Asymptotic Safety in Quantum Einstein Gravity

Martin Reuter

(I) The Effective Average Action approach to quantum gravity and Asymptotic Safety

(II) The importance of "Background Independence" for Asymptotic Safety

(or: What is the physical meaning of a coarse graining scale when the metric is quantized?)

Standard quantization of gravity $\hat{=}$

degrees of freedom carried by :	$g_{\mu\nu}(x)$
bare action:	$\int d^4x \sqrt{-g}\, R$
calculational method:	$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G}\, h_{\mu\nu}$, perturbative quantization, renormalization

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity ?

String Theory: d.o.f. , action , calc. meth.

Loop Quantum Gravity: d.o.f. , calc. meth.

Asymptotic Safety: calc. meth. , action

Asymptotic Safety Approach:

- degrees of freedom carried by $g_{\mu\nu}$
- quantization / renormalization is non-perturbative in an essential way
- bare action Γ_* is not an ad hoc assumption, but a prediction:

  $\Gamma_* \sim \int d^4x \sqrt{-g}\, R$ + "more" is a non-Gaussian fixed point of the (∞-dimensional, non-pert.) Wilsonian renormalization group flow
- fixed point "controls" UV divergences

The Effective Average Action $\Gamma_k[g_{\mu\nu}, \cdots]$

M.R. '96

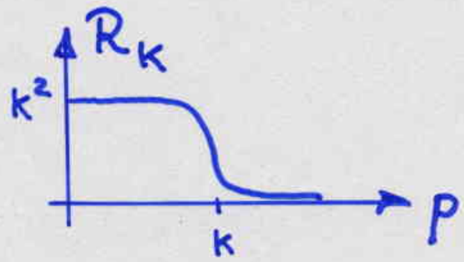
- Scale-dependent (coarse grained) effective action functional for the metric

- Defines family of effective field theories:
  $\{\Gamma_k \mid 0 \leq k < \infty\}$

- Built-in IR cutoff: Only metric fluctuations with cov. momentum $p > k$ are integrated out fully.

  Modes with $p < k$ are suppressed by "mass" term added to the bare action:

  $$(\text{mass})^2 = R_k(p^2)$$



- $\Gamma_{k \to \infty} = S$ = bare action

  $\Gamma_{k \to 0} = \Gamma$ = standard eff. action

- Γ_k satisfies a FRGE; symbolically:

  $$k\partial_k \Gamma_k = \frac{1}{2}\, \mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]$$

- Natural (nonperturbative) approximation scheme: project RG flow onto truncated theory space

Construction of Γ_k for Gravity

- starting point: $\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$
- decompose $\gamma_{\mu\nu} = \underbrace{\bar{g}_{\mu\nu}}_{\substack{\text{arbitrary}\\ \text{backgrd. metric}}} + h_{\mu\nu}$
- add background gauge fixing $S_{gf}[h;\bar{g}]$ + ghost terms
- expand $h_{\mu\nu}$ in $\bar{D}^2$-eigenmodes, and introduce IR cutoff k^2: only modes with generalized momenta ($\bar{D}^2$-eigenvalues) $> k$ are integrated out.
- add sources: generating fctl. $W_k[\text{sources};\bar{g}]$

$$\Big\downarrow \text{Legendre transf.}$$

$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$ $\qquad\qquad$ $\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- derive exact RG equation from path integral:

$$k\frac{\partial}{\partial k}\Gamma_k[g,\bar{g},\cdots] = \mathrm{Tr}(\cdots)$$

- "Ordinary" diffeomorphism invariant action:

$$\Gamma_k[g] = \Gamma_k[g, \bar{g}=g, \text{ghosts}=0]$$

Taking the UV-limit in QEG

If there exists a non-Gaussian Fixed Point Γ_*, $\beta_i(\Gamma_*) = 0$, Quantum Einstein Gravity is nonperturbatively renormalizable ("asymptotically safe").

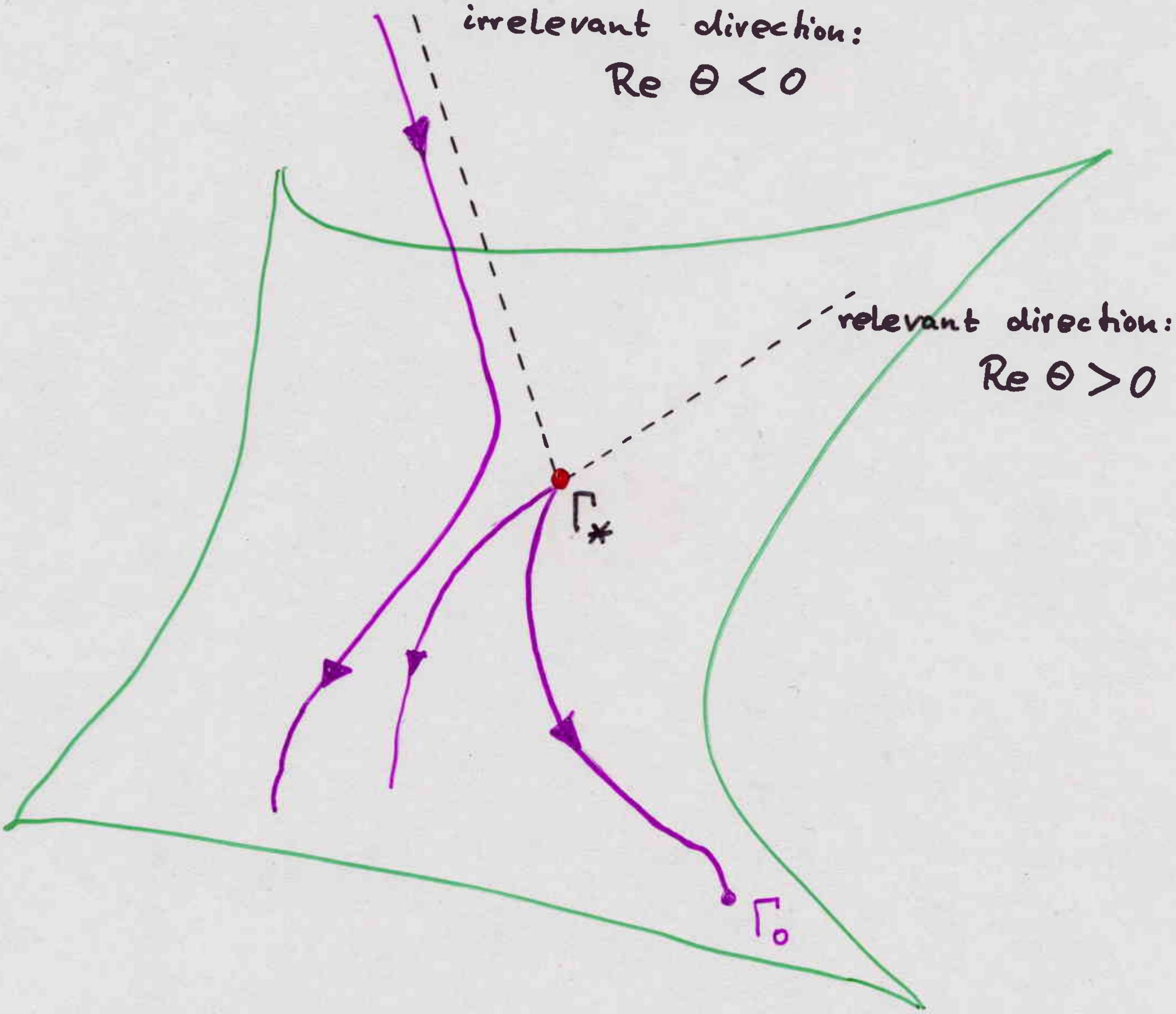
Weinberg 1979

Quantum theory is defined by a RG trajectory running inside the UV-critical hypersurface of the FP, with

initial point $= \Gamma_{k\to\infty} \equiv S$ = action infinitesimally close to Γ_*

end point $= \Gamma_0 \equiv \Gamma$

The UV-critical hypersurface $\mathscr{S}_{UV}$:



$\Delta_{UV} \equiv \dim \mathscr{S}_{UV}$ = # relevant directions

= # free parameters in the a.s. quantum field theory

UV $\longrightarrow$ IR

Θ : critical exponent (neg. eigenvalue of lin. flow)

Properties of QEG

- Background-independent quantization scheme:

No special <u>metric</u> plays any distinguished role!

<u>The background field method</u>:

a) Fix arbitrary $\bar{g}_{\mu\nu}$

b) Quantize (nonlinear) fluctuations $h_{\mu\nu} \equiv \gamma_{\mu\nu} - \bar{g}_{\mu\nu}$ in the backgrd. of $\bar{g}_{\mu\nu}$

c) Adjust $\bar{g}_{\mu\nu}$ such that $\langle h_{\mu\nu} \rangle = 0$

$$\rightsquigarrow \quad g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$$

- Fundamental action $S \approx \Gamma_*$ is a prediction:

No special <u>action</u> plays any distinguished role!

<u>The only input</u>: field contents + symmetries
$\hat{=}$ theory space

<u>The output</u>: $\Gamma_* = S_{\text{Einstein-Hilbert}} +$ "more"

Einstein-Hilbert action is often a reliable approximation, but not distinguished conceptually.

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$\Lambda_k \equiv \bar{\lambda}_k$

$$\Gamma_k = -\frac{1}{16\pi G_k}\int d^dx\,\sqrt{g}\,\left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k, dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k, dimensionless: $\lambda(k) = \Lambda_k / k^2$

insert ansatz into flow equation, expand

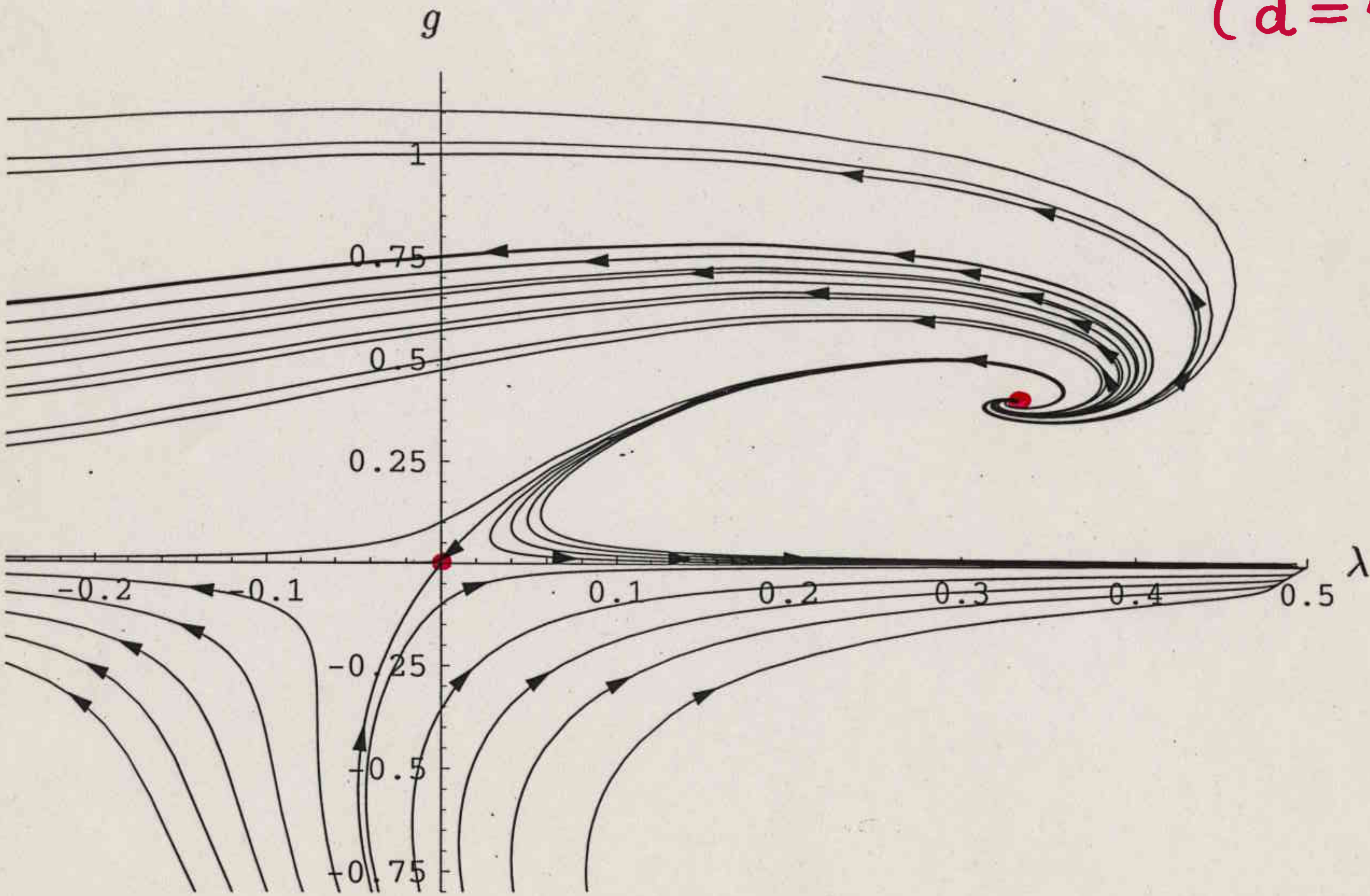
$$\mathrm{Tr}\left[\cdots\right] = (\cdots)\int\sqrt{g} + (\cdots)\int\sqrt{g}\,R + \cdots$$

$\rightsquigarrow$

$$\boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

RG - Flow in the Einstein - Hilbert Truncation

(d = 4)



M.R., F. Saueressig, hep-th/0110054

O. Lauscher, M.R.
F. Saueressig, M.R.
R. Percacci, ...
D. Litim, ...

Reliability checks (universality, ...),
more general truncations $\Longrightarrow$

The NGFP seems to exist in the full un-truncated theory beyond any reasonable doubt.

Conformally Reduced QEG

(M.R., H. Weyer, 2008)

- Simplified version of full QEG:
  only the conformal factor is quantized

- The same approach as in full QEG is used:
  effective average action,
  background field method

- Disentangles conceptual / technical problems

- Illustrates importance of "background independence" for the RG flow:

  scalar-like theory, but with RG behavior <u>very different</u> from that of a scalar matter field on a rigid spacetime

- Disentangles role of backgrd. field method for gauge invariance / "backgrd. independence"

- Has the same qualitative features as full QEG

  ⇝ play ground for gaining new conceptual insights

Conformally Reduced QEG

- quantize only the conformal factor:

$$\underbrace{\gamma_{\mu\nu}}_{\substack{\text{integration}\\ \text{variable}}} = \chi^2 \, \underbrace{\hat{g}_{\mu\nu}}_{\substack{\text{class. reference metric,}\\ \neq \text{ backgrd. metric !}}}$$

- treat scalar-like theory $\int \mathcal{D}\chi \, e^{-S[\chi]}$ in the same way as full QEG:
effective average action $\oplus$ backgrd. field method

- introduce background conf. factor:

$$\bar{g}_{\mu\nu} = \chi_B^2 \, \hat{g}_{\mu\nu}$$

- decompose quantum field:

$$\chi = \chi_B + \underbrace{f}_{\text{"fluctuation"}}$$

- expectation values:

$$\phi \equiv \langle \chi \rangle = \chi_B + \bar{f} \, , \quad \bar{f} \equiv \langle f \rangle$$

$$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \langle \chi^2 \rangle \, \hat{g}_{\mu\nu} = \langle (\chi_B + f)^2 \rangle \, \hat{g}_{\mu\nu}$$

The innocent first steps :

- define, formally ,
$$e^{W_k[J;\chi_B]} =$$
$$= \int \mathcal{D}f \; e^{-S[\chi_B+f]-\Delta_k S[f;\chi_B] + \int d^4x \sqrt{\hat{g}} \; J \, f}$$

with
$$\Delta_k S[f;\chi_B] = \frac{1}{2}\int d^4x \sqrt{\hat{g}} \; f(x)\, \mathcal{R}_k[\chi_B]\, f(x)$$

- define
$$\bar{f} \equiv \langle f \rangle_k = \frac{1}{\sqrt{\hat{g}}}\frac{\delta W_k}{\delta J}$$
$$\rightsquigarrow \quad J = \mathcal{J}_k[\bar{f};\chi_B]$$

- define effective average action:
$$\Gamma_k[\bar{f};\chi_B] = \int d^4x \sqrt{\hat{g}} \; \bar{f}\, \mathcal{J}_k - W_k[\mathcal{J}_k;\chi_B] - \Delta_k S[\bar{f};\chi_B]$$
$$\equiv \Gamma_k[\Phi,\chi_B] \qquad \Phi \equiv \chi_B + \bar{f}$$

- derive FRGE :

$$\boxed{\begin{aligned} &k\partial_k \Gamma_k[\bar{f};\chi_B] \\ &= \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)}[\bar{f};\chi_B] + \mathcal{R}_k[\chi_B]\right)^{-1} k\partial_k \mathcal{R}_k[\chi_B]\right]\end{aligned}}$$

Constructing $\mathcal{R}_k$:

"backgrd. independence" vs. rigid backgrd.

- require coarse graining scale k^{-1} of $\Gamma_k[g, \bar{g}]$ to be a <u>proper</u> (rather than coordinate) length

- "proper" w.r.t. which metric ?

- "backgrd. independence" $\Longrightarrow$ k^{-1} can be proper only w.r.t. metric given by the arguments $[g, \bar{g}]$, but not w.r.t. any rigid metric (such as $\hat{g}_{\mu\nu}$ in CR-QEG)

- <u>Our choice:</u> k^{-1} is proper w.r.t. $\bar{g}_{\mu\nu}$

  more precisely: $-k^2$ is a cutoff in the spectrum of

$$\boxed{\bar{\Box} \equiv (D^\mu D_\mu)(\bar{g})}$$

  $\Longrightarrow$ Typical structures (periods, ...) of $\bar{\Box}$- eigenfunction with eigenvalue $-k^2$ have $\bar{g}$-proper size of the order k^{-1}.

  $\Longrightarrow$ $\Gamma_k \mathrel{\hat{=}}$ "effective field theory valid near k"

- <u>Cf. rigid backgrd.</u> : $-k^2$ cutoff in $\hat{\Box}$-spectrum

Truncations employed :

- Conformally Reduced Einstein-Hilbert ("CREH") truncation:

$$\Gamma_k[\bar{f};\chi_B] \equiv \Gamma_k[\phi, \chi_B]$$

$$= -\frac{1}{16\pi G_k}\int d^4x\sqrt{g}\,\big(R(g)-2\Lambda_k\big)\Big|_{g_{\mu\nu}=\phi^2\hat{g}_{\mu\nu}}$$

$$= \frac{3}{4\pi G_k}\int d^4x\sqrt{\hat{g}}\left\{\frac{1}{2}\phi\hat{\Box}\phi - \frac{1}{12}\hat{R}\phi^2 + \frac{1}{6}\Lambda_k\phi^4\right\}$$

depends only on the combination $\phi \equiv \chi_B + \bar{f}$;

2-dimensional theory space : $\{g, \lambda\}$

- Local Potential Approximation (LPA):

$$\Gamma_k[\phi, \chi_B] = \frac{3}{4\pi G_k}\int d^4x\sqrt{\hat{g}}\left\{\frac{1}{2}\phi\hat{\Box}\phi - F_k(\phi)\right\}$$

infinite dimensional theory space :

$\{G, F(\cdot)\} \sim \{g, Y(\cdot)\}$

Flow equations and β-functions

$$Y_k(\varphi) \equiv k^2 F_k\left(\frac{\varphi}{k}\right), \quad \varphi \equiv k\phi \qquad \text{dim. less}$$

$$g_k \equiv k^2 G_k, \quad \lambda_k \equiv \Lambda_k / k^2$$

$$k\partial_k g_k = \left[2 + \eta_N\left(g_k, [Y_k]\right)\right] g_k$$

$$k\partial_k Y_k(\varphi) = (2+\eta_N) Y_k - \varphi Y_k' - \frac{g_k}{24\pi}\left(1 - \frac{1}{6}\eta_N\right) \frac{\varphi^6}{\varphi^2 + Y_k''(\varphi)}$$

anomalous dimension:

$$\eta_N(g, [Y]) = -\frac{g}{24\pi} \frac{\left[\varphi_1^3 Y'''(\varphi_1)\right]^2}{\left[\varphi_1^2 + Y''(\varphi_1)\right]^4}$$

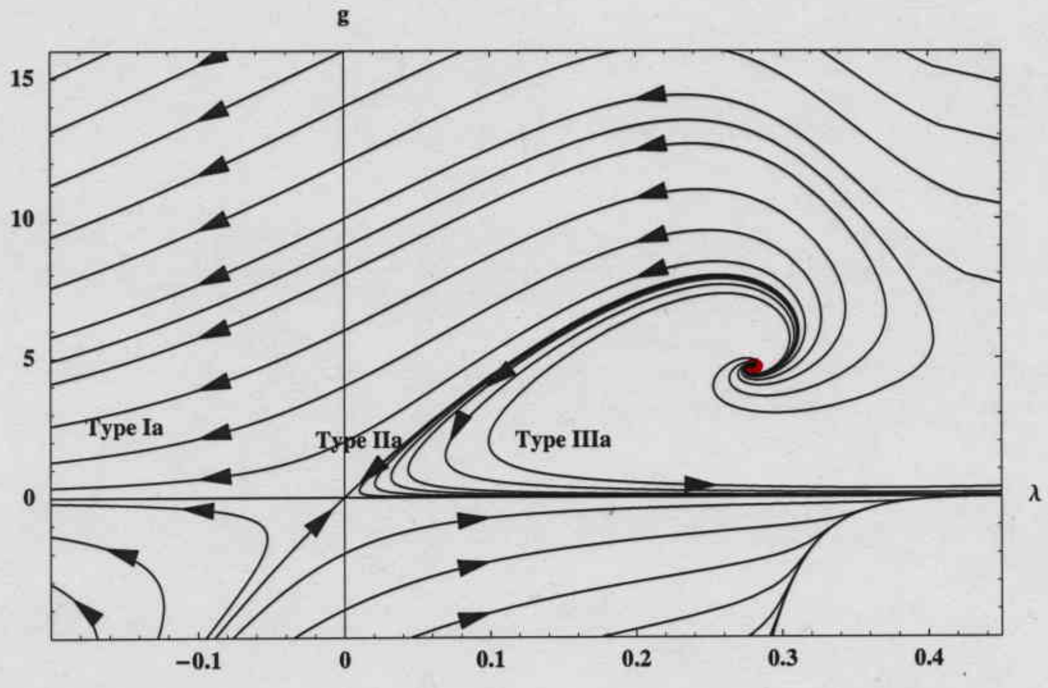
(R^4 topology)

φ_1 : normalization point ($\varphi_1 \to \infty$).

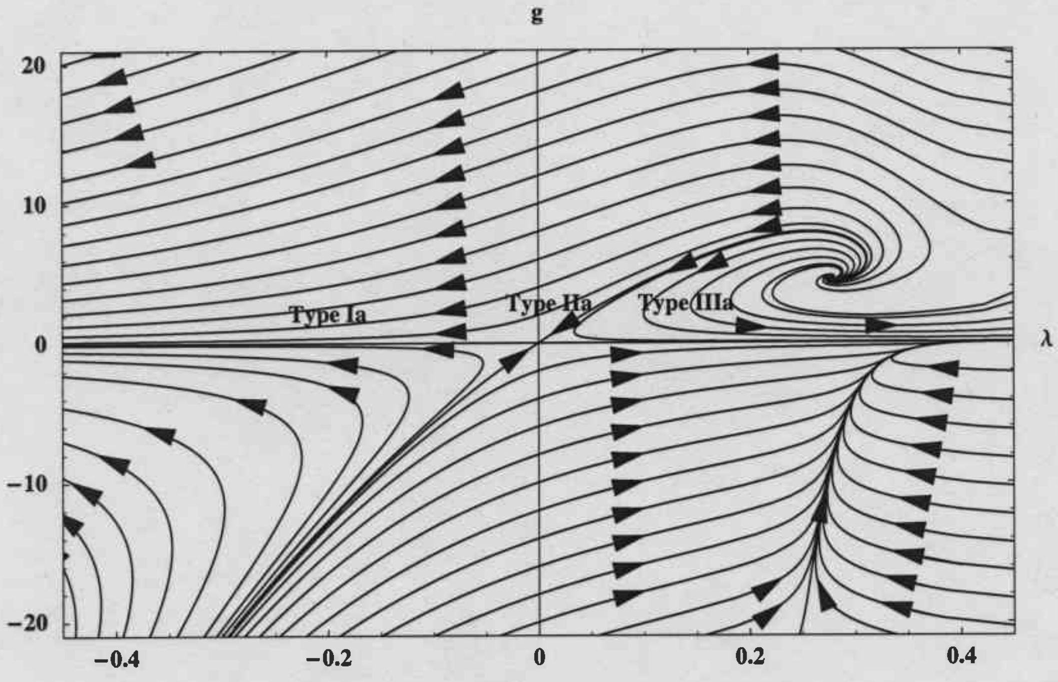
Compare to standard scalar on rigid backgrd.:

$$\frac{\phi^6 k^6}{\phi^2 k^2 + F_k''(\phi)} \longrightarrow \frac{k^6}{k^2 + F_k''(\phi)}$$

The CREH flow: $\Upsilon_k(\varphi) = -\frac{1}{6}\lambda_k \varphi^4$



(a)



(b)

Figure 1: The figures show the RG flow on the (g, λ)-plane which is obtained from the CREH truncation with $\eta_N^{(\mathrm{kin})}$. The arrows point in the direction of decreasing k.

M.R., H.Weyer, arXiv: 0801.3287

Results:

- Inequivalent quantization schemes:

  - rigid background - quantization
  - "backgrd. independent" quantization

- Resulting RG flows are very different:

  - standard $(-\phi^4)$ - theory : no NGFP (asym. free: Symanzik 1977)
  - NGFP exists : theory is asym. safe!

- Scaling dimensions at the GFP differ by 2 units, e.g.

$$\varphi^n \begin{cases} \Theta = n-4 \\ \Theta = n-2 \end{cases}$$

- Scaling fields / dimensions at the NGFP depend on choice of theory space, e.g.

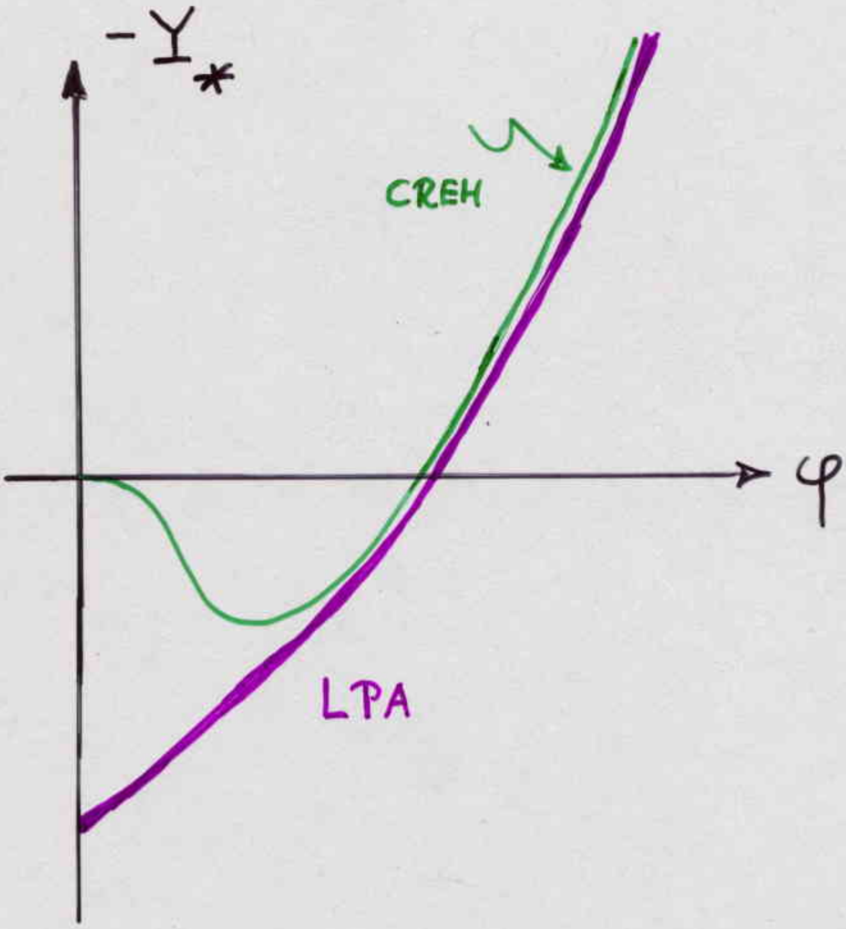
$$\{\varphi^m\}, \quad m \in \mathbb{N}, \; m \in \mathbb{Z}, \; m \in \mathbb{R}, \; m \in \mathbb{C}, \dots$$

- β - functions of LPA depend on <u>topology</u>: R^4, S^4, ...

Non-Gaussian Fixed Point (LPA)

$$\begin{cases} g_*^{NGFP} = g_*^{CREH} \\ Y_*^{NGFP}(\varphi) = y_* - \frac{1}{6}\lambda_*^{CREH}\varphi^4 \qquad (R^4) \end{cases}$$

Numerical solution for S^4 topology:



Corresponds to non-trivial fixed points of infinitely many couplings!

Phase Transitions to a new phase of gravity: Unbroken Diffeomorphism Invariance

- Solve full nonlinear PDE for Y_k numerically, search for trajectories inside $\mathcal{S}_{UV}$.

- Global minimum $\phi_0(k) \equiv k\,\varphi_0(k)$ of $F_k(\phi) \sim -Y_k(\varphi)$ determines expectation value

$$\langle \gamma_{\mu\nu} \rangle \equiv \langle g_{\mu\nu} \rangle_k = \phi_0^2(k)\, \hat{g}_{\mu\nu}$$

- $\phi_0 = 0$: phase with vanishing exp. val. of the metric (vielbein)

  $\phi_0 \neq 0$: exp. val. $\neq 0$, spontaneously breaks group of diffeo.'s to stability group of $\langle g_{\mu\nu} \rangle_k$

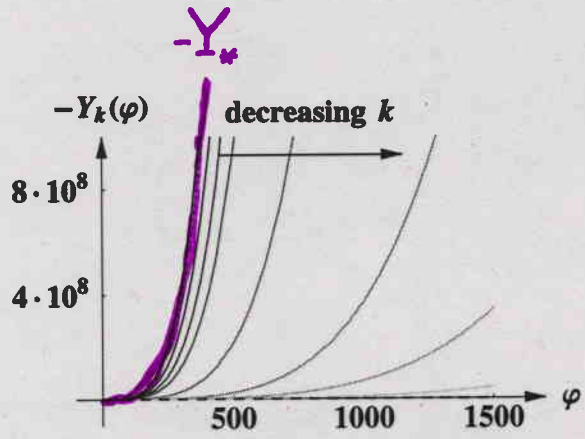
- Forms of phase transitions (w.r.t. scale k):

■ "1^{st} order" $\longleftrightarrow$ "2^{nd} order"

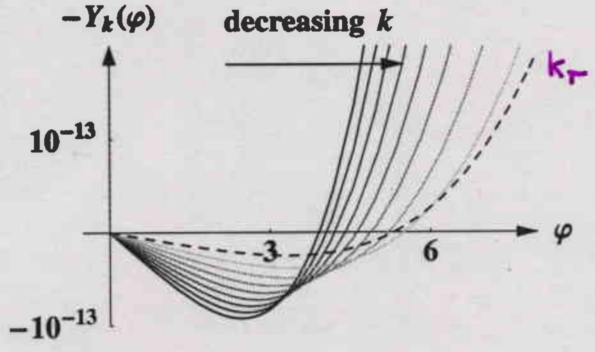
(φ_0 discontinuous) (φ_0 continuous)

■ at k "$=$" ∞ $\longleftrightarrow$ at $k = k_c < \infty$

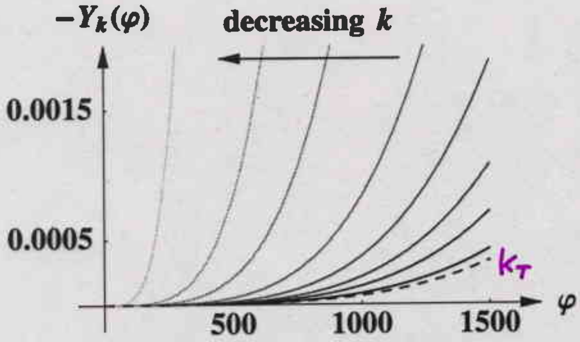
2^{nd} order transition at k "=" ∞ (R^4)



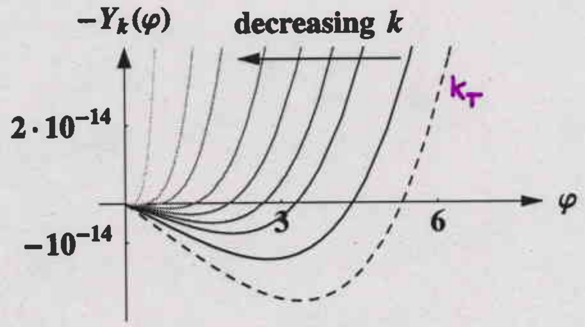
(a)



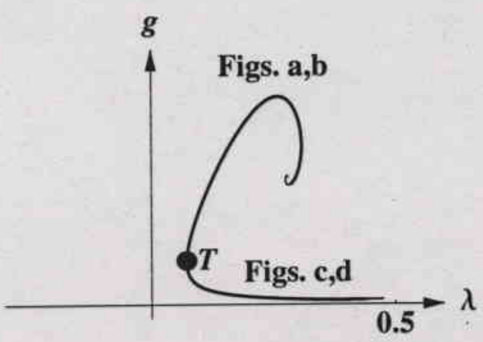
(b)



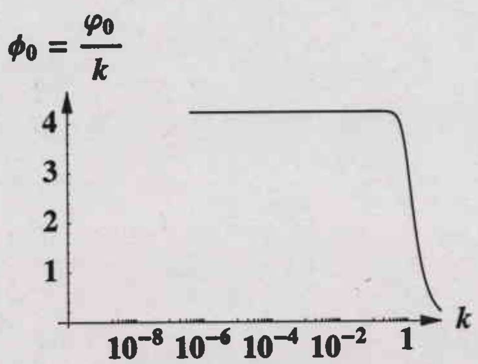
(c)



(d)



(e)



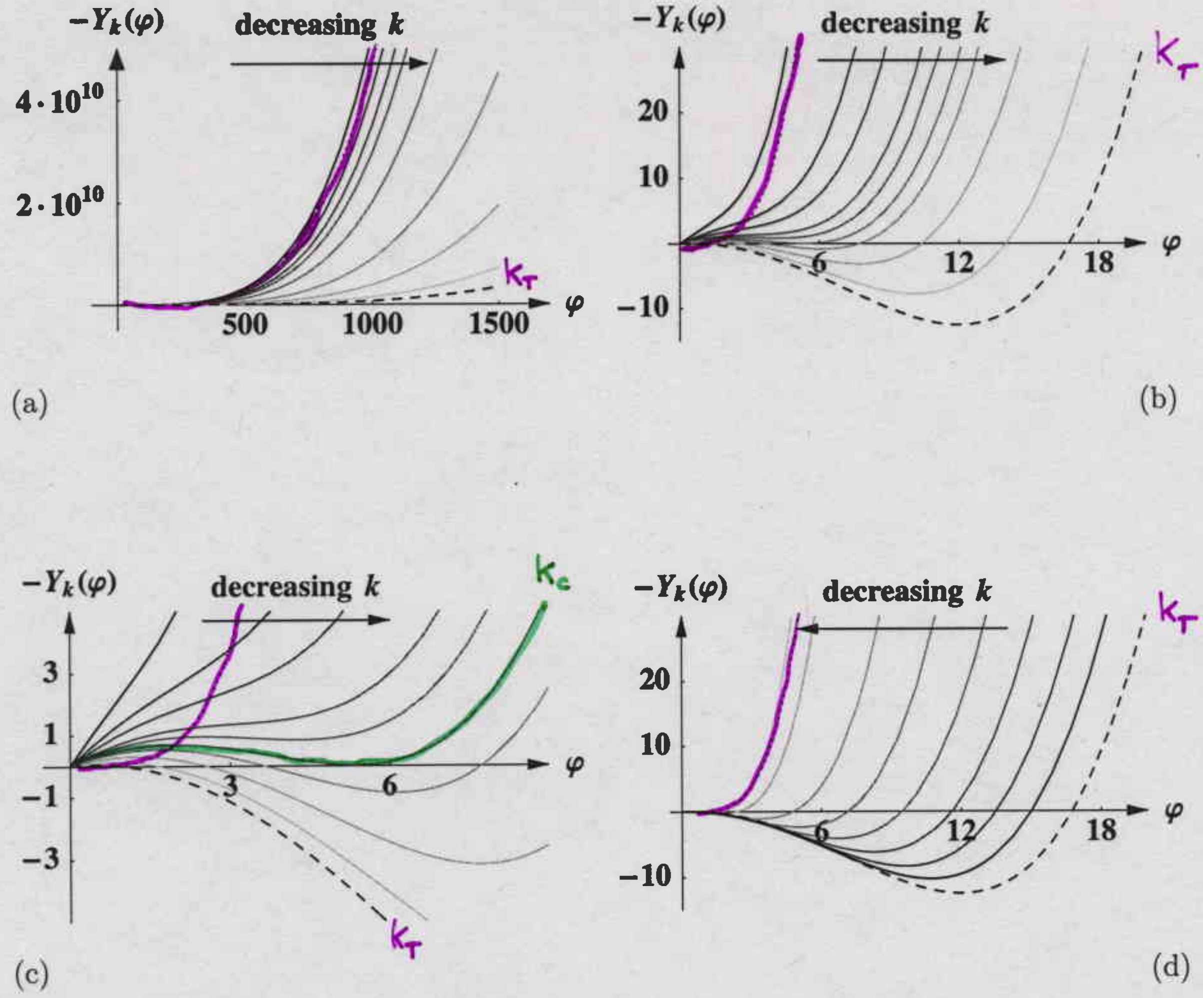
(f)

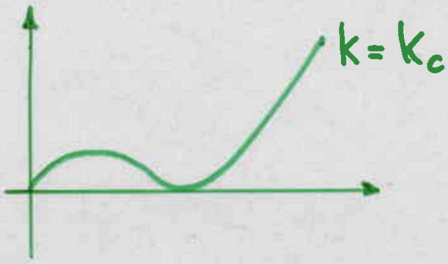
classical spacetime emerges:

$\phi_0 \approx \text{const} \neq 0$

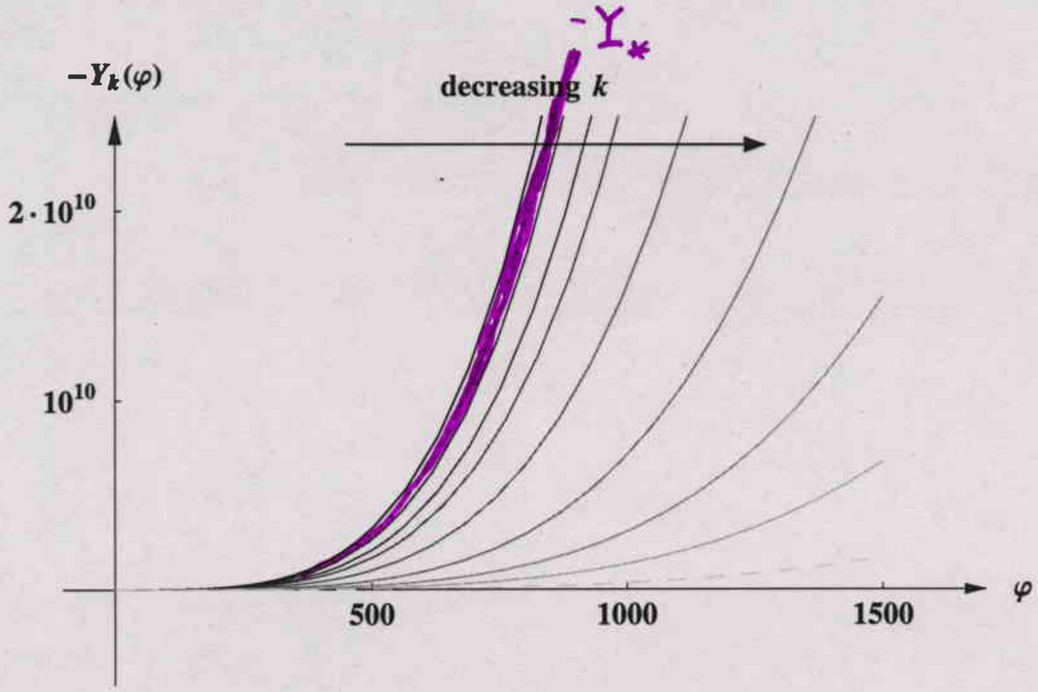
M. R., H. Weyer, arXiv: 0804.1475

1^{st} order transition at $k_c < \infty$ (R^4)

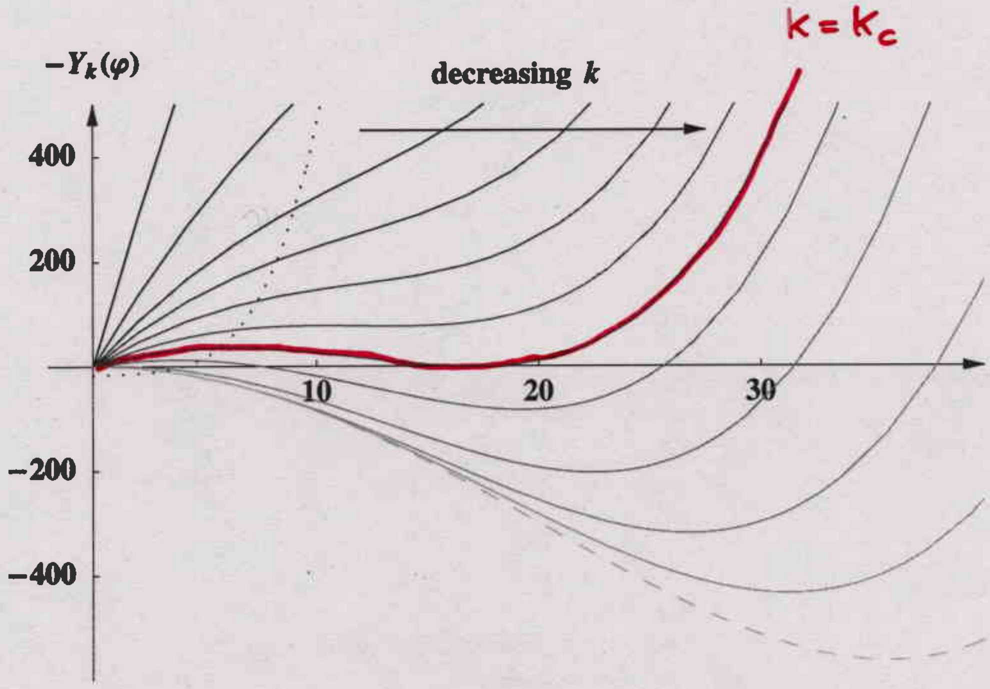




1^{st} order transition at $k_c < \infty$ (S^4)



(a)



(b)

Summary

- "Backgrd. independence" has a crucial impact on the RG flow of the eff. average action:

  - rigid backgrd: standard $-\phi^4$ theory (asymptotically free: Symanzik '73)

  - "backgrd. indep.": NGFP forms $\Rightarrow$ A.S.

- RG flow due to the conformal factor is <u>typical</u> of the full set of metric degrees of freedom:

  "backgrd. indep." seems to be more important to A.S. than spin-2 excitations, their complicated self-interactions, etc. !