Forget EFT —
what if New Physics is light, feeeeeeeeeeebly coupled...
...and DM = axion?

Sacha Davidson
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1. an axion review
   - why the axion in particle physics?
   - put an (invisible) axion: astrophysical constraints
   - the axion in cosmology: $COLD$ Dark Matter
2. how to distinguish **the axion vs the WIMP?**
   - (in)direct detection
   - non-linear structure formation : ingredients, scenarios ...and things to do?
3. (what we did) arXiv:1307.8024 with M Elmer

Why the axion: QCD is a model-builders nightmare

gauge boson sector of QCD:

$$-\frac{1}{4}G^{A}_{\mu\nu}G^{\mu\nu A} \qquad\qquad A:1..8$$

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$\vec{E}^2 + \vec{B}^2$ $\qquad\qquad$ $\vec{E}\cdot\vec{B}$

But... θ is CPV! neutron edm $\Rightarrow \theta \lesssim 10^{-10}$

Pich deRafael
Pospelov, Ritz

Why the axion: QCD is a model-builders nightmare

Model builders dream: theory that predicts SM = select particle content, and *dynamically generates couplings*

gauge boson sector of QCD:input one parameter g_s,

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and instantons dynamically generate $\theta \sim 1$!

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Model builders nightmare: a theory that *dynamically generates the wrong couplings*

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How to make θ unobservable? *Aha!* There are quarks and the axial anomaly: a chiral rotn through η contributes:

$$\delta\mathcal{L} \propto \eta\partial_\mu J_5^\mu = \eta\frac{g_s^2 N}{8\pi^2}G\widetilde{G} + \eta\sum_f m_f\overline{q}_f\gamma_5 q_f$$

(N $\Leftrightarrow$ coloured fermion reps)

a chiral phase rotn moves θ onto (coloured) fermion mass matrix...still CPV

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⇒ **solution**: add fields, such that "generalised" chiral rotns ($\equiv$ PQ sym) are a sym of classical theory.

Peccei Quinn

To build an (Invisible) axion model

ShifmanVainshteinZakharov
Srednicki NPB85

1. aim to obtain a "Peccei-Quinn" symmetry = a global symmetry of the classical Lagrangian, broken by colour anomalies ($\simeq$ some generalisation of chiral rotns)

2. for instance (SVZ), add a gauge-singlet scalar with $Q_{PQ} = 2$ and SU(2) singlet quarks $\Psi_{L,R}$ with $Q_{PQ} = \pm 1$, so

$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

3. arrange to break the PQ sym spontaneously, at high scale, such that all new particles are heavy except the goldstone = axion

4. so can rotate θ to the phase of Φ...which is a dynamical field...who will get a mass and want to sit at zero.

   ...so if CDM is an oscillating axion field, the nedm oscillates at $m_a \sim 10^{10}$ s^{-1}

The axion in particle physics (summary)

- strong CP problem of QCD: instantons choose θ — neutron edm $\Rightarrow \theta \lesssim 10^{-10}$
- solution : trade θ to a dynamical field a, with pot. min. at $\theta = 0$

phase of a complex SM-singlet scalar Φ, with big vev

Peccei Quinn
DineFischlerSrednicki,Zhitnitsky
Kim,ShifmanVainshteinZakharov

$$\Phi \to f_{PQ} e^{ia/f_{PQ}} \qquad f_{PQ} \sim 10^{11} \text{ GeV}$$

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$$\Phi \to f_{PQ} e^{ia/f_{PQ}} \qquad\qquad f_{PQ} \sim 10^{11}\ \text{GeV}$$

$\Rightarrow$ only new particle at low-energy is the (pseudo-) goldstone a

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f_{PQ}} \simeq\ 6 \times 10^{-5}\ \frac{10^{11}\ \text{GeV}}{f_{PQ}}\ \text{eV}$$

...
Srednicki NPB85

$$\text{couplings to SM} \quad \propto \quad \frac{1}{f_{PQ}} \propto m_a$$

always to gluons $\Leftrightarrow$ nucleon

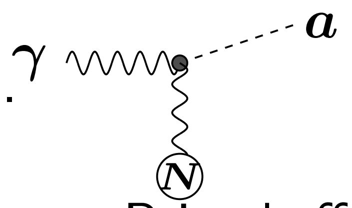
model − dep to fermions (electrons) at tree

generically $\sim \dfrac{\alpha}{\pi f_{PQ}}$ to 2γ (triangle, and mixing with π)

Astrophysical bounds

axion light and (feebly) coupled to SM $\propto \frac{1}{f_{PQ}} \propto m_a$

$\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...



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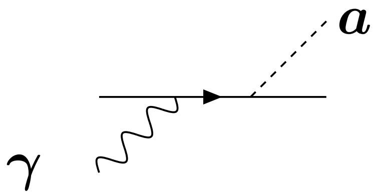

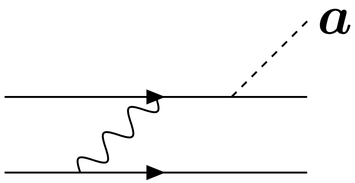

(axion couplings to e vs N vary across models by ~ 10)

upper bound on coupling to avoid rapid stellar energy loss:

$$m_a \lesssim 10^{-2} \text{ eV} \qquad\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$$

the axion in cosmology

1. non-thermal production (two populations) $\Rightarrow$ it redshifts like CDM
2. it grows density fluctuations like CDM
3. the *axion* vs the **WIMP**

Non-thermal axion production: the classical field is $Cold$ Dark Matter!

1. PQ phase transition : $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   a massless, random $-\pi f_{PQ} \le a_0 \le \pi f_{PQ}$ from one horizon to the next

2. QCD Phase Transition ($T \sim 200$ MeV): $m_a(t) : 0 \to f_\pi m_\pi / f_{PQ}$ (tilt mexican hat)
   $*$ ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum
   $*$ energy density $m_a^2 \langle a_0 \rangle^2 / R^3(t)$ $\Omega_a \lesssim 0.27 \Rightarrow m_a \langle a_0 \rangle = \ldots$

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Relate $\langle a_0 \rangle$ to f_{PQ}?
Scenario 1: PQPT before inflation
no: a_0 random ($10^{-7} f_{PQ}$ ok...so any $m_a \lesssim 10^{-2}$ eV ok...)
But, also $\delta a / a \sim H_I / (2\pi f_{PQ})$, gives isocurvature $\delta\rho/\rho$,
Planck $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV, or non-canonical kin.terms for a...

WantzShellard
HanannHRW
FolkertsCristianoRedondo

Scenario 2: PQPT after inflation
yes: $\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f_{PQ}^2 / 3$
density today higher for smaller mass $\Rightarrow$ correct Ω for $m_a \gtrsim 10^{-5}$eV)

Another contribution to CDM (if PQPT after inflation): cold axion particles

1. Suppose inflation before Peccei-Quinn Phase Trans.

   avoid CMB bounds on isocurvature fluctuations
   obtain varrying axion field across the U

2. then at PQPT $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   * a random in each horizon...
   * ...one string/horizon

3. QCD Phase Transition ($T \sim 200$ MeV):
   * strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$)

   Hiramatsu etal 1012.5502

if PQPT after inflation ⇒ **CDM = oscillating axion field + cold particles**

Axion density fluctuations

1. has adiabatic density fluctuations inherited from surroundings at the QCDPT

2. density fluctuations in the axion field, on LSS scales, have same linear growth equations as in WIMPs

   Ratra, Hwang+Noh

   - there is pressure and Jeans length $\sim 1/\sqrt{H(t) m_a}$ (and funny c_s on smaller scales?)
   - if PQPT after inflation, $\delta\rho_a/\rho_a \sim \mathcal{O}(1)$ on QCDPT horizon scale (5km then, 0.1 pc today)... axion "miniclusters"
     Physical size at QCDPT $\sim 5km$, today ~pc.

     Hogan,Rees

3. the axion field does not turn into particles by parametric resonance

   Kolb,Singh,Srednicki

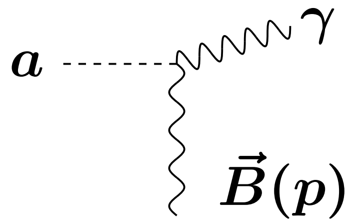
The *axion* vs the **WIMP**

1. direct detection?

2. *might axions differ from WIMPs during non-linear structure formation?*

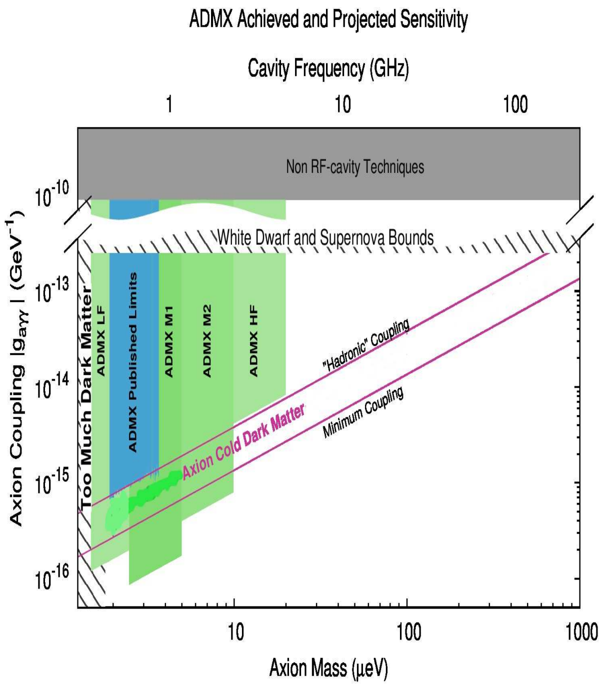
   (**Umm...** non-linear/N-body is hard!)

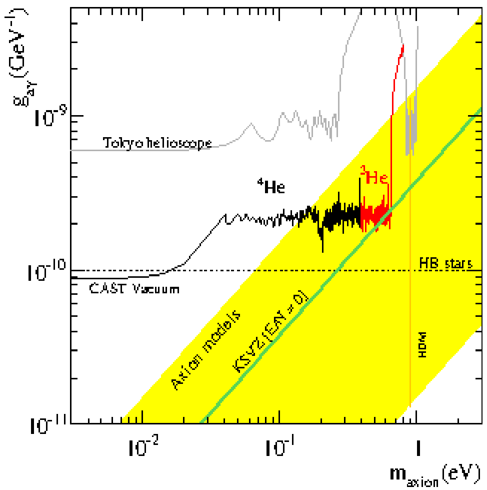
Direct detection (of axions)



1. $a \to \gamma$ conversion in $\vec{B}$ field. (with gradient, to transfer correct $\vec{p}$...a diff $\vec{B}$ for each m_a)
   (a) CernAxionSolarTel: LHC magnet, points at sun, convert solar a to γs (also Sumico)
   (b) ADMX: dark matter axions ($E_\gamma \sim m_a \sim$ microwave)
2. spherical mirror in $\vec{B}$ field: a convert in $\vec{B}$ to γ focused by mirror= antennae

Horns etal 1212.2970





The axion vs the WIMP — is non-linear structure formation different?

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$\Rightarrow$ write an axion DM code and compare to N-body?

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...there is diverse literature...

Sikivie: Erken,Sikivie,Tam,Yang

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a "Bose-Einstein Condensate"
2. BEC can support vortices, which allow caustics in the galactic DM distribution

⇔ **axion DM signature?**

The axion vs the WIMP — is non-linear structure formation different?

? non-perturbative dynamics...
⇒ write an axion DM code and compare to N-body?

...there is diverse literature...
....a variety of (hypothetical) scenarios...
...a gaggle of partial results...
...and a zoo of vocabulary and assumptions

So lets start from basics :)

The path integral should tell you everything...

The path integral (in Closed Time Path formalism) allows to compute:

1. the expectation value of the field ("classical field" of 1PI action)

2. the expectation value of the two pt function (propagator, number density)

... (higher point functions...)

hypothesis 1: the field and (particle) number density are the relevant variables.

Sacha's translation dictionary and assumptions

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1. the expectation value of the field ("classical field" of 1PI action)
   = coherent state   condensed regime /**Bose Einstein condensate** /

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   kinetic regime / cold particles / never a BE condensate

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hypothesis 2 structure formation cares about $T^{\mu\nu}$.

$\Rightarrow$ axions are $simple$: free scalar field, and/or non-rel. particles, coupled to gravity.
gravity is classical, cosmology is $\mathcal{O}(G_N)$

back to Sikivie's scenario **(with the hypotheses)**

Erken,Sikivie,Tam,Yang

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   what we asked: is that interaction rate a $thermalisation$ rate? leading order unitary eqns??

   what we did: look for entropy production...find at $\mathcal{O}(G_N p^2/m_a^2)$...negligeable rate

   DavidsonElmer

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   post scriptum: NB all these calns done for the field

   (for PQPT after inflation, $k \sim H_{QCDPT} < 10^{-6} m_a$)

   but no need to thermalise field modes ; field is a BE condensate!

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...sure, lab BECs have vortices, but they have self-interactions...

Rindler-Daller+Shapiro study a rotating galactic halo formed of classical scalar field. They show that for sufficiently strong $\lambda\phi^4$, with sufficient angular momentum, its energetically favourable to form a vortex. $\lambda \sim m_a^2/f_{PQ}^2 \lesssim 10^{-40}$ is to small.

Summary (review of well-known things)

The CPV θ parameter of QCD can be replaced by a light scalar field $\Leftrightarrow$ the *axion!*

The axion coupling to SM $\propto m_a$, so for $m_a \lesssim 10^{-2}$ eV, axion emission does not cool stars to fast.

Non-thermal production mechanisms in cosmology allow the axion to be a viable *Cold* DM candidate:

∗ redshifts like CDM
∗ grow density fluctuations like CDM
∗ not overclose U for $m_a \gtrsim 10^{-5} \to 10^{-4}$ eV (PQ before inflation can have smaller m_a by tuning $a \ll f_{PQ}$)

NB: there are (potentially) two axion contributions to CDM: classical field, and density of cold particles

$\Rightarrow$ how to distinguish axion from WIMP CDM?

Summary: to distinguish axions from WIMPs?

$1-$ **direct detection:** find WIMPs or axions in terrestrial searches (CAST, ADMX...)

$2-$ **during structure formation:** axions redshift like WIMPs, and linear density growth the same

$\Rightarrow$ *are axions different from WIMPs during non-linear structure formation ?*

- difficult dynamics : ask a N-body friend to write an axion code

- simple theory :free scalar (field and/or particles) coupled to gravity
  many interesting analytic proposals...vortices, caustics (Sikivie)... lots to do :)

We have some doubts about gravitational thermalisation in Sikivie's scenario. It remains to be shown that the gravitational interaction rate of axions is a thermalisation rate, or how it changes the axion distribution

$*$ leading order classical equations (no entropy generation?) for axion field in perturbed FRW reproduce the gravitational interaction rate Sikivie identifies as a thermalisation rate. Maybe its the gravitational interactions growing the density fluctuations?

$*$ We found some dissipative gravitational interactions, but suppressed by p_a^2/m_a^2...

Questions

1. Does a classical field form a galaxy differently from WIMPs?

2. Recall the two axion contributions to the DM density (field and particles)...can gravity move axions back and forth?

   - at what rate during which epochs does gravity condense axion particles into the classical field?
   - at what rate during which epochs does gravity evaporate the field into particles?

What we did

arXiv:1307.8024, with Martin Elmer

What does gravity do with the axion field?

consider early evolution of the Universe, until $\delta\rho \sim \rho$
Eqns of motion inside the horizon thermalisation is causal, so neglecting H^2/m_a^2,... for axion field in perturbed FRW (Newtonian gauge, $\phi =$ Newtonian potential comes from metric) can be obtained from

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad , \qquad \nabla^2\phi = 4\pi G_N \delta\rho$$

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1. other people get Eqns of motion for the axion field a:

$$\ddot{a} + 3H\dot{a} + k^2 a + m_a^2 a \sim \frac{Gm^2}{q^2} aaa$$

   ... non-linear...can obtain time evolution of number of axions of momentum q:

$$i\frac{\partial n_a(q)}{\partial t} \simeq 4\pi G m_a \sum_{\vec{k}} \frac{\delta\rho(k)}{k^2}\{n_a\}$$

   gives rate for axions to emit a graviton of any wavelength.

Interpretation of Sikivie : the rate to emit gravitons is a thermalisation rate.

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1. other people get Eqns of motion for the axion field (or its number density):
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non-linear... calculate rate for axions to emit a graviton of any wavelength.

2. Or (what we did), get Eqns of motion for a fluid with scalar perturbations
$$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta + \frac{c_s^2}{R^2(t)}\nabla^2\delta = 0$$
can solve (in fourier space); gives evolution of axion density fluctuations.

Interpretation: 1. and 2. describe the same physics. The gravitational interaction rate of 1. includes, or is, the growth if inhomogeneities given by 2.

$\Rightarrow$ Could part of the gravitational interactions be thermalising the axions? But this needs a bath? Or fluctuations to sum?? Can we find some entropy?

Looking for dissipation in the gravitational interactions of axions

1. Assume BE condensation requires dissipation

2. Assume no dissipation/thermalisation at leading order of classical equations of motion usual non-equilibrium field theory — must sum a bath of fluctuations to dissipate with t-reversal invariant eqns

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3. Look for gravitational interactions of axions that are neglected in obtaining expanding U with inhomogeneity growth:
   T_{ij}, $i \neq j$, is gauge invariant, of $\mathcal{O}(|\vec{p}|^2/m_a^2)$, and neglected in equations for density fluctuations.

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4. match axion in perturbed U onto imperfect fluid in FRW:

$$T^i{}_j(\vec{x},t) = -\frac{(1+2\phi)}{R^2(t)}\partial_i\phi\partial_j\phi \quad = \quad -\eta(t)(\partial_j U^i(\vec{x},t) + \partial^i U_j(\vec{x},t))$$

   $\eta =$ viscosity, $U = 4-$velocity. An imperfect fluid can grow density fluctuations, but contains dissipation...

5. estimate a dissipation scale:
   $<$ the Jeans length $1/\sqrt{m_a H}$, distance below which fluctuations oscillate due to axion pressure

Backup

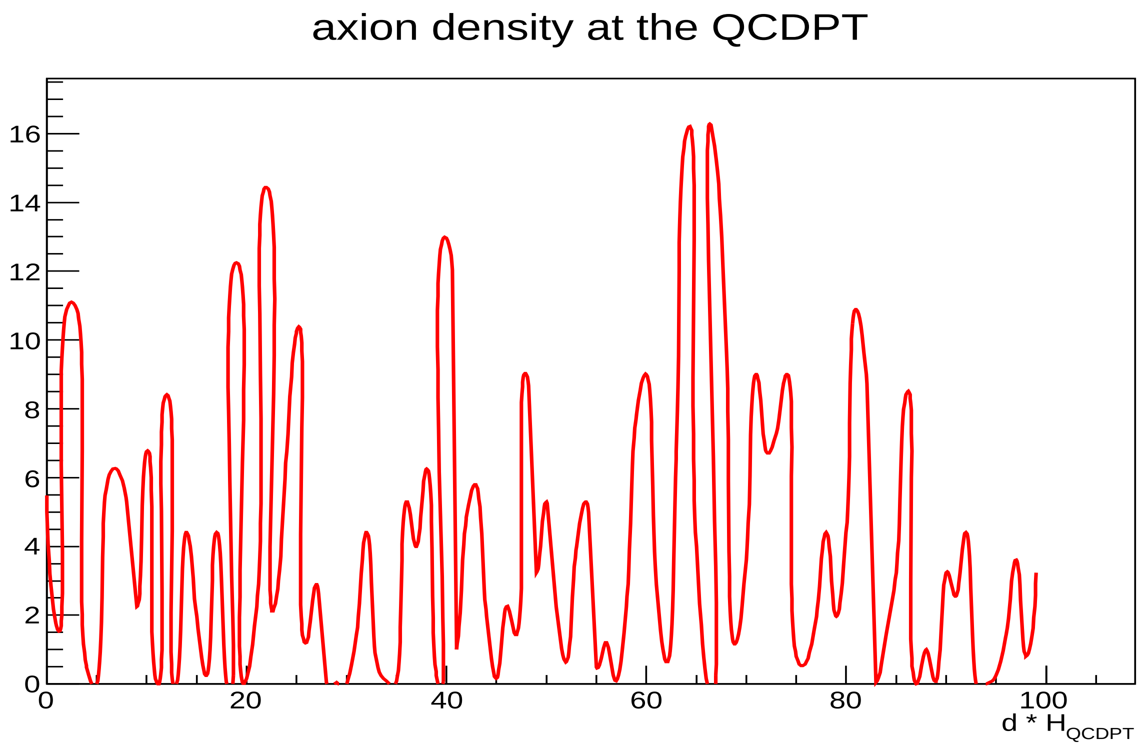
What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

1. in equilibrium stat mech: bosons pile into the $\vec{p} = 0$ mode
2. in equilibrium Finite Temp FT: a phase transition $\leftrightarrow$ form a vev
   store a density of conserved charge in a homogeneous $+$ isotropic classical field
3. for alkali gases in atomic traps: coherent collective behaviour (all the same $\vec{p}$; but not necc $\vec{p} = 0$)
4. Sikivie says: lowest energy state (not necc homogeneous)pragmatically, it needs to support vortices?

Is a BE condensate just a (non-relativistic) charge-carrying classical field? Or as well as being "coherent", does it need to be homogeneous + isotropic, ie, the $\vec{p} = 0$ *mode?*

Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale

$a(\vec{x}, t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x}, t) \simeq m_a^2 a^2(\vec{x}, t)$



$\Rightarrow$ its *not* a spatially homogeneous distribution of particles various momenta

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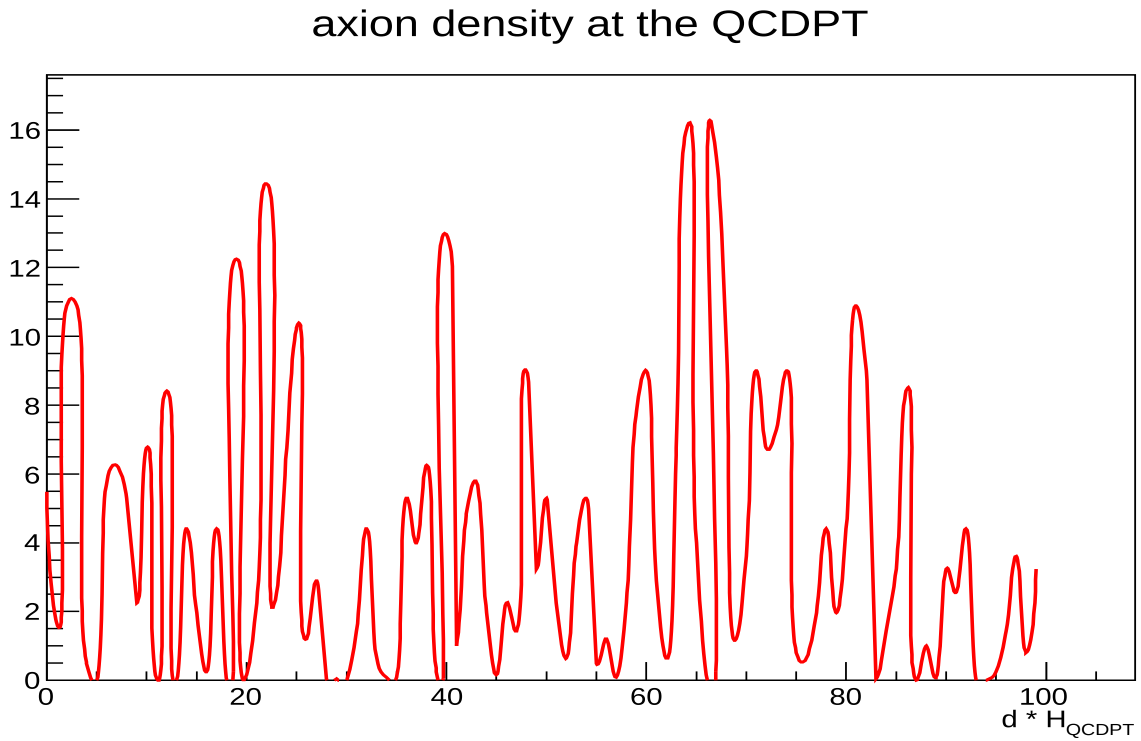

But how can axions form a *homogeneous-on-QCD-horizon-scale* bose-einstein condensate = zero mode of field? ??

$v = H_{QCDPT}/m_a \lesssim 10^{-6}c$...not "free-stream" QCD-horizon distance before t_{eq}:

$$d(t) = \int^t \frac{H_{QCDPT}}{m_a R(t')} dt' \sim \frac{H_{QCDPT}}{m_a} \frac{1}{H(t)R(t)} = \frac{R(t)}{m_a} \ll \frac{R(t)}{H_{QCDPT}}$$

(RD U, $R(t)$ =1@QCDPT)

The (beautiful) calculation of Saikawa and Yamaguchi

Suppose PQ PT after inflation. The classical axion field can be represented as a coherent state of axion particles (of momentum $\lesssim H_{QCDPT}$).
QFT rate for axions (momentum $\vec{k}$) to emit gravitons:

$$i\frac{\partial \hat{n}_k}{\partial t} = \left[\hat{H}_{int}, \hat{n}_k\right] \simeq \frac{G_E}{H(t)^2}\rho_a^2 \gg H(t)n_k$$

Saikawa+Yamaguchi

(evaluated in coherent state ⇔ classical field caln.)

Sikivie interprets as gravitational thermalisation rate: hugely occupied low-$\vec{p}$ modes, equilibrium after $T_\gamma \lesssim$ keV, → BE condensate.

But are some of those gravitons expanding the U, and some growing fluctuations?

Why is that a thermalisation rate??

thermalisation in closed unitary systems?

$$\text{entropy} = \sum_{states\ s} P_s \ln P_s \quad \text{increases}$$

- unitary evolution creates no entropy $\Leftrightarrow$ NO entropy generation in closed systems ... BUT... can calculate "effective" thermalisation: a subset of observables evolve towards equilibrium expectations
  $\Rightarrow$ the "rest" of the system is the bath??

- ex: couple two SHOs. Solve one, substitute into Eqns of second, and find dissipation.

- ...$K - \bar{K}$ evolution is non-unitatry, because not also follow 2π 3π states...

? $\Rightarrow$ divide axions+gravity into

1. U expansion + structure growth
2. other fluctuations which are the bath?

gravity and the second law

1. undergraduate memories say that gravitational collapse of a gas cloud to a star respects the second law...

2. story of $\Omega_{baryon} = 1$ U

   (a) quasi-homogeneous dust clouds collapse
   (b) ...generations of stars, supernovae, black holes...
   (c) ... ... proton decays...
   (d) venerable homogeneous and isotropic U full of photons and gravitons

3. so gravitational thermalisation of axions will happen.
   But does it happen before the U a year old?

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \Big\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \Big\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}} |a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \Big\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \Big\}$$

Brodsky+Heurer,Donoghue etal
Olive+Montonen...I+Z,C-T...

What is quantum?

Classical = saddle-point configurations of the path integral
$\Rightarrow$ attribute dimensions to fields/parameters $\ni$ [action]= E*t, and no $\hbar$ in selected classical limit (this is *not* unique)

Summary: particles or fields can be obtained in a "classical" (= no $\hbar$) limit. However, $\hbar$ is differently distributed in the Lagrangian in the two limits, so to get from one to another requires $\hbar$...

in particular, to define a number of quanta, in the field picture, requires $\hbar$.

ex 1: massive scalar electrodynamics

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \tilde{m}^2 \phi^\dagger \phi - \frac{1}{4} F F \qquad , \quad D_\mu = \partial_\mu - i\tilde{e} A_\mu$$

Classical field limit: $[\phi, A] = \sqrt{E/L}$, $[m] = 1/L$, $[\tilde{e}] = 1/\sqrt{EL}$.

No $\hbar$ in classical EoM. OK that $[m^2] = 1/L^2$ because gravity couples is the stress-energy tensor, function of the fields.

If in Maxwells Eqns, want $j^0 = i\tilde{e}(\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi})$ to be eN/V, then need number of charge-carrying quanta $\Rightarrow e = \tilde{e}\hbar$.

De même, if classically m a particle mass, need $m = \tilde{m}\hbar$.

ex 2: the SHO Hamiltonian is (no $\hbar$)

$$H = \frac{1}{2m} P^2 + \frac{m\nu^2}{2} X^2$$

where ν is the oscillator frequency.

But to $quantise$, = introduce creation and annihilation ops, requires $\hbar$.
To write the total energy as $\omega(N + 1/2)$, requires $\hbar$ to convert frequency to energy $\omega = \hbar\nu$, and downstairs in the defn of N, because its the number of $quanta$.

Erken,Sikivie,Tam,Yang

Sikivie's scenario—some questions and guesses

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a Bose-Einstein Condensate

Saikawa Yamaguchi

- which axions (field or particles) ?
  ESTY, S+Y consider the axion field
- what is grav thermalisation?
  ESTY,S+Y: grav interactions of axions
  SD+ME: entropy-producing grav interactions
- what is a BEC? /when does a classical field support vortices?
  DRS:classical field supports vortices for ranges of mass, coupling, not including axion.

2. BEC can support vortices, which allow caustics in the galactic DM distribution $\Leftrightarrow$ axion DM signature?

BEC galactic halos:
Rindler-Daller+Shapiro

- how do galaxies form in a "BEC" (?does it stay a "BEC"?)?
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