Backreaction from non-conformal fields in de Sitter spacetime

Albert Roura

Work in collaboration with Guillem Pérez-Nadal and Enric Verdaguer

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Los Alamos
NATIONAL LABORATORY
EST. 1943

Madrid
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Outline

1. Introduction

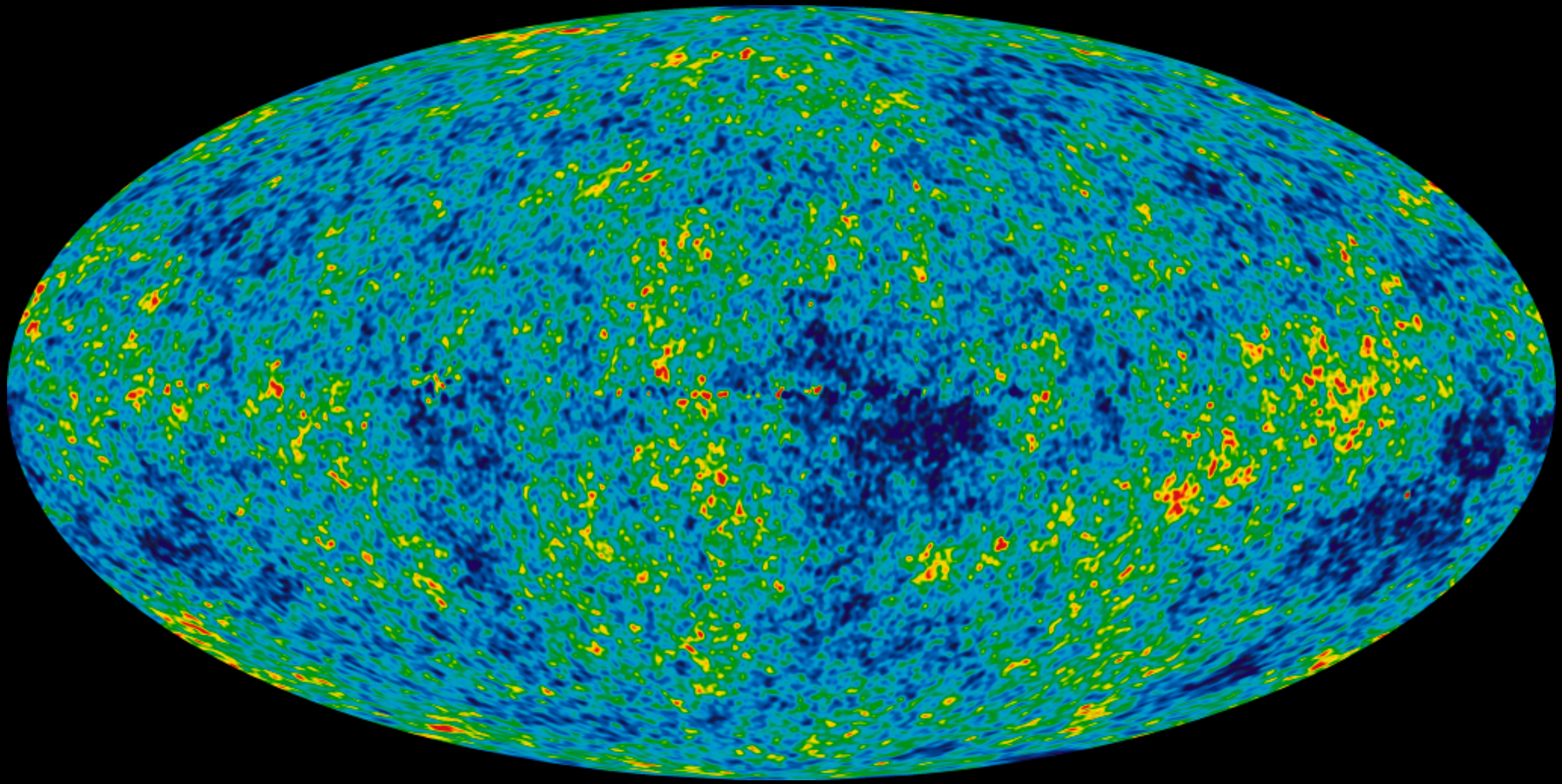
2. *Stability* of de Sitter spacetime under isotropic perturbations: *weakly non-conformal* fields

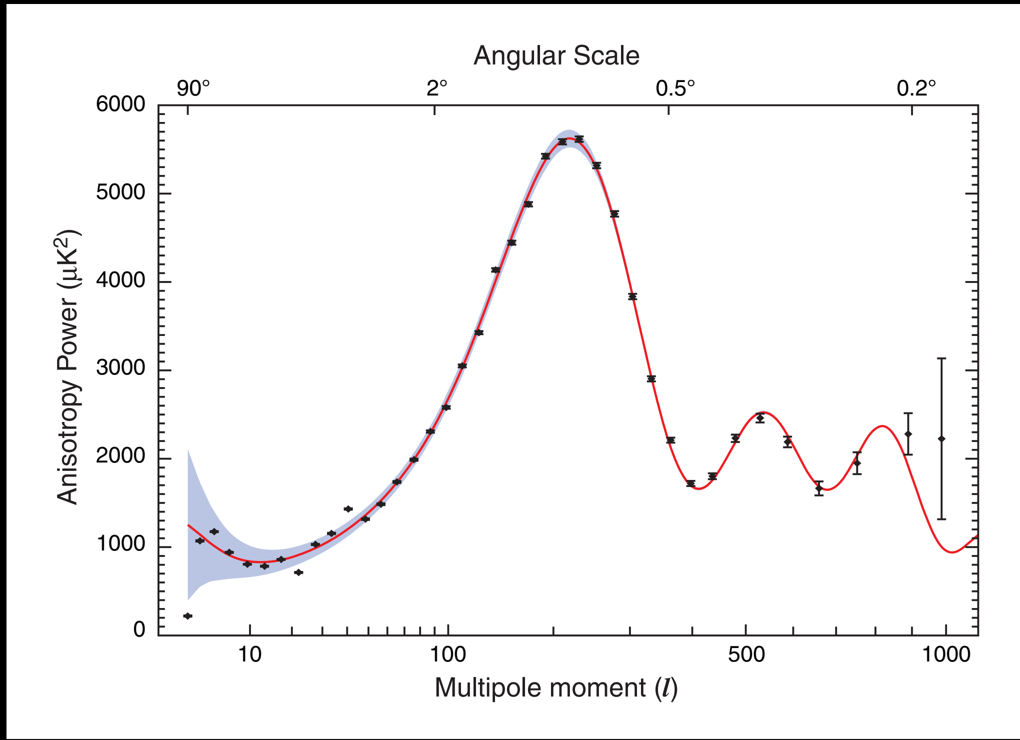
3. *Strongly non-conformal* fields (large mass) and RG running cosmological constant

4. Conclusions

Introduction

- Some key elements in our present understanding of cosmology:
  - inflation (predictions for CMB anisotropies & large scale structure formation)
  - present accelerated expansion (possibly due to Λ)
- Physics of **de Sitter** spacetime may be crucial to understand the early universe and its ultimate fate.





Angular Scale
90°
2°
0.5°
0.2°
Anisotropy Power (μK²)
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment (l)

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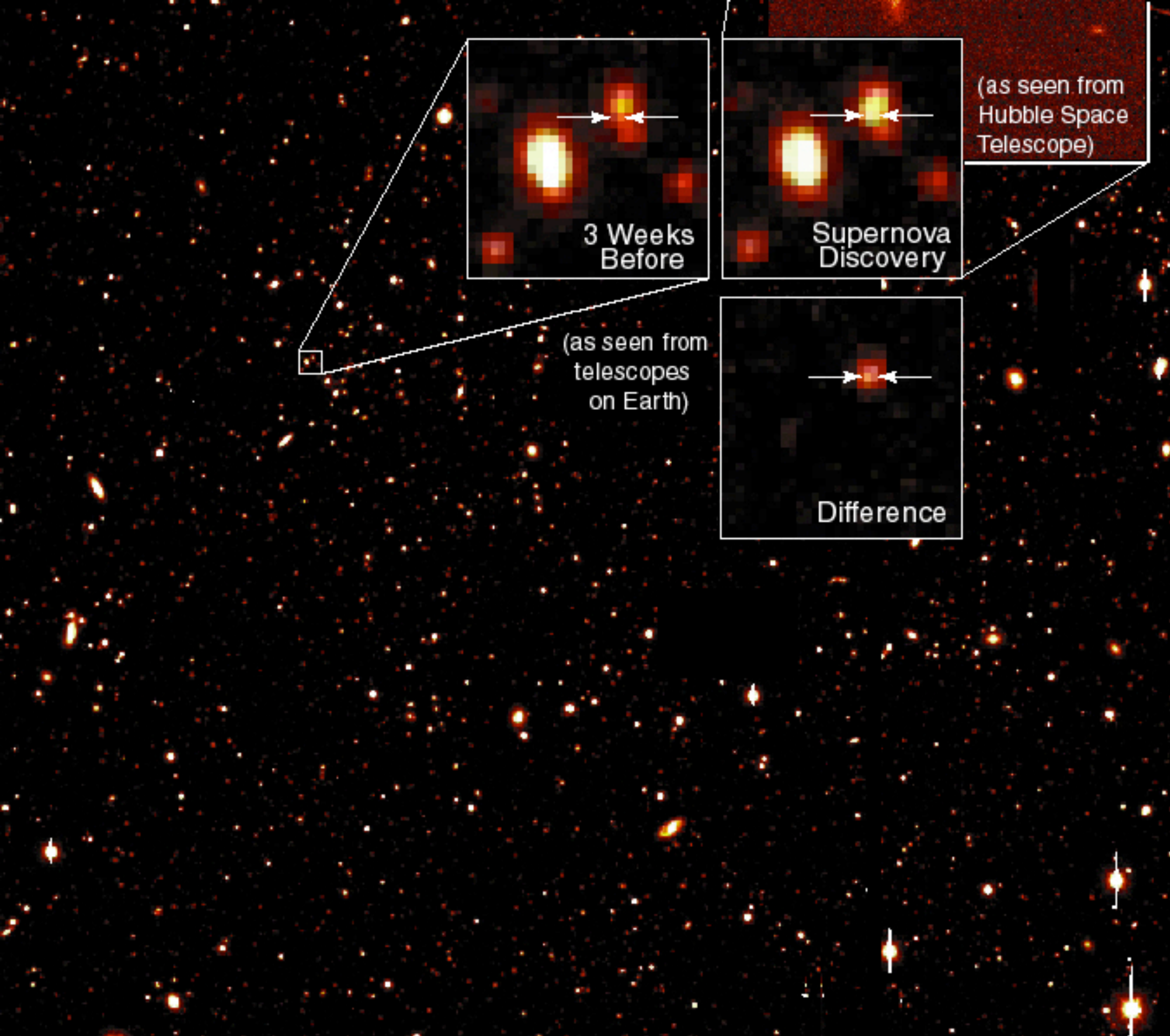
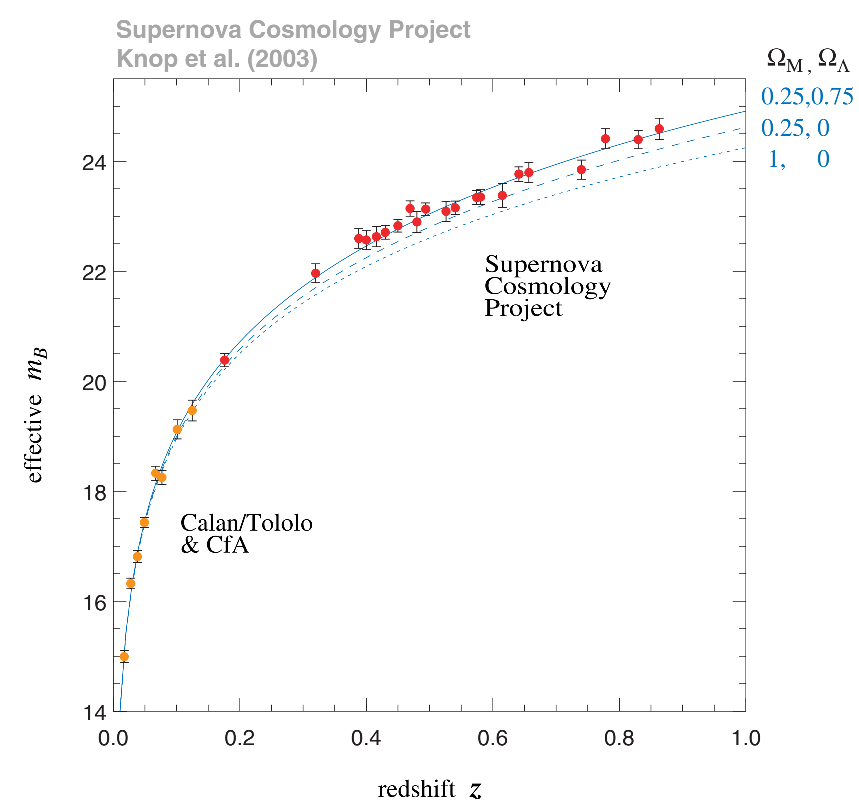

3 Weeks Before
Supernova Discovery
(as seen from Hubble Space Telescope)
(as seen from telescopes on Earth)
Difference


Supernova Cosmology Project
Knop et al. (2003)
Ω_M , Ω_Λ
0.25,0.75
0.25, 0
1, 0
effective m_B
redshift z
Supernova Cosmology Project
Calan/Tololo & CfA
14
16
18
20
22
24
0.0
0.2
0.4
0.6
0.8
1.0

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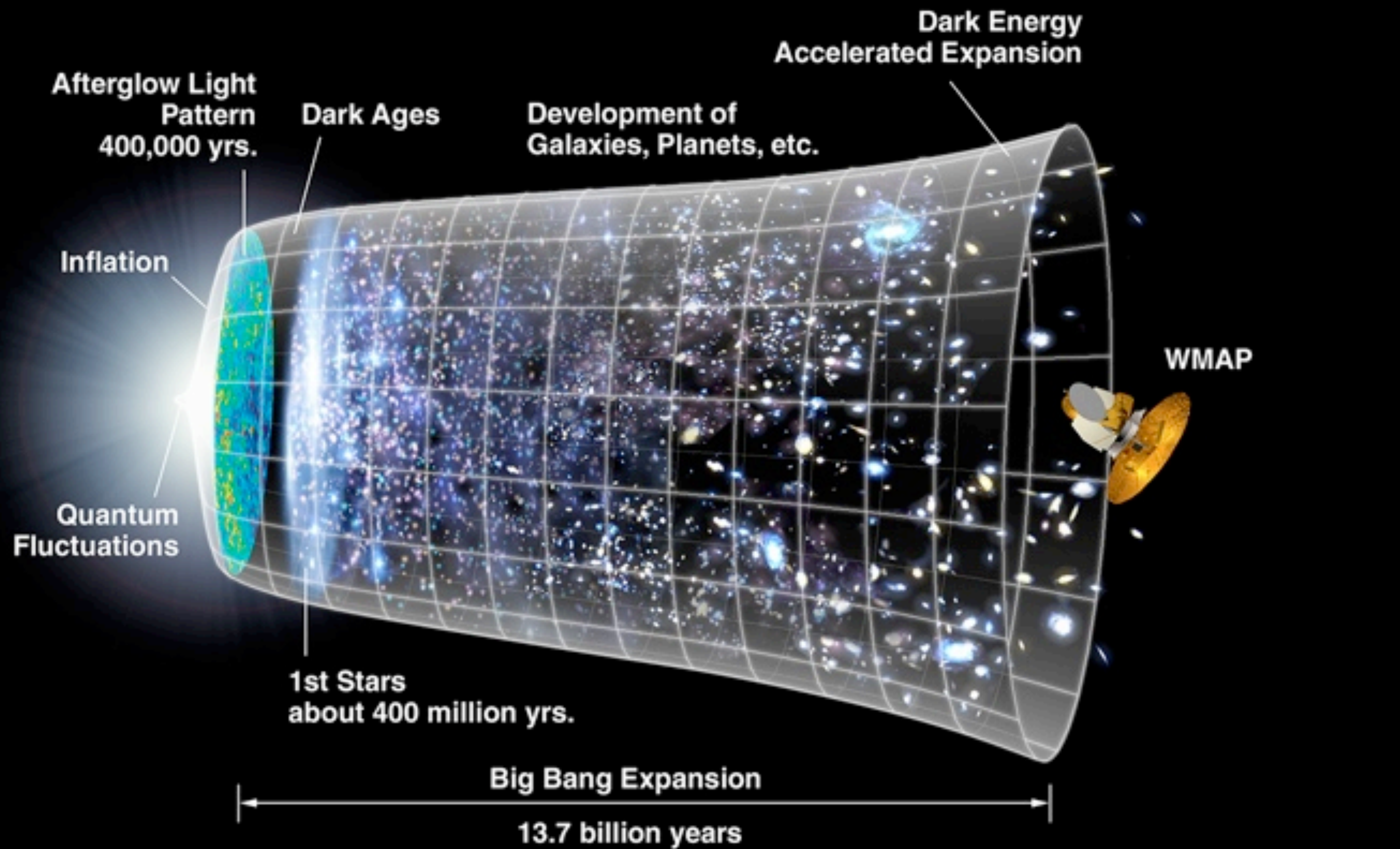

NASA/WMAP Scienc

Introduction

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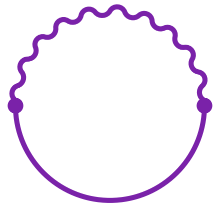
- Study the *backreaction* on the evolution of the background spacetime due to quantum effects.

- Low-energy EFT approach to QG ($L \gg l_{\rm p}$).

  Quantization of the metric perturbations around a mean field background geometry.

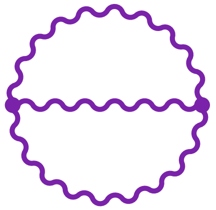
- Secular screening of Λ:
  - in *chaotic inflationary* models

    Mukhanov, Brandenberger, Abramo; Abramo, Woodard; Losic, Unruh



  - in *pure gravity*

    Tsamis & Woodard



- Quantization of metric perturbations → ambiguities unless *diffeomorph.-invariant observables* are considered.

- Gauge-invariant measure of the expansion rate → is secular screening a gauge artifact?
    - *chaotic inflation* — Abramo & Woodard
    - *pure gravity* (?) — Garriga & Tanaka; Tsamis & Woodard

- Interesting quantum effects have been suggested where the quantization of metric perturbations is not essential $\longrightarrow$ no *gauge ambiguities*.

- *Starobinsky inflation*: $\Lambda = 0$, entirely driven by the vacuum polarization (trace anomaly) of a large number of conformal fields.

Starobinsky

$$\langle \hat{T}^a_a[g] \rangle_{\rm ren} = \alpha\, E + \beta\, \Box R + \gamma\, C^{abcd} C_{abcd}$$

BUT it involves scales where the *EFT* breaks down (details of *UV completion* would be needed).

- Significant deviations from de Sitter due to *weakly* non-conformal fields (large *IR* effects):
  - non-local terms associated with *massless* fields

    $\ln(k^2/\mu_0^2)]$

    Espriu, Multamäki, Vagenas; Cabrer, Espriu

  - local terms associated with (light) *massive* fields

    $m^4\, a^4(\eta) \ln a(\eta)$ vs. $m^4\, a^4(\eta)$

    Shapiro, Solà

- RG running of Λ in the decoupling regime ($m \gg H$).

  Shapiro, Solà, *et al.*

Stability of de Sitter spacetime: weakly non-conformal fields

- de Sitter $\longrightarrow$ maximally symmetric spacetime

$$ds^2 = -dt^2 + e^{2Ht}\delta_{ij}dx^i dx^j = (1/H\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j)$$

- QFT on a *fixed* de Sitter spacetime:
  Bunch-Davies vacuum (Hadamard, dS invariant)

$$\left\langle \hat{T}_{ab}[g] \right\rangle_{\mathrm{ren}} = C\, g_{ab} \qquad C = \mathrm{ctnt.}$$

- For any isotropic regular initial state (finite high-frequency excitations) at late times

Anderson *et al.*

$$\left\langle \hat{T}_{ab}[g] \right\rangle_{\mathrm{ren}} \to C\, g_{ab}$$

- Including the back reaction on the mean geometry self-consistently (*semiclassical gravity*):

$$G_{ab}[g] = \kappa \left\langle \hat{T}_{ab}[g] \right\rangle_{\rm ren} - \Lambda g_{ab} \qquad \kappa = \frac{8\pi}{m_p^2}$$

- de Sitter + *BD* vacuum → *self-consistent* solution
- RW geometries: $ds^2 = a^2(\eta)(-d\eta^2 + \delta_{ij}dx^i dx^j)$
- Consider different **isotropic** initial states at η_i. Does the solution tend *asymptotically* to the self-consistent one?

CTP effective action

- Expectation values (rather than transition matrix elements) obtained from the ***CTP*** *effective action*:

$$e^{i\Gamma_{\rm CTP}[g,g']} = \langle \Psi_0 | \, \hat{U}^\dagger(\eta_{\rm f}, \eta_{\rm i}; g'] \hat{U}(\eta_{\rm f}, \eta_{\rm i}; g] \, | \Psi_0 \rangle$$

$$= \int \mathcal{D}\phi_{\rm f} \int^{\phi_{\rm f}(\vec{x})} \mathcal{D}\phi \, \Psi_0[\phi_{\rm i}(\vec{x})] \, e^{iS_{\rm g}[g] + iS_{\rm m}[\phi, g]} \int^{\phi_{\rm f}(\vec{x})} \mathcal{D}\phi' \, \Psi_0^* [\phi'_{\rm i}(\vec{x})] \, e^{-iS_{\rm g}[g'] - iS_{\rm m}[\phi', g']}$$

$$S_{\rm g}[g] = \frac{1}{2\kappa} \int_M d^4x \sqrt{-g}(R - 2\Lambda) + S_{\rm div}[g]$$

$$\Gamma_{\rm CTP}[g, g'] = S_{\rm g}^{\rm (ren)}[g] - S_{\rm g}^{\rm (ren)}[g'] + \Gamma_{\rm m}^{\rm (ren)}[g, g']$$

RG invariant (μ-independent)

- $\langle\hat{T}_{ab}[g]\rangle_{\rm ren}$ can be obtained from $\Gamma_{\rm m}^{(\rm ren)}[g,g']$:

$$T_{ab}[g,\phi] = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}[g,\phi]}{\delta g^{ab}}$$

$$\left\langle \hat{T}_{ab}[g] \right\rangle_{\rm ren} = -\frac{2}{\sqrt{-g}}\ \delta\Gamma_{\rm m}^{(\rm ren)}[g,g']/\delta g^{ab}\Big|_{g'=g}$$

- Similarly, from $\delta\Gamma_{\rm CTP}^{(\rm ren)}[g,g']/\delta g^{ab}\Big|_{g'=g} = 0$ one gets the *semiclassical Einstein* equation:

$$G_{ab}\left[g\right] = \kappa \left\langle \hat{T}_{ab}[g] \right\rangle_{\rm ren} - \Lambda g_{ab}$$

Our model

- Weakly non-conformal field: $\nu = (\xi - 1/6)$ $\quad m/H \ll 1$, $\nu \ll 1$

$$S_{\rm m}[g] = -\frac{1}{2}\int_M d^4x\sqrt{-g}\,\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2 + \xi R\phi^2\right)$$

- Conformal transformation $\longrightarrow$ flat space calculation

$$g_{ab} \to a^{-2}(\eta)\,g_{ab} = \eta_{ab} \qquad \phi(x) \to \varphi(x) = a(\eta)\,\phi(x)$$

- $\Gamma_{\rm m}^{(\rm ren)}[g,g']$ for $m=0$, $\xi = 1/6$ is **invariant** up to local terms corresponding to the *trace anomaly*.

$$S_{\rm div}[\Omega^2 g] \neq S_{\rm div}[g]$$

- *Massless free* field in flat space with

$$S_{\rm int} = -\frac{V}{2}\int_{\eta_{\rm i}}^{\eta} d\eta' a^2(\eta')\Big(m^2 + \nu R(\eta')\Big)|\varphi_k(\eta')|^2$$

- Perturbative treatment of m and ν.

- *Exact* result for $\Gamma_{\rm m}^{\rm (ren)}[a, a']$ through quadratic order in m and ν.

- "Dressed" initial state at $\eta = \eta_{\rm i}$:

$$|\Psi_0\rangle = \hat{U}_{a_\Psi}(\eta_{\rm i}, -\infty)\,|0, -\infty\rangle$$

determined by the auxiliary scale factor a_Ψ for $\eta_{\rm i} > \eta' > -\infty$

$$a_\psi(\eta') \to 0 \quad \text{when} \quad \eta' \to -\infty$$

smooth matching at $\eta_{\rm i}$

Back-reaction equations

$\cdot \equiv \frac{d}{d\eta}$

- Equation for the trace:

$$\left.\frac{\delta\Gamma_{\rm CTP}^{\rm (ren)}[a,a']}{\delta a}\right|_{a'=a} = 0 \quad \longrightarrow \quad -6\,\frac{\ddot{a}}{a^3} = G^\mu_\mu = \kappa\langle \hat{T}^\mu_\mu\rangle_{\rm ren} - 4\Lambda$$

- Friedmann equation:

$$3\,(\dot{a}/a)^2 = G_{00} = \kappa\langle\hat{T}_{00}\rangle_{\rm ren} + \Lambda$$

- Relation between $\langle\hat{T}^\mu_\mu\rangle_{\rm ren}$ and $\langle\hat{T}_{00}\rangle_{\rm ren}$:

$$\langle\hat{T}_{tt}(\eta)\rangle = a^{-2}(\eta)\langle\hat{T}_{00}(\eta)\rangle = -\frac{1}{a^4(\eta)}\int_{-\infty}^{\eta} d\eta'\, a^3\dot{a}\,\langle\hat{T}^\mu_\mu\rangle$$

The solutions

- Perturbative expansion in $(l_{\rm p} H)^2$. $H = \sqrt{\Lambda/3}$

- General solution for arbitrary times: $\epsilon_i^2 = \nu^2, m^2, \nu\, m$

$$a(\eta) = -\frac{1}{\tilde{H}\eta} + \sum_i \epsilon_i^2\, f_i(\eta)\, (l_{\rm p} H)^2 + O\big((l_{\rm p} H)^4\big)$$

$$f_i(\eta) \longrightarrow 0 \qquad \eta \longrightarrow 0$$

- Tends asymptotically to the *self-consistent* solution

$$a(\eta) = -\frac{1}{\tilde{H}\eta} \qquad \tilde{H} = H\left[1 + (l_{\rm p} H)^2 \left(\frac{1}{720\pi} + \sum_i A_i \epsilon_i^2\right)\right]$$

Discussion

- Semiclassical dS solution → *small* correction and *stable* (no large *IR* effects).

- Qualitative explanation:

  - massless case → power suppression

$$\omega^2 \left[1 + b \left(\omega^2 / m_{\rm p}^2 \right) \ln \left(\omega^2 / \mu_0^2 \right) \right]$$

  - (light) massive case → cancellation

$$m^4 \, a^2 \left[2 \ln a - \ln \left(\Box / \mu_0^2 \right) \right] a^2$$

Strongly non-conformal (large mass) regime

- For $m \gg H \longrightarrow$ **adiabatic** expansion in $1/m^{2n}$.

  Schwinger-DeWitt $\longrightarrow$ local *curvature* expansion

  (for the ground state)

- Result can be anticipated (up to dimensionless coeffs.) using low-energy ***EFT*** arguments based on *symmetries* and *power counting*:

$$\Gamma[g] = \int d^4x \sqrt{-g}\, \left[a\, m^4 + b\, m^2 R + c\, R^2 + d\, C_{abcd}C^{abcd} + O(1/m^2)\right]$$

- Excited state $\longrightarrow$ perfect fluid $+$ vacuum
- *Local* effective action independent of μ.
- Constant renormalization of $1/G$ and Λ (and coeffs. of R^2, $C^{abcd}C_{abcd}$).
- *No effect* from the quadratic terms on *small* perturbations around *dS*.
- Higher-order terms even more suppressed ($H^2/m^2 \ll 1$).

Conclusions

- Explicit calculation of *1-loop effective action* for *weakly* non-conformal fields in a RW metric.
- Backreaction on a Λ-driven spacetime:
  - crucial cancellation between *local* and *nonlocal* contrib.
  - self-consistent de Sitter solution (very small correction to Λ)
  - stable: all other solutions tend to it at late times.
- Effect of *strongly* non-conformal fields (large mass): constant renormalization of Λ and G.

Future directions

- Anisotropic and inhomogenous perturbations.
- Include *quantum* metric fluctuations (inhomog.) Significant backreaction effect?
- *Large* fluctuations $\longrightarrow$ *breakdown* of mean field approximation.