Madrid, WQG 2008

Bigravity and Λ

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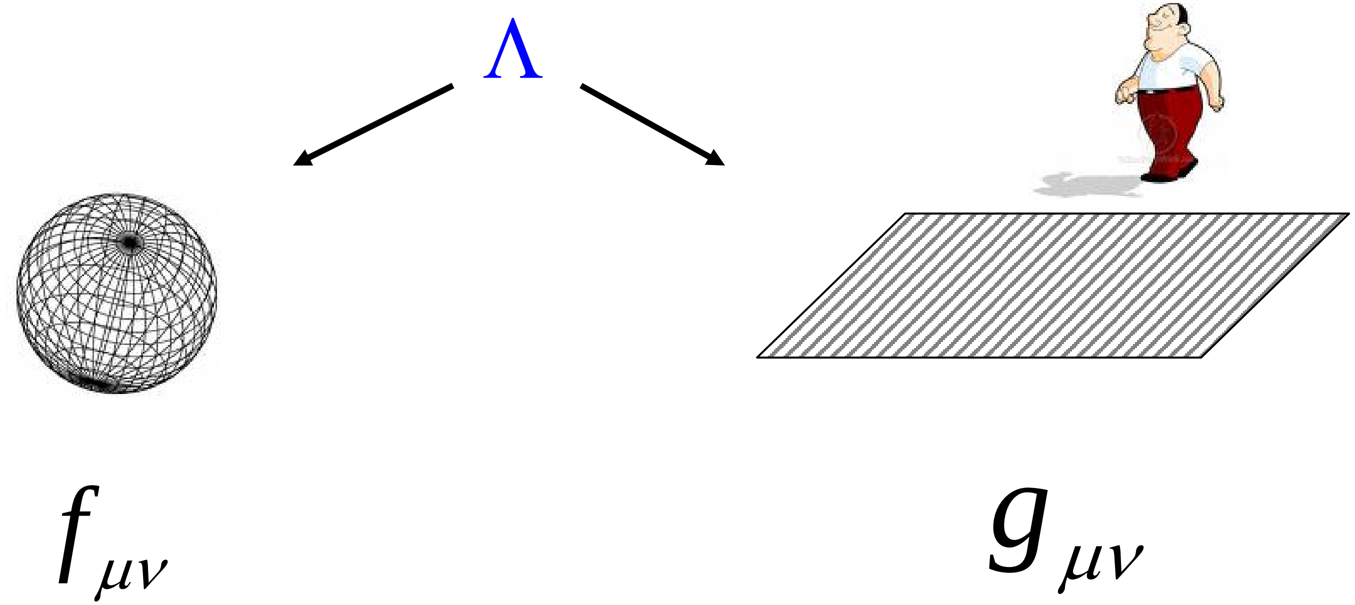
(with Blas and Deffayet, 07)



Λ

$f_{\mu\nu}$

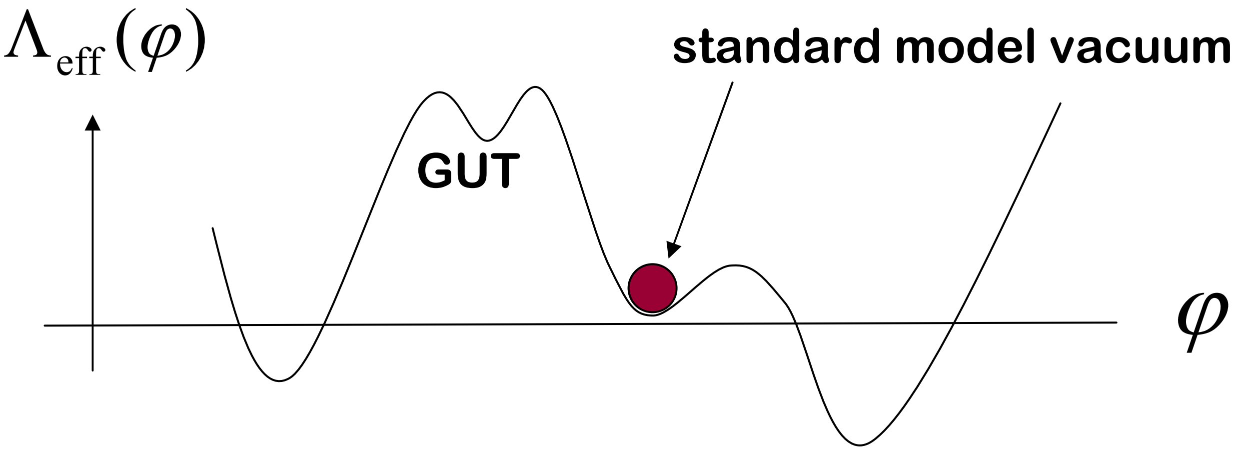
$g_{\mu\nu}$

The cc problem:

$|\Lambda_{\text{observed}}| \ll$ quantum corrections to it

(naturalness problem)

Also, $\Lambda_{\text{eff}} \equiv \Lambda_{\text{obs}} + \Delta V(\varphi)$ may change vastly in cosmic history



- Why is there a “SM” vacuum with $\Lambda_{\text{eff}} \sim 10^{-120}$?

Different approaches:

1- Symmetries
2- Adjustment mechanisms (no go)
3- Vast landscape of “vacua”
4- Changing gravity (in the IR).

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(e.g. Unimodular Gravity.)

Different approaches:

1- Symmetries

2- Adjustment mechanisms (no go)

3- Vast landscape of “vacua”

4- Changing gravity (in the IR).

Changing gravity leads sometimes
to a continuum of vacua (as in 3).
(e.g. Unimodular Gravity.)
And may also involve new symmetries.

Unimodular Gravity:

- **Any standard Lagrangian can be "unimodularized"**

$$g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} \equiv (-\mathrm{det}g)^{-1/n} g_{\mu\nu}$$

$$S = \frac{1}{2}\int R[\hat{g}]\ d^{\mathrm{n}}x + S_{matter}[\hat{g},\psi_{matter}]$$

Weinberg 83,
Buchmuller+Dragon 88
Alvarez et al. 07,
Blas 07

- **Different invariance group**

Weyl: $\delta g_{\mu\nu} = \phi\, g_{\mu\nu}$ **TDiff:** $\begin{cases} \delta g_{\mu\nu} = \xi_{(\mu;\nu)} \\ \partial_\mu \xi^\mu = 0 \end{cases}$

- **Equations of motion** in gauge $g=1$

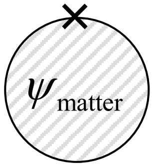
$$\frac{\delta S}{\delta g_{\mu\nu}} = 0 \quad \Rightarrow \quad \boxed{R_{\mu\nu} - \frac{1}{n} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{n} g_{\mu\nu} T}$$

Traceless Einstein's Eq.

- **The equation for the trace follows from the Bianchi identity**

Classical matter

$$R^{\mu\nu}{}_{;\nu} \equiv \frac{1}{2} R_{,\nu} \Rightarrow [(n-2)R + 2\,T\,]_{,\mu} = 2n\,T_{\mu}{}^{\nu}{}_{;\nu} = \begin{cases} \mathbf{0} \\ \vartheta_{,\mu} \end{cases}$$



anomaly

$$\Rightarrow \boxed{(n-2)R + 2\,T - \vartheta[R] = 2n\,\Lambda = \text{integration const.}}$$

- **Seems quite appealing (“fewer” equations)**
- **But nothing differs from Einstein’s theory at the level of phenomenology**

Bigravity

(Isham et al 71,
Kogan and Damour,
Damour et al 02)

- **Action**

$$S = M_g^2 \int g^{1/2} R_g + M_f^2 \int f^{1/2} R_f + \underbrace{\sum_{i=f,g} S_m^{(i)}}_{\text{Matter}} \underbrace{- M_*^4 \int g^{1/2} V(X)}_{\text{Interaction}}$$

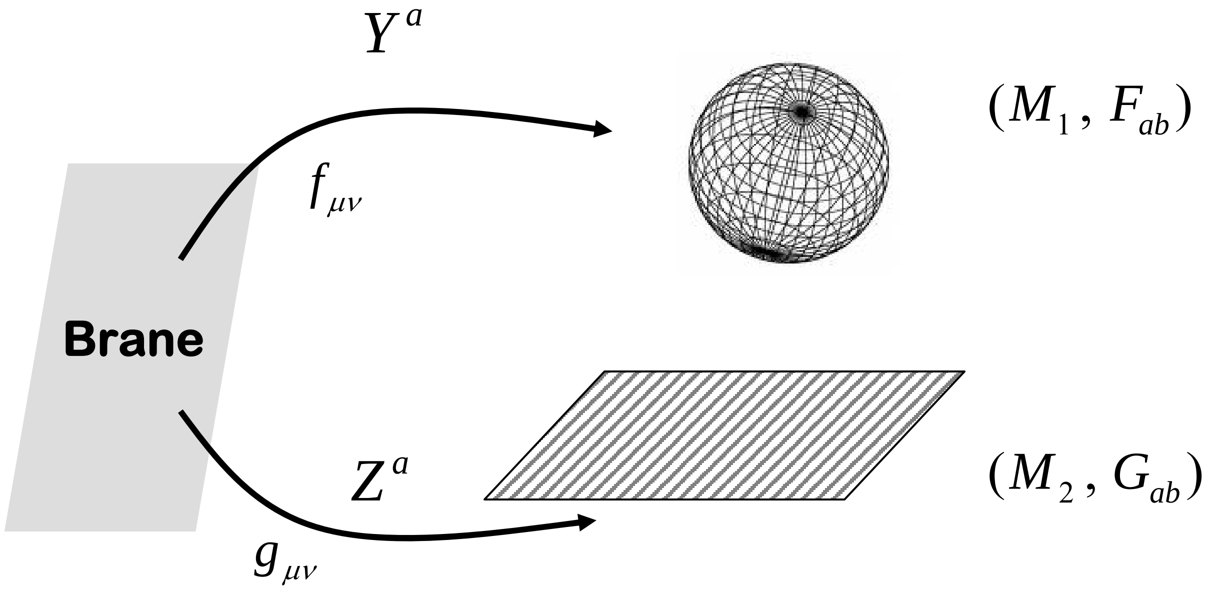
$$X^\mu_\nu \equiv f^{\mu\lambda} g_{\lambda\nu}$$

- $V(X)$ **Can only depend on** $\tau_n \equiv \mathrm{Tr}[X^n]$ $n = 1, \dots, 4$

- **Vintage requirements:**
  - $M_* \ll M_f, M_g$
  - $\tau_n \sim 1$
  - $\partial \tau_n \ll M_*$

- **Example: consider the double embedding**

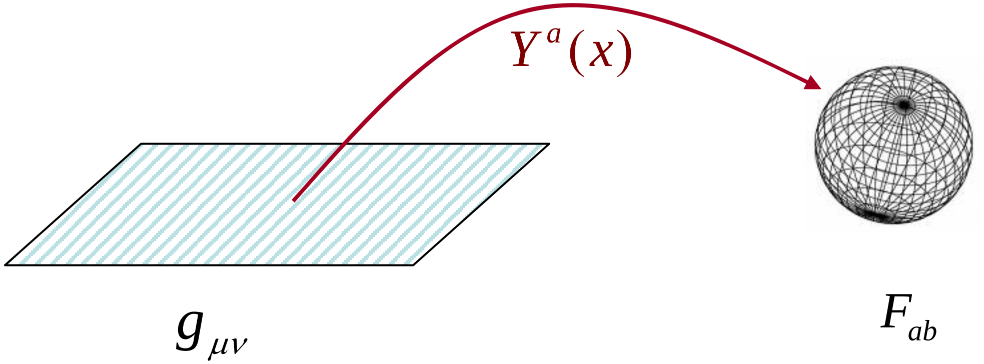
  J.G. , 08



- **Diff invariance in** M_1 **and** M_2
- **Reparametrization invariance on the brane**
- **Brane DBI-type action** $M_*^4 \int d^4\xi \sqrt{g_{\mu\nu} + f_{\mu\nu}}$
- **Standard EH kinetic terms for** F_{ab} **and** G_{ab} **in the target spaces**

- **In the gauge** $Z^a(x) = \delta^a_\mu \;\; x^\mu$ $\longrightarrow$ $V(X) = \sqrt{\det(\delta^\mu_\nu + (X^{-1})^\mu_\nu)}$

$$(X^{-1})^\mu_\nu = g^{\mu\lambda}(x)[\partial_\lambda Y^a(x)\partial_\nu Y^b(x) F_{ab}(Y(x))]$$



- $Y^a = x^a + \pi^a(x)$ **compensators to "restore"** $\mathrm{Diff}[g] \times \mathrm{Diff}[F]$

  (Arkani-Hamed et al 03, Dubovsky, **04)**

- **In "unitary" gauge (** $\pi = 0$ **) the diagonal invariance** $\mathrm{Diff}[f, g]$ **remains.**

Isn’t bigravity problematic?

- Bigravity is a theory of massive gravity, and as such it has some issues.
- On Lorentz-invariant backgrounds, we have the same problems as with Lorentz-invariant massive gravity
  - vDVZ discontinuity
  - Strong coupling near sources
  - Boulware Deser instability (6 propagating degrees of freedom)

Spectrum:
One massless graviton ← (Can be decoupled by $M_f^2 \to \infty$)
One massive graviton
+ possible ghostly junk…

- These problems can be understood in terms of the dynamics of the "pions" $\pi^a(x)$.

(Deffayet+Rombouts 05
Creminelli et al. 05)

- However, none of them need apply in Lorentz violating backgrounds.

(Dubovsky 04, Rubakov 04)

Lorentz invariant massive gravity
(Linearized theory)

- **Fierz-Pauli lagrangian**

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

$$\Big\downarrow$$

$$M_P^2 m^2\int d^4x\left(h_{ij}h_{ij} \underset{\Big\downarrow}{-2h_{0i}h_{0i}} - h_{ii}h_{jj} \underset{\Big\downarrow}{+2h_{ii}h_{00}}\right)$$

Quadratic in shift **Linear in the lapse**

- **Five propagating degrees of freedom**

- **Wrong prediction for the bending of light because of the additional Spin 0 polarization of the graviton (vDVZ discontinuity).**

Lorentz invariant massive gravity

(Linearized theory)

- **Fierz-Pauli lagrangian**

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \overset{\alpha \neq 1}{\eta^{\mu\nu}}\eta^{\alpha\beta}\right)$$

$$\Big\downarrow$$

$$M_P^2 m^2\int d^4x\left(h_{ij}h_{ij} - \underset{\substack{\Big\downarrow \\ \text{Quadratic in shift}}}{2h_{0i}h_{0i}} - h_{ii}h_{jj} + \underset{\substack{\Big\downarrow \\ \text{Linear in the lapse}}}{2h_{ii}h_{00}}\right)$$

- **Five propagating degrees of freedom**

- **Wrong prediction for the bending of light because of the additional Spin 0 polarization of the graviton (vDVZ discontinuity).**

- **This changes if we do not stick to the Fierz-Pauli form** $(\alpha \neq 1)$ **Then the spin 0 contribution cancels with a new ghostly contribution from the sixth polarization (the lapse is no longer a Lagrange multiplier)**

Lorentz invariant massive gravity

(Non-linear theory)

- Near sources, linear theory breaks down at distances smaller than

$$r_{\mathrm{v}} = (r_{Sch}^{-1}\ m^4)^{1/5}$$

Vainshtein 72

- At smaller distances, one can do another expansion which recovers the Schwarzschild solution in the limit $m^2 \to 0$ (although there is no guarantee that this matches the linearized solution far from the source).

- Boulware-Deser instability

$$M_P^2 \int d^4x \left\{ \left(\pi^{ij}\dot{g}_{ij} - N R^0 - N_i R^i\right) - m^2 \left(h_{ij}h_{ij} - 2N_iN_i - h_{ii}h_{jj} + 2h_{ii}\left(1 - N^2 + N_k g^{kl} N_l\right)\right)\right\}$$

↓

$$N \equiv \left(-g^{00}\right)^{-1/2}$$

$$N_i \equiv g_{0i},$$

- Lapse and shift are no longer Lag. multipliers
- The reduced Hamiltonian is unbounded below.

The “pion” description

(Arkani-Hamed et al. 03, Deffayet+Rombouts 05)

- **Action**

$$S_{AGS} = M_P^2 \int d^4x \sqrt{-g} R(g) + M_P^2 m^2 \int d^4x \sqrt{-g} h_{\mu\nu} h_{\alpha\beta} \left(g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}\right)$$

- **Replacing** $h_{\mu\nu} \equiv g_{\mu\nu} - g^{(0)}_{\mu\nu}$

$$\longrightarrow \quad H_{\mu\nu} = g_{\mu\nu} - g^{(0)}_{\alpha\beta}(Y) \partial_\mu Y^\alpha \partial_\nu Y^\beta = h_{\mu\nu} + \pi_{\mu,\nu} + \pi_{\nu,\mu} + \pi_{\alpha,\mu} \pi^{\alpha}_{,\nu}$$

- **Longitudinal mode** $\pi_\mu = \partial_\mu \phi$

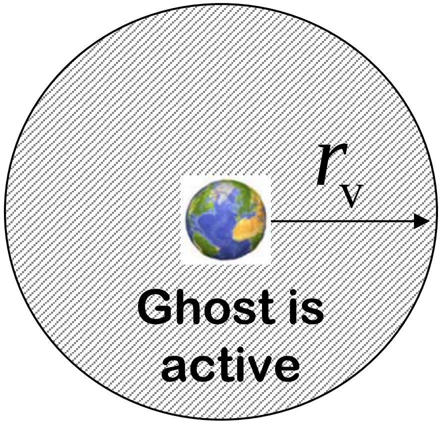
- **Action starts at cubic order (no kinetic term)**

$$2 M_P^2 m^2 \int d^4x \left((\Box\phi)^3 - (\Box\phi)(\partial^\mu \partial^\nu \phi)(\partial_\mu \partial_\nu \phi) \right)$$

This means one less degree of freedom, and hence the reason why there is no ghost in the linearized FP Lagrangian.

- **But there is a mixing with** $h_{\mu\nu} \longrightarrow \sqrt{g}M_{Pl}^2(1+m_g^2\phi)R$
- **In Einstein's frame** $\sim M_P^2 m^4 \phi \Box \phi$
- **After normalization** $\mathcal{L} = \frac{1}{2}\phi\Box\phi + \frac{1}{\Lambda^5}(\Box\phi)^3 - \frac{1}{\Lambda^5}(\Box\phi)(\partial^\mu\partial^\nu\phi)(\partial_\mu\partial_\nu\phi) - \frac{1}{M_P}\phi T$

  $\Lambda \equiv (m^4 M_P)^{1/5}$
- **Exact in the limit** $M_P \to \infty,\ T \to \infty$ and $m \to 0$ with $\Lambda = cst$
- **Strong coupling at the Vainshtein radius** $r_v = (r_{Sch}^{-1}\ m^4)^{1/5}$
- **Near the source, we are above cut-off , where we would have to take the higher derivative terms seriously** $\longrightarrow$ **Ghost mode kicks in.**



$r \gg r_v$

Linear theory applies

Lorentz-breaking mass term

(Damour, Kogan, Papazoglou 03, Rubakov 04)

$$L_m = \left(m_0^2 h_{00}h_{00} + 2m_1^2 h_{0i}h_{0i} - m_2^2 h_{ij}h_{ij} + m_3^2 h_{ii}h_{jj} - 2m_4^2 h_{00}h_{ii}\right)$$

- This will be the generic situation in the context of bigravity

- The previous discussion does not apply.

- In certain cases, due to residual gauge symmetry, we may have a healthy “pion” lagrangian in the decoupling limit (no ghosts) (Dubovsky 04)

- For instance, invariance under $\xi^i = \xi^i(t)$ leads to $m_1^2 = 0$

$\longrightarrow$ h_{0i} become Lagrange multipliers.

π^i Non-dynamical

π^0 Ghost “condensate”

- **Linearized spectrum:** One massive graviton of mass m_2 with just two polarizations.

- **Potentially interesting phenomenology**

$$\Phi_1 = -\frac{GM_1}{r} + GM_1 \underset{\overset{\uparrow}{\frac{m_2^2[3m_4^4 - m_0^2\left(3m_3^2 - m_2^2\right)]}{m_4^4 - m_0^2(m_3^2 - m_2^2)}}}{\mu}^2 r$$

slight modifications of Newton's law

(No vDVZ discontinuity)

(Dubovsky, Tinyakov, Tkachev, 05)

Scales:

$$S = M_g^2 \int g^{1/2} R_g + M_f^2 \int f^{1/2} R_f + \sum_{i=f,g} S_m^{(i)} - M_*^4 \int g^{1/2} V(X)$$

- Inverse grav. couplings

$$M_g^2 = 1$$

$$M_f^2 \gg 1$$

Binary pulsar

- Graviton mass

$$m_2 \sim m \equiv M_*^2 / M_g \qquad (< 10^{-48})$$

- Overall scale **(cut-off)**

$$M_* = (m M_g)^{1/2} \qquad (< 10^{-24})$$

- We would like to cancel

$$\rho_{\text{vac}}^{1/4} > \text{TeV} = 10^{-16}$$

(Much bigger than M_*)

Vacuum solutions

Generic $V = V(X)$

Field Eqs.

$$M_g^2 R_g{}^{\mu}{}_{\nu} = M_*^4 (V \delta^{\mu}{}_{\nu} + (V' X)^{\mu}{}_{\nu}) + \rho_{\text{vac}}^g \delta^{\mu}{}_{\nu}$$

$$M_f^2 R_f{}^{\mu}{}_{\nu} = M_*^4 (V \delta^{\mu}{}_{\nu} - \underbrace{(V' X)^{\mu}{}_{\nu})}) + \rho_{\text{vac}}^f \delta^{\mu}{}_{\nu}$$

Not automatically $\propto \delta^{\mu}{}_{\nu}$

- If we had a one parameter family of solutions satisfying $(V'X)^{\mu}_{\nu} \propto \delta^{\mu}_{\nu}$ with some free integration constant $\gamma \sim 1$, then:

$$M_g^2 \Lambda^g(\gamma) = M_*^4 F_1(\gamma, \ldots) + \rho_{\text{vac}}^g$$

$$M_f^2 \Lambda^f(\gamma) = M_*^4 F_2(\gamma, \ldots) + \rho_{\text{vac}}^f$$

We could choose the integ. constant so that

$$\Lambda^g(\gamma) \ll \rho_{\text{vac}}^g$$

Provided that $M_*^4 \geq \rho_{\text{vac}}^g$

- Our vacuum energy would be curving the "other" metric, but not ours !

Type I Ansatz (non-diagonal):

$$\begin{cases} g_{\mu\nu}\, dx^\mu dx^\nu = -(1-q)\,dt^2 + (1-q)^{-1}\,dr^2 + r^2\, d\Omega^2 \\ f_{\mu\nu}\, dx^\mu dx^\nu = -\,\gamma\,[\,\beta(1-p)\ dt^2 + \ A\,dr^2 + \ r^2\, d\Omega^2 + 2\,D\,dtdr\,] \end{cases}$$

Isham and Storey 78
Blas, Deffayet, J.G. 07
Berezhiani et al. 07

Overall constant γ

Lorentz breaking β

$$A = (1-q)^{-2}(\beta p - \beta q + 1 - q)$$

$$D^2 = (1-q)^{-2}(p-q)(\beta p + 1 - \beta - q)$$

- **With this ansatz** $\tau_{\mathrm{n}} = \mathrm{tr}\, X^n = \gamma^{-n}(3 + \beta^{-n}) = \mathrm{const.}$
- **Eigenvalues of** X^μ_ν : $\lambda_0(\beta,\gamma), \quad \lambda_1 = \lambda_2 = \lambda_3(\beta,\gamma)$

Generically, we obtain $X^\mu_\nu \propto \delta^\mu_\nu$ **for some** $\beta = \beta(\gamma)$

$$\boxed{p = \frac{\gamma\,\Lambda_f(\gamma)}{3}\, r^2 \qquad q = \frac{\Lambda_g(\gamma)}{3}\, r^2}$$

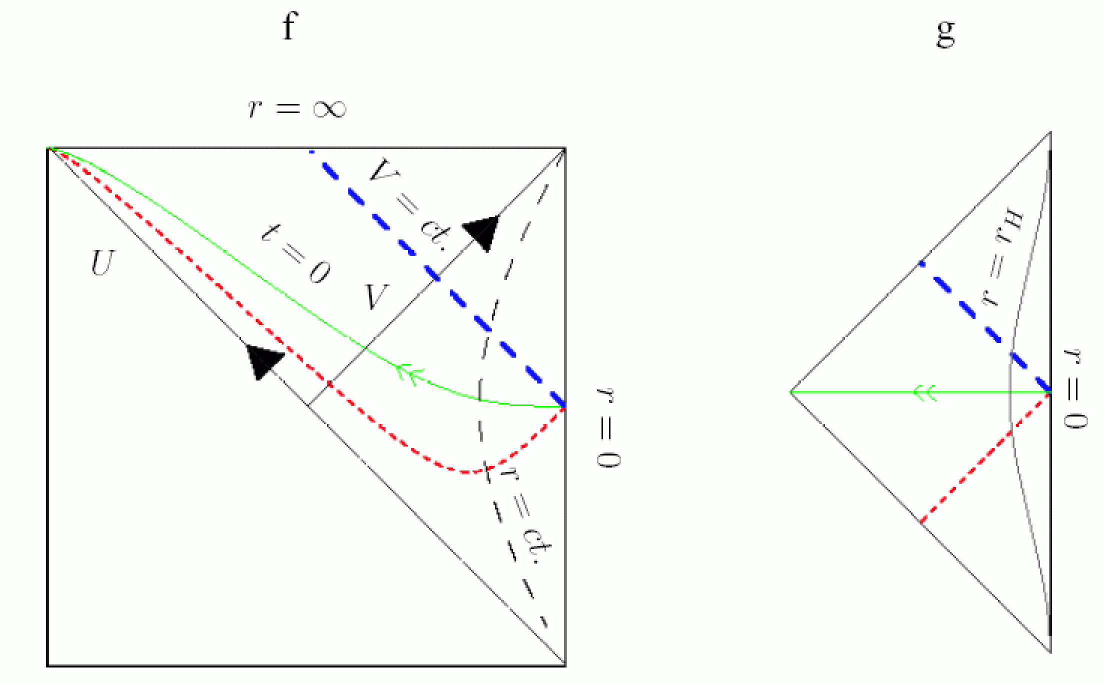
Free parameter

"Standard" vacuum solutions of GR

Global structure of a (de Sitter) – (Minkowski) bigravity solution

(Blas, Deffayet, J.G. 06)

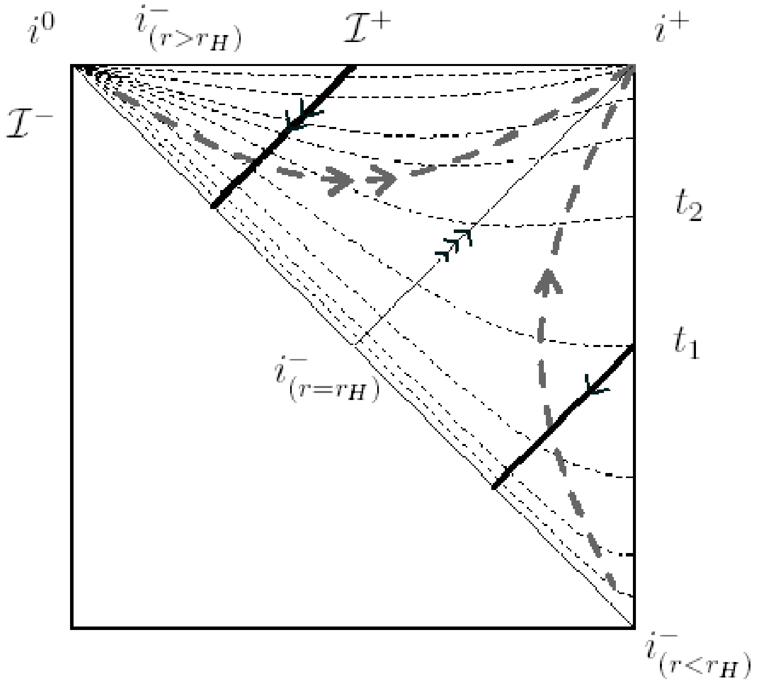
$$\begin{cases} g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dr^2 - r^2 d\Omega_2^2 \\ f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3}(1-p)dt^2 - \frac{4}{3}p\; dtdr - \frac{2}{3}(1+p)dr^2 - \frac{2}{3}r^2 d\Omega_2^2 \end{cases}$$



$$\pi^0{}_{,r} = \frac{p}{(1-p)}$$

The past timelike infinity of Minkowski is mapped into part of the de Sitter horizon

f



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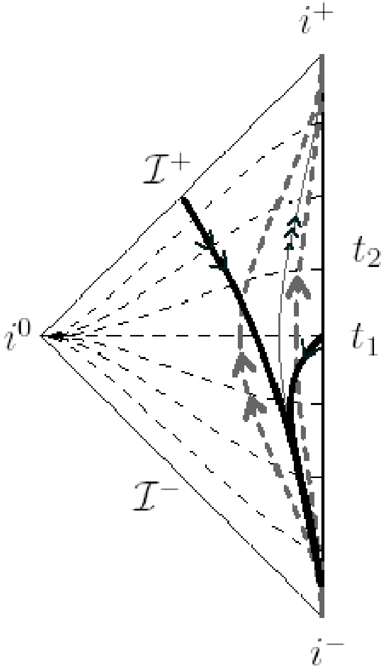
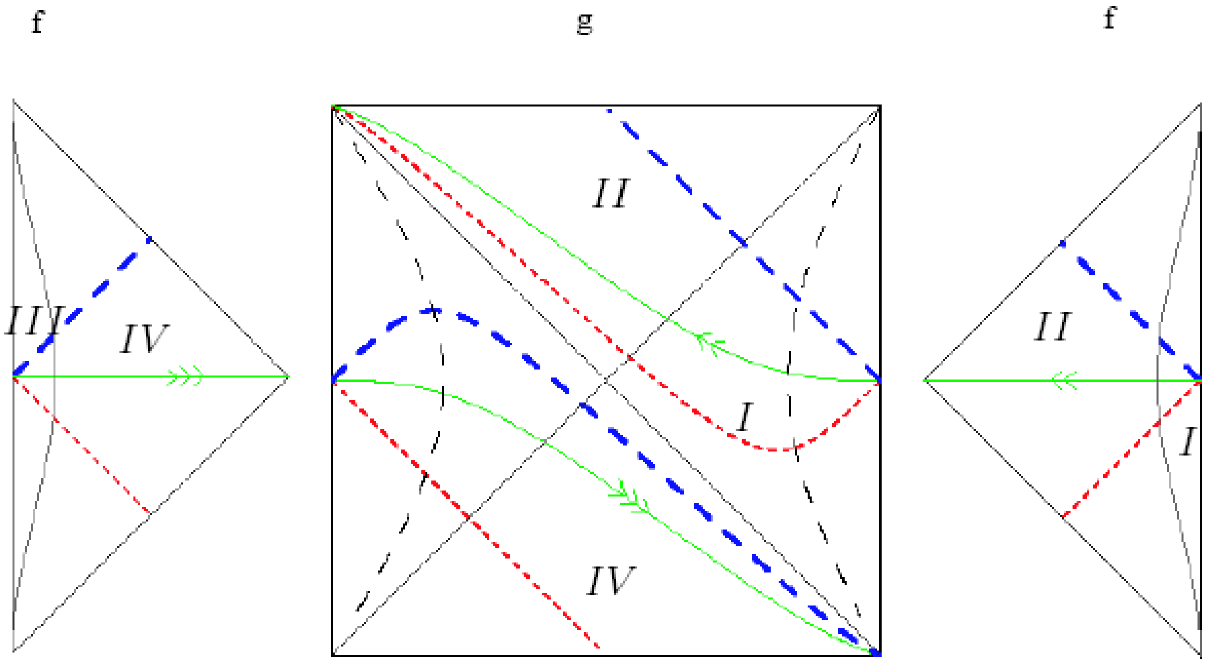
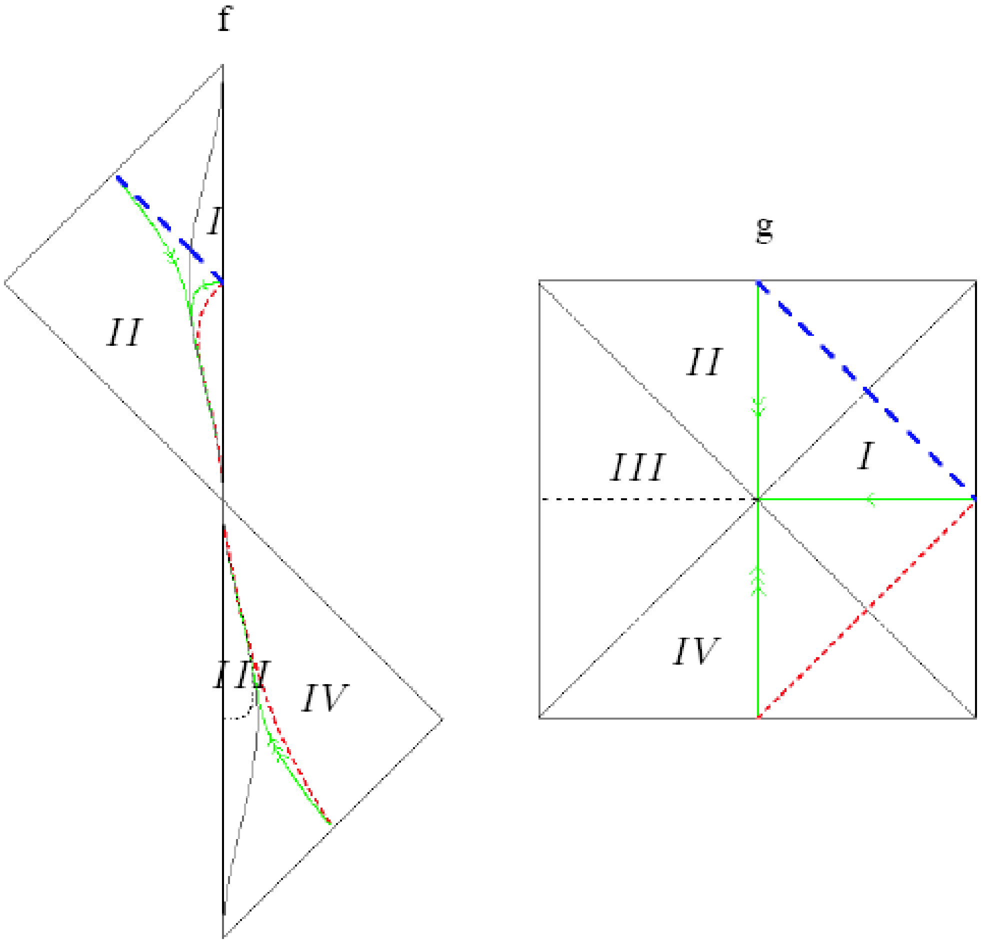


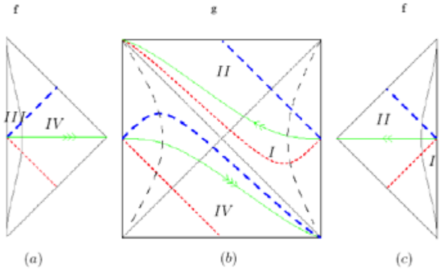


(a) (b) (c)

- Not necessarily unique extension
- Cauchy horizon (unstable?)

Global structure of a (de Sitter) – (Minkowski) bigravity solution





(de Sitter) – (de Sitter) solution

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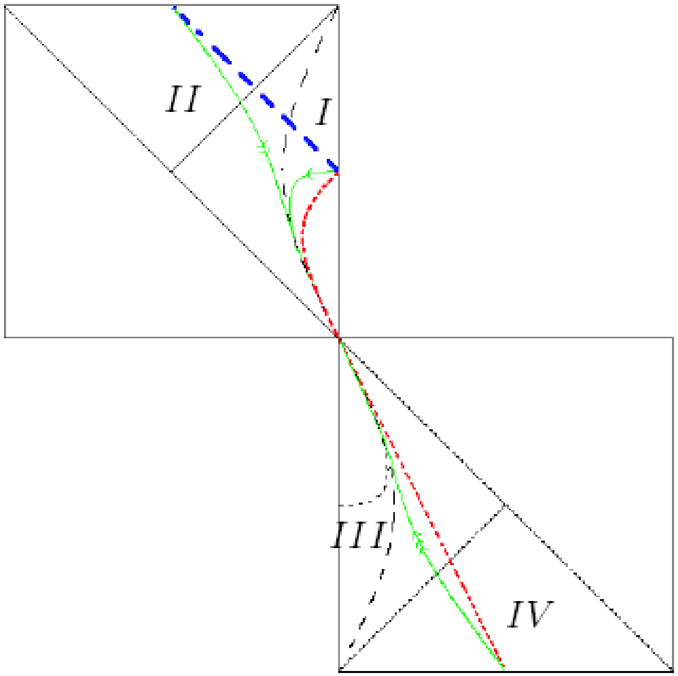

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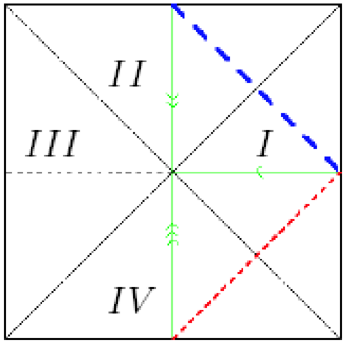


Diagonal Solutions:

Just take p=q in the Type I ansatz

$$g_{\mu\nu}\, dx^{\mu} dx^{\nu} = -(1-q)\, dt^2 + (1-q)^{-1}\, dr^2 + r^2\, d\Omega^2$$

$$f_{\mu\nu}\, dx^{\mu} dx^{\nu} = -\gamma\, [\, \beta (1-q)\; dt^2 + (1-q)^{-1}\; dr^2 + \; r^2\, d\Omega^2 \;]$$

Non-diagonal term $D^2 = (1-q)^{-2} (p-q)\, (\beta p + 1 - \beta - q) = 0$

Eigenvalues of X^{μ}_{ν} **:** $\lambda_0(\beta,\gamma), \quad \lambda_1 = \lambda_2 = \lambda_3(\beta,\gamma)$

We obtain $X^{\mu}_{\nu} \propto \delta^{\mu}_{\nu}$ **for** $\beta = 1$ **but, also for other** $\beta = \beta(\gamma)$

$$p = \frac{\gamma\, \Lambda_f(\gamma)}{3}\, r^2 \qquad q = \frac{\Lambda_g(\gamma)}{3}\, r^2$$

Was a free parameter for Type I sol.

But now the parameter is fixed
because of the additional requirement $\Lambda_g(\gamma) = \gamma\, \Lambda_f(\gamma)$

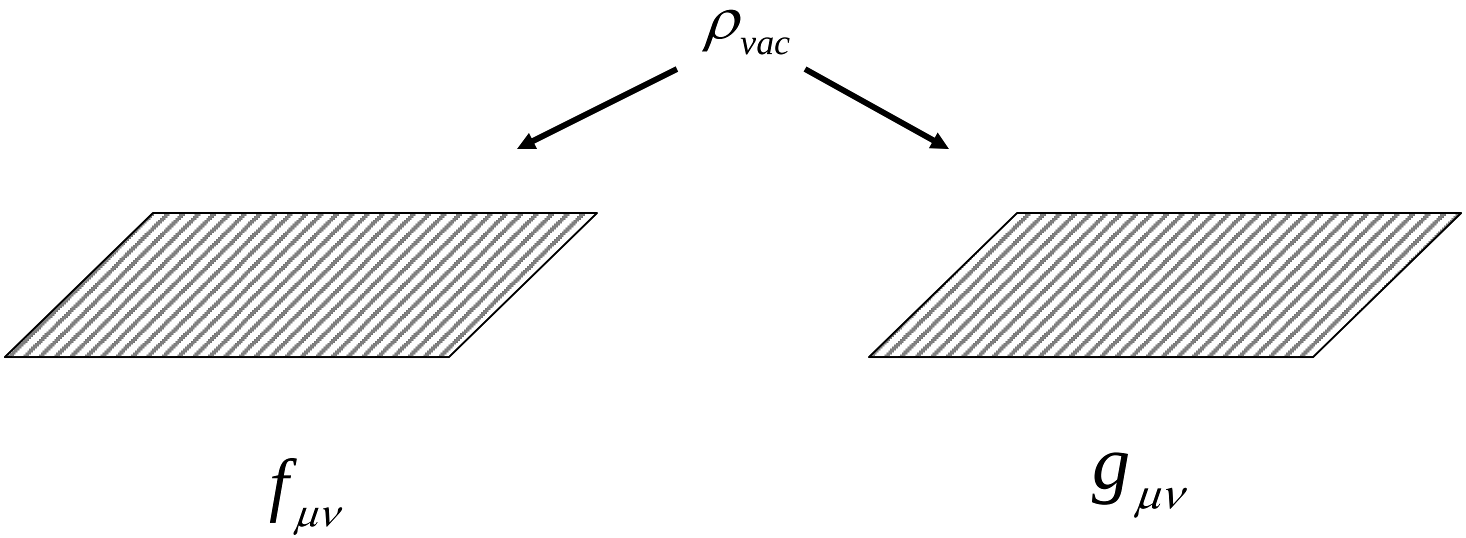
$$M_*^4\, F_1(\gamma) + \rho^{g}_{\mathrm{vac}} = \gamma\, M_f^{-2} [M_*^4\, F_2(\gamma) + \rho^{f}_{\mathrm{vac}}]$$

$$\Lambda_g(\gamma) = \gamma\,\Lambda_f(\gamma)$$

$$M_*^4\,F_1(\gamma) + \rho^g_{\text{vac}} \;=\; \gamma\,M_f^{-2}\left[M_*^4\,F_2(\gamma) + \rho^f_{\text{vac}}\right]$$

Still an acceptably low curvature vacuum provided that

$$\rho^g_{vac} \le M_*{}^4 \qquad\qquad M_f \gg M_*,$$



Perturbations around Lorentz-Breaking biflat solutions

$$-\frac{M^4}{8}\Big\{n_2(h^g{}_{ij}+h_f{}^{ij})(h^g{}_{ij}+h_f{}^{ij})+n_0(h^g{}_{00}+\beta^{-1}h_f{}^{00})(h^g{}_{00}+\beta^{-1}h_f{}^{00})$$
$$-2n_4(h^g{}_{00}+\beta^{-1}h_f{}^{00})(h^g{}_{ii}+h_f{}^{ii})+n_3(h^g{}_{ii}+h_f{}^{ii})^2\Big\}$$

n_i are potential V(X) dependent coefficients

$\xi^i(t)$

Residual Diff[f] invariance in this background

Peculiarity of this mass term: components $h_f{}^{0i}$ and $h^g{}_{0i}$ are absent

⇨ No propagating scalars and vectors, only tensors are propagating but stable perturbations

(3+1 split of the metric)

$$\begin{cases} h^X{}_{00} &= 2A^X \\ h^X{}_{0i} &= B^X_{,i}+V^X_i \\ h^X{}_{ij} &= 2\psi^X\delta_{ij}-2E^X_{,ij}-2F^X_{(i,j)}-t^X_{ij} \end{cases}$$

- **Linearized spectrum:** $\left\{ \begin{array}{l} \text{One massless graviton} \longleftarrow \\ \text{One massive graviton } m_2 \\ \quad \text{with just two polarizations.} \end{array} \right.$

(Can be decoupled by $M_f^2 \rightarrow \infty$, if need be)

$$\Phi_1 = -\frac{GM_1}{r} + GM_1 \underset{\underset{\frac{m_2^2[3m_4^4 - m_0^2\,(3m_3^2 - m_2^2)]}{m_4^4 - m_0^2(m_3^2 - m_2^2)}}{\uparrow}}{\mu^2} r$$

slight modifications of Newton's law

(No vDVZ discontinuity)

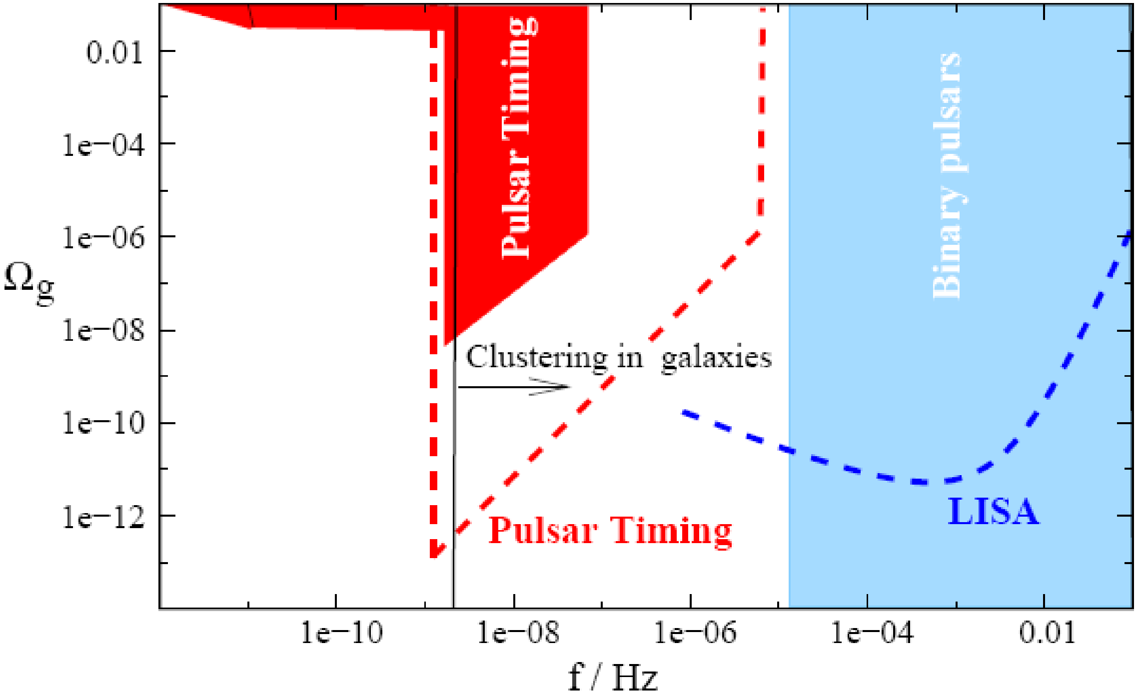
- Potentially interesting phenomenology

(Dubovsky, Tinyakov, Tkachev, 05)

- Asymptotically flat non-linear solutions with sources recently obtained

(Berezhiani, Comelli, Nesti, Pilo 08)

Relic massive gravitons detectable?



Dubovsky, Tinyakov, Tkachev, 2005

(See however Pshirkov et al. 2008)

Outlook

- Bigravity, or massive gravity, may perhaps play a role in “cancelling” the vacuum energy, by way of initial conditions

- If we do observe a background of relic massive gravitons, this possibility will of course become more popular.

- One drawback is that the generic energy scale in the massive graviton potential is not high enough to cancel a vacuum energy at, say, the TeV.

- Still, one may hope that the graviton mass $m_2 \ll m$, where m is the generic energy scale in the graviton potential. This may perhaps be due to some symmetry.

- An extreme example is the case when the graviton interaction potential is invariant under TDiff. This brings us back to the case of unimodular gravity, where we can cancel any vacuum energy.