The Dynamics of strings with junctions

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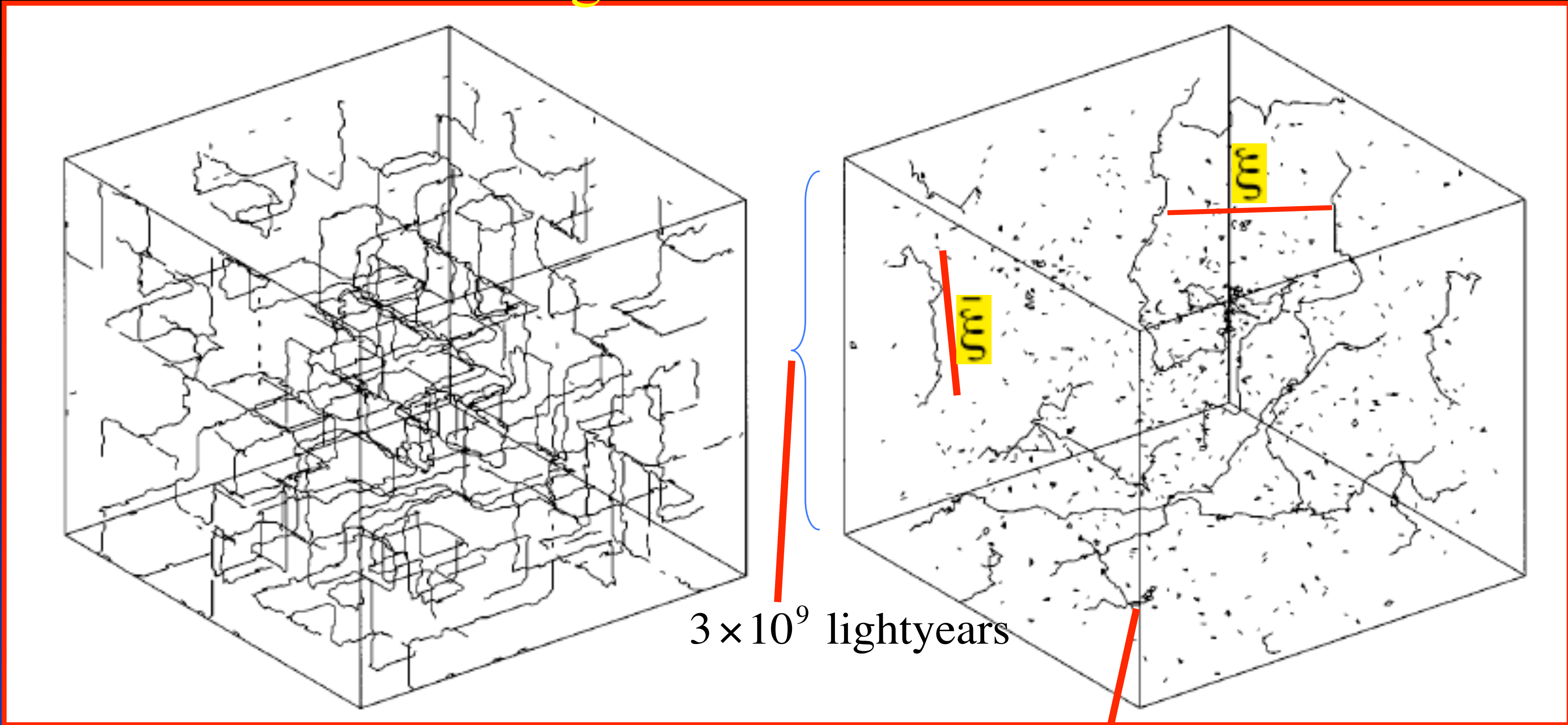
1. Why cosmic superstrings
2. Modelling strings with junctions.
3. Potential observational properties
4. Some details of strings with junctions.

Strings versus cosmology

Madrid - Nov 16 2006

Length scales on networks [Vincent et al]



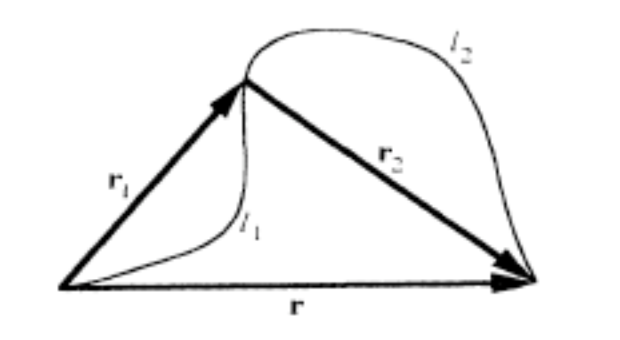
Initial

- $\bar{\xi}$ - persistence length of string
- ξ - interstring distance

Scaling

- ζ - small scale structure on network

Analytic modelling of networks [Kibble + many authors]



Approach: take random segment of string of length l and extension r. Write down evolution equations for the probability distribution p[r(l)] due to physical processes.

Probability:

$$\frac{\partial p}{\partial t} = \left(\frac{\partial p}{\partial t}\right)_{\rm str} + \left(\frac{\partial p}{\partial t}\right)_{\rm GR} + \left(\frac{\partial p}{\partial t}\right)_{\rm LSI} + \left(\frac{\partial p}{\partial t}\right)_{\rm loops},$$

Total length:

$$\frac{\partial L}{\partial t} = \left(\frac{\partial L}{\partial t}\right)_{\rm str} + \left(\frac{\partial L}{\partial t}\right)_{\rm GR} + \left(\frac{\partial L}{\partial t}\right)_{\rm loops}.$$

Gaussian ansatz:

$$p[\mathbf{r}(l)] = \left(\frac{3}{2\pi K(l)}\right)^{3/2} \exp\left(-\frac{3}{2}\frac{\mathbf{r}^2}{K(l)}\right).$$

Defns of length scales:

$$K(l,t) \sim 2\bar{\xi}(t)l, \qquad l \gg t,$$

$$\xi^2 = \frac{V}{L}.$$

$$K \approx l^2 - \frac{l^3}{3\zeta}.$$

Brownian

l << t

Evolution equations -- simplified ignoring expansion

$$\frac{\dot{\xi}}{\xi} = \frac{c}{2\,\bar{\xi}},$$

$$\frac{\dot{\bar{\xi}}}{\bar{\xi}} = \frac{-\chi\,\bar{\xi}}{\omega\xi^2} + \frac{I}{2\,\bar{\xi}},$$

$$\frac{\dot{\zeta}}{\zeta} = \frac{-\chi\zeta}{\xi^2} + \frac{kc}{\bar{\xi}}.$$

c,I -- related to loop production

χ-- related to intercommuting prob

k - related to removing small scales

Scaling solutions: lengths scale with H^{-1}

$x = \xi/\eta$, $\bar{x} = \bar{\xi}/\eta$, and $z = \zeta/\eta$

$$\bar{x}_* = \frac{c}{2},$$

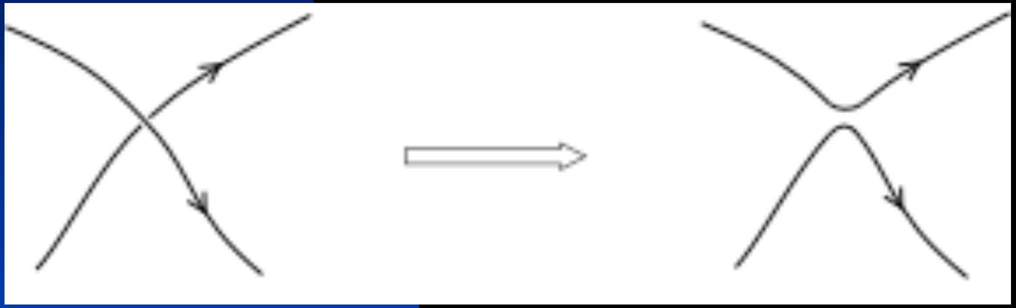
$$x_* = \sqrt{\frac{\chi c^2}{2\,\omega(I-c)}},$$

$$z_* = \begin{cases} (2k-1)x_*^2/\chi & \text{if } 2k-1>0, \\ 0 & \text{if } 2k-1\leq 0. \end{cases}$$

Note: formalism can in principle determine the contribution of loops to scaling solutions -- a source of recent debate.

Observational consequences : 1980's and 90's

Single string networks evolve with Nambu-Goto action, decaying primarily by forming loops through intercommutaion and emitting gravitational (or particle) radiation



For gauge strings, reconnection probability P~1

Scaling solutions are reached where energy density in long strings reaches constant fraction of background energy density:

$\rho_{string}/\rho_{rad} \sim 400G\mu$

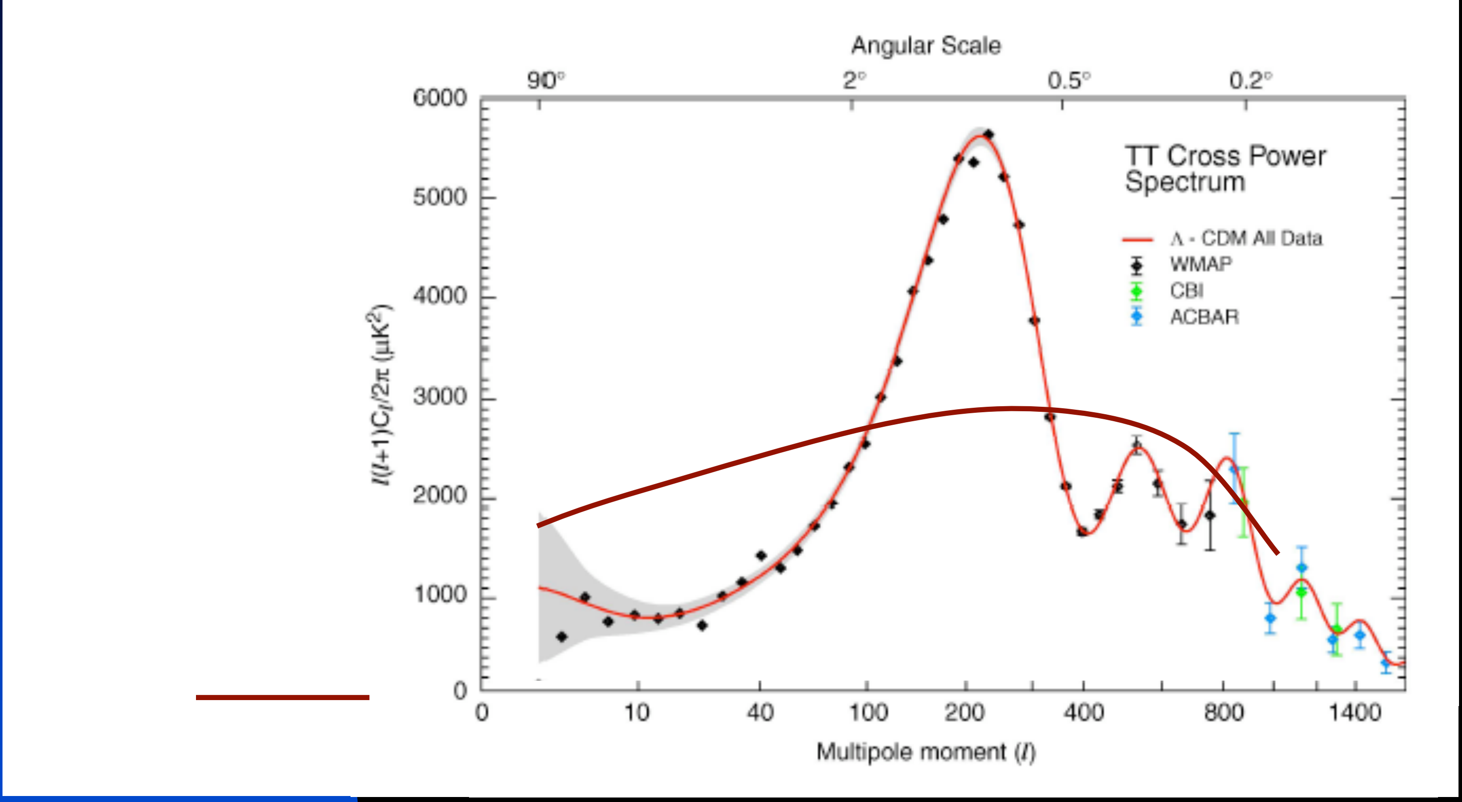
$\rho_{string}/\rho_{mat} \sim 60G\mu$

[Albrecht &Turok, Bennett & Bouchet, Allen & Shellard]

Density increases as P decreases because takes longer for network to lose energy to loops.

Unfortunately they didn't do the full job!

CMB power spectrum



Acoustic peaks come from temporal coherence. Inflation has it, strings don't. String contribution < 13% implies $G\mu < 10^{-6}$.

E.g. Pogosian et al 2004, Bevis et al 2004.

Pulsar bounds on gravitational wave emission also problematic for GUT scale strings:

Strings produce stochastic GW, $\Omega_{GW} \sim 10^{-1.5}\, G\mu$.
(Allen '95, Battye, Caldwell, Shellard '97)

Kaspi, Taylor, Ryba '94: $\Omega_{GW} < 1.2 \times 10^{-7}$, $G\mu < 10^{-5.5}$

Lommen, Backer '01: $\Omega_{GW} < 4 \times 10^{-9}$, $G\mu < 10^{-7}$

In relevant frequency range ~ 0.1 inverse year

Might need to reduce string tension

In 1980's Fundamental (F) strings excluded as being cosmic strings [Witten 85]:

1. F string tension close to Planck scale (e.g. Heterotic)

$$G\mu = \frac{\alpha_{GUT}}{16\pi} \geq 10^{-3}$$

Cosmic strings deflect light, hence constrained by CMB:

$$G\mu \propto \frac{\delta T}{T} \leq 10^{-6}$$

Consequently, cosmic strings had to be magnetic or electric flux tubes arising in low energy theory

2. Why no F strings of cosmic length?

a. Diluted by any period of inflation as with all defects.

b. They decay ! (Witten 85)

1990's: along came branes --> new one dimensional objects:

1. Still have F strings
2. D-strings
3. Higher dimensional D-, NS-, M- branes partly wrapped on compact cycles with only one non-compact dimension left.
4. Large compact dimensions and large warp factors allow for much lower string tensions.
5. Dualities relate strings and flux tubes, so can consider them as same object in different regions of parameter space.

What do they imply for cosmic strings?

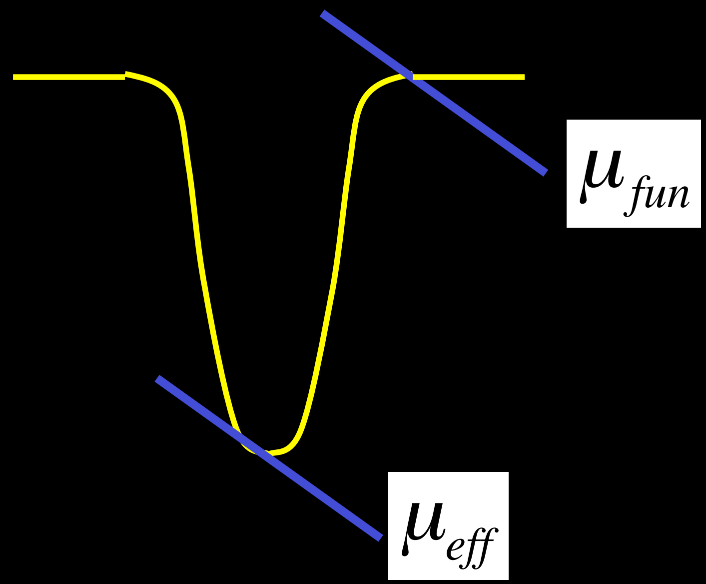
Ex: String tension reduced in "exotic" compactifications:

warped compactifications: tension is redshifted by internal warp factors

$$ds^2 = e^{2A(y)}\left(\eta_{\mu\nu}dx^\mu dx^\nu\right) + ds_\perp^2(y)$$

$$UV: \quad e^{2A} \approx 1$$

$$IR: \quad e^{2A} << 1$$



$$\mu_{eff} = \frac{e^{2A(IR)}}{e^{2A(UV)}}\mu_{fun} << \mu_{fun}$$

Strings surviving inflation:

D-brane-antibrane inflation leads to formation of D1 branes in non-compact space [**Burgess et al; Jones, Sarangi &Tye; Stoica & Tye**]

Form strings, not domain walls or monopoles.

$$10^{-11} \leq G\mu \leq 10^{-6}$$

In general for cosmic strings to be cosmologically interesting today we require that they are not too massive (from CMB constraints), are produced after inflation (or survive inflation) and are stable enough to survive until today [**Dvali and Vilenkin (2004); EJC,Myers and Polchinski (2004)**].

What sort of strings? Expect strings in non-compact dimensions where reheating will occur: F1-brane (fundamental IIB string) and D1 brane localised in throat.
[Jones,Stoica & Tye, Dvali & Vilenkin]

D1 branes - defects in tachyon field describing D3-anti D3 annihilation, so produced by Kibble mechanism.

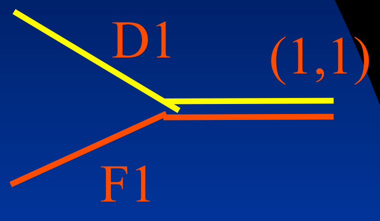
Strings created at end of inflation at bottom of inflationary throat. Remain there because of deep pot well. Eff 4d tensions can be reduced because they depend on warping and 10d tension $\bar{\mu}$

$$\mu = e^{2A(x_\perp)}\bar{\mu}$$

Depending on the model considered these strings can be metastable, with an age comparable to age of the universe

F1-branes and D1-branes --> also (p,q) strings for relatively prime integers p and q. [Harvey & Strominger; Schwarz]

Interpreted as bound states of p F1-branes and q D1-branes [Polchinski;Witten]



Tension in Minkowski 10d theory:

$$\bar{\mu}_{p,q} = \frac{1}{2\pi\alpha'}\sqrt{(p - Cq)^2 + e^{-2\Phi}q^2}$$

C- RR scalar, Φ - Dilaton -- evaluated at string. Fixed in terms of 3 form fluxes in model.

Tension in KLMT

$$\frac{G^2 e^{4A_0}}{(2\pi\alpha')^2 g_s} = \frac{\delta_H^3}{32\pi C_1^3 N_e^{5/2}}$$

Using: $\delta_H = 1.9 \times 10^{-5}, C_1 = 0.39, N_e = 60$

$$\frac{G^2 e^{4A_0}}{(2\pi\alpha')^2 g_s} = \frac{\delta_H^3}{32\pi C_1^3 N_e^{5/2}}$$

LHS: product of $G\mu$ for F and D string.

$$\delta_H = 1.9 \times 10^{-5}, C_1 = 0.39, N_e = 60$$

Find: $$\sqrt{G\mu_F \, G\mu_D} \sim 2 \times 10^{-10}, \qquad \frac{\mu_D}{\mu_F} = \frac{1}{g_s}$$

For $0.1 < g_s < 1$ have $G\mu \sim 10^{-9} - 10^{-10}$

Note: assumes all perturbations from inflation here.

Distinguishing cosmic superstrings

1. Intercommuting probability for gauged strings P~1 always ! In other words when two pieces of string cross each other, they reconnect. Not the case for superstrings -- model dependent probability.
2. Existence of new `defects' D-strings allows for existence of new hybrid networks of F and D strings which could have different scaling properties, and distinct observational effects.

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What are the probabilities for reconnection in this case?

Jackson, Jones and Polchinski [hep-th/0405229]

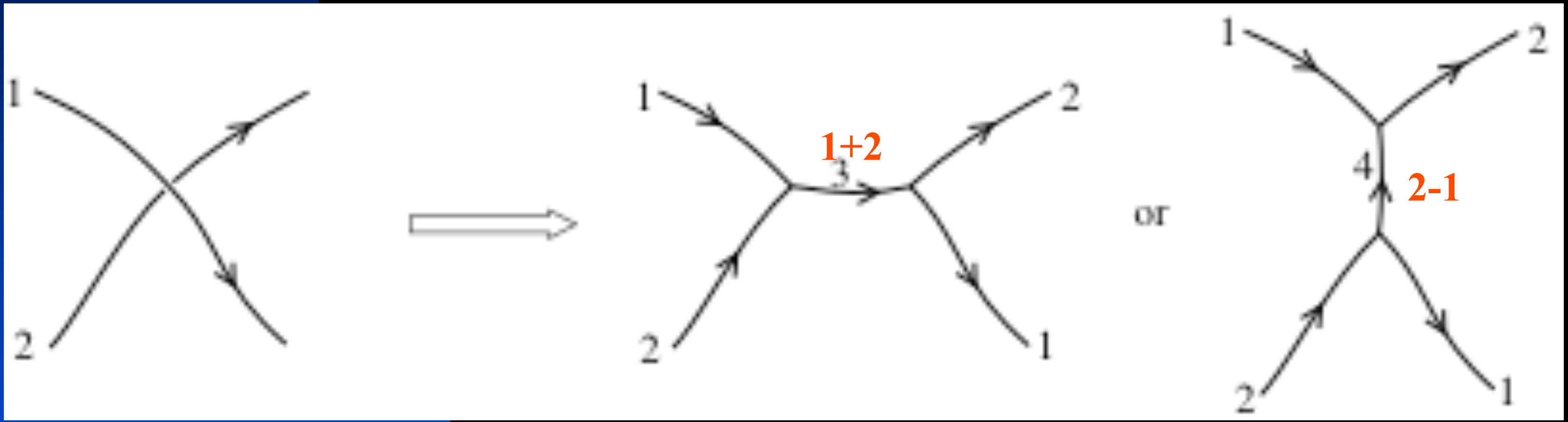
The results depend on the type of string, the string coupling, the details of the compactification

For example for F-F reconnection in KKLMMT depending on type of compactification obtain:

Summarise as $P_{FF} = 10^{-3} - 1$; $P_{DD} = 10^{-1} - 1$

(p,q) string networks -- exciting prospect.

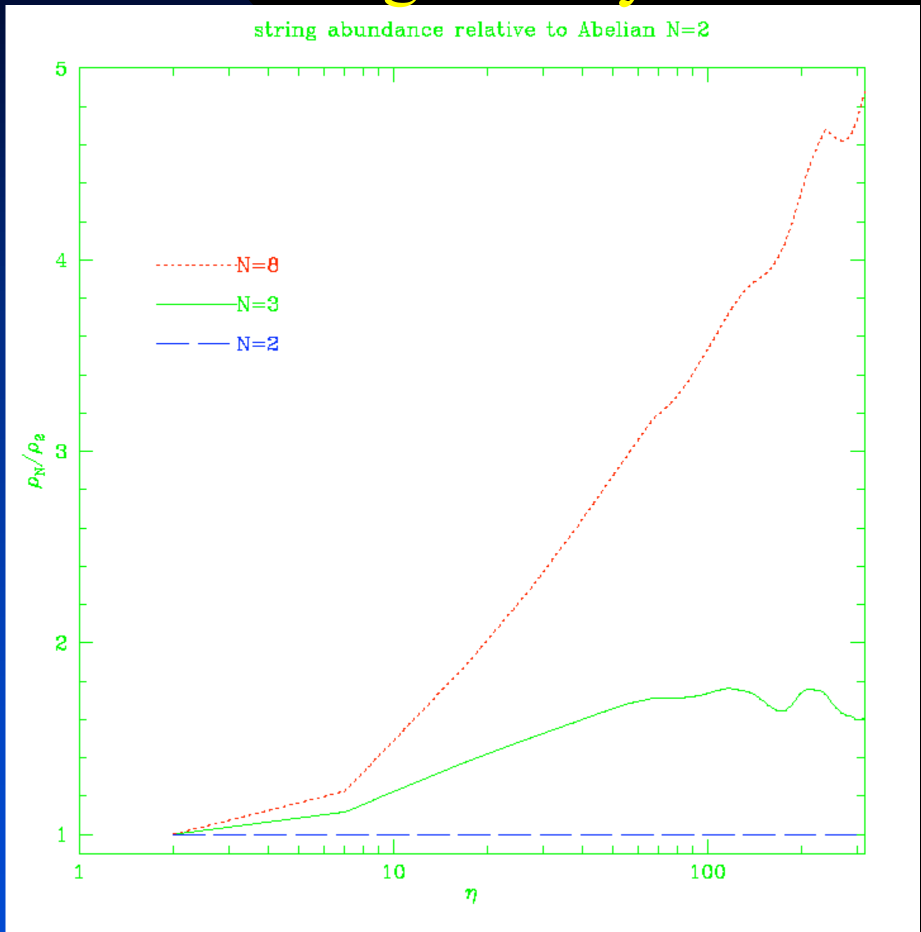
Two strings of different type cross, can not intercommute in general -- produce pair of trilinear vertices connected by segment of string.



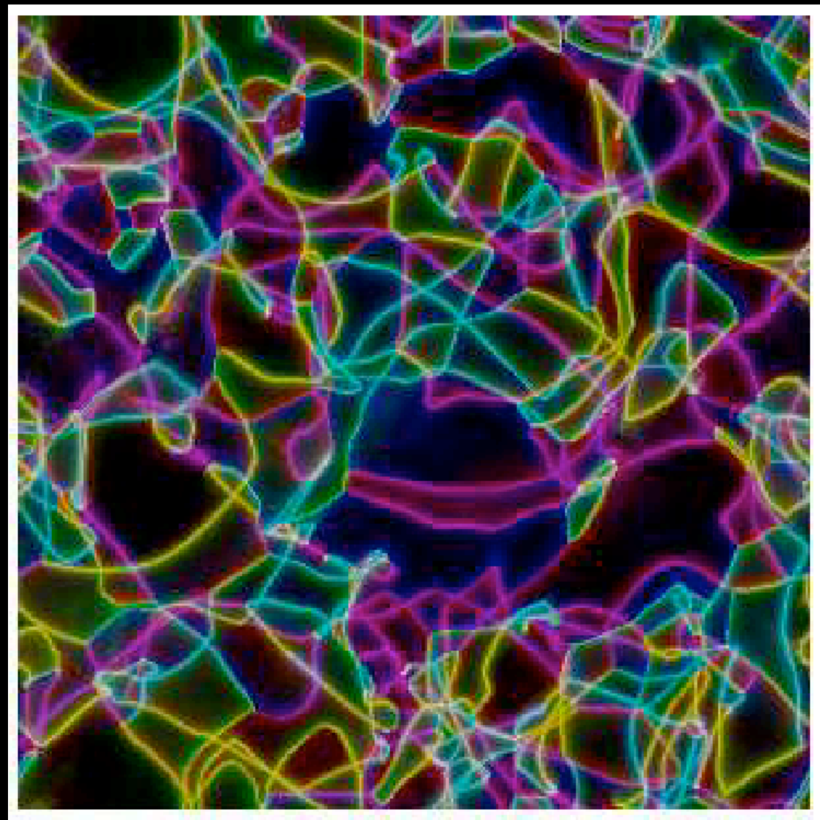
What happens to such a network in an expanding background? Does it scale or freeze out in a local minimum of its PE [Sen]?Then it could lead to a frustrated network scaling as w=-1/3 (p,q) string networks -- mimic with field theory. Under sym breaking G -->K (non-Abelian) find defects that do not intercommute.

K= S_3 and S_8 - [Spergel & Pen 96]

String density



N=3



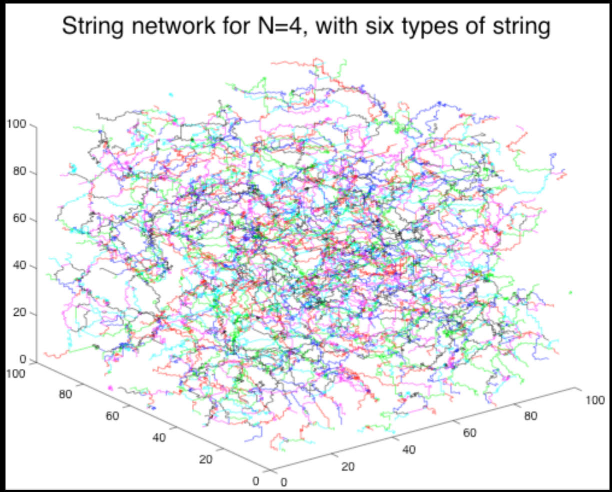
Enters scaling regime for N=3, no evidence for scaling for N=8. Not evolved for long enough?

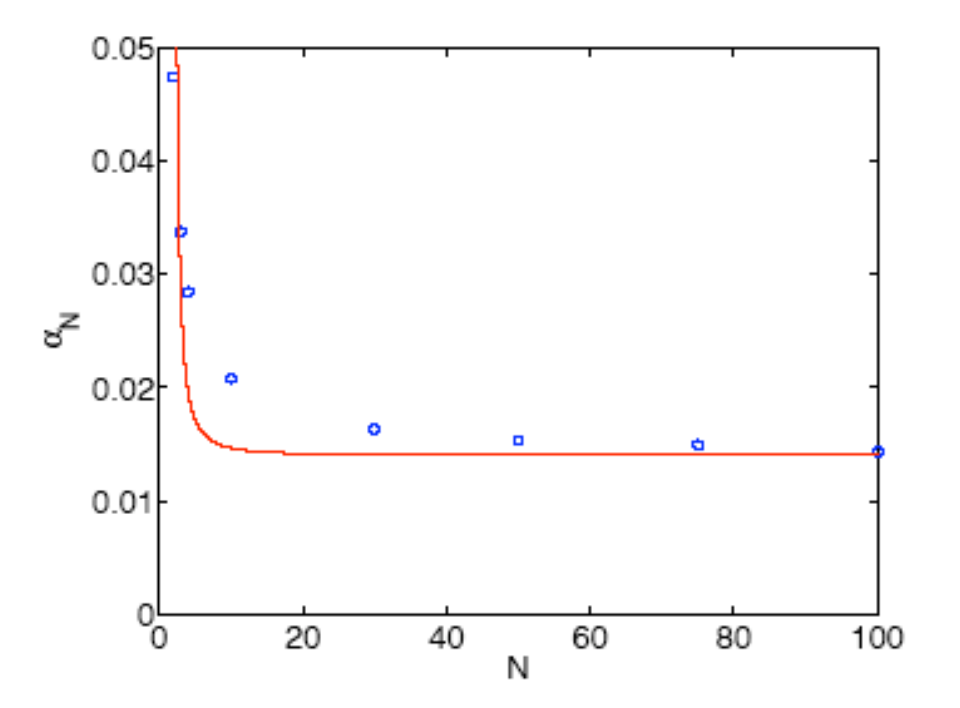
Modelling the case K= S_N

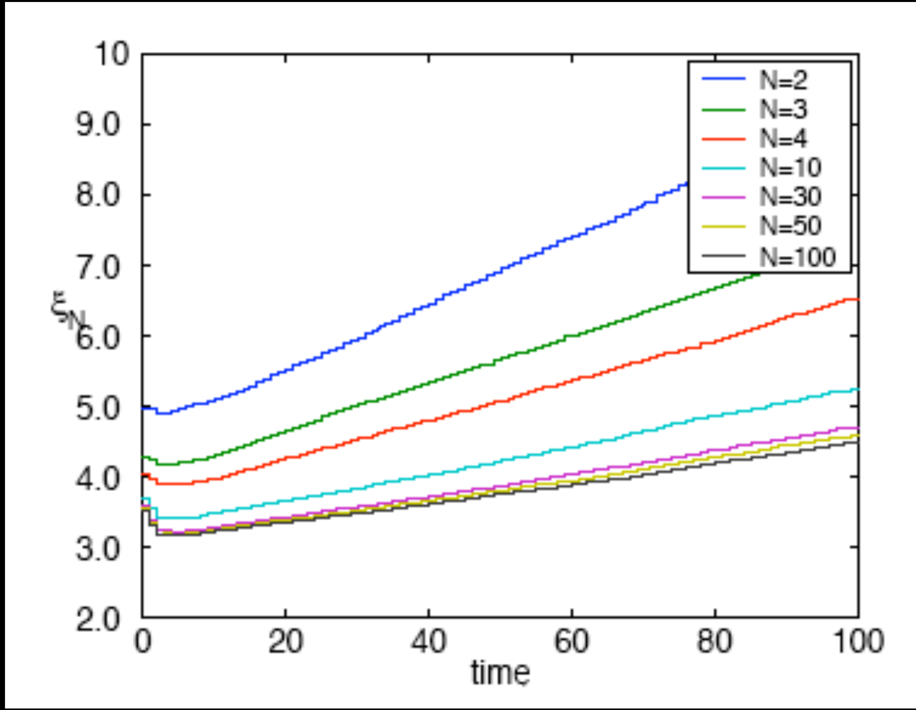
Numerically: Scaling solutions seem to exist for all N :

$\rho \sim \mu \xi^{-2}$

$\xi_N(t) = \xi_0(N) + \alpha_N\, t$

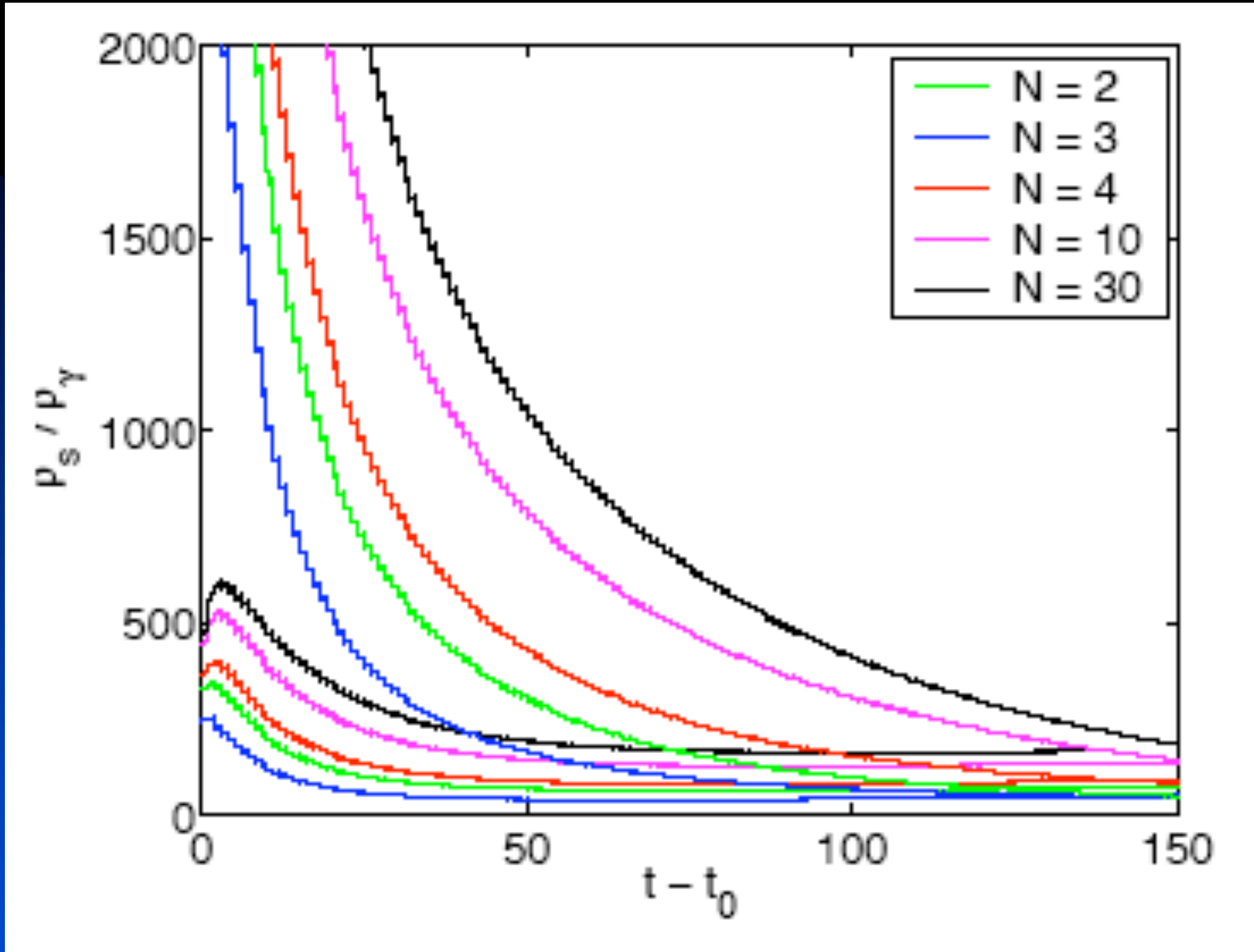






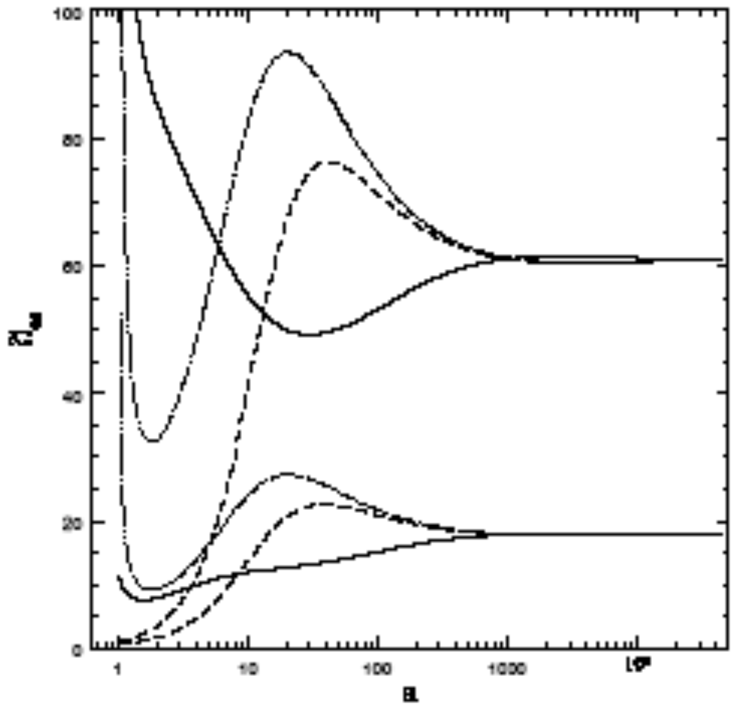
[EC and Saffin 05]

[Vachaspati and Vilenkin 87]



Scaling solutions in radiation as a function of N

Including multi-tension cosmic superstrings [Tye, Wasserman and Wyman 05].



Density of (p,q) cosmic strings.

Density of D1 strings.

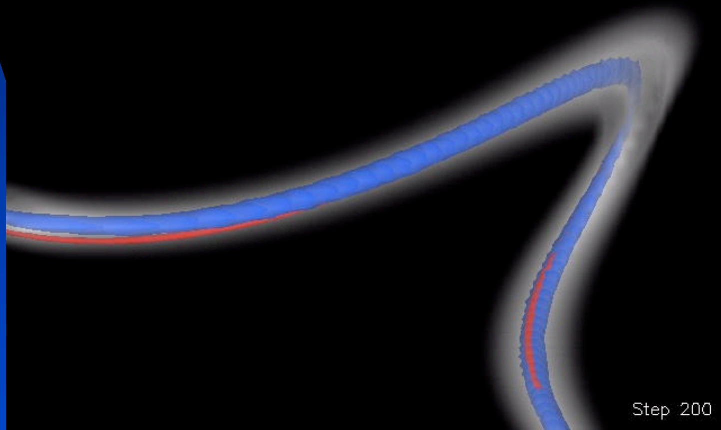
Scaling achieved indep of initial conditions, and indep of details of interactions.

Interesting feature: If turn off loop production, still reach scaling. Claim energy is lost through string binding and binding mediated annihilation.

Any smoking guns?

Possibly through strong non-gaussian nature of stochastic gravitational wave emission from loops which contain kinks and cusps. [Damour & Vilenkin 01 and 04]

[Blanco-Pillado]



Cusp: x’=0 for instant in an oscillation

Kink: x’ discontinuous, occurs every intercommuting -- common

Both produce beams of GW, cusps much more powerful

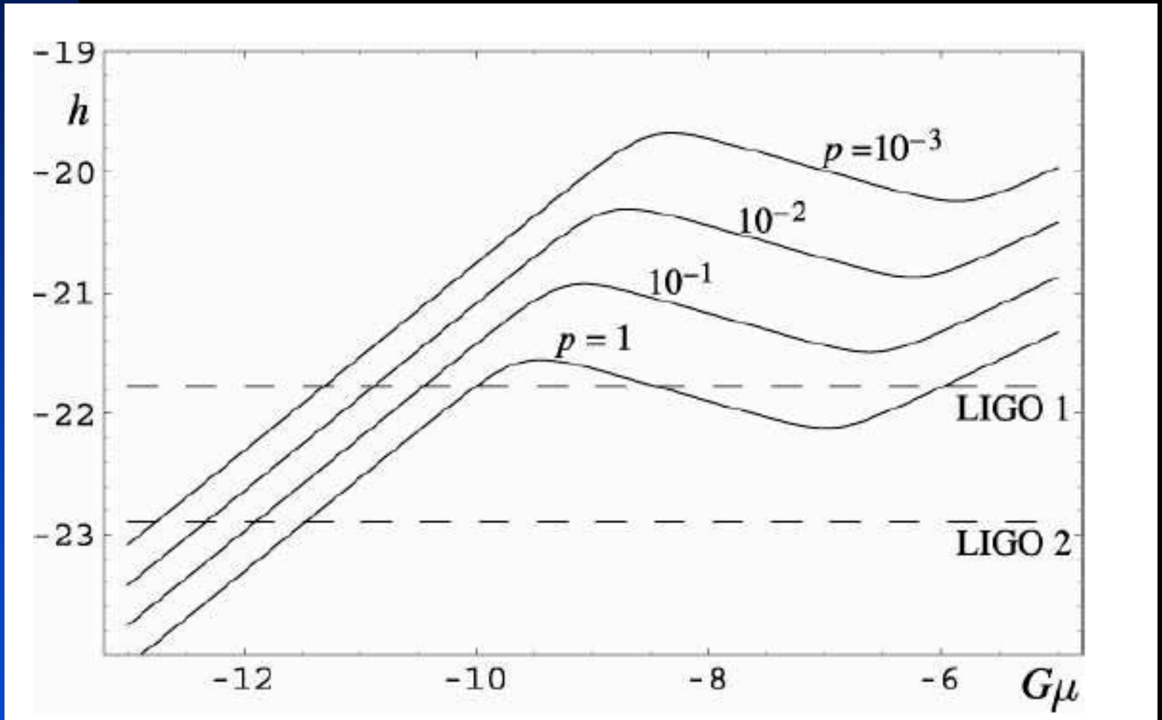
In loop network, if only 10% of loops have cusps, bursts of GW above `confusion' GW noise could be detected by LIGO and LISA for $G\mu \sim 10^{-12}$!

log$_{10}$h

strain

LIGO I

LIGO II



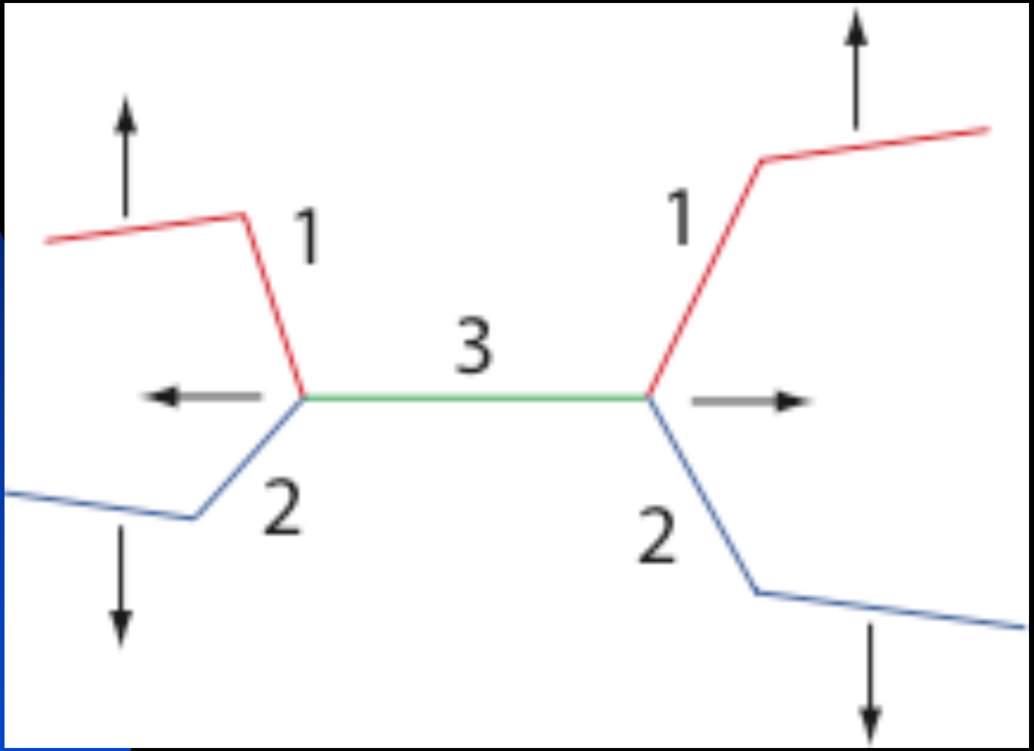
[Damour & Vilenkin 04]

Noise levels

Bursts emitted by cusps in LIGO frequency range f_{ligo}=150 Hz

New approach to strings with junctions -- solve the modified Nambu-Goto equations

EJC, Kibble and Steer: hep-th/0601153 (PRL 2006)



Need to account for the fact that there is a constraint -- three strings meet at a junction and evolve with that junction.

Nambu–Goto dynamics

Dynamics of relativistic string: action = area of world sheet

$$S = -\mu \int d\tau\, d\sigma \sqrt{(\dot{x}\cdot x')^2 - \dot{x}^2 x'^2}$$

with $\dot{x} = \partial_\tau x,\ x' = \partial_\sigma x$ $\mu =$ string tension

Gauge conditions: $\dot{x}^2 + x'^2 = 0,\ \dot{x}\cdot x' = 0$

(conformal gauge) and $\tau = t = x^0(\sigma,\tau),$

$\Rightarrow x(\sigma,t) = (t, \mathbf{x}(\sigma,t)),$ $\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1,$

Nambu–Goto action $\Rightarrow S = -\mu \int dt\, d\sigma \sqrt{(1-\dot{\mathbf{x}}^2)\mathbf{x}'^2}$

Equation of motion $\ddot{\mathbf{x}} - \mathbf{x}'' = \mathbf{0}$

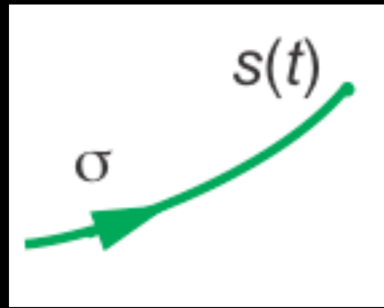
General solution $\mathbf{x}(\sigma,t) = \frac{1}{2}[\mathbf{a}(\sigma+t) + \mathbf{b}(\sigma-t)]$

where $\mathbf{a}'^2 = \mathbf{b}'^2 = 1$

Useful to recall Open strings

For string with free end at s(t),

$$S = -\mu \int dt\, d\sigma\, \theta(s(t) - \sigma)\sqrt{(1 - \dot{\mathbf{x}}^2)\mathbf{x}'^2}$$



Varying $\mathbf{x} \;\Rightarrow\; \ddot{\mathbf{x}} - \mathbf{x}'' = \mathbf{0},$

boundary terms $\Rightarrow\; \mathbf{x}' + \dot{s}\dot{\mathbf{x}} = \mathbf{0}$ at $(s(t), t)$

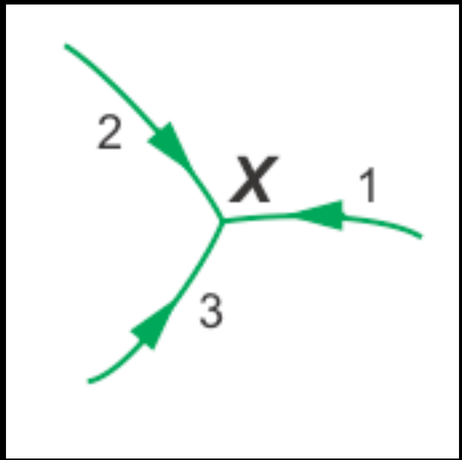
But $\mathbf{x}' \perp \dot{\mathbf{x}}$ so $\mathbf{x}' = \mathbf{0}$ and $\dot{s} = 0,\ |\dot{\mathbf{x}}| = 1$

$$\Rightarrow\; \mathbf{a}'(s + t) + \mathbf{b}'(s - t) = \mathbf{0}$$

If choose *s* = 0, then can take $\mathbf{a}(u) = \mathbf{b}(-u)$

Equations of motion for junction

Take σ on each leg j to increase towards the vertex, position $\boldsymbol{X}(t)$



$$S = -\sum_j \mu_j \int dt\, d\sigma\, \theta(s_j(t) - \sigma)\sqrt{\boldsymbol{x}_j'^2 (1 - \dot{\boldsymbol{x}}_j^2)}$$

$$+ \sum_j \int dt\, \boldsymbol{f}_j(t) \cdot [\boldsymbol{x}_j(s_j(t), t) - \boldsymbol{X}(t)]$$

Varying $\boldsymbol{x}_j \Rightarrow \ddot{\boldsymbol{x}}_j - \boldsymbol{x}_j'' = \mathbf{0},$

boundary terms $\Rightarrow \mu_j(\boldsymbol{x}_j' + \dot{s}_j \dot{\boldsymbol{x}}_j) = \boldsymbol{f}_j$ at $(s_j(t), t)$

Varying $\boldsymbol{X} \Rightarrow \sum_j \boldsymbol{f}_j = \mathbf{0}$ Varying $\boldsymbol{f}_j \Rightarrow \boldsymbol{x}_j(s_j(t), t) = \boldsymbol{X}(t)$

Varying $s_j \Rightarrow \boldsymbol{f}_j \cdot \boldsymbol{x}_j' = \boldsymbol{x}_j'^2$ (not independent of other eqns)

Obtain General solution

$$\boldsymbol{x}_j(\sigma,t) = \tfrac{1}{2}[\boldsymbol{a}_j(\sigma+t) + \boldsymbol{b}_j(\sigma-t)]$$ with $$\boldsymbol{a}_j'^2 = \boldsymbol{b}_j'^2 = 1$$

$$\boldsymbol{x}_j(s_j(t),t) = \boldsymbol{X}(t) \;\Rightarrow\; \boldsymbol{a}_j(s_j+t) + \boldsymbol{b}_j(s_j-t) = 2\boldsymbol{X}(t)$$

$$\sum_j \boldsymbol{f}_j = \boldsymbol{0} \;\Rightarrow\; \sum_j \mu_j[(1+\dot{s}_j)\boldsymbol{a}_j' + (1-\dot{s}_j)\boldsymbol{b}_j'] = \boldsymbol{0}$$

Initial conditions at $t=0 \Rightarrow$ values of $\boldsymbol{a}_j'(\sigma)$ and $\boldsymbol{b}_j'(\sigma)$ for $\sigma < s_j(0)$

So for $t > 0$, values of $\boldsymbol{b}_j'(s_j(t)-t)$ (ingoing wave) are known, but not those of $\boldsymbol{a}_j'(s_j(t)+t)$ (outgoing wave)

So use $(1+\dot{s}_j)\boldsymbol{a}_j' - (1-\dot{s}_j)\boldsymbol{b}_j' = 2\dot{\boldsymbol{X}}$ to eliminate $\boldsymbol{a}_j'$

$$\Rightarrow \sum_j \mu_j(1-\dot{s}_j)\boldsymbol{b}_j' = -(\mu_1+\mu_2+\mu_3)\dot{\boldsymbol{X}}$$

General soln --> Motion of vertex

Motion of vertex given by $(\mu_1+\mu_2+\mu_3)\dot{\boldsymbol{X}} = -\sum_j \mu_j(1-\dot{s}_j)\boldsymbol{b}'_j$

and outgoing wave by $(1+\dot{s}_j)\boldsymbol{a}'_j = 2\dot{\boldsymbol{X}} + (1-\dot{s}_j)\boldsymbol{b}'_j$

as long as we can find $\dot{s}_j$ To do this, impose $\boldsymbol{a}'^2_j = 1$

$\Longrightarrow$ simultaneous eqs for $\dot{s}_j$ in terms of:

$$c_{ij} = \boldsymbol{b}'_i(s_i - t)\cdot\boldsymbol{b}'_j(s_j - t)$$

e.g. $$(\sum_j \mu_j)^2(1+\dot{s}_1)^2 = (\sum_j \mu_j)^2(1-\dot{s}_1)^2 - 4(\sum_j \mu_j)(1-\dot{s}_1)\sum_k \mu_k(1-\dot{s}_k)c_{1k} + 4\sum_{j,k}\mu_j\mu_k(1-\dot{s}_j)(1-\dot{s}_k)c_{jk}$$

or $$(\sum_j \mu_j)^2\dot{s}_1 = -(\sum_j \mu_j)(1-\dot{s}_1)\sum_k \mu_k(1-\dot{s}_k)c_{1k} + \sum_{j,k}\mu_j\mu_k(1-\dot{s}_j)(1-\dot{s}_k)c_{jk}$$

As a check, summing 3 eqs $\Longrightarrow \mu_1\dot{s}_1 + \mu_2\dot{s}_2 + \mu_3\dot{s}_3 = 0$ (gives energy conservation.)

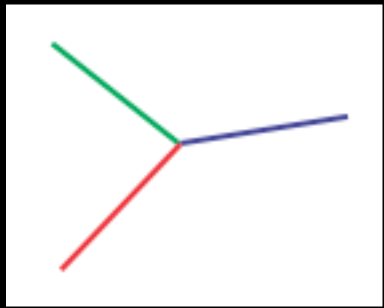
Hence eliminate $\dot{s}_3$ and solve for $\dot{s}_1$, $\dot{s}_2$

Final solution

Solve for $1-\dot{s}_j$ and define M_j

$M_1 = \mu_1^2 - (\mu_2 - \mu_3)^2$ etc. Then

$$\frac{\mu_1(1-\dot{s}_1)}{\mu_1+\mu_2+\mu_3} = \frac{M_1(1-c_{23})}{M_1(1-c_{23})+M_2(1-c_{31})+M_3(1-c_{12})}$$ etc.



Note: because $c_{ij} = \mathbf{b}_i'(s_i - t)\cdot\mathbf{b}_j'(s_j - t)$

these are **differential equations** for $s_j(t)$

Also since $\dot{s}_j < 1$ and $c_{ij} < 1$ all $M_j > 0$

i.e. μ_j satisfy **triangle inequalities** (obvious if $\dot{\mathbf{X}} = \mathbf{0}$)

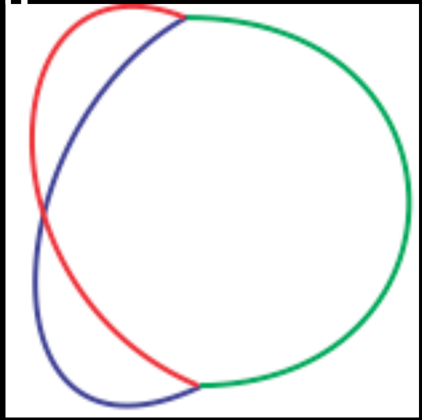
— e.g. if $\mu_3 > \mu_1 + \mu_2$ string 3 is unstable

Collapsing ring -- 3 semicircular arcs

Straight Static strings: $\boldsymbol{x}_j = \sigma \boldsymbol{e}_j, \quad \boldsymbol{e}_j = (\cos\theta_j, \sin\theta_j, 0)$

With eqm condition: $\sum_j \mu_j e^{i\theta_j} = 0$ (possible if Δ inequalities are satisfied)

Initially static loop configuration:



$$\boldsymbol{x}_j(\sigma,0) = (\cos\sigma\cos\theta_j, \cos\sigma\sin\theta_j, \sin\sigma)$$

for $-\pi/2 < \sigma < \pi/2$

$$\Rightarrow \boldsymbol{a}_j(\sigma) = \boldsymbol{b}_j(\sigma) = \boldsymbol{x}_j(\sigma,0)$$

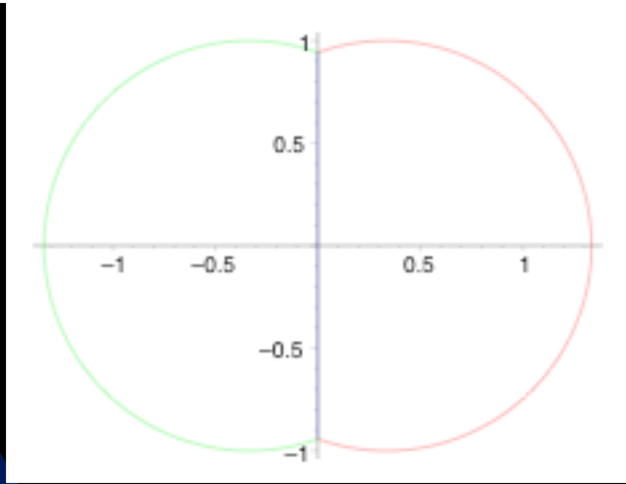
Find: $\dot{s}_j = 0 \Rightarrow s_j(t) = \pi/2$

$$\boldsymbol{x}_j(\sigma,t) = \cos t(\cos\sigma\cos\theta_j, \cos\sigma\sin\theta_j, \sin\sigma)$$

Semicircular rings all collapse to O at $t = \pi/2$

Heavy wings

1



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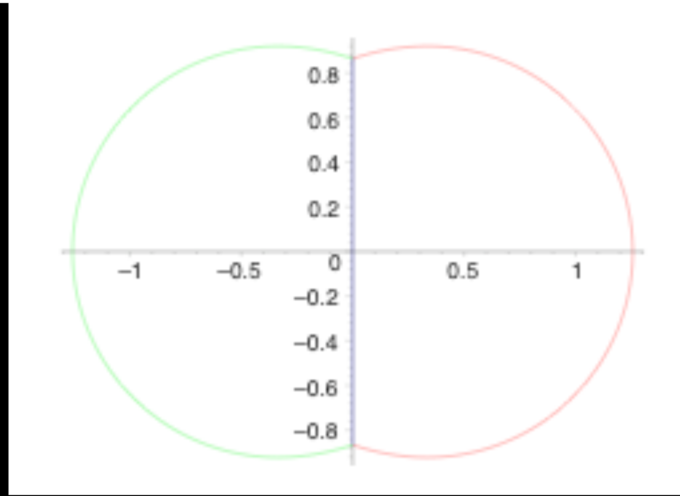

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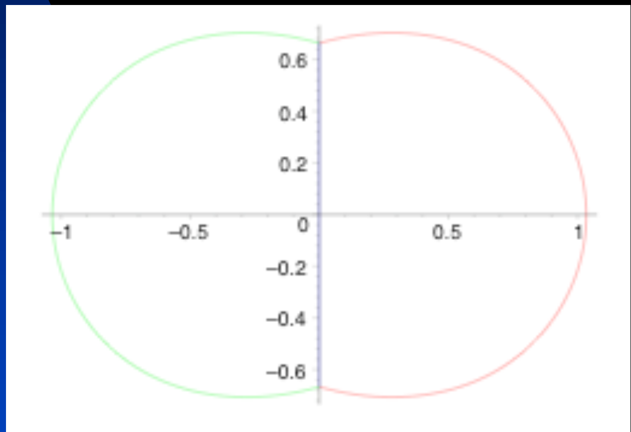

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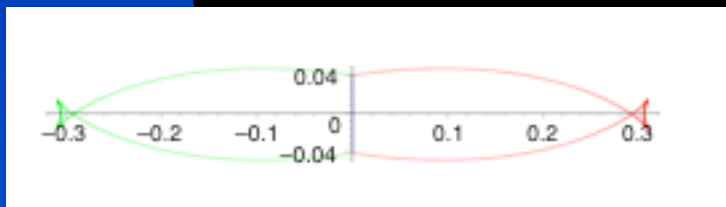
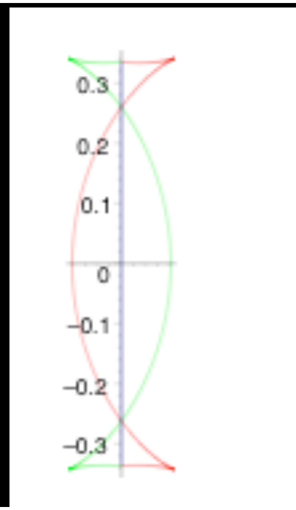


6



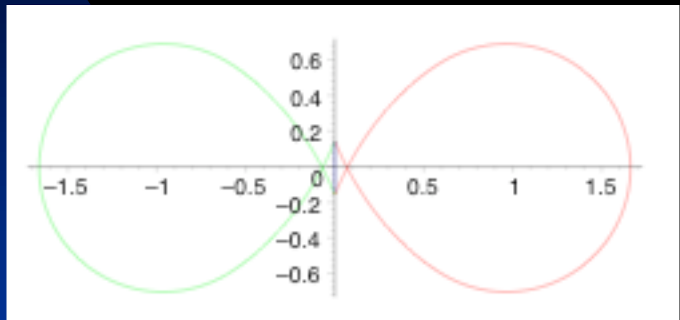
$$\mu_1 = 1; \quad \mu_2 = \mu_3 = 1.5$$

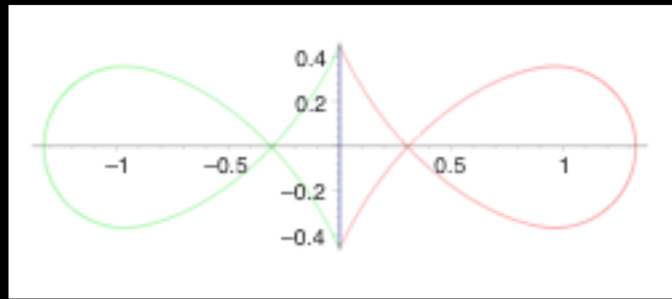
Light wings

1

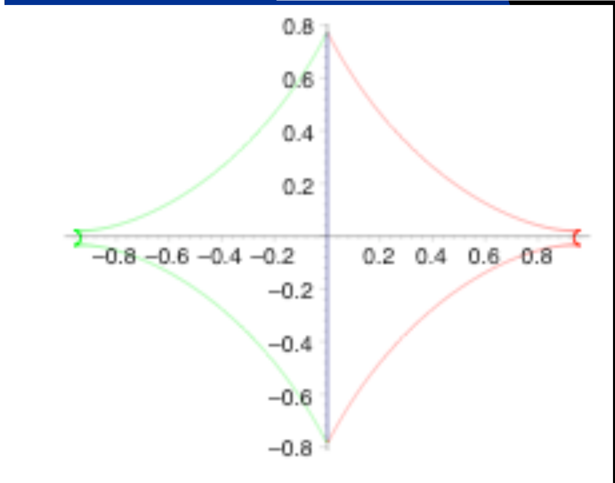
2

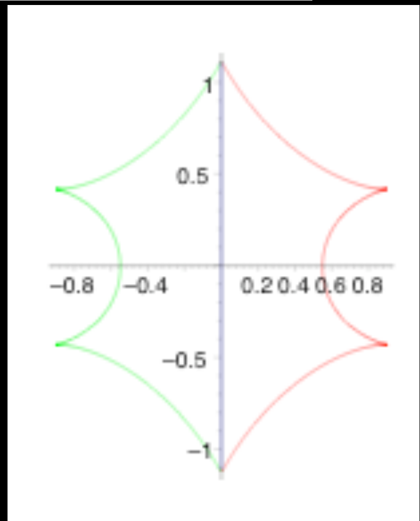
3

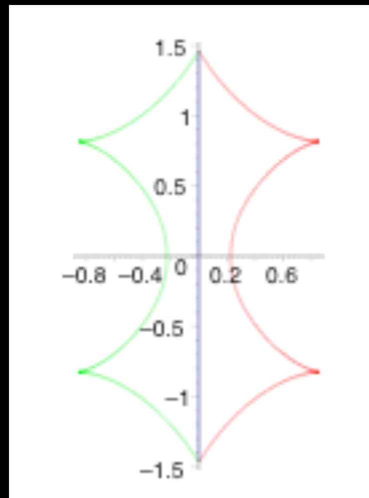


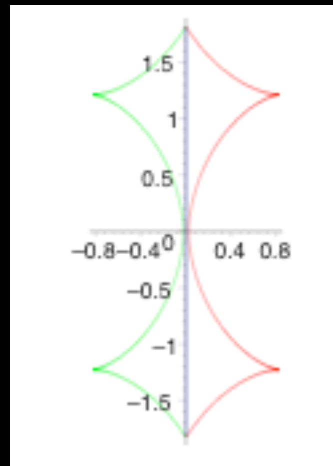


4





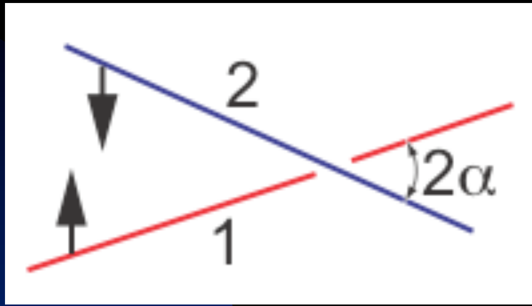




5 6 7 8

$$\mu_1 = 1; \quad \mu_2 = \mu_3 = 0.52$$

Collision of straight strings



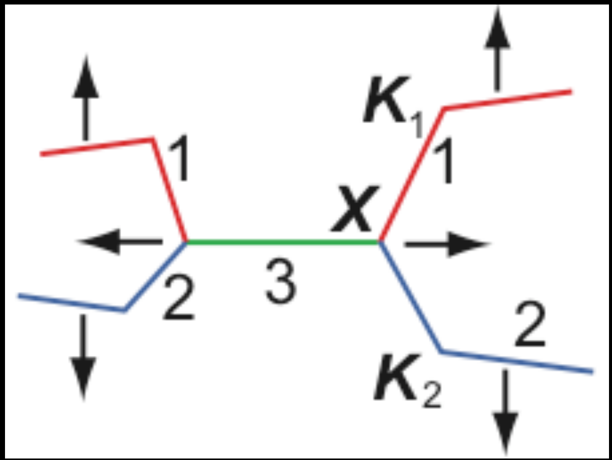
Take $\mu_1 = \mu_2$ and, for $t < 0$,

$$\boldsymbol{x}_{1,2}(\sigma,t) = (-\gamma^{-1}\sigma\cos\alpha, \mp\gamma^{-1}\sigma\sin\alpha, \pm vt)$$

$$\gamma^{-1} = \sqrt{1-v^2}$$

$$\Rightarrow \boldsymbol{a}'_{1,2} = (-\gamma^{-1}\cos\alpha, \mp\gamma^{-1}\sin\alpha, \pm v)$$

$$\boldsymbol{b}'_{1,2} = (-\gamma^{-1}\cos\alpha, \mp\gamma^{-1}\sin\alpha, \mp v)$$



If 1,2 exchange partners, and are joined by 3, it must lie on *x* or *y* axis (for small α or large α resp) Assume *x*-axis. Then for $t > 0$,

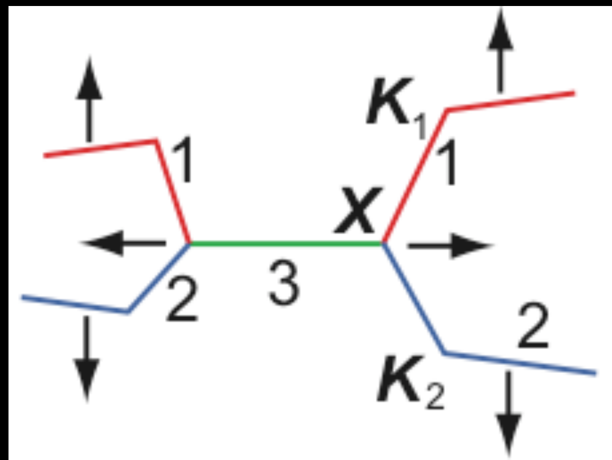
$$\boldsymbol{x}_3(\sigma,t) = (\sigma,0,0), \qquad \boldsymbol{a}'_3 = \boldsymbol{b}'_3 = (1,0,0)$$

Consider vertex ***X*** on right

Collision of straight strings

$$\boldsymbol{X}(t) = s_3(t)(1,0,0)$$

$$\boldsymbol{K}_{1,2}(t) = t(\gamma^{-1}\cos\alpha, \pm\gamma^{-1}\sin\alpha, \pm v)$$



$$\mu_1\dot{s}_1 + \mu_1\dot{s}_2 + \mu_3\dot{s}_3 = 0 \Rightarrow$$

$$\dot{s}_1 = \dot{s}_2 = -\frac{\mu_3}{2\mu_1}\dot{s}_3$$

Now

$$c_{12} = \boldsymbol{b}_1' \cdot \boldsymbol{b}_2' = \gamma^{-2}\cos 2\alpha - v^2$$

$$c_{13} = \boldsymbol{b}_1' \cdot \boldsymbol{b}_3' = -\gamma^{-1}\cos\alpha = c_{23}$$

$$\Rightarrow \dot{s}_3 = \frac{2\mu_1\gamma^{-1}\cos\alpha - \mu_3}{2\mu_1 - \mu_3\gamma^{-1}\cos\alpha}$$

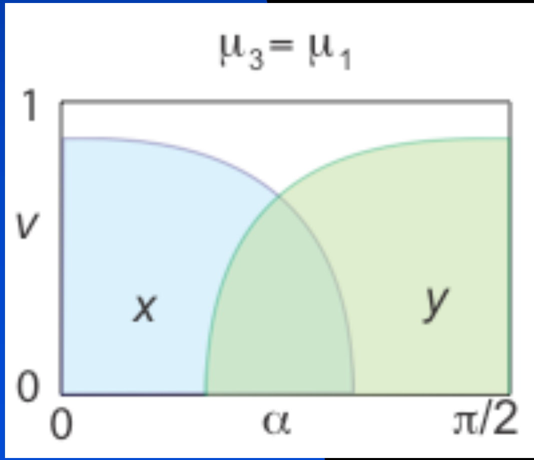
What does it imply?

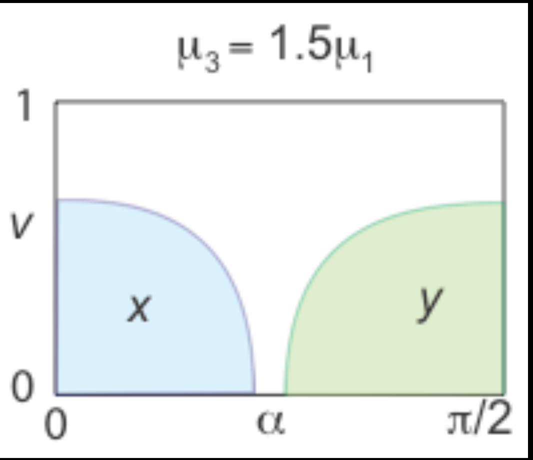
$$\dot{s}_3 = \frac{2\mu_1\gamma^{-1}\cos\alpha - \mu_3}{2\mu_1 - \mu_3\gamma^{-1}\cos\alpha}$$ with $\mu_3 < 2\mu_1$

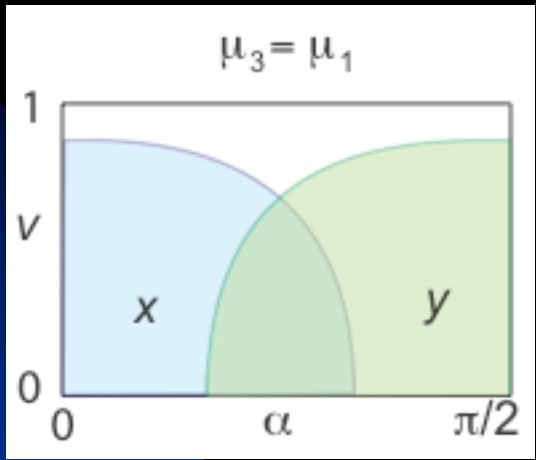
But $\dot{s}_3 > 0$, so for 3 along *x* axis, $\alpha < \arccos\left(\frac{\mu_3\gamma}{2\mu_1}\right)$

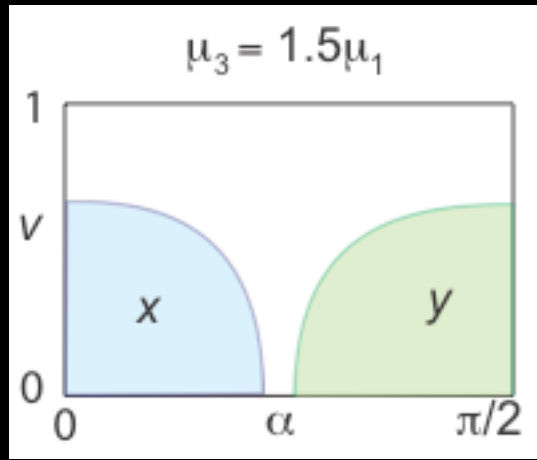
Similarly, for 3 along *y* axis, $\alpha > \arcsin\left(\frac{\mu_3\gamma}{2\mu_1}\right)$

Kinematically allowed regions are:









Note: **neither** is possible unless $\gamma < \dfrac{2\mu_1}{\mu_3}$

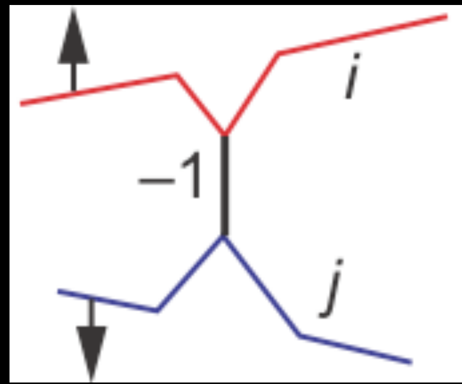
e.g., if $\mu_3 = \mu_1$, we require $v < \dfrac{\sqrt{3}}{2}$

What happens if this limit is violated?

For **abelian** strings, the only possibility is that they pass through each other without exchanging partners.

Linkage in z direction

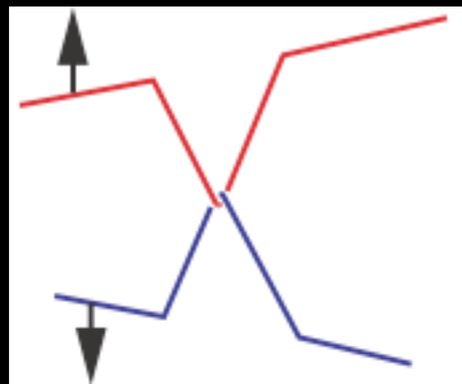
Non-abelian strings with $[\gamma_1, \gamma_2] \neq 0$ cannot pass through one another, and may become linked by a string along *z* axis.



Here $c_{12} = 2v^2 - 1,\ c_{13} = c_{23} = -v$

$$\Rightarrow \dot{s}_3 = \frac{2\mu_1 v - \mu_3}{2\mu_1 - \mu_3 v} \Rightarrow v > \frac{\mu_3}{2\mu_1}$$

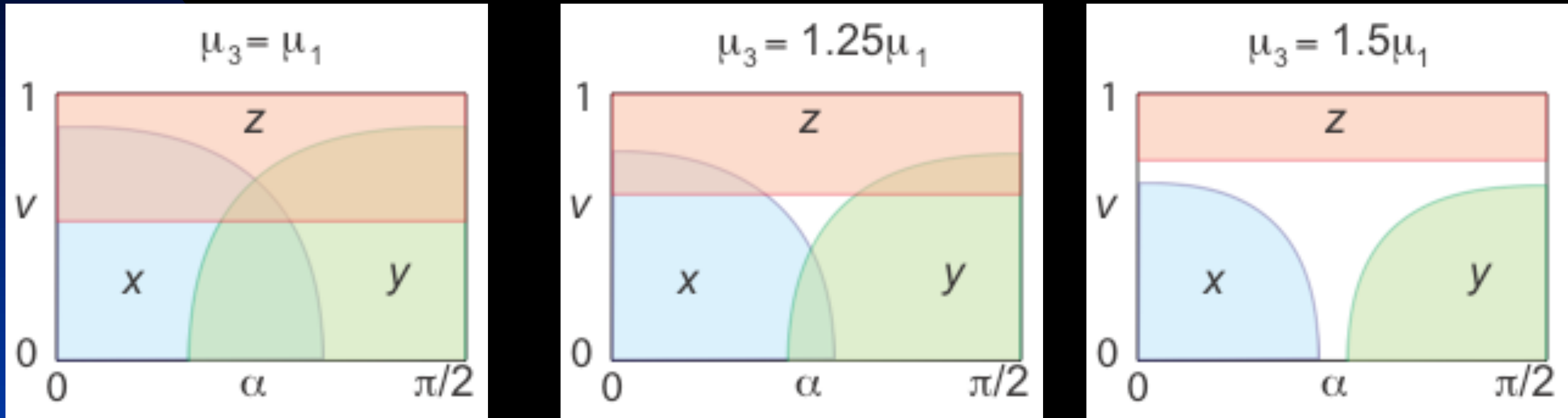
Linking in *x* or *y* dir. required $\gamma < \dfrac{2\mu_1}{\mu_3}$



So if $\mu_3 > \sqrt{2}\mu_1$ there is a range of velocities for which the strings cannot move apart, linked in **any** direction; they become locked.

Kinematic constraints

Allowed regions of the α-v plane for links along 3 axes:



$$\frac{\mu_3}{\mu_1} < \frac{2}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} < \frac{\mu_3}{\mu_1} < \sqrt{2}$$

$$\frac{\mu_3}{\mu_1} > \sqrt{2}$$

Abelian strings, in white or *z* region, must pass through one another.

Non-abelian-strings, in *z* region, may be linked along the *z* axis; in white region, they will be locked.

Rate of change of string lengths

- Would like to know about evolution of a network of such strings in the early universe.
- Ignore Hubble expansion and energy loss mechanisms.
- Energy in string network is then fixed, but some strings will shorten and others will grow.
- How fast, on average, is the growth or shortening?

Assume at string junction, unit vectors $\mathbf{b}'_j$ representing incoming waves are randomly distributed on unit sphere, and independent.

If tensions same, energy conservation --> $< \dot{s}_j >= 0$
Not so if tensions different.
Even for equal tension case, zero mean does not mean symmetrical distribution --> not the case that strings are as likely to grow as shrink.

Rate of change of string lengths cont.

Aim: calc prob distribution for the rate at which the first string grows - $\dot{s}_1$

$P(w_1)dw_1$-- prob $1-\dot{s}_1$ lies between w_1 and $w_1 + dw_1$

In terms of distribution of variables c_j if we choose z-axis along direction of b_3' --> can assume uniform distribution in c_1, c_2 and ϕ where

$$c_3 = c_1 c_2 + \sqrt{1-c_1^2}\sqrt{1-c_2^2}\cos\phi$$

Then
$$P(w_1) = \frac{1}{8\pi}\int_{-1}^{1} dc_1 \int_{-1}^{1} dc_2 \int_0^{2\pi} d\phi\,\delta\left(w_1 - \frac{\mu M_1(1-c_1)}{\mu_1 \mathcal{M}}\right)$$

where
$$1-\dot{s}_1 = \frac{\mu M_1(1-c_1)}{\mu_1 \mathcal{M}}$$
$$c_1 = \mathbf{b}_2' \cdot \mathbf{b}_3'$$

Kinks at the boundaries between the regions. For case of equal tensions have a single kink:

$$1. \quad P(\dot{s}_1) = \frac{27}{(\dot{s}_1+2)^3}\left(\frac{1+\dot{s}_1}{1-\dot{s}_1}\right)^2 \qquad \left(\dot{s}_1 < -\frac{1}{2}\right)$$

$$3. \quad P(\dot{s}_1) = \frac{4\dot{s}_1+5}{(\dot{s}_1+2)^3} \qquad \left(\dot{s}_1 > -\frac{1}{2}\right)$$

with $< \dot{s}_1 >= 0$

Rate of change of string lengths -- equal tension

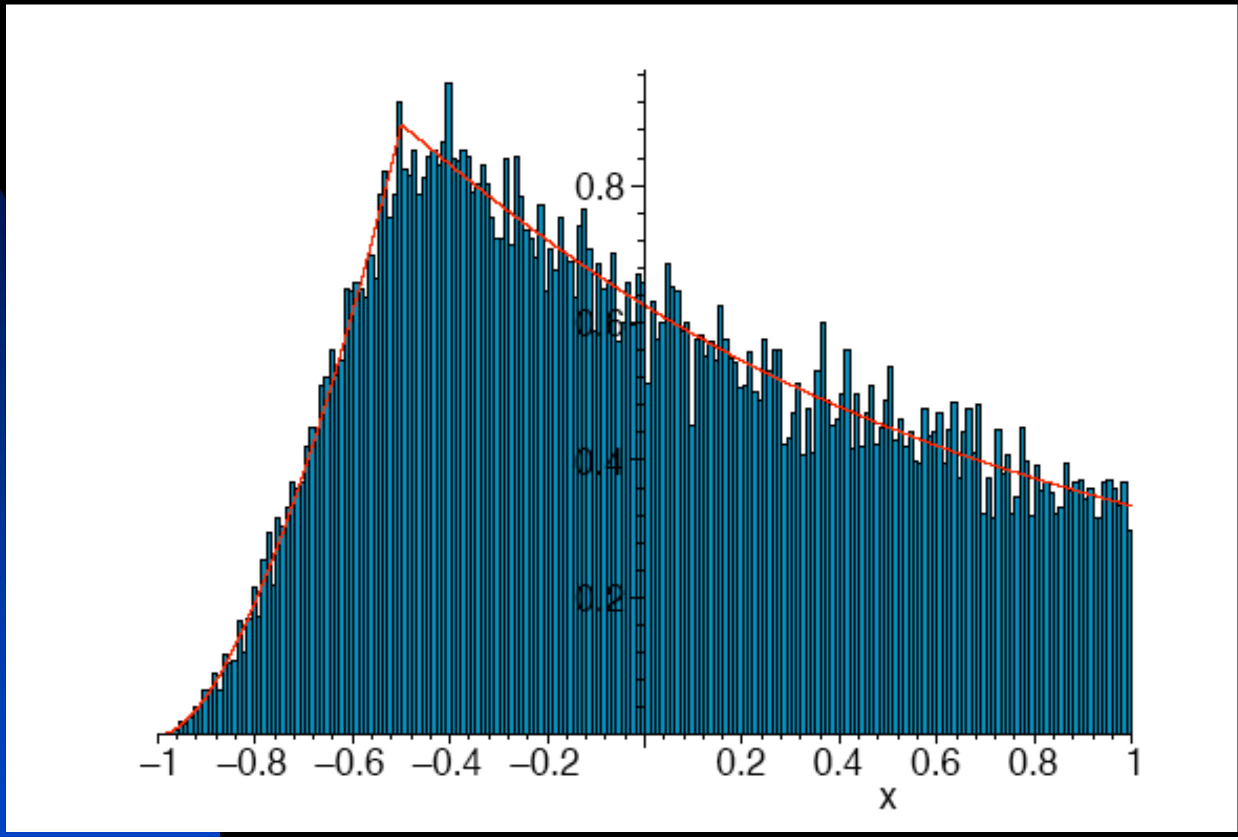


Figure 1: Analytic and numerical results for the distribution of $\dot{s}_i$ for $\mu_i = 1\ \forall i$. The mean is 0 and the standard deviation 0.48, according to Maple. 20000 samples.

At any given time it is most probable that one of the 3 legs is growing while the other two are shrinking at a slower rate.

General unequal tensions -- mean value:

$$\langle \dot{s}_1 \rangle = \frac{3\mu_1 - \mu}{3\mu_1} + \frac{Q}{\mu_1}\left[-(\mu_1 + \mu)\nu_1^2 \ln \frac{\mu\nu_1}{4\mu_2\mu_3} + (\mu_1 + \nu_3)\nu_2^2 \ln \frac{\mu\nu_2}{4\mu_1\mu_3} + (\mu_1 + \nu_2)\nu_3^2 \ln \frac{\mu\nu_3}{4\mu_1\mu_2} \right].$$

Symmetric as it should be. $\mu_2 \leftrightarrow \mu_3$

Also satisfies consistency condition:

$$\mu_1\langle \dot{s}_1 \rangle + \mu_2\langle \dot{s}_2 \rangle + \mu_3\langle \dot{s}_3 \rangle = 0.$$

Note: more likely to be positive if μ_1 small or if the other two tensions are very different. In a network of strings there may be a tendancy for the lighter strings to grow at the expense of the heavier ones.

Rate of change of string lengths -- unequal tension

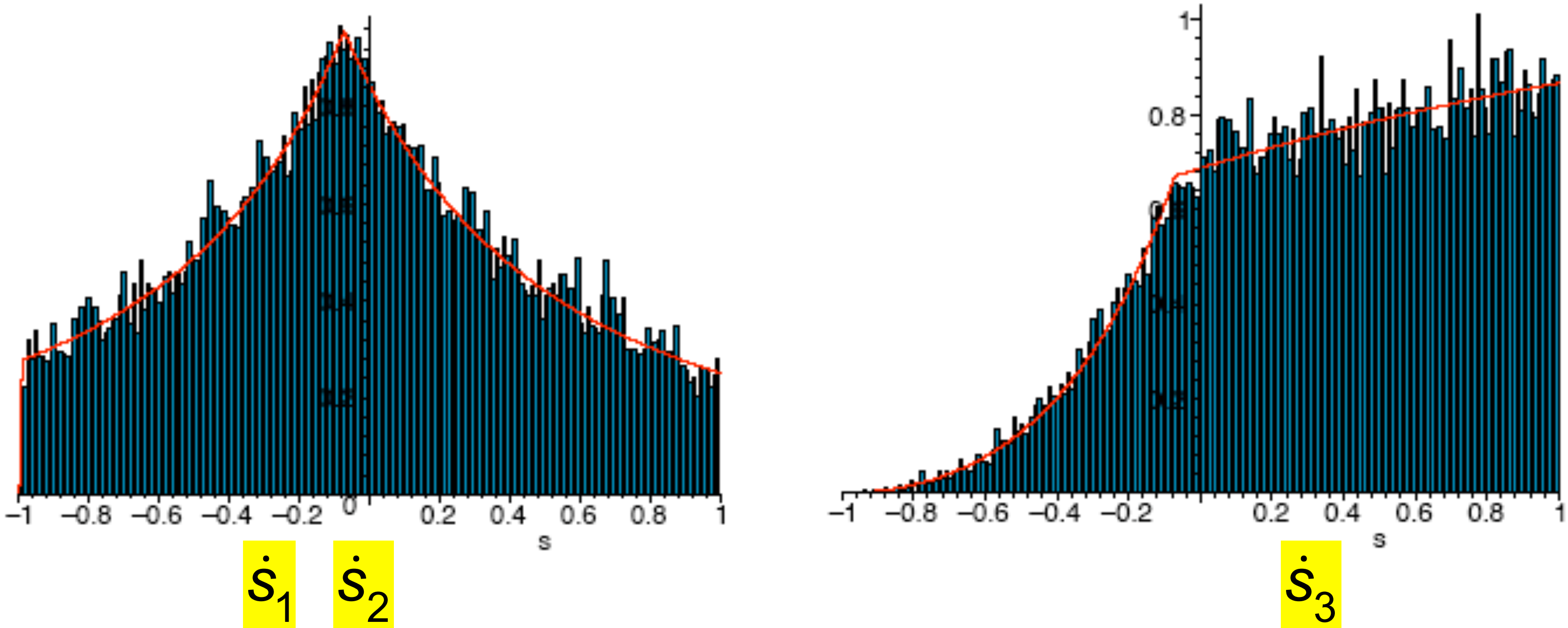


Figure 3: Analytic and numerical results for the distribution of $\dot{s}_1 = \dot{s}_2$ in LH panel, and $\dot{s}_3$ in RH panel for $\mu_1 = \mu_2 = 1.4$ and $\mu_3 = 0.2$. 20000 samples. $\langle \dot{s}_1 \rangle = -0.0243$, $\langle \dot{s}_2 \rangle = -0.0267$, $\langle \dot{s}_3 \rangle = 0.357$.

Looks like the larger tension strings want to shrink !

$$\mu_1 = \mu_2 = 1.4 \quad \mu_3 = 0.2$$

Conclusions

If we are lucky with inflation in string models, they may form metastable F and D strings which will survive long enough to be of interest.

What does a network of strings with junctions look like?

Observational signatures ?

This will have to be a combination of analytic and theoretical approaches, and should involve both field theory representations and phenomenolgical model building.

It leaves open the possibility that there is a window on string theory through cosmology!