"...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realize that these numbers and equations we play with at our desks have something to do with the real world..."

S. Weinberg, "The first three minutes"

Inflation and CMB

V. Mukhanov

LMU, München



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-Spatially flat Universe: $\Omega_{total} = 1 \pm 10^{-5}$

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-Gravity waves



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Which concrete scenario was realized???





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Today in galactic scales $h \sim 10^{-58}$ Can quantum fluctuations be amplified up to

"needed" value 10⁻⁵ in expanding Universe???

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$$c_s^2 = p_{,X}/\epsilon_{,X}$$

$$a''/a \approx z''/z \approx a^2 H^2$$



Purpose: Transfer these fluctuations to galactic scales $(10^{28}cm)$

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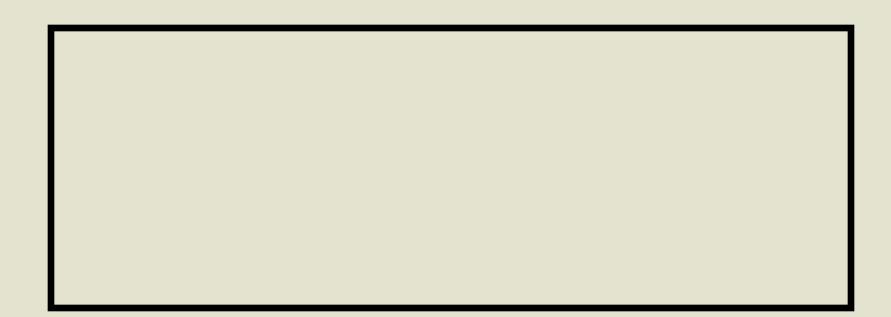
• Consider plane wave perturbation: $\delta \varphi, \Phi \propto \exp(i\vec{k}_{com}\vec{x})$

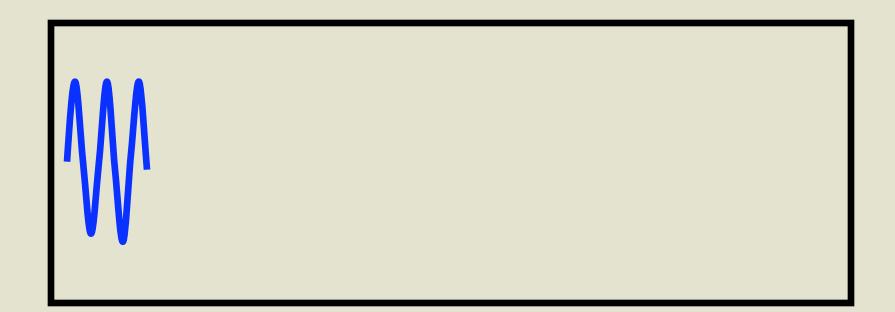
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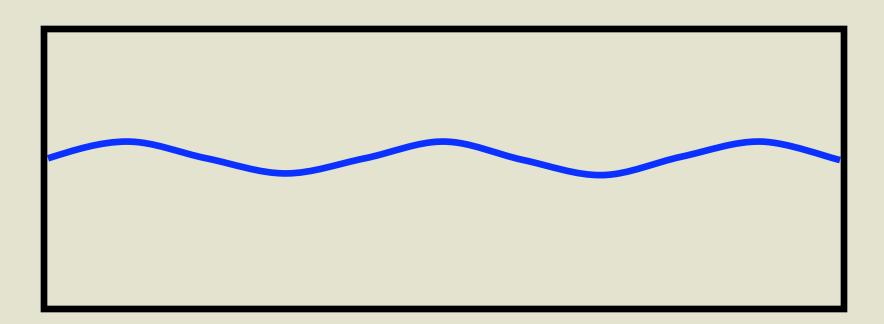
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For given k_{com} , $\lambda_{ph}(cm) \propto a/k_{com} \propto a(t)$ and the change of the amplitude with time depends on how big is λ_{phys} compared to the curvature scale (size of Einstein lift) $H^{-1} = a/\dot{a}$



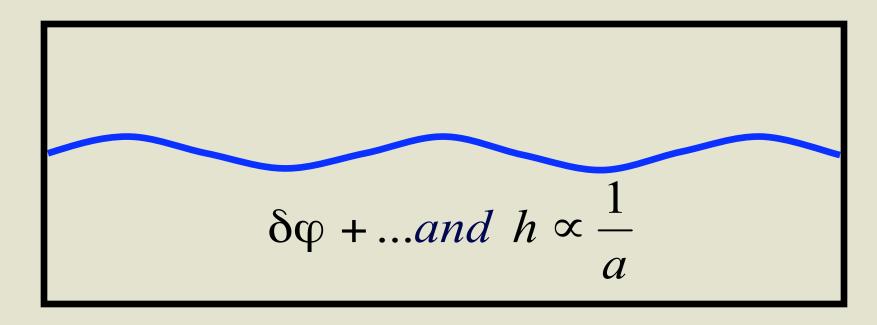




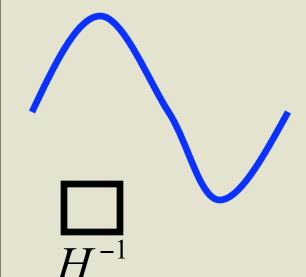


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$$\delta \varphi + ...and \ h \propto \frac{1}{a}$$



$$H^{-1}$$



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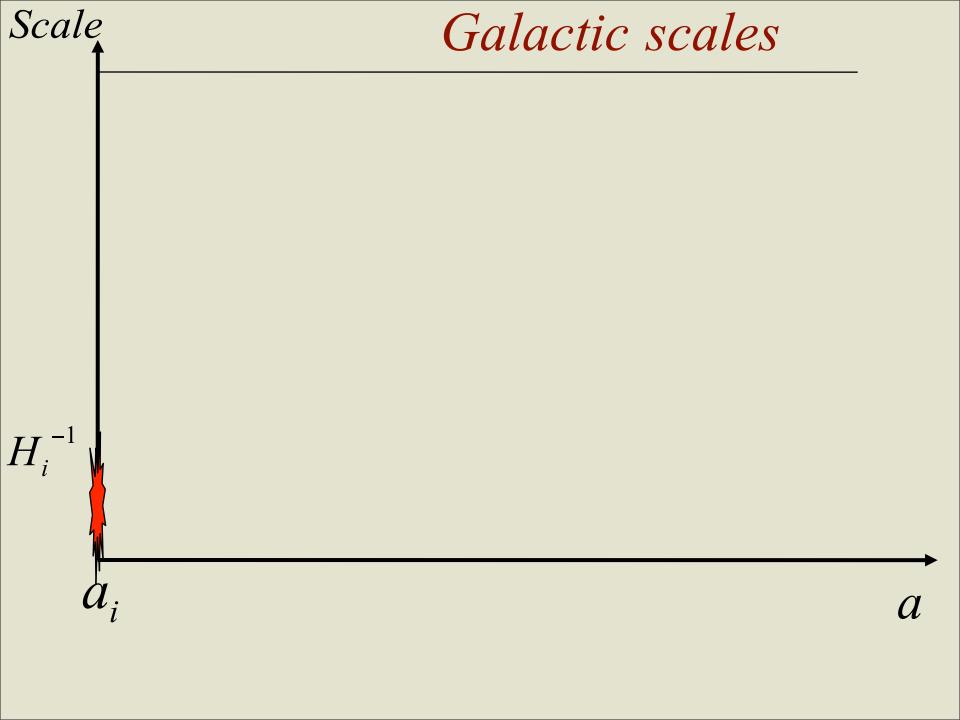


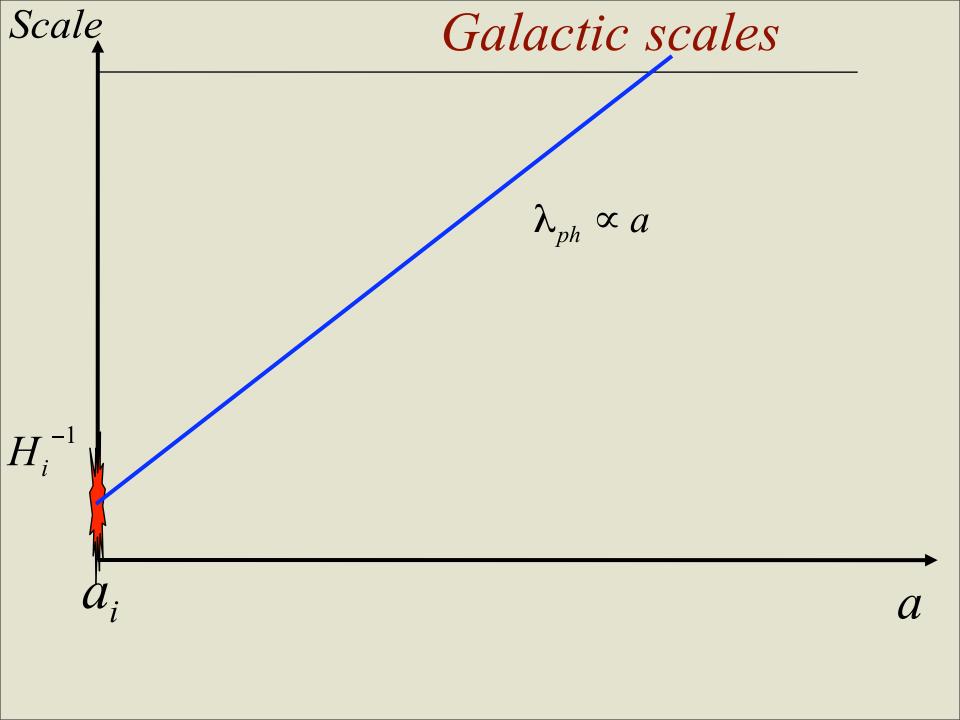
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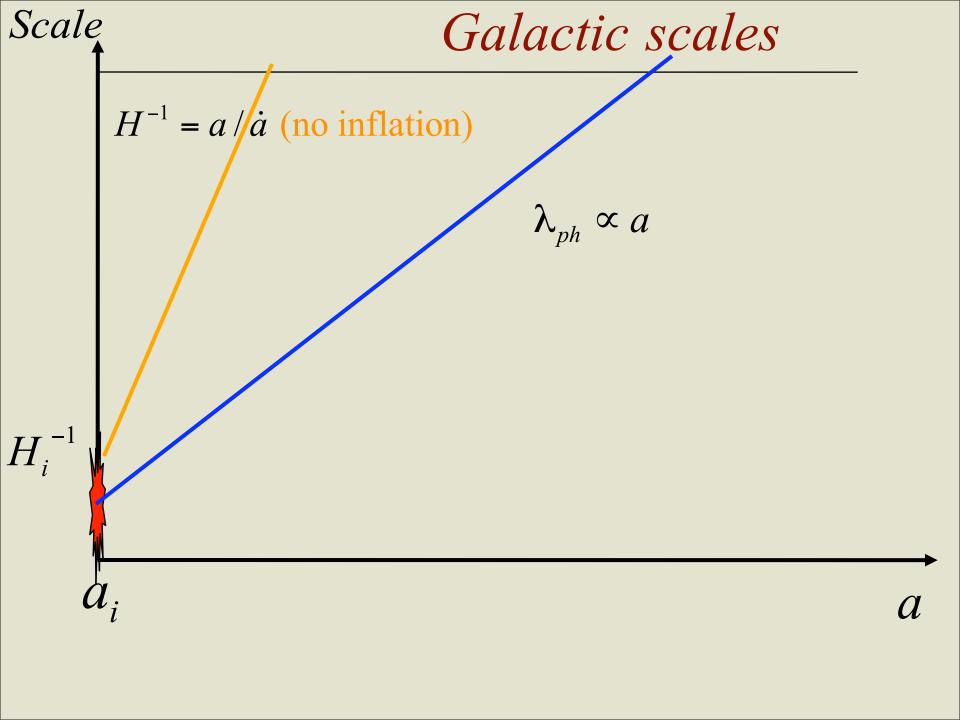
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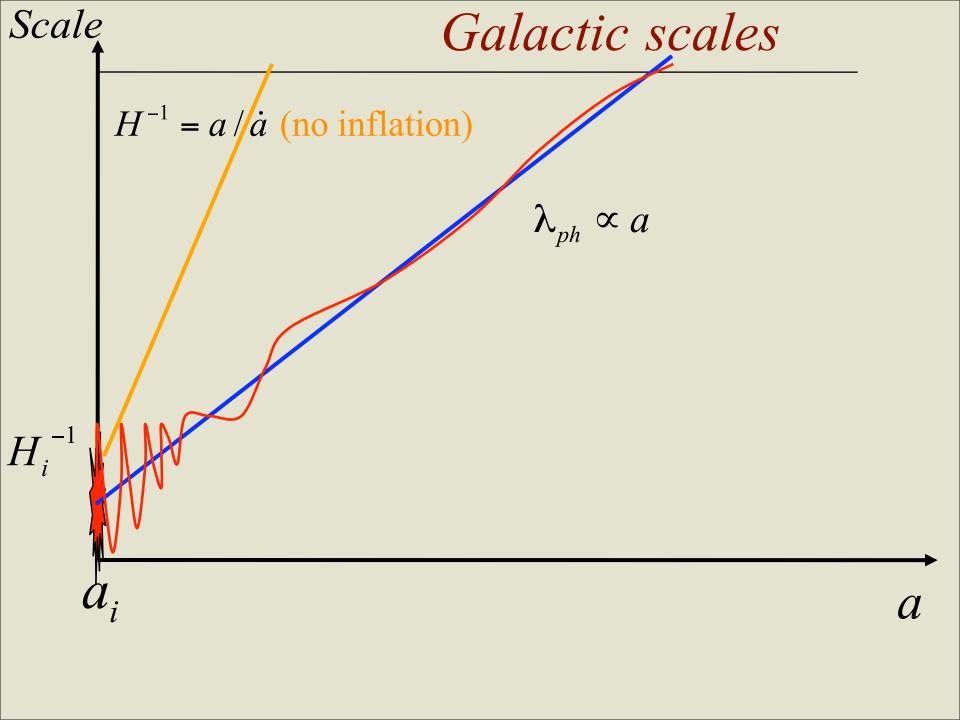
$$\delta \varphi + ... \propto \sqrt{1 + p/\epsilon}$$

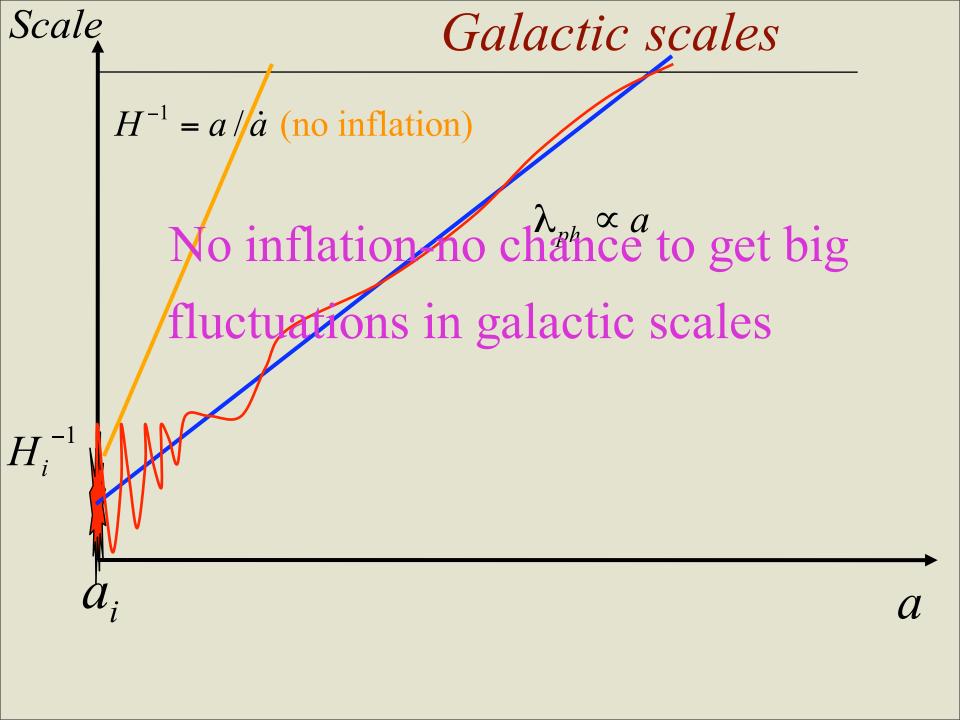




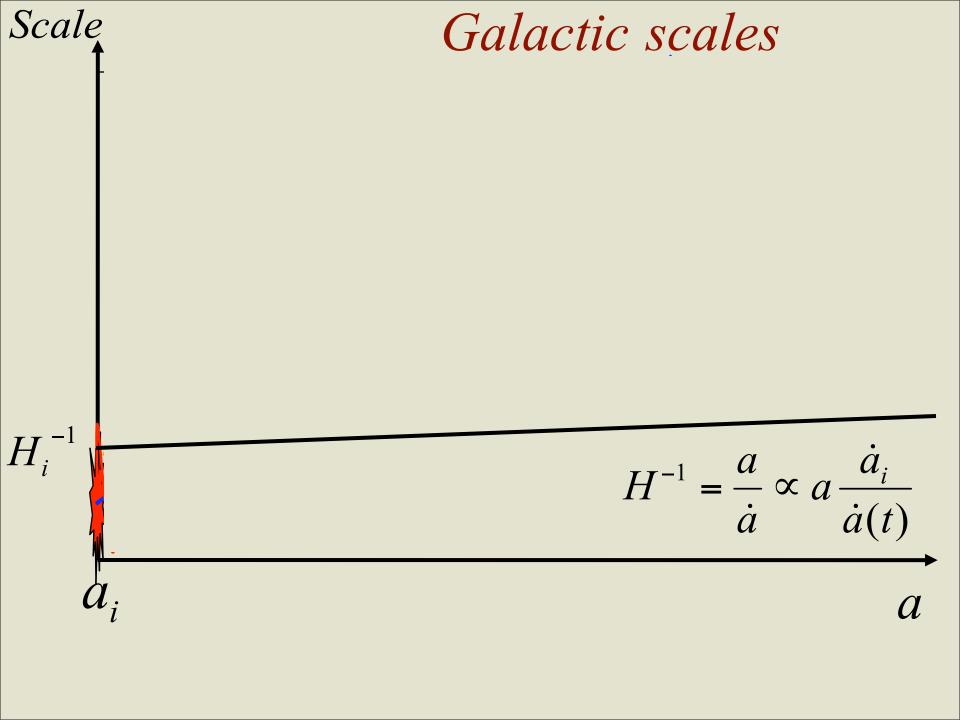


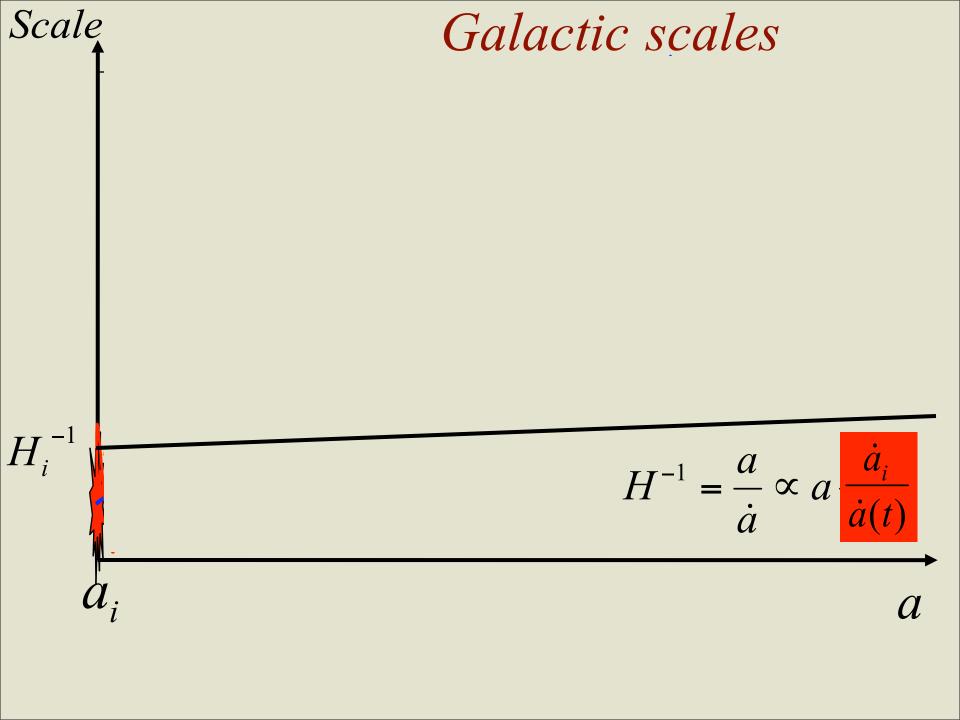


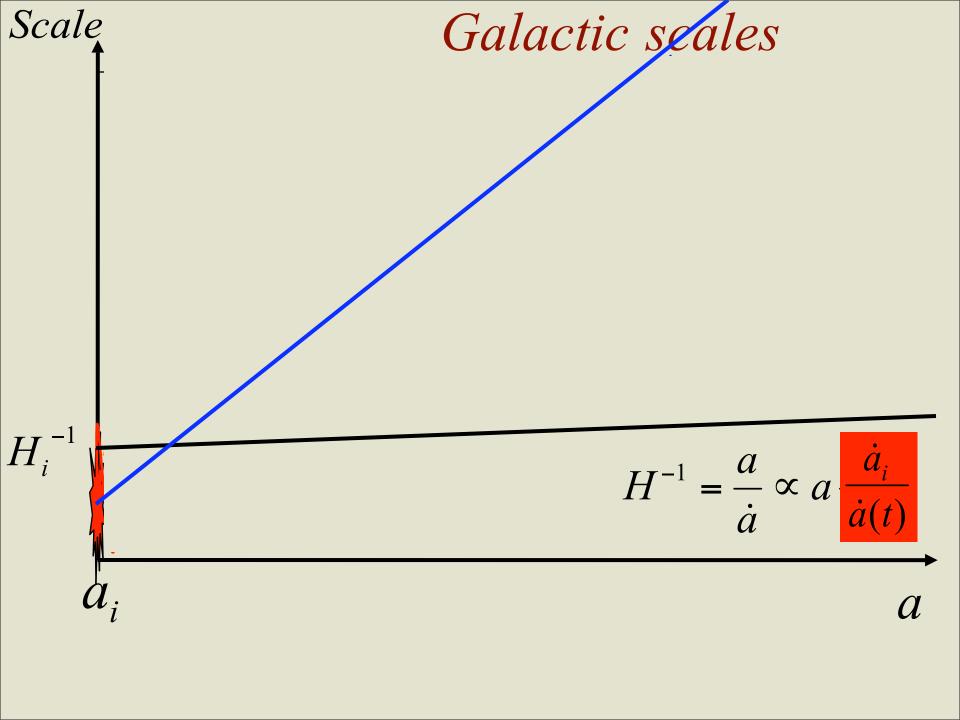


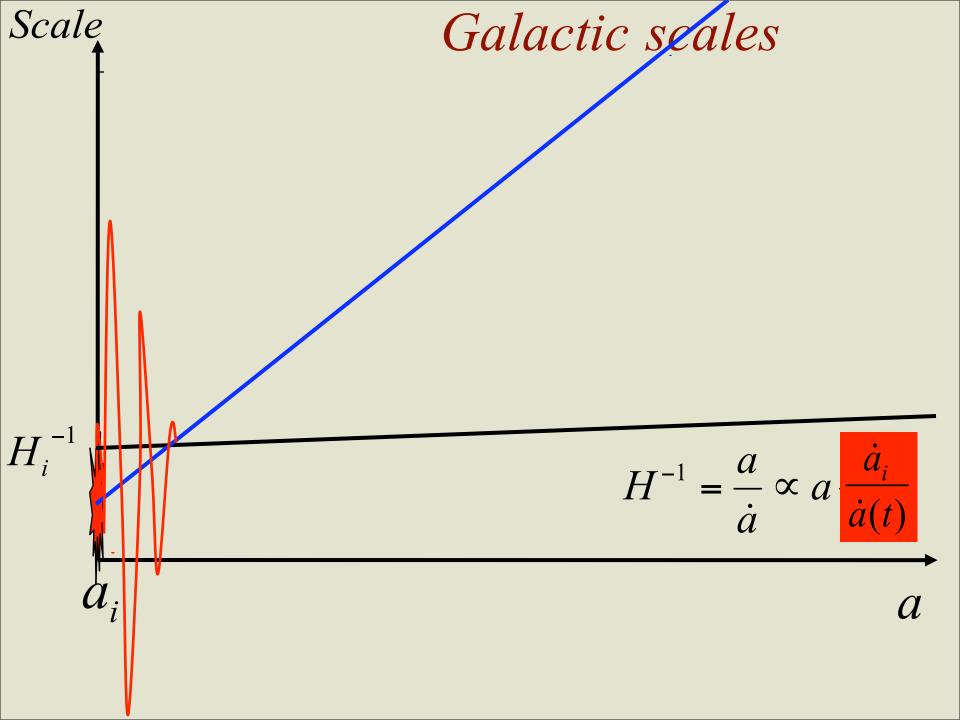


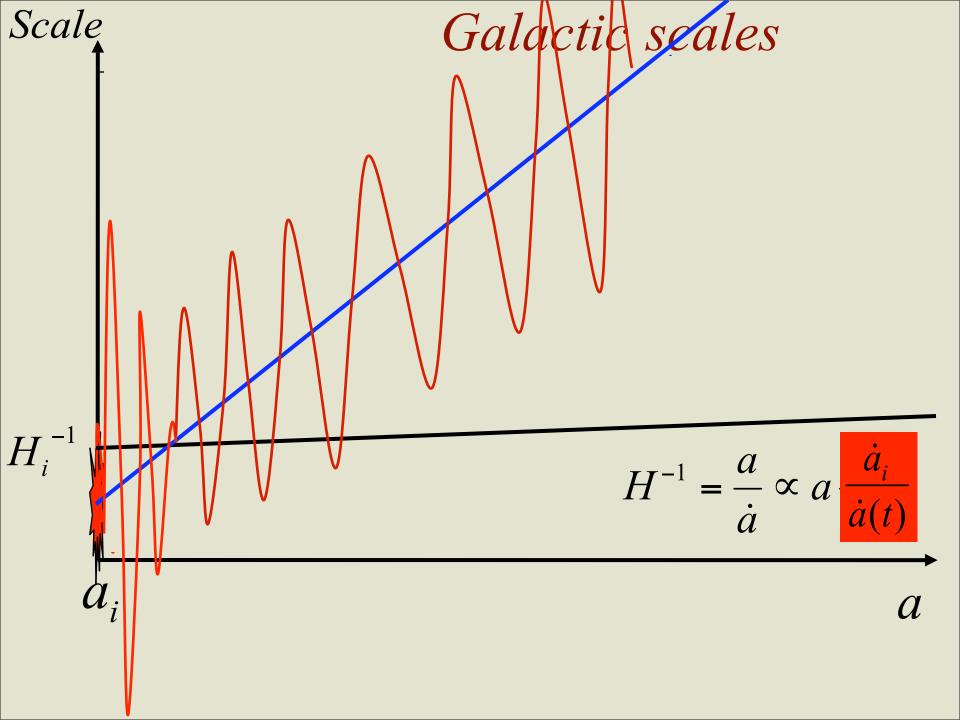


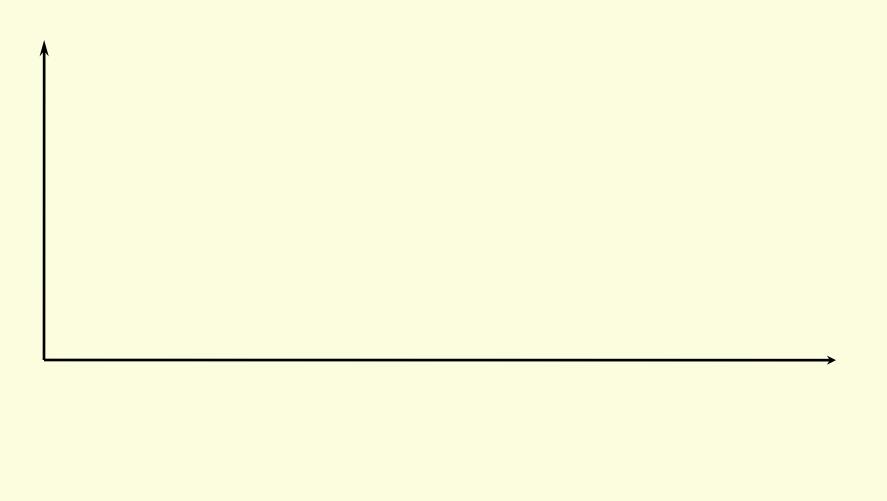


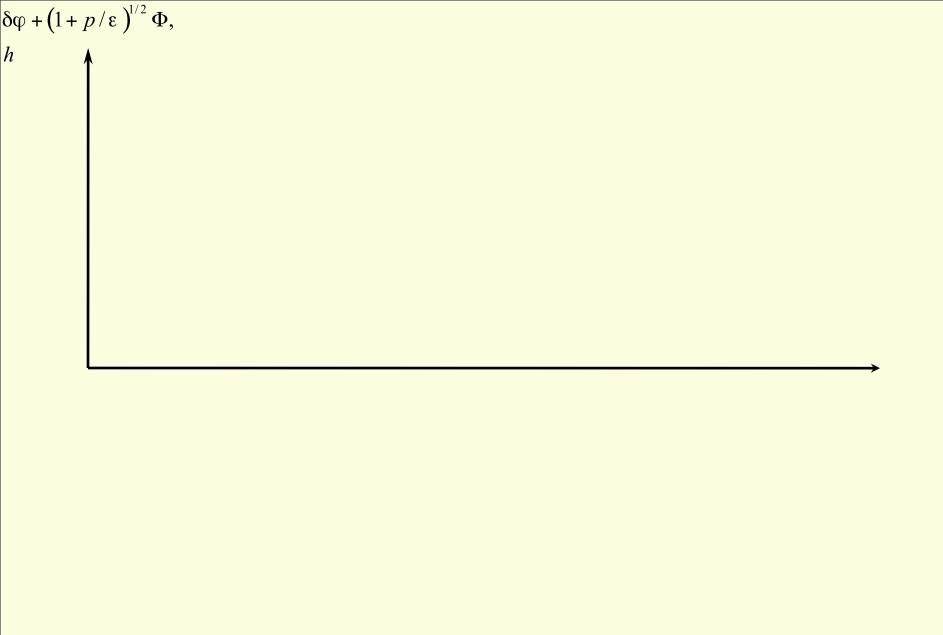


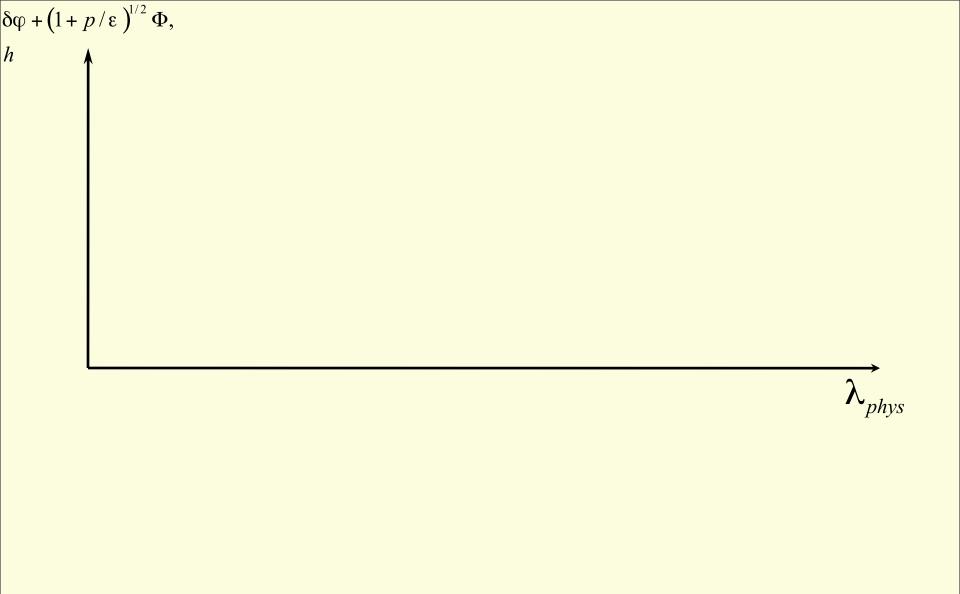




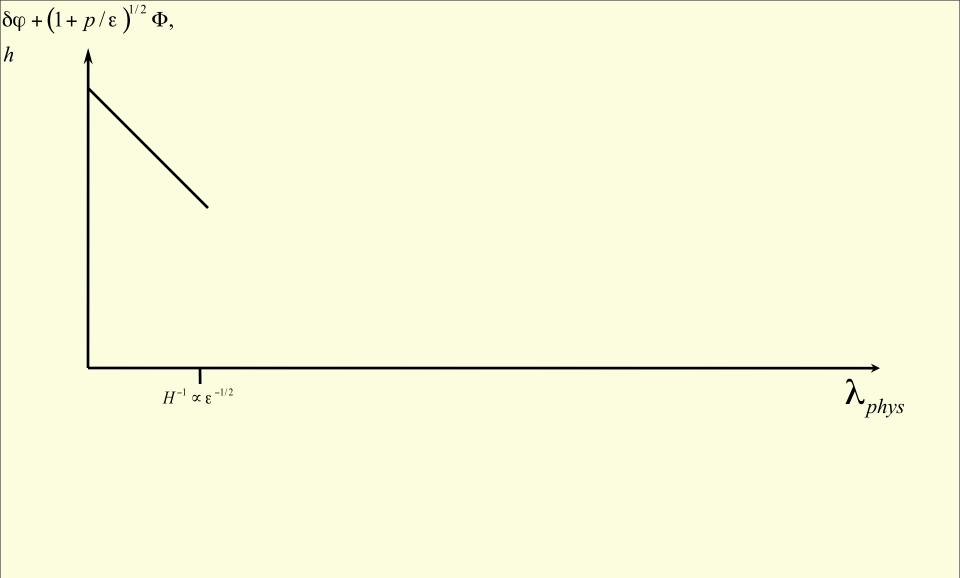


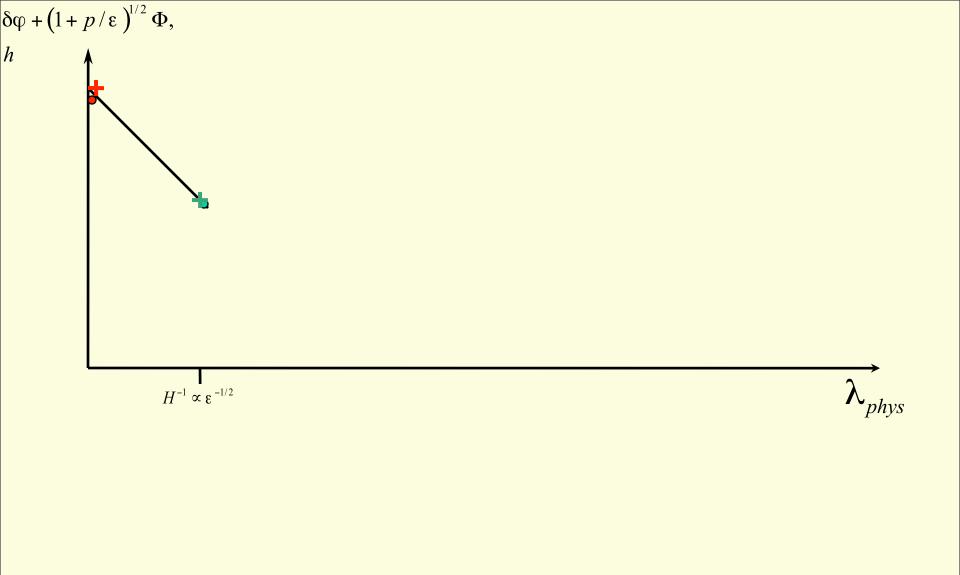


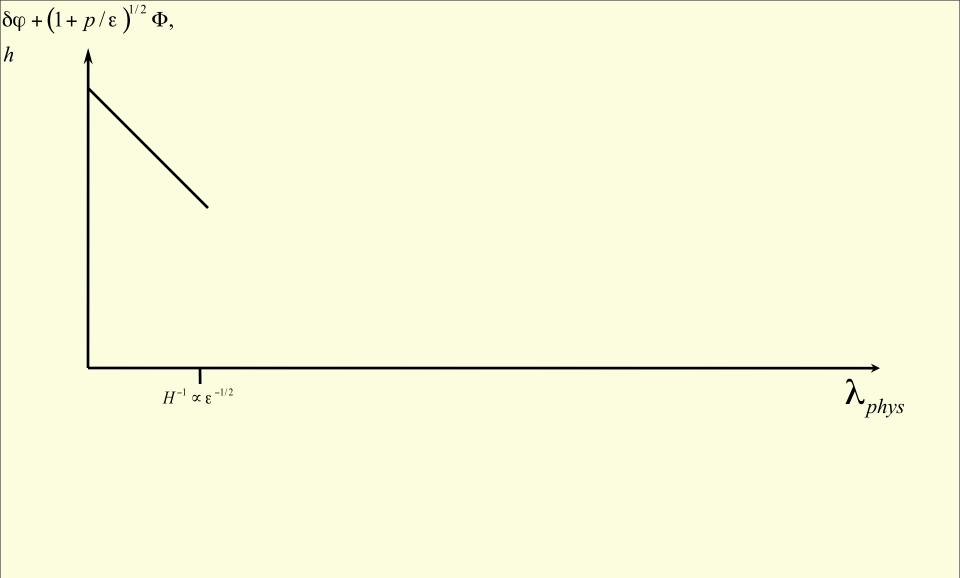


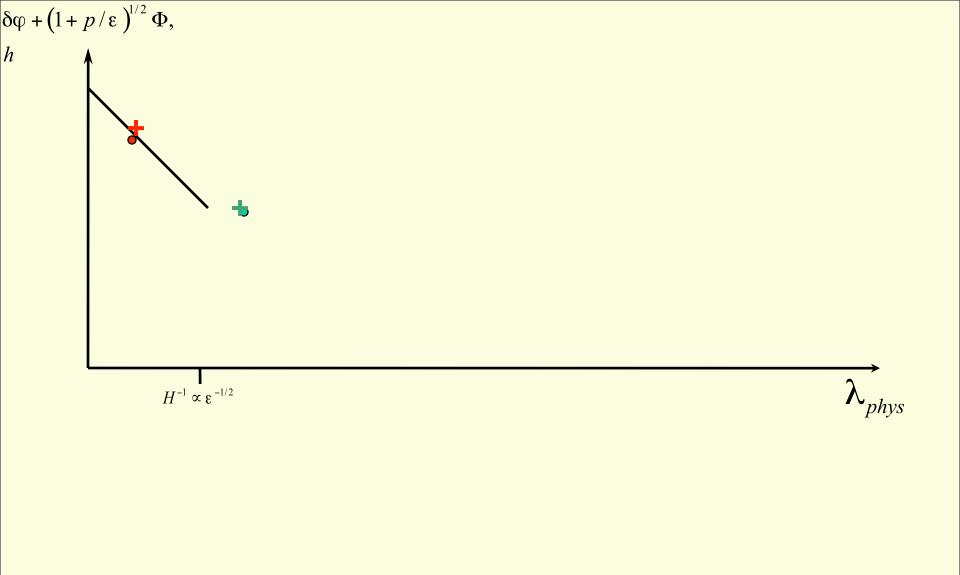


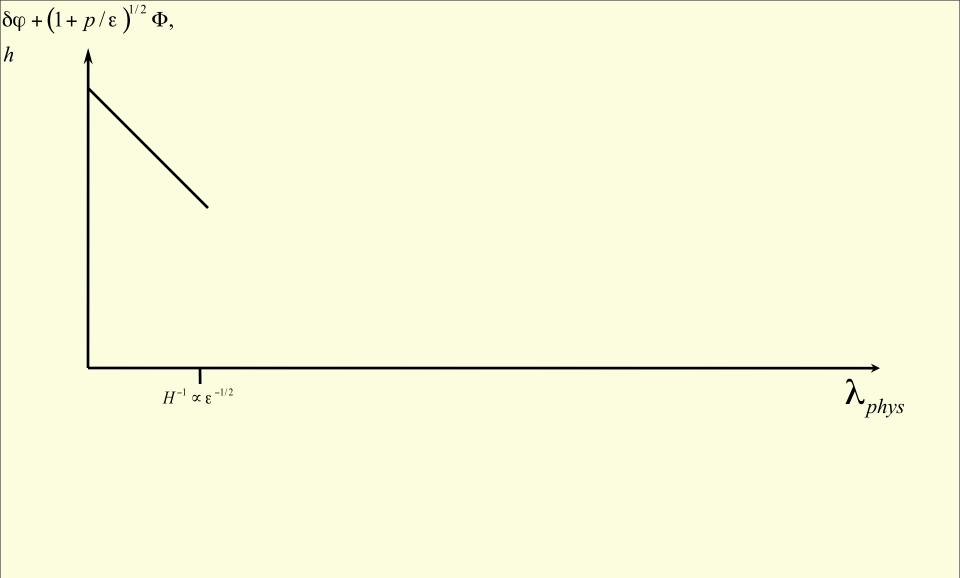


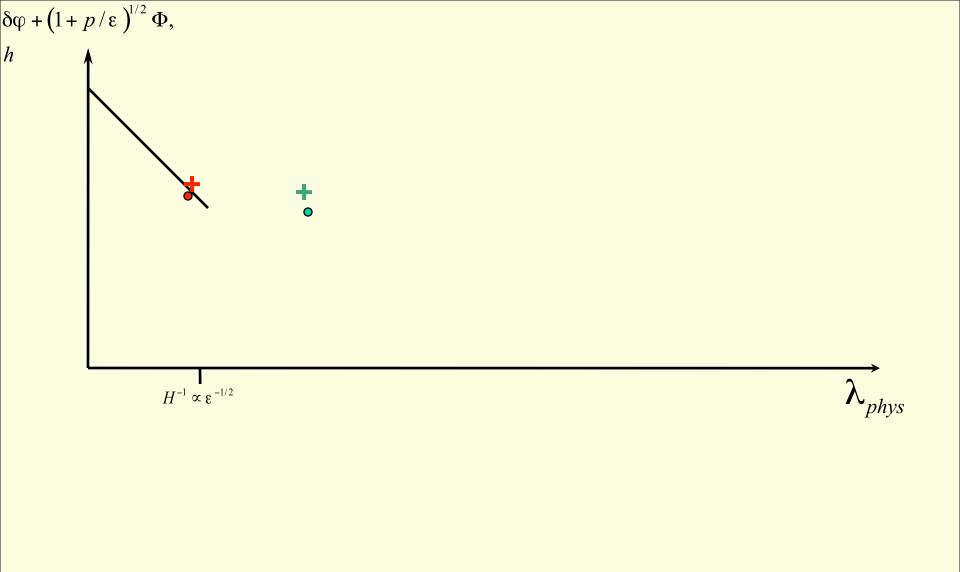


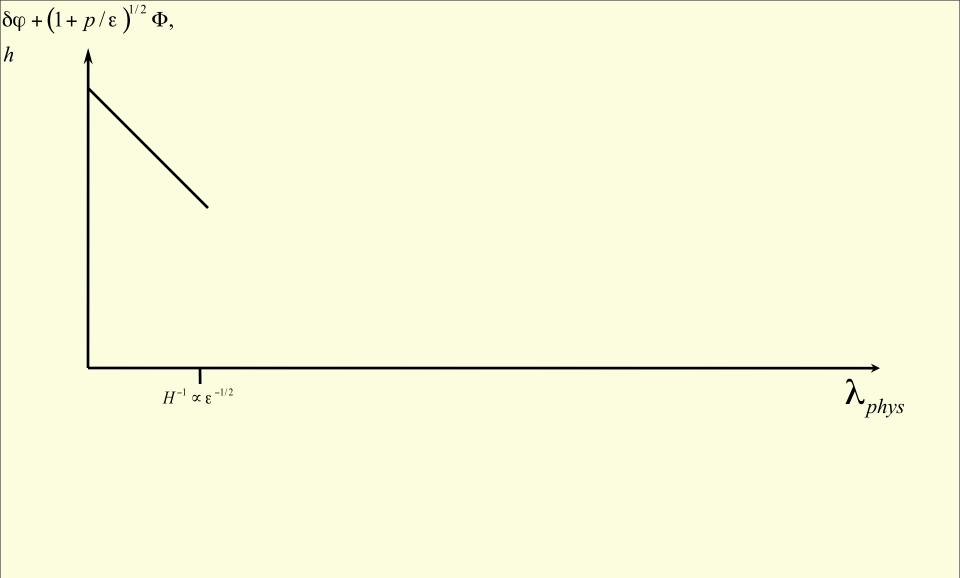


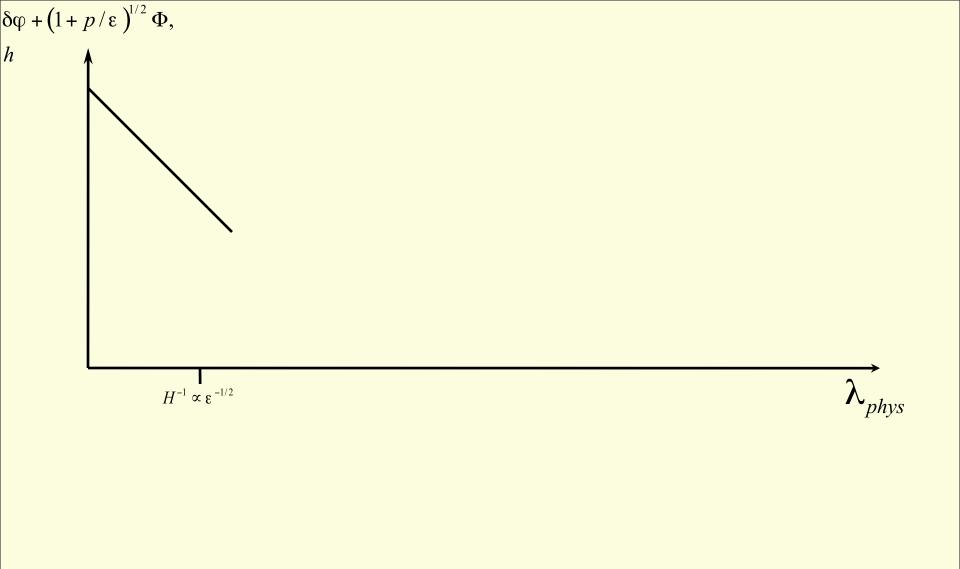


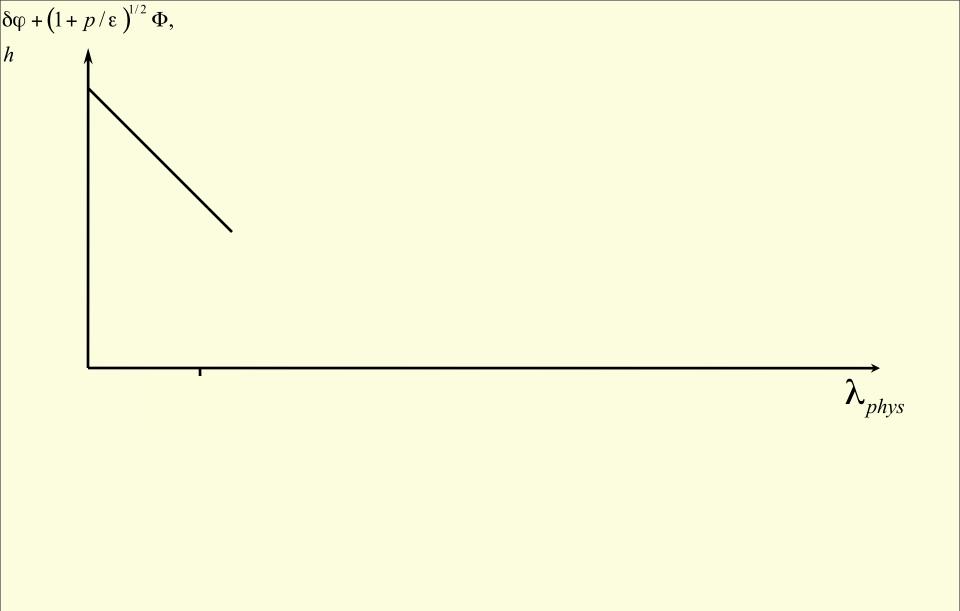


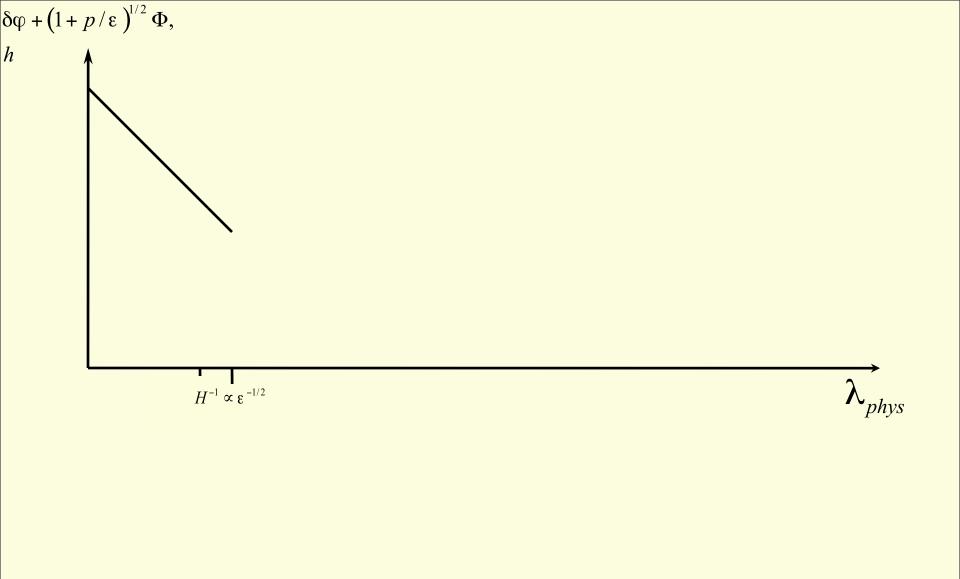


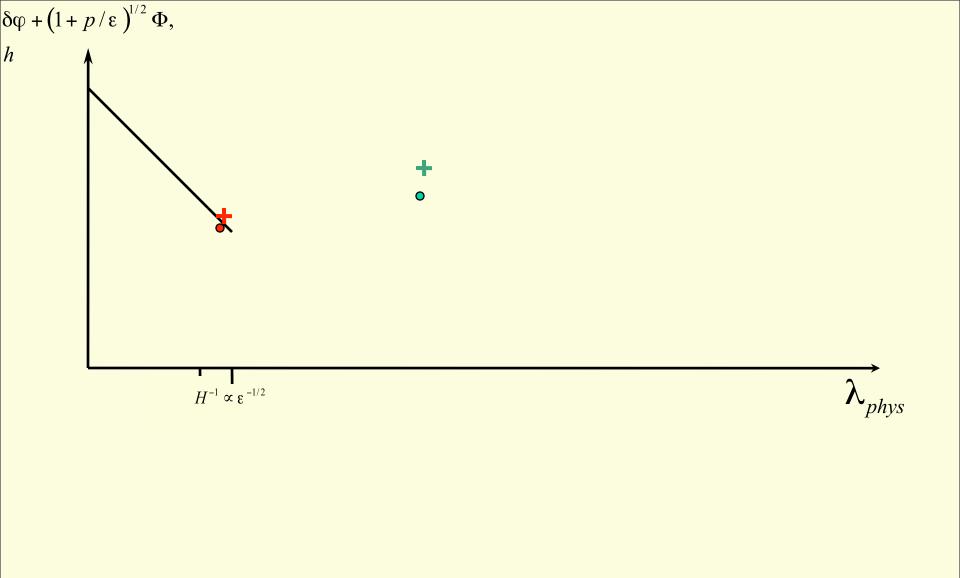


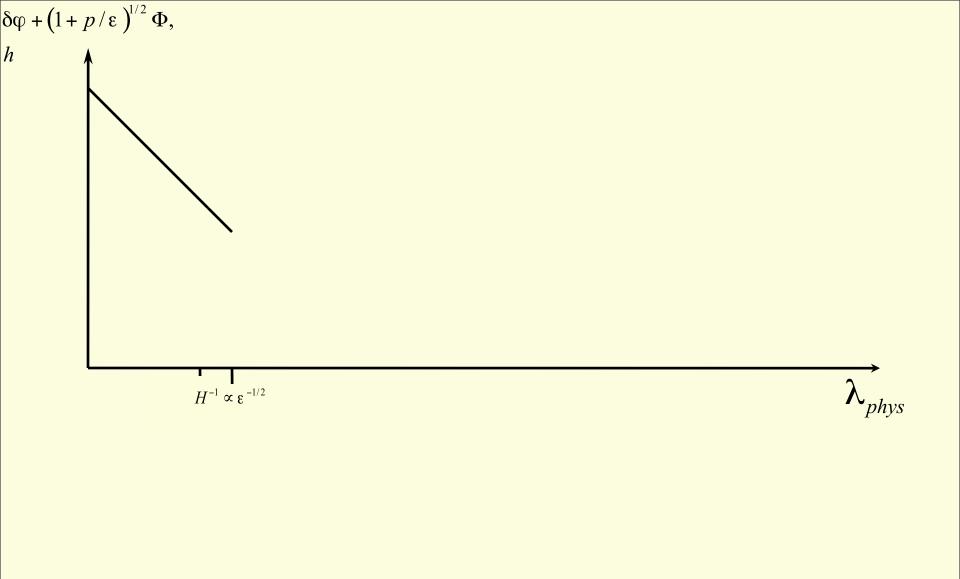


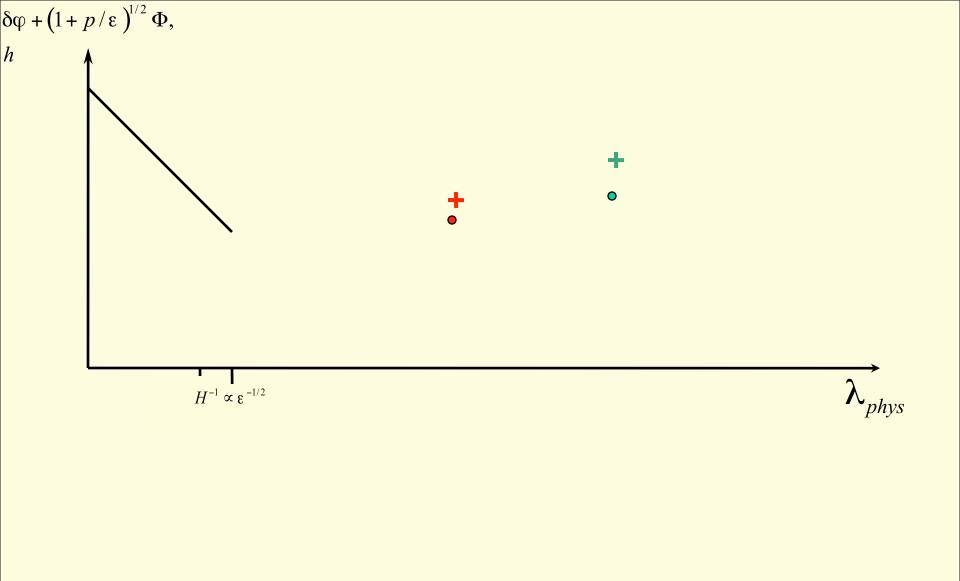


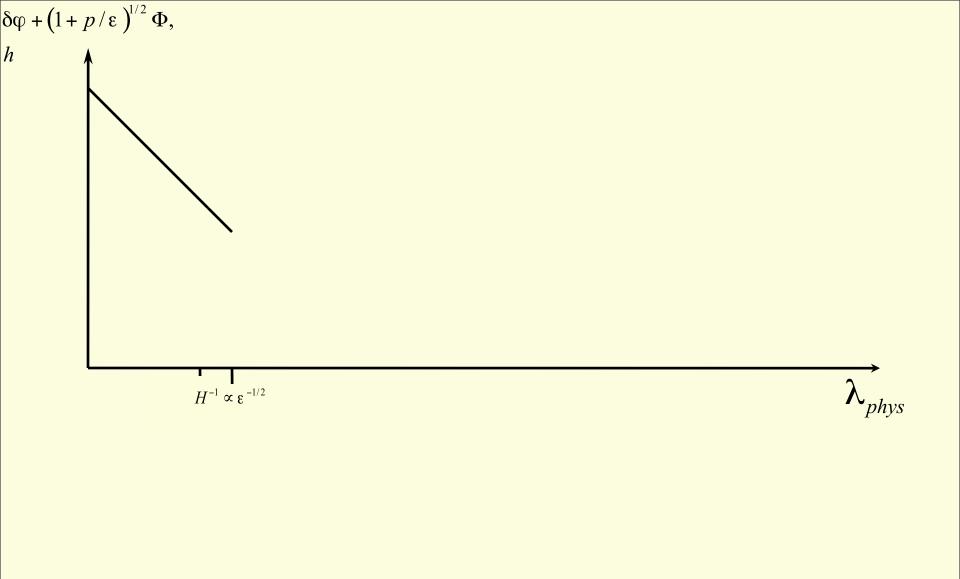


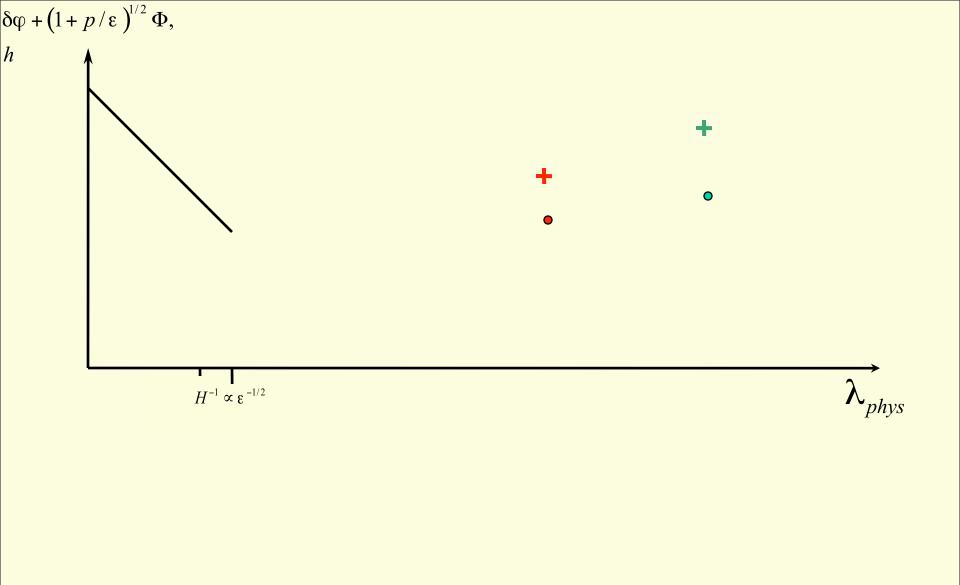


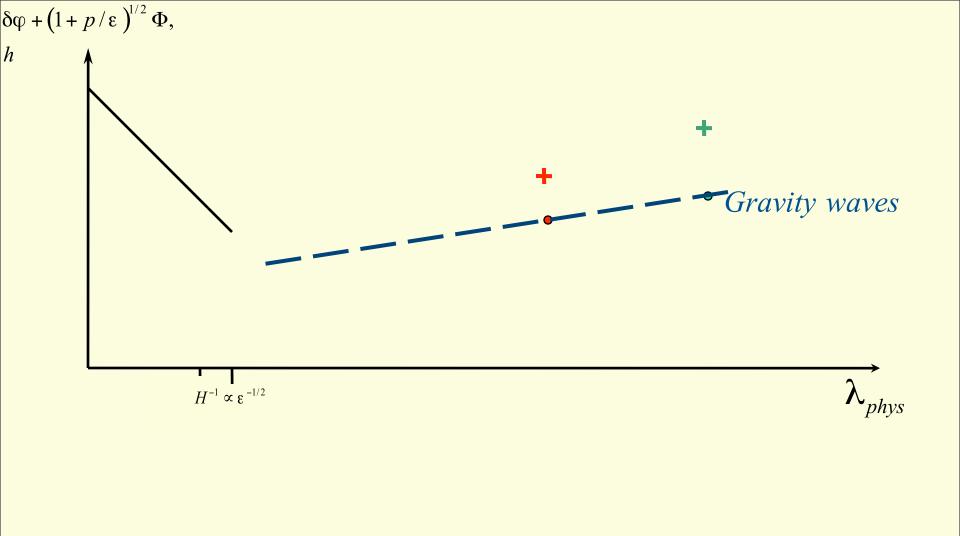


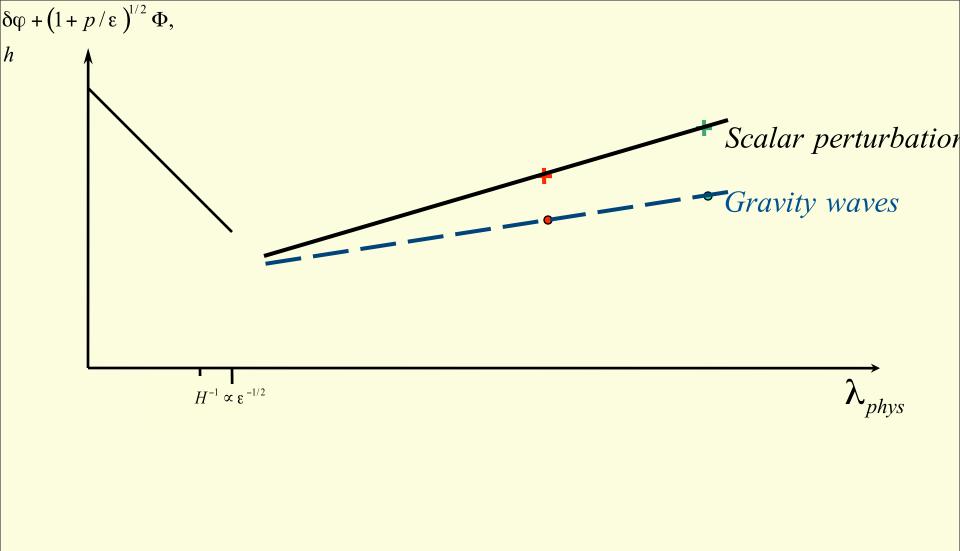


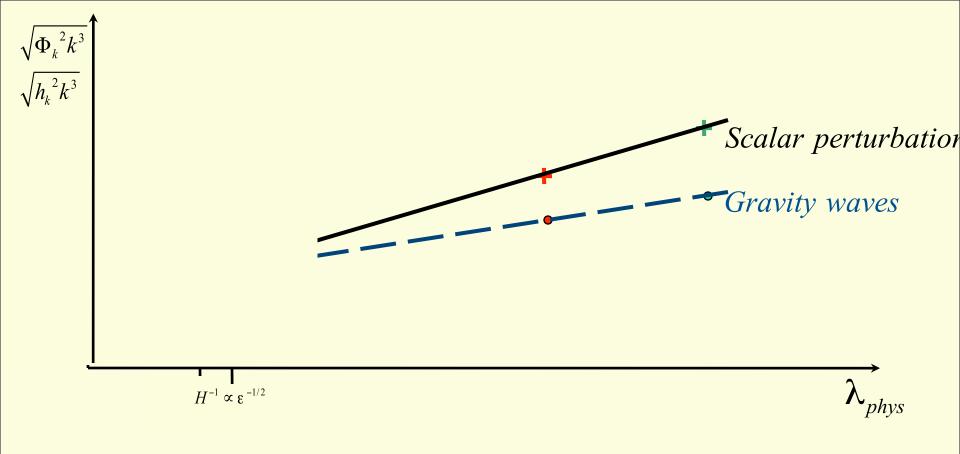


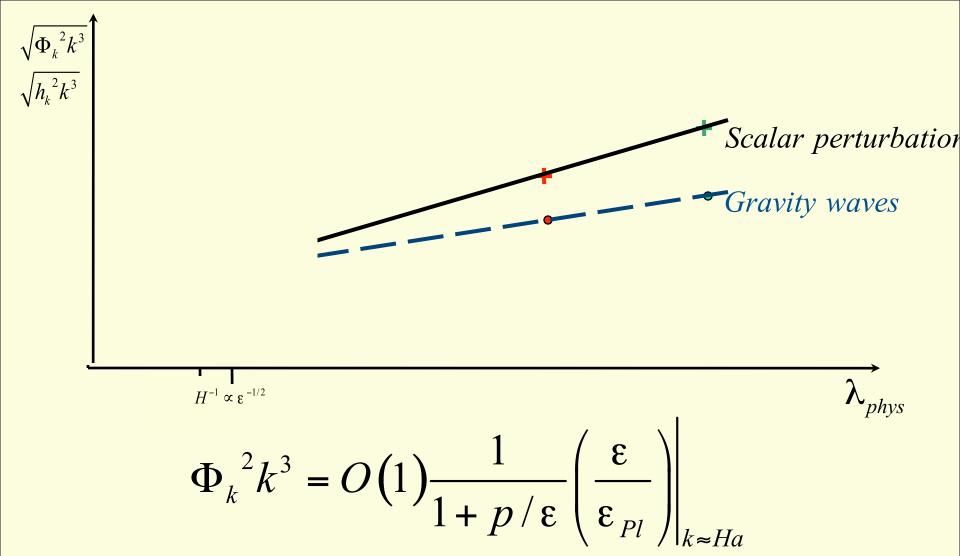


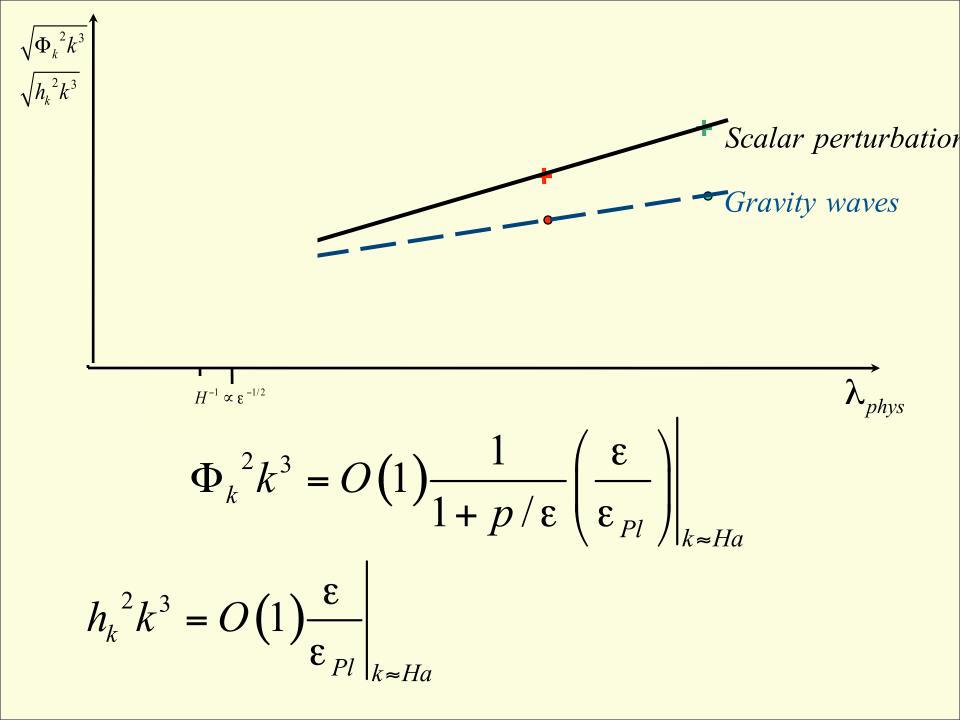


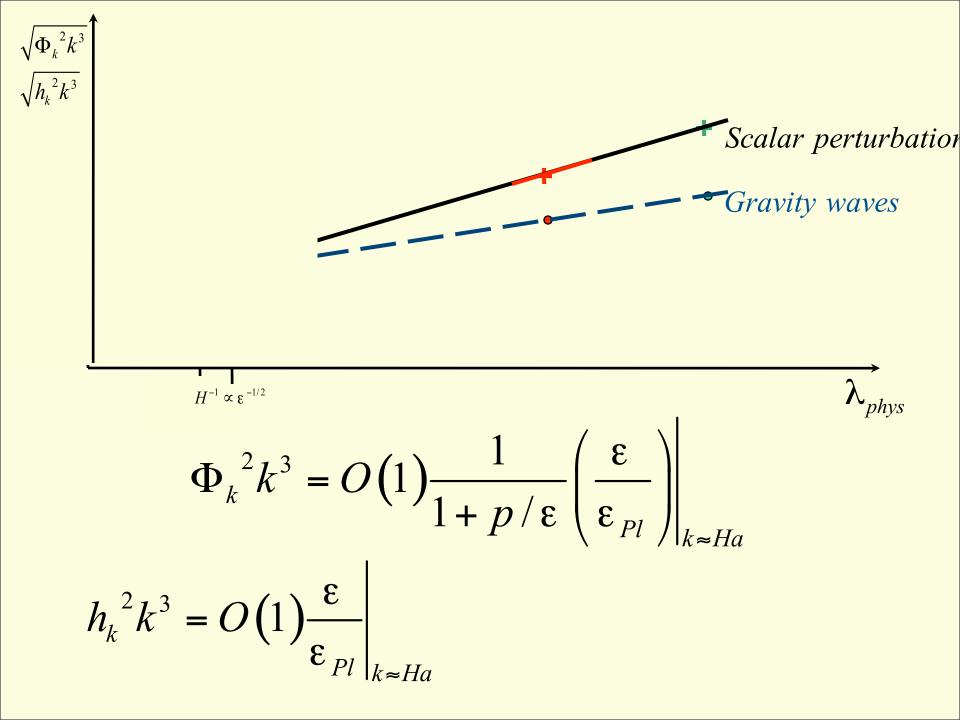








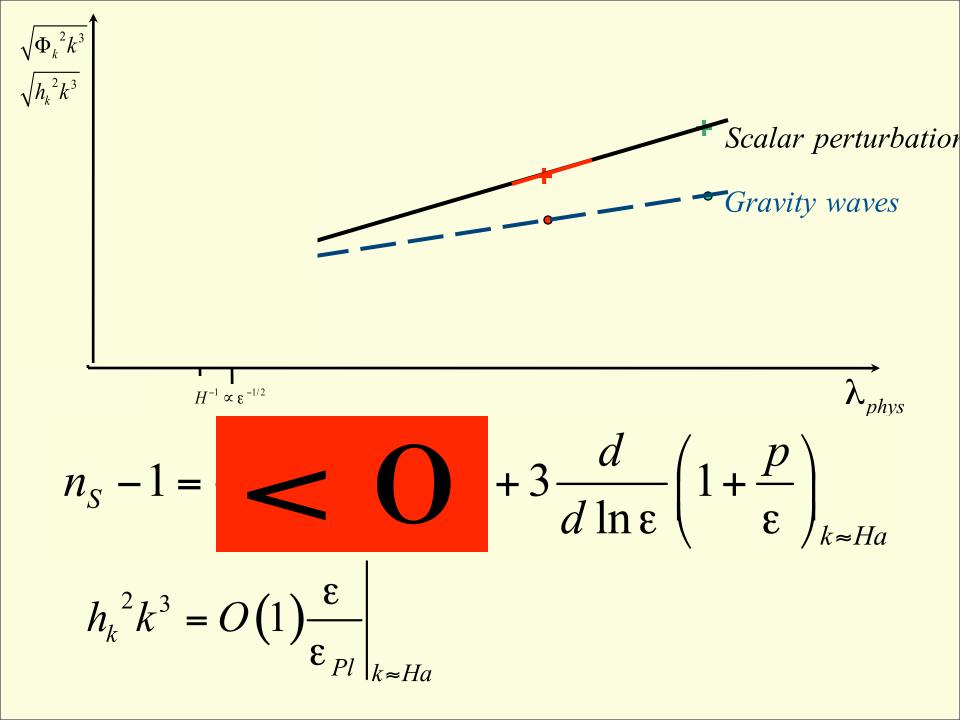


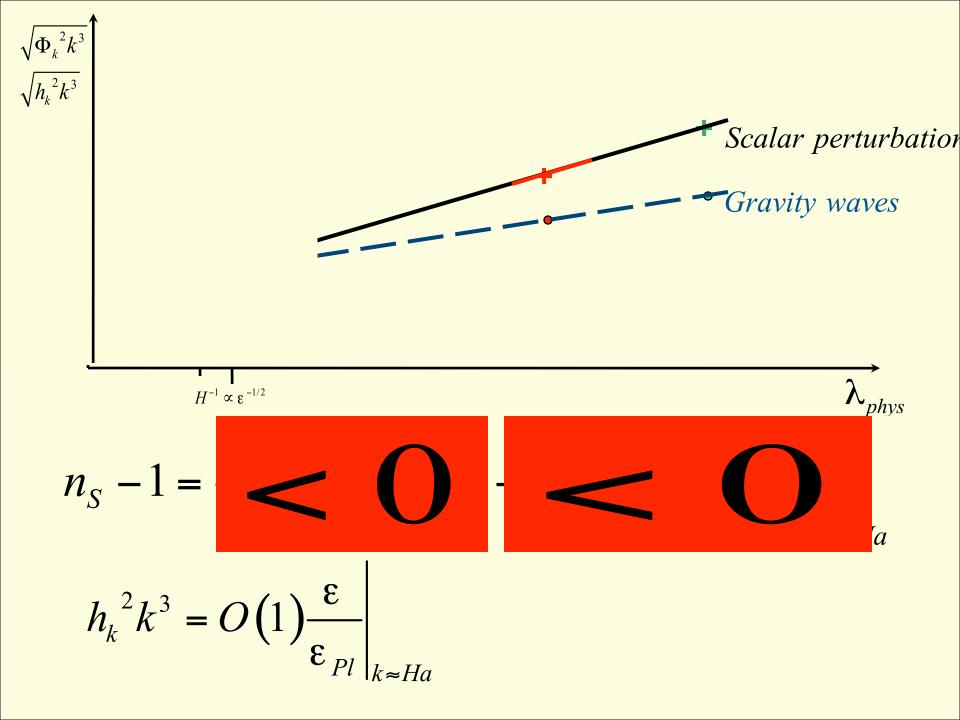


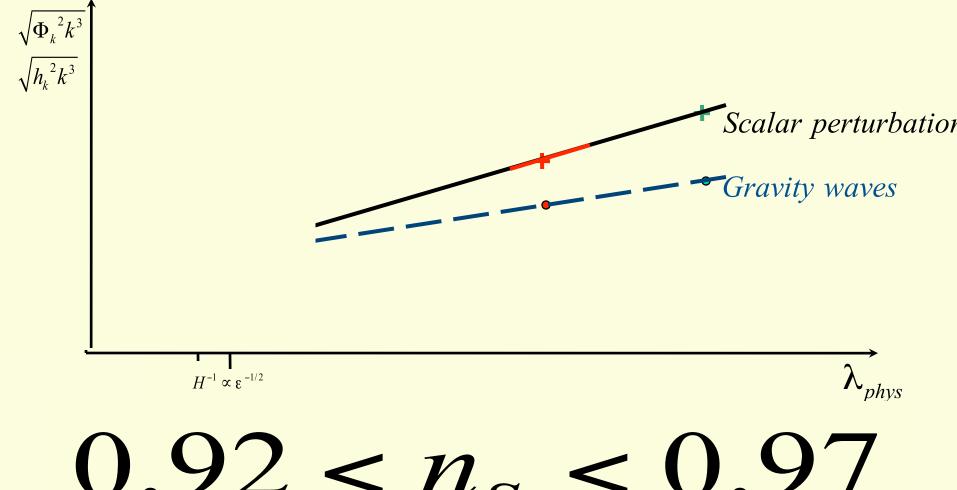
$$\frac{\sqrt{\Phi_k^2 k^3}}{\sqrt{h_k^2 k^3}}$$

$$N_S - 1 = -3 \left(1 + \frac{p}{\varepsilon} \right)_{k \approx Ha} + 3 \frac{d}{d \ln \varepsilon} \left(1 + \frac{p}{\varepsilon} \right)_{k \approx Ha}$$

$$h_k^2 k^3 = O\left(1\right) \frac{\varepsilon}{\varepsilon_{Pl}} \Big|_{k \approx Ha}$$

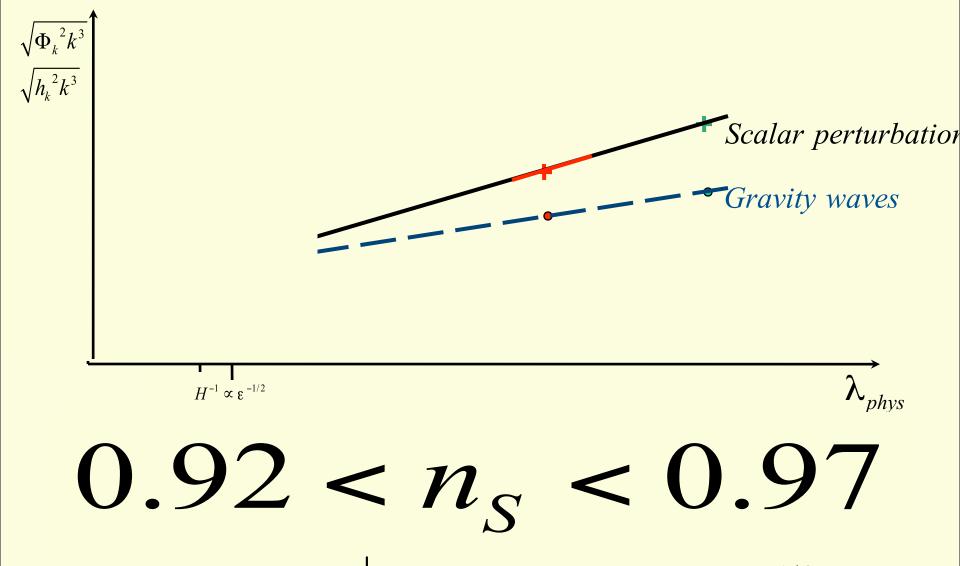




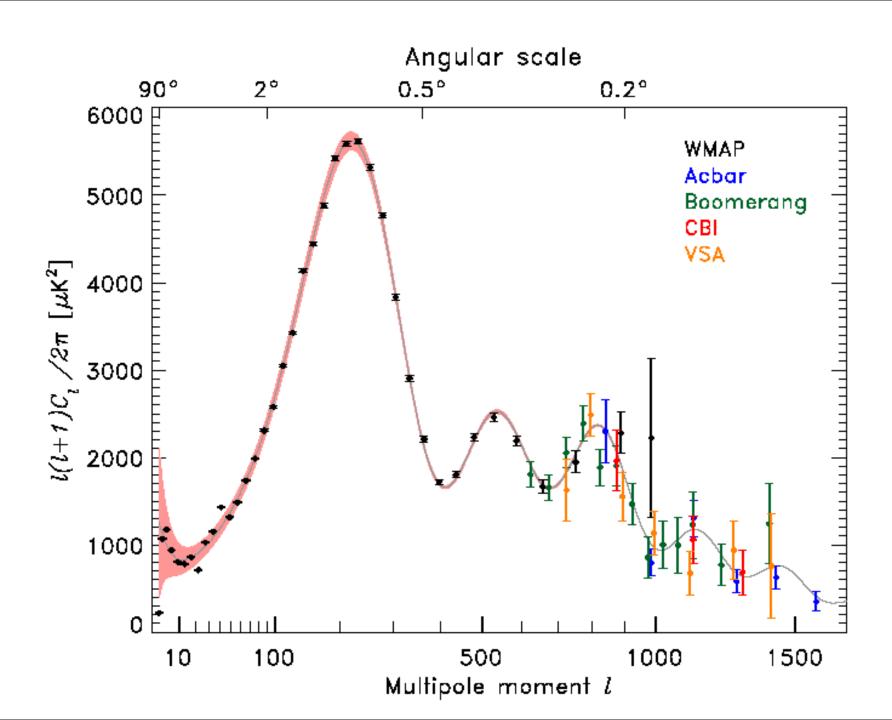


$$0.92 < n_S < 0.97$$

$$h_k^2 k^3 = O(1) \frac{\varepsilon}{\varepsilon_{Pl}} \Big|_{k \approx Ha}$$



$$h_k^2 k^3 = O(1) \frac{\varepsilon}{\varepsilon_{Pl}} \bigg|_{k \approx Ha} \frac{T}{S} = O(1) \left(1 + \frac{p}{\varepsilon}\right)^{1/2} k \approx Ha$$





$$Q(k) \approx 3lM \left(1 + \frac{1}{2} \ln \frac{H}{k}\right)$$

The fluctuation spectrum is... nearly flat...."

(Mukhanov, Chibiov, 1981)

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"In models with the initial superdense de Sitter state ... such a large amount of relic gravitational waves is generated ...that ... the very existence of this state can be experimentally" verified in the near future.

(Starobinsky, 1980)

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"What really interests me is whether God had any choice when he created the World"

A. Einstein

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Inflation was inevitable!!! (it is unique opportunity to create World starting from generic initial conditions with minimal efforts)

What was before inflation?

God creates new worlds constantly (Zohar)

In the beginning was the Word, and the Word was with God, and the Word was God (The Holy Gospel St.John)

Big Brain Theory: Have cosmologists lost theirs? (NYT, Jan. 15, 2008)