Brane Inflation : String Theory viewed from the Cosmos

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updated

A New View of the Cosmic Landscape

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How the universe can have such a small cosmological constant ?

- String theory has many meta-stable vacua, 10^{500} or more.
- They span a wide range of CC.
- Some of them have very small CC.
- This is the cosmic landscape.
- It is easy to convince ourselves that one of the string vacuum site in the landscape describes our universe.

Puzzle : of all sites, why we end up in one with such a small CC ?

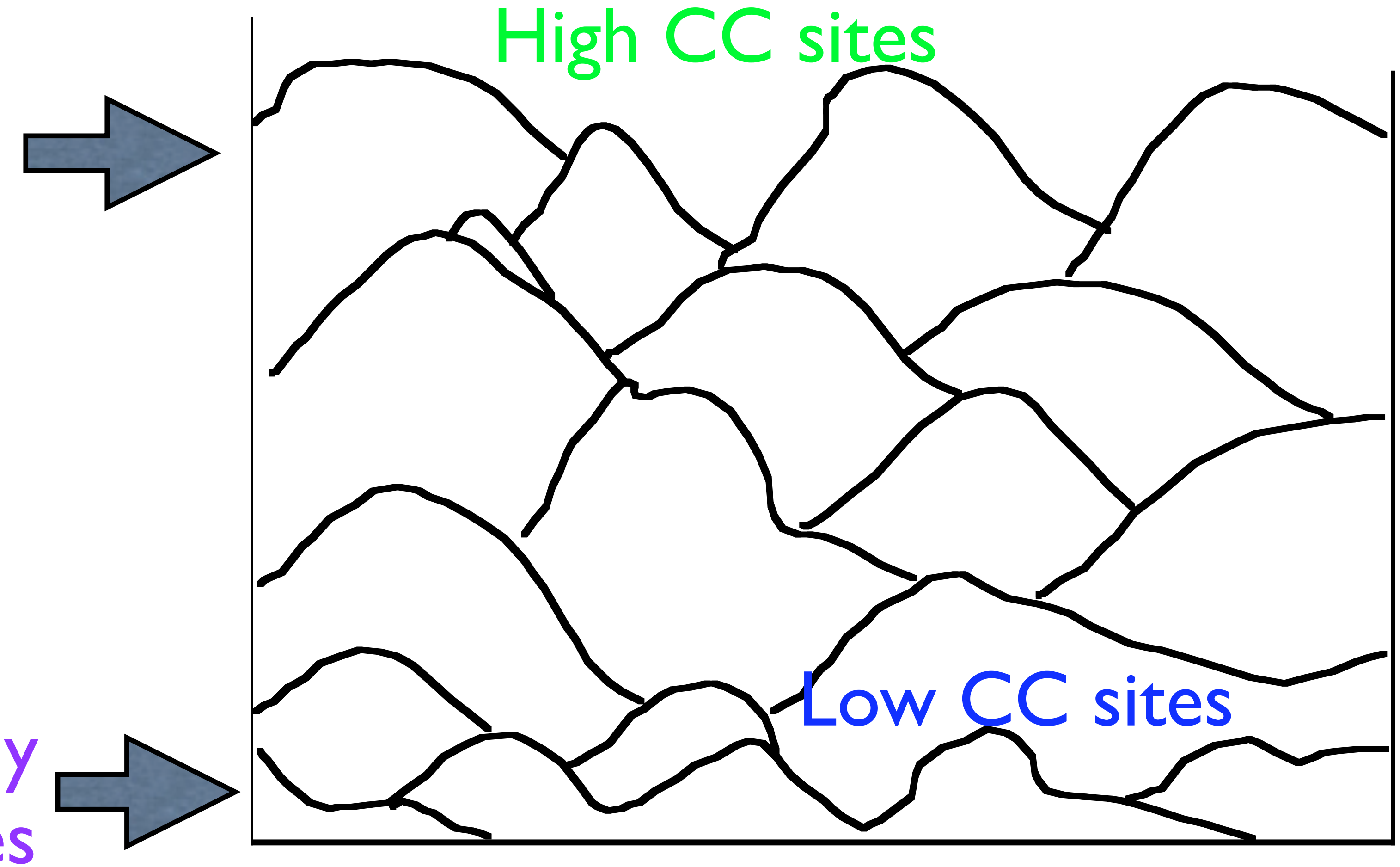
If we start at any meta-stable site with a long lifetime, we'll still be there now.

Eternal inflation !

What are the conditions under which a low CC universe will emerge naturally ?

very short lifetimes

exponentially long lifetimes



Suppose generic meta-stable dS sites have very short lifetimes while sites with small CC have long lifetimes.

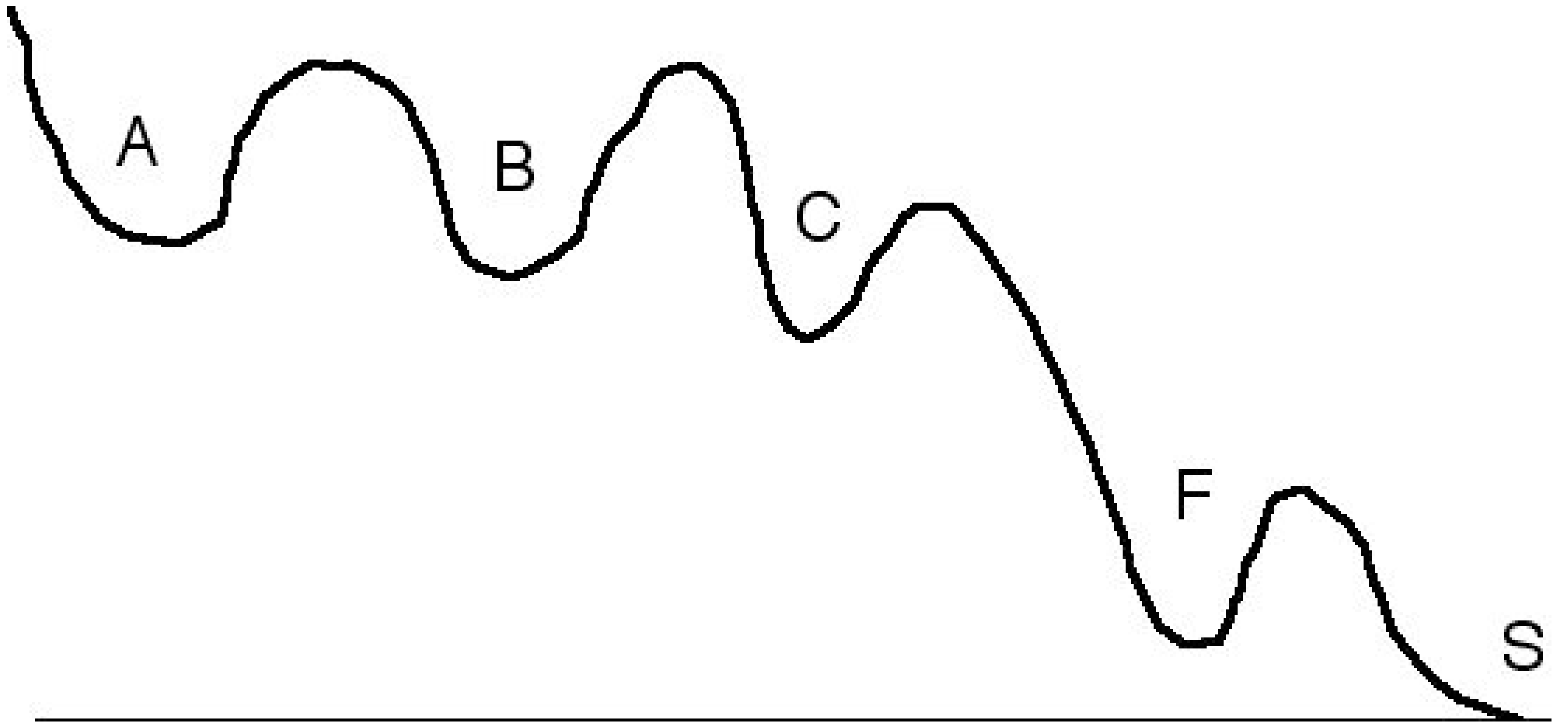
If we start at a generic site, it will quickly decay (and this decay process can repeat any number of times) until the universe reaches a site with a low CC and stays there.

EVEN BETTER :

Generic meta-stable dS sites have very short lifetimes (less than 1 second) while sites with CC below a certain value have exponentially long lifetimes (larger than the age of the universe).

Such a scenario is entirely possible.

- Vastness of the Landscape
- Resonance tunneling
- Topography of the Landscape



Resonance Tunneling :

Tunneling probabilities $T_{A \to B} \sim T_{B \to C}$ are exponentially small

When the condition is right : $T_{A \to C} \sim 1$

Resonance tunneling

Merzbacher, Quantum Mechanics, Chapter 7

$$\begin{pmatrix} \alpha_R \\ \alpha_L \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \Theta + \Theta^{-1} & i\left(\Theta - \Theta^{-1}\right) \\ -i\left(\Theta - \Theta^{-1}\right) & \Theta + \Theta^{-1} \end{pmatrix} \begin{pmatrix} \beta_R \\ \beta_L \end{pmatrix}$$

where, in the WKB approximation,

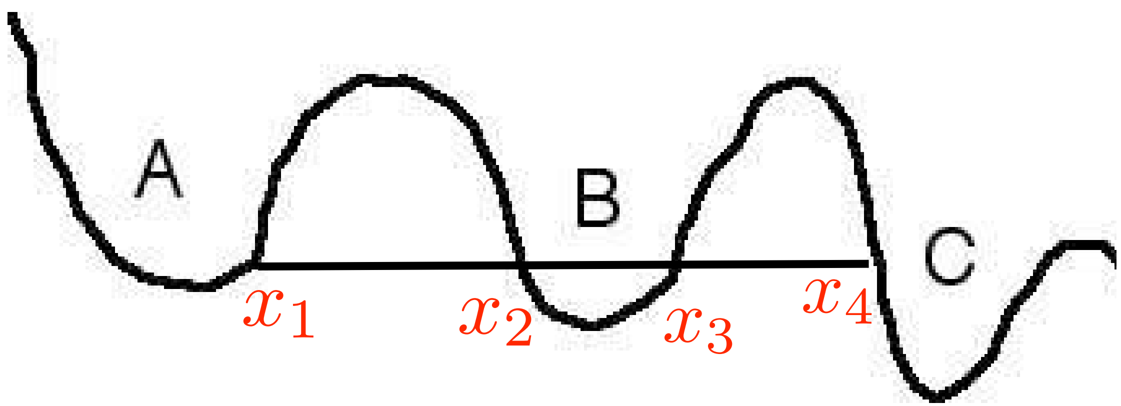
$$\Theta \quad \simeq \quad 2 \exp\left(\int_{x_1}^{x_2} dx \sqrt{2(V(x) - E)} \right) \; ,$$

$$T_{A \to B} = \left| \frac{\beta_R}{\alpha_R} \right|^2 = 4 \left(\Theta + \frac{1}{\Theta} \right)^{-2} \simeq \frac{4}{\Theta^2}$$

$$\Theta \simeq 2\exp\left(\int_{x_1}^{x_2} dx\sqrt{2(V(x)-E)}\right) \qquad \Phi = 2\exp\left(\int_{x_3}^{x_4} dx\sqrt{2(V(x)-E)}\right)$$

$$\frac{1}{4}\begin{pmatrix} \Theta+\Theta^{-1} & i\,(\Theta-\Theta^{-1}) \\ -i\,(\Theta-\Theta^{-1}) & \Theta+\Theta^{-1} \end{pmatrix}\begin{pmatrix} e^{-i\,W} & 0 \\ 0 & e^{i\,W} \end{pmatrix}\begin{pmatrix} \Phi+\Phi^{-1} & i\,(\Phi-\Phi^{-1}) \\ -i\,(\Phi-\Phi^{-1}) & \Phi+\Phi^{-1} \end{pmatrix}$$

$$W = \int_{x_2}^{x_3} \sqrt{2(E-V(x))}\,dx$$



WKB connection formula

$$\Gamma_{A\to C} = 4\left((\Theta\Phi + \frac{1}{\Theta\Phi})^2 \cos^2 W + (\Theta/\Phi + \Phi/\Theta)^2 \sin^2 W \right)^{-1}$$

In the absence of B, $W = 0$ so $T_{A\to C}$ is very small,

$$T_{A\to C} \simeq 4\Theta^{-2}\Phi^{-2} = T_{A\to B}T_{B\to C}/4$$

However, if W satisfies the quantum condition for bound states in B,

$$W = (n_B + 1/2)\pi$$

$$T_{A\to C} = \frac{4}{(\Theta/\Phi + \Phi/\Theta)^2}$$

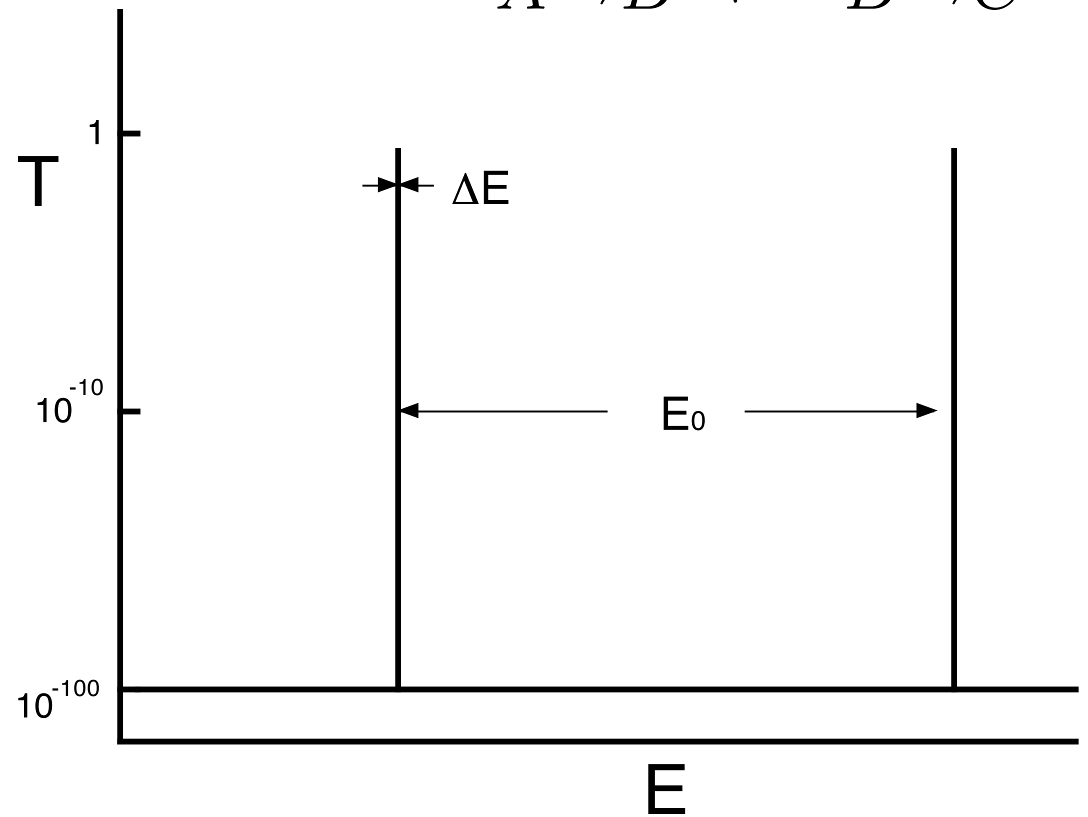
$$T_{A\to B} \to T_{B\to C} \quad \Rightarrow \quad T_{A\to C} \to 1$$

How likely to hit a resonance ?

$$P(A \to C) = \frac{\Delta E}{E_0} \simeq \frac{2}{\pi \Theta \Phi} \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta} \right) = \frac{1}{2\pi} \left(T_{A \to B} + T_{B \to C} \right)$$

$$< T_{A \to C} >= P(A \to C) T_{A \to C} \sim \frac{T_{A \to B} T_{B \to C}}{T_{A \to B} + T_{B \to C}}$$

Average tunneling probability is exponentially larger than the naive estimate.

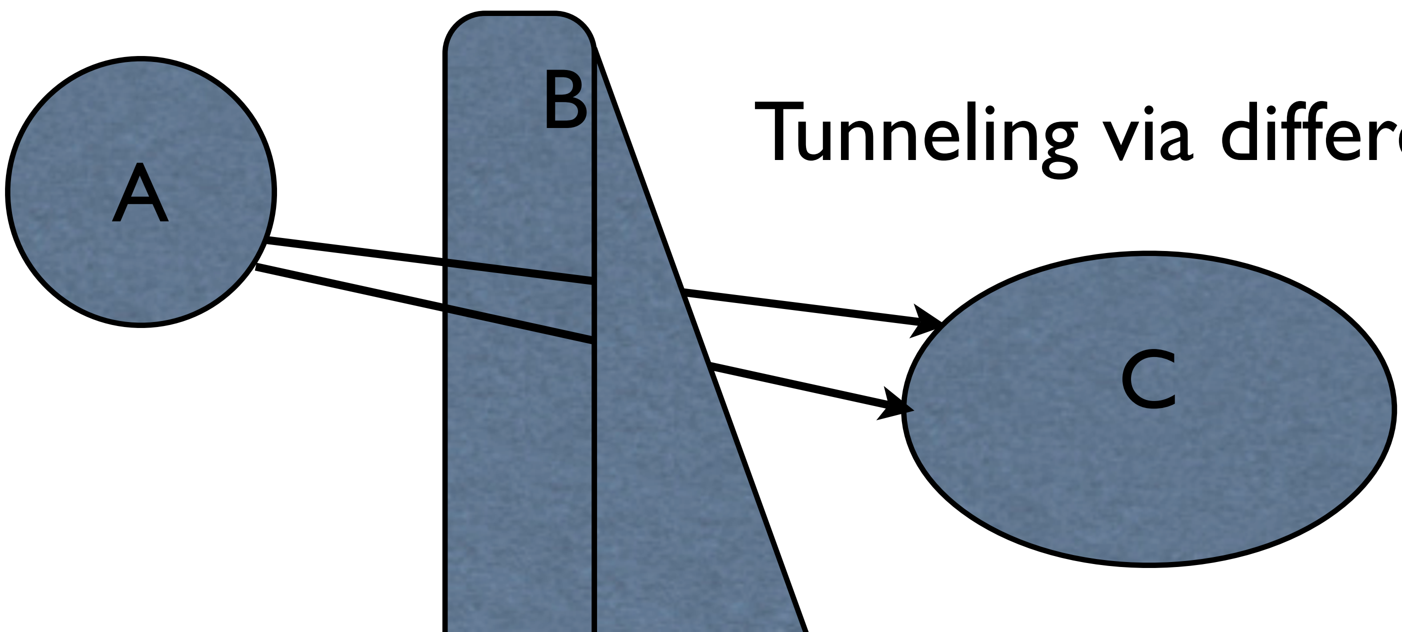


QM to QFT/Gravity

$$\phi_i(\mathbf{r}, t) \rightarrow \phi_i(t) = x_i(t)$$

$$U(\phi_i) \rightarrow V(x_i)$$

Resonance tunneling happens in QFT as well as in Gravity ?



Tunneling via different paths

Application to The Landscape

- Tunneling from a dS site to an AdS site is ignored.
- Tunneling from a dS site to another dS site with a larger CC is ignored.
- Evolution in a classically allowed region should be treated quantum mechanically.
- Sum over tunneling paths.

n-step tunneling channel :

$$A \;\rightarrow\; B_1 \;\rightarrow\; B_2 \;\rightarrow\; ..... \;\rightarrow\; B_{n-1} \;\rightarrow\; C$$

Naive :

$$\hat{T}_{A\rightarrow C}(n) \simeq T_{A\rightarrow B_1} T_{B_1\rightarrow B_2} .... T_{B_{n-1}\rightarrow C}$$

$$< T_{A\rightarrow C}(n) >= P(A\rightarrow C) T_{A\rightarrow C} \sim Min\left(T_{min}, \sqrt{\hat{T}_{A\rightarrow C}(n)}\right)$$

Tunneling probability is decreasing rapidly but still exponentially larger than the naive estimate.

Probability of hitting an efficient tunneling channel in n steps :

$$P(A \to C) \sim \left(T_{A\to B_1} \ldots . T_{B_{n-1}\to C}\right)^{1/2} \qquad n \geq 2$$

The number of efficient tunneling channels available to A :

$$N(A) = \sum_C P(A \to C) \qquad \Lambda_C < \Lambda_A$$

The number of sites C available to A is exponentially large.

Suppose the distribution of vacua is independent of CC.
A site with GUT scale CC has about 10^{100} more channels to tunnel to than our universe has.

$$N(A) = \sum_C P(A \to C) \qquad \Lambda_C < \Lambda_A$$

Both the number of decay sites C and the number of tunneling channels to C open to a site A decreases as its CC decreases.

$$\text{If } N(A) > 1, \text{ then } T_A \sim 1$$

$$\text{If } N(A) < 1$$

$$T_A \text{ would be exponentially small.}$$

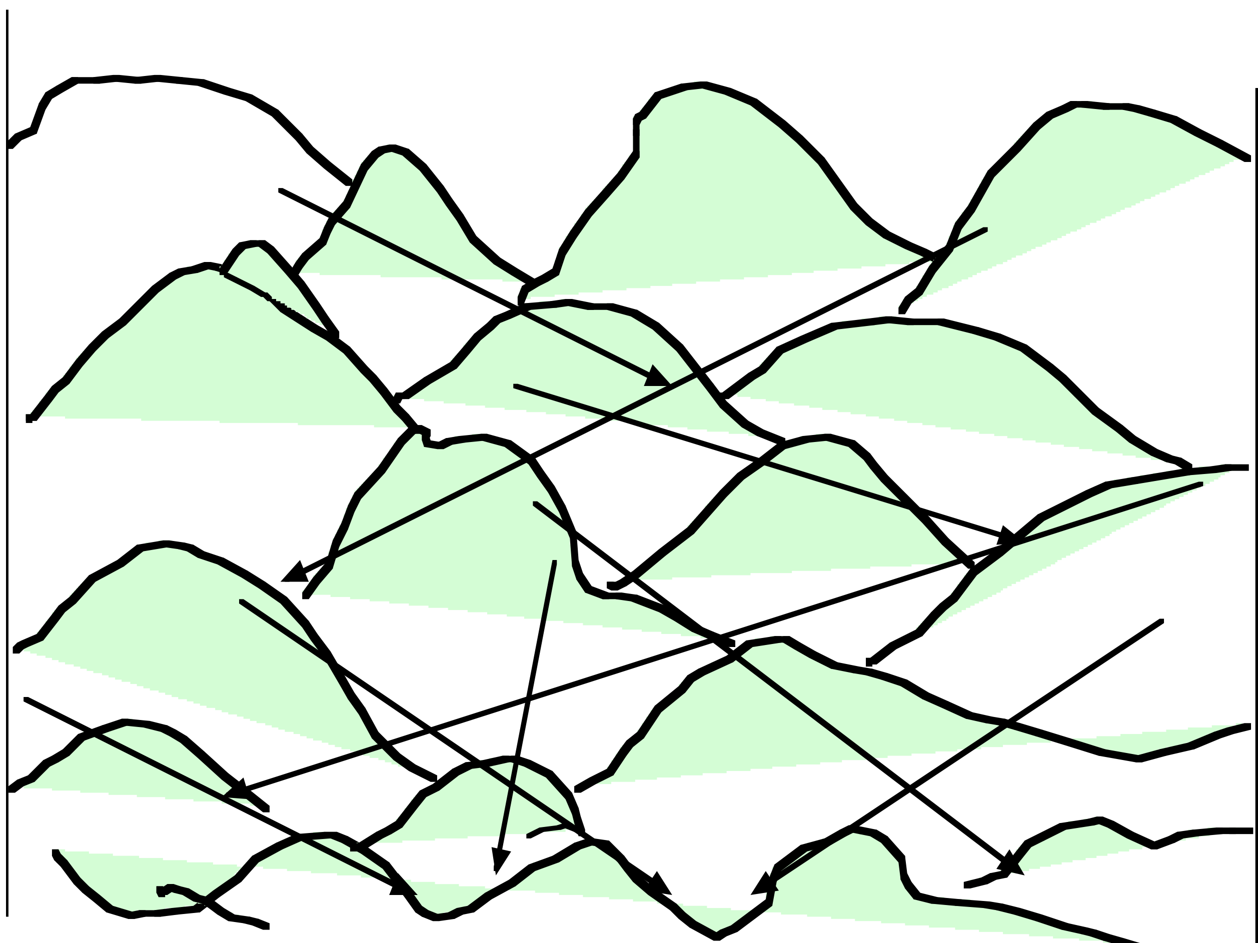
The Quantum Landscape

There is a critical CC :

$$N(A) > 1 \text{ for } \Lambda_A > \Lambda_c, \text{ and } N(A) < 1 \text{ for } \Lambda_A < \Lambda_c.$$

Decays quickly

very long lifetime



Eternal Inflation ?

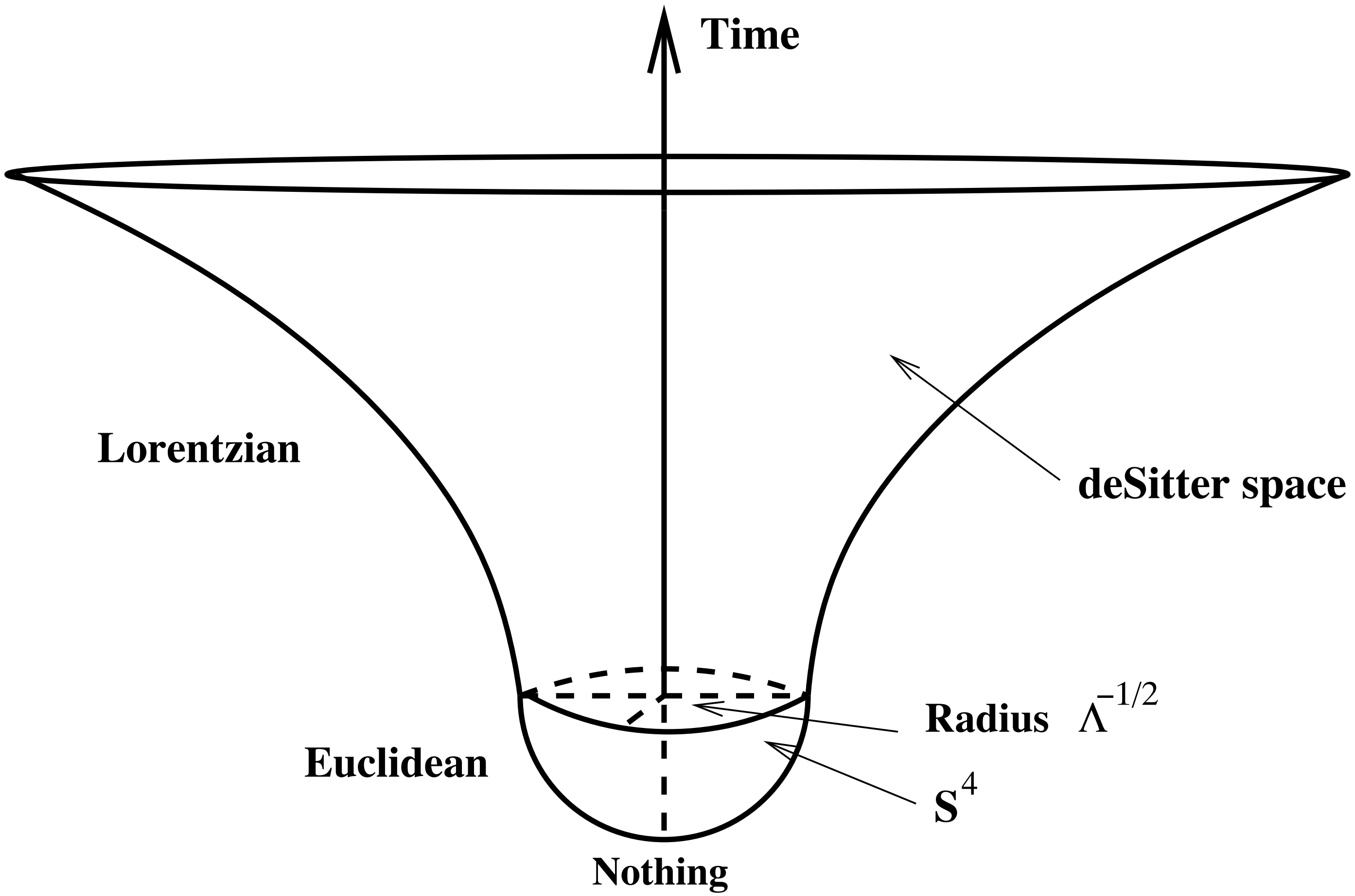
T ~ 1

$$\Gamma_A \sim M_s$$

$$H \sim M_s^2/M_P < \Gamma_A$$

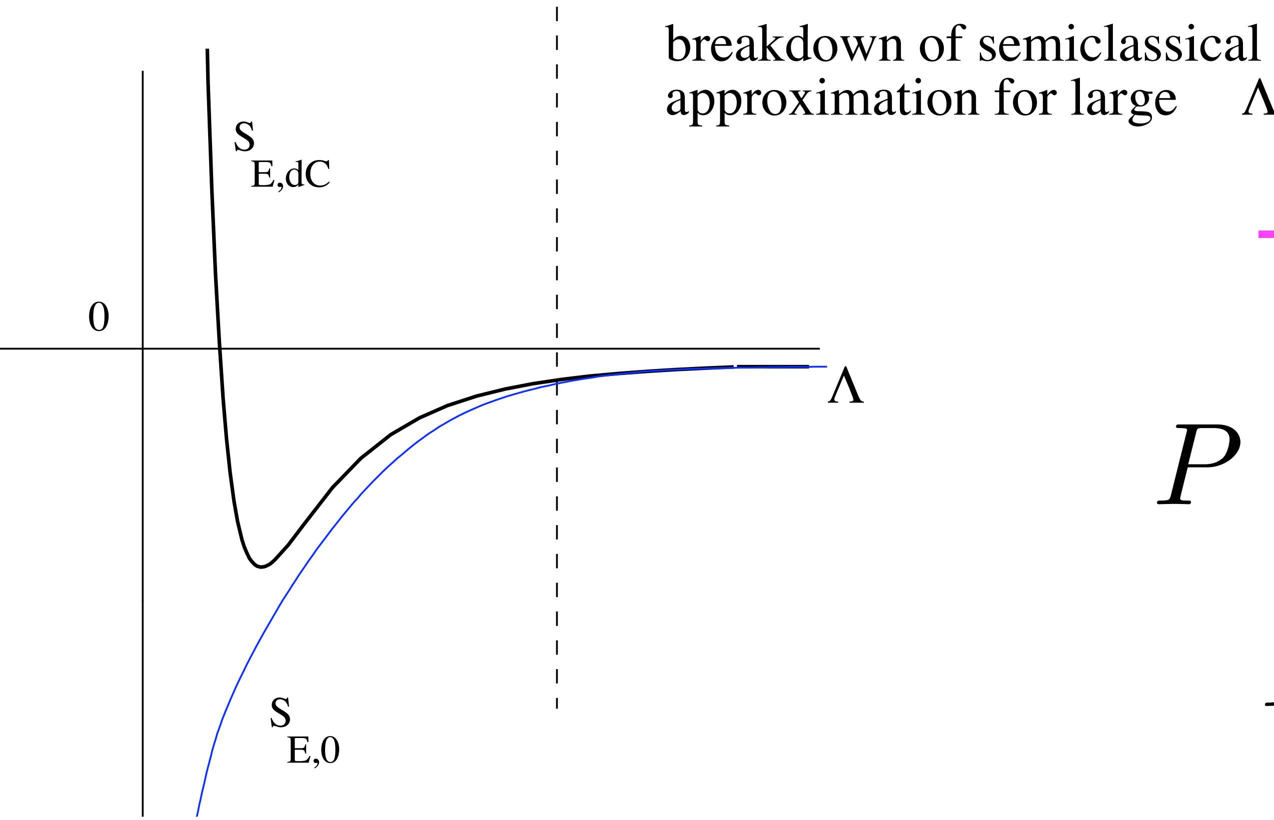
No eternal inflation !

Tunneling from Nothing



Vilenkin, 1982, 1983
Hartle and Hawking, 1983

The improved Euclidean action
$S_{E,dC}$ vs $S_{E,0}$



Tunneling probability

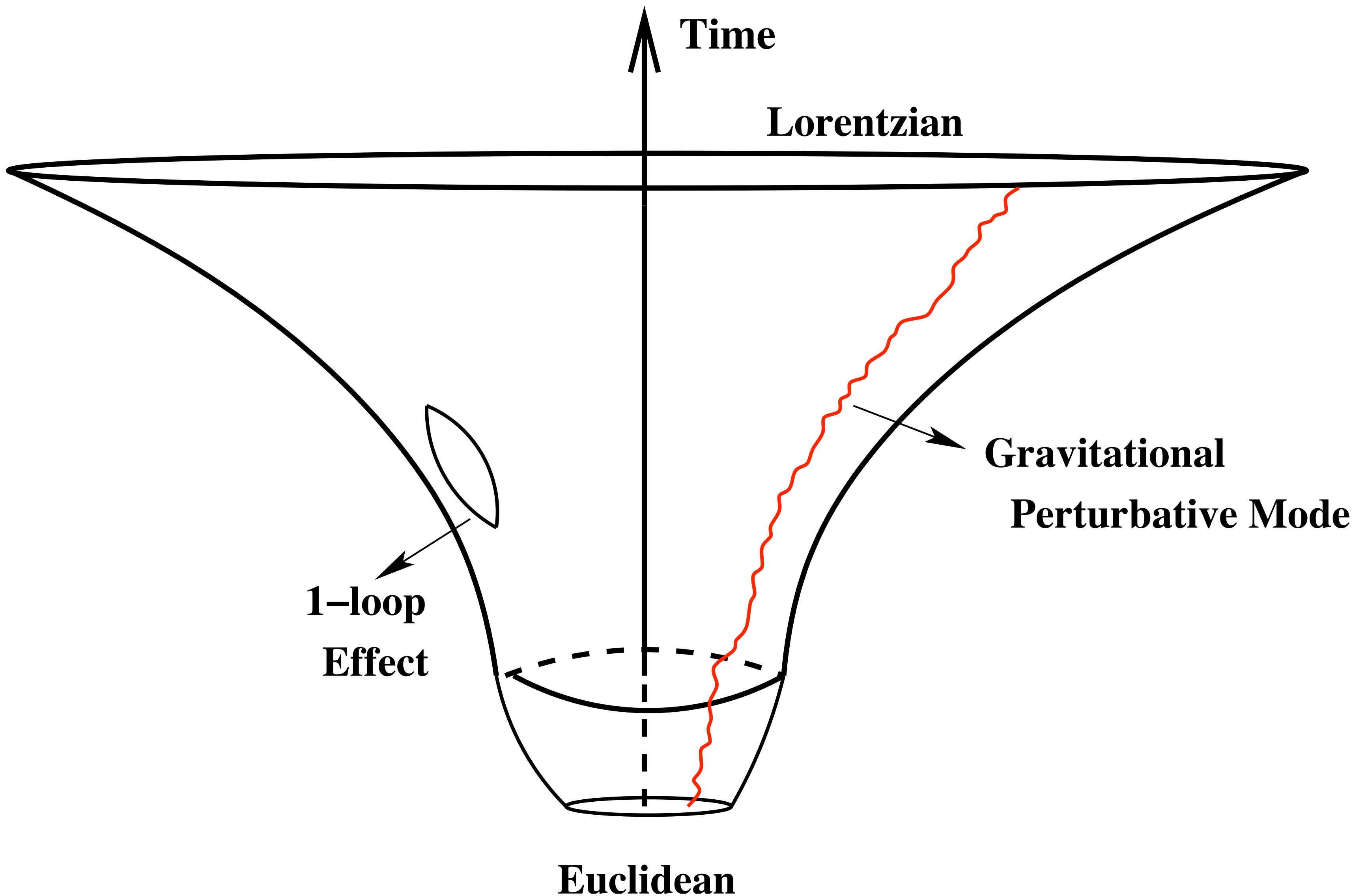
$$P \simeq e^{-S_{E,dC}} = e^{F}$$

$$F = \frac{3\pi}{G\Lambda} - \frac{12 n_d}{\Lambda^2 l_s^4}$$

$S_{E,0}$ is unbounded from below, but the interaction with the environment has made $S_{E,dC} = -F$ bounded from below.

Firouzjahi, Sarangi and H.T.

Tunneling from Nothing



Nothing

History

- The Universe is a spontaneous creation from NOTHING.
- It has a CC somewhere below the string scale.
- It then rapidly decays to a vacuum site in the landscape with an exponentially long lifetime.
- This is where we live.
- Resonance tunneling may happen both before and after inflation.

Conclusion

If the scenario is correct, we can appreciate string theory in this new light : it provides a vast landscape so that a small CC vacuum is among its solutions, and the same vastness destabilizes all vacua except ones with a very small CC, thus allowing our universe to emerge, survive and grow.

We still need to fold this scenario into the cosmological evolution of the universe. A proper treatment requires us to start with the wavefunction of the universe.
It is a challenging but tractable exercise to check the validity of this scenario.