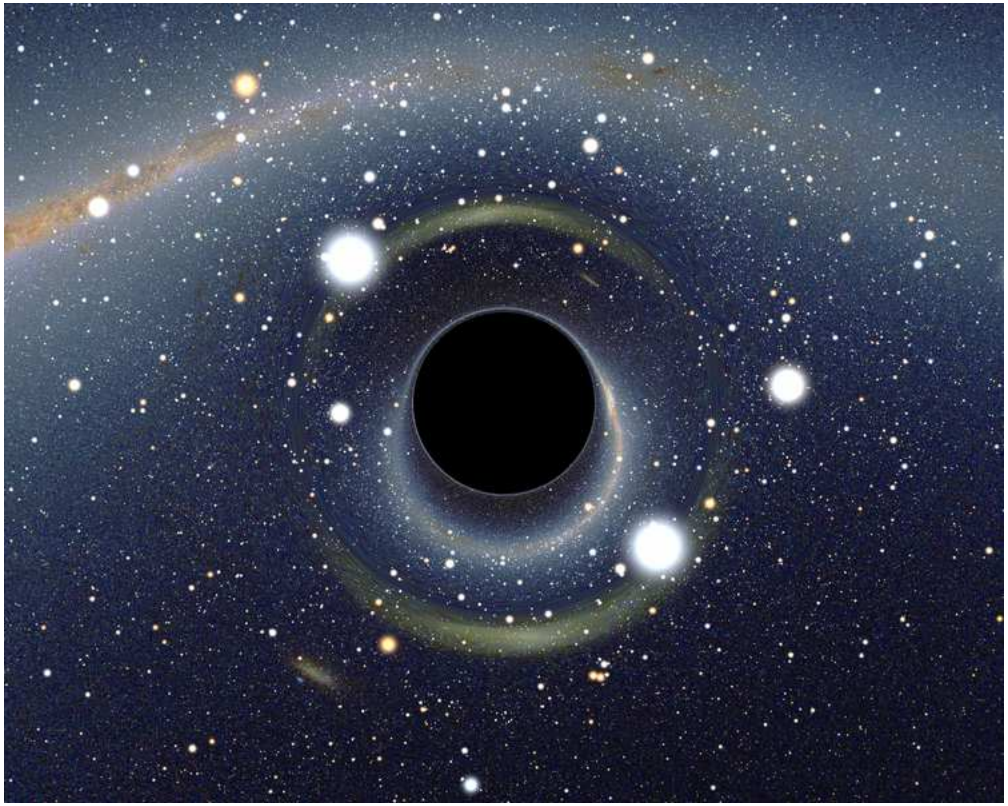
Falling into a Black Hole and the Information Paradox in AdS/CFT

Kyriakos Papadodimas
University of Groningen

IFT Xmas Workshop 2012
Madrid

December 20, 2012
based on arXiv:1211.6767, K.P and Suvrat Raju

Falling into a black hole

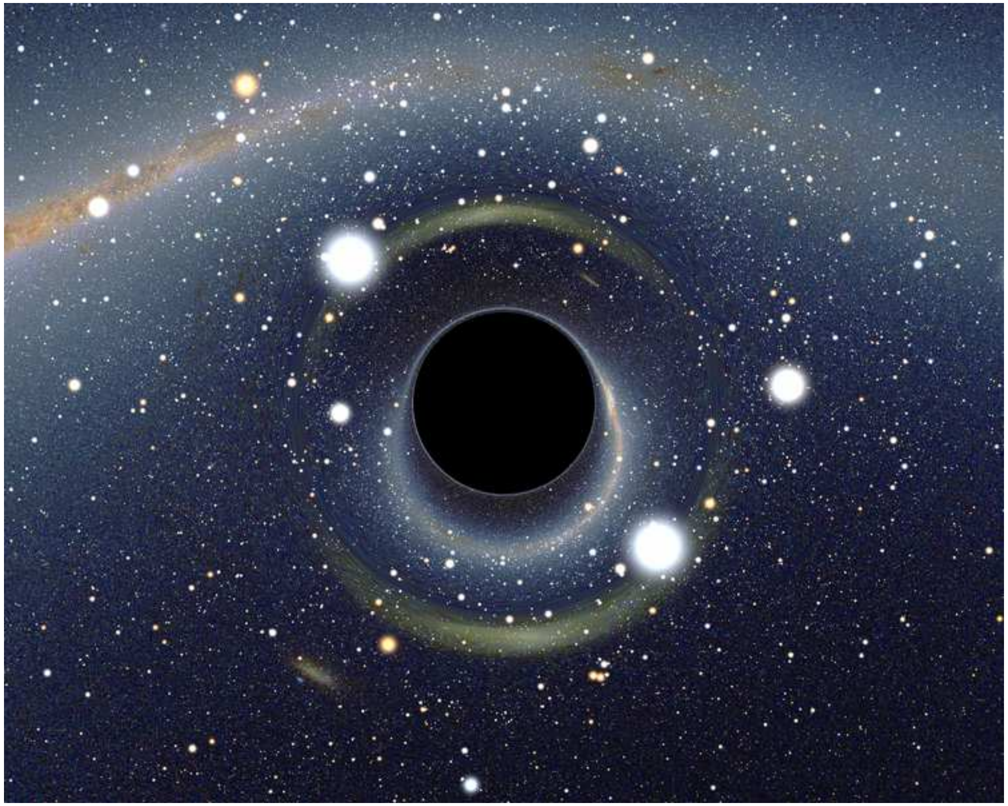


General Relativity (**equivalence principle**)

$\Downarrow$

For big black holes, nothing special happens at the horizon

Falling into a black hole



Curvature at horizon $\Rightarrow R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{3}{4}\frac{c^8}{(GM)^4}$

$$[\text{curvature}]_{\text{big black hole}} \sim 10^{-10} \times [\text{curvature}]_{\text{Earth}}$$

Including Quantum Mechanics

- Quantum Mechanically black holes radiate (Hawking)
- (Apparent) Conflict between Free Infall, Locality and Unitarity

$$\Downarrow$$

INFORMATION PARADOX

Hawking computation

- Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

- Consider scalar field:

$$\Box\phi = m^2\phi$$

- Quantize the scalar field
- Discover outgoing flux of ϕ-quanta from the black hole
- **Black holes radiate**
- **Outgoing radiation is thermal**

Hawking Radiation vs Unitarity

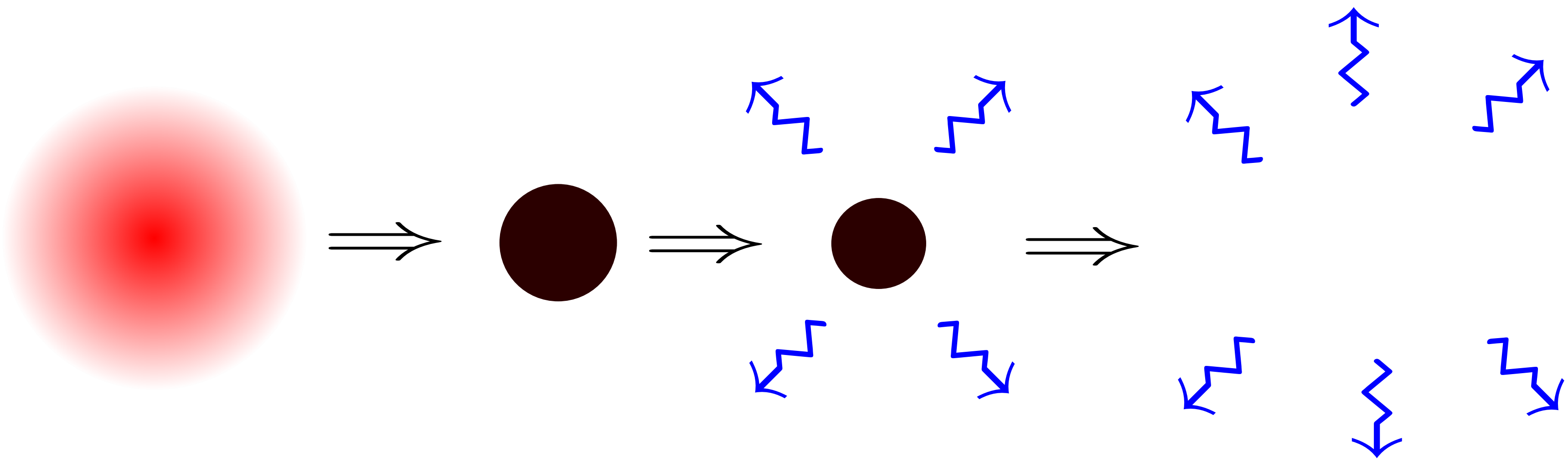
- Pure state $\Rightarrow$ vector $|\Psi\rangle$ in Hilbert space

- Thermal radiation $\Rightarrow$ Mixed state (density matrix)

$$\rho = Z^{-1} \sum_i e^{-\beta E_i} |\Psi_i\rangle\langle\Psi_i|$$

- **Under unitary evolution in Quantum Mechanics, a pure state cannot evolve into a mixed state**

Hawking Radiation vs Unitarity



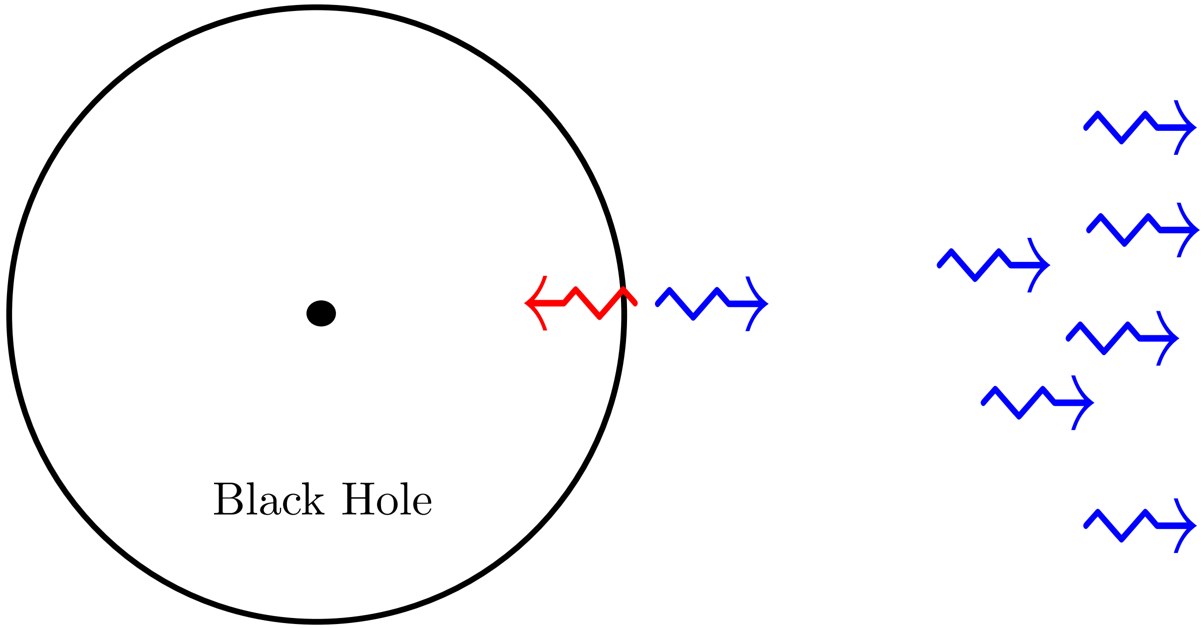
gas cloud
in pure state

Hawking radiation

$$|\Psi_0\rangle \quad \Longrightarrow \quad \cdots \quad \Longrightarrow \quad \rho_{\text{thermal}}$$

INCONSISTENT WITH UNITARY EVOLUTION

Hawking Radiation vs Unitarity



- For unitarity: final state must carry information of initial state
- (In some sense) Hawking quanta are created near the horizon
- **If horizon is featureless, how is information transfered to outgoing radiation?**

Modification of black hole geometry?

- Tension between Equivalence Principle (smooth horizon) and Unitarity
- (Many) Proposals to modify interior of black hole. Relevant for our discussion:

A. FUZZBALL PROPOSAL (Mathur et al.)
-quantum effects inside black hole spread all the way to horizon
-interior black hole geometry $\neq$ Schwarzschild solution
-black hole microstates have no horizon
-infalling observer (partly) feels deviations from semi-classical General Relativity

B. FIREWALLS (Almheiri, Marolf, Polchinski, Sully)
-horizon is not smooth
-infalling observer burns when crossing the horizon (firewall)

Modification of black hole geometry?

- Both the **Fuzzball** and the **Firewall** proposals predict that an infalling observer will notice something dramatic when crossing the horizon

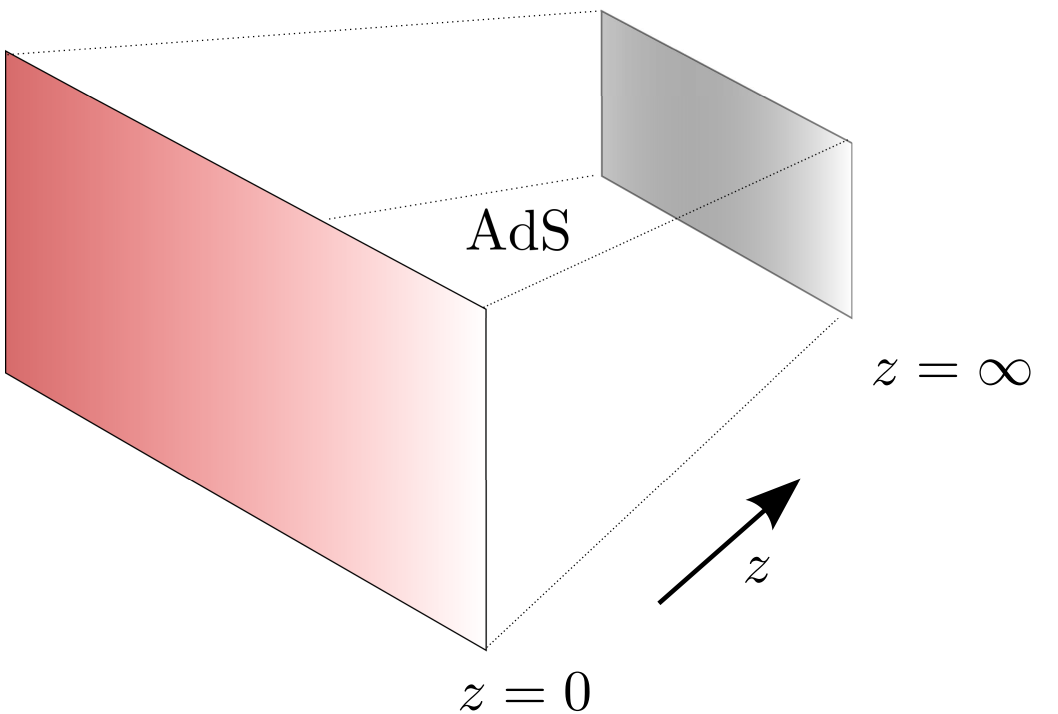
- In contradiction with General Relativity prediction

- Based on an **indirect** argument i.e. on the information paradox. **Do not directly address the experience of infalling observer**.

Free infall or not?

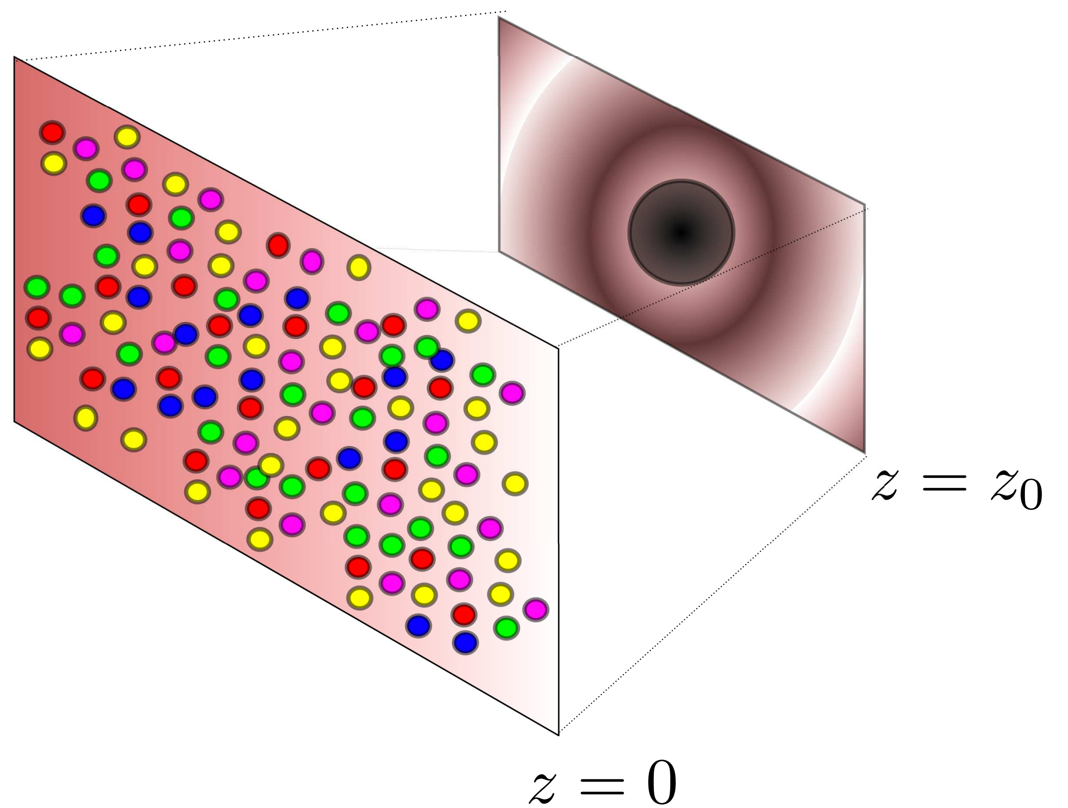
- So, does an infalling observer notice something or not?
- This is a question about Quantum Gravity effects
- To settle this question: should use a non-perturbative formulation of Quantum Gravity
- **AdS/CFT:** Non-perturbative description of Quantum Gravity in AdS via holographically dual gauge theory

AdS/CFT correspondence



Quantum Gravity in AdS $\Leftrightarrow$ large N gauge theory in lower dimensions

AdS/CFT correspondence



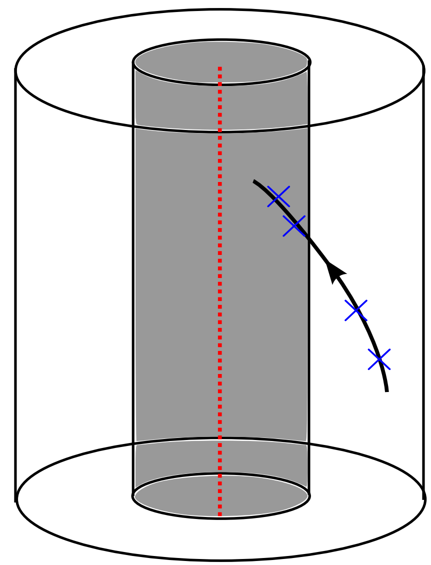
Black Hole in AdS $\Leftrightarrow$ Quark-Gluon Plasma (QGP) in gauge theory

Our approach

Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
- Understand what happens to an observer falling into a black hole
- Address the information paradox

An infalling observer in AdS



- Consider a big black hole in AdS and an observer freely falling towards it
- The observer performs local experiments (blue crosses in diagram)
- We will **reconstruct** these experiments from the boundary gauge theory
- We will show that the results of these experiments are the same as those of semi-classical GR and hence **AdS/CFT seems to be in conflict with the Fuzzball and Firewall proposals***

Reconstructing local observables in empty AdS

In AdS/CFT we know that

"S-matrix elements"

"S-matrix elements" in AdS	$\Leftrightarrow$	Correlation functions in CFT
Local bulk correlators in AdS	$\Leftrightarrow$	?

Reconstructing local observables in empty AdS

Our first goal:

Construct local bulk observables from CFT

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Hamilton, Kabat, Lifschytz, Lowe,...)

Reconstructing local observables in empty AdS

- Large N CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators $\mathcal{O}(x)$ whose correlators factorize

$$\langle \mathcal{O}(x_1)...\mathcal{O}(x_{2n})\rangle = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \; ... \; \langle \mathcal{O}(x_{2n-1})\mathcal{O}(x_{2n})\rangle + \ldots$$

- Factorization $\approx$ "superposition principle". However, the operators $\mathcal{O}$ **do not satisfy any linear equation of motion in the CFT**.

- Hence, they are not **free fields**, but rather **generalized free fields**

- Excitations created by $\mathcal{O}$ behave like **ordinary free particles** in a higher dimensional AdS spacetime!

(we can prove this, without even assuming AdS/CFT)

Reconstructing local observables in empty AdS

- First we define the Fourier modes of $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d\vec{x} \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.}\right)$$

- Conformal invariance fixes the 2-point function to be

$$\langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle = \left(\frac{-1}{t^2 - \vec{x}^2 - i\epsilon}\right)^{\Delta}$$

- From this we find

$$\mathcal{O}_{\omega,\vec{k}}|0\rangle = 0, \qquad \omega > 0$$

and

$$[\mathcal{O}_{\omega,\vec{k}}, \mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \mathcal{N}\theta(\omega^2 - \vec{k}^2)(\omega^2 - \vec{k}^2)^{\Delta - d/2}\delta(\omega - \omega')\delta(\vec{k} - \vec{k}')$$

Reconstructing local observables in empty AdS

- From this commutation relation we see that the modes $\mathcal{O}_{\omega,\vec{k}}$ create a **freely generated Fock space** of excitations.
- For an ordinary free field we have dispersion relation $\omega^2 = \vec{k}^2 + m^2$.
- For the generalized free fields, excitations labeled by the **independent** parameters ω and $\vec{k}$.
- $\Rightarrow$ excitations behave like higher dimensional excitations
- Behave like ordinary free particles in AdS
- These arguments hold for all values of the coupling. Also notice, **we did not use AdS/CFT anywhere above**

Reconstructing local observables in empty AdS

- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying $\Box\phi = m^2\phi$.

- We take m^2 to be related to the conformal dimension Δ of $\mathcal{O}$ by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of $\omega, \vec{k}$ we find a solution of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta-d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

Reconstructing local observables in empty AdS

- We construct non-local CFT operators as

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega\, d\vec{k}\, \left(\mathcal{O}_{\omega,\vec{k}}\; f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right)$$

Notice that while these are labeled by the coordinate z, they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

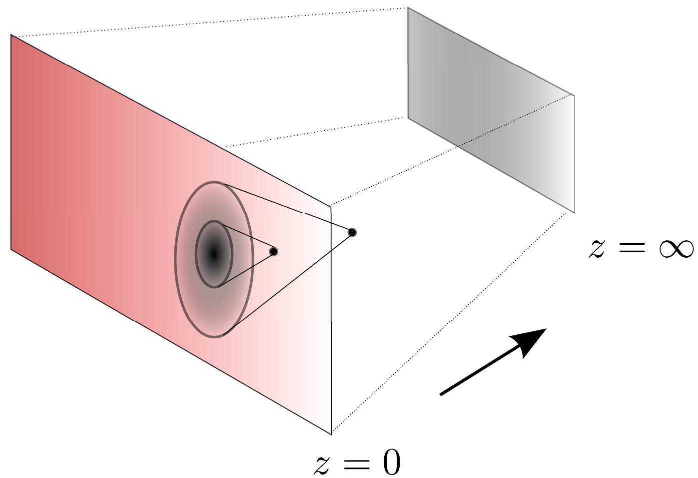
$$\Box_{\rm AdS}\phi_{\rm CFT} = m^2\,\phi_{\rm CFT}$$

and

$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0$$

for points $(t,\vec{x},z)$ and $(t',\vec{x}',z')$ spacelike **with respect to the AdS metric**.

Reconstructing local observables in empty AdS



- From the point of view of the CFT, coordinate z is an "auxiliary" parameter, which controls the smearing of the operators
- We can explicitly see how AdS space **emerges** from the lower dimensional CFT, as the combination of the coordinates $t, \vec{x}$ together with the extra parameter z
- **In particular, we can understand the $d + 1$ causal structure from the d-dimensional CFT!**

Reconstructing local observables in empty AdS

- We can also interchange the order of the Fourier transforms to write

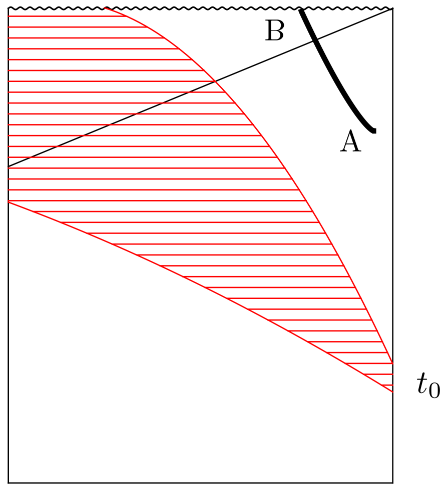
$$\phi_{\mathrm{CFT}}(t,\vec{x},z) = \int dt' d\vec{x}'\; K(\,t,\vec{x},z\;;\;t',\vec{x}')\mathcal{O}(t',\vec{x}')$$

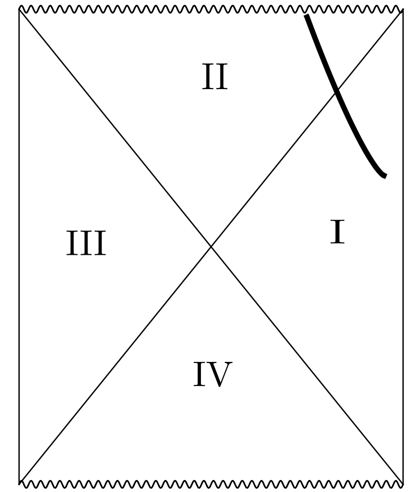
where K is some Kernel — sometimes called the *transfer function*. Notice: it is not the same thing as the bulk-to-boundary propagator.

- To summarize, using the operators $\phi_{\mathrm{CFT}}(t,\vec{x},z)$ we can study local bulk questions in AdS from the point of view of the dual gauge theory.

- We want to repeat the same construction in the presenct of a black hole in AdS, and then push the operators $\phi_{\mathrm{CFT}}(t,\vec{x},z)$ behind the horizon and see what we get

Black Holes in AdS





In AdS/CFT, Black Hole = Quark-Gluon Plasma (QGP) in gauge theory

BH formed by collapse $\approx$ Eternal Black Hole in AdS

Typical (QGP) pure state $|\Psi\rangle$ $\approx$ Thermal ensemble in gauge theory

We use the notation

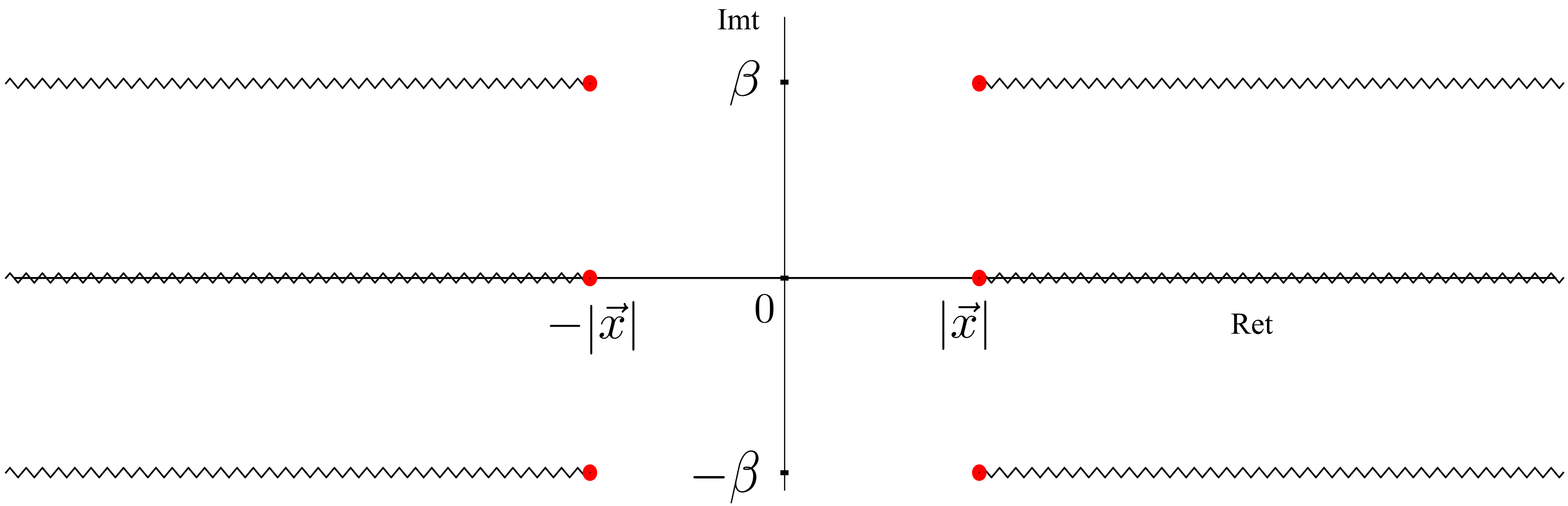
$$\langle A_1 ... A_n \rangle_\beta = \frac{1}{Z(\beta)} \mathrm{Tr} \left(e^{-\beta H} A_1 ... A_n \right)$$

- At large N thermal correlators factorize

$$\langle \mathcal{O}(x_1) ... \mathcal{O}(x_{2n}) \rangle_\beta = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \ldots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle_\beta + \ldots$$

- Of course $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_\beta \neq \langle 0 | \mathcal{O}(x_1) \mathcal{O}(x_2) | 0 \rangle$
- Factorization can fail if we scale the parameters of the correlator with N (for example: number of insertions, distances x_i, dimension of operators etc.)

CFT Correlators at finite temperature



- Consider the 2-point function $G_\beta(t,\vec{x}) = \langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle_\beta$
- Satisfies the **KMS condition**

$$G_\beta(t - i\beta, \vec{x}) = G_\beta(-t, -\vec{x})$$

- In Fourier space

$$G_\beta(-\omega, \vec{k}) = e^{-\beta\omega} G_\beta(\omega, \vec{k})$$

CFT Correlators at finite temperature

- If we again define the Fourier modes $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d^{d-1}x \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.}\right)$$

- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega,\vec{k}}\,,\,\mathcal{O}^\dagger_{\omega',\vec{k}'}] = \left(G_\beta(\omega,\vec{k}) - G_\beta(-\omega,\vec{k})\right)\delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle \hat{\mathcal{O}}^\dagger_{\omega,\vec{k}}\,\hat{\mathcal{O}}_{\omega,\vec{k}}\rangle_\beta = \frac{1}{e^{\beta\omega}-1}$$

(this is the CFT analogue of the “thermal atmosphere” of the black hole)

- Consider a black hole in AdS given by the metric

$$ds^2 = \frac{-h(z)dt^2 + dx^2 + h^{-1}(z)dz^2}{z^2} \qquad , \qquad h(z) = 1 - \frac{z^d}{z_0^d}$$

- Look for solutions of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}} \psi_{\omega,\vec{k}}(z)$$

- For every $(\omega, \vec{k})$ there is a unique solution, normalizable at the boundary $z = 0$.
- These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We identify

$$f_{\omega\vec{k}}(t,\vec{x},z) \qquad \Leftrightarrow \qquad \mathcal{O}_{\omega,\vec{k}}$$

Reconstructing the region outside the black hole

- As before, we can write nonlocal CFT operators

$$\phi_{\mathrm{CFT}}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left(\mathcal{O}_{\omega,\vec{k}}\, f_{\omega,\vec{k}}(t,\vec{x},z) + \mathrm{h.c.} \right)$$

- which behave like local fields around a black hole

$$(\Box - m^2)\phi_{\mathrm{CFT}} = 0$$

$$[\phi_{\mathrm{CFT}}(t,\vec{x},z)\,,\,\phi_{\mathrm{CFT}}(t',\vec{x}',z')] = 0 \qquad , \quad \text{for spacelike points}$$

- and more generally

$$\langle \phi_{\mathrm{CFT}}(P_1)...\phi_{\mathrm{CFT}}(P_n)\rangle_\beta = \langle \phi_{gravity}(P_1)...\phi_{gravity}(P_n)\rangle_{\text{Hartle Hawking}}$$

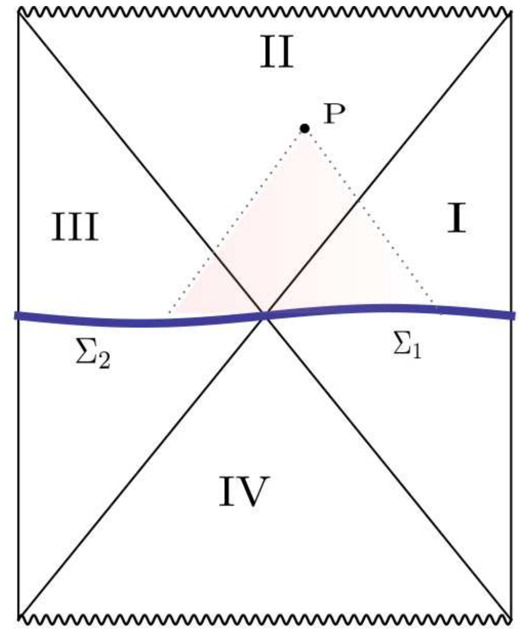
Reconstructing the region outside the black hole

- We have understood how to reconstruct the region outside the black hole from the point of view of the gauge theory

- We can write local observables in gravity as non-local operators in the gauge theory

Falling behind the horizon

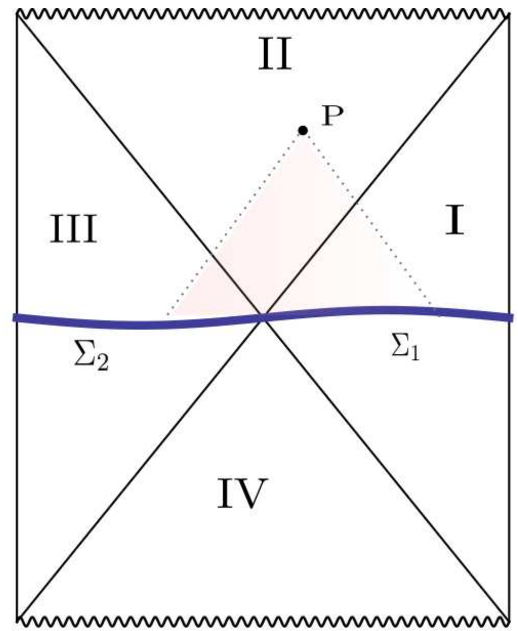
- Penrose diagram of (eternal) AdS black hole



- Cauchy slice for points in II is $\Sigma_1 \oplus \Sigma_2$
- To reconstruct local operator at P we need **both** modes on Σ_1 and Σ_2

Modes on Σ_1	$\Leftrightarrow$	$\mathcal{O}_{\omega,\vec{k}}$
Modes on Σ_2	$\Leftrightarrow$	$?$

Falling behind the horizon



- Maldacena: eternal black hole = 2 copies of CFT in entangled state
- In this formalism, modes on Σ_2 are the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ in the second copy of the CFT
- Do we really need the two entangled copies?
- If we work **with a single CFT**, what is the meaning of the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$?

Coarse-graining and doubling of operators

- Consider complicated (ergodic) system in pure state $|\Psi\rangle$
- Intuitive expectation $\Rightarrow$ system "thermalizes"
- For some observables $\{A_i\}$- called **coarse-grained observables**, their correlators on $|\Psi\rangle$ come close to thermal correlators

$$\langle\Psi|A_1..A_n|\Psi\rangle \approx \mathrm{Tr}\left(e^{-\beta H}A_1...A_n\right)$$

- This is not true for all observables, there are also **fine grained observables** which do not thermalize
- Hilbert space has the form

$$\mathcal{H} = \mathcal{H}_{\text{coarse}} \otimes \mathcal{H}_{\text{fine}}$$

- $\mathcal{H}_{\text{fine}}$ plays the role of a **heat bath** for $\mathcal{H}_{\text{coarse}}$

Coarse graining and doubling of operators

- Every state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

where $|\Psi_i^c\rangle$, $|\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\text{coarse}}$ and $\mathcal{H}_{\text{fine}}$ respectively

- If $\mathcal{H}_{coarse}$ thermalizes, it means that the reduced density matrix

$$\rho_{\text{coarse}} = Z_c^{-1} e^{-\beta H_{\text{coarse}}}$$

- which means we can redefine our orthonormal basis such that

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

Coarse graining and doubling of operators

- The state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- Consider a **coarse-grained** operator acting on $\mathcal{H}_{\mathrm{coarse}}$ as

$$A = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle\hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\widetilde{A} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes |\hat{\Psi}_j^f\rangle$$

acting on the fine-grained Hilbert space.

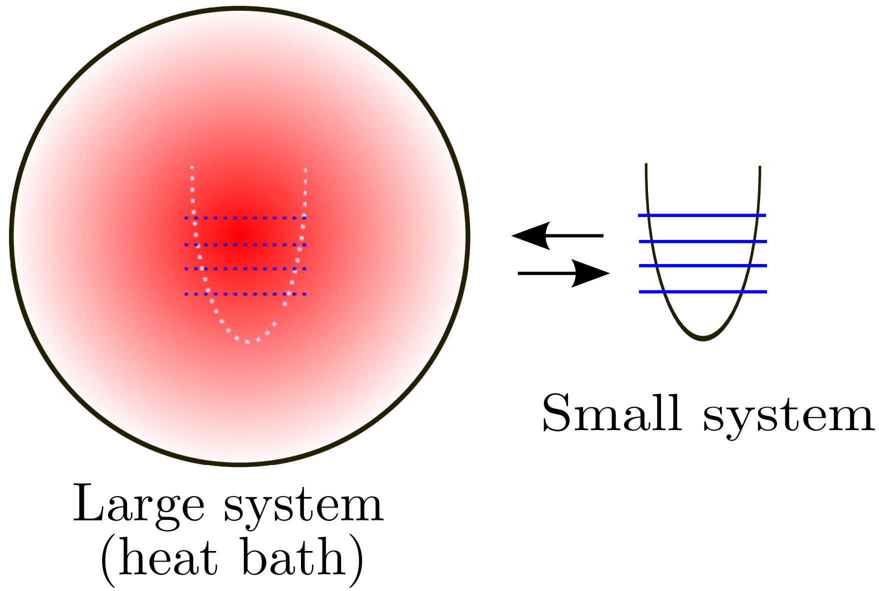
Coarse graining and doubling of operators

- We started with a set of coarse-grained operators A_i which thermalize.

- The operators $\widetilde{A}_i$ constructed as above, have the properties

1. They operator algebra $\widetilde{A}_i$ is isomorphic to that of A_i
2. Operators A_i commute with operators $\widetilde{A}_i$

Coarse graining and doubling of operators



SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH!

- For us the Quark-Gluon-Pasma is the heat bath
- The glueball operators $\mathcal{O}_i$ are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators $\widetilde{\mathcal{O}}_i$
- This mirroring involves the fine-degrees of freedom

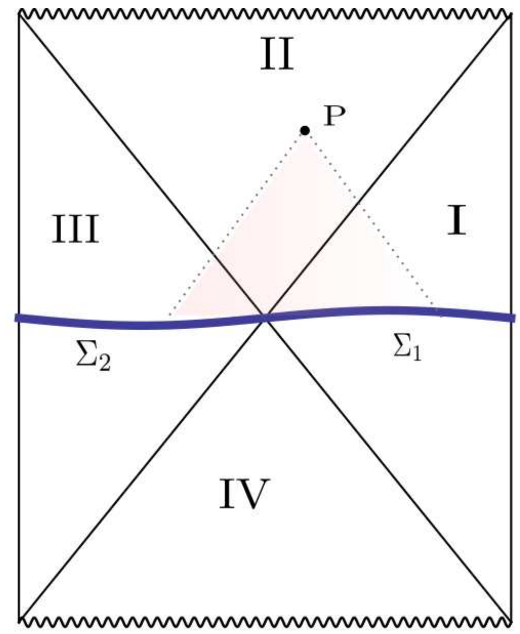
Coarse graining and doubling of operators

- At large N, correlation functions of the mirrored operators $\widetilde{\mathcal{O}}$ on a pure state, agree with those of analytically continued operators

$$\mathcal{O}(t + i\beta/2)$$

- However the $\widetilde{\mathcal{O}}$, **as operators acting on pure states**, were defined via the coarse/fine-grained decomposition

Falling behind the horizon



$$\text{Modes on } \Sigma_1 \quad \Leftrightarrow \quad \mathcal{O}_{\omega,\vec{k}}$$

$$\text{Modes on } \Sigma_2 \quad \Leftrightarrow \quad \widetilde{\mathcal{O}}_{\omega,\vec{k}}$$

where $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ are the Fourier transforms of the mirrored operators $\widetilde{\mathcal{O}}$

Local operators behind the horizon

Using both $\mathcal{O}_{\omega,\vec{k}}$ and $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ we can write local observables behind the horizon of the black hole.

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left[\mathcal{O}_{\omega,\vec{k}}\, g^{(1)}_{\omega,\vec{k}}(t,\vec{x},z) + \widetilde{\mathcal{O}}_{\omega,\vec{k}}\, g^{(2)}_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right]$$

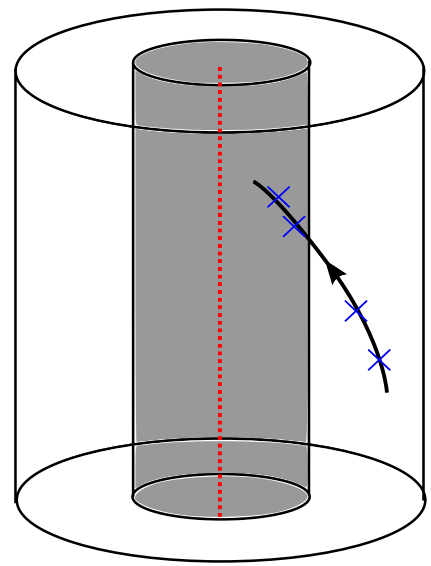
here $g^{(1),(2)}$ are solutions of the Klein-Gordon equation in region II

In the large N limit, correlators of $\phi_{\rm CFT}(t,\vec{x},z)$ on a typical pure state $|\Psi\rangle$ (corrseponding to a black hole microstate) agree with those computed in semiclassical gravity.

We have reconstructed both the exterior and the interior of the black hole from the dual gauge theory

Fate of the infalling observer

Using the operators ϕ_{CFT} we can reconstruct the experiments of the infalling observer



MAIN CONCLUSION: For a big black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon. In particular, he does not see a firewall or a fuzzball.

Various subtleties

- Sensitivity to pure state $|\Psi\rangle$?
- Including $1/N$ corrections?
- Spread of transfer function as we approach the horizon?
- Sensitivity to late times - Poincare recurrences?

None of these issues is really a problem. If large N expansion at finite temperature holds, we can show that they are all under control.

The information paradox

- We argued that infalling observer can use effective field theory (EFT) to describe physics near the horizon and does not notice anything special.

- Mathur and AMPS argue that free infall cannot be compatible with unitary evaporation (information paradox)

- Sharpened version of information paradox (strong subadditivity argument)

What does our construction teach us about the information paradox?

Naive version of the information paradox

- Hawking's computation $\Rightarrow$ mixed (thermal) state ρ_{Hawking}
- Starting from pure state $|\Psi\rangle$ we end up with mixed state ρ_{Hawking}
- Inconsistent with unitary evolution in Quantum Mechanics

Naive version of the information paradox

However, consider what happens when a normal object burns (say a piece of coal)

- Outgoing photons seem to be thermal to a very good approximation.
- How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
- ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
- While final state **looks like** a thermal density matrix ρ_{thermal} in reality it is a pure state.

SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY

Naive version of the information paradox

- Imagine that outgoing photons can be in 2 states. For N photons we have

$$2^N \qquad \text{states}$$

- Density matrix of outgoing radiation is of size $2^N \times 2^N$.
- Consider

$$\rho_{\text{exact}} = \rho_{\text{thermal}} + 2^{-N} \rho_{\text{correction}}$$

  where $\rho_{\text{correction}} \sim \mathcal{O}(1)$.
- Can easily check that ρ_{exact} of this form, can correspond to a **pure state**

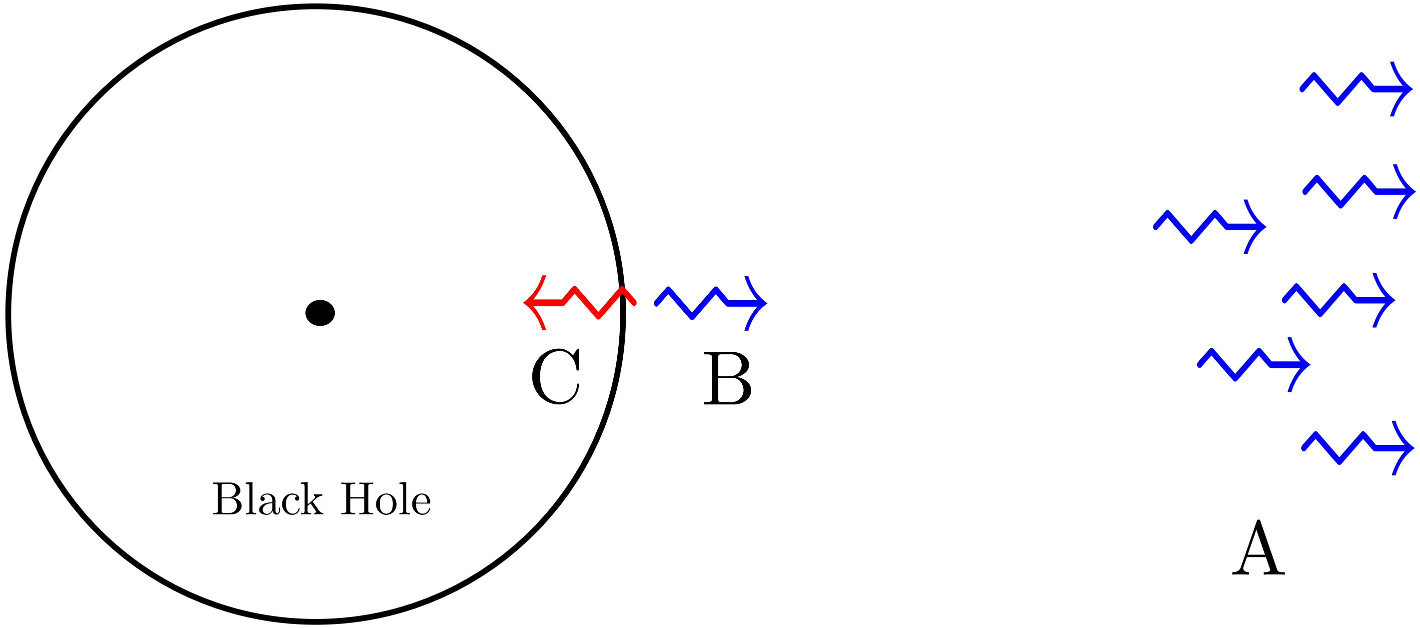
EXPONENTIALLY SMALL CORRECTIONS CAR RESTORE UNITARITY

Resolution of (naive version of) the information paradox

- Hawking's computation is only **the leading order** result
- We certainly expect corrections to the leading order computation, from quantum gravity effects (saddle points etc.)
- Even extremely small corrections are able to restore unitarity due to the large number of particles

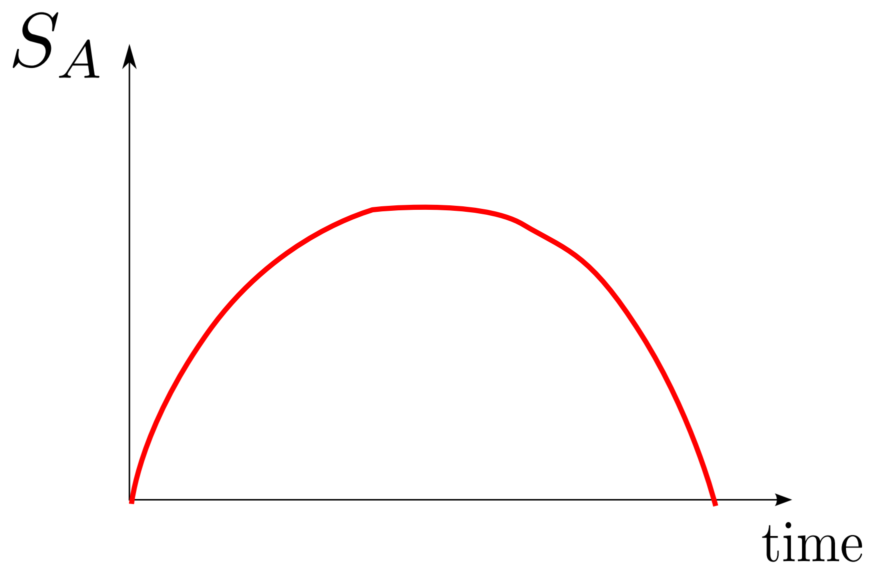
Sharpened version of the information paradox (Mathur, AMPS)

Consider the process of Hawking radiation



- A: old radiation, far from black hole
- B: newly created Hawking particle, outgoing
- C: ingoing partner of B

Sharpened version of the information paradox (Mathur, AMPS)

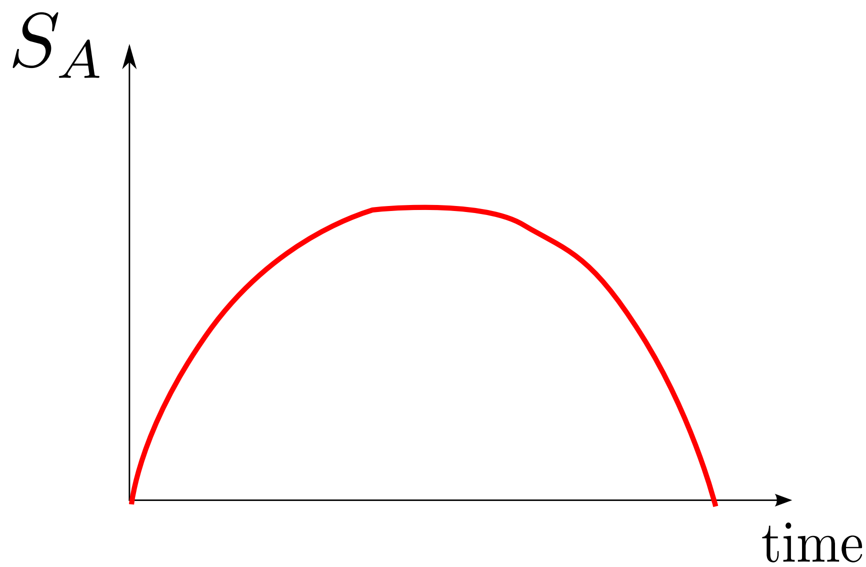


Consider the entropy of radiation A as a function of time

$$S_A = -\mathrm{Tr}\left(\rho_A \log \rho_A\right)$$

If initial state is pure then S_A must go to zero after complete evaporation of the black hole

Sharpened version of the information paradox (Mathur, AMPS)



In the beginning adding a B to A increases the entropy i.e. we expect

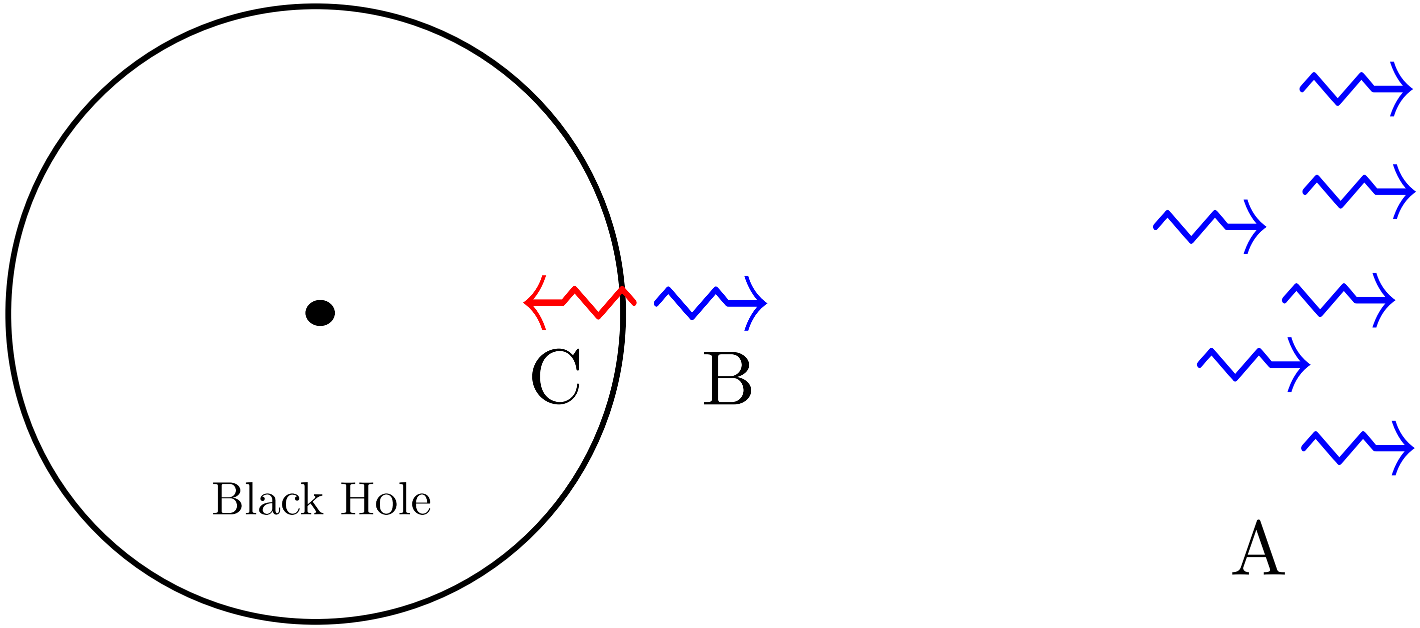
$$S_{AB} > S_A$$

but eventually this must turn around and for an **old black hole** we expect

$$S_{AB} < S_A$$

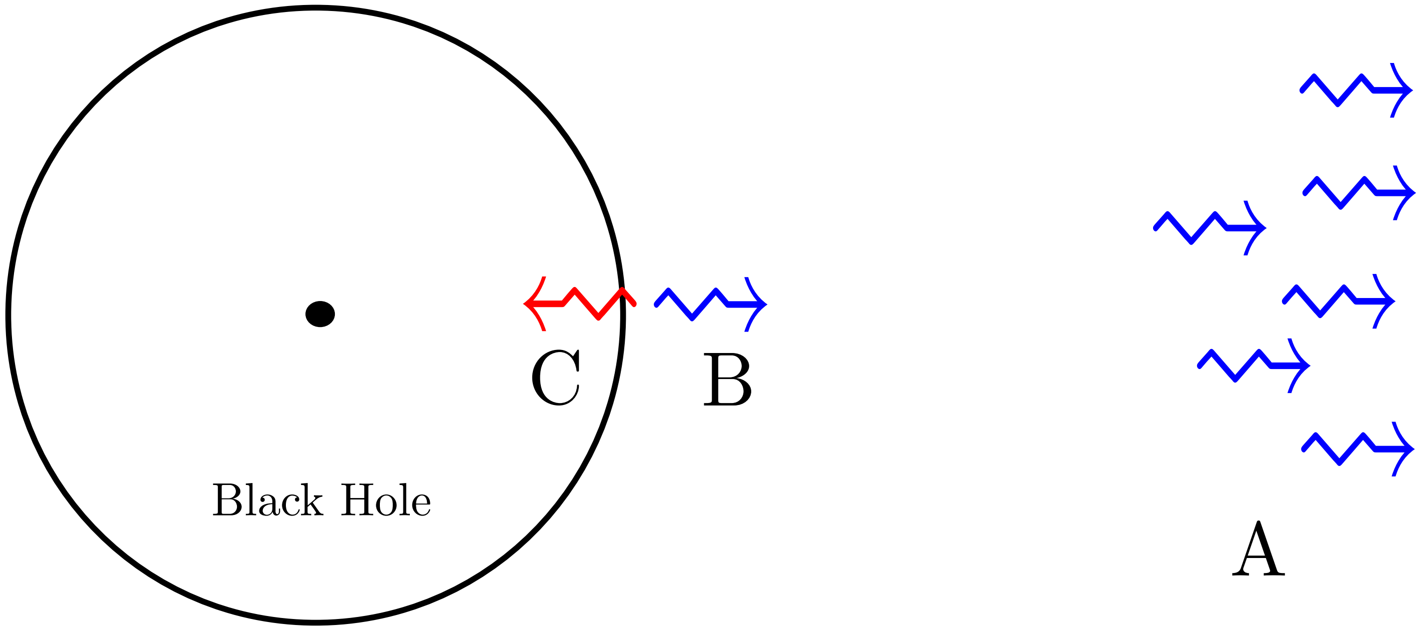
Sharpened version of the information paradox (Mathur, AMPS)

Consider the process of Hawking radiation



- For **free infall**: B must be fully entangled to C
- For **information recovery**: B must be very entangled to A
- Are these two statements compatible?

Sharpened version of the information paradox (Mathur, AMPS)



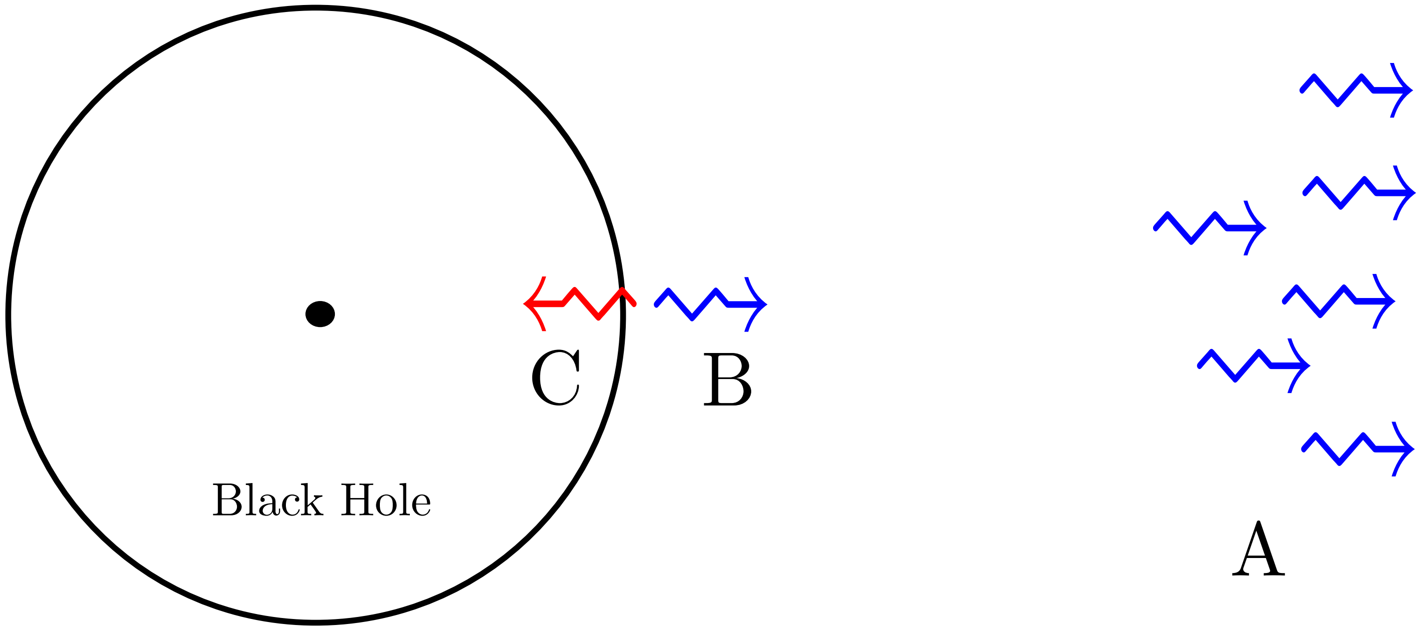
Strong subadditivity theorem: for 3 independent systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C \approx \log 2$ which would imply

$$S_{AB} > S_A$$

Sharpened version of the information paradox (Mathur, AMPS)



Strong subadditivity theorem: for 3 independent systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C \approx \log 2$ which would imply

$$S_{AB} > S_A$$

Sharpened version of the information paradox (Mathur, AMPS)

- **Refined version of information paradox:** If we have smooth horizon ($S_{BC} \approx 0$) then entropy of radiation keeps increasing.

- Critically based on theorem of strong subadditivity.

- What is the resolution suggested by our construction?

Resolution of the (refined) information paradox

Systems A,B,C are not really independent

$\Downarrow$

Strong subadditivity theorem cannot be applied to A,B,C

$\Downarrow$

There is no paradox

Resolution of the (refined) information paradox

- In our language the C is the tilde operator $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$
- These operator came to existence because of splitting the Hilbert space into coarse- and fine-grained components.
- The outgoing radiation A is the coarse-grained part described by the operators $\mathcal{O}_{\omega,\vec{k}}$.
- **However**, the splitting into fine-and coarse-graining makes sense only if

$$\dim\left(\mathcal{H}_{\text{coarse}}\right) < \dim\left(\mathcal{H}_{\text{fine}}\right)$$

- After Page time (half-evaporation), size of A is comparable to $\mathcal{H}_{\text{fine}}$
- Hence, it is not possible to indclude in our coarse-grained Hilbert space **AT THE SAME TIME** all the A **AND** reconstruct C as part of the fine grained Hilbert space

Other ways to say the same thing

- System C is not independent, but rather a **highly scrambled** version of A
- Locality for **simple measurements** is preserved. For P_1 inside horizon and P_2 outside

$$[\phi(P_1)\,,\,\phi(P_2)] \approx 0$$

  up to very small corrections.
- However if $\mathcal{W}$ is complicated operator acting outside the horizon which measures all of the As then

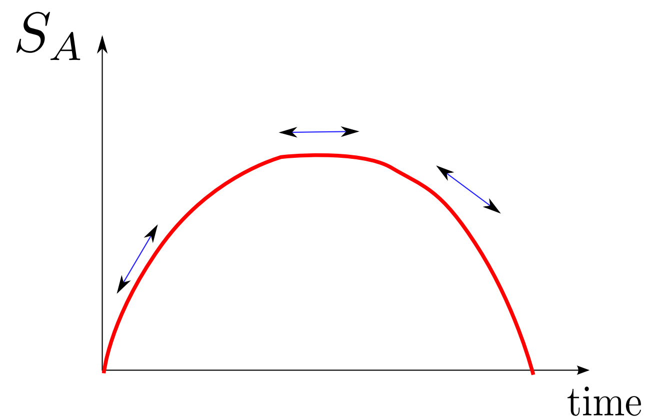
$$[\phi(P_1)\,,\,\mathcal{W}] \neq 0$$

- We can not reconstruct local n-point functions when n scales with N (or the entropy of the black hole)
- Consistent with the fact that boundary large N expansion does not hold when $n \sim N^{\#}$.

Toy model: spin chain

- Previous statements can be explicitly checked in toy model: N spins in random pure state
- Take $p \ll N$ of them as the coarse-grained Hilbert space
- For any operator $\mathcal{A}$ acting on the p spins, we can **explicitly** construct $\widetilde{\mathcal{A}}$ acting on the $N-p$ spins
- When $p \ll N$ we have $[\mathcal{A}, \widetilde{\mathcal{A}}] \approx 0$.
- Can see explicitly that this breaks down when $p \sim \frac{N}{2}$

Final conclusion about black hole evaporation



- Effective field theory (EFT) on evaporating black hole background works fine, as long as we do not try to compute **too complicated** correlation functions (for instance S-point functions, where S is the black hole entropy)
- By appropriately splitting the full Hilbert space into fine-and coarse-grained components, we can describe various parts of the evaporation process using EFT, but not all of them simultaneously.
- In spirit this is similar to the idea of complementarity.

Conclusions

- **Local bulk physics from CFT**: found local observables both outside and inside the black hole
- **Infalling observer:** does not notice anything special
- **Interior of black hole:** coarse-grained observables effectively doubled in fine grained degrees of freedom. Black hole interior is a combination of both.
- **Information paradox:** small corrections can restore unitarity.
- **Strong subadditivity argument (Mathur, AMPS)**: does not apply because A,B,C are not independent systems. C is a highly scrambled rewriting of A
- **Non-locality:** The amount of non-locality necessary is exponentially small.

Future directions

- Understand $1/N$ corrections in more details
- Concrete computation in $\mathcal{N}=4$ SYM at small λ?
- Dynamical toy model for thermalization/complementarity
- Use local operators to study black hole singularity from gauge theory

THANK YOU