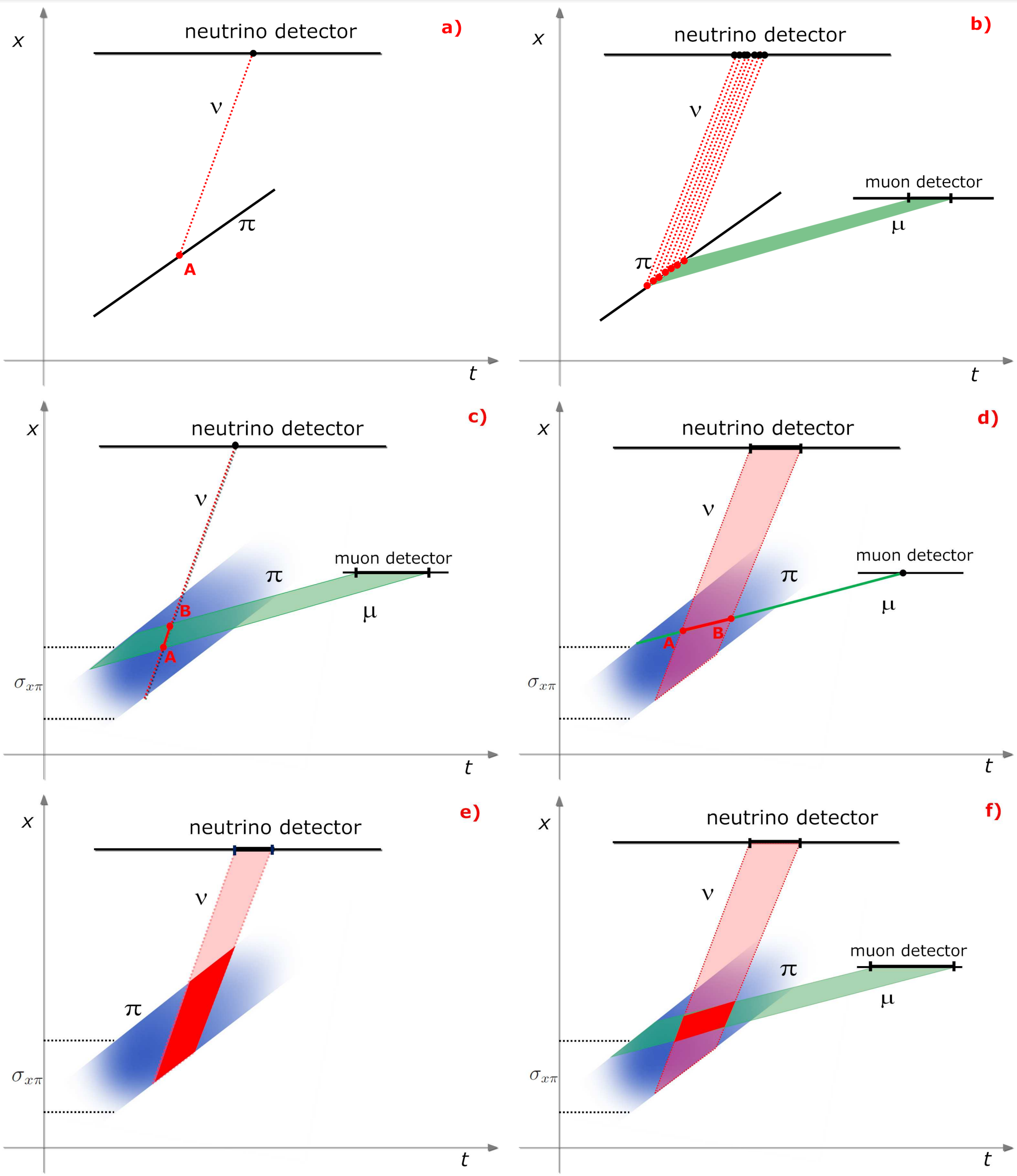
Neutrino production in extended sources: Amplitudes of neutrino emission in different points must be summed – a consistent QM procedure.

The standard approach: calculate the probability that neutrino produced at a fixed point x oscillates, and then integrate over all x in the source (probability summation procedure – classical in nature).

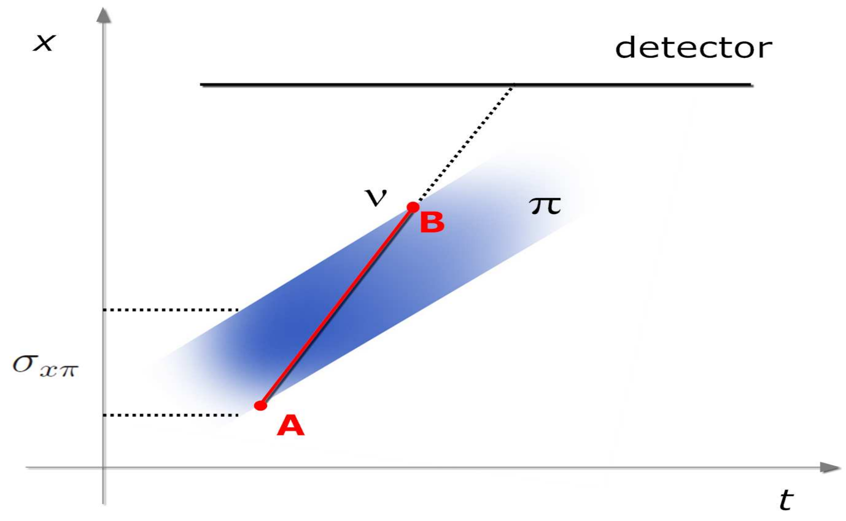
Both procedures give identical answers under realistic conditions!

The two approaches lead to different results whenever the localization properties of the parent particles at neutrino production and of the detection process are such that they prevent the precise localization of the point of neutrino emission – difficult to realize in practice.

Graphical interpretation



Finite-width pion WP



Additional phase for the segment AB:

$$\Delta\phi = -[E_j(P_j) - E_k(P_k)]\Delta t + (P_j - P_k)\Delta x\,.$$

Δt and Δx: projections of AB on the t and x axes. $\Rightarrow$

$$\Delta t = \frac{\sigma_{x\pi}}{v_g - v_\pi}\,, \qquad \Delta x = \sigma_{x\pi}\frac{v_g}{v_g - v_\pi}\,.$$

$$\Delta\phi \simeq -\frac{v_g}{v_g - v_\pi}\cdot\frac{\Delta m_{jk}^2}{2P}\sigma_{x\pi}$$

Finite-width pion WP – contd.

Are deviations between the results of the coherent amplitude summation and incoherent probability summation approaches experimentally observable? Requires extremely high energies of the parent pion:

$$2\,(E_\pi \sigma_{x\pi})\,\frac{\Delta m^2}{m_\pi^2} \gtrsim 1\,.$$

E.g. for $\sigma_{x\pi} \sim 10^{-4}$ cm and $\Delta m^2 \sim 1$ eV^2 $\Delta\phi$ would be ~ 1 for pion energies $E_\pi \gtrsim 10^3$ TeV – not feasible,

Another possibility: increase significantly the spatial width of w. packets of ancestor protons, which would increase the values of $\sigma_{x\pi}$. But: not clear how this could be achieved.

Other possibilities...

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Gives only order of magnitude estimate when decoherence parameters are of order one.

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The standard formula for osc. probability is stubbornly robust.

Validity conditions:

- Neutrinos are ultra-relativistic or quasi-degenerate in mass
- Coherence conditions for neutrino production, propagation and detection are satisfied.

Gives also the correct result in the case of strong coherence violation (complete averaging regime).

Gives only order of magnitude estimate when decoherence parameters are of order one.

But: Conditions for partial decoherence are difficult to realize

Summary

- QM and QFT wave packet formalisms provide consistent approaches to neutrino oscillations.

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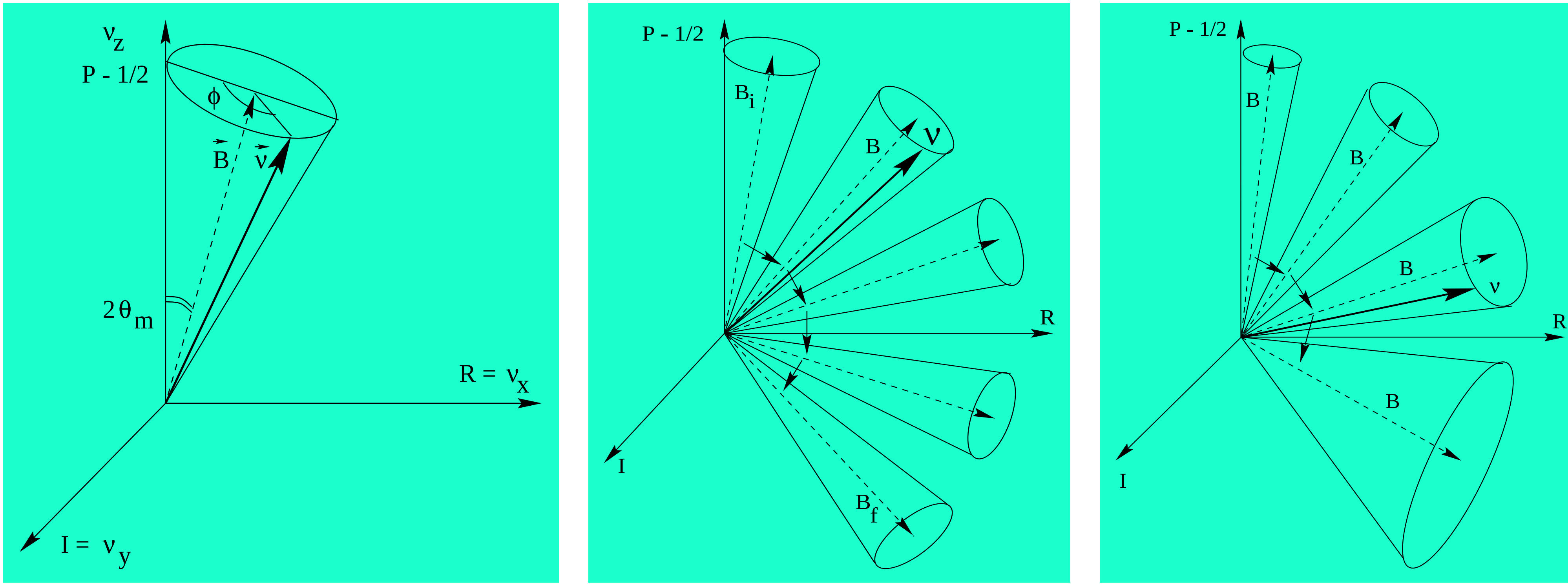
- QM and QFT wave packet formalisms provide consistent approaches to neutrino oscillations.
- The standard formula for $P_{\alpha\beta}$ obtains when neutrinos are relativistic and coherence conditions for neutrino production, propagation and detection are satisfied.
- QFT approach allows to justify and improve the simplistic QM wave packet one (e.g. allows to obtain the neutrino wave packets used in the QM approach instead of postulating them and justifies the normalization procedure).

Backup slides

Matter effects

Analogy: Spin precession in a magnetic field



$$\frac{d\vec{S}}{dt} = 2(\vec{B} \times \vec{S})$$

$$\vec{S} = \{\mathrm{Re}(\nu_e^* \nu_\mu)\,,\ \mathrm{Im}(\nu_e^* \nu_\mu)\,,\ \nu_e^* \nu_e - 1/2\}$$

$$\vec{B} = \{(\Delta m^2/4E)\sin 2\theta_m\,,\ \ 0\,,\ \ V/2 - (\Delta m^2/4E)\cos 2\theta_m\}$$

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in matter

Normal matter [(# of particles) $\neq$ (# of anti-particles)]:

The very presence of matter violates C, CP and CPT

$\Rightarrow$ Fake (extrinsic) $\not{C}\not{P}$. Exists even in 2f case. May complicate study of fundamental (intrinsic) $\not{C}\not{P}$

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- $\diamond$ May fake fundamental $\not{T}$ and complicate its study (extraction of $\delta_{\rm CP}$ from experiment)

Induced $\not{T}$: absent when either $U_{e3} = 0$ or $\Delta m^2_{\rm sol} = 0$ (2f limits)

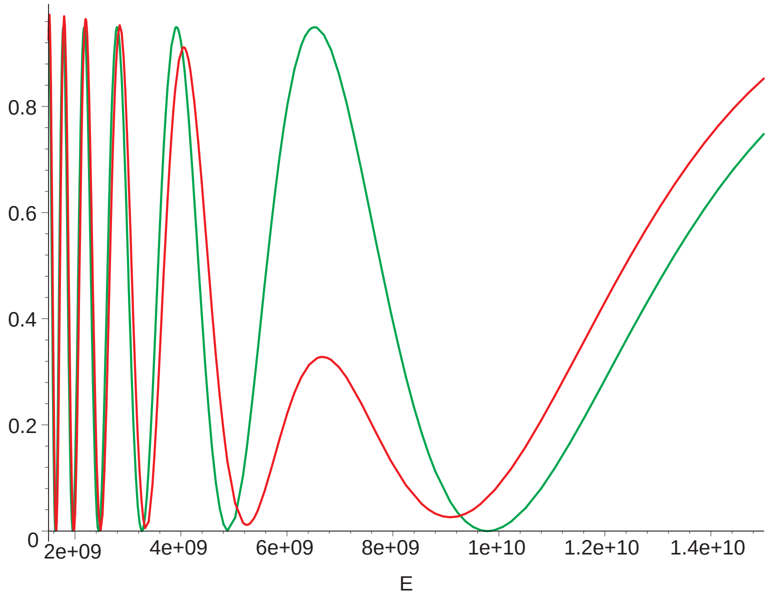
$\Rightarrow$ Doubly suppressed by both these small parameters

– effects in terrestrial experiments are small

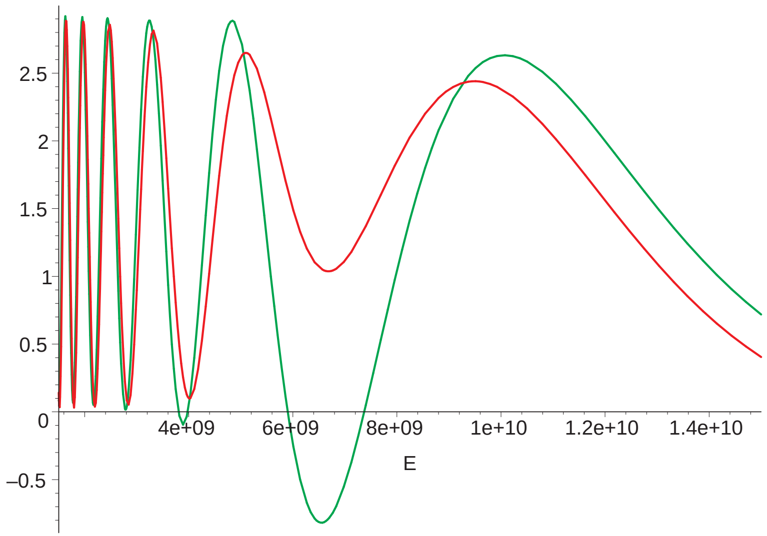
Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

In 2f approximation: no matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations [$V(\nu_\mu) = V(\nu_\tau)$ modulo tiny rad. corrections].

Not true in the full 3f framework! (E.A., 2002; Gandhi et al., 2004)



$P_{\mu\tau}$



Oscillated flux of atm. ν_μ

$\Delta m_{31}^2 = 2.5 \times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.026$, $\theta_{23} = \pi/4$, $\Delta m_{21}^2 = 0$, $L = 9400$ km

Red curves – w/ matter effects, green curves – w/o matter effects on $P_{\mu\tau}$

Another possible matter effect

Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves

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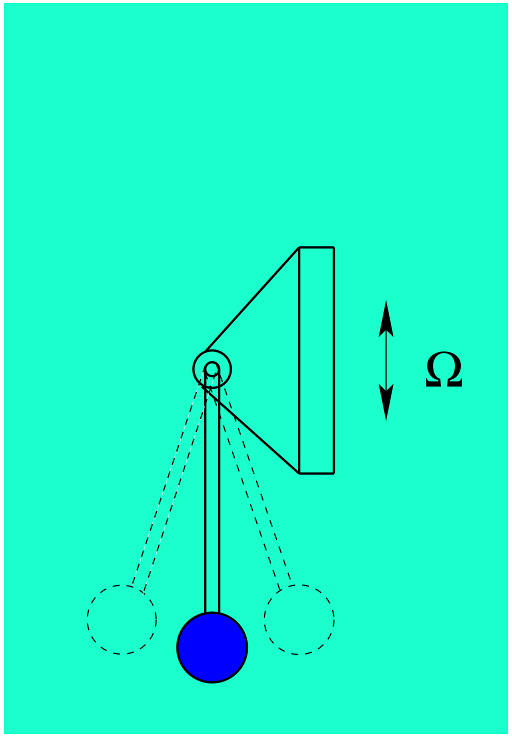
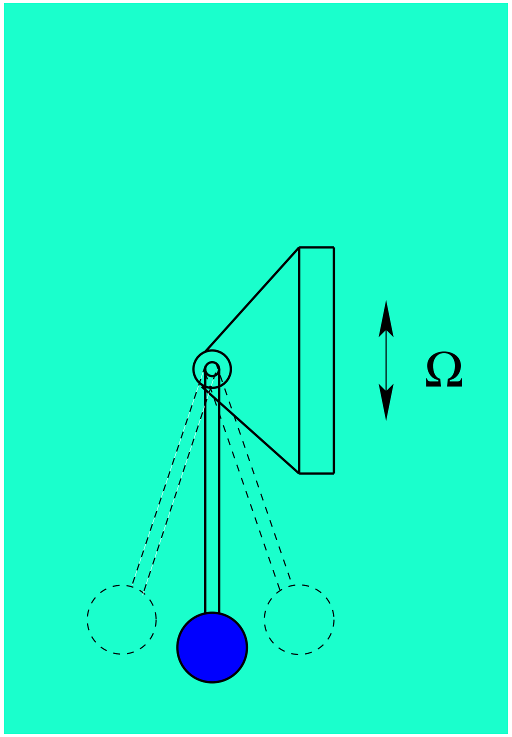
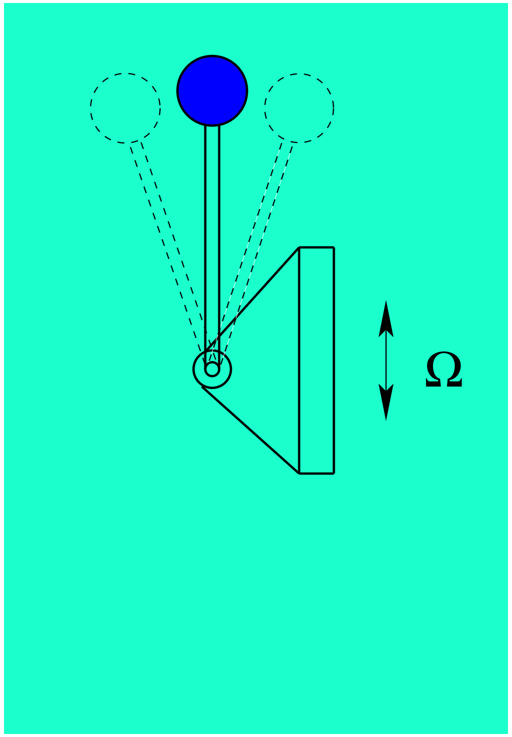


Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves





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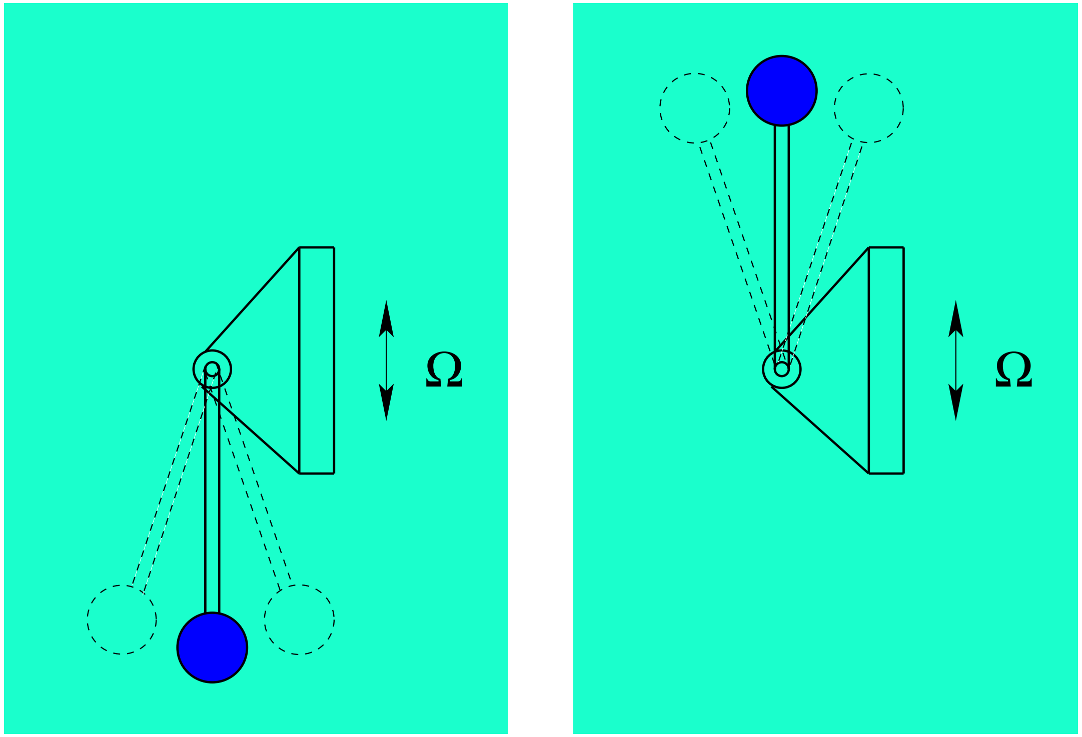


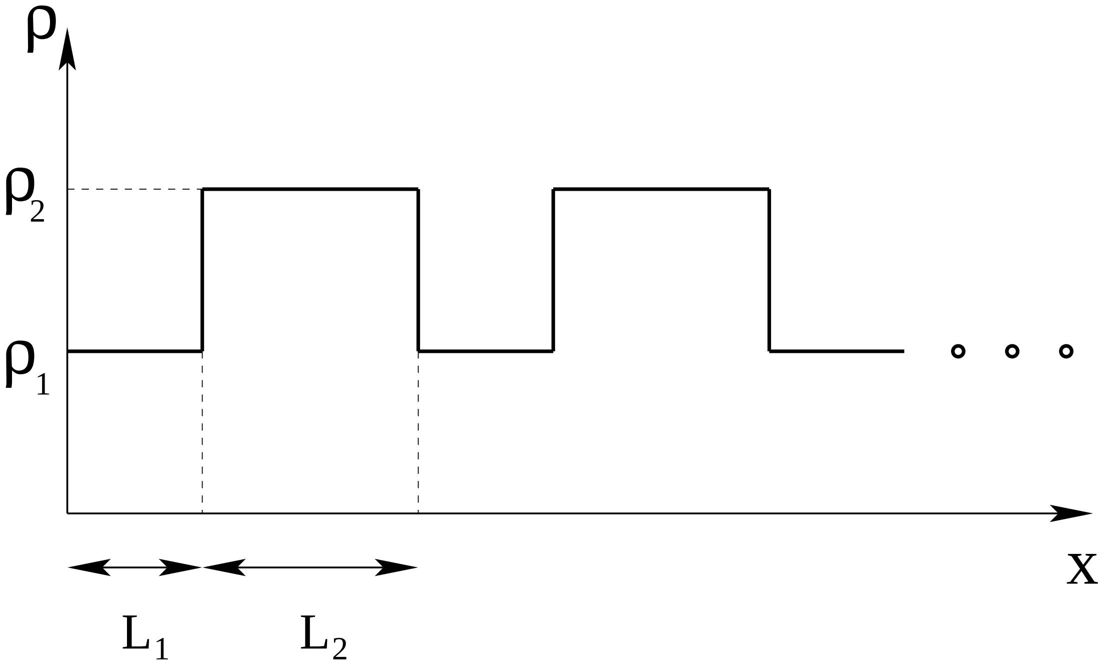
For small-ampl. osc.:

$$\Omega_{\rm res} = \frac{2\omega}{n}$$

$$n = 1, 2, 3...$$

Different from MSW eff. – no level crossing !

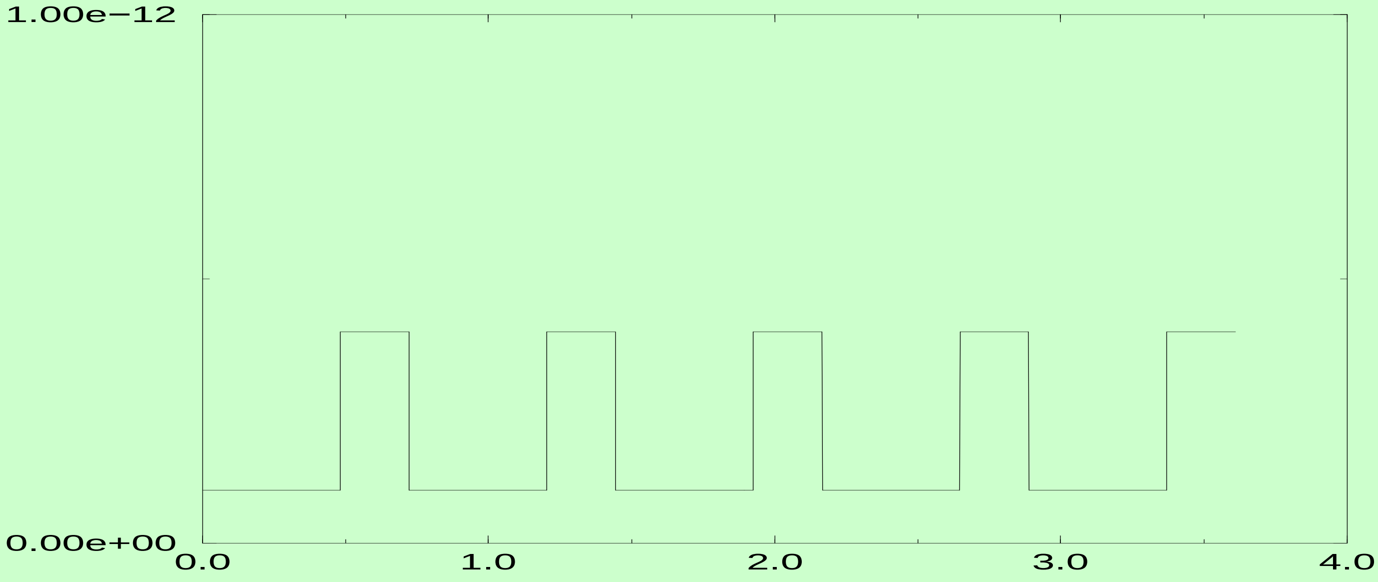
An example admitting an exact analytic solution – "castle wall" density profile (E.A., 1987, 1998):

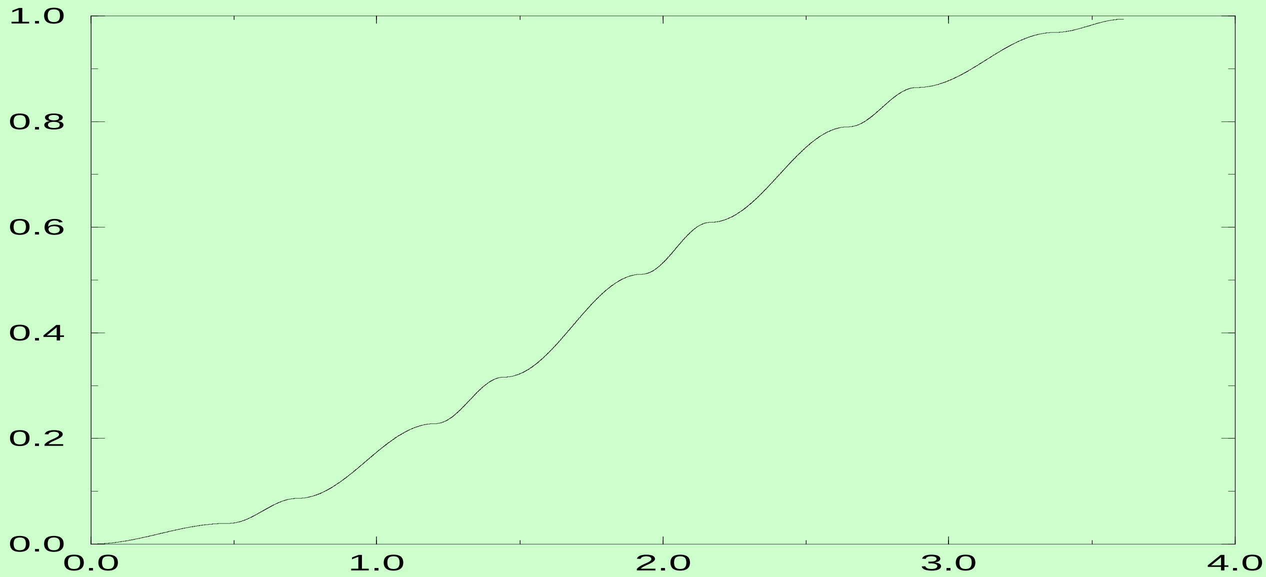


Resonance condition:

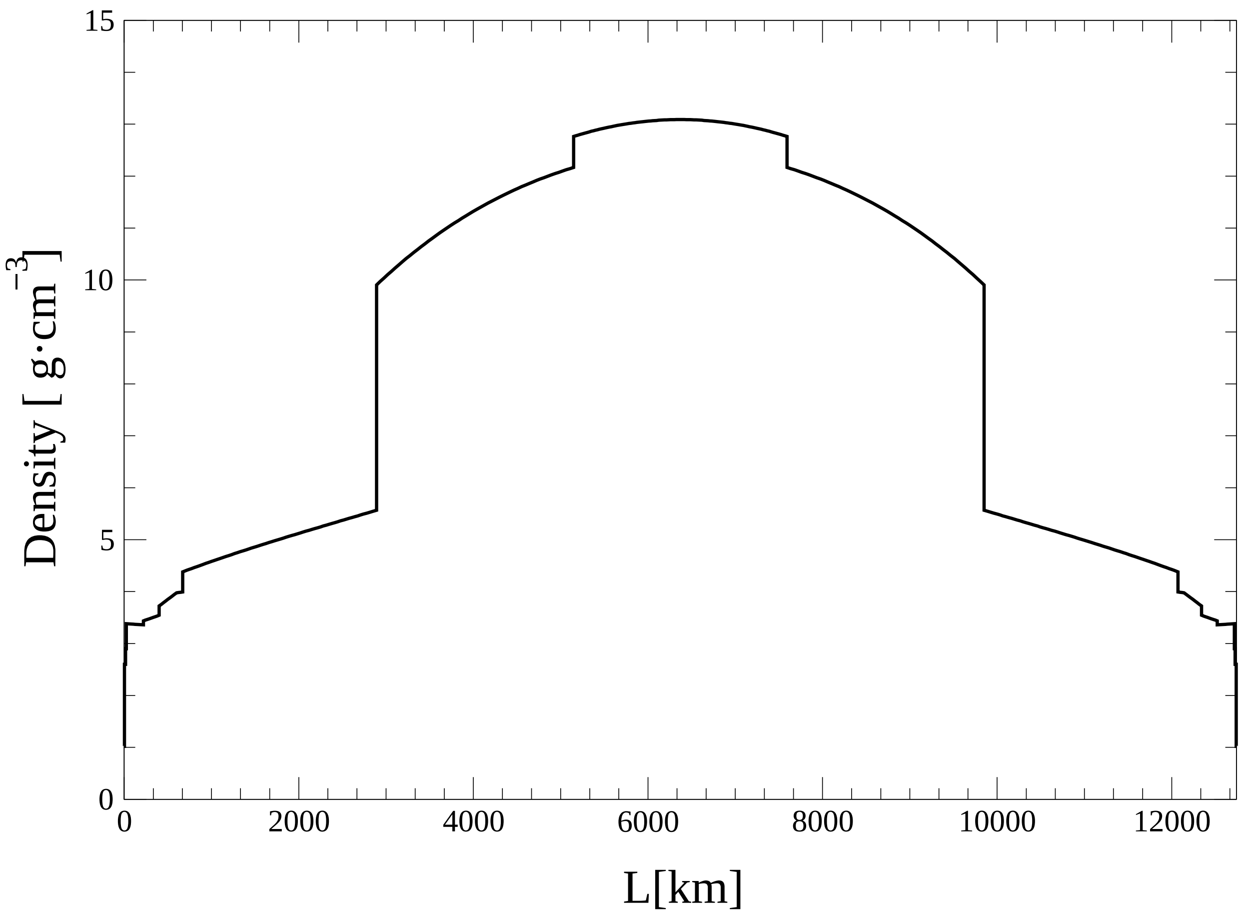
$$X_3 \equiv -(\sin\phi_1 \cos\phi_2 \cos 2\theta_{1m} + \cos\phi_1 \sin\phi_2 \cos 2\theta_{2m}) = 0$$

$\phi_{1,2}$ – oscillation phases acquired in layers 1, 2


1.00e−12
0.00e+00
0.0
1.0
2.0
3.0
4.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0

Earth's density profile (PREM model) :



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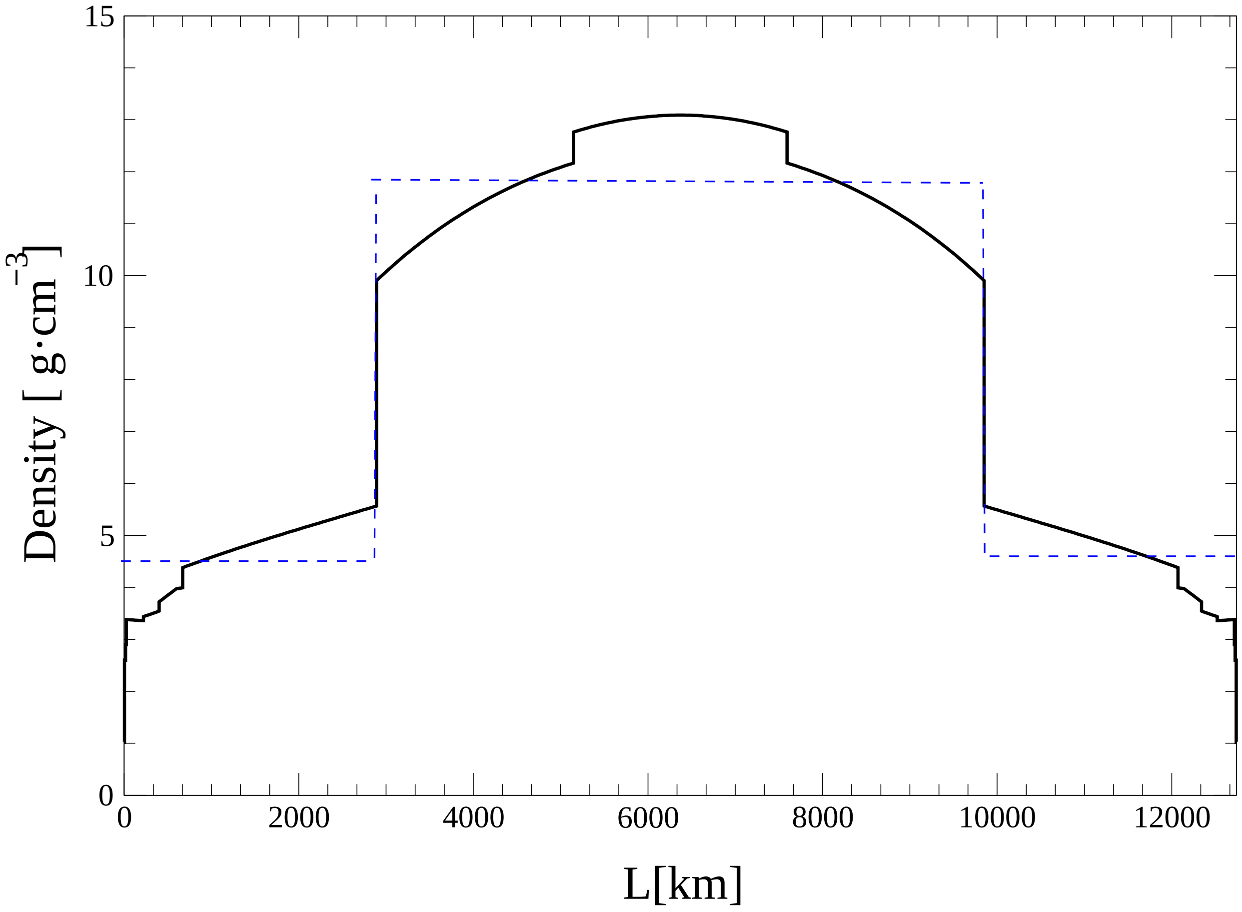
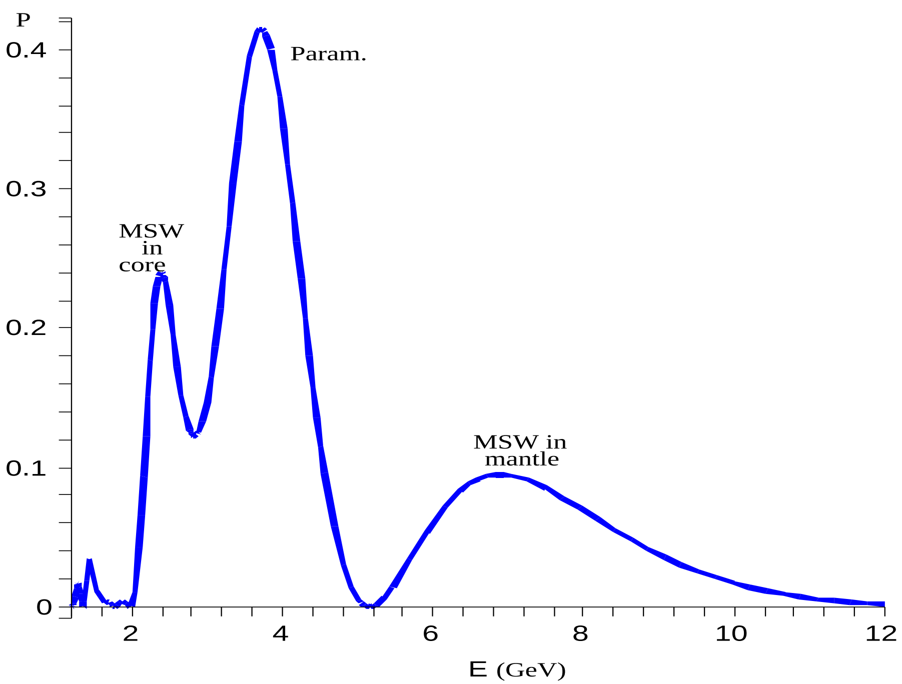
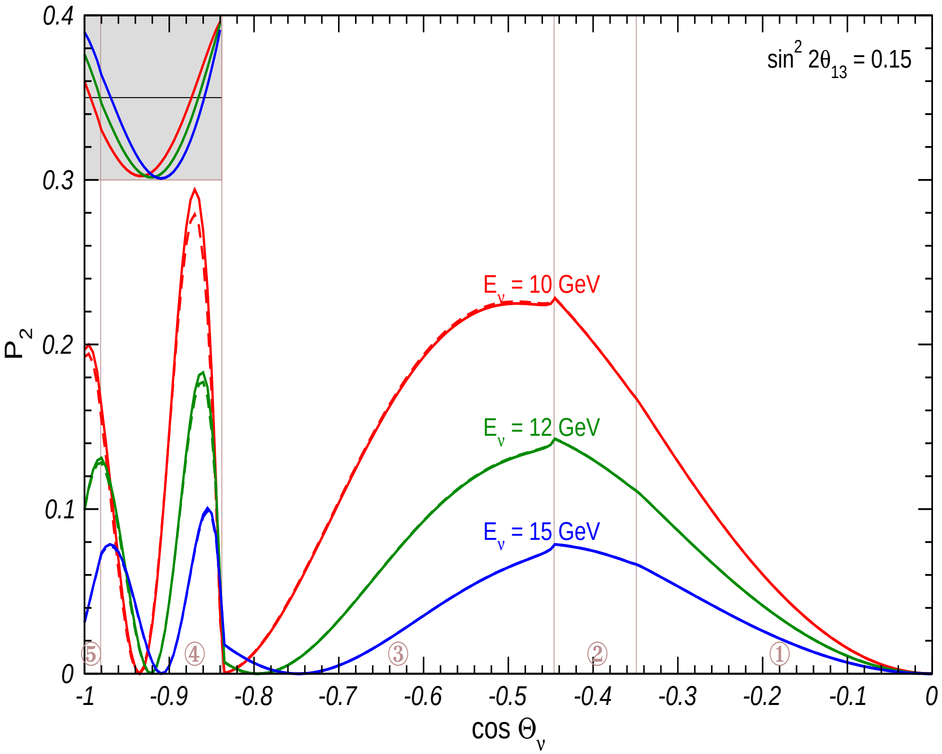


Param. res. condition: $(l_{osc})_{matt} \simeq l_{density\ mod.}$

Fulfilled for $\nu_e \leftrightarrow \nu_{\mu.\tau}$ oscillations of core-crossing ν's in the Earth for a wide range of energies and zenith angles !





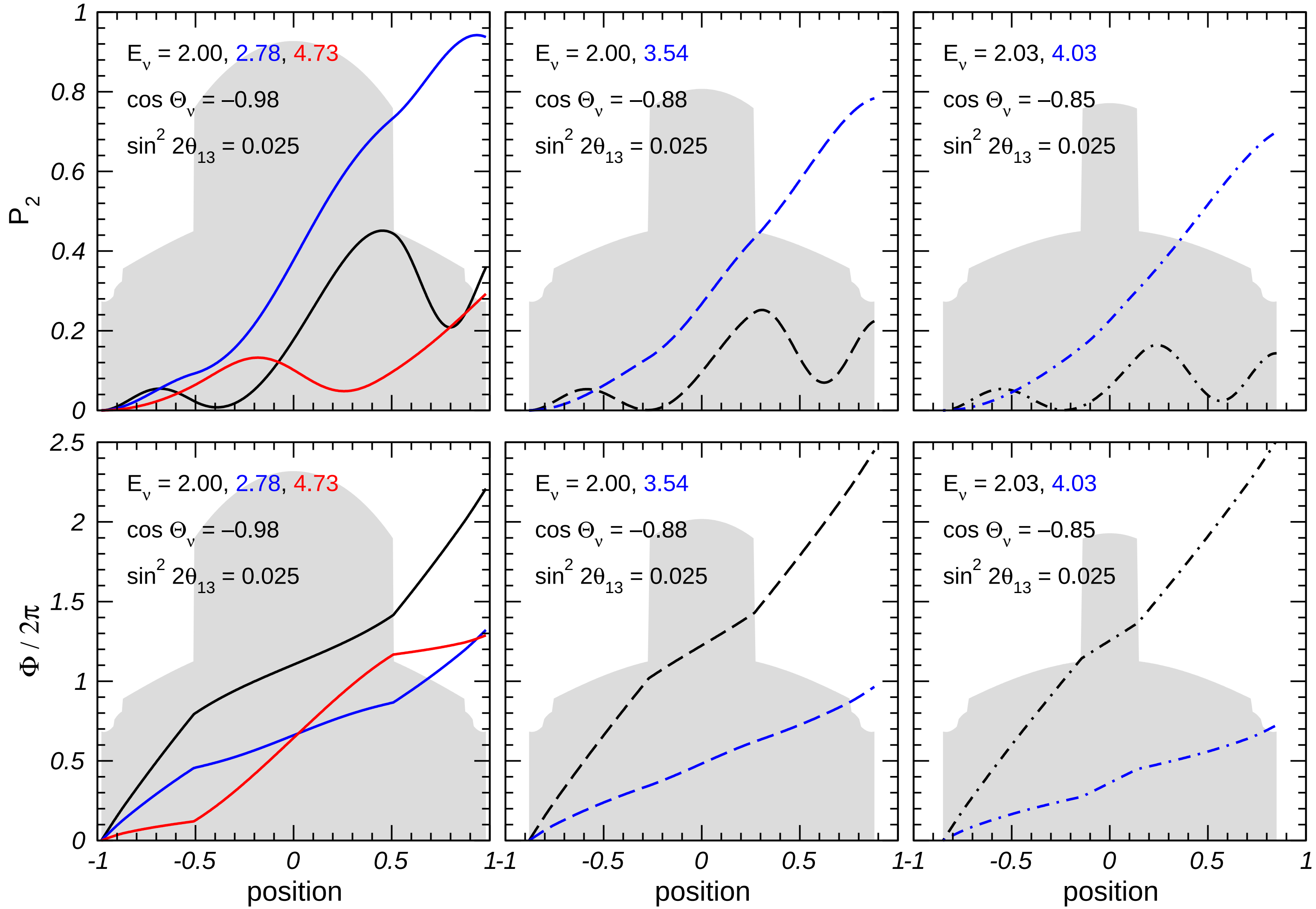
Intermed. energies

$\cos\Theta = -0.89$ $\sin^2 2\theta_{13} = 0.01$

(Liu, Smirnov, 1998; Petcov, 1998; EA 1998)

High energies, $\cos\Theta$ - dependence

(EA, Maltoni & Smirnov, 2005)



◇ Parametric resonance of ν oscillations in the Earth: can be observed in future atmospheric or accelerator experiments if θ_{13} is not much below its current upper limit

Genuine 3f effects

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in vacuum

- $\not{C}\not{P}$: $P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$
- $\not{T}$: $P(\nu_a \to \nu_b) \neq P(\nu_b \to \nu_a)$

CPT invariance: $\diamond\; P(\nu_a \to \nu_b) \to P(\bar{\nu}_b \to \bar{\nu}_a)$

$\not{C}\not{P} \;\Leftrightarrow\; \not{T}$ – *consequence of CPT*

Measures of $\not{C}\not{P}$ and $\not{T}$ – probability differences:

$$\Delta P_{ab}^{\mathrm{CP}} \equiv P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)$$

$$\Delta P_{ab}^{\mathrm{T}} \equiv P(\nu_a \to \nu_b) - P(\nu_b \to \nu_a)$$

From CPT:

$\diamond\quad \Delta P_{ab}^{\mathrm{CP}} = \Delta P_{ab}^{\mathrm{T}}\,; \qquad \Delta P_{aa}^{\mathrm{CP}} = 0$

3f case

One Dirac-type phase $\delta_{\rm CP}$ $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

- $\Delta P_{e\mu}^{\rm CP} = \Delta P_{\mu\tau}^{\rm CP} = \Delta P_{\tau e}^{\rm CP} \equiv \Delta P$

3f case

One Dirac-type phase $\delta_{\rm CP}$ $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

$$\diamond \quad \Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$

$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}\, L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}\, L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}\, L\right)\right]$$

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Vanishes when

- At least one $\Delta m_{ij}^2 = 0$
- At least one $\theta_{ij} = 0$ or 90°
- $\delta_{\rm CP} = 0$ or 180°
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

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- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

QM wave packet approach

QM wave packet approach

The evolved produced state:

$$|\nu_\alpha^{\rm fl}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,|\nu_i^{\rm mass}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,\Psi_i^S(\vec{x},t)|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\Psi_i^S(\vec{x},t) \;=\int\frac{d^3p}{(2\pi)^3}\,f_i^S(\vec{p})\,e^{i\vec{p}\vec{x}-iE_i(p)t}$$

Momentum distribution function $f_i^S(\vec{p})$: sharp maximum at $\vec{p}=\vec{P}$ (width of the peak $\sigma_{pP}\ll P$).

$$E_i(p) \;=\; E_i(P) + \frac{\partial E_i(p)}{\partial\vec{p}}\bigg|_{\vec{P}}(\vec{p}-\vec{P}) + \frac{1}{2}\frac{\partial^2 E_i(p)}{\partial\vec{p}^2}\bigg|_{\vec{p}_0}(\vec{p}-\vec{P})^2 + \ldots$$

$$\vec{v}_i \;=\; \frac{\partial E_i(p)}{\partial\vec{p}} \;=\; \frac{\vec{p}}{E_i}\,, \qquad\qquad \alpha\;\equiv\;\frac{\partial^2 E_i(p)}{\partial\vec{p}^2}\;=\;\frac{m_i^2}{E_i^2}$$

Evolved neutrino state

$$\Psi_i^S(\vec{x},\, t) \;\simeq\; e^{-iE_i(P)t+i\vec{P}\vec{x}}\, g_i^S(\vec{x}-\vec{v}_i t) \qquad (\alpha \;\to\; 0)$$

$$g_i^S(\vec{x}-\vec{v}_i t) \;\equiv\; \int \frac{d^3 p_1}{(2\pi)^3}\, f_i^S(\vec{p}_1)\, e^{i\vec{p}_1(\vec{x}-\vec{v}_g t)}$$

Center of the wave packet: $\vec{x}-\vec{v}_i t = 0$. Spatial length: $\sigma_{xP} \sim 1/\sigma_{pP}$ (g_i^S decreases quickly for $|\vec{x}-\vec{v}_i t| \gtrsim \sigma_{xP}$).

Detected state (centered at $\vec{x}=\vec{L}$):

$$|\nu_\beta^{\rm fl}(\vec{x})\rangle \;=\; \sum_k U_{\beta k}^*\, \Psi_k^D(\vec{x})|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\Psi_k^D(\vec{x}) \;=\; \int \frac{d^3 p}{(2\pi)^3}\, f_k^D(\vec{p})\, e^{i\vec{p}(\vec{x}-\vec{L})}$$

Oscillation probability

Transition amplitude:

$$\mathcal{A}_{\alpha\beta}(T,\vec{L}) \;=\; \langle\nu_\beta^{\rm fl}|\nu_\alpha^{\rm fl}(T,\vec{L})\rangle \;=\; \sum_i U_{\alpha i}^* U_{\beta i}\,\mathcal{A}_i(T,\vec{L})$$

$$\boxed{\mathcal{A}_i(T,\vec{L}) \;=\; \int\frac{d^3p}{(2\pi)^3}\,f_i^S(\vec{p})\,f_i^{D*}(\vec{p})\,e^{-iE_i(p)T+i\vec{p}\vec{L}}}$$

Strongly suppressed unless $|\vec{L}-\vec{v}_i T| \lesssim \sigma_x$. E.g., for Gaussian wave packets:

$$\mathcal{A}_i(T,\vec{L}) \;\propto\; \exp\left[-\frac{(\vec{L}-\vec{v}_i T)^2}{4\sigma_x^2}\right], \qquad \sigma_x^2 \;\equiv\; \sigma_{xP}^2+\sigma_{xD}^2$$

Oscillation probability:

◇ $$P(\nu_\alpha\to\nu_\beta;T,\vec{L}) \;=\; |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\,\mathcal{A}_i(T,\vec{L})\mathcal{A}_k^*(T,\vec{L})$$

When are neutrino oscillations observable?

Another source of decoherence: wave packet separation due to the difference of group velocities Δv of different mass eigenstates.

If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \propto \sum_i P_{\rm prod}(\mu\,\nu_i)P_{\rm det}(e\,\nu_i) \propto \sum_i |U_{\mu i}|^2|U_{ei}|^2$$

– the same result as for averaged oscillations.

How are the oscillations destroyed? Suppose by measuring momenta and energies of particles at neutrino production (or detection) we can determine its energy E and momentum p with uncertainties σ_E and σ_p. From $E_i^2 = p_i^2 + m_i^2$:

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2\right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 \;=\; |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \;\simeq\; l_{\rm osc}^{-1}$.

But: To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{\rm x,\,prod} \;\gtrsim\; \sigma_p^{-1} \;>\; l_{\rm osc}$$

$\Rightarrow$ Oscillations washed out. Similarly for neutrino detection.

Natural necessary condition for coherence (observability of oscillations):

$$L_{\rm source} \;\ll\; l_{\rm osc}\,, \qquad\qquad L_{\rm det} \;\ll\; l_{\rm osc}$$

No averaging of oscillations in the source and detector

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\rm coh} \simeq \sigma_x \,; \qquad l_{\rm coh} \simeq v t_{\rm coh}$$

$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\, v\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

Neutrino oscillations: *Coherence at macroscopic distances* –
$L >$ 10,000 km in atmospheric neutrino experiments !

Do charged leptons oscillate?

Do charged leptons oscillate?

What do we mean by charged leptons?

The usual $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$ are mass eigenstates $\Rightarrow$ do not oscillate.

[Also: unlike neutrinos, they participate also in EM interactions (and are normally detected via these interactions) which are flavour-blind.]

Assume we create a muon at $t_0 = 0$ and $\vec{x}_0 = 0$. Neglecting muon decay, we have

$$|\Psi(0)\rangle = |\mu\rangle\,; \quad |\Psi(\vec{x},t)\rangle = e^{-ip_\mu x}|\mu\rangle \quad \Rightarrow \quad P_{\mu\mu} = |\langle\mu|\Psi(\vec{x},t)\rangle|^2 = 1$$

Assume now we manage to create a coherent superposition of μ and e:

$$|\Psi(0)\rangle = \cos\theta|\mu\rangle + \sin\theta|e\rangle$$

The weights of μ and e in the initial state: $\cos^2\theta$ and $\sin^2\theta$.

Do charged leptons oscillate?

Evolved state:

$$|\Psi(\vec{x},\, t)\rangle \;=\; e^{-ip_\mu x}\cos\theta|\mu\rangle \;+\; e^{-ip_e x}\sin\theta|e\rangle$$

The probabilities of finding μ and e:

$$P_\mu \;=\; |\langle\mu|\Psi(\vec{x}\,,t)\rangle|^2 \;=\; |e^{-ip_\mu x}\cos\theta|^2 \;=\; \cos^2\theta$$
$$P_e \;=\; |\langle e|\Psi(\vec{x}\,,t)\rangle|^2 \;=\; |e^{-ip_e x}\sin\theta|^2 \;=\; \sin^2\theta$$

– are the same! $\Rightarrow$ There are no oscillations between mass eigenstates, no matter if the initial state is pure or (coherently) mixed

$$\Downarrow$$

There are no oscillations between e, μ and τ !

[NB: The same for neutrinos – initially produced ν_e can with some probability oscillate into ν_μ or ν_τ, but the weights of ν_1, ν_2 and ν_3 that were in the initial state will remain the same!]

Is that the full answer?

Can we imagine a situation when one creates a coherent superposition of e, μ and τ and then also detects their coherent superposition (the same or different) rather than individual mass-eigenstate charged leptons?

Charged - current weak interactions look completely symmetric w.r.t. neutrinos and charged leptons!

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}(\bar{e}_{aL}\gamma^\mu U_{ai}\nu_{iL})W_\mu^- + h.c., \qquad U = V_L^\dagger V_\nu$$

Why do we say that charged leptons are emitted and detected in mass eigenstates and neutrinos in flavour states (superpositions of mass eigenstates) and not vice versa? Or not both as some superpositions of mass eigenstates? E.g.

$|e_1\rangle = U_{1e}|e\rangle + U_{1\mu}|\mu\rangle + U_{1\tau}|\tau\rangle$ is emitted or detected together with ν_1,

$|e_2\rangle = U_{2e}|e\rangle + U_{2\mu}|\mu\rangle + U_{2\tau}|\tau\rangle$ is emitted or detected together with ν_2,

$|e_3\rangle = U_{3e}|e\rangle + U_{3\mu}|\mu\rangle + U_{3\tau}|\tau\rangle$ is emitted or detected together with ν_3.

Why do neutrinos oscillate?

Because they are emitted (and absorbed) alongside charged leptons of definite mass $e^{\pm}$, $\mu^{\pm}$ or $\tau^{\pm}$. (This “measures” the flavour of neutrinos). How do we know that charged leptons are in mass eigenstates?

(1) Beta decay: only electrons are emitted together with neutrinos. Emission of $\mu^{\pm}$ and $\tau^{\pm}$ is forbidden by energy conservation.

(2) Decays $\pi^{\pm} \to \mu^{\pm}\nu$, $\pi^{\pm} \to e^{\pm}\nu$ (or $K^{\pm} \to \mu^{\pm}\nu$, $K^{\pm} \to e^{\pm}\nu$). Here emission of both muons and electrons is allowed.

Assume a coherent superposition of e and μ is produced in pion decay (nearly) at rest. The energy uncertainty of the charged lepton:

$$\sigma_E \simeq \Gamma_\pi = 2.5 \cdot 10^{-8}\ \mathrm{eV}$$

Uncertainty in the mass determination $\left(\sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2]} \simeq 2\sqrt{2}E\sigma_E\right)$:

$$\sigma_{m^2} \ \sim\ 2\sqrt{2}E\sigma_E \ \simeq\ 2\sqrt{2}\cdot(90\ \mathrm{MeV})\cdot(2.5\cdot 10^{-8}\ \mathrm{eV}) \ \simeq\ 6.4\ \mathrm{eV}^2$$

Do charged leptons oscillate?

This has to be compared with $m_\mu^2 - m_e^2 \simeq (106 \text{ MeV})^2 \quad \Rightarrow$
Different mass-eigenstate charged leptons are emitted incoherently!

This provides a "measurement" of the flavour of the emitted neutrino

For pion decay in flight: assume pion's energy is E_0. The energies of the produced charged leptons are rescaled as $E \to E\,(E_0/m_\pi)$, but the pion decay width (and so σ_E) is rescaled as $\Gamma_\pi \to \Gamma_\pi(m_\pi/E_0) \quad \Rightarrow$ $[(2E\sigma_E)^2 + (2p\sigma_p)^2]^{1/2}$ remains the same (σ_{m^2} a Lorentz invariant quantity).

$$\Downarrow$$

♢ Charged leptons produced in $\pi^\pm \to l^\pm \nu$ and $K^\pm \to l^\pm \nu$ decays are always emitted as mass eigenstates and not as coherent superpositions of different mass eigenstates because of their very large Δm^2.

♢ Therefore even oscillations between e_1, e_1 and e_3 (or any other superpositions of e, μ and τ) are not possible.

Do charged leptons oscillate?

The masses and decay widths of $\pi^\pm$, $K^\pm$ are rather small $\Rightarrow$ σ_{m^2} small.

How about decays of $W^\pm$? For $W^\pm \to l^\pm \nu$ decays at rest:

$$\Gamma^0_{W\to l_a \nu} \simeq \frac{G_F m_W^3}{6\sqrt{2}\pi} \simeq 230 \;\; \mathrm{MeV}$$

$$\Rightarrow \qquad \sigma_{m^2} \;\sim\; 2\sqrt{2}\, E\sigma_E \;\simeq\; 2\sqrt{2}\cdot 40\ \mathrm{GeV}\cdot 230\ \mathrm{MeV} \;\simeq\; (5\ \mathrm{GeV})^2\,.$$

Thus

$$\sigma_{m^2} \gg m_\mu^2 - m_e^2\,, \qquad \sigma_{m^2} > m_\tau^2 - m_\mu^2 \simeq (1.77\ \mathrm{GeV})^2\,,$$

$\Rightarrow$ all three charged leptons are produced *coherently* in $W^\pm$ decays.

Can one then observe oscillations between their different coh. superpositions?

Coherence length $l_{\rm coh} \simeq \sigma_x/\Delta v_g$:

$$(l_{\rm coh})_{\max} \simeq [\Gamma^0_{W\to l_a \nu}(\Delta v_g)_{\min}]^{-1} \simeq \frac{3\sqrt{2}\pi}{G_F m_W (m_\mu^2 - m_e^2)} \simeq 2.5\times 10^{-8}\ \mathrm{cm}\,.$$

$\Rightarrow$ $l^\pm$ loose their coherence almost immediately after their production

Do charged leptons oscillate?

What about $W^\pm \to l^\pm \nu$ decays in flight? Let γ be the Lorentz factor of $W^\pm$. $(\Delta v_g)_{\rm min} \simeq \Delta m_{\mu e}^2/2E^2 \equiv (m_\mu^2 - m_e^2)/2E^2$ and the partial decay width of $W^\pm$ scale with γ as

$$(\Delta v_g)_{\rm min} \to \gamma^{-2} (\Delta v_g)_{\rm min}\,, \qquad \Gamma^0_{W\to l_a \nu} \to \gamma^{-1} \Gamma^0_{W\to l_a \nu}\,.$$

Therefore the maximum coherence length $(l_{\rm coh})_{\rm max} \simeq \sigma_x/(\Delta v_g)_{\rm min} \simeq 1/[\Gamma^0_{W\to l_a \nu}(\Delta v_g)_{\rm min}]$ scales as

$$(l_{\rm coh})_{\rm max} \to \gamma^3 (l_{\rm coh})_{\rm max}\,.$$

In order for $(l_{\rm coh})_{\rm max}$ to be larger than e.g. 1 m, one would need $\gamma \gtrsim 1600$, or $E_W \gtrsim 130$ TeV – far above presently feasible energies.

Do charged leptons oscillate?

What about $W^{\pm} \to l^{\pm}\nu$ decays in flight? Let γ be the Lorentz factor of $W^{\pm}$. $(\Delta v_g)_{\rm min} \simeq \Delta m^2_{\mu e}/2E^2 \equiv (m_\mu^2 - m_e^2)/2E^2$ and the partial decay width of $W^{\pm}$ scale with γ as

$$(\Delta v_g)_{\rm min} \to \gamma^{-2}(\Delta v_g)_{\rm min}\,, \qquad \Gamma^0_{W\to l_a\nu} \to \gamma^{-1}\Gamma^0_{W\to l_a\nu}\,.$$

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In order for $(l_{\rm coh})_{\rm max}$ to be larger than e.g. 1 m, one would need $\gamma \gtrsim 1600$, or $E_W \gtrsim 130$ TeV – far above presently feasible energies.

N.B.: Even if coherence was satisfied for charged leptons, to fix the composition of the mixed $l^{\pm}$ state in terms of e, μ and τ one would have to detect the accompanying neutrino as a state different from $\nu_{\rm fl}$ – e.g. as a mass eigenstate. Not possible within the standard model!

Extensions of the standard model?

Consider the SM amended by three heavy RH neutrinos N_i (seesaw model) plus an extra Higgs doublet. In this model N_i can decay into a charged lepton and charged Higgs boson:

$$N_i \to e_i^- + \Phi^+ \,.$$

Decays are caused by the Yukawa coupling Lagrangian

$$\mathcal{L}_Y = Y_{ai} \bar{L}_a N_{Ri} \Phi \; + \; h.c. \,,$$

In the basis where the mass matrices of N_i and $l^\pm$ have been diagonalized, the Yukawa coupling matrix Y_{ai} is in general not diagonal $\Rightarrow$ in the decay of a mass-eigenstate sterile neutrino N_i any of the three charged leptons $e_a = e,\, \mu,\, \tau$ can be produced.

What are the conditions for the produced charged lepton state e_i to be a coherent superposition of the mass eigenstates e_a:

$$|e_i\rangle = [(Y^\dagger Y)_{ii}]^{-1/2} \sum_a Y^\dagger_{ia} \, |e_a\rangle \,,$$

and how long this state can maintain its coherence?

Extensions of the standard model?

Neglecting the masses of $\Phi^{\pm}$ and $l^{\pm}$ compared to the mass M_i of the sterile neutrino:

$$\Gamma_i^0 \simeq \alpha_i M_i\,, \qquad \text{where} \qquad \alpha_i \equiv \frac{(Y^\dagger Y)_{ii}}{16\pi}\,.$$

Coherent production condition:

$$2\sqrt{2}\,E\,\Gamma_i^0 \simeq 2\sqrt{2}\,(M_i/2)\,\alpha_i M_i > \max\{m_\mu^2 - m_e^2,\; m_\tau^2 - m_\mu^2\}\,,$$

or

$$\alpha_i > 2.2\,(\mathrm{GeV}/M_i)^2\,.$$

From $l_{\rm coh} = \sigma_x v_g/\Delta v_g$ the coherence length for the emitted charged lepton state:

$$l_{\rm coh} \simeq \frac{M_i^2}{2\Gamma_i^0(m_\tau^2 - m_\mu^2)} \simeq 3.1\times 10^{-15}\,\alpha_i^{-1}\frac{M_i}{\mathrm{GeV}}\;\;\mathrm{cm}\,.$$

$\Rightarrow$

Extensions of the standard model?

$$l_{\rm coh} < 1.4\times 10^{-15}\ {\rm cm}\ (M_i/{\rm GeV})^3\,.$$

For N_i decays in flight the r.h.s. has to be multiplied by $\gamma^3 \;\;\Rightarrow\;\; (M_i/{\rm GeV})^3$ has to be replaced by $(E_i/{\rm GeV})^3$.

The charged lepton state will maintain its coherence over the distance $\sim 1\ m$ if

$$E_i \gtrsim 400\ {\rm TeV} \quad\Rightarrow\quad (Y^\dagger Y)_{ii} \gtrsim 1.3\times 10^{-11}\,.$$

If only e and μ are to be produced coherently, a milder lower limit on E_i results:

$$E_i \gtrsim 10\ {\rm TeV}\,, \qquad (Y^\dagger Y)_{ii} \gtrsim 8.5\times 10^{-11}\,.$$

Extensions of the standard model?

If the condition for coherent creation of the charged lepton state is satisfied and this state is detected through the inverse decay process before it loses its coherence, it may exhibit oscillations: a mass eigenstate sterile neutrino N_j different from N_i can be produced in the detection process $\Rightarrow$ the state e_i has oscillated into e_j.

Charged leptons would be able to oscillate, leading to a non-zero probability of the emission or absorption of a different sterile neutrino mass eigenstate N_j in the processes $e_j^{\pm} + \Phi^{\mp} \to N_j$ or $e_j^{\pm} + N_j \to \Phi^{\pm}$.

$\Rightarrow$ The roles of neutrinos and charged leptons reversed compared to the usual situation because of sterile neutrinos being much heavier than the charged leptons.

Shortcomings of the QM w. packet approach

- Neutrino wave packet postulated rather than derived, widths estimated
- Production and detection processes are not considered
- Inadequate normalization procedure. Normalization “by hand” is unavoidable.

Advantage: simplicity

QFT approach

Calc. from 1st principles – QFT approach

Production - propagation - detection treated as a single inseparable process. External particles are described by wave packets, neutrinos – by propagators

One-particle states of external particles:

$$|A\rangle = \int [dp]\, f_A(\vec{p}, \vec{P})\, |A, \vec{p}\rangle\,, \qquad [dp] \equiv \frac{d^3p}{(2\pi)^3 \sqrt{2E_A(\vec{p})}}$$

$|A, \vec{p}\rangle$ – one-particle momentum eigenstate corresponding to momentum $\vec{p}$ and energy $E_A(\vec{p})$ (free particles: $E_A(\vec{p}) = \sqrt{\vec{p}^2 + m_A^2}$). The normalization condition for the plane wave states $|A, \vec{p}\rangle$:

$$\langle A, \vec{p}\,' | A, \vec{p}\rangle = 2E_A(\vec{p})\,(2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}\,')\,.$$

$f_A(\vec{p}, \vec{P})$ – momentum distribution function with the mean momentum $\vec{P}$.
Normalization condition: $\langle A|A\rangle = 1 \quad \Rightarrow \quad \int d^3p\, |f_A(\vec{p})|^2/(2\pi)^3 = 1$.

QFT approach – contd.

Coordinate-space wave packet with maximum at $\vec{x} = \vec{x}_0$ at the time $t - t_0$:

$$\Psi_A(x) = \int [dp]\, f_A(\vec{p}) e^{-iE_A(\vec{p})(t-t_0)+i\vec{p}(\vec{x}-\vec{x}_0)}$$

Consistent with the usual QFT definition of the wave function:

$$\Psi_A(x) = \langle 0|\hat{\Psi}_A(x)|A\rangle\,.$$

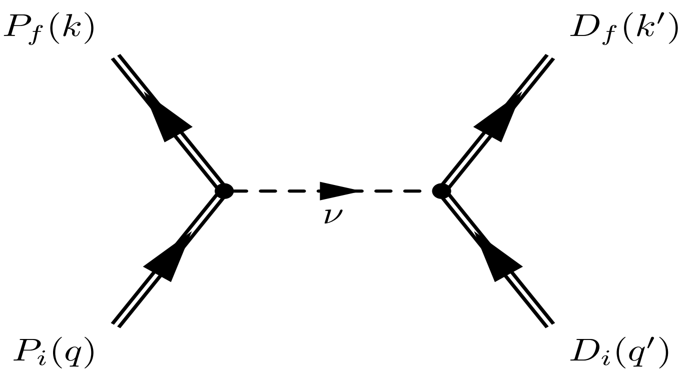
Transition amplitude:

$$\mathcal{A}_{\alpha\beta} \;=\; \sum_j U_{\alpha j}^* U_{\beta j} \mathcal{A}_j\,.$$

Use the Feynman rules in the configuration space. In lowest (2nd) order in weak interaction:

$$\mathcal{A}_j = \int d^4 x_1 \int d^4 x_1\, A_j^P(x_1) S_{Fj}(x_1 - x_2) A_j^D(x_2)\,.$$

How is it obtained?



$$|P_i\rangle = \int [dq]\, f_{Pi}(\vec{q}, \vec{Q})\, |P_i, \vec{q}\rangle\,, \qquad |P_f\rangle = \int [dk]\, f_{Pf}(\vec{k}, \vec{K})\, |P_f, \vec{k}\rangle\,,$$

$$|D_i\rangle = \int [dq']\, f_{Di}(\vec{q}', \vec{Q}')\, |D_i, \vec{q}'\rangle\,, \qquad |D_f\rangle = \int [dk']\, f_{Df}(\vec{k}', \vec{K}')\, |D_f, \vec{k}'\rangle\,.$$

The transition amplitude:

$$i\mathcal{A}_{\alpha\beta} = \langle P_f\, D_f | \hat{T} \exp\left[-i \int d^4x\, \mathcal{H}_I(x) \right] - \mathbb{1} | P_i\, D_i \rangle\,,$$

QFT approach – contd.

In the second order in weak interaction:

$$i\mathcal{A}_{\alpha\beta} \;=\; \sum_j U_{\alpha j}^* U_{\beta j} \int [dq]\, f_{Pi}(\vec{q},\vec{Q}) \int [dk]\, f_{Pf}^*(\vec{k},\vec{K})$$
$$\times \int [dq']\, f_{Di}(\vec{q}\,',\vec{Q}\,') \int [dk']\, f_{Df}^*(\vec{k}\,',\vec{K}\,')\, i\mathcal{A}_j^{p.w.}(q,k;q',k')\,.$$

Plane-wave amplitude:

$$i\mathcal{A}_j^{p.w.}(q,k;q',k') = \int d^4x_1 \int d^4x_2\, \tilde{M}_D(q',k')\, e^{-i(q'-k')(x_2-x_D)}$$
$$\times\, i \int \frac{d^4p}{(2\pi)^4} \frac{\not{p}+m_j}{p^2-m_j^2+i\epsilon}\, e^{-ip(x_2-x_1)}\, \tilde{M}_P(q,k)\, e^{-i(q-k)(x_1-x_P)}$$

$\tilde{M}_{jP}$, $\tilde{M}_{jD}$ – production and detection amplitudes with neutrino spinors excluded. Full amplitudes:

$$M_{jP}(q,k) \equiv \frac{\bar{u}_{jL}(p)}{\sqrt{2p_0}}\tilde{M}_P(q,k)\,, \qquad M_{jD}(q',k') \equiv \tilde{M}_D(q',k')\frac{u_{jL}(p)}{\sqrt{2p_0}}$$

QFT approach – contd.

Neutrino prod. and det. regions: the overlap regions of the wave packets of participating external particles. 4-coordinates of the "central points" of these regions (points of the maximal overlap of external w. packets): x_P and x_D. It will be convenient to go to shifted 4-coordinates:

$$x_1' = x_1 - x_P\,, \qquad x_2' = x_2 - x_D\,.$$

Also define

$$T\ =\ t_D - t_P\,, \qquad \vec{L}\ =\ \vec{x}_D - \vec{x}_P\,.$$

A useful formula:

$$\not{p} + m_j\ =\ \sum_\sigma u_{j\sigma}(p)\bar{u}_{j\sigma}(p)\,.$$

For neutrinos only one chirality contributes $(\sigma = L$ for ν and $\sigma = R$ for $\bar{\nu})$ because of the chiral nature of weak interactions $\Rightarrow$ the sum over σ can be dropped; $u_{j\sigma}(p)$ and $\bar{u}_{j\sigma}(p)$ can then be merged with $\tilde{M}_{P,D}$ to produce M_{jP} and M_{jD}.

QFT approach – contd.

$$i\mathcal{A}_{\alpha\beta} = i\sum_j U^*_{\alpha j}U_{\beta j}\int\frac{d^4p}{(2\pi)^4}\,\Phi_{jP}(p^0,\vec{p})\Phi_{jD}(p^0,\vec{p})\,\frac{2p_0\,e^{-ip^0T+i\vec{p}\vec{L}}}{p^2-m_j^2+i\epsilon}\,.$$

$$\Phi_{jP}(p^0,\vec{p}) = \int d^4x'_1 e^{ipx'_1}\int[dq]\int[dk]\,f_{Pi}(\vec{q},\vec{Q})\,f^*_{Pf}(\vec{k},\vec{K})\,e^{-i(q-k)x'_1}M_{jP}(q,k)$$

$$\Phi_{jD}(p^0,\vec{p}) = \int d^4x'_2 e^{-ipx'_2}\int[dq']\int[dk']\,f_{Di}(\vec{q}',\vec{Q}')f^*_{Df}(\vec{k}',\vec{K}')\,e^{-i(q'-k')x'_2}M_{jD}(q',k')$$

For $L \gg 1/p$ – fast oscillating factor in $i\mathcal{A}_{\alpha\beta}$ $\Rightarrow$ main contribution to integral over p^0 from the pole at $p^0 = E_j(\vec{p}) - i\epsilon$ (on-shell neutrinos).

$$\Downarrow$$

$$i\mathcal{A}_{\alpha\beta} = \Theta(T)\sum_j U^*_{\alpha j}U_{\beta j}\int\frac{d^3p}{(2\pi)^3}\,\Phi_{jP}(E_j(\vec{p}),\vec{p})\Phi_{jD}(E_j(\vec{p}),\vec{p})\,e^{-iE_j(\vec{p})T+i\vec{p}\vec{L}}$$

In the QM w.packet approach we had:

Transition amplitude

$$\mathcal{A}_{\alpha\beta}(T,\vec{L}) \;=\; \langle\nu_\beta^{\rm fl}|\nu_\alpha^{\rm fl}(T,\vec{L})\rangle \;=\; \sum_j U_{\alpha j}^* U_{\beta j}\,\mathcal{A}_j(T,\vec{L})$$

$$\boxed{\mathcal{A}_j(T,\vec{L}) \;=\; \int\frac{d^3p}{(2\pi)^3}\,f_j^S(\vec{p})\,f_j^{D*}(\vec{p})\,e^{-iE_j(p)T+i\vec{p}\vec{L}}}$$

The QM and QFT expressions have exactly the same form !

QFT approach – contd.

Comparing with $\mathcal{A}_{ab}(T,\vec{L})$ obtained in the QM w. packet approach: the two amplitudes coincide if

$$f_{jP}(\vec{p}) = \Phi_{jP}(E_j(\vec{p}),\vec{p})\,, \qquad f_{jD}(\vec{p}) = \Phi^*_{jD}(E_j(\vec{p}),\vec{p})\,,$$

QFT approach – contd.

Comparing with $\mathcal{A}_{ab}(T, \vec{L})$ obtained in the QM w. packet approach: the two amplitudes coincide if

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Easy to understand: $\Phi_{jP}(E_j(p), \vec{p})$ is the probability amplitude of ν production process in which ν_j is emitted with momentum $\vec{p}$

$\Rightarrow$ Φ_{jP} is momentum distribution function of the produced neutrino, i.e. the momentum-state wave packet $f_{jP}(\vec{p})$. Similarly for neutrino detection.

N.B.: $f_{jP}(\vec{p})$ and $f_{jD}(\vec{p})$ are not “canonically” normalized.

Alternative approaches:

QFT approach – contd.

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Alternative approaches:

- $|P_f \nu_j\rangle \;=\; (S - \mathbb{1})|P_i\rangle\,, \qquad |\nu_j\rangle \;=\; \langle P_f|P_f \nu_j\rangle$

QFT approach – contd.

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QFT approach – contd.

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All three approaches give the same results.

General properties of ν w. packets in QFT

$$f_{jP}(\vec{p}) \simeq M_{jP}(Q,K)\int d^4x\, e^{iE_j(\vec{p})t-i\vec{p}\vec{x}}\int [dq]\int [dk] f_{Pi}(\vec{q},\vec{Q}) f^*_{Pf}(\vec{k},\vec{K}) e^{-i(q-k)x}$$

Integral over $\vec{x}$ gives $\sim \delta^{(3)}(\vec{q}-\vec{k}-\vec{p})$. Since $f_{Pi}(\vec{q},\vec{Q})$, $f_{Pf}(\vec{k},\vec{K})$ are sharply peaked at $\vec{Q}$ and $\vec{K}$ $\Rightarrow$ $f_{jP}(\vec{p})$ is sharply peaked at

$\vec{P} \equiv \vec{Q}-\vec{K}$. Width of the peak: $\sigma_{pP} \simeq \max\{\sigma_{P_i}, \sigma_{P_f}\}$

For external particles described by plane waves:

$$f_{jP}(\vec{p}) = \frac{M_{jP}(Q,K)}{\sqrt{2E_{Pi}V\cdot 2E_{Pf}V}}\,\delta^{(4)}(Q-K-p)$$

In general: $f_{jP}(\vec{p}) \Rightarrow M_{jP}(Q,K)\times$ ("smeared δ-functions") representing approx. conservation of mean energies and mean momenta.

Matching QM & QFT expressions for ν w. p.

Example – Gaussian wave packets for external particles. QFT gives

$$f_{jP}(\vec{p}) \;\propto\; [M_{jP}(Q,K)]/(\sigma_{eP}\sigma_{pP}^3)\;\exp\big[-g_P(E_j(\vec{p}),\,\vec{p})\big],$$

$$g_P(E_j(\vec{p}),\,\vec{p}) = \frac{(\vec{p}-\vec{P})^2}{4\sigma_{pP}^2} + \frac{[E_j(\vec{p}) - E_P - \vec{v}_P(\vec{p}-\vec{P})]^2}{4\sigma_{eP}^2}\,.$$

Here $\vec{P} \equiv \vec{Q} - \vec{K}\,, \quad E_P \equiv E_{Pi}(\vec{Q}) - E_{Pf}(\vec{K})\,,$

$$\sigma_{pP}^2 = \sigma_{pPi}^2 + \sigma_{pPf}^2\,, \qquad \sigma_{xP}\sigma_{pP} = \frac{1}{2}\,,$$

$$\vec{v}_P \equiv \sigma_{xP}^2\left(\frac{\vec{v}_{Pi}}{\sigma_{xPi}^2} + \frac{\vec{v}_{Pf}}{\sigma_{xPf}^2}\right), \qquad \Sigma_P \equiv \sigma_{xP}^2\left(\frac{\vec{v}_{Pi}^2}{\sigma_{xPi}^2} + \frac{\vec{v}_{Pf}^2}{\sigma_{xPf}^2}\right),$$

$$\sigma_{eP}^2 = \sigma_{pP}^2(\Sigma_P - \vec{v}_P^2) \equiv \sigma_{pP}^2\,\lambda_P\,, \qquad 0 \le \lambda_P \le 1\,.$$

For 2 ext. particles at production: $\sigma_{eP} = |\vec{v}_{Pi} - \vec{v}_{Pf}|/2\sqrt{\sigma_{xPi}^2 + \sigma_{xPf}^2}$ $\sim$ inverse overlap time

Matching QM & QFT expressions for ν w. p.

Compare with Gaussian wave packet in QM approach:

$$f_{jP}(\vec{p},\vec{P}) = \left(\frac{2\pi}{\sigma_{pP}^2}\right)^{3/4} \exp\left[-\frac{(\vec{p}-\vec{P})^2}{4\sigma_{pP}^2}\right]$$

To match the QM and QFT expression: expand $E_j(\vec{p})$ around $\vec{p}=\vec{P}$ and subst. into $g_P(E_j(\vec{p}),\vec{p})$:

$\diamondsuit$ $$\boxed{g_P(E_j(\vec{p}),\vec{p}) = (p-P)^k\,\alpha^{kl}\,(p-P)^l - \beta^k(p-P)^k + \gamma_j}$$

$$\alpha^{kl} = \frac{1}{4\sigma_{eP}^2}\left[\lambda_P\,\delta^{kl} + (v_j-v_P)^k\,(v_j-v_P)^l + \frac{E_j-E_P}{E_j}(\delta^{kl}-v_j^k v_j^l)\right],$$

$$\beta^k = -\frac{1}{2\sigma_{eP}^2}(E_j-E_P)(v_j-v_P)^k\,, \qquad \gamma_j = \frac{(E_j-E_P)^2}{4\sigma_{eP}^2}\,.$$

Try to represent $g_P(E_j(\vec{p}),\vec{p})$ in the form

$\diamondsuit$ $$\boxed{g_P(E_j(\vec{p}),\vec{p}) = (p-P_{\rm eff})^k\,\alpha^{kl}\,(p-P_{\rm eff})^l + \tilde{\gamma}_j}\,, \qquad \vec{P}_{\rm eff} \equiv \vec{P}+\vec{\delta}$$

Matching QM & QFT expressions for ν w. p.

$$\delta^k = -\frac{(E_j - E_P)(v_j - v_P)^k}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,, \qquad\qquad \tilde{\gamma}_j = \frac{(E_j - E_P)^2}{4\sigma_{eP}^2}\,\frac{\lambda_P}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,.$$

Diagonalization of α^{kl} gives ($OZ||(\vec{v}_j - \vec{v}_P)$):

$$(\sigma^x_{pP\,\mathrm{eff}})^2 = (\sigma^y_{pP\,\mathrm{eff}})^2 = \sigma^2_{pP}\,, \qquad\qquad \frac{1}{(\sigma^z_{pP\,\mathrm{eff}})^2} = \frac{1}{\sigma^2_{pP}} + \frac{(\vec{v}_j - \vec{v}_P)^2}{\sigma^2_{eP}}\,,$$

$\Rightarrow$ QM neutrino wave packets can match those obtained QFT if

Matching QM & QFT expressions for ν w. p.

$$\delta^k = -\frac{(E_j - E_P)(v_j - v_P)^k}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,, \qquad \tilde{\gamma}_j = \frac{(E_j - E_P)^2}{4\sigma_{eP}^2}\frac{\lambda_P}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,.$$

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$\Rightarrow$ QM neutrino wave packets can match those obtained QFT if

- Momentum uncertainties of the neutrino mass eigenstates are replaced (anisotropic) effective ones: $-(\vec{p} - \vec{P})^2/(4\sigma_{pP}^2) \;\to$

$-[(p^x - P^x_{\mathrm{eff}})^2/4(\sigma^x_{pP})^2 + (p^y - P^y_{\mathrm{eff}})^2/4(\sigma^y_{pP})^2 + (p^z - P^z_{\mathrm{eff}})^2/4(\sigma^z_{pP})^2]$.

Matching QM & QFT expressions for ν w. p.

$$\delta^k = -\frac{(E_j - E_P)(v_j - v_P)^k}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,, \qquad \tilde{\gamma}_j = \frac{(E_j - E_P)^2}{4\sigma_{eP}^2}\,\frac{\lambda_P}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,.$$

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$$(\sigma^x_{pP\,\mathrm{eff}})^2 = (\sigma^y_{pP\,\mathrm{eff}})^2 = \sigma^2_{pP}\,, \qquad \frac{1}{(\sigma^z_{pP\,\mathrm{eff}})^2} = \frac{1}{\sigma^2_{pP}} + \frac{(\vec{v}_j - \vec{v}_P)^2}{\sigma^2_{eP}}\,,$$

$\Rightarrow$ QM neutrino wave packets can match those obtained QFT if

- Momentum uncertainties of the neutrino mass eigenstates are replaced (anisotropic) effective ones: $-(\vec{p} - \vec{P})^2/(4\sigma^2_{pP}) \;\rightarrow$

  $-[(p^x - P^x_{\mathrm{eff}})^2/4(\sigma^x_{pP})^2 + (p^y - P^y_{\mathrm{eff}})^2/4(\sigma^y_{pP})^2 + (p^z - P^z_{\mathrm{eff}})^2/4(\sigma^z_{pP})^2]$.
- The mean momentum $\vec{P}$ is shifted according to $\vec{P} \rightarrow \vec{P}_{\mathrm{eff}} = \vec{P} + \vec{\delta}$.

Matching QM & QFT expressions for ν w. p.

$$\delta^k = -\frac{(E_j - E_P)(v_j - v_P)^k}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,, \qquad \tilde{\gamma}_j = \frac{(E_j - E_P)^2}{4\sigma_{eP}^2}\frac{\lambda_P}{\lambda_P + (\vec{v}_j - \vec{v}_P)^2}\,.$$

Diagonalization of α^{kl} gives $(OZ||(\vec{v}_j - \vec{v}_P))$:

$$(\sigma_{pP\,\mathrm{eff}}^x)^2 = (\sigma_{pP\,\mathrm{eff}}^y)^2 = \sigma_{pP}^2\,, \qquad \frac{1}{(\sigma_{pP\,\mathrm{eff}}^z)^2} = \frac{1}{\sigma_{pP}^2} + \frac{(\vec{v}_j - \vec{v}_P)^2}{\sigma_{eP}^2}\,,$$

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- The mean momentum $\vec{P}$ is shifted according to $\vec{P} \rightarrow \vec{P}_{\mathrm{eff}} = \vec{P} + \vec{\delta}$.
- The wave packet of each neutrino mass eigenstate gets an extra factor $N_j = \exp[-\tilde{\gamma}_j]$.

Matching QM & QFT expressions for ν w. p.

If $$|E_i - E_j| \ll \sigma_{eP} \qquad \Rightarrow$$

factors N_j are the same for all ν mass eigenstates, can be included in common normalization factor. In the opposite case – coherence of different neutrino mass eigenstates is lost.

$\sigma_{eP} \le \sigma_{pP} \quad \Rightarrow$ except for $\vec{v}_j \approx \vec{v}_P$ momentum uncertainty along $(\vec{v}_j - \vec{v}_P)$ is dominated by σ_{eP}.

In the stationary neutrino source limit $(\sigma_{eP}\,, \vec{v}_P \to 0)$, effective longitudinal mom. uncertainty $\sigma^z_{pP\,\mathrm{eff}} = 0$ even though the true mom. uncertainty $\sigma_{pP} \neq 0$.

$$\Downarrow$$

Coherence length $l_{\mathrm{coh}} \;\to\; \infty$

Oscillation probability in QFT

What is calculated in QFT is the probability of the overall production-propagation-detection process. How to extract from it the oscillation probability $P_{\alpha\beta}(L)$?

1. Recall the operational definition of $P_{\alpha\beta}(L)$. Detection rate for ν_β:

$$\Gamma_\beta^{\rm det} = \int dE\, j_\beta(E)\sigma_\beta(E)\,,$$

If a source at a distance L from the detector emits ν_α with the energy spectrum $d\Gamma_\alpha^{\rm prod}(E)/dE$:

$$j_\beta(E) = \frac{1}{4\pi L^2}\frac{d\Gamma_\alpha^{\rm prod}(E)}{dE}P_{\alpha\beta}(L,E)\,,$$

$\Rightarrow$ substitute into $\Gamma_\beta^{\rm det}$:

Oscillation probability in QFT

$$\Gamma^{\rm tot}_{\alpha\beta} \equiv \int dE\, \frac{d\Gamma^{\rm tot}_{\alpha\beta}(E)}{dE} = \frac{1}{4\pi L^2}\int dE\, \frac{d\Gamma^{\rm prod}_{\alpha}(E)}{dE}\, P_{\alpha\beta}(L,E)\,\sigma_\beta(E)$$

$$P_{\alpha\beta}(L,E) = \frac{d\Gamma^{\rm tot}_{\alpha\beta}(E)/dE}{\frac{1}{4\pi L^2}\,[d\Gamma^{\rm prod}_{\alpha}(E)/dE]\,\sigma_\beta(E)}\,.$$

An important ingredient: the assumption that the overall rate factorizes into the production rate, propagation (oscillation) probability and detection cross section.

If this does not hold, oscillation probability is undefined $\Rightarrow$

Need to deal instead with the overall rate of neutrino production, propagation and detection.

Oscillation probability in QFT

Try to cast $P_{\alpha\beta}^{\rm tot}$ in the same form (check if the factorization condition holds !)

$$iA_{\alpha\beta} = i\sum_j U_{\alpha j}^* U_{\beta j}\int \frac{d^4p}{(2\pi)^4}\,\Phi_{jP}(p^0,\vec{p})\Phi_{jD}(p^0,\vec{p})\,\frac{2p_0\, e^{-ip^0T+i\vec{p}\vec{L}}}{p^2-m_j^2+i\epsilon}$$

Integrate first over $\vec{p}$, then over $p^0\equiv E$. Make use of Grimus-Stockinger theorem: for a large L ($L\gg p/\sigma_p^2$), $A>0$ and a sufficiently smooth $\psi(\vec{p})$,

$$\int d^3p\,\frac{\psi(\vec{p})\,e^{i\vec{p}\vec{L}}}{A-\vec{p}^2+i\epsilon} = -\frac{2\pi^2}{L}\psi(\sqrt{A}\tfrac{\vec{L}}{L})e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}}) \quad \Rightarrow$$

$$iA_{\alpha\beta}(T,\vec{L}) = \frac{-i}{8\pi^2 L}\sum_j U_{\alpha j}^* U_{\beta j}\int dE\,\Phi_P(E,p_j\vec{l})\Phi_D(E,p_j\vec{l})\;2E\,e^{-iE\,T+ip_jL}$$

where

$$p_j \equiv \sqrt{E^2-m_j^2}\,, \qquad \vec{l}\equiv\frac{\vec{L}}{L}\,,$$

Oscillation probability in QFT

Introduce

$$\tilde{P}^{\rm tot}_{\alpha\beta}(\vec{L}) = \int dT\, P_{\alpha\beta}(T,\vec{L}) = \frac{1}{8\pi^2}\frac{1}{4\pi L^2}\sum_{j,k} U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta k}$$

$$\times \int dE\, \Phi_P(E,p_j\vec{l})\Phi_D(E,p_j\vec{l})\,\Phi^*_P(E,p_k\vec{l})\Phi^*_D(E,p_k\vec{l})\;(2E)^2\, e^{i(p_j-p_k)L}$$

Neutrino production probability:

$$P^{\rm prod}_\alpha = \sum_j |U_{\alpha j}|^2 \int \frac{d^3p_j}{(2\pi)^3}\left|\Phi_P(E,p_j)\right|^2 = \sum_j |U_{\alpha j}|^2 \frac{1}{8\pi^2}\int dE\left|\Phi_P(E,p_j)\right|^2 4Ep_j$$

Detection probability:

$$P^{\rm det}_\beta(E) = \sum_k |U_{\beta k}|^2 |\Phi_D(E,p_k)|^2 \frac{1}{V}\,,$$

Oscillation probability in QFT

Let the number of particles P_i entering the production region during time interval T_0 be N_P and number of D_i entering the detection region be N_D. Probability of neutrino emission during the finite interval of time t:

$$\mathcal{P}_\alpha^{\rm prod}(t) = N_P \int_0^t \frac{dt_P}{T_0}\, P_\alpha^{\rm prod} = N_P\, P_\alpha^{\rm prod} \frac{t}{T_0}\,, \quad \text{rate:} \quad \Gamma_\alpha^{\rm prod} = N_P\, P_\alpha^{\rm prod} \frac{1}{T_0}$$

Detection cross section:

$$\sigma_\beta(E) = \frac{N_D}{T_0} \sum_k |U_{\beta k}|^2 |\Phi_{kD}(E)|^2 \frac{E}{p_k}$$

Probability of the overall production-propagation-detection process:

$$\mathcal{P}_{\alpha\beta}^{\rm tot}(t, L) = \frac{N_P N_D}{T_0^2} \int_0^t dt_D \int_0^t dt_P\, P_{\alpha\beta}^{\rm tot}(T, L) \quad \Rightarrow$$

Oscillation probability in QFT

New integration variables $\tilde{T} \equiv (t_P + t_D)/2$ and $T = t_D - t_P \quad \Rightarrow$

$$\mathcal{P}^{\rm tot}_{\alpha\beta}(t,L) = \frac{N_P N_D}{T_0^2}\left[\int_0^t dT\, P^{\rm tot}_{\alpha\beta}(T,L)(t-T) + \int_{-t}^0 dT\, P^{\rm tot}_{\alpha\beta}(T,L)(t+T)\right]$$

$$= \frac{N_P N_D}{T_0^2}\left[t\int_{-t}^t dT\, P^{\rm tot}_{\alpha\beta}(T,L) - \int_0^t dT\, T\, P^{\rm tot}_{\alpha\beta}(T,L) + \int_{-t}^0 dT\, T\, P^{\rm tot}_{\alpha\beta}(T,L)\right]$$

$$\equiv \frac{N_P N_D}{T_0^2}\left[t I_1(t) - I_2(t) + I_3(t)\right].$$

For large t (much larger than the time scales of the neutrino production and detection processes) $I_1 = \tilde{P}^{\rm tot}_{\alpha\beta}(L)$ whereas $I_2 = I_3 = 0 \quad \Rightarrow$

$$\mathcal{P}^{\rm tot}_{\alpha\beta}(t,L) = \frac{N_P N_D}{T_0^2}\, t\, \tilde{P}^{\rm tot}_{\alpha\beta}(L)\,, \qquad \Gamma^{\rm tot}_{\alpha\beta}(L) = \frac{N_P N_D}{T_0^2}\, \tilde{P}^{\rm tot}_{\alpha\beta}$$

Oscillation probability in QFT

$$\text{“}P_{\alpha\beta}(L,E)\text{”} = \frac{\sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \Phi_P(E,p_j)\Phi_D(E,p_j)\Phi^*_P(E,p_k)\Phi^*_D(E,p_k) e^{i(p_j-p_k)L}}{\sum_j |U_{\alpha j}|^2\,|\Phi_P(E,p_j)|^2\,p_j\;\sum_k |U_{\beta k}|^2\,|\Phi_D(E,p_k)|^2 p_k^{-1}}$$

For $|p_j - p_k| \ll p_j, p_k$ (ultra-relativistic or quasi-degenerate in mass ν's):
In expressions for $\Gamma_\alpha^{\rm prod}$ and σ_β can replace

$$p_j \to p\,, \qquad \Phi_P(E,p_j) \to \Phi_P(E,p) \qquad (p\ -\ \text{average momentum})$$

$\Rightarrow$ in the denominator of “$P_{\alpha\beta}(L,E)$”:

$$\sum_j |U_{\alpha j}|^2\,|\Phi_P(E,p_j)|^2\,p_j \to |\Phi_P(E,p)|^2\,p\sum_j |U_{\alpha j}|^2 = |\Phi_P(E,p)|^2\,p\,,$$

$$\sum_k |U_{\beta j}|^2\,|\Phi_D(E,p_k)|^2\,p_k^{-1} \to |\Phi_D(E,p)|^2\,p^{-1}\sum_k |U_{\beta k}|^2 = |\Phi_D(E,p)|^2\,p^{-1}\,,$$

Cannot in general be done in the numerator of “$P_{\alpha\beta}(L,E)$” !

Oscillation probability in QFT

For $|p_j - p_k| \ll p_j, p_k$ $\Gamma_\alpha^{\rm prod}$ and σ_β do not depend on the elements of the mixing matrix $\Rightarrow$ factorization holds. $P_{\alpha\beta}(E,L)$ can be defined as a sensible quantity:

$$P_{\alpha\beta}(L,E) = \frac{\sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* \Phi_P(E,p_j)\Phi_D(E,p_j)\Phi_P^*(E,p_k)\Phi_D^*(E,p_k) e^{i(p_j-p_k)L}}{|\Phi_P(E,p)|^2\,|\Phi_D(E,p)|^2}$$

Automatically satisfies unitarity, i.e. is properly normalized.

For $|p_j - p_k| \gg \sigma_p$ $(\Leftrightarrow \Delta m_{jk}^2/(2p) \gg \sigma_p)$ – interf. terms strongly suppressed. In the opposite case

$$\frac{\Delta m_{jk}^2}{2p} \ll \sigma_p\,,$$

(production & detection coherence cond. satisfied) – $\Phi_P(E,p_{j,k})$, $\Phi_D(E,p_{j,k})$ can be pulled out of the sums in the numerator $\Rightarrow$ stand. osc. probability:

$$P_{\alpha\beta}(L,E) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{jk}^2}{2p}L}$$

Oscillation probability in QFT

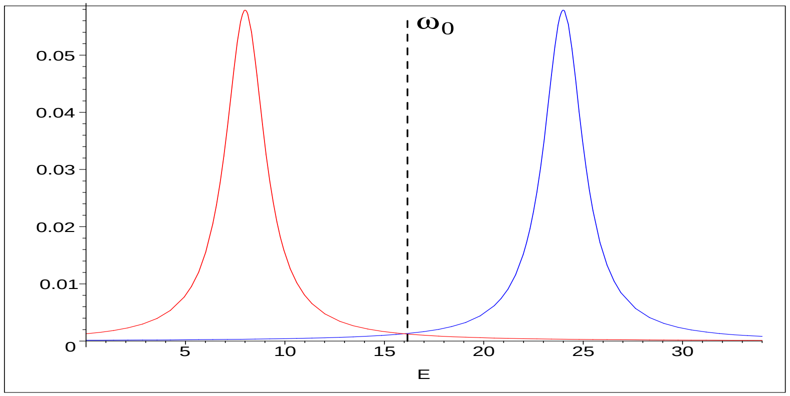
The condition for the existence of well-defined oscillation probabilities is that neutrinos are either ultra-relativistic or nearly degenerate in mass.

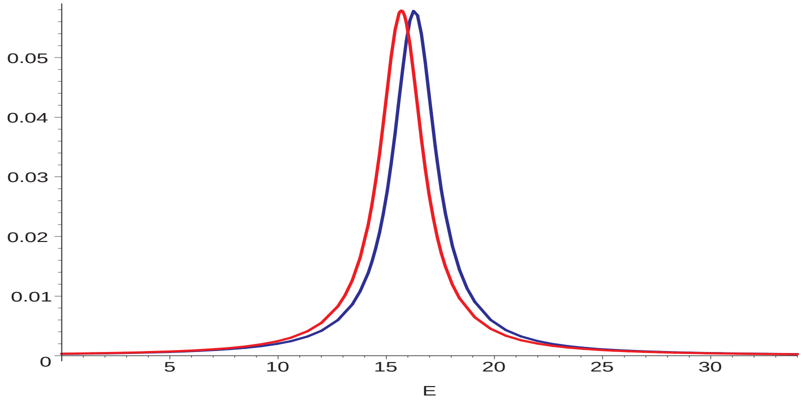
The QFT-based consideration clarifies the QM wave packet normalization prescription. QM and QFT approaches can be matched if the QM quantities f_{jP} and f_{jD} are identified with the QFT functions $\Phi_{jP}(E_j, \vec{p})$ and $\Phi^*_{jD}(E_j, \vec{p})$, respectively. But: the latter bear information not only on the properties of the emitted and absorbed neutrinos, but also on the production and detection processes. The QM normalization procedure is equivalent, in the limit $|p_j - p_k| \ll p_j, p_k$, to the division of the overall rate of the process by the production rate and detection cross section, as in QFT approach.

Mössbauer effect

Conventional Mössbauer effect – Res. absorption of γ quanta:

$$A^* \rightarrow A + \gamma; \qquad A + \gamma \rightarrow A^*$$





Nuclear exc. energy: ω_0.

Recoil energy: $R = \frac{\omega_0^2}{2M}$

$$E_e = \omega_0 - \frac{\omega_0^2}{2M}$$

$$E_a = \omega_0 + \frac{\omega_0^2}{2M}$$

Recoilless emission and absorption (Mössb. eff.):

$$E_e \simeq E_a \simeq \omega_0$$

Strong enhancement of absorption

Mössbauer effect with neutrinos?

Beta decay with 2 - body final state:

$$A(N,Z) \to A(N-1,Z+1) \ + e^-_B \ + \ \bar{\nu}_e$$

Inverse process:

$$\bar{\nu}_e \ + \ e^-_B \ + \ A(N-1,Z+1) \to A(N,Z)$$

If the neuclei are embedded in solid state lattice, recoilless emission and absorption in principle possible.

Possibility of Mössbauer effect with neutrinos:

Visscher, 1959; Kells & Schiffer, 1983; Raghavan, 2005, 2006

Relevant processes considered:

Bahcall, 1961 – bound state β decay;

Mikaelyan, Tsinoev & Borovoi, 1967 – inverse process (stimulated K-electron capture)

Mössbauer effect with neutrinos?

Mössbauer effect with neutrinos on ^{3}H – ^{3}He system:

$$^3\mathrm{H} \;\to\; (^3\mathrm{He} \;+\; e^-_B) + \;\bar{\nu}_e \,; \qquad \bar{\nu}_e \;+\; (^3\mathrm{He} \;+\; e^-_B) \;\to\; ^3\mathrm{H}$$

Energy release: $Q = 18.6$ keV. Mean lifetime of ^{3}H is 17.8 yr $\Rightarrow$

Nat. linewidth $\Gamma_{^3\mathrm{H}} = 1.17\times 10^{-24}$ eV – extremely small: $\Delta E/E \sim 10^{-28}$!

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Various (homogeneous and inhomogeneous) broadening effects exist. By suppressing them probably an effective linewidth $\Gamma_{\rm eff} \sim 10^{-11}$ eV can be achieved (W. Potzel) $\Rightarrow$ $\Delta E/E \sim 10^{-15}$ – still very small.

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Number of ^{3}H atoms produced in the target can be counted by detecting their decay or using mass spectroscopy.

Very serious technical difficulties exist, but apparently realization of a Mössbauer experiment with neutrinos is not impossible (Raghavan, Potzel).
If realized: for $\Gamma \sim 10^{-11}$ eV, $\sigma \sim 10^{-33}\ cm^2$!

Mössbauer effect with neutrinos?

If a Mössbauer neutrino experiment is realized $\Rightarrow$ a unique source of extremely monochromatic low energy neutrinos. Would open up possibilities

- to detect for the first time keV neutrinos
- to detect neutrinos with g or 100 g scale (rather than t or kt scale) detectors
- to observe gravitational redshift of neutrinos
- to study neutrino oscillations at distances ~ 10 m rather than km or hundreds/thousands of km
- to search for the effects of yet unmeasured mixing angle θ_{13} and possibly measure it
- to discriminate between the normal and inverted neutrino mass hierarchies without using matter effects
- to study possible oscillations into sterile neutrino states

Will Mössbauer neutrinos oscillate?

Arguments in the literature (Bilenky et al.):

Mössbauer neutrinos may not oscillate because of their extremely small linewidth

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Is that true?

Will Mössbauer neutrinos oscillate?

Neutrino oscillations require some intrinsic uncertainty of energy and momentum of the emitted and detected neutrino states !

If E and p were known precisely, from $E^2 = p^2 + m_i^2$ one would determine which mass eihenstate has been emitted $\Rightarrow$ neutrinos of different mass would not be emitted coherently.

For Mössbauer effect with neutrinos in ^{3}H – ^{3}He system:

♢ $$\frac{\Delta m^2}{2E} = \frac{2.5 \times 10^{-3}\,\mathrm{eV}^2}{2 \cdot 18.6\,\mathrm{keV}} \simeq 6.7 \cdot 10^{-8}\ \mathrm{eV} \gg \Gamma \sim 10^{-11}\ \mathrm{eV}\,!$$

Can neutrinos of different mass be accommodated within such a small energy uncertainty?

Will neutrinos with such small energy uncertainty oscillate ?

Two “standard” approaches to ν oscillations

The oscillation phase: $\phi = p_\mu x^\mu = E \cdot t - p \cdot x \quad \Rightarrow$

$$\Delta\phi \;=\; \Delta E \cdot t \,-\, \Delta p \cdot L$$

I. Same momentum approach $(\Delta p \;=\; 0)$. The oscillation phase

$$\Delta\phi \;=\; \Delta E \cdot t \;-\; \Delta p \cdot L \;\;\Rightarrow\;\; \Delta E \cdot t$$

– evolution in time; needs to use $L \;\simeq\; t$.

II. Same energy approach $(\Delta E \;=\; 0)$:

$$\Delta\phi \;=\; -\,\Delta p \cdot L$$

– evolution in space.

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Our point of view: in general, there is no reason to believe that ν_i have either same energy or same momentum. No need to perform Mössbauer ν experiment to decide which approach is correct – it is sufficent to carefully examine the validity of the approximations used.

How about Mössbauer neutrinos?

Very small effective linewidth Γ $\Rightarrow$ small energy uncertainty of the emitted neutrino state. Can different neutrino mass eigenstates be emitted coherently?

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2\right]^{1/2}$$

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– not the case. σ_x is certainly smaller than the size of the crystal (a few cm). In reality it is of the order of interatomic distances (one can destroy the crystal and find out which tritium atom turned into helium).

$\Rightarrow \quad \sigma_p \sim 10\ {\rm keV}$, i.e. $\sigma_m^2 \simeq 2p\sigma_p \sim 4\times 10^8\ {\rm eV}^2 \gg \Delta m^2$

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$\Rightarrow$ $\sigma_p \sim 10\ {\rm keV}$, i.e. $\sigma_m^2 \simeq 2p\sigma_p \sim 4\times 10^8\ {\rm eV}^2 \gg \Delta m^2$

$\Rightarrow$ Oscillations must occur !

QFT calculation

Inhomogeneous line broadening: Calculate the probability of the overall process for zero linewidths and then average the result over the energy distribution of ^{3}H and ^{3}He nuclei in the source and detector.

Homogeneous line broadening: modify the amplitude of the process and apply a proper averaging procedure to take into account the stochastic nature of the processes leading to homog. broadening. $\Rightarrow$ Results in both cases are

formally very similar. Mössbauer res. condition:

$$|E_S - E_D| \ll \gamma_S + \gamma_D$$

If it is satisfied $\Rightarrow$ neutrino detection cross section enhanced by a factor

$$\sim (\alpha Z m_e)^3/[p_e E_e(\gamma_S + \gamma_D)] \sim 10^{12}$$

compared to non-resonance $\sigma(\bar{\nu}_e + A \to A' + e^+)$ for neutrinos of same energy (assuming recoil-free fraction ~ 1).

QFT calculation – contd.

The amplitude for zero linewidths:

$$i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2\, \Psi^*_{He,S}(\vec{x}_1) e^{+iE_{He,S}\, t_1}\; \Psi_{H,S}(\vec{x}_1) e^{-iE_{H,S}\, t_1}$$

$$\cdot\, \Psi^*_{H,D}(\vec{x}_2) e^{+iE_{H,D}\, t_2}\; \Psi_{He,S}(\vec{x}_2) e^{-iE_{He,D}\, t_2}$$

$$\cdot \sum_j \mathcal{M}^\mu_S \mathcal{M}^{\nu *}_D |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)}$$

$$\cdot\, \bar{u}_{e,S}\gamma_\mu(1-\gamma_5)\, \frac{i(\not{p}+m_j)}{p_0^2-\vec{p}^2-m_j^2+i\epsilon}\, \gamma_\nu(1-\gamma_5) u_{e,D}$$

Here

$$\mathcal{M}^\mu_{S,D} = \frac{G_F\cos\theta_c}{\sqrt{2}}\, \psi_e(R)\, \bar{u}_{He}\, (M_V\, \delta^\mu_0 - g_A M_A \sigma_i\, \delta^\mu_i/\sqrt{3})\, u_H\, \kappa^{1/2}_{S,D}$$

QFT calculation – contd.

The overall process rate:

$$\Gamma = \frac{\Gamma_0\, B_0}{4\pi L^2}\, Y_S Y_D \int_0^\infty dE_{H,S}\, dE_{He,S}\, dE_{He,D}\, dE_{H,D}$$

$$\cdot\, \delta(E_S - E_D)\rho_{H,S}(E_{H,S})\, \rho_{He,D}(E_{He,D})\, \rho_{He,S}(E_{He,S})\, \rho_{H,D}(E_{H,D})$$

$$\cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right] e^{i\left(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2}\right)L}$$

σ_p – effective momentum uncertainty of the emission/absorption processes:

$$\frac{1}{\sigma_p^2} = \frac{1}{m_H \omega_{H,S} + m_{He}\omega_{He,S}} + \frac{1}{m_H \omega_{H,D} + m_{He}\omega_{He,D}}\,,$$

An analogue of the Debye - Waller (Lamb - Mössbauer) factor:

♢ $\exp[-(2E_S^2 - m_j^2 - m_k^2)/2\sigma_p^2] \;=\; \exp[-(p_j^2 + p_k^2)/2\sigma_p^2]$

QFT calculation – contd.

For Lorentzian energy distributions of external particles:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

$$\left(A = \{H, He\},\ \ B = \{S, D\},\ \ E_{A,B,0} = m_A + \tfrac{1}{2}\omega_{A,B}\right) \quad \Rightarrow$$

$$\Gamma \simeq \frac{\Gamma_0\, B_0}{4\pi L^2}\, Y_S Y_D \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-\frac{(p_{jk}^{\rm min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

$$\cdot\ \frac{1}{2}\left(e^{-L/L_{jk,S}^{\rm coh}} + e^{-L/L_{jk,D}^{\rm coh}}\right) \exp\left[-\, i\,\frac{\Delta m_{jk}^2}{2\bar{E}}\, L\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}}$$

$L_{jk,B}^{\rm coh}$ – coherence lengths:

$$L_{jk,B}^{\rm coh} = \frac{4\bar{E}^2}{\gamma_B |\Delta m_{jk}^2|} = \frac{\sigma_x}{\Delta v_g}\,, \qquad \sigma_x = \frac{2}{\gamma_B} \qquad (B = S, D)$$

QFT calculation – contd.

Generalized Lamb – Mössbauer (Debye – Waller) factor

$$\exp\left[-\frac{p_j^2+p_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

First factor $\Rightarrow$ suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor $\Rightarrow$ suppression of oscillations.

QFT calculation – contd.

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$|\Delta m_{jk}^2| \lesssim 2\sigma_p^2$ $\Rightarrow$ localization condition: Spatial localization $\sigma_x \sim 1/\sigma_p$.
Oscillations would be suppressed only if $|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2$.

QFT calculation – contd.

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In reality: $|\Delta m_{jk}^2|_{\max} \simeq 2.5\cdot 10^{-3}\ \mathrm{eV}^2$; $\sigma_p^2 \sim (10\ \mathrm{keV})^2$ $\Rightarrow$
oscillations will not be suppressed.

QFT calculation – contd.

For realistic values of parameters – just the expected result: the rate of no-oscillation production-detection process times the standard oscillation probability (probability of $\bar{\nu}_e$ survival). Decoherence and delocalization can be neglected.

Conclusion:

If a Mössbauer neutrino experiment is realized – recoillessly emitted and absorbed neutrinos will oscillate.

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E\,, \qquad\qquad |\Delta p| \ll \sigma_p\,.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \quad \Rightarrow$

$$\left| \frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E} \right| \ll 1. \qquad\qquad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$ $v_g|\Delta p| \ll \sigma_E \le \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

Finite-width pion WP

Two models of finite-size pion WP, Gaussian and box-type. For $\Gamma l_p/v_\pi \gg 1$:

$$\diamondsuit \quad P_{\mu\mu}^{\text{eff}} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2+1}\left[(\cos\phi + \xi\sin\phi) - A_\pi \xi(\xi\cos\phi - \sin\phi)\right]$$

The parameter A_π:

$$A_{\pi\text{box}} = \frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,, \qquad A_{\pi\text{Gauss}} = \frac{2}{\sqrt{2\pi}}\frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,.$$

i.e. $A_\pi \sim (v_g/v_\pi)\sigma_{x\pi}/\sigma_{x\nu}$. The correction is of order

$$A_\pi\xi \sim \left[\frac{\Delta m^2}{2P}\sigma_{x\pi}\right]\cdot\frac{v_g}{v_g - v_\pi} = 2\pi\frac{\sigma_{x\pi}}{l_{\text{osc}}}\cdot\frac{v_g}{v_g - v_\pi}$$

– small since $\sigma_{x\pi} \lll l_{\text{osc}}$ (unless $v_\pi \simeq v_g$ to a very high accuracy).

An interesting point: summation at the probabilities level for finite-thickness ($= d$) proton target and point-like neutrino production gives similar expression, but with $A_\pi\xi = (\Delta m^2/2P)d$ (no factor $[v_g/(v_g - v_\pi)]$).

Effects of muon detection (for pointlike pion)

If muons is detected: plane wave $\rightarrow$ wave packet

$$\psi_\mu(x,t) = e^{iKx - iE_\mu(K)t}\, g_\mu[(x - x_S) - v_\mu(t - t_S)]\,.$$

Shape factor $g_\mu[(x - x_S) - v_\mu(t - t_S)]$ determined by the muon detection process. The argument of g_μ: initial condition that at time $t = t_S$ the peak of the w. packet is at $x = x_S$. Choose x_S as the coordinate of the center of the muon w. packet at the neutrino production time. For pointlike pions x_S should lie on the pion's trajectory $\Rightarrow$ $x_S = v_\pi t_S$.

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu\Big((x - x_S)\frac{v_\pi - v_\mu}{v_\pi}\Big)\right|^2 e^{-i\frac{\Delta m_{jk}^2}{2P}(L-x) - \Gamma\frac{x}{v_\pi}}\,.$$

When the muon is undetected: $g_\mu \rightarrow 1$. Eff. width of the muon w. packet:

$$\tilde{\sigma}_{x\mu} \equiv \sigma_{x\mu}\frac{v_\pi}{v_\pi - v_\mu}\,.$$

The results of amplitude summation and probability summation approaches again coincide.

Limiting cases

(1) $\tilde{\sigma}_{x\mu} \to \infty$: plane wave limit. $g_\mu \to const$ – previous results recovered.

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$$I_{jk}(L) = const.\, e^{-\Gamma \frac{x_S}{v_\pi}}\, e^{-i\frac{\Delta m_{jk}^2}{2P}(L-x_S)} \qquad \Rightarrow \qquad P_{\alpha\beta}^{\rm stand}(L - x_S).$$

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For $\tilde{\sigma}_{x\mu} \not\to \infty$ – oscillations of a "tagged" neutrino, i.e. of a neutrino produced together with the muon which was detected and whose production coordinate was found to be x_S with the accuracy $\tilde{\sigma}_{x\mu}$.

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$$I_{jk}(L) = const.\, e^{-\Gamma \frac{x_S}{v_\pi}}\, e^{-i\frac{\Delta m^2_{jk}}{2P}(L-x_S)} \quad \Rightarrow \quad P^{\rm stand}_{\alpha\beta}(L - x_S).$$

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$$P_{\mu\mu} = c^4 + s^4 + 2s^2c^2\, e^{-\frac{1}{2}\left(\frac{\Delta m^2}{2P}\right)^2 \tilde{\sigma}^2_{x\mu}} \cos\left(\frac{\Delta m^2}{2P}(L - x_S)\right).$$

Limiting cases

(1) $\tilde{\sigma}_{x\mu} \to \infty$: plane wave limit. $g_\mu \to const$ – previous results recovered.

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$$P_{\mu\mu} = c^4 + s^4 + 2s^2c^2\, e^{-\frac{1}{2}\left(\frac{\Delta m^2}{2P}\right)^2 \tilde{\sigma}_{x\mu}^2} \cos\left(\frac{\Delta m^2}{2P}(L - x_S)\right).$$

$\Rightarrow$ the decoherence parameter is $\frac{\Delta m^2}{2P}\tilde{\sigma}_{x\mu}$. For $\tilde{\sigma}_{x\mu} \ll l_{\rm osc}/2\pi$ the stand. probability is recovered.

The case of muon interacting with medium

The case when the muon interacts with the medium but there are no muon detectors (the muon's position not measured). Neutrinos are not tagged $\Rightarrow$ one has to integrate

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu\big((x - x_S)\frac{v_\pi - v_\mu}{v_\pi}\big)\right|^2 e^{-i\frac{\Delta m_{jk}^2}{2P}(L-x)-\Gamma\frac{x}{v_\pi}} \,.$$

over x_S.

Integration of $|g_\mu|^2$ gives the normalization constant of this function which does not influence the oscillation probabilities. The results obtained in the case when the muon is not detected are recovered.

Pion interactions

Interaction of the pions in the bunch btw themselves or with other particles may identify the individual pion whose decay produces a given neutrino.
E.g. pion decay may lead to some recoil of the neighbouring particles which may be detected. $\Rightarrow$

Would localize the neutrino production point up to an uncertainty of order of the inter-pionic distance (or the distance between the pion and the other particles in the source) r_0 $\Rightarrow$ neutrino tagging.
Production decoherence parameter: $(\Delta m^2/2P)r_0$.

If the information about the interaction between the decaying pion and the surrounding particles is not recorded and not used for neutrino tagging, the oscillations occur in exactly the same way as if pions did not interact with each other or with other particles.

Production coherence for some experiments

Unless otherwise specified, $\Delta m^2 = 2$ eV^2. For β-beams $E_0 = 2$ MeV, $\tau_0 = 1$s, $\gamma = 100$.

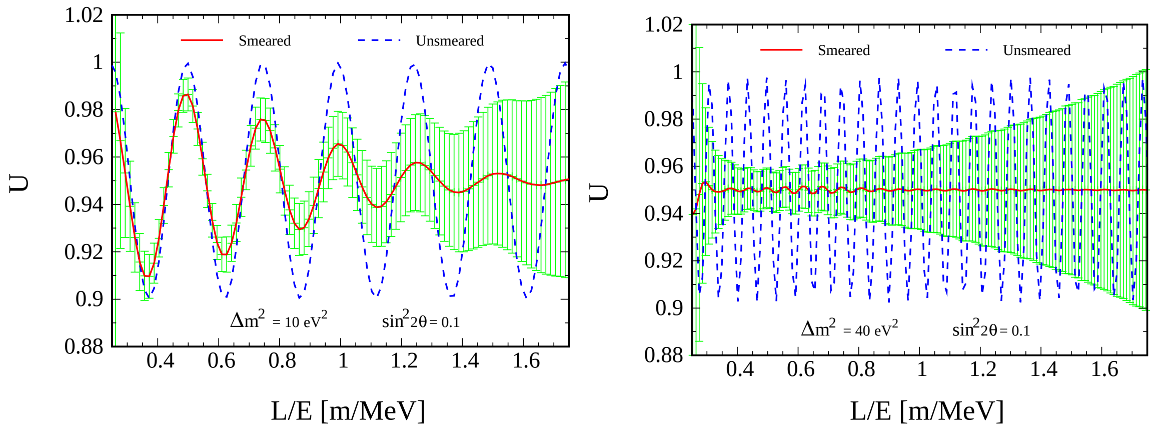
Experiment	$\langle E_\nu \rangle$(MeV)	L(m)	$l_p(m)$	$l_{\rm dec}$(m)	$l_{\rm osc}$(m)	ϕ	$\Gamma l_p/v_P$	ϕ_p	ξ
LSND	$\sim$40	30	0	0	50	3.8	-	0	0
KARMEN	$\sim$40	17.7	0	0	50	2.24	-	0	0
MiniBooNE	$\sim$800	541	50	89	992	3.43	0.56	0.32	0.56
NOMAD	$2.7 \cdot 10^3$	770	290	3009	33480	0.145	0.1	0.054	0.56
(20 eV^2)					3348	1.45	0.1	0.54	5.64
CCFR(10^2 eV^2)	$5 \cdot 10^4$	891	352	5570	1240	4.51	0.06	1.78	28.2
CDHS	3000	130	52	334	3720	0.22	0.155	0.088	0.56
(20 eV^2)					372	2.2	0.155	0.878	5.64
K2K	1500	300	200	167	1861	1.01	1.2	0.68	0.56
T2K	600	280	96	66.4	744	2.36	1.45	0.81	0.56
Minos	3300	1040	675	368	4092	1.6	1.84	1.04	0.56
NOνA	2000	1040	675	223	2480	2.64	3.03	1.71	0.56
β-beams	400	$1.3 \cdot 10^5$	2500	$3 \cdot 10^{10}$	496	1647	$8.3 \cdot 10^{-8}$	31.7	$3.8 \cdot 10^8$

Noticeable effects for MiniBooNE, NOMAD (20 eV^2), CCFR (100 eV^2), CDHS (20 eV^2), K2K, T2K, MINOS, NOνA, very large effects for β-beams

Examples of prod. coherence violation

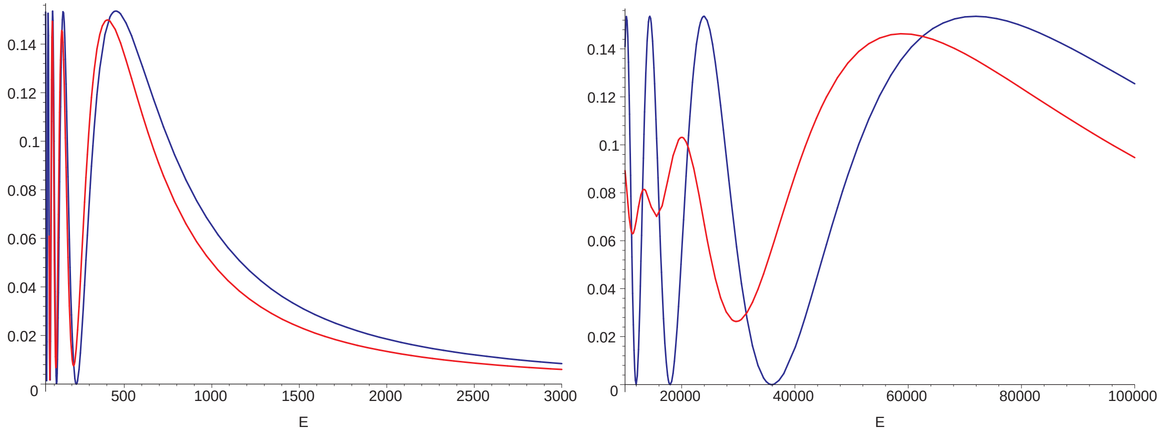
$\nu_e \to \nu_s$ oscillations in β-beam expts. (Agarwalla, Huber & Link, arXiv:0907.3145).

Ratio of oscillated and unoscillated fluxes ($\gamma = 30,\ l_p = 10$m, $L = 50$ m):



T2K

CCFR



Longitudinal *vs.* transversal w.p. dispersion

Spreading of the wave packets: consequence of the fact that the there is a spread of momenta inside of the wave packets and of the p-dependence of the group velocity.

$$v^i_{spr} \simeq \frac{\partial v_i}{\partial p^j}\sigma_p^j = \frac{1}{E}(\delta_{ij} - v_i v_j)\sigma_p^j = \frac{1}{E}[\sigma_p^i - v_i(\vec{v}\vec{\sigma_p})]$$

This gives

$$v^{\perp}_{spr.} = \frac{\sigma_p}{E}\,, \qquad v^{||}_{spr.} = \frac{\sigma_p}{E}(1-v^2) = \frac{\sigma_p}{E}\frac{m^2}{E^2}$$

$t_{transv} \sim E/\sigma_p^2, \;\; t_{long.} \sim E^3/\sigma_p^2 m^2$.