A measure for the multiverse?

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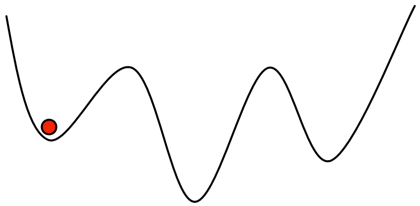
(with D. Schwartz-Perlov, A. Vilenkin and S. Winitzki)

Motivation



"FUNDAMENTAL THEORY"

With few vacua,
a few observations determine which one is ours.

Everything else can be predicted.

However…

"Landscape" → Many vacua to check through $(\sim 10^{100's})$.

Many may look like our own,
except for small variations of the "constants".
Can we predict the values of such constants?

Statistical approach → Do all vacua carry the same weight ?
What is the measure?

Perhaps the answer lies in cosmology.

Eternal inflation: → **All vacua** are realized in the **multiverse**.

PLAN

1- Eternal inflation

2- Counting bubbles

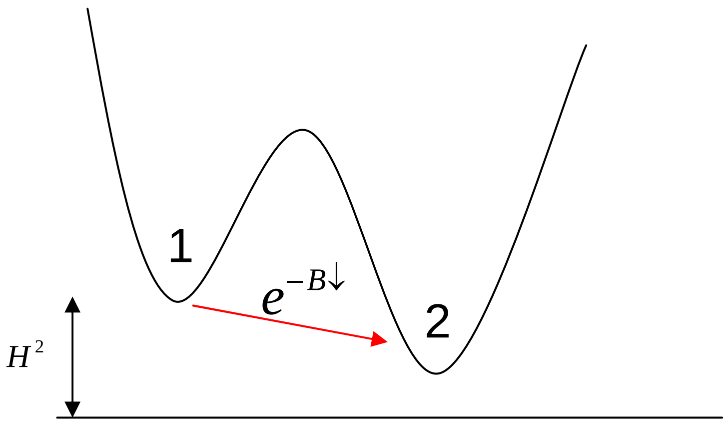
3- Counting objects inside bubbles

4- Bubble universes and signals from before

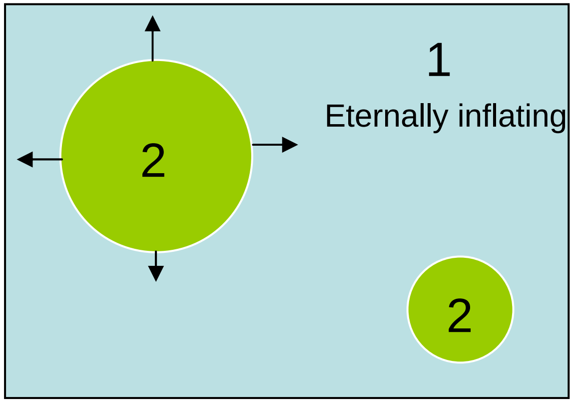
1-ETERNAL INFLATION

dS vacua generically lead to eternal inflation
(even if they are metastable)

$\Gamma \equiv$ Nucleation rate per unit physical spacetime volume



For $\Gamma \sim A\, e^{-B} \ll H^4$
bubbles of the new phase
do not percolate



$$\frac{dV_1}{dt} \approx (3H - a\,H^{-3}\Gamma)\;V_1$$

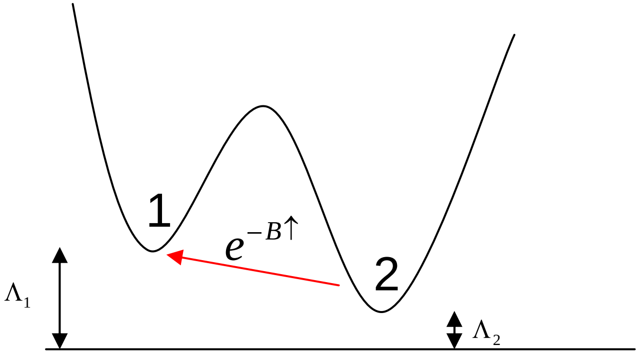
$$V_1 \sim C\; e^{(3 - aH^{-4}\Gamma)\,Ht}$$

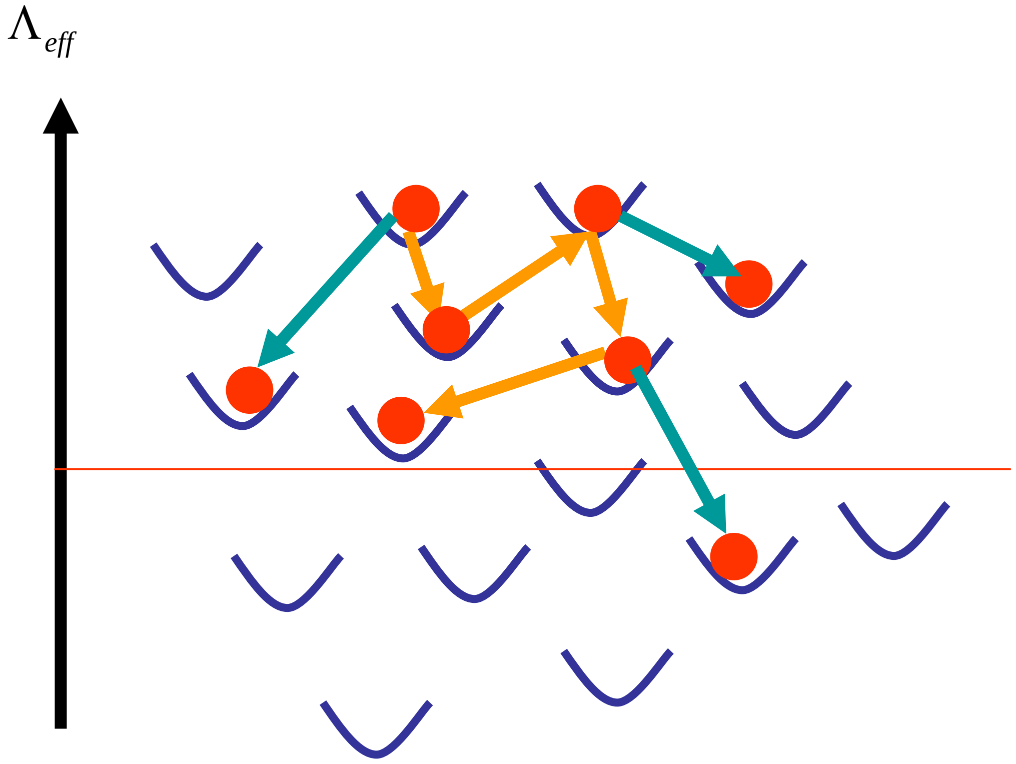
Tunneling uphill is also possible

(Lee and Weinberg, 87)

$$\Gamma^{\uparrow}/\Gamma^{\downarrow} \sim e^{-S(2)+S(1)}$$

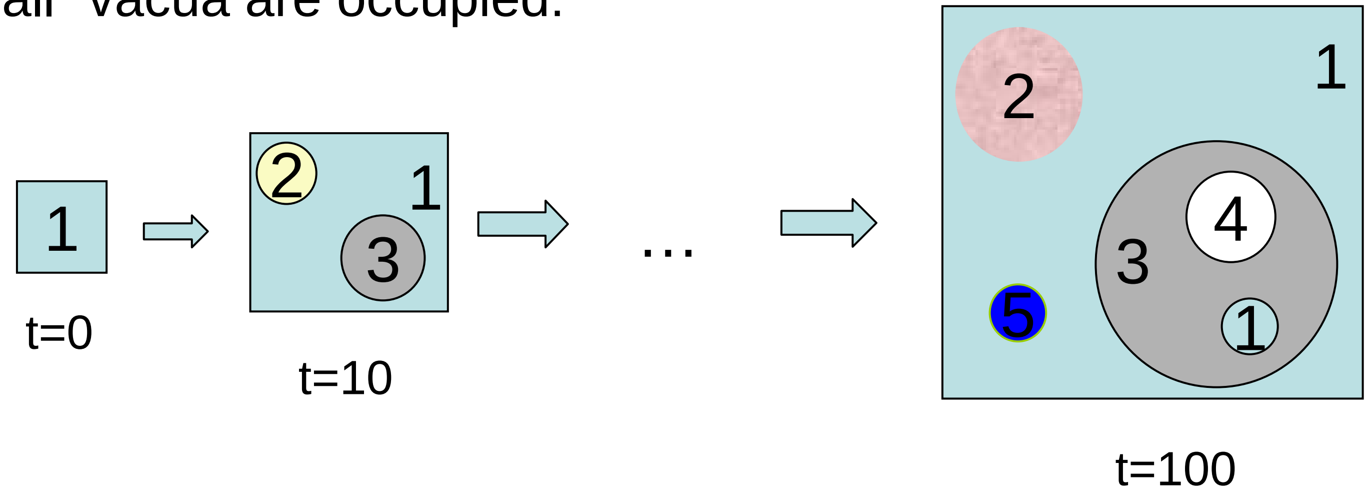
Entropy difference

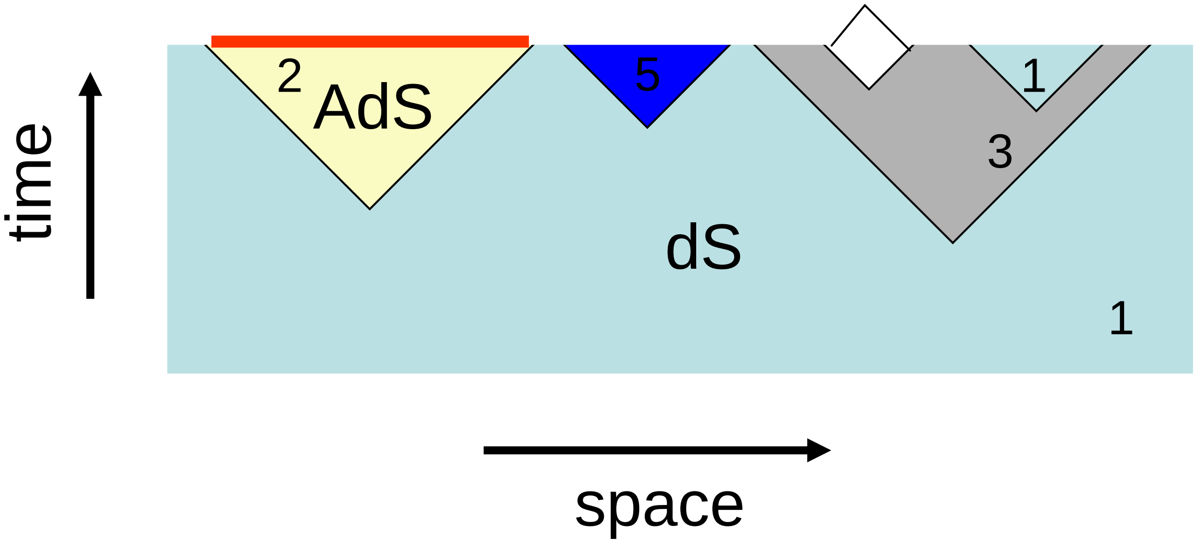


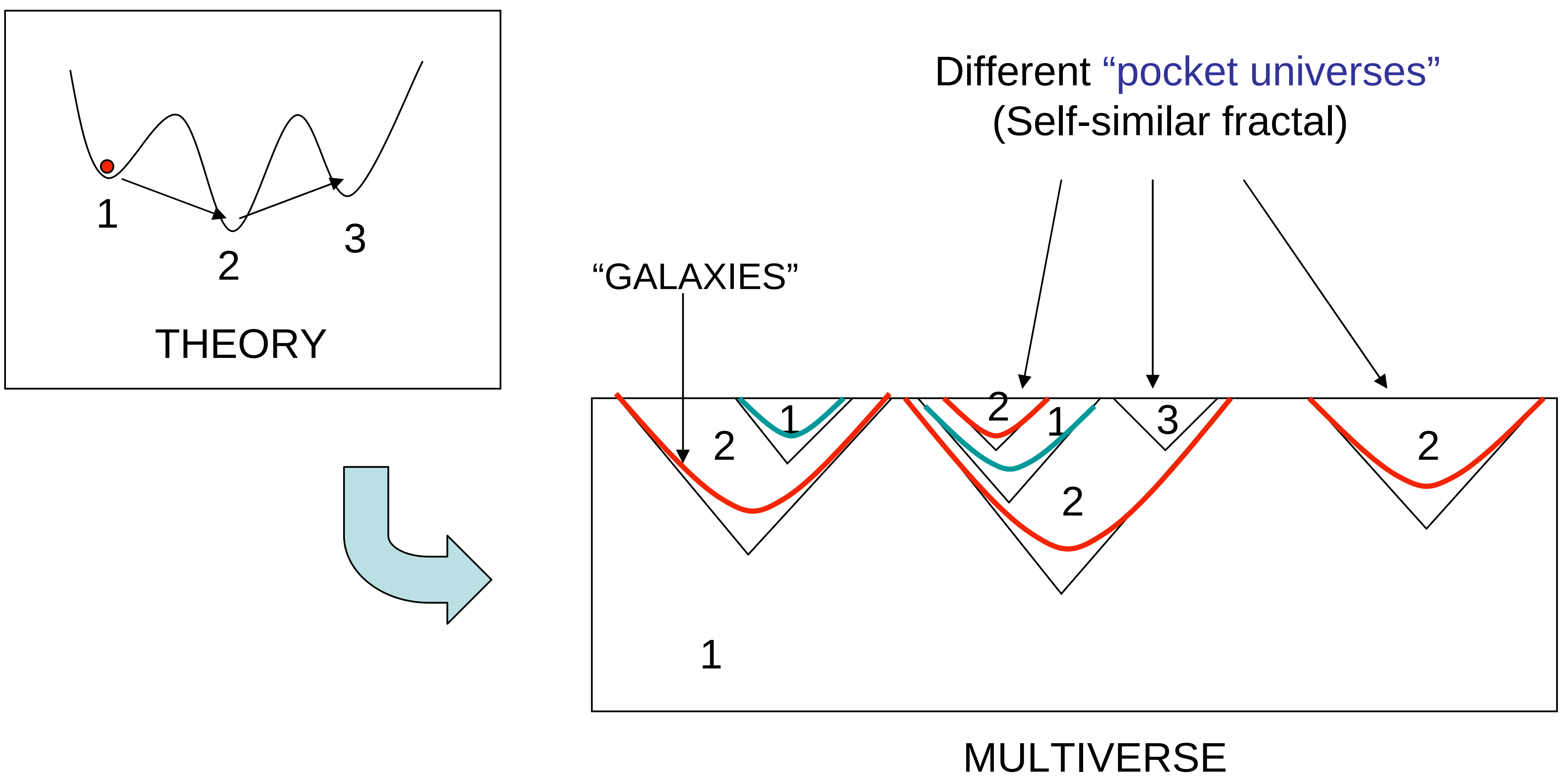


Λ_{eff}

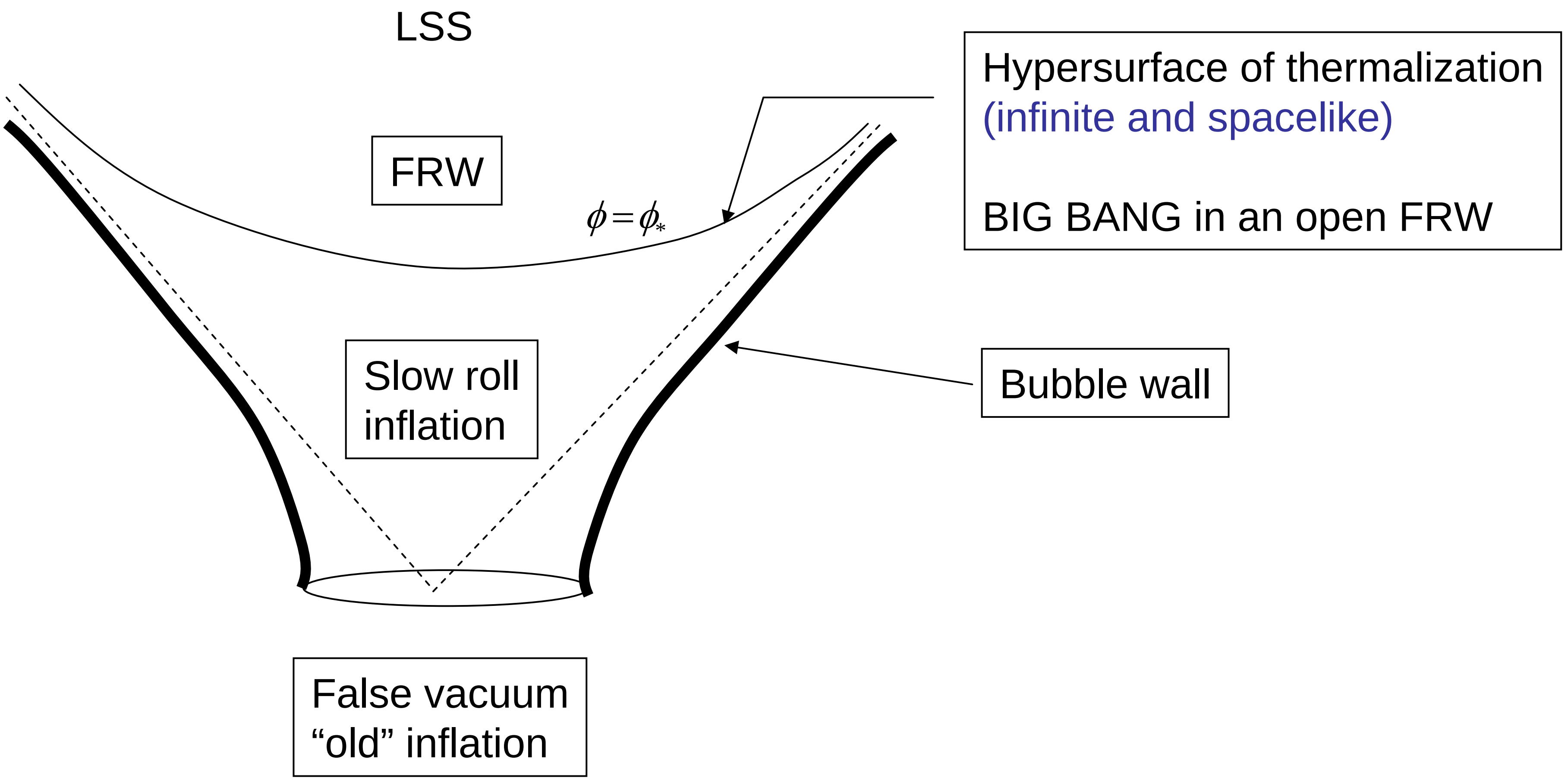
Eventually, "all" vacua are occupied:






1
2
3
THEORY
Different “pocket universes”
(Self-similar fractal)
“GALAXIES”
2
1
2
1
3
2
2
1
MULTIVERSE

Structure of a “pocket” universe



Quantitative description

V_i Fraction of volume in inflating vacuum of type i

$$\boxed{\frac{dV_i}{d\eta} = M_{ij} V_j + 3 H_i^{1-\alpha} V_i}$$

$$M_{ij} = \underbrace{\kappa_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij} \sum_r \kappa_{ri}}_{\text{Lost to other vacua}}$$

$$\kappa_{ij} = \frac{4\pi}{3} H_j^{-3-\alpha} \Gamma_{ij}$$

From bubbles of type "i" in vacuum "j".

$$d\eta = H^{\alpha}\, dt$$

$\alpha = 0$ Proper time gauge

$\alpha = 1$ Scale factor gauge $\eta = \log a$

In scale factor gauge, the solution is of the form

$$V_i(a) = V_i^{(0)} a^{\gamma_0} + \sum_n V_i^{(n)} a^{\gamma_n} \to \boxed{V_i^{(0)}} a^{\gamma_0}$$

$$a \to \infty$$

$\gamma_0 = 3 - \kappa$

Dominant eigenvalue

$$\kappa \sim -\sum_i M_{ij}$$

$$0 < \kappa \ll 1$$

Volume in all inflating vacua grows unbounded (**that's eternal inflation**).

Volume fraction in vacuum of type i on a "late" constant "time" slice becomes

$$\frac{V_i(a)}{V_{total}(a)} \propto V_i^{(0)} \quad (a \to \infty)$$

$\Longrightarrow$ Independent of initial conditions

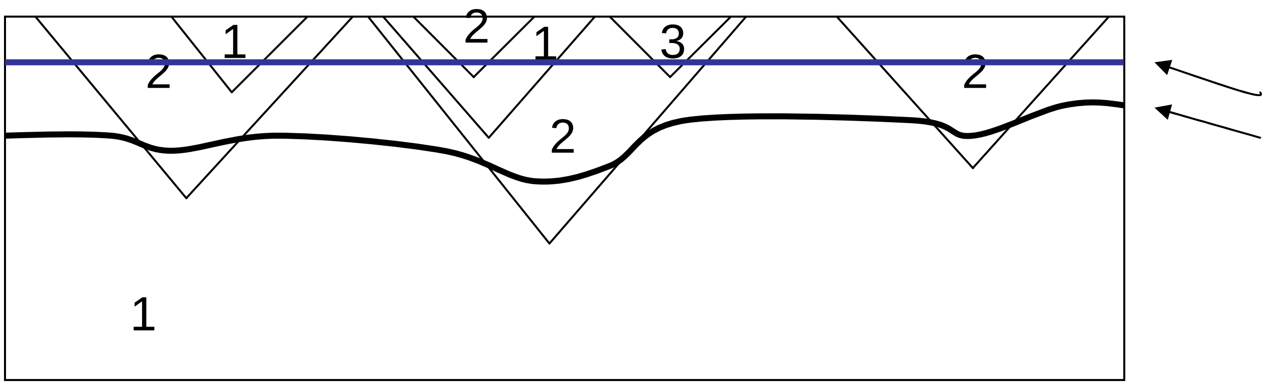
(it is just an eigenvector)

Independence on initial conditions (attractor behaviour)

$$\frac{V_i(\eta)}{V_{total}(\eta)} \propto V_i^{(0)} \qquad (\eta \to \infty)$$ is true in any gauge.

However, the dominant eigenvector $V_i^{(0)}$ **does depend on gauge** !!

⇒ **Volume fractions with a constant time cut-off cannot be used** in order to define meaningful probabilities.



Different choices of the time slicing give different volume fractions

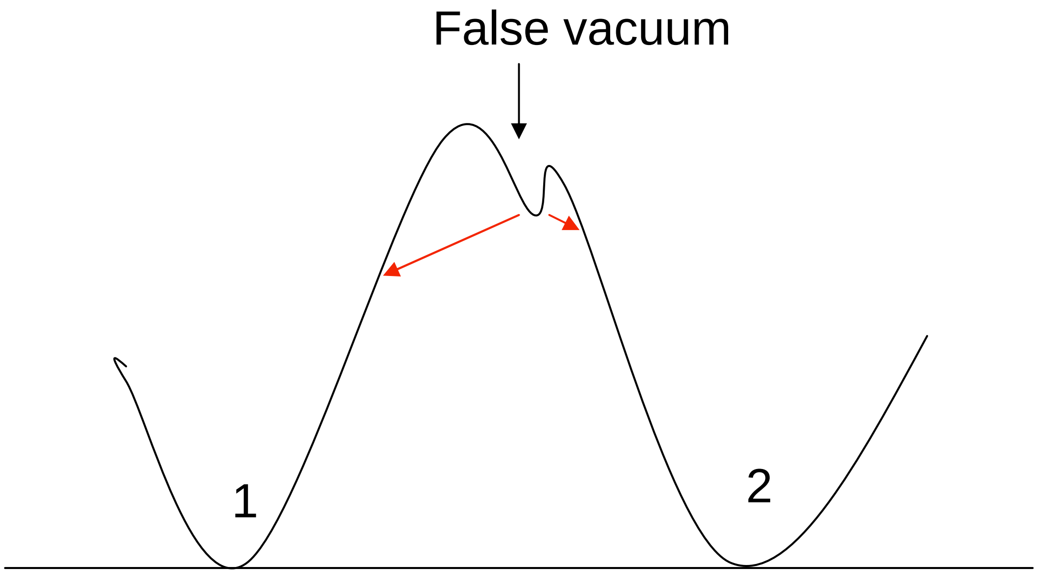
The problem of slicing at some constant “global” time.

2- Counting relative numbers of bubbles $\equiv p_j$

TRICKY BUSINESS: the number of pockets of any given type is infinite.

SOME BASIC REQUIREMENTS FOR THE COUNTING PROCEDURE

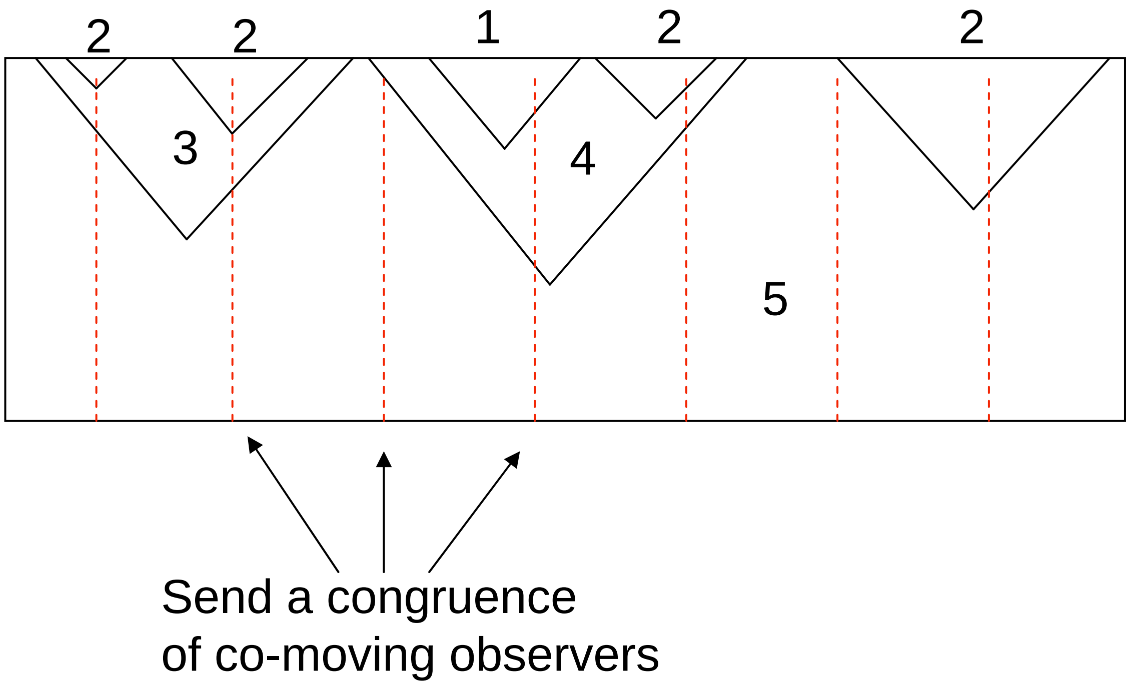
- Independent of initial conditions.
- Geometrically defined (gauge independent).
- Intuitively reasonable (it should be an ordinary counting with some cut-off).



$$\Gamma_1 \ll \Gamma_2$$

Intuitively, there should be more bubbles of type 2 than of type 1.

A PRESCRIPTION THAT FAILS TO SATISFY OUR BASIC REQUIREMENTS

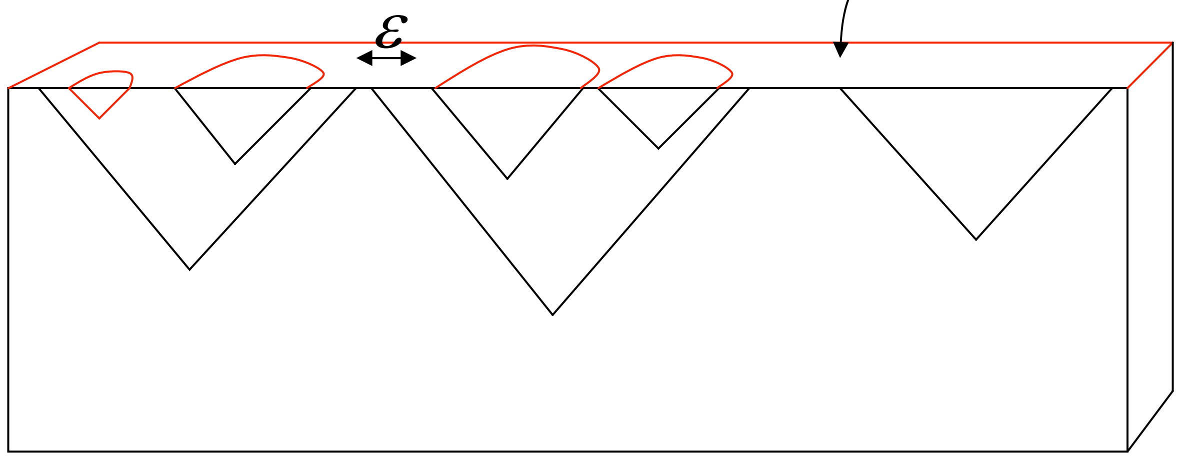


$p_j^C \propto$ Number of obs. who end up in pockets of type j

The result depends on initial conditions: No good.

OUR PROPOSAL

p_j RELATIVE NUMBER OF POCKETS OF TYPE j.
COUNT THEM AT THE FUTURE BOUNDARY.



Number is infinite: count only the ones which are bigger than some small co-moving size ε and then let $\varepsilon \to 0$

Result is independent on initial conditions
(number is dominated by bubbles formed at late times).

Result is invariant under coordinate transformations.

f_i Fraction of co-moving volume in vacuum of type i

$$\boxed{\frac{df_i}{dt} = M_{ij}\, f_j}$$

$$M_{ij} = \underbrace{\kappa_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij} \sum_r \kappa_{ri}}_{\text{Lost to other vacua}}$$

$$\kappa_{ij} = \frac{4\pi}{3} H_j^{-4}\, \Gamma_{ij}$$

$$f_i(t) \approx f_i(0) + s_i\, e^{-qt} + \ldots$$

$-q < 0$ Highest nonvanishing eigenvalue of M_{ij} (it can be shown to be negative).

s_i Corresponding eigenvector (it is non-degenerate).

$$\frac{dN_{j\alpha}(t)}{dt} = \frac{3}{4\pi} H_\alpha^3 \exp(3t) \frac{df_{j\alpha}}{dt}.$$

It can be shown that

$$\boxed{p_i \propto \sum_j H_j^q\, \Gamma_{ij}\, s_j}$$

E.g., if all vacua emanate from a single false vacuum, then

$$p_i \propto \Gamma_i$$

(WE HAVE IGNORED BUBBLE COLLISIONS)

This counting agrees with subsequent alternative proposals,

(at least in the limit where the “desirable” requirements are met by those proposals.)

1-Lim et al. 05: Send a dense congruence of geodesics, and let their number N go to infinity. Then count all bubbles which have been hit by the congruence at least once.

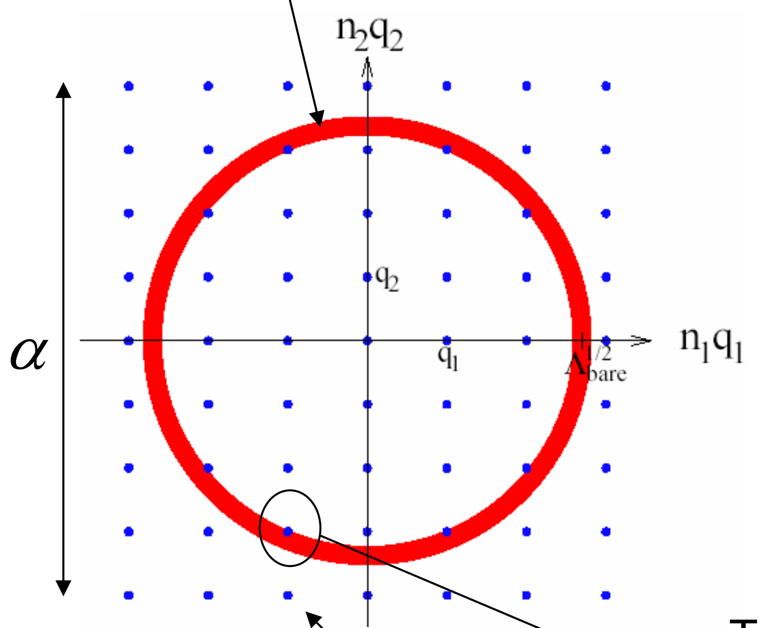
2- Bousso 06: Send a congruence of observers and record the vacua they visit. Give probabilities according to the total number of visits.
This suffers from a problem of dependence on initial conditions, except in the case where there are no sinks of probability. In that case, the result given by our prescription is recovered (Vanchurin, Vilenkin 06).

AN EXAMPLE WHERE THIS HAS BEEN APPLIED (Schwartz-Perlov, Vilenkin 05)

Spherical shell with a given range of the vacuum energy

Bousso+Polchinski 00

The 4D theory has in fact **many different fluxes**



$$F_a = n_a\, q_a \qquad a = 1, \ldots, J \qquad (J \sim 100)$$

$$E(n_1, \ldots, n_J) = \frac{1}{2} \sum_a (n_a q_a)^2$$

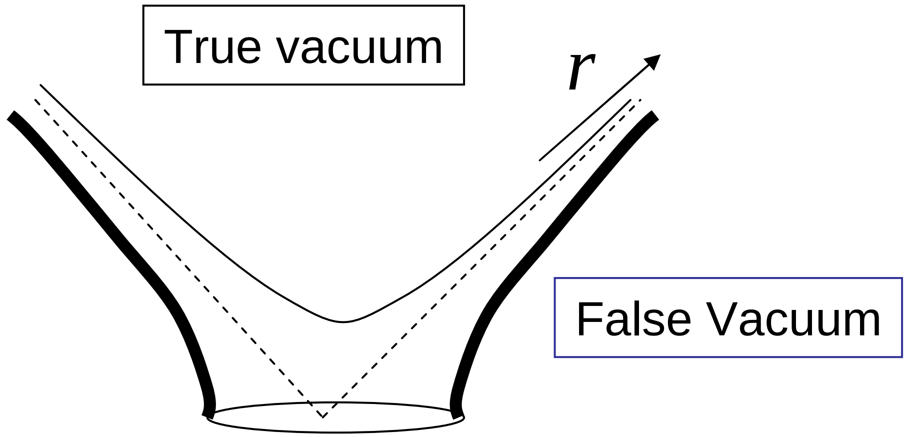
$$\boxed{\#vacua \sim \alpha^J \sim G^\beta \quad (\beta \gtrsim 1)}$$

$$G \equiv 10^{100} \quad (Google)$$

Tends to dominate the dynamics of bubble production (Schwartz-Perlov, Vilenkin), producing a **jagged probability distribution** for Lambda

Each dots represents a metastable vacuum, with a different value of the vacuum energy

3-Counting objects within a pocket



FRW universe inside the bubble is infinite, but homogeneous on large scales.
Need new regulator.

We may take a region of co-moving size equal to a multiple of the curvature scale
The number of objects in that scale is given by:

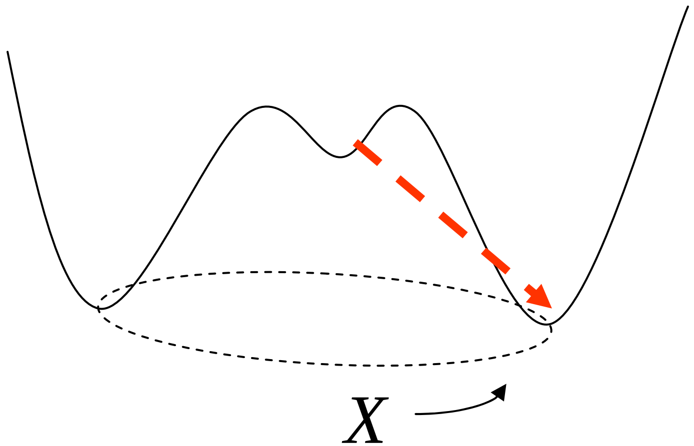
$$N_{objects} \sim r^3 \int a^3(\tau)\, n_{obj}(\tau)\, d\tau$$

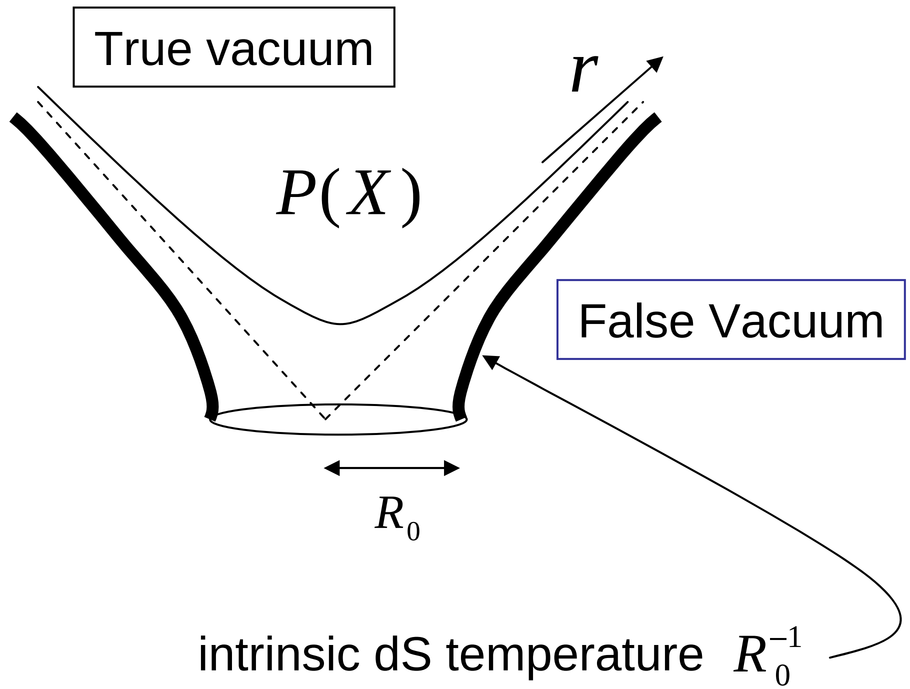
DENSITY OF OBJECTS THAT WILL FORM PER UNIT VOLUME AT TIME τ

*BOLZMAN BRAINS MUST BE SUBTRACTED OUT $n_{obj} \to n_{obj} - n_{obj}^{equilibrium}(T = H/2\pi)$

(Vilenkin, to appear)

Distribution of constants inside a bubble (flat space, no gravity)





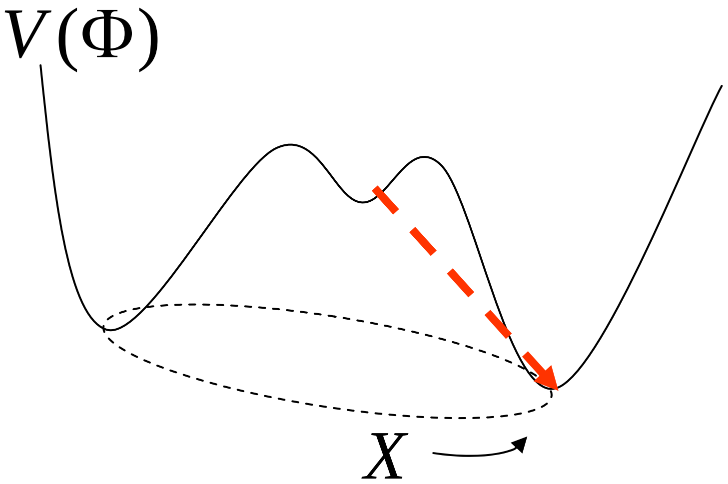
X does a random walk of step $\delta X \sim R_0^{-1}$ each proper time interval R_0 as the bubble wall eats false vacuum.

$$\boxed{P(X) = const.}$$

$$\boxed{\Delta X \sim N^{1/2} R_0^{-1} \sim (\Delta r / r_K)^{1/2} R_0^{-1}}$$

(r_K curvature scale)

Consider now a tilted potential



$$\Phi(x) = f(x)\, e^{iX(x)}$$

Preferred tunneling direction (saddle point).

$$X = 0, \quad f = f_0(\vec{x}^2 - t^2)$$

O(3,1) symmetry

(in thin wall approx.)

$$< X^2 >_{O(3,1)} = m_X^{-2} R_0^{-4} = \gamma^{-1} R_0^{-2} \quad >> \quad R_0^{-2},$$

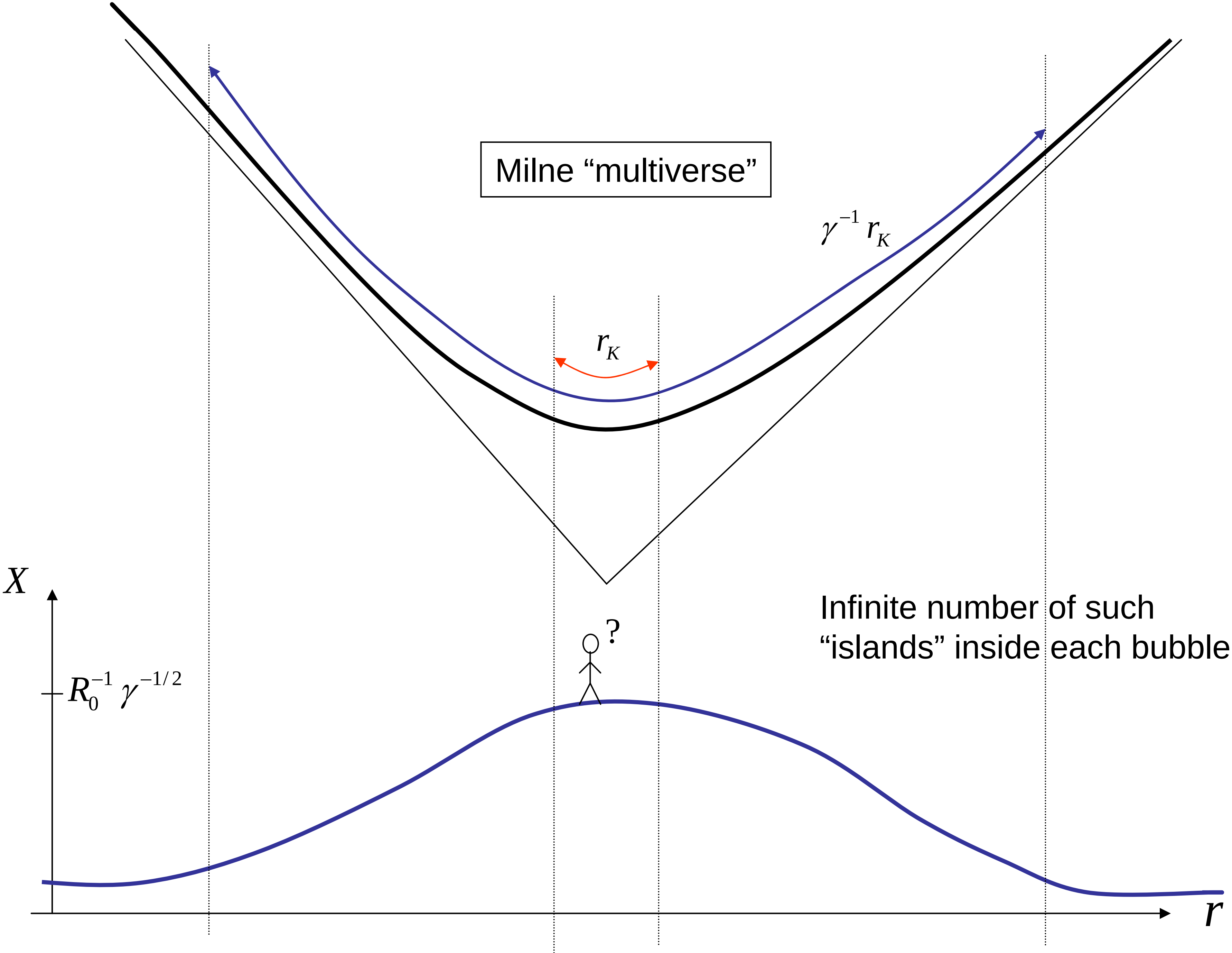
$$< X(r)\, X(r') > \; \sim \exp(-\gamma\, r / r_K)$$

Correlation length

$$\gamma^{-1} \equiv (m_X R_0)^{-2} \;\; >> 1$$

$$\boxed{P(X; j) = e^{-X^2/\left\langle X^2 \right\rangle} = e^{-I_E(X;j)}}$$

$I_E(X; j)$ Euclidean action at constant phase (adiabatic approximation)



Milne "multiverse"
$\gamma^{-1} r_K$
r_K
X
?
$R_0^{-1}\gamma^{-1/2}$
Infinite number of such "islands" inside each bubble
r

Including gravity: one of the fields ϕ becomes inflaton for slow roll inflation inside the bubble

VOLUME DISTRIBUTION (AT THERMALIZATION) WITHIN A POCKET OF TYPE j

$$\boxed{P_*(X;j) \sim \underbrace{e^{-I_E(X;j)}}_{\text{Volume distribution right after quantum tunneling}} \; \underbrace{Z^3(X;j)}_{\text{Classical slow roll expansion factor inside the bubble, up to the time of thermalization.}}}$$

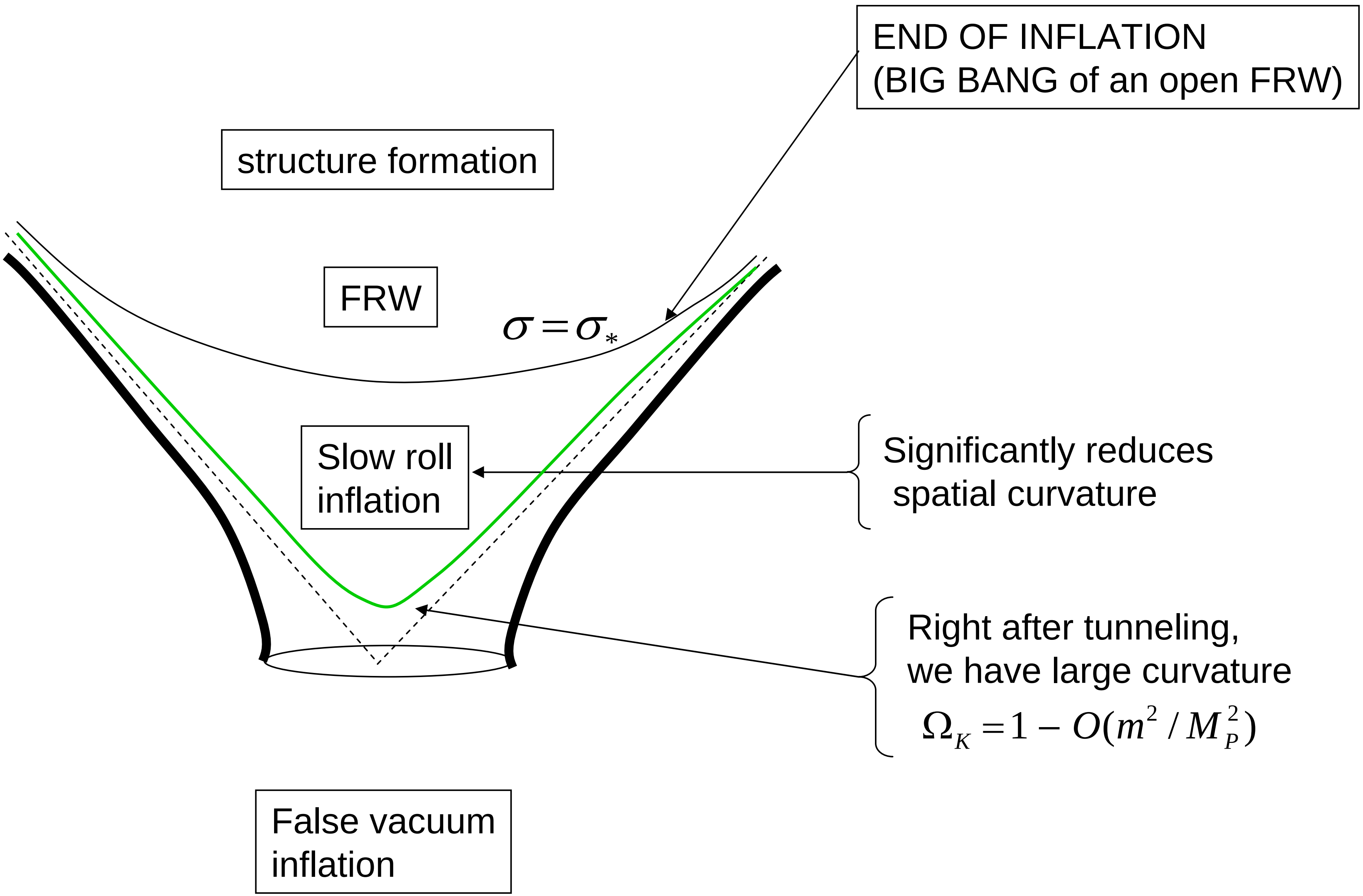
$$\begin{cases} Z(X;j) = \exp\left[\int_{\phi_i}^{\phi_*} \dfrac{H(\phi,X)}{H'(\phi,X)}\,d\phi\right] \\ \\ I_E(X;j) \quad \text{Euclidean action for the CdL instanton} \end{cases}$$

(the corresponding formula for the case of quantum diffusion will be discussed later)

4-Bubble universes, and signals from before

(Palau de la Musica)

Structure of a “pocket” universe



$$\Omega_K^* = e^{-2N_{\rm inf}}$$

e.g. $$\Omega_K^{dec} \sim e^{-2N_{\rm inf}} \left(\frac{T_*}{T_{eq}}\right)^2 \left(\frac{T_{eq}}{T_{dec}}\right)$$

curvature "dilutes" as a^{-2}

OBSERVATIONS

$\Omega_K^{now} = -.01 \pm .01$

(WMAP + HST)

TAKE

$$|\Omega_K^{now}| < 0.02$$

(more stringent for negative K)

$$i.e. \quad |\Omega_K^{dec}| < 5\times 10^{-5} \qquad (\ll 10^{-3})$$

COMPARE WITH ANTHROPIC BOUNDS
(curvature interferes with structure formation)

$$|\Omega_K^{dec}| \le \begin{array}{l} 10^{-3} \text{ (for giant galaxies)} \\ 10^{-2} \text{ (dwarf)} \end{array}$$

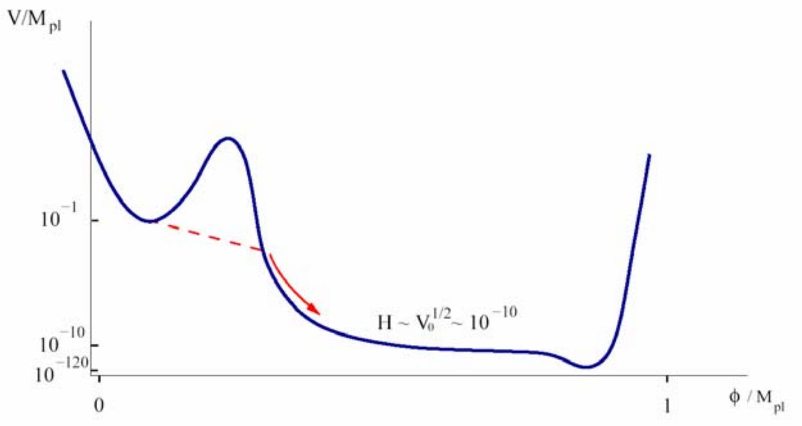
Vilenkin+Winitzki 97

$$\Rightarrow \quad N_{\rm inf}^{obs} - N_{\rm inf}^{Anth} > \begin{array}{l} 1.5 \\ 2.5 \end{array}$$

Freivogel, Kleban, Rodriguez, Susskind, 05

WITH A BIT OF LANDSCAPING …

Freivogel, Kleban, Rodriguez, Susskind, 05



Integrating over length, height and slope of the plateau (with flat priors)

and keeping $(\delta\rho/\rho) \sim (V^{3/2}/V')$ fixed,

$$\Rightarrow \quad dP(N) \propto N^{-4}\, dN$$

$$\Rightarrow \quad P(\Delta N < 2.5) \sim 3\Delta N/N \sim 10\%$$

…SATURATING THE OBSERVATIONAL BOUND DOESN'T LOOK SO UNLIKELY ANYMORE.

(there are, however, many however's)

What is it that we might expect to observe?

(Assuming Ω_K is close to its observational bound)

1- Signatures from the beginning of slow roll.
Perhaps a smaller quadrupole than normal? (a " post-diction")

2- Signatures from BEFORE slow roll.

a. Bubble wall fluctuations
b. Gravity waves from the false vacuum phase
c. Scalar perturbations from the false vacuum phase?

3- And, of course $\Omega_K \neq 0$

The wobbly bubble (and the waves it makes)

J.G.'96

Consider a light bubble

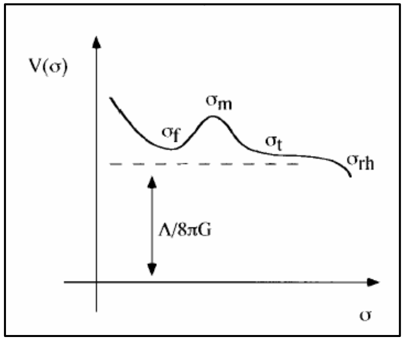
μ wall tension

$R_0^{-1} \sim \Delta V \, \mu^{-1}$ wall acceleration

$G\mu \sim$ repulsive grav. acc.

assume

$$\begin{cases} G\mu \ll R_0^{-1} \qquad (G\mu^2 \ll \Delta V) \\ \Delta V \ll V \end{cases}$$



In this case, we can use field theory in external dS background

Effect on CMB

$$\frac{\delta T}{T} \sim \int_0^{r_{ls}} h'_{rr}(r_{ls} - r)\, dr$$

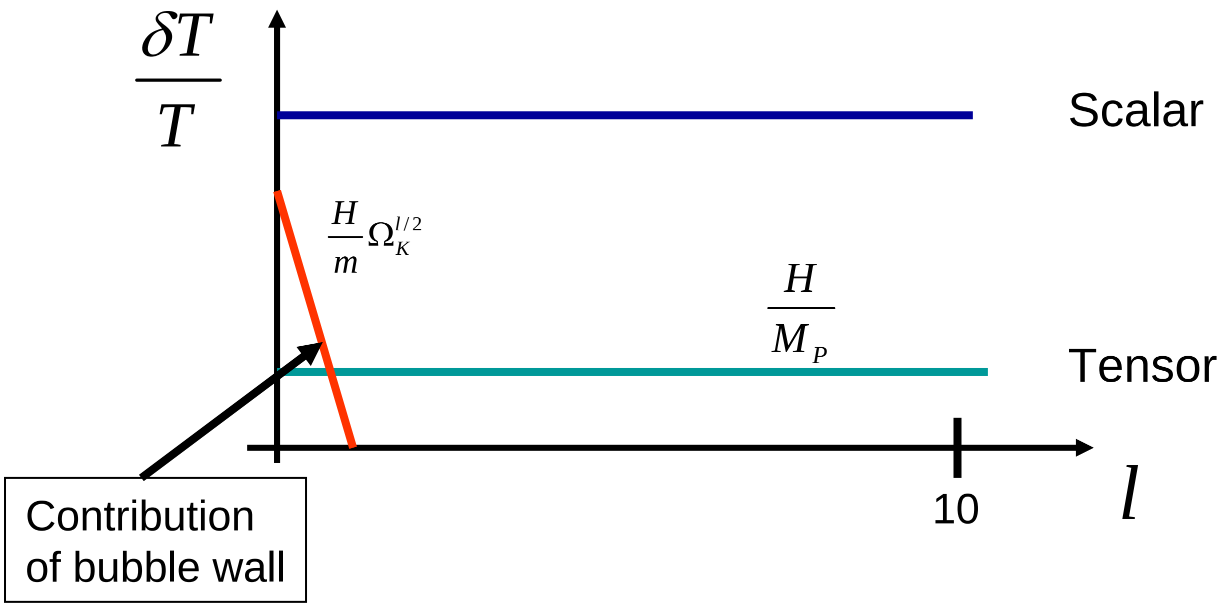
$$Y_{-3lm} \propto r^l \; Y_{lm}$$

$$r_{ls} \propto \Omega_K^{1/2}$$

$$\frac{\delta T}{T} \sim \frac{H}{M_P}\left(\frac{1}{G\mu R_0}\right)^{1/2} \Omega_K^{l/2}$$

$$\boxed{\Omega_K^{l/2} \sim 10^{-l}}$$

Exponentially decaying with multipole



We know it's small.
We may just hope it is not too small…

SUMMARY

1-We have proposed a regulator for **counting relative numbers of bubbles**, which satisfies certain desirable requirements.

2-This agrees with two other subsequent proposals (where applicable)

3-We have proposed a regulator for **counting objects within bubbles**

4-Boltzmann brains must be subtracted (they are irrelevant compared with fluctuations to higher or lower vacua, which produce an infinite number of objects.)

5-The measure we adopt determines what we might expect to observe. Such proposals may in principle be checked against observatios. (e.g. volume factors seem to preclude any curvature to be seen, or at least they make it very unlikely).