Madrid, WQG 2008

Can infrared gravitons screen Λ ?

Jaume Garriga,
(U. Barcelona)

(with T. Tanaka, 2007)

Plan

- **Gauge dependence of** $H\left[\left\langle h_{\mu\nu}\right\rangle\right]$

- **Consider a different observable which shows no infrared screening:**

$$\left\langle R\right\rangle_W \equiv \left\langle \int \left(\sqrt{g}\; R\right)_{ren} W(x)\; d^4x \right\rangle$$

$$= const. \left\langle \int \left(\sqrt{g}\right)_{ren} W(x)\; d^4x \right\rangle$$

- **Comments**

Setup

- **Action**

$$S_{gr} = \frac{1}{2\kappa}\int \sqrt{-g}\,(\mathcal{R} - 2\Lambda)\, d^4x$$

- **Metric**

$$g_{\mu\nu}(x) = a^2(\eta)[\eta_{\mu\nu} + h_{\mu\nu}(x)]$$

- **Gauge fixing**

$$S_{gf} = -(1/2)\eta^{\mu\nu} F_\mu[h] F_\nu[h]$$

$$F_\mu[h] \equiv a\left(h^\nu_{\mu,\nu} - \frac{1}{2} h_{,\mu} + 2 h^\nu_\mu \frac{a_{,\nu}}{a}\right)$$

$$S_{tot} = S_{gr} + S_{gf} + S_{FP} + S_{ct}$$

- **Tadpole**

$$\langle h_{\mu\nu}\rangle_F = \int_{CTP} \mathcal{D}\psi^+\ \mathcal{D}\psi^-\ h^+_{\mu\nu}\ e^{iS_{tot}[\psi^+]}\, e^{-iS_{tot}[\psi^-]}$$

Tsamis and Woodard

- **From symmetries of initial state**

$$\langle h_{\mu\nu}\rangle_F = A_F(\eta)\ \eta_{\mu\nu} + B_F(\eta)\ t_\mu t_\nu$$

- **Expansion rate of the averaged metric**

$$H_F \equiv H(\langle h_{\mu\nu}\rangle_F) = \frac{d\ln[a(1+A_F)^{1/2}]}{a\ (1+A_F-B_F)^{1/2}d\eta} = \frac{H_0}{(1+A_F-B_F)^{1/2}}\left[1-\frac{1}{2}\frac{\eta A_F'}{(1+A_F)}\right]$$

- **Tadpole at two loops**

$$H_F = H_0\left[1-\underbrace{4\kappa^2\left(\frac{H_0}{4\pi}\right)^4\left[\frac{1}{6}(H_0t)^2+\mathcal{O}(H_0t)\right]}+0(\kappa^6)\right] \longrightarrow$$

"Secular screening"

- **However, something is peculiar:**

$$\frac{dH_F}{dt} \propto -\frac{H_0^6}{M_P^4}\quad H_0\, t$$

Looks different from a typical screening effect due to pair creation. e.g. :

$$\frac{dE}{dt} \propto \exp\left[-\frac{m^2}{(eE)^2}\right]$$

Changing the gauge

- **Original gauge fixing condition** $F[h] = 0$
- **New g.f. condition** $G[\tilde{h}] = 0$
- **Metrics related by** $\tilde{h}_{\mu\nu} = h_{\mu\nu} + \delta_\chi h_{\mu\nu}$

$$\delta_\chi h_{\mu\nu} = 2a^{-2}\nabla_{(\mu}\chi_{\nu)} + \mathcal{O}(\chi^2)$$

- **In general** $\boxed{\chi^\mu = \chi^\mu[h]}$ $F[h] = G[h + \delta_\chi h]$
- **For instance**

$$F_\mu^{(\alpha)} = F_\mu[h] \equiv a\left(h^\nu_{\mu,\nu} + \alpha h_{,\mu} + 2h^\nu_\mu \frac{a_{,\nu}}{a}\right)$$

A change of α corresponds to $\delta F_\mu = a\, h_{,\mu}\, \delta\alpha$

$$\int dx' \frac{\delta F_\mu[h]}{\delta h_{\rho\sigma}(x')}\, \delta_\chi h_{\rho\sigma}(x') = a\; h_{,\mu}\, \delta\alpha \longrightarrow \mathcal{O}_{\mu\nu}[h]\chi^\nu = h_{,\mu}$$

Gauge dependence

- **Under** $\chi^\mu = \chi^\mu[h]$ $\quad F[h] = G[h + \delta_\chi h]$

$$S_{gr}[h] = S_{gr}[h + \delta_\chi h] \qquad S_{ct}[h] = S_{ct}[h + \delta_\chi h]$$

$$(S_{gf})_F[h] = (S_{gf})_G[h + \delta_\chi h] \qquad (S_{FP})_F[h] = (S_{FP})_G[h + \delta_\chi h]$$

- **We have**

$$\langle h_{\mu\nu} \rangle_G = \langle h_{\mu\nu} + \delta_\chi h_{\mu\nu} \rangle_F$$

- **Tadpole correction**

$$\langle \delta_\chi h_{\mu\nu} \rangle = \langle [\eta_{\lambda\nu} + h_{\lambda\nu}]\chi^\lambda_{,\mu} + [\eta_{\lambda\mu} + h_{\lambda\mu}]\chi^\lambda_{,\nu} + h_{\mu\nu,\lambda}\chi^\lambda - \frac{2}{\eta}[\eta_{\mu\nu} + h_{\mu\nu}]\chi^0 \rangle + O(\chi^2)$$

- **Trouble is that** $\langle \delta_\chi h_{\mu\nu} \rangle \neq \delta_{\langle \chi \rangle} \langle h_{\mu\nu} \rangle + \cdots$ $\quad \langle \delta_\chi h_{\mu\nu} \rangle \neq \delta_\xi \langle h_{\mu\nu} \rangle + \cdots$

- **And therefore** $\delta H(\eta) \neq \frac{dH}{d\eta}\, \delta\eta$ → **The "secular screening" depends on gauge.**

Observables

- **Consider adiabatic change** $|\dot{H}| \ll H^2$

$$\mathcal{R} = 12H^2(t) + 6\dot{H}(t) \approx 12H^2(t)$$

→ Scalar curvature is a good tracer of expansion rate

- **NB: this would not be true for changes which are not adiabatic (e.g. if the universe is filled with radiation). In general**

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1 + w_{\rm eff}) \sim 1$$

But here we are assuming $(1 + w_{\rm eff}) \ll 1$

- **In the adiabatic regime, there is not much difference between** $\mathcal{R}$ **and** $12H^2(t)$

Probing R

- e.g. test scalar field $$L \sim \sqrt{-g}\left\{(\partial_\mu \phi)^2 - (m^2 + \frac{1}{6}R)\phi^2 + J\phi\right\}$$

  Abramo + Woodard

- Equation of motion in massless case $$\sqrt{g}\left[\Box + \frac{1}{6}R\right]\phi = \sqrt{g}\ J$$

- In the adiabatic limit, and for a constant source J we obtain the late time solution

$$\phi \approx \frac{6\sqrt{g}\ J}{\sqrt{g}\ R(t)} \approx \frac{J}{2H^2(t)}$$

$\longrightarrow$ ϕ probes the expansion rate at late times

- In the massive case, the sign of $\sqrt{-g}\,(m^2 + \xi R)\phi^2$ determines whether the field is stable or not.

Observables

- We are interested in comparing $\sqrt{g}\,R$ with $\sqrt{g}\;m^2$

- Define gauge invariant expectation values of renormalized operators

$$\langle R_{ren} \rangle_W \equiv \left\langle \int \left(\sqrt{g}\; R \right)_{ren} W(x) d^4 x \right\rangle$$

$$m^2 \langle 1_{ren} \rangle_W \equiv m^2 \left\langle \int \left(\sqrt{g} \right)_{ren} W(x) d^4 x \right\rangle$$

- $W(x)$ arbitrary scalar window function $\delta W = W,_\mu \; \chi^\mu$

- Recall that $S_{tot} = S_{gr} + S_{gf} + S_{FP} + S_{ct}$

  S_{ct} will make the graviton n-point functions finite, but not at coincident points.

- Need "insertions"

$$\left(\sqrt{g}\; R \right)_{ren} \equiv \sqrt{g}\; (R + \delta R)$$

$$m^2 \left(\sqrt{g}\; \right)_{ren} \equiv \sqrt{g}\, m^2 (1 + \delta)$$

Definition of $\delta R,\ \delta$ is somewhat ambiguous (but this cannot be helped).

- We take $\delta R \equiv 2\kappa \dfrac{g_{\mu\nu}}{\sqrt{g}} \dfrac{\delta S_{ct}}{\delta g_{\mu\nu}} + 4\, \delta\Lambda$, and $\delta\Lambda \equiv \Lambda\, \delta$

- **Identity**

$$0 = -i\int D\psi \int d^4x\, W \,\frac{\delta}{\delta g_{\mu\nu}} g_{\mu\nu}\, e^{iS_{tot}} = \left\langle \frac{1}{2\kappa}(R-4\Lambda) + \frac{g_{\mu\nu}}{\sqrt{g}} \frac{\delta(S_{gf+FP}+S_{ct})}{\delta g_{\mu\nu}} \right\rangle_W$$

$$\longrightarrow \quad \langle R_{ren} \rangle_W = 4\,\Lambda \langle 1_{ren} \rangle_W - 2\kappa \left\langle \frac{g_{\mu\nu}}{\sqrt{g}} \frac{\delta S_{gf+FP}}{\delta g_{\mu\nu}} \right\rangle_W$$

Shown to vanish by a BRS technique

- **We were interested in comparing** $\sqrt{g}\,R$ **with** $\sqrt{g}\; m^2$

- **Defined corresponding gauge invariant expectation values**

$$\langle R_{ren} \rangle_W \equiv \left\langle \int \left(\sqrt{g}\;R\right)_{ren} W(x)\, d^4x \right\rangle \qquad m^2 \langle 1_{ren} \rangle_W \equiv m^2 \left\langle \int \left(\sqrt{g}\right)_{ren} W(x)\, d^4x \right\rangle$$

- **We find that** $\dfrac{\langle R_{ren} \rangle_W}{m^2 \langle 1_{ren} \rangle_W} = \dfrac{\Lambda \langle 1_{ren} \rangle_W}{m^2 \langle 1_{ren} \rangle_W} = \text{const.}$ $\longrightarrow$ No evidence for “secular screening”

Summary

- The two-loop result of TW looks peculiar:

$$\frac{dH_F}{dt} \propto -\frac{H_0^6}{M_P^4} \; H_0 t$$

different from a typical screening effect due to pair creation. e.g. :

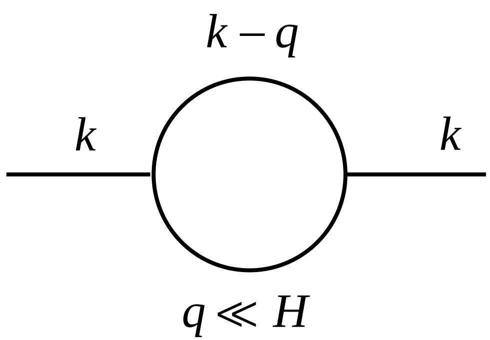
$$\frac{dE}{dt} \propto \exp\left[-\frac{m^2}{(eE)^2}\right]$$

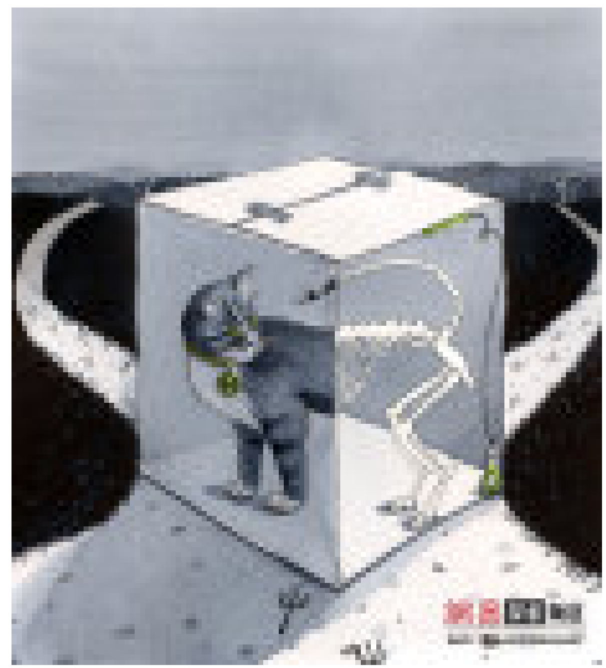
- $H\left[\langle h_{\mu\nu} \rangle\right]$ is gauge dependent

- We find $\dfrac{\langle R_{ren} \rangle_W}{m^2 \langle 1_{ren} \rangle_W} = \dfrac{\Lambda \langle 1_{ren} \rangle_W}{m^2 \langle 1_{ren} \rangle_W} = \text{const.}$ ⟶ No evidence for "secular screening" in this observable.

One more thing:

- It doesn't make much sense to include infrared contributions in loop integrals
- Modes bigger than the horizon behave classically. Circulating them in loops is analogous averaging over live and dead cats.

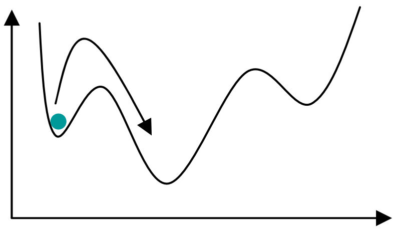




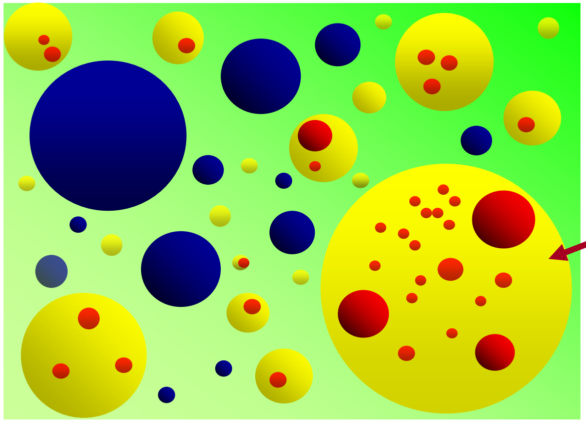
And yet another thing:

- De Sitter space *is* unstable.
- It will fry an egg if you put one in it,
  and it will fry itself into another vacuum
  if there is an adjacent one

$$T_{GH} \sim H$$



- This *is* an infrared instability, but it has nothing to do with the screening of the effective cosmological constant.
- The initial de Sitter decays into a fractal multiverse of bubbles within bubbles



Effective cosmological constant stays constant within each vacuum

Globally, all vacua are realized
(very different from screening)