From Unified Theories to Precision Neutrino Experiments

Talk by Stefan Antusch
University of Southampton

at the

IFT Mini Workshop on Neutrino Physics 2005

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Content

RG running of neutrino parameters in see-saw models

- RGEs and running for the neutrino mass operator
- RG evolution above and between the see-saw scales
- Analytic approximations: understanding and estimating RG effecs

Consequences for model building

- Possibilities from large RG effects

Consequences with respect to future precision neutrino experiments

- RG corrections to θ_{23} = 45°, θ_{13} = 0° and $\theta_{12} + \theta_C$ = 45° at M_U

Summary

Motivation

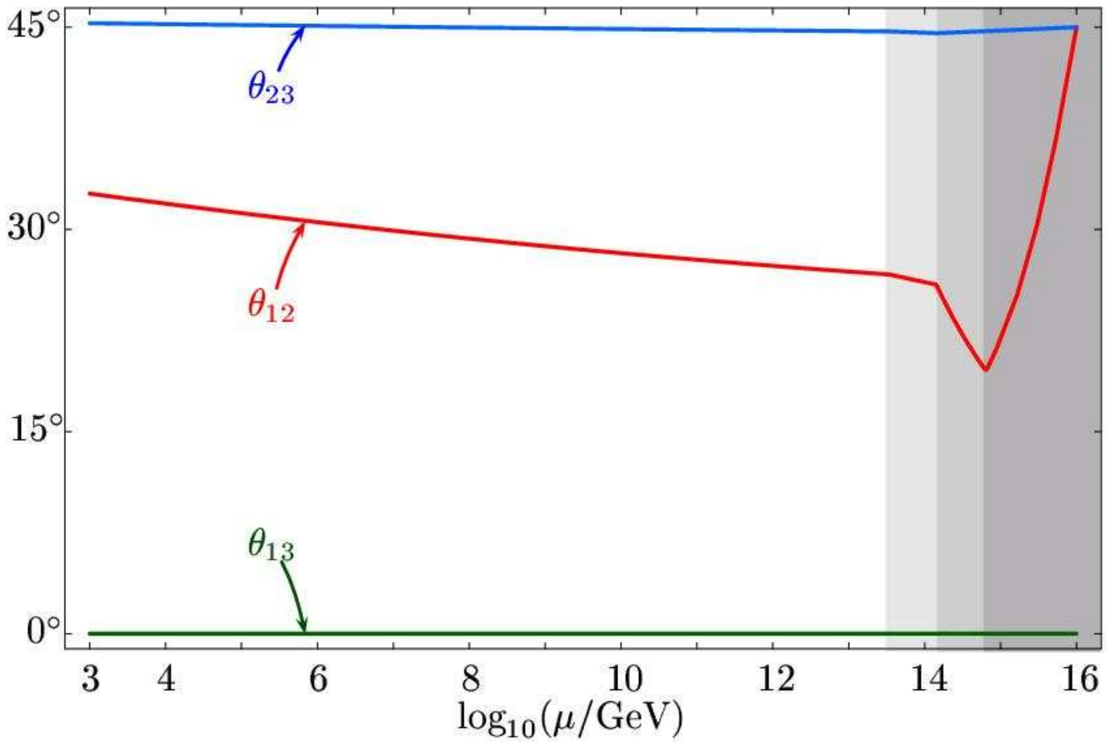
Low energy:

- Future: precision ν exp.!
  - θ_{13}? θ_{23} maximal?
    $\theta_{12} + \theta_C = 45°$? δ?
  - ν mass scale/ scheme?

RG Running ⟷

High energy:

- Origin of ν masses?
- Unified Theories?
- Flavour symmetries?
- ...



Mixing Angles and CP Phases in the Lepton Sector

Definition:

Charged lepton masses — ν-masses (assumed: 3 Majorana ν's)

$$U_{e_L} M_e U_{e_R}^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_{\nu_L} m_{LL}^\nu U_{\nu_L}^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Charged EW current

$$\overline{e'_L}^f \gamma^\mu \nu'^f_L W_\mu^- \stackrel{!}{=} \overline{e_L}^f \gamma^\mu U_{MNS} \nu_L^f W_\mu^- \quad \Rightarrow \quad \boxed{U_{MNS} = U_{e_L} U_{\nu_L}^\dagger}$$

Mixing matrix in the lepton sector

Parametrization:

$$U_{MNS} = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}) \cdot U \cdot \mathrm{diag}(e^{i\frac{\varphi_1}{2}}, e^{i\frac{\varphi_2}{2}}, 1)$$

Majorana CP phases

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Dirac CP phase for the leptons

Known:

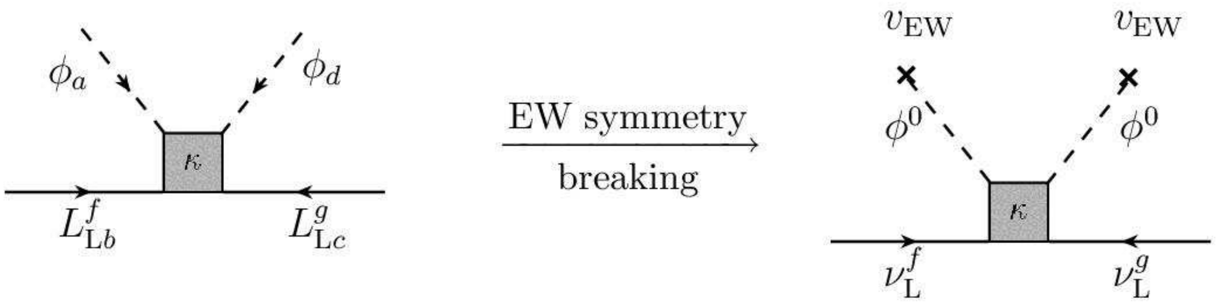
- $\theta_{12} \approx 33°$, Δm^2_{21}
- $\theta_{23} \approx 45°$, Δm^2_{31}
- $\theta_{13} < 13°$

Unknown:

- ν mass scale and mass scheme
- CP phases, ...

The Lowest Dimensional Neutrino Mass Operator

No renormalizable mass term for neutrinos in the SM (MSSM)



$$\mathscr{L}_\kappa^{SM} = \frac{1}{4}\kappa_{gf}\,\overline{L^{C}_{Lc}}^{g}\varepsilon^{cd}\phi_d\, L^f_{Lb}\varepsilon^{ba}\phi_a + \text{h.c.}\,,$$

$$\mathscr{L}_\kappa^{MSSM} = -\frac{1}{4}\kappa_{gf}\,\hat{L}^g_c\varepsilon^{cd}(\hat{H}_u)_d\,\hat{L}^f_b\varepsilon^{ba}(\hat{H}_u)_a\big|_{\theta\theta} + \text{h.c.}$$

$$\mathscr{L}_{\nu_L\nu_L} = \frac{1}{2}(m^\nu_{LL})_{gf}\,\overline{\nu}^g_L\nu^{Cf}_L$$

Neutrino mass matrix of Majorana type

Smallness of neutrino masses:

$$\kappa \sim \frac{1}{\Lambda}$$

High realisation scale Λ (scale of new physics)

Neutrinos provide a window to physics at very high energies!

Beyond the SM: LR Symmetric Unified Theories

Pati-Salam Unification: $G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$

RH neutrinos

$$f_L^f = \begin{pmatrix} u_{Lr}^f & u_{Ly}^f & u_{Lb}^f & \nu_L^f \\ d_{Lr}^f & d_{Ly}^f & d_{Lb}^f & e_L^f \end{pmatrix}, \quad f_R^f = \begin{pmatrix} u_{Rr}^f & u_{Ry}^f & u_{Rb}^f & \nu_R^f \\ d_{Rr}^f & d_{Ry}^f & d_{Rb}^f & e_R^f \end{pmatrix}$$

SU(2)$_L$ triplet Higgs (neutrino masses from coupling $(y_L)_{fg}\, L^f\, \Delta_L\, L^g$)

Field	f_L^f	f_R^{Cf}	Φ	Φ'	χ_L	χ_R^*	Δ_L	Δ_R^*
$SU(4)_C$	$\mathbf{4}$	$\overline{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{15}$	$\mathbf{4}$	$\overline{\mathbf{4}}$	$\mathbf{10}$	$\overline{\mathbf{10}}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$
$SU(2)_R$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{3}$

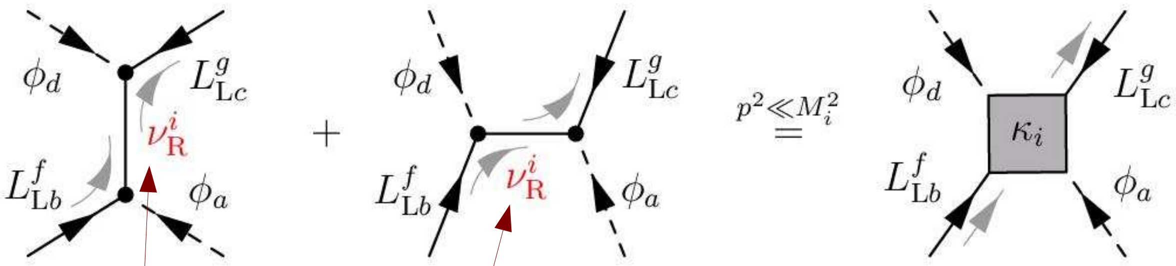
$$G_{422} \xrightarrow{\langle\Delta_R\rangle \text{ or } \langle\chi_R\rangle} G_{321} \xrightarrow{\langle\Phi\rangle,\, \langle\Phi'\rangle} SU(3)_C \times U(1)_e$$

Large masses for the RH neutrinos (from ren. coupling $(y_R)_{fg}\, R^f\, \Delta_R\, R^g$)

SO(10) GUTs: $\mathbf{16}^f_{SO(10)} = (\mathbf{4},\mathbf{2},\mathbf{1})^f_{G_{422}} + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})^f_{G_{422}} = f_L^f + f_R^{Cf}$

Generating the Neutrino Mass Operator

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)



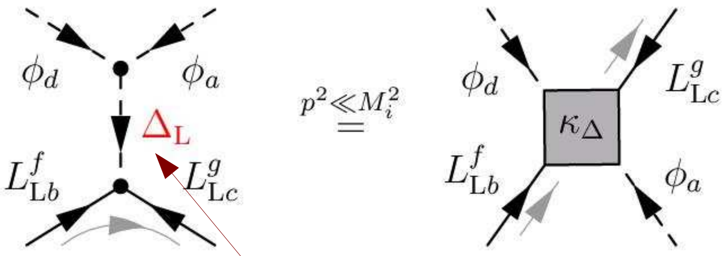
heavy RH neutrinos

Type I see-saw

$$\overset{(1)}{\kappa} = \sum_i \kappa_i = 2(Y_\nu^T)_{gi}(M^{-1})_{ij}(Y_\nu)_{jf}$$

Lazarides, Magg, Mohapatra, Schechter, Senjanovic, Shafi, Valle, Wetterich (1981)

Type I +



heavy $SU(2)_L$-triplet

Type II see-saw

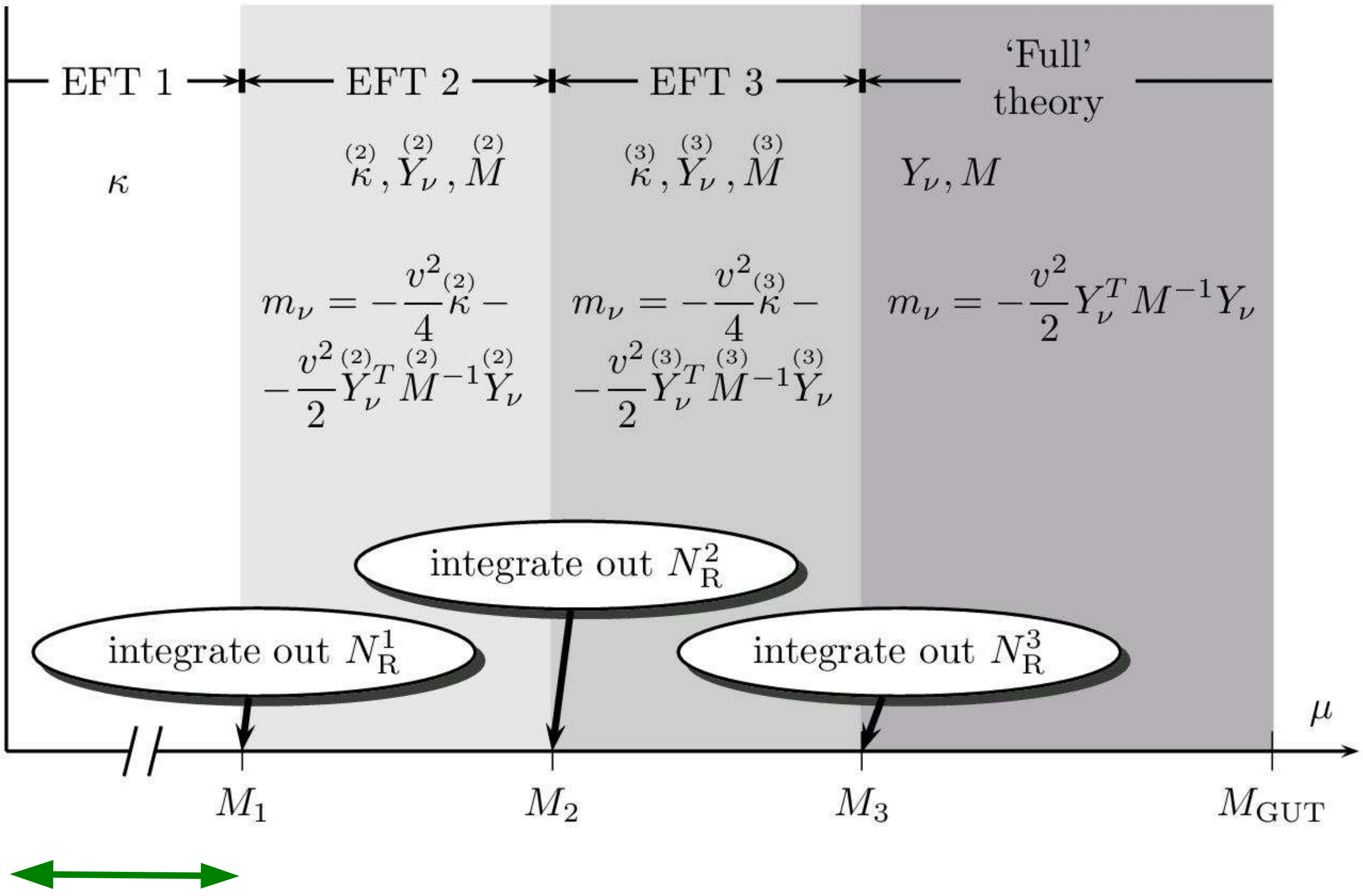
$$\overset{(1)}{\kappa} = \kappa_\Delta + \sum_i \kappa_i$$
$$= -m^{II} + 2(Y_\nu^T)_{gi}(M^{-1})_{ij}(Y_\nu)_{jf}$$

- We will mostly discuss RG effects for ν parameters in the type I framework (example!)
- Note that the running has important consequences in many other scenarios as well
  (other variants of the see-saw, Dirac neutrinos, ν masses from the Kähler potential, ...)

RG effects (large mixing can be fixed point): J.A. Casas, J.R. Espinosa, I. Navarro ('02,'03)

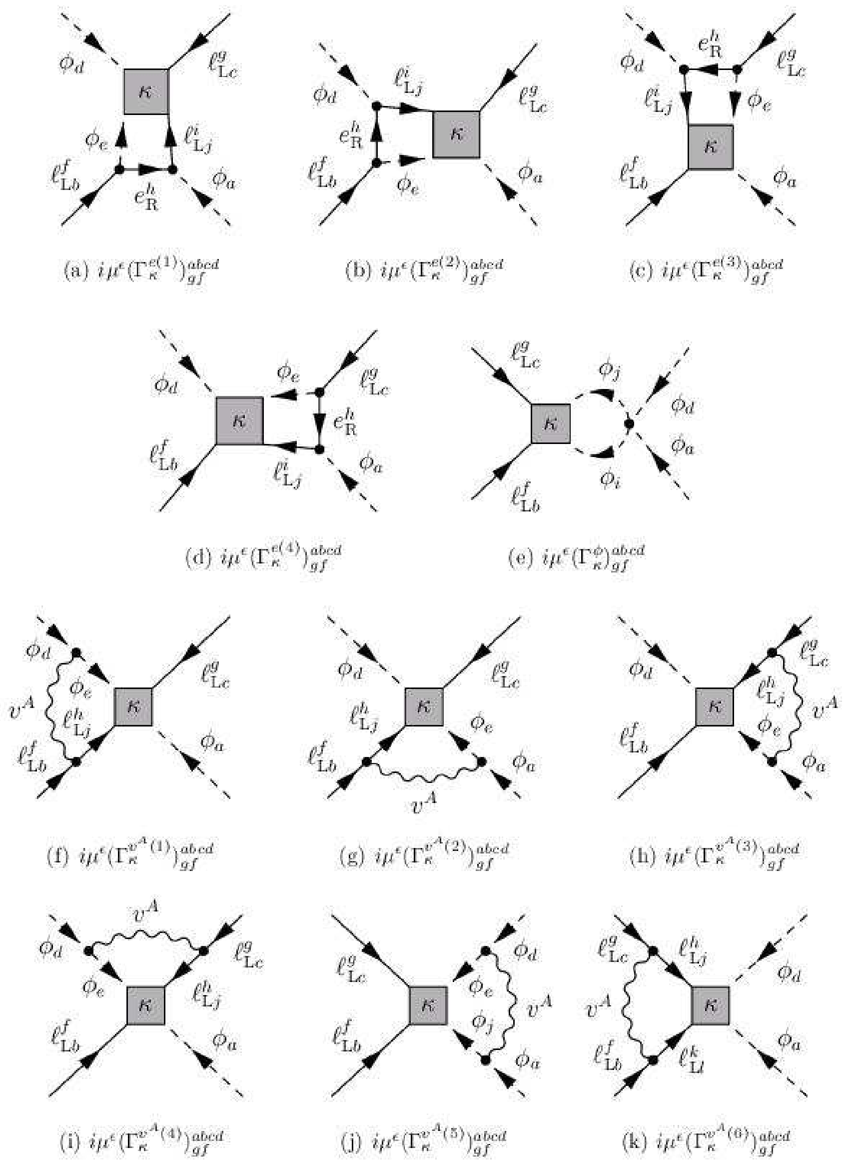
RG Running in 'Minimal' See-Saw Scenarios

(Type I see-saw)



Running of the dim. 5 operator: 'model independent'

β-Functions for the dim. 5 Neutrino Mass Operator



β-Function in the Standard Model:

$$16\pi^2\beta_\kappa = -\frac{3}{2}\kappa\,(Y_e^\dagger Y_e) - \frac{3}{2}(Y_e^\dagger Y_e)^T\kappa + \lambda\kappa - 3g_2^2\kappa$$
$$+2\mathrm{Tr}\,(Y_e^\dagger Y_e)\,\kappa + 6\mathrm{Tr}\,(Y_u^\dagger Y_u)\,\kappa + 6\mathrm{Tr}\,(Y_d^\dagger Y_d)\,\kappa$$

S.A., M. Drees, J. Kersten, M. Lindner, M. Ratz
Phys. Lett. B 519, (2001), (hep-ph/0108005)

β-Functions in 2-Higgs-Doublet Models:

[redacted]

S.A., M. Drees, J. Kersten, M. Lindner, M. Ratz
Phys. Lett. B 525, (2001), (hep-ph/0110366);
For multi-Higgs models:
W. Grimus, L. Lavoura, hep-ph/0409231

β-Functions for the dim. 5 Neutrino Mass Operator in the MSSM

1-loop part:

$$16\pi^2 \beta_\kappa^{(1)} = \kappa\,(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa - 2g_1^2\kappa - 6g_2^2\kappa + 6\,\mathrm{Tr}\,(Y_u^\dagger Y_u)\,\kappa$$

Chankowski, Pluciennik, Phys. Lett. B316 (1993);
Babu, Leung, Pantaleone, Phys. Lett. B319, (1993).

2-loop part:

$$\begin{aligned}(4\pi)^4 \beta_\kappa^{(2)} = &\Big[-6\,\mathrm{Tr}\,(Y_u^\dagger Y_d Y_d^\dagger Y_u) - 18\,\mathrm{Tr}\,(Y_u^\dagger Y_u Y_u^\dagger Y_u) + \frac{8}{5} g_1^2\,\mathrm{Tr}\,(Y_u^\dagger Y_u) \\ &+ 32\, g_3^2\,\mathrm{Tr}\,(Y_u^\dagger Y_u) + \frac{207}{25} g_1^4 + \frac{18}{5} g_1^2 g_2^2 + 15\, g_2^4\Big]\,\kappa \\ &- \left[2\,(Y_e^\dagger Y_e Y_e^\dagger Y_e)^T - \left(\frac{6}{5} g_1^2 - \mathrm{Tr}\,(Y_e Y_e^\dagger) - 3\,\mathrm{Tr}\,(Y_d Y_d^\dagger)\right)(Y_e^\dagger Y_e)^T\right]\kappa \\ &- \kappa\left[2\,Y_e^\dagger Y_e Y_e^\dagger Y_e - \left(\frac{6}{5} g_1^2 - \mathrm{Tr}\,(Y_e Y_e^\dagger) - 3\,\mathrm{Tr}\,(Y_d Y_d^\dagger)\right) Y_e^\dagger Y_e\right]\end{aligned}$$

S.A., M. Ratz, JHEP 0207 (2002) 059

Analytical Approximations: Below M_{R1}

Example: Formulae for the mixing angles

J.A. Casas, J.R. Espinosa, A. Ibarra I. Navarro ('99)
P.H. Chankowski, W. Krolikowski, S. Pokorski ('99)
S.A., J. Kersten, M. Lindner, M. Ratz ('03)

Strong enhancement possible due to small mass squared difference

Dependence on CP phases

$t := \ln(\mu/\mu_0)$

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2 \frac{|m_1\, e^{i\varphi_1} + m_2\, e^{i\varphi_2}|^2}{\Delta m^2_{\text{sol}}} + \mathscr{O}(\theta_{13})$$

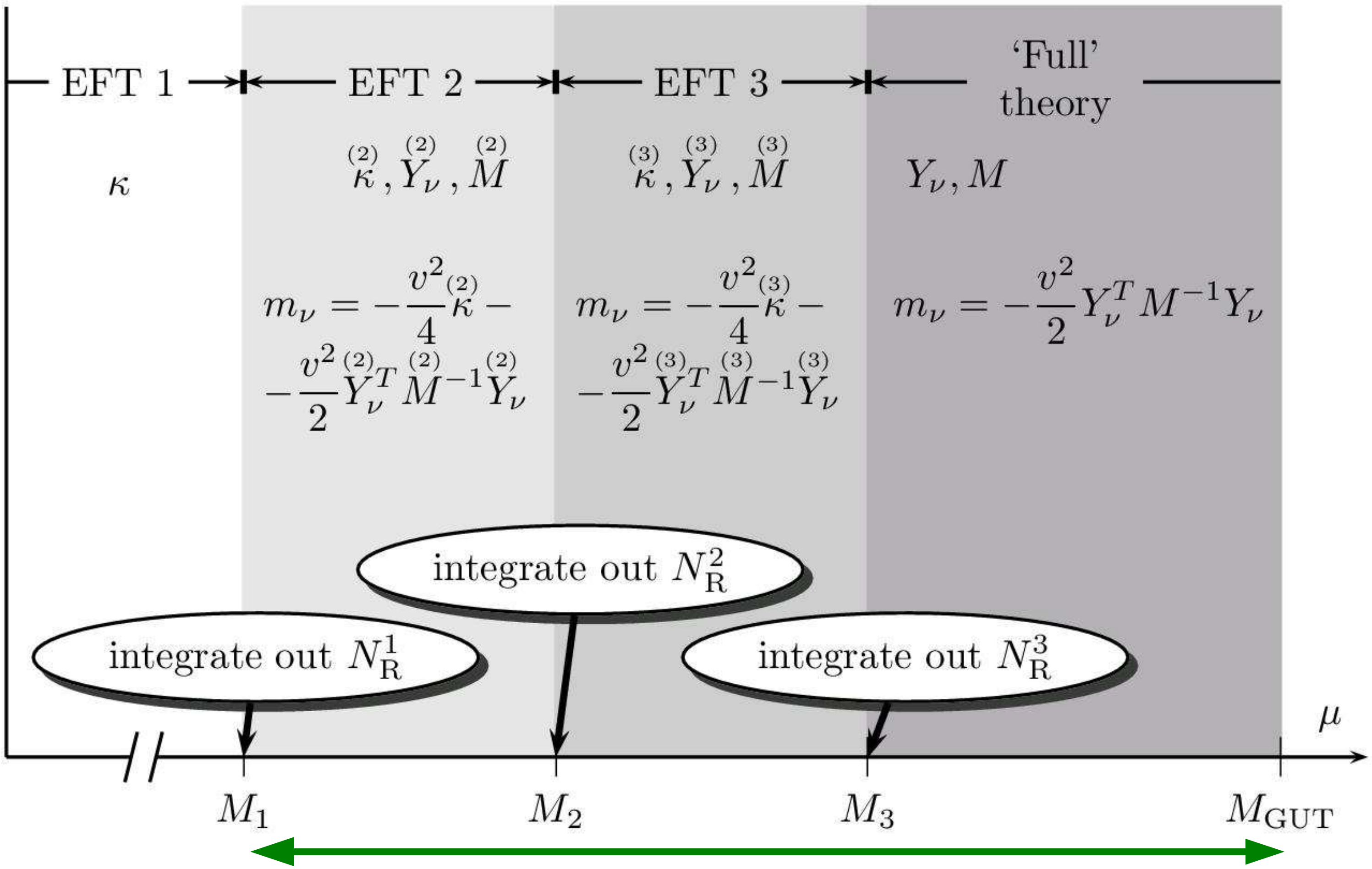
$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\text{atm}}} \left[m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)\right] + \mathscr{O}(\theta_{13})$$

$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{\text{atm}}} \left[c_{12}^2\, |m_2\, e^{i\varphi_2} + m_3|^2 + s_{12}^2\, |m_1\, e^{i\varphi_1} + m_3|^2\right] + \mathscr{O}(\theta_{13})$$

Generically:
- Running enhanced for larger neutrino masses
- Running in the MSSM enhanced for large tan β

(shown: MSSM, leading order in θ_{13} and in $\Delta m^2_{sol} / \Delta m^2_{atm}$)

RG Running in 'Minimal' See-Saw Scenarios

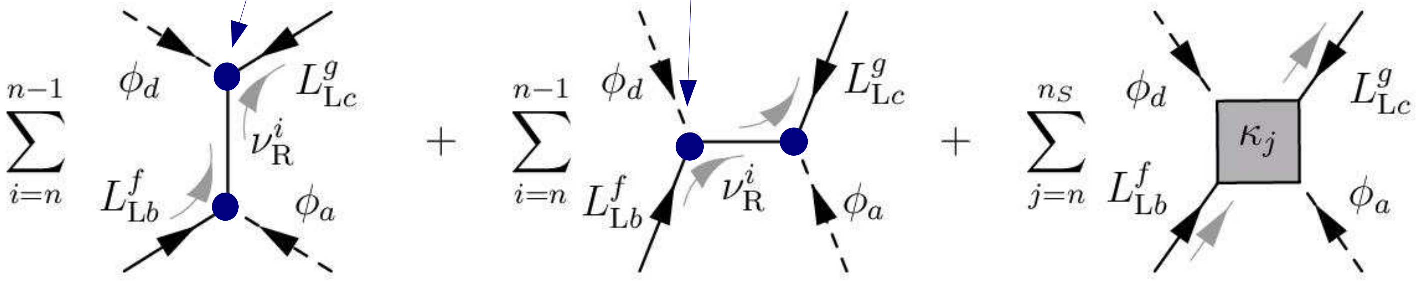


Importance of running above the see-saw thresholds:

J.A. Casas, J.R. Espinosa, A. Ibarra I. Navarro ('99); S.F. King, M. Singh ('00); S.A., J. Kersten, M. Lindner, M. Ratz ('02); S.A., M. Ratz ('02); T. Shindou, E. Takasugi ('04); Jian-wei Mei, Zhi-zhong Xing ('04); S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt ('05); Jian-wei Mei ('05);

Between the See-Saw Scales: ν_{Ri} partly integrated out

Neutrino Yukawa couplings



$M_i < \mu$
(ν_{Ri} propagate
in the theory)

$M_i > \mu$
(ν_{Ri} effectively
integrated out)

$$
Y_\nu \to \left(\begin{array}{ccc}
(Y_\nu)_{1,1} & \cdots & (Y_\nu)_{1,n_F} \\
\vdots & & \vdots \\
(Y_\nu)_{n-1,1} & \cdots & (Y_\nu)_{n-1,n_F} \\
\hline
0 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & 0
\end{array}\right)
\quad \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} =: \overset{(n)}{Y}_\nu
$$

← Neutrino Yukawa matrix between the see-saw scales

(n): below n-th see-saw scale

S.A., J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B538 (2002)

β-Functions above and between the See-Saw Scales

Effective neutrino mass $m_\nu = \frac{v_{\mathrm{EW}}^2}{4}\left(\overset{(n)}{\kappa} + 2\,\overset{(n)}{Y}{}_\nu^T\,\overset{(n)}{M}{}^{-1}\,\overset{(n)}{Y}_\nu\right)$

Effects from the neutrino Yukawa couplings

$$
\begin{aligned}
16\pi^2\,\overset{(n)}{\beta}_X = {} & C_e (Y_e^\dagger Y_e)^T\,\overset{(n)}{X} + C_e\,\overset{(n)}{X}\,(Y_e^\dagger Y_e) + C_\nu \left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right)^T \overset{(n)}{X} + C_\nu\,\overset{(n)}{X}\left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right) \\
& + \alpha_e\,\mathrm{Tr}(Y_e^\dagger Y_e)\,\overset{(n)}{X} + \alpha_\nu\,\mathrm{Tr}\left(\overset{(n)}{Y}{}_\nu^\dagger \overset{(n)}{Y}_\nu\right)\overset{(n)}{X} + \alpha_d\,\mathrm{Tr}(Y_d^\dagger Y_d)\,\overset{(n)}{X} \\
& + \alpha_u\,\mathrm{Tr}(Y_u^\dagger Y_u)\,\overset{(n)}{X} + \alpha_{g_1} g_1^2\,\overset{(n)}{X} + \alpha_{g_2} g_2^2\,\overset{(n)}{X} + \alpha_\lambda \lambda\,\overset{(n)}{X}
\end{aligned}
$$

Different coefficients for κ and $2\,Y_\nu^T M^{-1} Y_\nu$

⇒ Running of mixing angles

(note: not in SUSY, non-ren. theorem!)

(1-loop)

Model	Quantity $\overset{(n)}{X}$	C_e	C_ν	α_e	α_ν	α_d	α_u	α_{g_1}	α_{g_2}	α_λ
SM	$\overset{(n)}{\kappa}$	$-\frac{3}{2}$	$\frac{1}{2}$	2	2	6	6	0	-3	1
SM	$2\,\overset{(n)}{Y}{}_\nu^T\,\overset{(n)}{M}{}^{-1}\,\overset{(n)}{Y}_\nu$	$-\frac{3}{2}$	$\frac{1}{2}$	2	2	6	6	$-\frac{3}{2}$	$-\frac{9}{2}$	0
MSSM	$\overset{(n)}{\kappa}$	1	1	0	2	0	6	-2	-6	0
MSSM	$2\,\overset{(n)}{Y}{}_\nu^T\,\overset{(n)}{M}{}^{-1}\,\overset{(n)}{Y}_\nu$	1	1	0	2	0	6	-2	-6	0

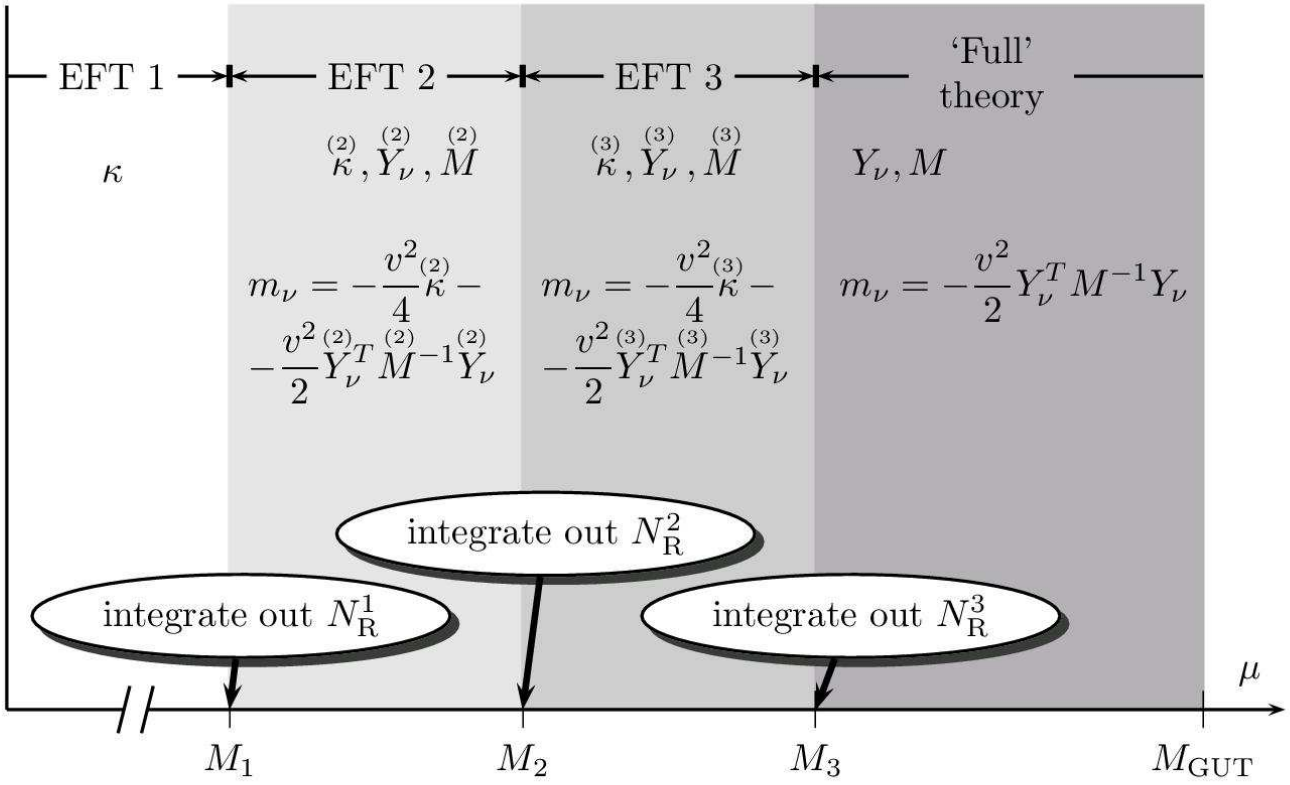
S.A., J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B538 (2002)

Analytical approximations (above M_{Ri}): RGEs for θ_{ij}, δ, φ_1, φ_2, m_i and Δm^2's

S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt, hep-ph/0501272; see also: Jian-wei Mei, hep-ph/0502015

Numerical RG Evolution: REAP/MPT Software Packages

introduced in: S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt JHEP 0503 (2005) 024, (hep-ph/0501272)



- REAP (Renormalization group Evolution of Angles and Phases): Mathematica package for numerical RG evolution (heavy RH neutrinos successively integrated out)
- MPT (Mixing Parameter Tools): Mathematica package for extracting masses, mixing angles and CP phases from the quark and lepton mass matrices

public, download: www.ph.tum.de/~rge

'Corrections' to the Minimal Scenario

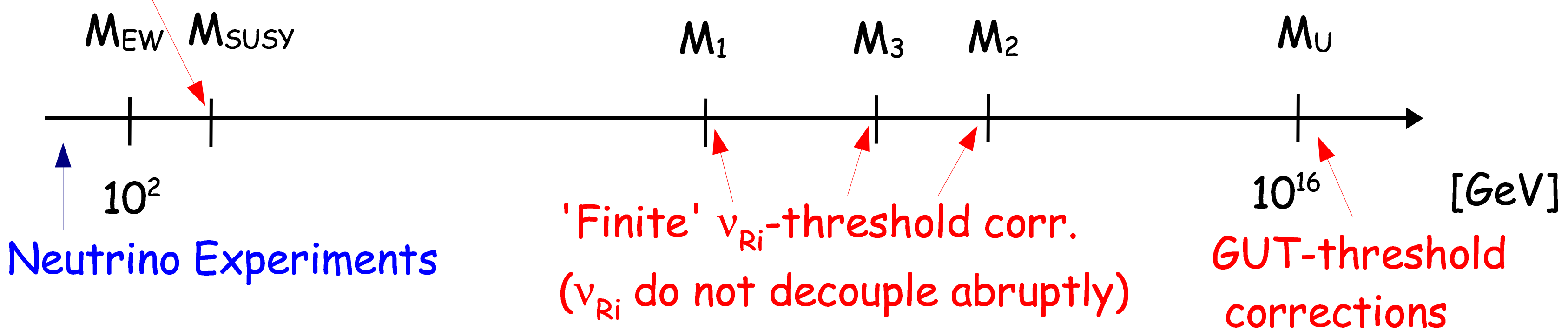
- For higher accuracy: 2-loop running   RGE's in the MSSM + ν_{Ri}: S.A., M. Ratz ('02)

- Additional (1-loop) threshold corrections

SUSY-threshold corrections
see e.g. : E.J. Chun, S. Pokorski ('99), P.H. Chankowski, P. Wasowicz ('01), ...

Eventually: type II see-saw scale $M_\Delta < M_U$ (for: $M_\Delta \sim M_U$, $M_\Delta > M_U$: additional contribution to κ at M_U)

Unification Scale



- Effects of dim. 6 (or higher) operators   A. Broncano, M. B. Gavela, E. Jenkins ('05)

- Additional particles between M_{SUSY} and M_U could introduce new scales and alter RGEs

Implications/Possiblities for Model Building from RG Running

Some possibilities for model building from RG running:

- Radiative magnification of mixing angles

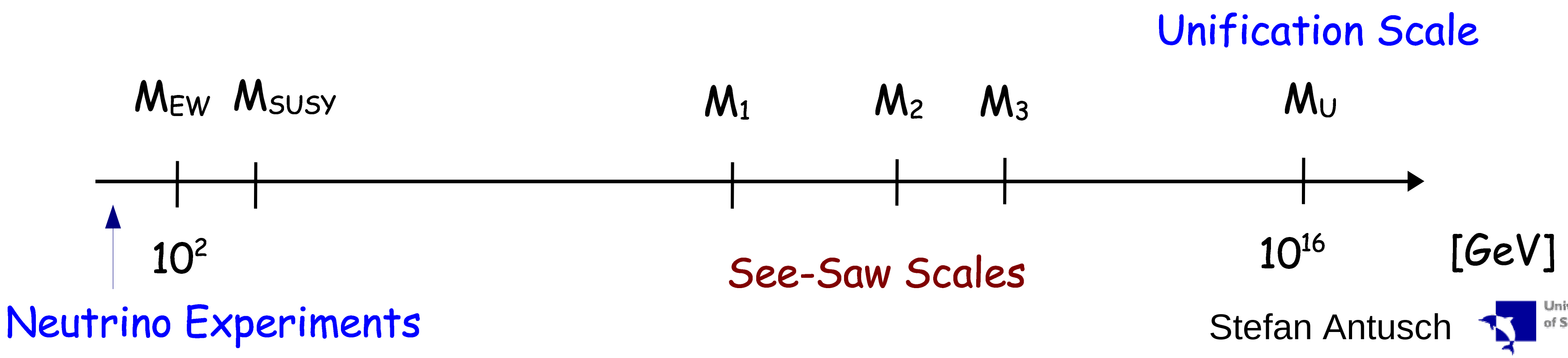
  see e.g.: M. Tanimoto ('95); K. R. S. Balaji, A.S. Dighe, R.N. Mohapatra, M.K. Parida (''00); T. Miura, E. Takasugi, M. Yoshimura ('00); S.A., M. Ratz ('02); H. S. Goh, R. N. Mohapatra ('03); ...

- Bi-maximal mixing scenarios (at M_{GUT}) can be compatible with experiments

  S.A., J. Kersten, M. Lindner, M. Ratz ('02),T. Miura, T. Shindou, E. Takasugi ('03), T. Shindou, E. Takasugi ('04), ...

- Radiative generation of small mass splittings

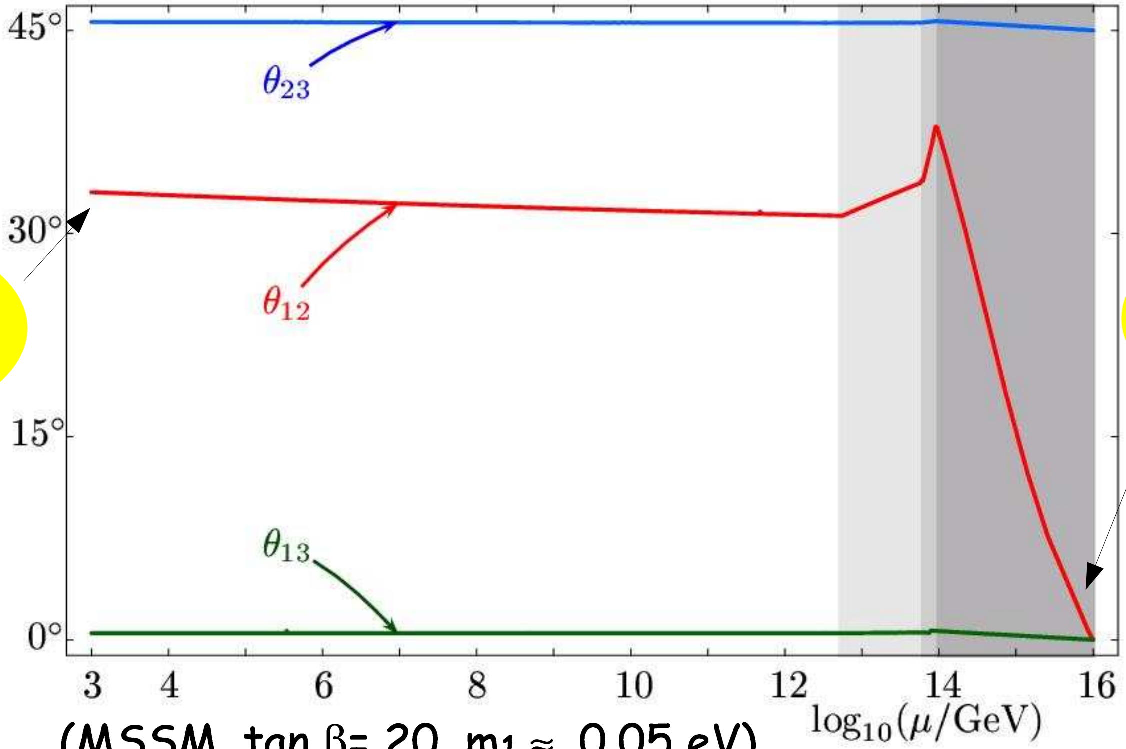
  P.H. Chankowski, A. Ioannisian, S. Pokorski, J. W. F. Valle ('00); M.-C. Chen, K.T. Mahanthappa ('01); A.S. Joshipura, S.D. Rindani, N.N. Singh ('02), A. S. Joshipura, S.D. Rindani ('03), N.N. Singh, M.K. Das ('04); ...



Large Solar Mixing from θ_{12} = 0 at the GUT Scale

S.A., M. Ratz, JHEP 0211 (2002) 010

LMA solution at low energy



θ_{12} = 0° at the GUT scale

(MSSM, tan β= 20, $m_1 \approx$ 0.05 eV)

Example:

$$Y_\nu|_{M_U} = X \cdot \begin{pmatrix} \varepsilon^2 & \varepsilon^3 & 0 \\ \varepsilon^3 & \varepsilon & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

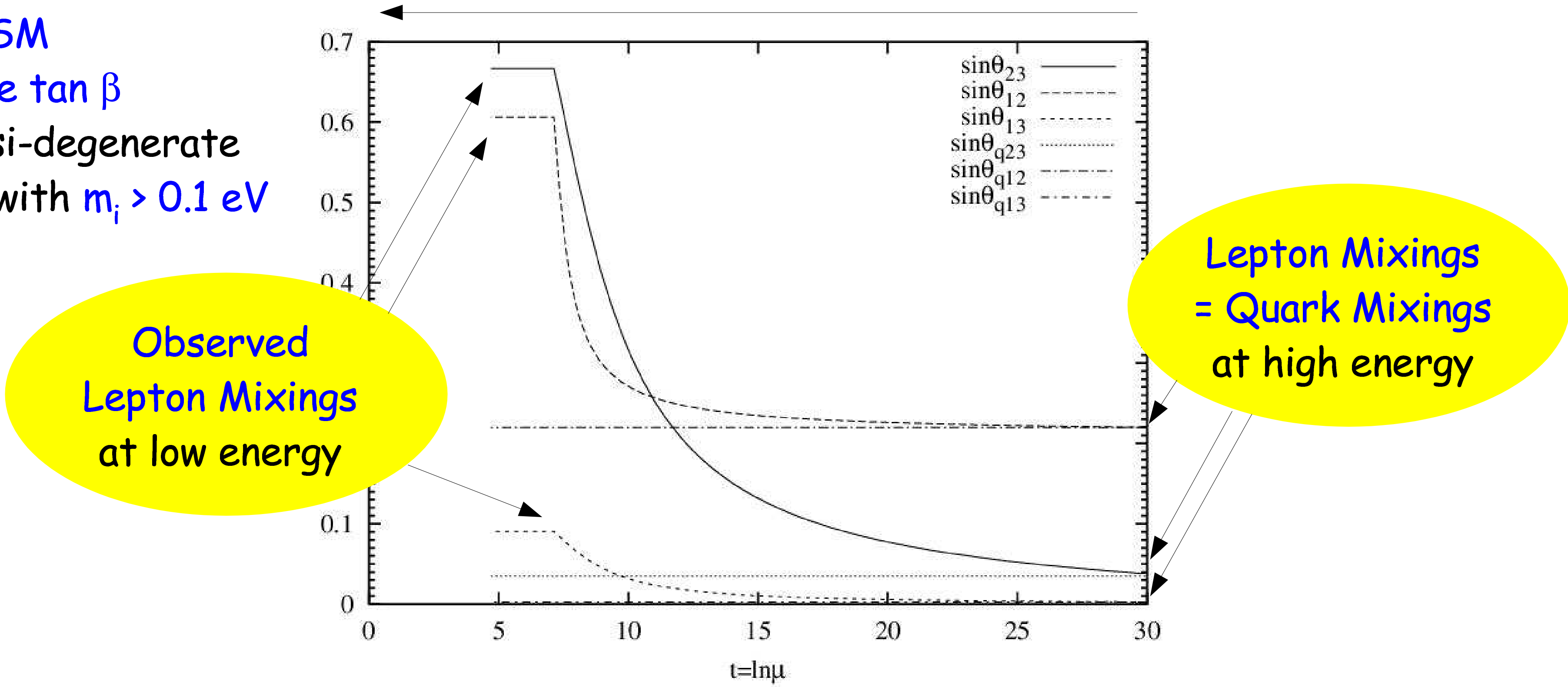
~ 0.5

Quark-Lepton Mixing Unification

R. N. Mohapatra, M. K. Parida, G. Rajasekaran, Phys. Rev. D69 (2004) 053007

MSSM
large tan β
quasi-degenerate
ν's with m_i > 0.1 eV

Running of lepton and quark mixing angles

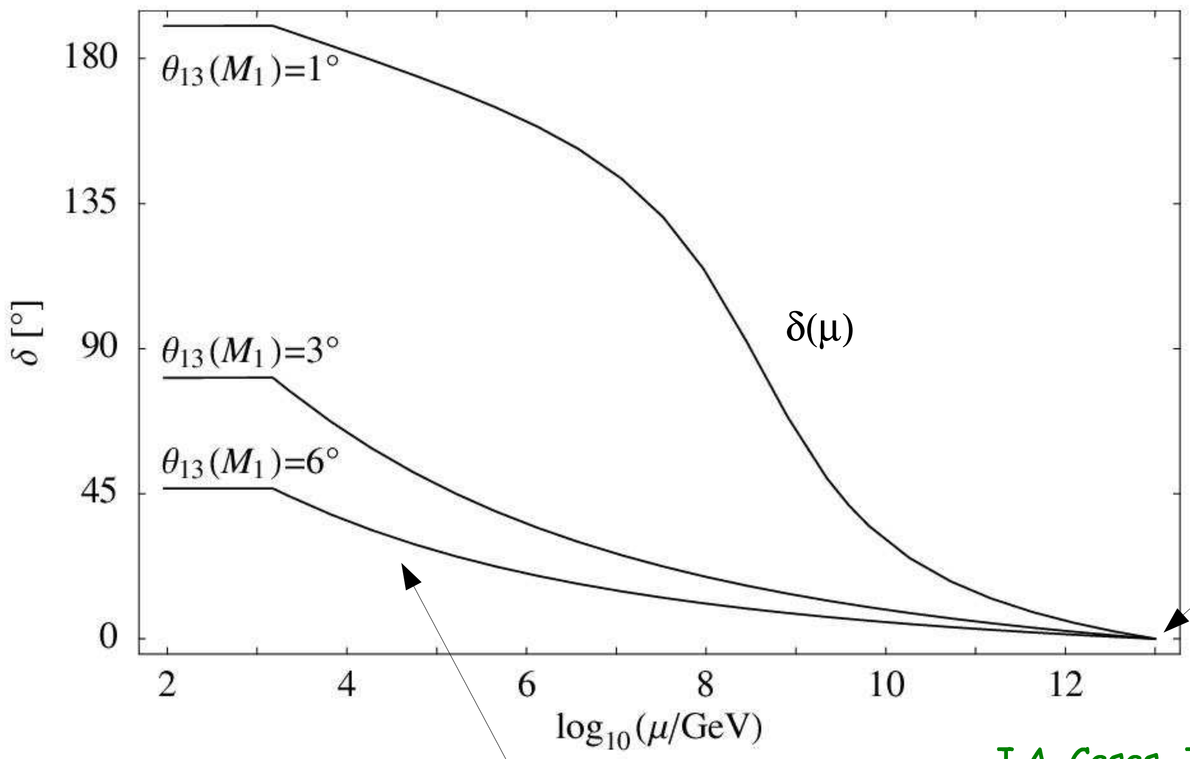


Type II see-saw scenario with discrete LR symmetry:

$$m_\nu = f v_L - M_D (f v_R)^{-1} M_D^T$$

f ~ unit matrix

Radiative Generation of the Dirac CP Phase δ



(MSSM, tan β= 50, m_1= 0.17eV)

Initial condition at Λ:

δ = 0°, $\varphi_1 - \varphi_2$ = 120°

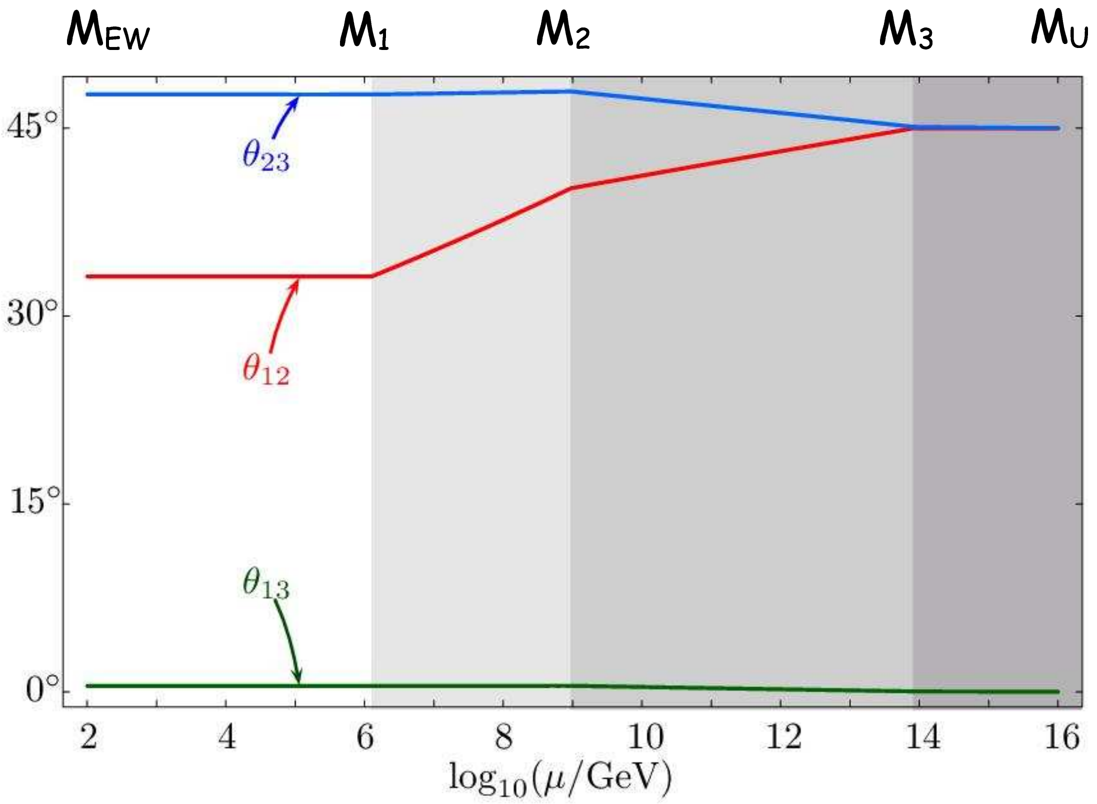
J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro ('99)
S.A., J. Kersten, M. Lindner, M. Ratz ('03)

Dirac CP phase δ = 0 at M_{GUT},
non-zero δ at low energy

From Bi-maximal Mixing to LMA solution by Running

S.A., J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B544 (2002)

LMA solution
($\theta_{12} \approx 33°$)



Bi-maximal mixing

normal neutrino mass ordering

$m_1 = 4 \cdot 10^{-3}$ eV

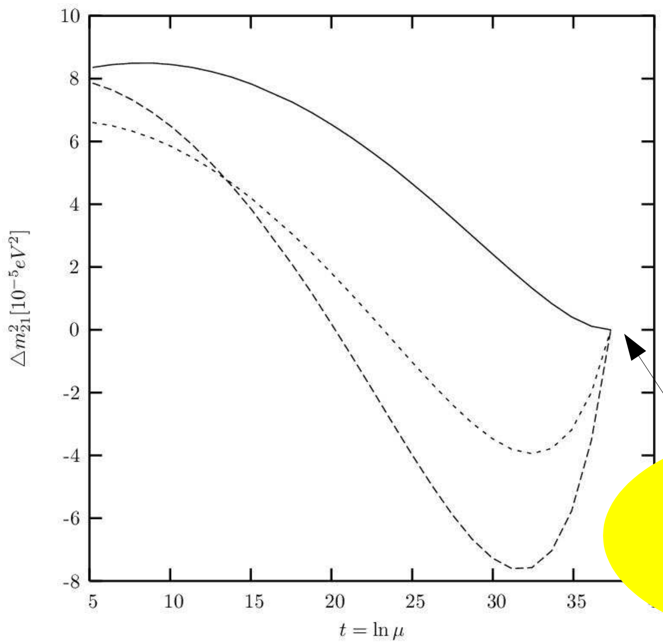
Example: SM,

$Y_\nu = \mathrm{diag}(\varepsilon^2, \varepsilon, 1)$

Radiative Generation of Neutrino Mass Splittings

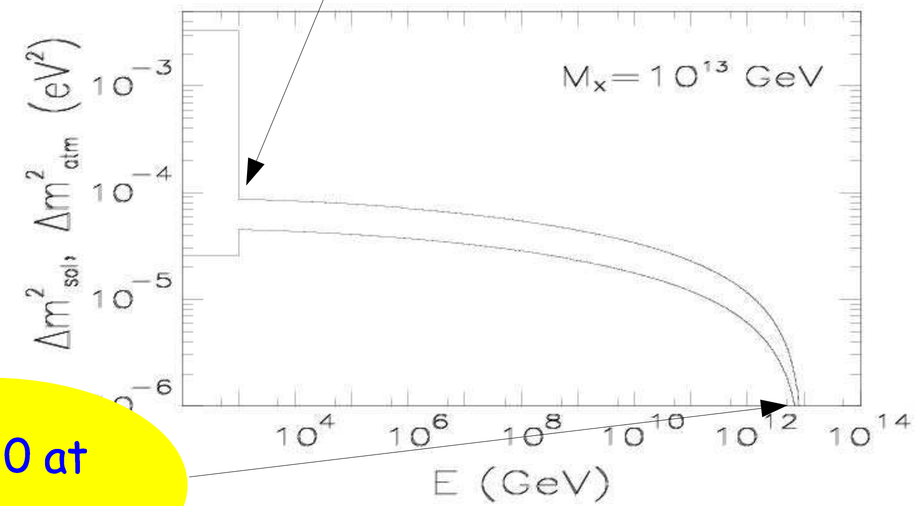
Rad. generation of Δm^2_{12}:

(quasi-degenerate ν's, large tan β)



Rad. generation of Δm^2_{12} and Δm^2_{13}:

(large SUSY threshold corrections, large tan β, and quasi-degenerate ν's)



$\Delta m^2 = 0$ at high energy

Plot from: N. Nimai Singh, M. K. Das, hep-ph/0407206; see also: Joshipura, Rindani, Singh ('02)

P. H. Chankowski, A. Ioannisian, S. Pokorski, J. W. F Valle, Phys. Rev. Lett. 86 (2001) 3488-3491

RG Corrections and Future Precision Neutrino Experiments

- RG corrections to θ_{13} vs. experimental sensitivities: special case: $\theta_{13} = 0$.

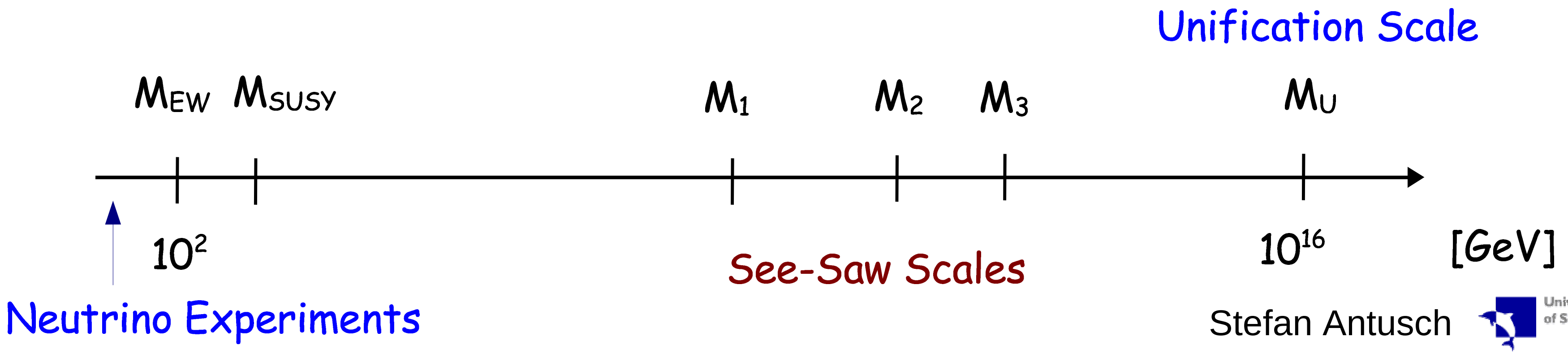
  S.A., J. Kersten, M. Lindner, M. Ratz ('03); Jian-wei Mei, Zhi-zhong Xing ('04);
  S.A., J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt ('05);

- How 'maximal' is θ_{23}? Deviation $\theta_{23} - \pi/4$ induced by RG running.

  S.A., J. Kersten, M. Lindner, M. Ratz ('03); S.A., M. Huber, J. Kersten, T. Schwetz, W. Winter ('04)

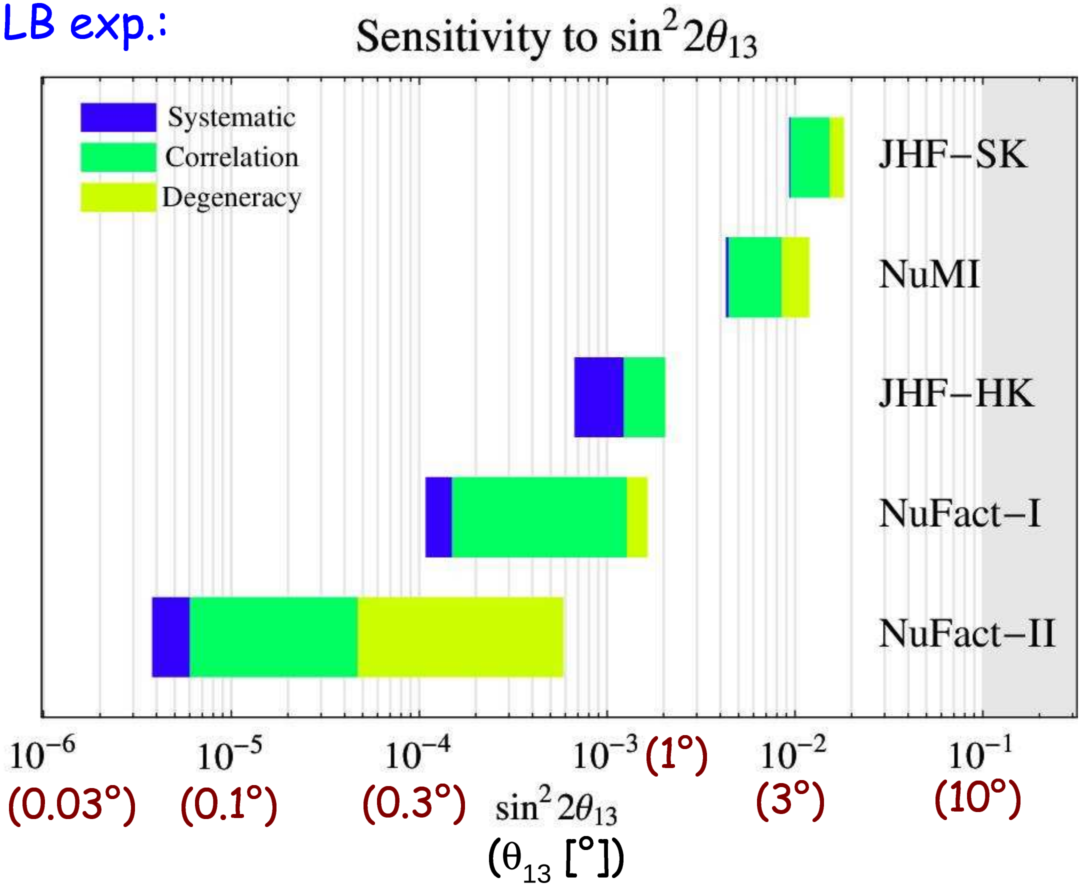
- RG corrections to θ_{12} and precision tests of relations like $\theta_{12} + \theta_C = 45°$ (QLC)?

  Recent papers on QLC: M. Raidal ('04), H. Minakata, A. Y. Smirnov ('04), ...



Future Exp. Sensitivity on θ_{13}

LB exp.:



Also: reactor exp.!
- Double-CHOOZ (sensitivity ≈ 0.03)
- ...

P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645 (2002)

RG Corrections to θ_{13}

Analytical Approximation (below see-saw scales): $\Delta\theta_{13} = \dot{\theta}_{13} \ln(\Lambda/M_{EW})$

quasi-degenerate ν's: $\varphi_1 = \varphi_2$ can damp large RG effects

$$\dot{\theta}_{13} \approx \frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\rm atm}} \left[m_1 \cos(\varphi_1 - \delta) - m_2 \cos(\varphi_2 - \delta)\right] + \mathscr{O}(\theta_{13})$$

$\dot{\theta}_{13} = 0$ (for $\theta_{13} = 0$) possible for $m_3 = 0$ or for 'conspiracy' of CP phases

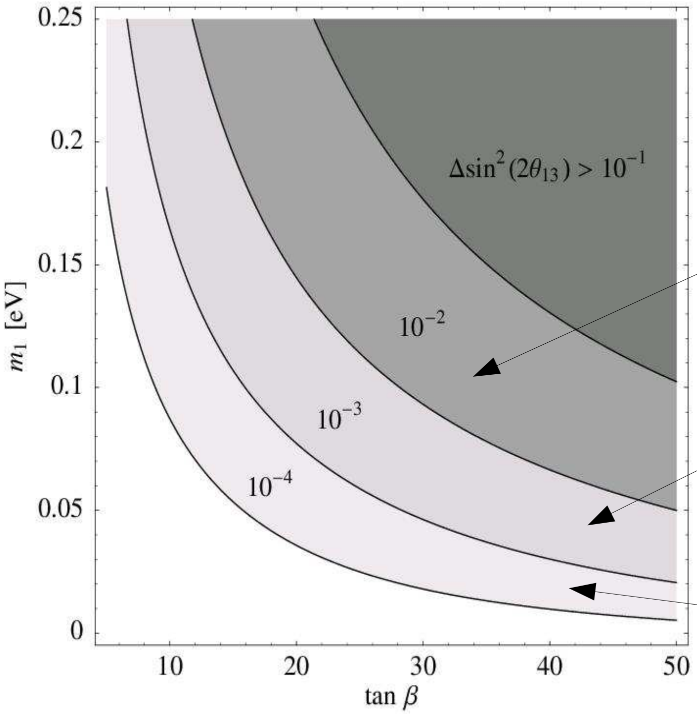
- In general: RG effects larger for tan $\beta \uparrow$ and for $m_1 \uparrow$

S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)

RG Corrections to θ_{13} = 0 @ High Energy

Graphical illustration of RG corrections ($\Delta\sin^2(2\theta_{13})$)

(running of dim. 5 operator between $\Lambda = 10^{12}$ GeV and M_{EW} in the MSSM)



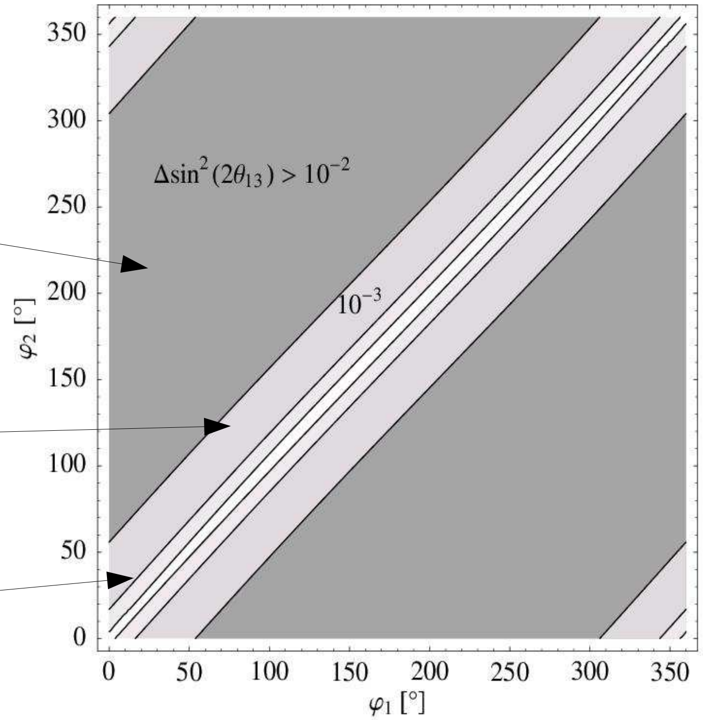
($\varphi_1 - \varphi_2$ = 180°, δ = 0, normal scheme)

Expected sensitivities:

Reactor and superbeam exp.

Superbeam exp. (upgrade)

Neutrino factory



(tan β = 50, m_1= 0.08eV, δ = 0)

Even for θ_{13} = 0 @ high energy, RG running $\Rightarrow$ in general non-zero θ_{13} @ low energy

S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)

Estimated Exp. Sensitivties for Excluding θ_{23} = 45°

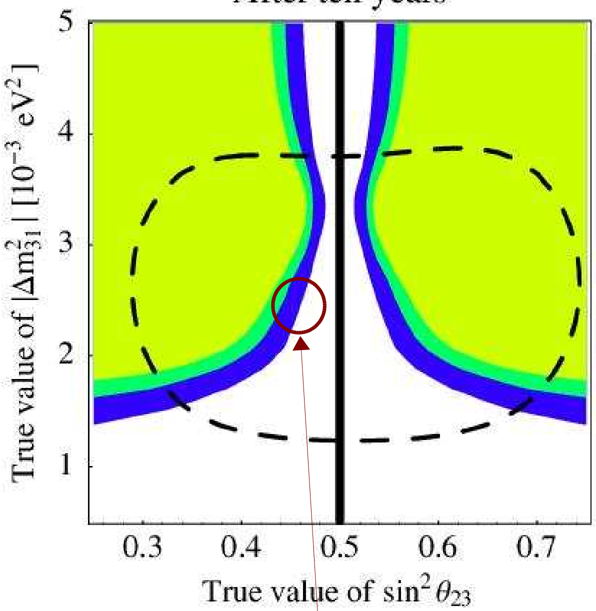
Combined: MINOS, ICARUS, OPERA, T2K, NuMI

Long Baseline Experiments

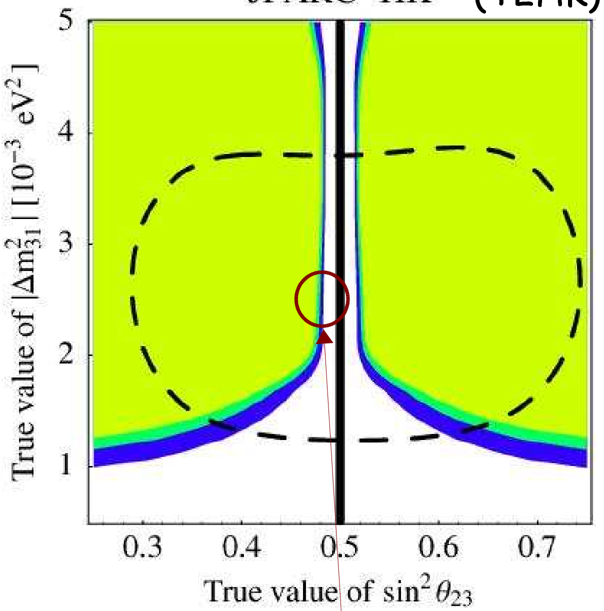
4 MW, 1 Mt

(T2HK)

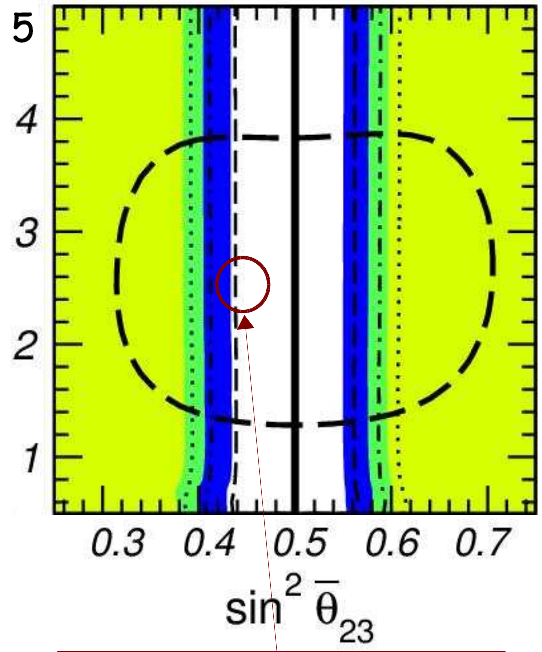
Atm. neutrino experiment with statistics SK x 20



$\Delta\theta_{23} \approx 3°$ @ 90% CL



$\Delta\theta_{23} \approx 1°$ @ 90% CL



$\Delta\theta_{23} \approx 3°$ @ 90% CL

and θ_{23} > or < $\pi/4$

S.A., P. Huber, J. Kersten, T. Schwetz, W. Winter (hep-ph/0404268)

M.C. Gonzalez-Garcia, M. Maltoni, A.Yu. Smirnov (hep-ph/0408170)

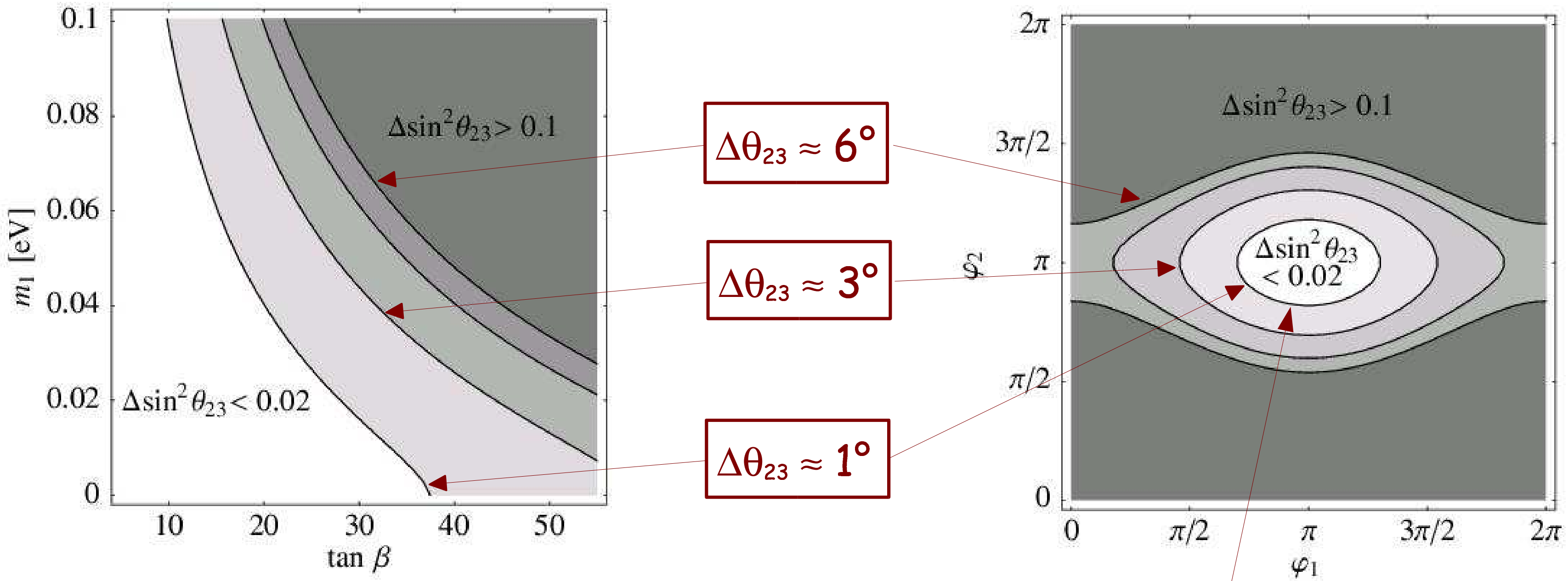
RG Corrections to Maximal Mixing $\theta_{23} = 45°$

Analytical Approximation (below see-saw scales): $\Delta\theta_{23} = \dot{\theta}_{23} \ln(\Lambda/M_{EW})$

$$\dot{\theta}_{23} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{\text{atm}}} \left[c_{12}^2 \left| m_2 e^{i\varphi_2} + m_3 \right|^2 + s_{12}^2 \left| m_1 e^{i\varphi_1} + m_3 \right|^2 \right] + \mathscr{O}(\theta_{13})$$

Note: $\Delta\theta_{23} > (<)\ 0$ for $\Delta m^2_{atm} < (>)\ 0$

Example: Conservative estimate (ignore Y_ν contributions) of RG corrections (MSSM)



(MSSM, $\varphi_1 = \varphi_2 = 0$, $\theta_{13} = 0$)

Majorana CP phases ($m_1 = 0.075$ eV)
(can damp RG effects, but $\Delta\theta_{23}$ always $\neq 0$!)

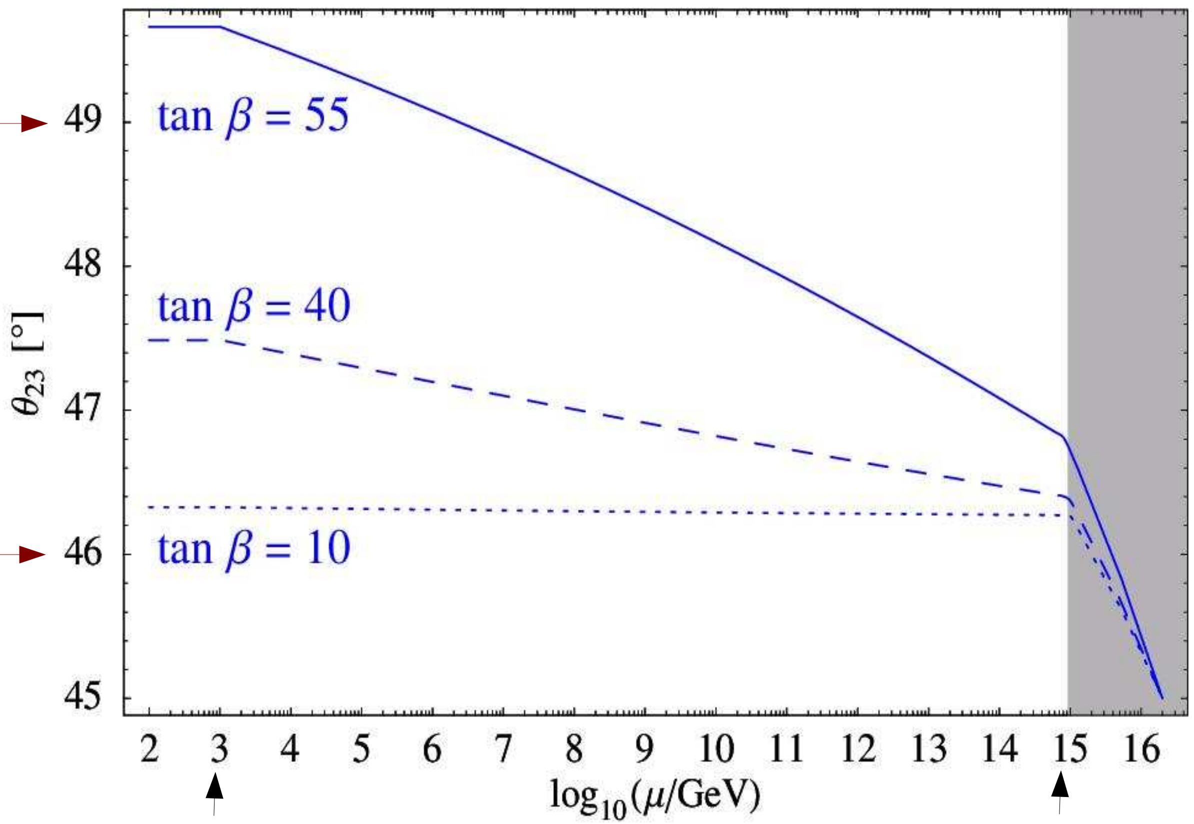
S.A., J. Kersten, M. Lindner, M. Ratz (hep-ph/0305273)
S.A., M. Huber, J. Kersten, T. Schwetz, W. Winter (hep-ph/0404268)

RG Corrections to Maximal Mixing θ_{23} = 45° @ M_{GUT}

Even if maximal mixing is predicted @ high energy, RG running will lead to deviation:

$\Delta\theta_{23} \approx 4°$

$\Delta\theta_{23} \approx 1°$



M_{SUSY} (MSSM)

M_3

$$Y_\nu \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

leading order

(RL convention)

θ_{23} = 45° at M_U

n. hierarchy

$m_1 \approx 0$ eV

S.A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt (hep-ph/0501272)

Is there a relation between θ_{12} and θ_C?

at present: large uncertainty for θ_{12}

Lepton mixings

Parameter	Best-fit value	3σ range
θ_{12}	33.2°	28.7° .. 38.1°
θ_{23}	45.0°	35.7° .. 55.6°
θ_{13}	2.6°	0° .. 12.5°

Quark mixings

Parameter	Best-fit value	2σ range
θ^q_{12}	12.88°	12.75° .. 13.01°
θ^q_{23}	0.21°	0.17° .. 0.25°
θ^q_{13}	2.36°	2.25° .. 2.48°

Is there a relation $\theta_{12} + \theta_C = 45°$ hidden behind the large uncertainties?

'Quark-lepton complementarity'

Recent papers: Raidal ('04), Smirnov, Minakata ('04), Mohapatra, Frampton ('04), Ferrandis, Pakvasa ('04), ...

- Future: few % accuracy for $\sin^2\theta_{12}$ possible, e.g. from reactor experiments; could test of 'quark-lepton complementarity' to high precision
  see e.g.: Minakata *et al*, hep-ph/0407326
- If 'confirmed' (not accidental):
  - Relation between quark and lepton mixing angles would point towards unification
  - Challenging for model building. However, we could learn a great deal ...
    (predictions at M_U modified by RG running!)

Challenges for Quark-Lepton Complementarity

- Approach: $\theta^\nu_{12} = \pi/4$ from the neutrino sector, deviation induced by θ^e_{12} related to θ_C
- Using a Georgi Jarlskog (Clebsch) factor of -3 for $(Y_e)_{22}$: no QLC predicted

Note: same operators lead to entries in Y_e and Y_d!

Assume: $\theta^\nu_{12} = \pi/4$ (and large θ^ν_{23}), e.g. with inverted neutrino mass hierarchy:

$\theta^d_{12} = \lambda = \theta_C$

$$Y_d \sim \begin{pmatrix} 0 & 1\lambda^4 & \lambda^4 \\ * & 1\lambda^3 & \lambda^2 \\ * & * & 1 \end{pmatrix}, \quad Y_e \sim \begin{pmatrix} 0 & \lambda^4 & \lambda^4 \\ * & -3\,\lambda^3 & \lambda^2 \\ * & * & 1 \end{pmatrix}, \quad m_{LL} \sim \begin{pmatrix} 0 & m & m' \\ m & 0 & 0 \\ m' & 0 & 0 \end{pmatrix}$$

$\theta^e_{12} = 1/3\ \lambda = 1/3\ \theta_C$

- In addition: one has to 'shift' R^e_{12} to the right $\Delta\theta_{12} \Rightarrow$ additional factor $1/\sqrt{2}$

$$U_{MNS} \approx R^{e\dagger}_{12} R^\nu_{23} U^\nu_{13} R^\nu_{12} P^\nu_0 \quad \Rightarrow \quad s_{12} \approx \frac{1}{\sqrt{2}} - \frac{1}{2}\theta^e_{12} \rightarrow \theta_{12} \approx \frac{\pi}{4} - \frac{1}{\sqrt{2}}\theta^e_{12}$$

$$\theta_{12} \approx \frac{\pi}{4} - \frac{1}{\sqrt{2}}\frac{1}{3}\theta_C \Rightarrow \Delta\theta_{12} = \frac{1}{\sqrt{2}}\frac{1}{3}\theta_C \approx 3^\circ$$

Too small for explaining observed deviation from $\theta_{12} = \pi/4$

- in particular: no QLC!

Two Routes to QLC in Unified Models: Scenario 1

- Inverted hierarchy + modified Clebsch factors for $Y_e \leftrightarrow Y_d$:

Note: same operators lead to shown entries of Y_e and Y_d!

$\theta^d_{12} = \lambda = \theta_C$

$\theta^\nu_{12} = \pi/4$ and large θ^ν_{23}

$$Y_\mathrm{d} \sim \begin{pmatrix} 0 & 1\lambda^4 & \lambda^4 \\ * & 1\lambda^3 & \lambda^2 \\ * & * & 1 \end{pmatrix}, \quad Y_\mathrm{e} \sim \begin{pmatrix} 0 & -3\lambda^4 & \lambda^4 \\ * & 2\lambda^3 & \lambda^2 \\ * & * & 1 \end{pmatrix}, \quad m_\mathrm{LL} \sim \begin{pmatrix} 0 & m & m' \\ m & 0 & 0 \\ m' & 0 & 0 \end{pmatrix}$$

$$\theta^\mathrm{e}_{12} \approx \tfrac{3}{2}\lambda \approx \tfrac{3}{2}\theta_\mathrm{C} \Rightarrow \boxed{\theta_{12} \approx \frac{\pi}{4} - \frac{1}{\sqrt{2}}\theta^\mathrm{e}_{12} \approx \frac{\pi}{4} - 1.06\,\theta_\mathrm{C}}$$

(Factor 1.06: 3/2 nearly compensates factor $1/\sqrt{2}$!)

(approximate QLC @ M_U)

- In addition: predicts $\theta_{13} = 1.06\ \theta_C$

- Summary of predictions (with some theoretical error): (calculated with REAP)

quantity	θ_{12}	θ_{13}
prediction at M_U	$\frac{\pi}{4} - 1.06\,\theta_\mathrm{C}$	$1.06\,\theta_\mathrm{C}$
prediction at M_{EW}	$\frac{\pi}{4} - 1.06\,\theta_\mathrm{C} - 0.8^\circ \approx 30.5^\circ$	$1.06\,\theta_\mathrm{C} - 0.5^\circ \approx 13.2^\circ$

RG corrections

S.A., S.F. King, R.N. Mohapatra (hep-ph/0504007)

Two Routes to QLC in Unified Models: Scenario 2

S.A., S.F. King, R. N. Mohapatra (hep-ph/0504007)

- Inverted hierarchy + lopsided Y_e + modified Clebsch factors for $Y_e \leftrightarrow Y_d$:

$\theta^d_{12} = \lambda = \theta_C$ 2 x Clebsch factors -3: $\theta^\nu_{12} = \pi/4$ and $\theta^\nu_{23} = 0$

$$Y_d = \begin{pmatrix} 0 & 1\lambda^4 & \lambda^4 \\ * & 1\lambda^3 & \lambda^2 \\ * & * & 1 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & -3\lambda^4 & \lambda \\ * & -3\lambda^3 & 1 \\ * & 0 & 1 \end{pmatrix}, \quad m_{LL} \approx \begin{pmatrix} 0 & m & 0 \\ m & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\theta^e_{12} = \theta_C \Rightarrow \theta_{12} = \pi/4 - \theta_C$

(QLC @ M_U)

$\theta^e_{23} = \pi/4$ (no 'shifting of R^e_{12} to the right' required $\Rightarrow$ no factor $1/\sqrt{2}$!)

Typically (but no prediction): $\theta_{13} \approx \theta_C$

- Note: In both scenarios for QLC via inverted hierarchy, RG corrections are dominated by $O(\theta_{13})$-effects (zeroth order in θ_{13} suppressed by Majorana parity between m_1 and m_2) $\Rightarrow$ smaller θ_{13} would imply smaller RG corrections to QLC!

$$\dot{\theta}_{12} \approx -\frac{y_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2 \frac{|m_1\, e^{i\varphi_1} + m_2\, e^{i\varphi_2}|^2}{\Delta m^2_{\mathrm{sol}}} + \mathcal{O}(\theta_{13})$$

here: dominant!

Summary

RG running of neutrino parameters in see-saw scenarios

- RG effects between and above the see-saw scales can be important (even dominant)
- Analytical approximations: estimating & understanding RG corrections
- Public software packages REAP/MPT: convenient numerical RG evolution

Consequences for model building: non-hierarchical neutrinos (large RG effects)

- Radiative magnification of lepton mixings
- Bi-maximal mixing at M_U possible due to RG effects
- Radiative generation of neutrino mass splittings

RG corrections and future precision neutrino exeriments

- Even for $\theta_{13} = 0$ @ M_U: (in general) non-zero θ_{13} by RG effects
- Deviation of θ_{23} from $\pi/4$: induced by RG running even for hierarchical ν's
- RG corrections to relation like 'Quark-Lepton Complementarity' $\theta_{12} + \theta_C = 45°$

Conclusions

What can we learn from future precision neutrino experiments?

One interesting aspect:

Precision tests of flavour models

Requires:

- High precision experiments
- High accuracy of model predictions
- Inclusion of RG corrections